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Upper and Lower Almost Contra-(τ_1, τ_2)-continuity

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Abstract. This paper introduces new classes of multifunctions between bitopological spaces, namely upper almost contra-(τ_1, τ_2)-continuous multifunctions and lower almost contra-(τ_1, τ_2)-continuous multifunctions. Furthermore, several characterizations of upper almost contra-(τ_1, τ_2)-continuous multifunctions and lower almost contra-(τ_1, τ_2)-continuous multifunctions are considered.

2020 Mathematics Subject Classifications: 54C08, 54C60

Key Words and Phrases: Upper almost contra-(τ_1, τ_2)-continuous multifunction, lower almost contra-(τ_1, τ_2)-continuous multifunction

1. Introduction

It is well-known that the branch of mathematics called topology is concerned with all questions directly or indirectly related to continuity. Stronger and weaker forms of open sets play an important role in the generalization of different forms of continuity. Using different forms of open sets, many authors have introduced and studied various types of continuity for functions and multifunctions. In [1], the present authors studied some properties of (Λ, sp)-open sets, $r(\Lambda, sp)$-open sets, $s(\Lambda, sp)$-open sets, $p(\Lambda, sp)$-open sets, $\alpha(\Lambda, sp)$-open sets, $\beta(\Lambda, sp)$-open sets and $b(\Lambda, sp)$-open sets. Viriyapong and Boonpok [2] investigated some characterizations of (Λ, sp)-continuous functions by utilizing the notions of (Λ, sp)-open sets and (Λ, sp)-closed sets. Dungthaisong et al. [3] introduced and studied the concept of $g_{(m,n)}$-continuous functions. Duangphui et al. [4] introduced and investigated the notion of almost $(\mu, \mu')^{(m,n)}$-continuous functions. Furthermore, several characterizations of almost (Λ, p)-continuous functions, strongly $\theta(\Lambda, p)$-continuous functions, almost strongly $\theta(\Lambda, p)$-continuous functions, $\theta(\Lambda, p)$-continuous functions, weakly (Λ, b)-continuous functions, $\theta(\star)$-precontinuous functions, $(\Lambda, p(\star))$-continuous functions,

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$\star$-continuous functions, θ-$\mathscr{I}$-continuous functions, almost (g, m)-continuous functions, pairwise almost M-continuous functions, (τ_1, τ_2)-continuous functions, almost (τ_1, τ_2)-continuous functions, weakly (τ_1, τ_2)-continuous functions and slightly $(\tau_1, \tau_2)s$-continuous functions were presented in [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18] and [19], respectively. Kong-ied at al. [20] introduced and studied the concept of almost quasi (τ_1, τ_2)-continuous functions. Chiangpradit et al. [21] introduced and investigated the notion of weakly quasi (τ_1, τ_2)-continuous functions. Thongmoon et al. [22] introduced and studied the notion of rarely (τ_1, τ_2)-continuous functions. Srisarakham et al. [23] introduced and investigated the concept of faintly (τ_1, τ_2)-continuous functions. On the other hand, the present authors introduced and studied the notions of $\delta(\tau_1, \tau_2)$-continuous functions [24], quasi $\theta(\tau_1, \tau_2)$-continuous functions [25], almost weakly (τ_1, τ_2)-continuous functions [26] and almost nearly (τ_1, τ_2)-continuous functions [27]. The notion of contra-continuity in topological spaces was introduced by Dontchev [28]. Dontchev and Noiri [29] introduced and studied the concept of RC-continuity between topological spaces which is weaker than contra-continuity. Jafari and Noiri [30] introduced a new class of functions called contra-precontinuous functions which is weaker than contra-continuous functions and studied several basic properties of contra-precontinuous functions. Ekici [31] introduced and studied a new class of functions called almost contra-precontinuous functions which generalize classes of regular set-connected functions [32], contra-precontinuous functions [30], contra-continuous functions [28], almost s-continuous functions [33] and perfectly continuous functions [34].

In 2008, Ekici et al. [35] extended the notion of contra-continuous functions to the setting of multifunctions. Noiri and Popa [36] introduced the notion of weakly precontinuous multifunctions. Ekici et al. [37] introduced and studied two new concepts namely contra-precontinuous multifunctions and almost contra-precontinuous multifunctions which are containing the class of contra-continuous multifunctions [35] and contained in the class of weakly precontinuous multifunctions. Laprom et al. [38] introduced and investigated the notion of almost $\beta(\tau_1, \tau_2)$-continuous multifunctions. Moreover, some characterizations of $(\tau_1, \tau_2)\delta$-semicontinuous multifunctions, almost weakly (τ_1, τ_2)-continuous multifunctions, weakly quasi (Λ, sp)-continuous multifunctions, $\star$-continuous multifunctions, $\beta(\star)$-continuous multifunctions, α-$\star$-continuous multifunctions, almost α-$\star$-continuous multifunctions, almost quasi $\star$-continuous multifunctions, weakly α-$\star$-continuous multifunctions, $s\beta(\star)$-continuous multifunctions, weakly $s\beta(\star)$-continuous multifunctions, $\theta(\star)$-quasi continuous multifunctions, almost $\imath^\star$-continuous multifunctions, weakly (Λ, sp)-continuous multifunctions, $\alpha(\Lambda, sp)$-continuous multifunctions, almost $\alpha(\Lambda, sp)$-continuous multifunctions, weakly $\alpha(\Lambda, sp)$-continuous multifunctions, almost $\beta(\Lambda, sp)$-continuous multifunctions, slightly (Λ, sp)-continuous multifunctions, (τ_1, τ_2)-continuous multifunctions, almost (τ_1, τ_2)-continuous multifunctions, weakly (τ_1, τ_2)-continuous multifunctions, weakly quasi (τ_1, τ_2)-continuous multifunctions, almost quasi (τ_1, τ_2)-continuous multifunctions, c-(τ_1, τ_2)-continuous multifunctions, c-quasi (τ_1, τ_2)-continuous multifunctions, s-$(\tau_1, \tau_2)p$-continuous multifunctions, slightly $\alpha(\tau_1, \tau_2)$-continuous multifunctions and slightly $(\tau_1, \tau_2)p$-continuous multifunctions were established in [39], [40], [41], [42], [43], [44], [45], [46], [47], [48], [49], [50], [51], [52], [53], [54], [55], [56], [57], [58], [59], [60], [61], [62], [63], [64], [65],

[66] and [67], respectively. On the other hand, the present authors introduced and investigated the notions of rarely s-$(\tau_1, \tau_2)p$-continuous multifunctions [68], almost nearly (τ_1, τ_2)-continuous multifunctions [69], s-(τ_1, τ_2)-continuous multifunctions [70], quasi $\theta(\tau_1, \tau_2)$-continuous multifunctions [71], almost nearly quasi (τ_1, τ_2)-continuous multifunctions [72], weakly s-(τ_1, τ_2)-continuous multifunctions [73], nearly (τ_1, τ_2)-continuous multifunctions [74] and almost quasi (τ_1, τ_2)-continuous multifunctions [75]. Ekici et al. [76] introduced and studied a new generalization of contra-continuous multifunctions called almost contra-continuous multifunctions. Boonpok and Khampakdee [77] introduced and investigated the notions of upper almost contra-(Λ, sp)-continuous multifunctions and lower almost contra-(Λ, sp)-continuous multifunctions. In this paper, we introduce the concepts of upper almost contra-(τ_1, τ_2)-continuous multifunctions and lower almost contra-(τ_1, τ_2)-continuous multifunctions. We also investigate some characterizations of upper almost contra-(τ_1, τ_2)-continuous multifunctions and lower almost contra-(τ_1, τ_2)-continuous multifunctions.

2. Preliminaries

Throughout the present paper, spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or simply X and Y) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a bitopological space (X, τ_1, τ_2). The closure of A and the interior of A with respect to τ_i are denoted by τ_i-Cl(A) and τ_i-Int(A), respectively, for $i = 1, 2$. A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-*closed* [78] if $A = \tau_1$-Cl$(\tau_2$-Cl$(A))$. The complement of a $\tau_1\tau_2$-closed set is called $\tau_1\tau_2$-*open*. The intersection of all $\tau_1\tau_2$-closed sets of X containing A is called the $\tau_1\tau_2$-*closure* [78] of A and is denoted by $\tau_1\tau_2$-Cl(A). The union of all $\tau_1\tau_2$-open sets of X contained in A is called the $\tau_1\tau_2$-*interior* [78] of A and is denoted by $\tau_1\tau_2$-Int(A).

Lemma 1. [78] *Let A and B be subsets of a bitopological space (X, τ_1, τ_2). For the $\tau_1\tau_2$-closure, the following properties hold:*

(1) $A \subseteq \tau_1\tau_2$-Cl(A) and $\tau_1\tau_2$-Cl$(\tau_1\tau_2$-Cl$(A)) = \tau_1\tau_2$-Cl(A).

(2) If $A \subseteq B$, then $\tau_1\tau_2$-Cl$(A) \subseteq \tau_1\tau_2$-Cl$(B)$.

(3) $\tau_1\tau_2$-Cl(A) is $\tau_1\tau_2$-closed.

(4) A is $\tau_1\tau_2$-closed if and only if $A = \tau_1\tau_2$-Cl(A).

(5) $\tau_1\tau_2$-Cl$(X - A) = X - \tau_1\tau_2$-Int$(A)$.

A subset A of a bitopological space (X, τ_1, τ_2) is called $(\tau_1, \tau_2)r$-*open* [79] (resp. $(\tau_1, \tau_2)s$-*open* [39], $(\tau_1, \tau_2)p$-*open* [39], $(\tau_1, \tau_2)\beta$-*open* [39]) if $A = \tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A))$ (resp. $A \subseteq \tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(A))$, $A \subseteq \tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A))$, $A \subseteq \tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A))))$. The complement of a $(\tau_1, \tau_2)r$-open (resp. $(\tau_1, \tau_2)s$-open, $(\tau_1, \tau_2)p$-open, $(\tau_1, \tau_2)\beta$-open) set is called $(\tau_1, \tau_2)r$-*closed* (resp. $(\tau_1, \tau_2)s$-*closed*, $(\tau_1, \tau_2)p$-*closed*, $(\tau_1, \tau_2)\beta$-*closed*). Let A be a subset of a bitopological space (X, τ_1, τ_2). The set $\cap\{V \mid V$ is $(\tau_1, \tau_2)r$-open and $A \subseteq V\}$ is called the $(\tau_1, \tau_2)r$-*kernel* of A and is denoted by $(\tau_1, \tau_2)r$-*ker*(A).

Lemma 2. *For subsets A, B of a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) $A \subseteq (\tau_1, \tau_2)r\text{-}ker(A)$.

(2) If $A \subseteq B$, *then* $(\tau_1, \tau_2)r\text{-}ker(A) \subseteq (\tau_1, \tau_2)r\text{-}ker(B)$.

(3) If A is $(\tau_1, \tau_2)r$-open, then $(\tau_1, \tau_2)r\text{-}ker(A) = A$.

(4) $x \in (\tau_1, \tau_2)r\text{-}ker(A)$ *if and only if* $A \cap K \neq \emptyset$ *for every $(\tau_1, \tau_2)r$-closed set K containing x.*

A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\alpha(\tau_1, \tau_2)$-*open* [80] if $A \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A)))$. The complement of an $\alpha(\tau_1, \tau_2)$-open set is called $\alpha(\tau_1, \tau_2)$-*closed*. Let A be a subset of a bitopological space (X, τ_1, τ_2). The intersection of all $(\tau_1, \tau_2)p$-closed (resp. $(\tau_1, \tau_2)s$-closed, $\alpha(\tau_1, \tau_2)$-closed) sets of X containing A is called the $(\tau_1, \tau_2)p$-*closure* [65] (resp. $(\tau_1, \tau_2)s$-*closure* [39], $\alpha(\tau_1, \tau_2)$-*closure* [66]) of A and is denoted by $(\tau_1, \tau_2)\text{-pCl}(A)$ (resp. $(\tau_1, \tau_2)\text{-sCl}(A)$, $\alpha(\tau_1, \tau_2)\text{-Cl}(A)$). The union of all $(\tau_1, \tau_2)p$-open (resp. $(\tau_1, \tau_2)s$-open, $\alpha(\tau_1, \tau_2)$-open) sets of X contained in A is called the $(\tau_1, \tau_2)p$-*interior* [65] (resp. $(\tau_1, \tau_2)s$-*interior* [39], $\alpha(\tau_1, \tau_2)$-*interior* [66]) of A and is denoted by $(\tau_1, \tau_2)\text{-pInt}(A)$ (resp. $(\tau_1, \tau_2)\text{-sInt}(A)$, $\alpha(\tau_1, \tau_2)\text{-Int}(A)$).

Lemma 3. *For a subset A of a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) $(\tau_1, \tau_2)\text{-}pCl(A) = \tau_1\tau_2\text{-}Cl(\tau_1\tau_2\text{-}Int(A)) \cup A$ [65].

(2) $(\tau_1, \tau_2)\text{-}pInt(A) = \tau_1\tau_2\text{-}Int(\tau_1\tau_2\text{-}Cl(A)) \cap A$ [26].

(3) $(\tau_1, \tau_2)\text{-}sCl(A) = \tau_1\tau_2\text{-}Int(\tau_1\tau_2\text{-}Cl(A)) \cup A$ [39].

(4) $(\tau_1, \tau_2)\text{-}sInt(A) = \tau_1\tau_2\text{-}Cl(\tau_1\tau_2\text{-}Int(A)) \cap A$ [62].

For a subset A of a bitopological space (X, τ_1, τ_2), a point $x \in X$ is called a $s(\tau_1, \tau_2)\theta$-*cluster point* [81] of A if $\tau_1\tau_2\text{-Cl}(U) \cap A \neq \emptyset$ for every $(\tau_1, \tau_2)s$-open set U containing x. The set of all $s(\tau_1, \tau_2)\theta$-cluster points of A is called the $s(\tau_1, \tau_2)\theta$-*closure* [81] of A and is denoted by $s(\tau_1, \tau_2)\theta\text{-Cl}(A)$. A subset A of a bitopological space (X, τ_1, τ_2) is called $s(\tau_1, \tau_2)\theta$-*closed* [81] if $s(\tau_1, \tau_2)\theta\text{-Cl}(A) = A$. The complement of a $s(\tau_1, \tau_2)\theta$-closed set is said to be $s(\tau_1, \tau_2)\theta$-*open* [81]. The union of all $s(\tau_1, \tau_2)\theta$-open sets of X contained in A is called the $s(\tau_1, \tau_2)\theta$-*interior* [81] of A and is denoted by $s(\tau_1, \tau_2)\theta\text{-Int}(A)$.

By a multifunction $F : X \to Y$, we mean a point-to-set correspondence from X into Y, and we always assume that $F(x) \neq \emptyset$ for all $x \in X$. For a multifunction $F : X \to Y$, we shall denote the upper and lower inverse of a set B of Y by $F^+(B)$ and $F^-(B)$, respectively, that is, $F^+(B) = \{x \in X \mid F(x) \subseteq B\}$ and $F^-(B) = \{x \in X \mid F(x) \cap B \neq \emptyset\}$. In particular, $F^-(y) = \{x \in X \mid y \in F(x)\}$ for each point $y \in Y$. For each $A \subseteq X$, $F(A) = \cup_{x \in A} F(x)$.

3. Upper and lower almost contra-(τ_1, τ_2)-continuous multifunctions

In this section, we introduce the concepts of upper almost contra-(τ_1, τ_2)-continuous multifunctions and lower almost contra-(τ_1, τ_2)-continuous multifunctions. Moreover, some characterizations of upper almost contra-(τ_1, τ_2)-continuous multifunctions and lower almost contra-(τ_1, τ_2)-continuous multifunctions are considered.

Definition 1. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be upper almost contra-(τ_1, τ_2)-continuous at a point $x \in X$ if for each $(\sigma_1, \sigma_2)r$-closed set K of Y with $x \in F^+(K)$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $U \subseteq F^+(K)$. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be upper almost contra-(τ_1, τ_2)-continuous if F is upper almost contra-(τ_1, τ_2)-continuous at each point x of X.*

Theorem 1. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is upper almost contra-(τ_1, τ_2)-continuous;

(2) $F^+(K)$ is $\tau_1\tau_2$-open in X for every $(\sigma_1, \sigma_2)r$-closed set K of Y;

(3) $F^-(V)$ is $\tau_1\tau_2$-closed in X for every $(\sigma_1, \sigma_2)r$-open set V of Y;

(4) $F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$ is $\tau_1\tau_2$-closed in X for every $\sigma_1\sigma_2$-open set V of Y;

(5) $F^+(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K)))$ is $\tau_1\tau_2$-open in X for every $\sigma_1\sigma_2$-closed set K of Y;

(6) for each $x \in X$ and each $(\sigma_1, \sigma_2)s$-open set V of Y with $F(x) \subseteq V$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $F(U) \subseteq \sigma_1\sigma_2\text{-Cl}(V)$;

(7) $F^+(V) \subseteq \tau_1\tau_2\text{-Int}(F^+(\sigma_1\sigma_2\text{-Cl}(V)))$ for every $(\sigma_1, \sigma_2)s$-open set V of Y.

Proof. (1) ⇒ (2): Let K be any $(\sigma_1, \sigma_2)r$-closed set of Y and $x \in F^+(K)$. Since F is is upper almost contra-(τ_1, τ_2)-continuous, there exists a $\tau_1\tau_2$-open set U of X containing x such that $U \subseteq F^+(K)$. It follows that $F^+(K)$ is $\tau_1\tau_2$-open in X.

(2) ⇒ (1): The proof is obvious.

(2) ⇔ (3): It follows from the fact that $F^+(Y - B) = X - F^-(B)$ for every subset B of Y.

(3) ⇔ (4): Let V be any $\sigma_1\sigma_2$-open set of Y. Since $\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))$ is $(\sigma_1, \sigma_2)r$-open in Y, by (3) we have $F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$ is $\tau_1\tau_2$-closed in X. The converse is obvious.

(4) ⇔ (5): It follows from the fact that $F^+(Y - B) = X - F^-(B)$ for every subset B of Y.

(5) ⇔ (2): It is similar to that of (3) ⇔ (4).

(6) ⇒ (7): Let V be any $(\sigma_1, \sigma_2)s$-open set of Y and $x \in F^+(V)$. Then, $F(x) \subseteq V$. By (6), there exists a $\tau_1\tau_2$-open set U of X containing x such that $F(U) \subseteq \sigma_1\sigma_2\text{-Cl}(V)$. Thus, $U \subseteq F^+(\sigma_1\sigma_2\text{-Cl}(V))$ and hence $x \in \tau_1\tau_2\text{-Int}(F^+(\sigma_1\sigma_2\text{-Cl}(V)))$. This implies that $F^+(V) \subseteq \tau_1\tau_2\text{-Int}(F^+(\sigma_1\sigma_2\text{-Cl}(V)))$.

(7) ⇒ (2): Let K be any $(\sigma_1, \sigma_2)r$-closed set of Y. Since K is $(\sigma_1, \sigma_2)s$-open in Y, by (7) we have $F^+(V) \subseteq \tau_1\tau_2\text{-Int}(F^+(\sigma_1\sigma_2\text{-Cl}(V))) = \tau_1\tau_2\text{-Int}(F^+(V))$. Thus, $F^+(K)$ is $\tau_1\tau_2$-open in X.

(2) ⇒ (6): Let $x \in X$ and V be any $(\sigma_1, \sigma_2)s$-open set of Y with $F(x) \subseteq V$. Since $\sigma_1\sigma_2\text{-Cl}(V)$ is $(\sigma_1, \sigma_2)r$-closed in Y, by (2) we have $F^+(\sigma_1\sigma_2\text{-Cl}(V))$ is $\tau_1\tau_2$-open in X. Then, there exists a $\tau_1\tau_2$-open set U of X containing x such that $U \subseteq F^+(\sigma_1\sigma_2\text{-Cl}(V))$. Thus, $F(U) \subseteq \sigma_1\sigma_2\text{-Cl}(V)$.

Definition 2. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be lower almost contra-(τ_1, τ_2)-continuous at a point $x \in X$ if for each $(\sigma_1, \sigma_2)r$-closed set K of Y with $x \in F^-(K)$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $U \subseteq F^-(K)$. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be lower almost contra-(τ_1, τ_2)-continuous if F is lower almost contra-(τ_1, τ_2)-continuous at each point x of X.*

Theorem 2. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is lower almost contra-(τ_1, τ_2)-continuous;

(2) $F^-(K)$ is $\tau_1\tau_2$-open in X for every $(\sigma_1, \sigma_2)r$-closed set K of Y;

(3) $F^+(V)$ is $\tau_1\tau_2$-closed in X for every $(\sigma_1, \sigma_2)r$-open set V of Y;

(4) $F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$ is $\tau_1\tau_2$-closed in X for every $\sigma_1\sigma_2$-open set V of Y;

(5) $F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K)))$ is $\tau_1\tau_2$-open in X for every $\sigma_1\sigma_2$-closed set K of Y;

(6) for each $x \in X$ and each $(\sigma_1, \sigma_2)s$-open set V of Y with $F(x) \cap V \neq \emptyset$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $F(z) \cap \sigma_1\sigma_2\text{-Cl}(V) \neq \emptyset$ for each $z \in U$;

(7) $F^-(V) \subseteq \tau_1\tau_2\text{-Int}(F^-(\sigma_1\sigma_2\text{-Cl}(V)))$ for every $(\sigma_1, \sigma_2)s$-open set V of Y.

Proof. The proof is similar to that of Theorem 1.

Theorem 3. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is upper almost contra-(τ_1, τ_2)-continuous;

(2) $F^+(\sigma_1\sigma_2\text{-Cl}(V))$ is $\tau_1\tau_2$-open in X for every $(\sigma_1, \sigma_2)\beta$-open set V of Y;

(3) $F^+(\sigma_1\sigma_2\text{-Cl}(V))$ is $\tau_1\tau_2$-open in X for every $(\sigma_1, \sigma_2)s$-open set V of Y;

(4) $F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$ is $\tau_1\tau_2$-closed in X for every $(\sigma_1, \sigma_2)p$-open set V of Y.

Proof. (1) ⇒ (2): Let V be any $(\sigma_1, \sigma_2)\beta$-open set of Y. Then, $\sigma_1\sigma_2$-Cl(V) is $(\sigma_1, \sigma_2)r$-closed in Y, by Theorem 1 we have $F^+(\sigma_1\sigma_2\text{-Cl}(V))$ is $\tau_1\tau_2$-open in X.

(2) ⇒ (3): This is obvious since every $(\sigma_1, \sigma_2)s$-open set is $(\sigma_1, \sigma_2)\beta$-open.

(3) ⇒ (4): Let V be any $(\sigma_1, \sigma_2)p$-open set of Y. Thus, $Y - \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))$ is $(\sigma_1, \sigma_2)r$-closed in Y and hence $Y - \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))$ is $(\sigma_1, \sigma_2)s$-open in Y. By (3),

$$\begin{aligned} X - F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))) &= F^+(Y - \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))) \\ &= F^+(\sigma_1\sigma_2\text{-Cl}(Y - \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) \end{aligned}$$

is $\tau_1\tau_2$-open in X. Thus, $F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$ is $\tau_1\tau_2$-closed in X.

(4) ⇒ (1): Let V be any $(\sigma_1, \sigma_2)r$-open set of Y. Then, V is $(\sigma_1, \sigma_2)p$-open in Y and by (4), $F^-(V) = F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$ is $\tau_1\tau_2$-closed in X. By Theorem 1, F is upper almost almost contra-(τ_1, τ_2)-continuous.

Theorem 4. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is lower almost contra-(τ_1, τ_2)-continuous;

(2) $F^-(\sigma_1\sigma_2\text{-Cl}(V))$ is $\tau_1\tau_2$-open in X for every $(\sigma_1, \sigma_2)\beta$-open set V of Y;

(3) $F^-(\sigma_1\sigma_2\text{-Cl}(V))$ is $\tau_1\tau_2$-open in X for every $(\sigma_1, \sigma_2)s$-open set V of Y;

(4) $F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$ is $\tau_1\tau_2$-closed in X for every $(\sigma_1, \sigma_2)p$-open set V of Y.

Proof. The proof is similar to that of Theorem 3.

Lemma 4. [81] *For a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) $\alpha(\tau_1, \tau_2)\text{-Cl}(V) = \tau_1\tau_2\text{-Cl}(V)$ for every $(\tau_1, \tau_2)\beta$-open set V of X;

(2) $(\tau_1, \tau_2)\text{-pCl}(V) = \tau_1\tau_2\text{-Cl}(V)$ for every $(\tau_1, \tau_2)s$-open set V of X;

(3) $(\tau_1, \tau_2)\text{-sCl}(V) = \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(V))$ for every $(\tau_1, \tau_2)p$-open set V of X.

Corollary 1. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is upper almost contra-(τ_1, τ_2)-continuous;

(2) $F^+(\alpha(\sigma_1, \sigma_2)\text{-Cl}(V))$ is $\tau_1\tau_2$-open in X for every $(\sigma_1, \sigma_2)\beta$-open set V of Y;

(3) $F^+((\sigma_1, \sigma_2)\text{-pCl}(V))$ is $\tau_1\tau_2$-open in X for every $(\sigma_1, \sigma_2)s$-open set V of Y;

(4) $F^-((\sigma_1, \sigma_2)\text{-sCl}(V))$ is τ_1, τ_2-closed in X for every $(\sigma_1, \sigma_2)p$-open set V of Y.

Corollary 2. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is lower almost contra-(τ_1, τ_2)-continuous;

(2) $F^-(\alpha(\sigma_1, \sigma_2)\text{-}Cl(V))$ is $\tau_1\tau_2$-open in X for every $(\sigma_1, \sigma_2)\beta$-open set V of Y;

(3) $F^-((\sigma_1, \sigma_2)\text{-}pCl(V))$ is $\tau_1\tau_2$-open in X for every $(\sigma_1, \sigma_2)s$-open set V of Y;

(4) $F^+((\sigma_1, \sigma_2)\text{-}sCl(V))$ is $\tau_1\tau_2$-closed in X for every $(\sigma_1, \sigma_2)p$-open set V of Y.

Theorem 5. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is upper almost contra-(τ_1, τ_2)-continuous;

(2) $\tau_1\tau_2\text{-}Cl(F^-(V)) \subseteq F^-(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))$ for every $\sigma_1\sigma_2$-open set V of Y;

(3) $\tau_1\tau_2\text{-}Cl(F^-(V)) \subseteq F^-((\sigma_1, \sigma_2)\text{-}sCl(V))$ for every $\sigma_1\sigma_2$-open set V of Y.

Proof. (1) $\Rightarrow$ (2): Let V be any $\sigma_1\sigma_2$-open set of Y. Then, $\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))$ is $(\sigma_1, \sigma_2)r$-open in Y. Thus by (1), $F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$ is $\tau_1\tau_2$-closed in X. Since $V \subseteq \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))$, we have $F^-(V) \subseteq F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$ and hence $\tau_1\tau_2\text{-Cl}(F^-(V)) \subseteq F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$.

(2) $\Rightarrow$ (1): Let V be any $(\sigma_1, \sigma_2)r$-open set of Y. By (2), we have

$$\tau_1\tau_2\text{-Cl}(F^-(V)) \subseteq F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))) = F^-(V)$$

and hence $F^-(V)$ is $\tau_1\tau_2$-closed in X, by Theorem 1 we have F is upper almost contra-(τ_1, τ_2)-continuous.

(2) $\Leftrightarrow$ (3): It follows from Lemma 4.

Theorem 6. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is lower almost contra-(τ_1, τ_2)-continuous;

(2) $\tau_1\tau_2\text{-}Cl(F^+(V)) \subseteq F^+(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))$ for every $\sigma_1\sigma_2$-open set V of Y;

(3) $\tau_1\tau_2\text{-}Cl(F^+(V)) \subseteq F^+((\sigma_1, \sigma_2)\text{-}sCl(V))$ for every $\sigma_1\sigma_2$-open set V of Y.

Proof. The proof is similar to that of Theorem 5.

Theorem 7. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is lower almost contra-(τ_1, τ_2)-continuous;

(2) $F^-(V)$ is $\tau_1\tau_2$-open in X for every $s(\sigma_1, \sigma_2)\theta$-open set V of Y;

(3) $F^+(K)$ is $\tau_1\tau_2$-closed in X for every $s(\sigma_1, \sigma_2)\theta$-closed set K of Y;

(4) $\tau_1\tau_2$*-Cl*$(F^+(\sigma_1\sigma_2$*-Int*$(\sigma_1\sigma_2$*-Cl*$(B)))) \subseteq F^+((\sigma_1,\sigma_2)$*-sCl*$(B))$ *for every subset* B *of* Y*;*

(5) $\tau_1\tau_2$*-Cl*$(F^+(B)) \subseteq F^+(s(\sigma_1,\sigma_2)\theta$*-Cl*$(B))$ *for every subset* B *of* Y*;*

(6) $F(\tau_1\tau_2$*-Cl*$(A)) \subseteq s(\sigma_1,\sigma_2)\theta$*-Cl*$(F(A))$ *for every subset* A *of* X*.*

Proof. (1) ⇒ (2): Let V be any $s(\sigma_1,\sigma_2)\theta$-open set of Y. There exists a family of $(\sigma_1,\sigma_2)r$-closed sets $\{K_\gamma \mid \gamma \in \nabla\}$ such that $V = \cup\{K_\gamma \mid \gamma \in \nabla\}$. It follows from Theorem 2 that $F^-(V) = \cup\{F^-(K_\gamma) \mid \gamma \in \nabla\}$ is $\tau_1\tau_2$-open in X.

(2) ⇒ (3): The proof is obvious.

(3) ⇒ (4): Let B be any subset of Y. Then, $\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(B))$ is $(\sigma_1,\sigma_2)r$-open and hence $\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(B))$ is $s(\sigma_1,\sigma_2)\theta$-closed in Y. By (3), $F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(B)))$ is $\tau_1\tau_2$-closed in X. Thus,

$$\begin{aligned}\tau_1\tau_2\text{-Cl}(F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(B)))) &= F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(B))) \\ &\subseteq F^+((\sigma_1,\sigma_2)\text{-sCl}(B)).\end{aligned}$$

(4) ⇒ (5): Let B be any subset of Y. For any $(\sigma_1,\sigma_2)r$-open set V of Y with $B \subseteq V$, by (4) and Lemma 4 we have

$$\begin{aligned}\tau_1\tau_2\text{-Cl}(F^+(B)) &\subseteq \tau_1\tau_2\text{-Cl}(F^+(V)) \\ &= \tau_1\tau_2\text{-Cl}(F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) \\ &\subseteq F^+((\sigma_1,\sigma_2)\text{-sCl}(V)) \\ &= F^+(V).\end{aligned}$$

Thus,

$$\begin{aligned}\tau_1\tau_2\text{-Cl}(F^+(B)) &\subseteq F^+(\cap\{V \mid V \text{ is } (\sigma_1,\sigma_2)r\text{-open in } Y \text{ and } B \subseteq V\}) \\ &= F^+(s(\sigma_1,\sigma_2)\theta\text{-Cl}(B)).\end{aligned}$$

(5) ⇒ (1): Let V be any $(\sigma_1,\sigma_2)s$-open set of Y. By (5), we have

$$\begin{aligned}X - \tau_1\tau_2\text{-Int}(F^-(\sigma_1\sigma_2\text{-Cl}(V))) &= \tau_1\tau_2\text{-Cl}(F^+(Y - \sigma_1\sigma_2\text{-Cl}(V))) \\ &\subseteq F^+((\sigma_1,\sigma_2)\text{-sCl}(Y - \sigma_1\sigma_2\text{-Cl}(V))) \\ &= F^+(Y - \sigma_1\sigma_2\text{-Cl}(V)) \\ &= X - F^-(\sigma_1\sigma_2\text{-Cl}(V))\end{aligned}$$

and hence $F^-(V) \subseteq F^-(\sigma_1\sigma_2\text{-Cl}(V)) \subseteq \tau_1\tau_2\text{-Int}(F^-(\sigma_1\sigma_2\text{-Cl}(V)))$. By Theorem 2, F is lower almost contra-(τ_1,τ_2)-continuous.

(5) ⇒ (6): Let A be any subset of X and $B = F(A)$. Then, $A \subseteq F^+(B)$ and by (5), $\tau_1\tau_2\text{-Cl}(A) \subseteq \tau_1\tau_2\text{-Cl}(F^+(B)) \subseteq F^+(s(\sigma_1,\sigma_2)\theta\text{-Cl}(B))$. Thus,

$$\begin{aligned}F(\tau_1\tau_2\text{-Cl}(A)) &\subseteq F(F^+(s(\sigma_1,\sigma_2)\theta\text{-Cl}(B))) \\ &\subseteq s(\sigma_1,\sigma_2)\theta\text{-Cl}(B)\end{aligned}$$

$$= s(\sigma_1, \sigma_2)\theta\text{-Cl}(F(A)).$$

(6) ⇒ (5): Let B be any subset of Y. By (6), we have

$$F(\tau_1\tau_2\text{-Cl}(F^+(B))) \subseteq s(\sigma_1, \sigma_2)\theta\text{-Cl}(F(F^+(B))) \subseteq s(\sigma_1, \sigma_2)\theta\text{-Cl}(B)$$

and hence $\tau_1\tau_2\text{-Cl}(F^+(B)) \subseteq F^+(s(\sigma_1, \sigma_2)\theta\text{-Cl}(B))$.

Theorem 8. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is upper almost contra-(τ_1, τ_2)-continuous;

(2) $\tau_1\tau_2$-Cl$(F^-(\sigma_1\sigma_2\text{-Int}(K))) \subseteq F^-(K)$ for every $(\sigma_1, \sigma_2)s$-closed set K of Y;

(3) $\tau_1\tau_2$-Cl$(F^-(\sigma_1\sigma_2\text{-Int}((\sigma_1, \sigma_2)\text{-sCl}(B)))) \subseteq F^-((\sigma_1, \sigma_2)\text{-sCl}(B))$ for every subset B of Y;

(4) $F^+((\sigma_1, \sigma_2)\text{-sInt}(B)) \subseteq \tau_1\tau_2\text{-Int}(F^+(\sigma_1\sigma_2\text{-Cl}((\sigma_1, \sigma_2)\text{-sInt}(B))))$ for every subset B of Y.

Proof. (1) ⇒ (2): Let K be any $(\sigma_1, \sigma_2)s$-closed set of Y. Then, $Y - K$ is $(\sigma_1, \sigma_2)s$-open in Y. By Theorem 1, $F^+(Y - K) \subseteq \tau_1\tau_2)\text{-Int}(F^+(Y - \sigma_1\sigma_2\text{-Int}(K)))$. Thus,

$$\begin{aligned} X - F^-(K) &\subseteq \tau_1\tau_2\text{-Int}(F^+(Y - \sigma_1\sigma_2\text{-Int}(K))) \\ &= \tau_1\tau_2\text{-Int}(X - F^-(\sigma_1\sigma_2\text{-Int}(K))) \\ &= X - \tau_1\tau_2\text{-Cl}(F^-(\sigma_1\sigma_2\text{-Int}(K))) \end{aligned}$$

and hence $\tau_1\tau_2\text{-Cl}(F^-(\sigma_1\sigma_2\text{-Int}(K))) \subseteq F^-(K)$.

(2) ⇒ (3): Let B be any subset of Y. Then, $(\sigma_1, \sigma_2)\text{-sCl}(B))$ is $(\sigma_1, \sigma_2)s$-closed in Y, by (2) we have $\tau_1\tau_2\text{-Cl}(F^-(\sigma_1\sigma_2\text{-Int}((\sigma_1, \sigma_2)\text{-sCl}(B)))) \subseteq F^-((\sigma_1, \sigma_2)\text{-sCl}(B))$.

(3) ⇒ (4): Let B be any subset of Y. By (3), we have

$$\begin{aligned} X - F^+((\sigma_1, \sigma_2)\text{-sInt}(B)) &= F^-((\sigma_1, \sigma_2)\text{-sCl}(Y - B)) \\ &\supseteq \tau_1\tau_2\text{-Cl}(F^-(\sigma_1\sigma_2\text{-Int}((\sigma_1, \sigma_2)\text{-sCl}(Y - B)))) \\ &= \tau_1\tau_2\text{-Cl}(F^-(\sigma_1\sigma_2\text{-Int}(Y - (\sigma_1, \sigma_2)\text{-sInt}(B)))) \\ &= \tau_1\tau_2\text{-Cl}(F^-(Y - \sigma_1\sigma_2\text{-Cl}((\sigma_1, \sigma_2)\text{-sInt}(B)))) \\ &= \tau_1\tau_2\text{-Cl}(X - F^+(\sigma_1\sigma_2\text{-Cl}((\sigma_1, \sigma_2)\text{-sInt}(B)))) \\ &= X - \tau_1\tau_2\text{-Int}(F^+(\sigma_1\sigma_2\text{-Cl}((\sigma_1, \sigma_2)\text{-sInt}(B)))) \end{aligned}$$

and hence $F^+((\sigma_1, \sigma_2)\text{-sInt}(B)) \subseteq \tau_1\tau_2\text{-Int}(F^+(\sigma_1\sigma_2\text{-Cl}((\sigma_1, \sigma_2)\text{-sInt}(B))))$.

(4) ⇒ (1): Let V be any $(\sigma_1, \sigma_2)s$-open set of Y. Then, $V = (\sigma_1, \sigma_2)\text{-sInt}(V)$ and by (4), $F^+(V) \subseteq \tau_1\tau_2\text{-Int}(F^+(\sigma_1\sigma_2\text{-Cl}(V)))$. By Theorem 5, F is upper almost contra-(τ_1, τ_2)-continuous.

Theorem 9. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is lower almost contra-(τ_1, τ_2)-continuous;

(2) $\tau_1\tau_2$-$Cl(F^+(\sigma_1\sigma_2$-$Int(K))) \subseteq F^+(K)$ for every $(\sigma_1, \sigma_2)s$-closed set K of Y;

(3) $\tau_1\tau_2$-$Cl(F^+(\sigma_1\sigma_2$-$Int((\sigma_1, \sigma_2)$-$sCl(B)))) \subseteq F^+((\sigma_1, \sigma_2)$-$sCl(B))$ for every subset B of Y;

(4) $F^-((\sigma_1, \sigma_2)$-$sInt(B)) \subseteq \tau_1\tau_2$-$Int(F^-(\sigma_1\sigma_2$-$Cl((\sigma_1, \sigma_2)$-$sInt(B))))$ for every subset B of Y.

Proof. The proof is similar to that of Theorem 8.

Theorem 10. *Let $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a multifunction. If $\tau_1\tau_2$-$Cl(F^-(B)) \subseteq F^-((\sigma_1, \sigma_2)r$-$ker(B))$ for every subset B of Y, then F is upper almost contra-(τ_1, τ_2)-continuous.*

Proof. Suppose that $\tau_1\tau_2$-Cl$(F^-(B)) \subseteq F^-((\sigma_1, \sigma_2)r$-$ker(B))$ for every subset B of Y. Let V be any $(\sigma_1, \sigma_2)r$-open set of Y. By Lemma 2, we have

$$\tau_1\tau_2\text{-Cl}(F^-(V)) \subseteq F^-((\sigma_1, \sigma_2)r\text{-}ker(V)) = F^-(V)$$

and hence $F^-(V)$ is $\tau_1\tau_2$-closed in X. By Theorem 1, F is upper almost contra-(τ_1, τ_2)-continuous.

Theorem 11. *Let $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a multifunction. If $F(\tau_1\tau_2$-$Cl(A)) \subseteq (\sigma_1, \sigma_2)r$-$ker(F(A))$ for every subset A of X, then F is lower almost contra-(τ_1, τ_2)-continuous.*

Proof. Let V be any $(\sigma_1, \sigma_2)r$-open set of Y. Then, we have

$$F(\tau_1\tau_2\text{-Cl}(F^+(V))) \subseteq (\sigma_1, \sigma_2)r\text{-}ker(V)$$

and hence $\tau_1\tau_2$-Cl$(F^+(V)) \subseteq F^+((\sigma_1, \sigma_2)r$-$ker(V))$. Thus by Lemma 2,

$$\tau_1\tau_2\text{-Cl}(F^+(V)) \subseteq F^+((\sigma_1, \sigma_2)r\text{-}ker(V)) = F^+(V)$$

and so $F^+(V)$ is $\tau_1\tau_2$-closed in X. By Theorem 2, F is lower contra-(τ_1, τ_2)-continuous.

Theorem 12. *Let $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a multifunction. If $\tau_1\tau_2$-$Cl(F^+(B)) \subseteq F^+((\sigma_1, \sigma_2)r$-$ker(B))$ for every subset B of Y, then F is lower almost contra-(τ_1, τ_2)-continuous.*

Proof. Let V be any $(\sigma_1, \sigma_2)r$-open set of Y. Then,

$$\tau_1\tau_2\text{-Cl}(F^+(V)) \subseteq F^+((\sigma_1, \sigma_2)r\text{-}ker(V))$$

and by Lemma 2, $\tau_1\tau_2$-Cl$(F^+(V)) \subseteq F^+((\sigma_1, \sigma_2)r\text{-}ker(V)) = F^+(V)$. This implies that $F^+(V)$ is $\tau_1\tau_2$-closed in X. By Theorem 2, F is lower almost contra-(τ_1, τ_2)-continuous.

Definition 3. [60] *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be upper weakly (τ_1, τ_2)-continuous if for each $x \in X$ and each $\sigma_1\sigma_2$-open set V of Y containing $F(x)$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $F(U) \subseteq \sigma_1\sigma_2$-Cl$(V)$.*

Theorem 13. *If $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is an upper almost contra-(τ_1, τ_2)-continuous multifunction, then F is upper weakly (τ_1, τ_2)-continuous.*

Proof. Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y containing $F(x)$. Then, $\sigma_1\sigma_2$-Cl(V) is a $(\sigma_1, \sigma_2)r$-closed set Y containing $F(x)$. Since F is upper almost contra-(τ_1, τ_2)-continuous, there exists a $\tau_1\tau_2$-open set U of X containing x such that $U \subseteq F^+(\sigma_1\sigma_2\text{-Cl}(V))$; hence $F(U) \subseteq \sigma_1\sigma_2$-Cl$(V)$. Thus, F is upper weakly (τ_1, τ_2)-continuous.

Definition 4. [60] *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be lower weakly (τ_1, τ_2)-continuous if for each $x \in X$ and each $\sigma_1\sigma_2$-open set V of Y such that $F(x) \cap V \neq \emptyset$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $\sigma_1\sigma_2$-Cl$(V) \cap F(z) \neq \emptyset$ for each $z \in U$.*

Theorem 14. *If $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a lower almost contra-(τ_1, τ_2)-continuous multifunction, then F is lower weakly (τ_1, τ_2)-continuous.*

Proof. It is similar to that of Theorem 13.

Definition 5. [82] *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called upper contra-(τ_1, τ_2)-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-closed set K of Y with $x \in F^+(K)$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $U \subseteq F^+(K)$. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called upper contra-(τ_1, τ_2)-continuous if F is upper contra-(τ_1, τ_2)-continuous at each point x of X.*

Theorem 15. *If $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is an upper contra-(τ_1, τ_2)-continuous multifunction, then F is upper almost contra-(τ_1, τ_2)-continuous.*

Proof. Let $x \in X$ and K be any $(\sigma_1, \sigma_2)r$-closed set of Y with $x \in F^+(K)$. Then, K is $\sigma_1\sigma_2$-closed in Y. Since F is upper contra-(τ_1, τ_2)-continuous, there exists a $\tau_1\tau_2$-open set U of X containing x such that $U \subseteq F^+(K)$. Thus, F is upper almost contra-(τ_1, τ_2)-continuous.

The converse of Theorem 15 is not true in general as shown in the following example.

Example 1. *Let* $X = \{a, b, c, d\}$ *with topologies*

$$\tau_1 = \{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}, X\}$$

and $\tau_2 = \{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, X\}$. *Let* $Y = \{1, 2, 3, 4\}$ *with topologies* $\sigma_1 = \{\emptyset, \{1\}, \{3\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}, \{1, 3, 4\}, Y\}$ *and*

$$\sigma_2 = \{\emptyset, \{1\}, \{3\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 2, 3\}, \{1, 3, 4\}, \{1, 2, 4\}, Y\}.$$

A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is defined as follows:* $F(a) = \{4\}$, $F(b) = \{3\}$, $F(c) = \{1\}$ *and* $F(d) = \{2\}$. *Then,* F *is upper almost* (τ_1, τ_2)*-continuous but* F *is not upper contra-*(τ_1, τ_2)*-continuous.*

Definition 6. [82] *A multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is said to be lower contra-*(τ_1, τ_2)*-continuous at a point* $x \in X$ *if for each* $\sigma_1\sigma_2$*-closed set* K *of* Y *such that* $x \in F^-(K)$, *there exists a* $\tau_1\tau_2$*-open set* U *of* X *containing* x *with* $U \subseteq F^-(K)$. *A multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is called lower contra-*(τ_1, τ_2)*-continuous if* F *is lower contra-*(τ_1, τ_2)*-continuous at each point* x *of* X.

Theorem 16. *If* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is a lower contra-*(τ_1, τ_2)*-continuous multifunction, then* F *is lower almost contra-*(τ_1, τ_2)*-continuous.*

Proof. It is similar to that of Theorem 15.

Definition 7. [58] *A multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is said to be:*

(1) upper (τ_1, τ_2)*-continuous if for each* $x \in X$ *and each* $\sigma_1\sigma_2$*-open set* V *of* Y *such that* $F(x) \subseteq V$, *there exists a* $\tau_1\tau_2$*-open set* U *of* X *containing* x *such that* $F(U) \subseteq V$;

(2) lower (τ_1, τ_2)*-continuous if for each* $x \in X$ *and each* $\sigma_1\sigma_2$*-open set* V *of* Y *such that* $F(x) \cap V \neq \emptyset$, *there exists a* $\tau_1\tau_2$*-open set* U *of* X *containing* x *such that* $F(z) \cap V \neq \emptyset$ *for each* $z \in U$.

Lemma 5. [58] *For a multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, *the following properties are equivalent:*

(1) F *is upper* (τ_1, τ_2)*-continuous;*

(2) $F^+(V)$ *is* $\tau_1\tau_2$*-open in* X *for every* $\sigma_1\sigma_2$*-open set* V *of* Y;

(3) $F^-(K)$ *is* $\tau_1\tau_2$*-closed in* X *for every* $\sigma_1\sigma_2$*-closed set* K *of* Y;

(4) $\tau_1\tau_2\text{-}Cl(F^-(B)) \subseteq F^-(\sigma_1\sigma_2\text{-}Cl(B))$ *for every subset* B *of* Y;

(5) $F^+(\sigma_1\sigma_2\text{-}Int(B)) \subseteq \tau_1\tau_2\text{-}Int(F^+(B))$ *for every subset* B *of* Y.

Lemma 6. [58] *For a multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, *the following properties are equivalent:*

(1) F is lower (τ_1, τ_2)-continuous;

(2) $F^-(V)$ is $\tau_1\tau_2$-open in X for every $\sigma_1\sigma_2$-open set V of Y;

(3) $F^+(K)$ is $\tau_1\tau_2$-closed in X for every $\sigma_1\sigma_2$-closed set K of Y;

(4) $\tau_1\tau_2$-$Cl(F^+(B)) \subseteq F^+(\sigma_1\sigma_2$-$Cl(B))$ for every subset B of Y;

(5) $F(\tau_1\tau_2$-$Cl(A)) \subseteq \sigma_1\sigma_2$-$Cl(F(A))$ for every subset A of X;

(6) $F^-(\sigma_1\sigma_2$-$Int(B)) \subseteq \tau_1\tau_2$-$Int(F^-(B))$ for every subset B of Y.

Theorem 17. *If $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is an upper (τ_1, τ_2)-continuous multifunction and $G : (Y, \sigma_1, \sigma_2) \to (Z, \rho_1, \rho_2)$ is an upper almost contra-(σ_1, σ_2)-continuous multifunction, then $G \circ F : (X, \tau_1, \tau_2) \to (Z, \rho_1, \rho_2)$ is upper almost contra-(τ_1, τ_2)-continuous.*

Proof. Let K be any $(\rho_1, \rho_2)r$-closed set of Z. Since G is upper almost contra-(σ_1, σ_2)-continuous, by Theorem 1 we have $F^+(K)$ is $\sigma_1\sigma_2$-open in Y. Since F is upper (τ_1, τ_2)-continuous, by Lemma 5 we have $(G \circ F)^+(K) = F^+(G^+(K))$ is $\tau_1\tau_2$-open in X. Thus by Theorem 1, $G \circ F$ is upper almost contra-(τ_1, τ_2)-continuous.

Theorem 18. *If $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is an lower (τ_1, τ_2)-continuous multifunction and $G : (Y, \sigma_1, \sigma_2) \to (Z, \rho_1, \rho_2)$ is an lower almost contra-(σ_1, σ_2)-continuous multifunction, then $G \circ F : (X, \tau_1, \tau_2) \to (Z, \rho_1, \rho_2)$ is lower almost contra-(τ_1, τ_2)-continuous.*

Proof. It is similar to that of Theorem 17.

For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, a multifunction

$$\mathrm{Cl}F_{\circledast} : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$$

is defined in [78] as follows: $\mathrm{Cl}F_{\circledast}(x) = \sigma_1\sigma_2$-$\mathrm{Cl}(F(x))$ for each $x \in X$.

Definition 8. [78] *A subset A of a bitopological space (X, τ_1, τ_2) is said to be:*

(1) $\tau_1\tau_2$-paracompact if every cover of A by $\tau_1\tau_2$-open sets of X is refined by a cover of A which consists of $\tau_1\tau_2$-open sets of X and is $\tau_1\tau_2$-locally finite in X;

(2) $\tau_1\tau_2$-regular if for each $x \in A$ and each $\tau_1\tau_2$-open set U of X containing x, there exists a $\tau_1\tau_2$-open set V of X such that $x \in V \subseteq \tau_1\tau_2$-$Cl(V) \subseteq U$.

Lemma 7. [78] *If $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a multifunction such that $F(x)$ is $\sigma_1\sigma_2$-regular and $\sigma_1\sigma_2$-paracompact for each $x \in X$, then $ClF^+_{\circledast}(V) = F^+(V)$ for each $\sigma_1\sigma_2$-open set V of Y.*

Lemma 8. [82] *If $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a multifunction such that $F(x)$ is $\sigma_1\sigma_2$-regular and $\sigma_1\sigma_2$-paracompact for each $x \in X$, then $ClF^-_{\circledast}(K) = F^-(K)$ for each $\sigma_1\sigma_2$-closed set K of Y.*

Lemma 9. [78] *For a multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$*,* $ClF^-_{\circledast}(V) = F^-(V)$ *for each* $\sigma_1\sigma_2$*-open set* V *of* Y*.*

Lemma 10. [82] *For a multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$*,* $ClF^+_{\circledast}(K) = F^+(K)$ *for each* $\sigma_1\sigma_2$*-closed set* K *of* Y*.*

Theorem 19. *A multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is upper almost contra-*(τ_1, τ_2)*-continuous if and only if* $ClF_{\circledast} : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is upper almost contra-*(τ_1, τ_2)*-continuous.*

Proof. Suppose that F is upper almost contra-(τ_1, τ_2)-continuous. Let K be any $(\sigma_1, \sigma_2)r$-closed set of Y. It follows from Lemma 9, Lemma 10 and Theorem 1, $\text{Cl}F^+_{\circledast}(K) = F^+(K)$ is $\tau_1\tau_2$-open in X. Thus, $\text{Cl}F_{\circledast}$ is upper almost contra-(τ_1, τ_2)-continuous.

Conversely, suppose that $\text{Cl}F_{\circledast}$ is upper almost contra-(τ_1, τ_2)-continuous. Let K be any $(\sigma_1, \sigma_2)r$-closed set of Y. By Lemma 9, Lemma 10 and Theorem 1, $F^+(K) = \text{Cl}F^+_{\circledast}(K)$ is $\tau_1\tau_2$-open in X. Thus, F is upper almost contra-(τ_1, τ_2)-continuous.

Theorem 20. *Let* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *be a multifunction such that* $F(x)$ *is* $\sigma_1\sigma_2$*-paracompact and* $\sigma_1\sigma_2$*-regular for each* $x \in X$*. Then,* F *is lower almost contra-*(τ_1, τ_2)*-continuous if and only if* $ClF_{\circledast} : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is lower almost contra-*(τ_1, τ_2)*-continuous.*

Proof. Suppose that F is lower almost contra-(τ_1, τ_2)-continuous. Let K be any $(\sigma_1, \sigma_2)r$-closed set of Y. By Lemma 7, Lemma 8 and Theorem 2, $\text{Cl}F^-_{\circledast}(K) = F^-(K)$ is $\tau_1\tau_2$-open in X. This shows that $\text{Cl}F_{\circledast}$ is lower almost contra-(τ_1, τ_2)-continuous.

Conversely, suppose that $\text{Cl}F_{\circledast}$ is lower almost contra-(τ_1, τ_2)-continuous. Let K be any $(\sigma_1, \sigma_2)r$-closed set of Y. By Lemma 7, Lemma 8 and Theorem 2, $F^-(K) = \text{Cl}F^-_{\circledast}(K)$ is $\tau_1\tau_2$-open in X. This shows that F is lower almost contra-(τ_1, τ_2)-continuous.

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