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On R-(τ_1, τ_2)-continuous Functions

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Abstract. This paper presents a new class of functions between bitopological spaces called R-(τ_1, τ_2)-continuous functions. Furthermore, several characterizations and some properties concerning R-(τ_1, τ_2)-continuous functions are established.

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1. Introduction

The field of the mathematical science which goes under the name of topology is concerned with all questions directly or indirectly related to continuity. Preopen sets, semi-open sets, α-open sets, β-open sets, δ-open sets and θ-open sets play an important role in the research of generalizations of continuity. By using these sets many authors introduced and investigated various types of continuity. In [1], the present authors studied some properties of (Λ, sp)-open sets, $s(\Lambda, sp)$-open sets, $p(\Lambda, sp)$-open sets, $\alpha(\Lambda, sp)$-open sets and $\beta(\Lambda, sp)$-open sets. Viriyapong and Boonpok [2] investigated several characterizations of (Λ, sp)-continuous functions by utilizing the notions of (Λ, sp)-open sets and (Λ, sp)-closed sets. Dungthaisong et al. [3] introduced and studied the concept of $g_{(m,n)}$-continuous functions. Duangphui et al. [4] introduced and investigated the notion of $(\mu, \mu')^{(m,n)}$-continuous functions. Moreover, some characterizations of almost (Λ, p)-continuous functions, almost strongly $\theta(\Lambda, p)$-continuous functions, weakly (Λ, b)-continuous functions, $\theta(\star)$-precontinuous functions, $(\Lambda, p(\star))$-continuous functions, $\star$-continuous functions, θ-$\mathscr{I}$-continuous functions, almost (g, m)-continuous functions, pairwise almost M-continuous functions were presented in [5], [6], [7], [8], [9], [10], [11], [12] and [13], respectively. Konstadilaki-Savvopoulou and Janković [14] introduced and studied a strong form of continuity of functions between topological spaces called R-continuous functions. Crossley and

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Hildebrand [15] introduced and investigated the concept of irresolute functions. Reilly and Vamanamurthy [16] introduced and studied the notion of preirresolute functions. Baker [17] introduced and investigated the concept of R-irrsolute functions. Furthermore, the present author [18] introduced a strong form of preirresolute functions called R-preirresolute functions. These four classes of functions have properties similar to the class of R-continuous functions. Beceren and Noiri [19] introduced and studied the notions of new classes of functions, namely α-preirresolute functions and β-preirresolute functions. A new class of α-preirresolute functions which is stronger than preirresolute functions [17] is a generalization of strongly M-precontinuous functions [20]. A new class of β-irresolute functions which is stronger than almost α-irresolute functions [19] is a generalization of preirresolute functions [17]. Noiri and Popa [21] introduced a new class of functions called R-M-continuous functions as functions defined between sets satisfying some minimal conditions and obtained several characterizations of R-M-continuous functions. Noiri and Popa [21] investigated the relationship between R-M-continuity and some low separation axioms (m-T_1, m-T_2, m-R_0). The notion of (τ_1, τ_2)-continuous functions was introduced in [22]. Moreover, several characterizations of almost (τ_1, τ_2)-continuous functions and weakly (τ_1, τ_2)-continuous functions were studied in [23] and [24], respectively. In this paper, we introduce the concept of R-(τ_1, τ_2)-continuous functions. We also investigate some characterizations of R-(τ_1, τ_2)-continuous functions.

2. Preliminaries

Throughout the present paper, spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or simply X and Y) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a bitopological space (X, τ_1, τ_2). The closure of A and the interior of A with respect to τ_i are denoted by τ_i-Cl(A) and τ_i-Int(A), respectively, for $i = 1, 2$. A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-*closed* [25] if $A = \tau_1$-Cl$(\tau_2$-Cl$(A))$. The complement of a $\tau_1\tau_2$-closed set is called $\tau_1\tau_2$-*open*. The intersection of all $\tau_1\tau_2$-closed sets of X containing A is called the $\tau_1\tau_2$-*closure* [25] of A and is denoted by $\tau_1\tau_2$-Cl(A). The union of all $\tau_1\tau_2$-open sets of X contained in A is called the $\tau_1\tau_2$-*interior* [25] of A and is denoted by $\tau_1\tau_2$-Int(A).

Lemma 1. [25] *Let A and B be subsets of a bitopological space (X, τ_1, τ_2). For the $\tau_1\tau_2$-closure, the following properties hold:*

(1) $A \subseteq \tau_1\tau_2$-*Cl*(A) *and* $\tau_1\tau_2$-*Cl*$(\tau_1\tau_2$-*Cl*$(A)) = \tau_1\tau_2$-*Cl*(A).

(2) If $A \subseteq B$, *then* $\tau_1\tau_2$-*Cl*$(A) \subseteq \tau_1\tau_2$-*Cl*$(B)$.

(3) $\tau_1\tau_2$-*Cl*(A) *is* $\tau_1\tau_2$-*closed.*

(4) A *is* $\tau_1\tau_2$-*closed if and only if* $A = \tau_1\tau_2$-*Cl*(A).

(5) $\tau_1\tau_2$-*Cl*$(X - A) = X - \tau_1\tau_2$-*Int*$(A)$.

A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)r$*-open* [26] (resp. $(\tau_1, \tau_2)s$*-open* [27], $(\tau_1, \tau_2)p$*-open* [27], $(\tau_1, \tau_2)\beta$*-open* [27]) if $A = \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A))$ (resp. $A \subseteq \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A))$, $A \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A))$, $A \subseteq \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A)))$). The complement of a $(\tau_1, \tau_2)r$-open (resp. $(\tau_1, \tau_2)s$-open, $(\tau_1, \tau_2)p$-open, $(\tau_1, \tau_2)\beta$-open) set is called $(\tau_1, \tau_2)r$*-closed* (resp. $(\tau_1, \tau_2)s$*-closed*, $(\tau_1, \tau_2)p$*-closed*, $(\tau_1, \tau_2)\beta$*-closed*). A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\alpha(\tau_1, \tau_2)$*-open* [28] if $A \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A)))$. The complement of an $\alpha(\tau_1, \tau_2)$-open set is said to be $\alpha(\tau_1, \tau_2)$*-closed*. Let A be a subset of a bitopological space (X, τ_1, τ_2). A point $x \in X$ is called a $(\tau_1, \tau_2)\theta$*-cluster point* [26] of A if $\tau_1\tau_2\text{-Cl}(U) \cap A \neq \emptyset$ for every $\tau_1\tau_2$-open set U containing x. The set of all $(\tau_1, \tau_2)\theta$-cluster points of A is called the $(\tau_1, \tau_2)\theta$*-closure* [26] of A and is denoted by $(\tau_1, \tau_2)\theta\text{-Cl}(A)$. A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)\theta$*-closed* [26] if $A = (\tau_1, \tau_2)\theta\text{-Cl}(A)$. The complement of a $(\tau_1, \tau_2)\theta$-closed set is said to be $(\tau_1, \tau_2)\theta$*-open*. The union of all $(\tau_1, \tau_2)\theta$-open sets contained in A is called the $(\tau_1, \tau_2)\theta$*-interior* [26] of A and is denoted by $(\tau_1, \tau_2)\theta\text{-Int}(A)$.

Lemma 2. [26] *For a subset A of a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) If A is $\tau_2\tau_2$-open in X, then $\tau_1\tau_2\text{-Cl}(A) = (\tau_1, \tau_2)\theta\text{-Cl}(A)$.

(2) $(\tau_1, \tau_2)\theta\text{-Cl}(A)$ is $\tau_1\tau_2$-closed in X.

3. R-(τ_1, τ_2)-continuous functions

In this section, we introduce the concept of R-(τ_1, τ_2)-continuous functions. Furthermore, several characterizations of R-(τ_1, τ_2)-continuous functions are discussed.

Definition 1. *A functions $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be R-(τ_1, τ_2)-continuous if for each $x \in X$ and for each $\sigma_1\sigma_2$-open set V of Y containing $f(x)$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $\sigma_1\sigma_2\text{-Cl}(f(U)) \subseteq V$.*

Definition 2. [22] *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called (τ_1, τ_2)-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open set V of Y containing $f(x)$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $f(U) \subseteq V$. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called (τ_1, τ_2)-continuous if f has this property at each point of X.*

Lemma 3. [22] *For a function $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) f is (τ_1, τ_2)-continuous;

(2) $f^{-1}(V)$ is $\tau_1\tau_2$-open in X for every $\sigma_1\sigma_2$-open set V of Y;

(3) $f(\tau_1\tau_2\text{-Cl}(A)) \subseteq \sigma_1\sigma_2\text{-Cl}(f(A))$ for every subset A of X;

(4) $\tau_1\tau_2\text{-Cl}(f^{-1}(B)) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(B))$ for every subset B of Y;

(5) $f^{-1}(\sigma_1\sigma_2\text{-}Int(B)) \subseteq \tau_1\tau_2\text{-}Int(f^{-1}(B))$ *for every subset* B *of* Y*;*

(6) $f^{-1}(K)$ *is* $\tau_1\tau_2$*-closed in* X *for every* $\sigma_1\sigma_2$*-closed set* K *of* Y*.*

Lemma 4. *If a function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is* R*-*(τ_1, τ_2)*-continuous, then* f *is* (τ_1, τ_2)*-continuous.*

Proof. Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$. Since f is R-(τ_1, τ_2)-continuous, there exists a $\tau_1\tau_2$-open set U of X containing x such that $\sigma_1\sigma_2$-Cl$(f(U)) \subseteq V$. This implies that $f(U) \subseteq V$. Thus, f is (τ_1, τ_2)-continuous.

Theorem 1. *For a function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$*, the following properties are equivalent:*

(1) f *is* R*-*(τ_1, τ_2)*-continuous;*

(2) for each point $x \in X$ *and each* $\sigma_1\sigma_2$*-open set* V *of* Y *containing* $f(x)$*, exists a* $\tau_1\tau_2$*-open set* U *of* X *containing* x *such that* $\sigma_1\sigma_2\text{-}Cl(f(\tau_1\tau_2\text{-}Cl(U))) \subseteq V$*;*

(3) for each point $x \in X$ *and each* $\sigma_1\sigma_2$*-closed set* F *of* Y *with* $f(x) \notin F$*, exists a* $\tau_1\tau_2$*-open set* U *of* X *containing* x *and a* $\sigma_1\sigma_2$*-open set* V *of* Y *such that* $F \subseteq V$ *and* $f(\tau_1\tau_2\text{-}Cl(U)) \cap V = \emptyset$*;*

(4) for each point $x \in X$ *and each* $\sigma_1\sigma_2$*-closed set* F *of* Y *with* $f(x) \notin F$*, exists a* $\tau_1\tau_2$*-open set* U *of* X *containing* x *and a* $\sigma_1\sigma_2$*-open set* V *of* Y *such that* $F \subseteq V$ *and* $f(U) \cap V = \emptyset$*.*

Proof. (1) $\Rightarrow$ (2): Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$. Since f is R-(τ_1, τ_2)-continuous, there exists a $\tau_1\tau_2$-open set U of X containing x such that $\sigma_1\sigma_2$-Cl$(f(U)) \subseteq V$. By Lemma 4, we have f is (τ_1, τ_2)-continuous and by Lemma 3, $f(\tau_1\tau_2\text{-Cl}(U)) \subseteq \sigma_1\sigma_2\text{-Cl}(f(U)) \subseteq V$. Thus,

$$\sigma_1\sigma_2\text{-Cl}(f(\tau_1\tau_2\text{-Cl}(U))) \subseteq \sigma_1\sigma_2\text{-Cl}(f(U)) \subseteq V.$$

(2) $\Rightarrow$ (3): Let $x \in X$ and F be any $\sigma_1\sigma_2$-closed set of Y with $f(x) \notin F$. Then, we have $f(x) \in Y - F$ and $Y - F$ is $\sigma_1\sigma_2$-open in Y. By (2), there exists a $\tau_1\tau_2$-open set U of X containing x such that $\sigma_1\sigma_2\text{-Cl}(f(\tau_1\tau_2\text{-Cl}(U))) \subseteq Y - F$. Put

$$V = Y - \sigma_1\sigma_2\text{-Cl}(f(\tau_1\tau_2\text{-Cl}(U))).$$

Then, V is $\sigma_1\sigma_2$-open and $F \subseteq V$. Furthermore, $f(\tau_1\tau_2\text{-Cl}(U)) \cap V = \emptyset$.

(3) $\Rightarrow$ (4): The proof is obvious.

(4) $\Rightarrow$ (1): Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$. Then, $Y - V$ is $\sigma_1\sigma_2$-closed in Y and $f(x) \notin Y - V$. By (4), there exists a $\tau_1\tau_2$-open set U of X containing x and a $\sigma_1\sigma_2$-open set W of Y such that $Y - V \subseteq W$ and $f(U) \cap W = \emptyset$. Since $f(U) \subseteq Y - W$ and $Y - W$ is $\sigma_1\sigma_2$-closed, $\sigma_1\sigma_2\text{-Cl}(f(U)) \subseteq \sigma_1\sigma_2\text{-Cl}(Y - W) = Y - W \subseteq V$. This shows that f is R-(τ_1, τ_2)-continuous.

Definition 3. [29] *A function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is said to be strongly* $\theta(\tau_1, \tau_2)$*-continuous at a point* $x \in X$ *if for each* $\sigma_1\sigma_2$*-open set* V *of* Y *containing* $f(x)$*, there exists a* $\tau_1\tau_2$*-open set* U *of* X *containing* x *such that* $f(\tau_1\tau_2\text{-}Cl(U)) \subseteq V$*. A function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is said to be strongly* $\theta(\tau_1, \tau_2)$*-continuous if* f *is strongly* $\theta(\tau_1, \tau_2)$*-continuous at each point* x *of* X*.*

Theorem 2. *If a function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is* R*-*(τ_1, τ_2)*-continuous, then* f *is strongly* $\theta(\tau_1, \tau_2)$*-continuous.*

Proof. It follows from Theorem 1(2).

Definition 4. *A function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is said to be weakly* (τ_1, τ_2)*-closed if for each* $\tau_1\tau_2$*-closed set* F *of* X*,* $\sigma_1\sigma_2\text{-}Cl(f(\tau_1\tau_2\text{-}Int(F))) \subseteq f(F)$*.*

Theorem 3. *For a function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$*, the following properties are equivalent:*

(1) f *is weakly* (τ_1, τ_2)*-closed;*

(2) $\sigma_1\sigma_2\text{-}Cl(f(U)) \subseteq f(\tau_1\tau_2\text{-}Cl(U))$ *for each* $\tau_1\tau_2$*-open set* U *of* X*;*

(3) for each subset B *of* Y *and each* $\tau_1\tau_2$*-open set* U *of* X *with* $f^{-1}(B) \subseteq U$*, there exists a* $\sigma_1\sigma_2$*-open set* V *of* Y *such that* $B \subseteq V$ *and* $f^{-1}(V) \subseteq \tau_1\tau_2\text{-}Cl(U)$*;*

(4) for each $y \in Y$ *and each* $\tau_1\tau_2$*-open set* U *of* X *with* $f^{-1}(y) \subseteq U$*, there exists a* $\sigma_1\sigma_2$*-open set* V *of* Y *containing* y *such that* $f^{-1}(y) \subseteq \tau_1\tau_2\text{-}Cl(U)$*;*

(5) $\sigma_1\sigma_2\text{-}Cl(f(\tau_1\tau_2\text{-}Int(\tau_1\tau_2\text{-}Cl(A)))) \subseteq f(\tau_1\tau_2\text{-}Cl(A))$ *for each subset* A *of* X*;*

(6) $\sigma_1\sigma_2\text{-}Cl(f(\tau_1\tau_2\text{-}Int((\tau_1, \tau_2)\theta\text{-}Cl(A)))) \subseteq f((\tau_1, \tau_2)\theta\text{-}Cl(A))$ *for each subset* A *of* X*.*

Proof. (1) ⇒ (2): Let U be any $\tau_1\tau_2$-open set of X. Since f is weakly (τ_1, τ_2)-closed, we have $\sigma_1\sigma_2\text{-Cl}(f(U)) \subseteq \sigma_1\sigma_2\text{-Cl}(f(\tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(U)))) \subseteq f(\tau_1\tau_2\text{-Cl}(U))$.

(2) ⇒ (3): Let B be any subset of Y and U be any $\tau_1\tau_2$-open set of X with $f^{-1}(B) \subseteq U$. Since U is $\tau_1\tau_2$-open,

$$\begin{aligned} f^{-1}(B) \cap \tau_1\tau_2\text{-Cl}(X - \tau_1\tau_2\text{-Cl}(U)) &= f^{-1}(B) \cap (X - \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(U))) \\ &\subseteq f^{-1}(B) \cap (X - \tau_1\tau_2\text{-Int}(U)) \\ &= f^{-1}(B) \cap (X - U) = \emptyset. \end{aligned}$$

Thus, $f^{-1}(B) \cap \tau_1\tau_2\text{-Cl}(X - \tau_1\tau_2\text{-Cl}(U)) = \emptyset$ and hence

$$B \cap f(\tau_1\tau_2\text{-Cl}(X - \tau_1\tau_2\text{-Cl}(U))) = \emptyset.$$

On the other hand, we have $\tau_1\tau_2\text{-Cl}(U)$ is $\tau_1\tau_2$-closed and $X - \tau_1\tau_2\text{-Cl}(U)$ is $\tau_1\tau_2$-open. Thus by (2), $\sigma_1\sigma_2\text{-Cl}(f(X - \tau_1\tau_2\text{-Cl}(U))) \subseteq f(\tau_1\tau_2\text{-Cl}(X - \tau_1\tau_2\text{-Cl}(U)))$ and so

$$B \cap \sigma_1\sigma_2\text{-Cl}(f(X - \tau_1\tau_2\text{-Cl}(U))) \subseteq B \cap f(\tau_1\tau_2\text{-Cl}(X - \tau_1\tau_2\text{-Cl}(U))).$$

Let $V = Y - \sigma_1\sigma_2\text{-Cl}(f(X - \tau_1\tau_2\text{-Cl}(U)))$. Then, V is $\sigma_1\sigma_2$-open in Y, $B \subseteq V$ and

$$\begin{aligned} f^{-1}(V) &= f^{-1}(Y - \sigma_1\sigma_2\text{-Cl}(f(X - \tau_1\tau_2\text{-Cl}(U)))) \\ &\subseteq X - f^{-1}(f(X - \tau_1\tau_2\text{-Cl}(U))) \\ &\subseteq \tau_1\tau_2\text{-Cl}(U). \end{aligned}$$

(3) ⇒ (4): The proof is obvious.

(4) ⇒ (1): Let F be any $\tau_1\tau_2$-closed set of X and $y \in Y - f(F)$. Since $Y - F$ is $\sigma_1\sigma_2$-open and $f^{-1}(y) \subseteq X - F$, by (4) there exists a $\sigma_1\sigma_2$-open set V of Y with $y \in V$ and $f^{-1}(V) \subseteq \tau_1\tau_2\text{-Cl}(X - F) = X - \tau_1\tau_2\text{-Int}(F)$. Thus, $V \cap f(\tau_1\tau_2\text{-Int}(F)) = \emptyset$ and hence $y \in Y - \sigma_1\sigma_2\text{-Cl}(f(\tau_1\tau_2\text{-Int}(F)))$. Therefore, $\sigma_1\sigma_2\text{-Cl}(f(\tau_1\tau_2\text{-Int}(F))) \subseteq f(F)$ which implies that f is weakly (τ_1, τ_2)-closed.

(1) ⇒ (5): Let A be any subset of X. Then, $\tau_1\tau_2\text{-Cl}(A)$ is $\tau_1\tau_2$-closed and by (1), $\sigma_1\sigma_2\text{-Cl}(f(\tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A)))) \subseteq f(\tau_1\tau_2\text{-Cl}(A))$.

(5) ⇒ (2): Let U be any $\tau_1\tau_2$-open set of X. Then by (5),

$$\sigma_1\sigma_2\text{-Cl}(f(U)) \subseteq \sigma_1\sigma_2\text{-Cl}(f(\tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(U)))) \subseteq f(\tau_1\tau_2\text{-Cl}(U)).$$

(1) ⇒ (6): Let A be any subset of X. Thus by Lemma 2, $(\tau_1, \tau_2)\theta\text{-Cl}(A)$ is $\tau_1\tau_2$-closed and by (1), we have $\sigma_1\sigma_2\text{-Cl}(f(\tau_1\tau_2\text{-Int}((\tau_1, \tau_2)\theta\text{-Cl}(A)))) \subseteq f((\tau_1, \tau_2)\theta\text{-Cl}(A))$.

(6) ⇒ (2): Let U be any $\tau_1\tau_2$-open set of X. By Lemma 2, we have

$$\tau_1\tau_2\text{-Cl}(U) = (\tau_1, \tau_2)\theta\text{-Cl}(U)$$

and by (6),

$$\begin{aligned} \sigma_1\sigma_2\text{-Cl}(f(U)) &\subseteq \sigma_1\sigma_2\text{-Cl}(f(\tau_1\tau_2\text{-Int}((\tau_1, \tau_2)\theta\text{-Cl}(U)))) \\ &\subseteq f((\tau_1, \tau_2)\theta\text{-Cl}(U)) \\ &= f(\tau_1\tau_2\text{-Cl}(U)). \end{aligned}$$

Theorem 4. *If a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is strongly $\theta(\tau_1, \tau_2)$-continuous and weakly (τ_1, τ_2)-closed, then f is R-(τ_1, τ_2)-continuous.*

Proof. Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$. Since f is strongly $\theta(\tau_1, \tau_2)$-continuous, there exists a $\tau_1\tau_2$-open set U of X containing x such that $f(\tau_1\tau_2\text{-Cl}(U)) \subseteq V$. Since f is weakly (τ_1, τ_2)-closed, by Theorem 3 we have

$$\sigma_1\sigma_2\text{-Cl}(f(U)) \subseteq f(\tau_1\tau_2\text{-Cl}(U)) \subseteq V.$$

This shows that f is R-(τ_1, τ_2)-continuous.

Definition 5. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be contra-(τ_1, τ_2)-open if $f(U)$ is $\sigma_1\sigma_2$-closed in Y for every $\tau_1\tau_2$-open set U of X.*

Theorem 5. *If a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is contra-(τ_1, τ_2)-open, then f is weakly (τ_1, τ_2)-closed.*

Proof. Let F be any $\tau_1\tau_2$-closed set of X. Then, $\tau_1\tau_2$-Int(F) is a $\tau_1\tau_2$-open set of X. Since f is contra-(τ_1, τ_2)-open, we have $f(\tau_1\tau_2\text{-Int}(F))$ is $\sigma_1\sigma_2$-closed in Y and hence $\sigma_1\sigma_2\text{-Cl}(f(\tau_1\tau_2\text{-Int}(F))) = f(\tau_1\tau_2\text{-Int}(F)) \subseteq f(F)$. This shows that f is weakly (τ_1, τ_2)-closed.

Theorem 6. *If a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (τ_1, τ_2)-continuous and contra-(τ_1, τ_2)-open, then f is R-(τ_1, τ_2)-continuous.*

Proof. Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$. Since f is (τ_1, τ_2)-continuous, by Lemma 3 we have $f^{-1}(V)$ is $\tau_1\tau_2$-open in X. Since f is contra-(τ_1, τ_2)-open, $f(f^{-1}(V))$ is $\sigma_1\sigma_2$-closed in Y and $\sigma_1\sigma_2\text{-Cl}(f(f^{-1}(V))) = f(f^{-1}(V)) \subseteq V$. Put $U = f^{-1}(V)$. Then, U is a $\tau_1\tau_2$-open set of X containing x and $\sigma_1\sigma_2\text{-Cl}(f(U)) \subseteq V$. This shows that f is *R*-(τ_1, τ_2)-continuous.

Definition 6. [24] *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be weakly (τ_1, τ_2)-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open set V of Y containing $f(x)$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $f(U) \subseteq \sigma_1\sigma_2\text{-}Cl(V)$. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be weakly (τ_1, τ_2)-continuous if f has this property at each point of X.*

Lemma 5. [24] *For a function $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) f is weakly (τ_1, τ_2)-continuous;

(2) $f(\tau_1\tau_2\text{-}Cl(A)) \subseteq (\sigma_1, \sigma_2)\theta\text{-}Cl(f(A))$ for every subset A of X;

(3) $\tau_1\tau_2\text{-}Cl(f^{-1}(B)) \subseteq f^{-1}((\sigma_1, \sigma_2)\theta\text{-}Cl(B))$ for every subset B of Y.

Recall that a bitopological space (X, τ_1, τ_2) is said to be (τ_1, τ_2)-*regular* [30] if for each $\tau_1\tau_2$-closed set F and each $x \notin F$, there exist disjoint $\tau_1\tau_2$-open sets U and V such that $x \in U$ and $F \subseteq V$.

Lemma 6. [31] *A bitopological space (X, τ_1, τ_2) is (τ_1, τ_2)-regular if and only if for each $x \in X$ and each $\tau_1\tau_2$-open set U containing x, there exists a $\tau_1\tau_2$-open set V such that $x \in V \subseteq \tau_1\tau_2\text{-}Cl(V) \subseteq U$.*

Lemma 7. [31] *Let (X, τ_1, τ_2) be a (τ_1, τ_2)-regular space. Then, the following properties hold:*

(1) $\tau_1\tau_2\text{-}Cl(A) = (\tau_1, \tau_2)\theta\text{-}Cl(A)$ for every subset A of X.

(2) Every $\tau_1\tau_2$-open set is $(\tau_1, \tau_2)\theta$-open.

Theorem 7. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is weakly (τ_1, τ_2)-continuous and (Y, σ_1, σ_2) is (σ_1, σ_2)-regular, then f is R-(τ_1, τ_2)-continuous.*

Proof. Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$. Since (Y, σ_1, σ_2) is (σ_1, σ_2)-regular, by Lemma 6 there exists a $\sigma_1\sigma_2$-open set W of Y such that

$$f(x) \in W \subseteq \sigma_1\sigma_2\text{-Cl}(W) \subseteq V.$$

Since f is weakly (τ_1, τ_2)-continuous, there exists a $\tau_1\tau_2$-open set U of X containing x such that $f(U) \subseteq \sigma_1\sigma_2$-Cl$(W)$. Thus, $\sigma_1\sigma_2$-Cl$(f(U)) \subseteq \sigma_1\sigma_2$-Cl$(W) \subseteq V$. This shows that f is R-(τ_1, τ_2)-continuous.

Definition 7. [32] *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called faintly (τ_1, τ_2)-continuous at a point $x \in X$ if for each $(\sigma_1, \sigma_2)\theta$-open set V of Y containing $f(x)$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $f(U) \subseteq V$. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called faintly (τ_1, τ_2)-continuous if f has this property at every point of X.*

Lemma 8. [32] *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) f is faintly (τ_1, τ_2)-continuous;

(2) $f^{-1}(V)$ is $\tau_1\tau_2$-open in X for each $(\sigma_1, \sigma_2)\theta$-open set V of Y;

(3) $f^{-1}(K)$ is $\tau_1\tau_2$-closed in X for each $(\sigma_1, \sigma_2)\theta$-closed set K of Y.

Theorem 8. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, where (Y, σ_1, σ_2) is (σ_1, σ_2)-regular, the following properties are equivalent:*

(1) f is R-(τ_1, τ_2)-continuous;

(2) f is strongly $\theta(\tau_1, \tau_2)$-continuous;

(3) f is (τ_1, τ_2)-continuous;

(4) f is weakly (τ_1, τ_2)-continuous;

(5) f is faintly (τ_1, τ_2)-continuous.

Proof. (1) ⇒ (2): It follows from Theorem 2.

(2) ⇒ (3) and (3) ⇒ (4): The proofs are obvious.

(4) ⇒ (5): Let F be any $\theta(\tau_1, \tau_2)$-closed set of Y. Since f is weakly (τ_1, τ_2)-continuous, by Lemma 5 we have $\sigma_1\sigma_2$-Cl$(f^{-1}(F)) \subseteq f^{-1}((\sigma_1, \sigma_2)\theta\text{-Cl}(F)) = f^{-1}(F)$ and hence $f^{-1}(F)$ is $\tau_1\tau_2$-closed in X. Thus by Lemma 8, f is faintly (τ_1, τ_2)-continuous.

(5) ⇒ (1): Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$. Since (Y, σ_1, σ_2) is (σ_1, σ_2)-regular, by Lemma 7 we have V is a $\theta(\tau_1, \tau_2)$-open set of Y. Since f is faintly (τ_1, τ_2)-continuous, by Lemma 8 we have $f^{-1}(V)$ is $\tau_1\tau_2$-open in X. Then by Lemma 3, f is (τ_1, τ_2)-continuous.

Definition 8. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called (τ_1, τ_2)-open if $f(V)$ is $\sigma_1\sigma_2$-open in Y for every $\tau_1\tau_2$-open set V of X.*

Theorem 9. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a R-(τ_1, τ_2)-continuous and (τ_1, τ_2)-open surjection, then (Y, σ_1, σ_2) is (σ_1, σ_2)-regular.*

Proof. Let $y \in Y$ and V be any $\sigma_1\sigma_2$-open set of Y containing y. Let $x \in f^{-1}(V)$. Since f is R-(τ_1, τ_2)-continuous, there exists a $\tau_1\tau_2$-open set U of X containing x such that $\sigma_1\sigma_2\text{-Cl}(f(U)) \subseteq V$. Since f is (τ_1, τ_2)-open, we have $f(U)$ is $\sigma_1\sigma_2$-open in Y and hence $y \in f(U) \subseteq \sigma_1\sigma_2\text{-Cl}(f(U)) \subseteq V$. It follows from Lemma 6 that (Y, σ_1, σ_2) is (σ_1, σ_2)-regular.

Corollary 1. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a (τ_1, τ_2)-continuous and (τ_1, τ_2)-open surjection, then f is R-(τ_1, τ_2)-continuous if and only if (Y, σ_1, σ_2) is (σ_1, σ_2)-regular.*

Proof. This is an immediate consequence of Theorem 7 and Theorem 9.

Lemma 9. [33] *Let (X, τ_1, τ_2) be (τ_1, τ_2)-regular. Then, a function*

$$f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$$

is strongly $\theta(\tau_1, \tau_2)$-continuous if and only if f is (τ_1, τ_2)-continuous.

Theorem 10. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a (τ_1, τ_2)-continuous, (τ_1, τ_2)-open and weakly (τ_1, τ_2)-closed surjection and (X, τ_1, τ_2) is (τ_1, τ_2)-regular, then (Y, σ_1, σ_2) is (σ_1, σ_2)-regular.*

Proof. Since f is (τ_1, τ_2)-continuous and (X, τ_1, τ_2) is (τ_1, τ_2)-regular, by Lemma 9 we have f is strongly $\theta(\tau_1, \tau_2)$-continuous. Furthermore, since f is weakly (τ_1, τ_2)-closed, by Theorem 4 we have f is R-(τ_1, τ_2)-continuous. It follows from Theorem 9 that (Y, σ_1, σ_2) is (σ_1, σ_2)-regular.

Definition 9. [29] *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to have a strongly $\theta(\tau_1, \tau_2)$-closed graph with respect to X if for each $(x, y) \in (X \times Y) - G(f)$, there exist a $\tau_1\tau_2$-open set U of X containing x and a $\sigma_1\sigma_2$-open set V of Y containing y such that $[\tau_1\tau_2\text{-}Cl(U) \times V] \cap G(f) = \emptyset$.*

Lemma 10. [29] *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ has a strongly $\theta(\tau_1, \tau_2)$-closed graph with respect to X if and only if for each $(x, y) \in (X \times Y) - G(f)$, there exist a $\tau_1\tau_2$-open set U of X containing x and a $\sigma_1\sigma_2$-open set V of Y containing y such that*

$$f(\tau_1\tau_2\text{-}Cl(U)) \cap V = \emptyset.$$

Recall that a bitopological space (X, τ_1, τ_2) is said to be (τ_1, τ_2)-T_1 [34] if for any pair of distinct points x, y in X, there exist $\tau_1\tau_2$-open sets U and V of X such that $x \in U$, $y \notin U$ and $y \in V$, $x \notin V$.

Theorem 11. *If a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is R-(τ_1, τ_2)-continuous and (Y, σ_1, σ_2) is (σ_1, σ_2)-T_1, then $G(f)$ is strongly $\theta(\tau_1, \tau_2)$-closed with respect to X.*

Proof. Let $(x, y) \in (X \times Y) - G(f)$. Then, $y \neq f(x)$. Since (Y, σ_1, σ_2) is (σ_1, σ_2)-T_1, there exists a $\sigma_1\sigma_2$-open set V of Y such that $f(x) \in V$ and $y \notin V$. Since f is R-(τ_1, τ_2)-continuous, there exists a $\tau_1\tau_2$-open set U of X containing x such that $\sigma_1\sigma_2$-Cl$(f(U)) \subseteq V$. Since $y \notin V$, we have $y \notin \sigma_1\sigma_2$-Cl$(f(U))$ and so $y \in Y - \sigma_1\sigma_2$-Cl$(f(U))$. By Lemma 1, $\sigma_1\sigma_2$-Cl$(f(U))$ is $\sigma_1\sigma_2$-closed and $Y - \sigma_1\sigma_2$-Cl$(f(U))$ is $\sigma_1\sigma_2$-open. Since f is R-(τ_1, τ_2)-continuous, f is (τ_1, τ_2)-continuous and by Lemma 3, $f(\tau_1\tau_2$-Cl$(U)) \subseteq \sigma_1\sigma_2$-Cl$(f(U))$. Then, $f(\tau_1\tau_2$-Cl$(U)) \cap (Y - \sigma_1\sigma_2$-Cl$(f(U))) = \emptyset$ and by Lemma 10, $G(f)$ is strongly $\theta(\tau_1, \tau_2)$-closed with respect to X.

Recall that a bitopological space (X, τ_1, τ_2) is said to be (τ_1, τ_2)-T_2 [35] if for any pair of distinct points x, y in X, there exist disjoint $\tau_1\tau_2$-open sets U and V of X containing x and y, respectively.

Theorem 12. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a (τ_1, τ_2)-continuous injection and (Y, σ_1, σ_2) is (σ_1, σ_2)-T_2, then (X, τ_1, τ_2) is (τ_1, τ_2)-T_2.*

Proof. Suppose that (Y, σ_1, σ_2) is (σ_1, σ_2)-T_2. Let x, y be any distinct points of X. Since f is injective, $f(x) \neq f(y)$. Since (Y, σ_1, σ_2) is (σ_1, σ_2)-T_2, there exist $\sigma_1\sigma_2$-open sets V and W of Y containing $f(x)$ and $f(y)$, respectively, such that $V \cap W = \emptyset$. Since f is (τ_1, τ_2)-continuous, there exist $\tau_1\tau_2$-open sets U and G of X containing x and y, respectively, such that $f(U) \subseteq V$ and $f(G) \subseteq W$. It follows that $U \cap G = \emptyset$. Thus, (X, τ_1, τ_2) is (τ_1, τ_2)-T_2.

Corollary 2. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a R-(τ_1, τ_2)-continuous injection and (Y, σ_1, σ_2) is (σ_1, σ_2)-T_2, then (X, τ_1, τ_2) is (τ_1, τ_2)-T_2.*

Proof. This is an immediate consequence of Lemma 4 and Theorem 12.

Recall that a bitopological space (X, τ_1, τ_2) is said to be (τ_1, τ_2)-R_0 [36] if for each $\tau_1\tau_2$-open set U and each $x \in U$, $\tau_1\tau_2$-Cl$(\{x\}) \subseteq U$.

Theorem 13. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a R-(τ_1, τ_2)-continuous surjection, then (Y, σ_1, σ_2) is (σ_1, σ_2)-R_0.*

Proof. Let V be any $\sigma_1\sigma_2$-open set of Y and $y \in V$. Let $x \in X$ such that $y = f(x)$. Since f is R-(τ_1, τ_2)-continuous, there exists a $\tau_1\tau_2$-open set U of X containing x such that $\sigma_1\sigma_2$-Cl$(f(U)) \subseteq V$. Thus, $\sigma_1\sigma_2$-Cl$(\{y\}) = \sigma_1\sigma_2$-Cl$(\{f(x)\}) \subseteq \sigma_1\sigma_2$-Cl$(f(U)) \subseteq V$ and hence (Y, σ_1, σ_2) is (σ_1, σ_2)-R_0.

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