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Novel Types of Supra Functions Inspired by Supra ϵ-Open Sets

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Abstract. Using the concept of supra ϵ-open sets, this manuscript discusses and investigates new forms of supra continuity. More specifically, we introduce the concept of supra ϵ-continuous functions, which built upon the previous types of weaker forms of such notions. The relationships between our new class and existing previous supra continuity notions were examined using the diagram in Figure 1. Furthermore, the essential features of this concept are analyzed, as well as its analogous circumstances. Additionally, the notions of supra ϵ-irresolute functions and supra ϵ^*-cts functions were introduced. Moreover, we prove that the composition of supra ϵ-irresolute function and supra ϵ-cts function (respectively, supra ϵ-cts function and cts function is supra ϵ-cts, two supra ϵ-irresolute functions) is supra ϵ-cts (respectively, supra ϵ-cts, supra ϵ-irresolute). Also, we provide three new approaches for supra functions named supra ϵ-open functions, supra ϵ-closed functions, and supra ϵ-homeomorphism functions. We conclude with a detailed discussion of their key characteristics and provide several essential examples.

2020 Mathematics Subject Classifications: 54A05, 54C10, 54C08

Key Words and Phrases: Supra ϵ-open set, Supra ϵ-continuous functions, Supra ϵ-irresolute functions, supra ϵ-homeomorphism functions

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1. Introduction

The study of different types of generalized continuous functions, supra continuous functions, and soft continuous functions and their structural properties has been a major area of topological, supra topological, and soft topological research in the last several decades. Levine [1] initially introduced semi-open sets and semi-continuity of functions in 1963. Then, in 1965, Njasta [2] introduced his α-open sets approach. The concept of pre-open set was proposed by Mashhour et al. [3] in order to investigate pre-continuous functions. Abd-El-Monsef et al. [4] introduced the concept of β-open sets in 1983 as a means of studying β-continuous functions. The concept of b-open sets was studied in detail in [5, 6]. According to [7], Piotrowski [8] defined relatively open sets to present somewhat continuity. In [9, 10], the concept of somewhere dense sets was presented. Other aspects of this concept were studied in [11]. In 2024, Alqahtani and Abd El-latif [12] introduced the $\mathcal{N}$-open sets approach, which generalized nearly all of the previously proposed concepts.

The concept of supra open sets, which take into account the fundamental components of supra topology (abbreviated, STS), was introduced by Mashhour et al. [13]. The continuity and separation axioms, as well as interior and closure operators, were among the fundamental topological notions they elaborated on. The concepts of supra α [14] (pre- [15], b- [16], β- [17], R- [18], and semi- [19]) open sets have been introduced, along with their main features.

Several types of soft open sets and soft continuity have been provided in the field of generalized soft open sets [20, 21], generalized soft continuity [22], soft semi-open sets [23, 24], several types of soft continuity [25], soft somewhere dense sets [26], and nearly soft β-open sets [27]. More research on soft continuity was later conducted [28, 29]. In [30], the notion of the soft ideal was first introduced. Later, Fatouh and Abd El-latif [31] generalized this notion using soft semi-open sets. After that, this concept is used to generalize several types of topological properties, involving soft compactness [32], soft connectedness [33], soft generalized open sets [34–36], soft separation axioms [37], and generalized soft rough sets [38, 39]. Recently, some lower soft separation axioms [40] and Some applications of soft δ-closed sets [41] were presented.

The notion of supra soft topological spaces (SSTs) was put forth by El-Sheikh et al. [42]. Additionally, the concepts of supra soft pre- (respectively, α-, semi, β, and γ-) open sets were presented. The approach of supra ϵ-open sets in supra topological spaces (STSs) was introduced by Abd El-latif et al.[43]. They also discussed the relationships between their novel approach and previous relevant research. Additionally, they supplied this new category's primary characteristics. Furthermore, in general, the intersection of finite numbers of supra ϵ-open sets is not such. They then used their previously defined category of supra open sets to study new kinds of operators called supra ϵ-interior (closure, accumulation, exterior, and boundary, respectively).

Several generalized supra soft operators have been studied in later studies using supra soft-b-open sets [44], supra generalized closed soft sets inspired by soft ideals [45], supra soft sw-open sets [46], supra soft δ_i-open sets [47, 48], supra soft somewhere dense sets [49], and separation axioms via supra soft topological spaces [50].

We continue studying the features of supra topological spaces in this paper. In particular, we present and discuss novel types of supra continuity. Building on the earlier types of weaker forms of such conceptions, we established the notion of supra ϵ-continuous functions. Figure 1's diagram was used to analyze the connections between our new class and earlier supra continuity concepts.

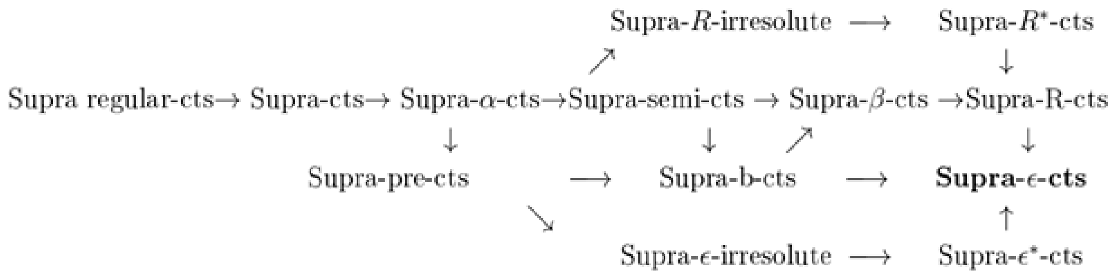

Figure 1: The connections between Supra-ϵ-cts functions and other preceding studies

We also introduced the concepts of supra ϵ-irresolute functions and supra ϵ^*-cts functions and provide their essential features in detail. Furthermore, we present novel approaches for supra functions, which we call supra ϵ-open functions, supra ϵ-closed functions, and supra ϵ-homeomorphism functions. Finally, several essential examples were provided with a detailed discussion of their key characteristics.

2. Preliminaries and background

Let (λ, ϑ) be an STS, the classes of supra (respectively, regular-, pre-, semi, β-, α-, b-, and R-) open sets will represented by $SO(\lambda)$ (respectively, $SO_{regular}(\lambda)$, $SPO(\lambda)$, $SSO(\lambda)$, $S\beta O(\lambda)$, $S\alpha O(\lambda)$, $SBO(\lambda)$, and $SRO(\lambda)$). Also, the classes of supra (respectively, regular-, semi, pre-, β-, b-, α-, and R-) continuous functions will represented by supra (respectively, regular-, semi, pre-, β-, b-, α-, and R-) cts, through this paper.

Definition 1. *[13] The collection $\vartheta \subseteq P(\lambda)$ is called supra topology (or STS) on λ if ϑ contains λ and $\emptyset$ and closed under arbitrary union. Also, if $G \in \vartheta$, then G is called supra open set and G^c is called supra closed set. Moreover, if $\nu \subset \vartheta$ for a given topology ν, then ϑ is called an associated STS with ν.*

Definition 2. *[13] Regarding a subset K of an STS (λ, ϑ), the $int^s(K)$ or K° (respectively, $cl^s(K)$ or $\overline{K}$, and $b(K)$) will refer to the supra interior (respectively, closure, and boundary) of K, where*
$int^s(K) = \cup\{G : G \in \vartheta \text{ and } G \subseteq K\}$, $cl^s(K) = \cap\{N : N \in \vartheta^c \text{ and } K \subseteq N\}$, *and* $b(K) = cl^s(K) \backslash int^s(K)$.

Theorem 1. *[13] Regarding a subset T of an STS (λ, ϑ), we have*

(1) $cl^s(T^c) = [int^s(T)]^c$.

(2) $int^s(T^c) = [cl^s(T)]^c$.

Definition 3. *[15–19] Let H be a subset of an STS (λ, ϑ). Then,*

(1) *If $H = int^s(cl^s(H))$, then $H \in SO_{regular}(\lambda)$.*

(2) *If $H \subseteq int^s(cl^s(H))$, then $H \in SPO(\lambda)$.*

(3) *If $H \subseteq cl^s(int^s(H))$, then $H \in SSO(\lambda)$.*

(4) *If $H \subseteq int^s(cl^s(int^s(H)))$, then $H \in S\alpha O(\lambda)$.*

(5) *If $H \subseteq cl^s(int^s(cl^s(H)))$, then $H \in S\beta O(\lambda)$.*

(6) *If $H \subseteq cl^s(int^s(H))\tilde{\cup} int^s(cl^s(H))$, then $H \in SBO(\lambda)$.*

(7) *If $int^s(cl^s(H)) \neq \emptyset$, then $H \in SRO(\lambda)$.*

(8) *If $int^s(cl^s(H)) = \emptyset$, then $H \in SND(\lambda)$.*

Definition 4. *[13] Regarding the subset K of an STS (λ, ϑ), the class*

$$\vartheta_K = \{K \cap G : G \in \vartheta\}$$

defines an STS on K, and it is called a subspace of (λ, ϑ).

Definition 5. *[43] Let H be a subset of an STS (λ, ϑ). Then, H is called supra ϵ-open set if either $H = \emptyset$ or*

$$H \subseteq \begin{cases} b(H) \cup \overline{H}^{\circ}, & H \in SRO(\lambda), \\ b(H), & H \in SND(\lambda) \text{ and } b(H) \text{ is infinite.} \end{cases}$$

Also, H^c is called supra ϵ-closed-set. The category of all supra ϵ-open (respectively, supra ϵ-closed) sets will be indicated by $SO_\epsilon(\lambda)$ (respectively, $SC_\epsilon(\lambda)$).

Theorem 2. *[43] Every supra (respectively, α-, semi-, b-, regular, pre-, β-, R-) open set is supra ϵ-open.*

Definition 6. *[43] For the subset K of an STS (λ, ϑ), the $int^s_\epsilon(K)$ will denote the supra ϵ-interior of K, where*

$$int^s_\epsilon(K) = \cup\{G : G \in SO_\epsilon(\lambda) \text{ and } G \subseteq K\}.$$

Theorem 3. *[43] For the supra ϵ-interior operator $int^s_\epsilon : P(\lambda) \longrightarrow P(\lambda)$ and $E \in P(\lambda)$, we have*

$$int^s_\epsilon(E) = \begin{cases} \emptyset, & E \in SND(\lambda) \text{ and } b(E) \text{ is finite.} \\ E \cap b(E), & E \in SND(\lambda) \text{ and } b(E) \text{ is infinite.} \\ E, & E \in SRO(\lambda). \end{cases}$$

Definition 7. *[43] Let $C \in P(\lambda)$ be a subset of an STS (λ, ϑ), then $cl^s_\epsilon(C)$ will denote the supra ϵ-closure of C, where*

$$cl^s_\epsilon(C) = \cap\{N : N \in SC_\epsilon(\lambda) \; and \; C \subseteq N\}.$$

Theorem 4. *[43] For the supra ϵ-closure operator $cl^s_\epsilon : P(\lambda) \longrightarrow P(\lambda)$ and $E \in P(\lambda)$, we have*

$$cl^s_\epsilon(E) = \begin{cases} \lambda, & E^c \in SND(\lambda) \; and \; b(E^c) \; is \; finite. \\ E, & E^c \in SND(\lambda) \; and \; b(E^c) \; is \; infinite. \\ E, & E^c \in SRO(\lambda). \end{cases}$$

Theorem 5. *[43] Regarding a subset T of an STS (λ, ϑ), we have*

(1) $cl^s_\epsilon(T^c) = [int^s_\epsilon(T)]^c$.

(2) $int^s_\epsilon(T^c) = [cl^s_\epsilon(T)]^c$.

(3) $int(T) \subseteq int^s(T) \subseteq int^s_\epsilon(T)$.

(4) $cl^s_\epsilon(T) \subseteq cl^s(T) \subseteq cl(T)$.

Definition 8. *[43] Given a subset T of an STS (λ, ϑ) with arbitrary point $s \in \lambda$. Then, s called a supra ϵ-accumulation point of T if all each supra ϵ-open set G_s, we have*

$$[T \backslash \{s\}] \cap G \neq \emptyset.$$

The set of all supra ϵ-accumulation points of T will denoted by $acc_\epsilon(T)$.

Definition 9. *[43] If $s \in [cl^s_\epsilon(Z) \backslash int^s_\epsilon(Z)]$ for an arbitrary point s and oft subset Z of an STS (λ, ϑ), then s is called a supra-ϵ-boundary point of Z. The supra-ϵ-boundary set of (Z) is the set of all upper-so-boundary points of Z, and it is represented by $b_\epsilon(Z)$.*
Also, the upper-so-exterior of Z is also represented by $ext_\epsilon(Z)$, where $ext_\epsilon(Z) = int^s_\epsilon(Z^c)$.

3. New types of supra continuous functions based on supra ϵ-open sets

This section refers to supra continuity using the concept of supra ϵ-open sets. To be more precise, we expanded on the earlier kinds of weaker forms of such conceptions by introducing the concept of supra ϵ-continuous functions. Figure 1, shows a diagram that was used to examine the connections between our new class and other earlier supra continuity concepts. Additionally, The fundamental characteristics of this notion are examined, along with its comparable conditions. Furthermore, we presented the concepts of supra ϵ-irresolute functions and supra ϵ^*-cts functions. In addition, we prove that the composition of supra ϵ-irresolute function and supra ϵ-cts function (respectively, supra ϵ-cts function and cts function is supra ϵ-cts, two supra ϵ-irresolute functions) is supra ϵ-cts (respectively, supra ϵ-cts, supra ϵ-irresolute). Finally, several essential examples are provided.

Definition 10. *A function $\pi_\epsilon : (\lambda_1, \nu_1) \rightarrow (\lambda_2, \nu_2)$ with ϑ_1 as an associated STS with ν_1 is said to be a supra ϵ-continuous (abbreviate: supra ϵ-cts) if $\pi_\epsilon^{-1}(G) \in SO_\epsilon(\lambda_1)$ for each $G \in \nu_2$.*

Theorem 6. *Every supra (respectively, semi-, α-, b-, pre, regular, β-, and R-) cts function is supra ϵ-cts.*

Proof. It is inferred from Theorem 2.

Remark 1. *In general, the following example demonstrates that the contrary of Theorem 6 is not valid.*

Example 1. *Consider the two topologies $\nu_1 = \{\emptyset, A \subseteq \mathbb{R} : -1 \in A\}$, $\nu_2 = \{\emptyset, \mathbb{R}, \mathbb{N}\}$ and $\lambda = \{\emptyset, T \subseteq \mathbb{R} : -1 \in T$ or $0 \in T\}$ be an associated STS with ν_1 on the set of real numbers $\mathbb{R}$. Consider the identity function $\pi_\epsilon : (\mathbb{R}, \nu_1) \rightarrow (\mathbb{R}, \nu_2)$. Regarding the set of natural numbers $\mathbb{N}$, we have $\pi_\epsilon^{-1}(\mathbb{N}) = \mathbb{N}$ is a supra ϵ-open subset of $\mathbb{R}$, but it is not supra R-open. Hence, π_ϵ is supra ϵ-cts, but it is not supra R-cts.*

Theorem 7. *Let $\pi_\epsilon : (\lambda_1, \nu_1) \rightarrow (\lambda_2, \nu_2)$ be a function with ϑ_1 as an associated STS with ν_1, then the next assertions are equivalent:*

(1) *π_ϵ is supra ϵ-cts.*

(2) *For each $Z \in \nu_2^c$, $\pi_\epsilon^{-1}(Z) \in SC_\epsilon(\lambda_1)$.*

(3) $cl_\epsilon^s(\pi_\epsilon^{-1}(Z)) \subseteq \pi_\epsilon^{-1}(cl(Z)) \ \forall \ Z \subseteq \lambda_2$.

(4) $\pi_\epsilon(cl_\epsilon^s(Y)) \subseteq cl(\pi_\epsilon(Y)) \ \forall \ Y \subseteq \lambda_1$.

(5) $\pi_\epsilon^{-1}(int(Z)) \subseteq int_\epsilon^s(\pi_\epsilon^{-1}(Z)) \ \forall \ Z \subseteq \lambda_2$.

Proof.

(1) $\Rightarrow$ (2) Let $Z \in \nu_2^c$, then $Z^c \in \nu_2$. Given (1), $\pi_\epsilon^{-1}(Z^c) = [\pi_\epsilon^{-1}(Z)]^c \in SO_\epsilon(\lambda_1)$. Hence,

$$\pi_\epsilon^{-1}(Z) \in SC_\epsilon(\lambda_1).$$

(2) $\Rightarrow$ (3) Let $Z \subseteq \lambda_2$. Since $cl(Z) \in \nu_2^c$ and given (2),

$$\pi_\epsilon^{-1}(Z) \in SC_\epsilon(\lambda_1), \text{ which implies}$$

$$cl_\epsilon^s(\pi_\epsilon^{-1}(Z)) \subseteq cl_\epsilon^s(\pi_\epsilon^{-1}(cl(Z))) = \pi_\epsilon^{-1}(cl(Z)).$$

Consequently, the proof is acquired.

(3) $\Rightarrow$ (4) Regarding $\pi_\epsilon(Y) \subseteq \lambda_2$ for a subset $Y \subseteq \lambda_1$, we have $Y \subseteq \pi_\epsilon^{-1}(\pi_\epsilon(Y))$. Given (3), we obtain

$$cl_\epsilon^s(\pi_\epsilon^{-1}(\pi_\epsilon(Y))) \subseteq \pi_\epsilon^{-1}(cl(\pi_\epsilon(Y))).$$

Hence,

$$\pi_\epsilon[cl_\epsilon^s(\pi_\epsilon^{-1}(\pi_\epsilon(Y)))] \subseteq \pi_\epsilon[\pi_\epsilon^{-1}(cl(\pi_\epsilon(Y)))] \subseteq cl(\pi_\epsilon(Y)).$$

Therefore,

$$\pi_\epsilon(cl_\epsilon^s(Y)) \subseteq cl(\pi_\epsilon(Y)).$$

(4) $\Rightarrow$ (5) Regarding $\pi_\epsilon^{-1}(Z^c) \subseteq \lambda_1$ for a subset $Z^c \subseteq \lambda_2$, and by utilizing (4), we obtain that

$$\pi_\epsilon[cl_\epsilon^s[\pi_\epsilon^{-1}(Z^c)]] \subseteq cl(\pi_\epsilon[\pi_\epsilon^{-1}(Z^c)]) \subseteq cl(Z^c) = [int(Z)]^c, \ from\ Theorem\ 5.$$

Hence,

$$\pi_\epsilon^{-1}[\pi_\epsilon(cl_\epsilon^s[\pi_\epsilon^{-1}(Z^c)])] \subseteq \pi_\epsilon^{-1}[[int(Z)]^c] = [\pi_\epsilon^{-1}(int(Z))]^c.$$

Therefore,

$$cl_\epsilon^s[(\pi_\epsilon^{-1}(Z))]^c \subseteq [\pi_\epsilon^{-1}(int(Z))]^c.$$

Thus,

$$\pi_\epsilon^{-1}(int(Z)) \subseteq [cl_\epsilon^s[(\pi_\epsilon^{-1}(Z))]^c]^c = int_\epsilon^s(\pi_\epsilon^{-1}(Z)).$$

(5) $\Rightarrow$ (1) Regarding $Z = int(Z)$ for a supra open set Z. Given (5),

$$\pi_\epsilon^{-1}(Z) \subseteq int_\epsilon^s(\pi_\epsilon^{-1}(Z)).$$

However,

$$int_\epsilon^s(\pi_\epsilon^{-1}(Z)) \subseteq \pi_\epsilon^{-1}(Z).$$

Therefore,

$$int_\epsilon^s(\pi_\epsilon^{-1}(Z)) = \pi_\epsilon^{-1}(Z) \in SO_\epsilon(\lambda_1).$$

Thus, π_ϵ is a supra ϵ-cts.

Definition 11. *A function $\pi_\epsilon : (\lambda_1, \nu_1) \rightarrow (\lambda_2, \nu_2)$ with ϑ_1, ϑ_2 associated STSs with ν_1, ν_2, respectively, is said to be supra ϵ-irresolute (supra ϵ^*-cts) if $\pi_\epsilon^{-1}(D) \in SO_\epsilon(\lambda_1)$ for each $D \in SO_\epsilon(\lambda_2)$ ($D \in \vartheta_2$).*

Theorem 8. **(1)** *Every supra ϵ-irresolute function is supra ϵ^*-cts.*

(2) *Every supra ϵ^*-cts function is supra ϵ-cts.*

Proof. It is immediately obvious from Theorem 2.

Remark 2. *The following examples demonstrate that the contrary of Theorem 8 is generally untrue.*

Examples 1. **(1)** *Consider the two topologies $\nu_1 = \{\emptyset, A \subseteq \mathbb{R} : -2 \in A\}$, $\nu_2 = \{\emptyset, \mathbb{R}, \mathbb{N}\}\}$ on the set of real numbers $\mathbb{R}$. Let $\vartheta_1 = \{\emptyset, T \subseteq \mathbb{R} : -2 \in T\ or\ 0 \in T\}$ and $\vartheta_2 = \{\emptyset, \mathbb{R}, \mathbb{N}, \{0,1\}, \{0,2\}, \{0,1,2\}\}$ be associated STSs with ν_1 and ν_1, respectively, and let $\pi_\epsilon : (\mathbb{R}, \nu_1) \rightarrow (\mathbb{R}, \nu_2)$ be the identity function. Then, we have $\pi_\epsilon^{-1}(D) \in SO_\epsilon(\mathbb{R})$ for each $D \in \vartheta_2$, and hence π_ϵ is supra ϵ^*-cts. However, π_ϵ is not supra ϵ-irresolute, since $\{-3, -4\}$ is supra ϵ-open set over ϑ_2, but $\pi_\epsilon^{-1}(\{-3, -4\}) = \{-3, -4\}$ is not supra ϵ-open over ϑ_1. Therefore, π_ϵ is supra ϵ^*-cts, but it is not supra ϵ-irresolute.*

(2) *Consider the two topologies* $\nu_1 = \{\lambda_1, \emptyset, \{1\}, \{1,2\}\}$, $\nu_2 = \{\lambda_2, \emptyset, \{y,z\}\}$ *on* $\lambda_1 = \{1,2,3\}$ *and* $\lambda_2 = \{x,y,z\}$, *respectively. Let* $\vartheta_1 = \{\lambda_1, \emptyset, \{1\}, \{1,2\}, \{2,3\}\}$ *and* $\vartheta_2 = \{\lambda_2, \emptyset, \{z\}, \{y,z\}, \{x,y\}\}$ *be associated STSs with* ν_1 *and* ν_1, *respectively, and let* $\pi_\epsilon : (\lambda_1, \nu_1) \rightarrow (\lambda_2, \nu_2)$ *be a function defined as follows:* $\pi_\epsilon(\{1\}) = \{y\}$, $\pi_\epsilon(\{2\}) = \{x\}$, *and* $\pi_\epsilon(\{3\}) = \{z\}$. *Then, we have* $\pi_\epsilon^{-1}(D) \in SO_\epsilon(\mathbb{R})$ *for each* $D \in \nu_2$, *and hence* π_ϵ *is supra* ϵ*-cts. However,* π_ϵ *is not supra* ϵ^**-cts, since* $\{z\} \in \vartheta_2$, *but* $\pi_\epsilon^{-1}(\{z\}) = \{3\} \notin SO_\epsilon(\lambda_1)$. *Therefore,* π_ϵ *is supra* ϵ*-cts, but it is not supra* ϵ^**-cts.*

Corollary 1. *It follows from Theorem 6, Theorem 8 and [18, Reamrk 2] that we have the following implications for an STS* (λ, ν), *which are not reversible.*

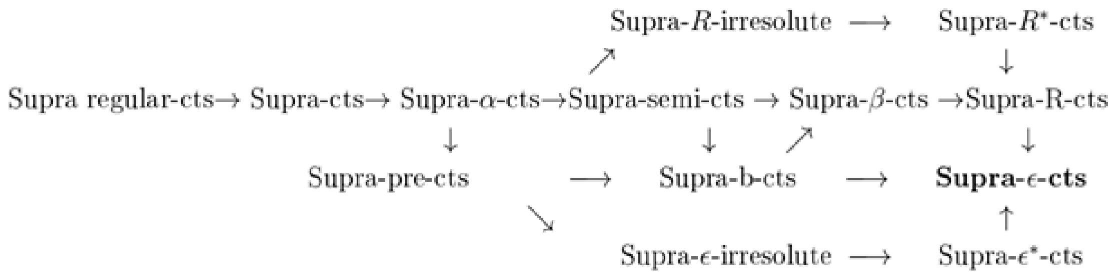

Figure 1: The connections between Supra-ϵ-cts functions and other preceding studies

The proofs for the next two theorems are eliminated since they could be demonstrated similarly to Theorem 7.

Theorem 9. *Let* $\pi_\epsilon : (\lambda_1, \nu_1) \rightarrow (\lambda_2, \nu_2)$ *be a function with* ϑ_1 *and* ϑ_2 *as associated STSs with* ν_1 *and* ν_1, *respectively, then the next assertions are equivalent:*

(1) π_ϵ *is supra* ϵ^**-cts.*

(2) *For each* $Z \in \vartheta_2^c$, $\pi_\epsilon^{-1}(Z) \in SC_\epsilon(\lambda_1)$.

(3) $cl_\epsilon^s(\pi_\epsilon^{-1}(Z)) \subseteq \pi_\epsilon^{-1}(cl^s(Z)) \; \forall \; Z \subseteq \lambda_2$.

(4) $\pi_\epsilon(cl_\epsilon^s(Y)) \subseteq cl_\epsilon^s(\pi_\epsilon(Y)) \; \forall \; Y \subseteq \lambda_1$.

(5) $\pi_\epsilon^{-1}(int_\epsilon^s(Z)) \subseteq int_\epsilon^s(\pi_\epsilon^{-1}(Z)) \; \forall \; Z \subseteq \lambda_2$.

Theorem 10. *Let* $\pi_\epsilon : (\lambda_1, \nu_1) \rightarrow (\lambda_2, \nu_2)$ *be a function with* ϑ_1 *and* ϑ_2 *as associated STSs with* ν_1 *and* ν_1, *respectively, then the next assertions are equivalent:*

(1) π_ϵ *is supra* ϵ*-irresolute.*

(2) *For each* $Z \in SC_\epsilon(\lambda_2)$, $\pi_\epsilon^{-1}(Z) \in SC_\epsilon(\lambda_1)$.

(3) $cl_\epsilon^s(\pi_\epsilon^{-1}(Z)) \subseteq \pi_\epsilon^{-1}(cl_\epsilon^s(Z)) \; \forall \; Z \subseteq \lambda_2$.

(4) $\pi_\epsilon(cl_\epsilon^s(Y)) \subseteq cl_\epsilon^s(\pi_\epsilon(Y)) \; \forall \; Y \subseteq \lambda_1$.

(5) $\pi_\epsilon^{-1}(int_\epsilon^s(Z)) \subseteq int_\epsilon^s(\pi_\epsilon^{-1}(Z))\ \forall\ Z \subseteq \lambda_2$.

Theorem 11. *Let $\pi_\epsilon : (\lambda_1, \nu_1) \to (\lambda_2, \nu_2)$ be a supra ϵ-irresolute with ϑ_1, ϑ_2 associated STSs with ν_1, ν_2, respectively, and $\psi_\epsilon : (\lambda_2, \nu_2) \to (\lambda_3, \nu_3)$ be a supra ϵ-cts with ϑ_3 as an associated STS with ν_3, then the composition $\psi_\epsilon \circ \pi_\epsilon : (\lambda_1, \nu_1) \to (\lambda_3, \nu_3)$ is supra ϵ-cts.*

Proof. Let $E \in \vartheta_3$. Since ψ_ϵ is supra ϵ-cts, $\psi_\epsilon^{-1}(E) \in SO_\epsilon(\lambda_2)$. Given π_ϵ is supra ϵ-irresolute, then $[\psi_\epsilon \circ \pi_\epsilon]^{-1}(E) = \pi_\epsilon^{-1}[\psi_\epsilon^{-1}(E)] \in SO_\epsilon(\lambda_1)$. Hence, $\psi_\epsilon \circ \pi_\epsilon$ is supra ϵ-cts.

The proof of the upcoming two Corollaries is straightforward from Theorem 11.

Corollary 2. *The composition of supra ϵ-cts function and cts function is supra ϵ-cts.*

Corollary 3. *The composition of two supra ϵ-irresolute functions is also supra ϵ-irresolute.*

Theorem 12. *A function $\pi_\epsilon : (\lambda_1, \nu_1) \to (\lambda_2, \nu_2)$ with with ϑ_1, ϑ_2 associated STSs with ν_1, ν_2, respectively, is supra ϵ-irresolute if*

$$cl^s(\pi_\epsilon^{-1}(W)) \subseteq \pi_\epsilon^{-1}(cl_\epsilon^s(W))\ \forall\ W \subseteq \lambda_2.$$

Proof. Assume that $W \subseteq \lambda_2$. For $\pi_\epsilon^{-1}(W)$, taking into account the specified condition and Theorem 5 (4), we get
$cl_\epsilon^s(\pi_\epsilon^{-1}(W)) \subseteq cl^s(\pi_\epsilon^{-1}(W)) \subseteq \pi_\epsilon^{-1}(cl_\epsilon^s(W))$.
Given Theorem 10 (3), π_ϵ is supra ϵ-irresolute.

Theorem 13. *A function $\pi_\epsilon : (\lambda_1, \nu_1) \to (\lambda_2, \nu_2)$ with ϑ_1, ϑ_2 associated STSs with ν_1, ν_2 , respectively, is supra ϵ-cts in the event that one of the subsequent conditions is fulfilled:*

(1) $\pi_\epsilon(cl(Y)) \subseteq cl_\epsilon^s(\pi_\epsilon(Y))\ \forall\ Y \subseteq \lambda_1$.

(2) $cl(\pi_\epsilon^{-1}(Z)) \subseteq \pi_\epsilon^{-1}(cl_\epsilon^s(Z))\ \forall\ Z \subseteq \lambda_2$.

(3) $\pi_\epsilon^{-1}(int_\epsilon^s(Z)) \subseteq int(\pi_\epsilon^{-1}(Z))\ \forall\ Z \subseteq \lambda_2$.

Proof. If the first condition is fulfilled, then

$$\pi_\epsilon(cl(Y)) \subseteq cl_\epsilon^s(\pi_\epsilon(Y))\ \forall\ Y \subseteq \lambda_1.$$

Since $cl_\epsilon^s(Y, \Theta) \subseteq cl(Y, \Theta)$ from Theorem 5 (4),

$$\pi_\epsilon(cl_\epsilon^s(Y)) \subseteq \pi_\epsilon(cl(Y)) \subseteq cl_\epsilon^s(\pi_\epsilon(Y)) \subseteq cl(\pi_\epsilon(Y)).$$

Therefore, π_ϵ is supra ϵ-cts according to Theorem 7 (4).
If the second condition is fulfilled, then $\forall\ Z \subseteq \lambda_2$, then
$cl_\epsilon^s(\pi_\epsilon^{-1}(Z)) \subseteq cl(\pi_\epsilon^{-1}(Z)) \subseteq \pi_\epsilon^{-1}(cl_\epsilon^s(Z)) \subseteq \pi_\epsilon^{-1}(cl(Z))$, given Theorem 5 (4). This implies,
$cl_\epsilon^s(\pi_\epsilon^{-1}(Z)) \subseteq \pi_\epsilon^{-1}(cl(Z))\ \forall\ Z \subseteq \lambda_2$, and therefore Therefore, π_ϵ is supra ϵ-cts according to Theorem 7 (3).
If the third condition is fulfilled, and given Theorem 5 (3)
$\pi_\epsilon^{-1}(int(Z)) \subseteq \pi_\epsilon^{-1}(int_\epsilon^s(Z)) \subseteq int(\pi_\epsilon^{-1}(Z)) \subseteq int_\epsilon^s(\pi_\epsilon^{-1}(Z))\ \forall\ Z \subseteq \lambda_2$. Hence,

$$\pi_\epsilon^{-1}(int(Z)) \subseteq int_\epsilon^s(\pi_\epsilon^{-1}(Z))\ \forall\ Z \subseteq \lambda_2.$$

Hence, π_ϵ is supra ϵ-cts according to Theorem 7 (5).

The proofs for the next two theorems are eliminated since they could be demonstrated similarly to Theorem 13.

Theorem 14. *A function $\pi_\epsilon : (\lambda_1, \nu_1) \to (\lambda_2, \nu_2)$ with ϑ_1, ϑ_2 associated STSs with ν_1, ν_2 , respectively, is supra ϵ^*-cts in the event that one of the subsequent conditions is fulfilled:*

(1) $\pi_\epsilon(cl^s(Y)) \subseteq cl^s_\epsilon(\pi_\epsilon(Y)) \; \forall \; Y \subseteq \lambda_1$.

(2) $cl^s(\pi_\epsilon^{-1}(Z)) \subseteq \pi_\epsilon^{-1}(cl^s_\epsilon(Z)) \; \forall \; Z \subseteq \lambda_2$.

(3) $\pi_\epsilon^{-1}(int^s_\epsilon(Z)) \subseteq int^s(\pi_\epsilon^{-1}(Z)) \; \forall \; Z \subseteq \lambda_2$.

Theorem 15. *A function $\pi_\epsilon : (\lambda_1, \nu_1) \to (\lambda_2, \nu_2)$ with ϑ_1, ϑ_2 associated STSs with ν_1, ν_2, respectively, is supra ϵ-irresolute in the event that one of the subsequent conditions is fulfilled:*

(1) $\pi_\epsilon(cl^s_\epsilon(Y)) \subseteq cl^s_\epsilon(\pi_\epsilon(Y)) \; \forall \; Y \subseteq \lambda_1$.

(2) $cl^s_\epsilon(\pi_\epsilon^{-1}(Z)) \subseteq \pi_\epsilon^{-1}(cl^s_\epsilon(Z)) \; \forall \; Z \subseteq \lambda_2$.

(3) $\pi_\epsilon^{-1}(int^s_\epsilon(Z)) \subseteq int^s_\epsilon(\pi_\epsilon^{-1}(Z)) \; \forall \; Z \subseteq \lambda_2$.

4. Supra ϵ-homeomorphism functions

We present new approaches for supra functions in this section, which we call supra ϵ-open functions, supra ϵ-closed functions, and supra ϵ-homeomorphism functions. Furthermore, we show their corresponding properties in a transparent way. Moreover, for every notion, we give the analogous conditions that are required.

Definition 12. *A function $\pi_\epsilon : (\lambda_1, \nu_1) \to (\lambda_2, \nu_2)$ with ϑ_2 as an associated STS with ν_2 is said to be:*

(1) *Supra ϵ-open if $\pi_\epsilon(U) \in SO_\epsilon(\lambda_2)$ for each $U \in \nu_1$.*

(2) *Supra ϵ-closed if $\pi_\epsilon(C) \in SC_\epsilon(\lambda_2)$ for each $C \in \nu_1^c$.*

Theorem 16. *Let $\pi_\epsilon : (\lambda_1, \nu_1) \to (\lambda_2, \nu_2)$ be a function with ϑ_2 as an associated STS with ν_2 and $Y \subseteq \lambda_1$, then*

$$\pi_\epsilon \text{ is supra } \epsilon\text{-open if and only if } \pi_\epsilon(int(U)) \subseteq int^s_\epsilon[\pi_\epsilon(U)] \; \forall \; U \subseteq \lambda_1.$$

Proof. " $\Rightarrow$ " *Let π_ϵ be a supra ϵ-open function and $U \subseteq \lambda_1$. Since $int(U) \subseteq U$, $\pi_\epsilon(int(U)) \subseteq \pi_\epsilon((U))$, which implies*

$$\pi_\epsilon(int(U)) = int^s_\epsilon[\pi_\epsilon(int(U))] \subseteq int^s_\epsilon[\pi_\epsilon((U))].$$

" $\Leftarrow$ " *Assume that $U \in \nu_1$. Based on the presumption,*

$$\pi_\epsilon(U) = \pi_\epsilon(int(U)) \subseteq int^s_\epsilon[\pi_\epsilon(U)] \; .$$

But, we have

$$int^s_\epsilon[\pi_\epsilon(U)] \subseteq \pi_\epsilon(U).$$

Hence,

$$int^s_\epsilon[\pi_\epsilon(U)] = \pi_\epsilon(U).$$

Therefore,

$$\pi_\epsilon(U) \in SO_\epsilon(\lambda_2), \text{ and thus } \pi_\epsilon \text{ is a supra } \epsilon\text{-open function.}$$

Theorem 17. *Let $\pi_\epsilon : (\lambda_1, \nu_1) \rightarrow (\lambda_2, \nu_2)$ be a function with ϑ_2 as an associated STS with ν_2 and $Y \subseteq \lambda_1$, then*

$$\pi_\epsilon \text{ is supra } \epsilon\text{-closed if and only if } cl^s_\epsilon[\pi_\epsilon(H)] \subseteq \pi_\epsilon(cl(H)).$$

Proof. *" $\Rightarrow$ " Assume that π_ϵ is supra ϵ-closed function and $H \subseteq \lambda_1$. Since $\pi_\epsilon(H) \subseteq \pi_\epsilon(cl(H))$, $cl^s_\epsilon[\pi_\epsilon(H)] \subseteq cl^s_\epsilon[\pi_\epsilon(cl(H))] = \pi_\epsilon(cl(H))$, given π_ϵ is supra ϵ-closed function.*
" $\Leftarrow$ " Let $H \in \nu_1^c$. Based on the presumption,

$$\pi_\epsilon(H) \subseteq cl^s_\epsilon[\pi_\epsilon(H)] \subseteq \pi_\epsilon(cl(H)) = \pi_\epsilon(H).$$

Hence,

$$cl^s_\epsilon[\pi_\epsilon(H)] = \pi_\epsilon(H).$$

Therefore,

$$\pi_\epsilon(H) \in SC(\lambda_2)_{\Theta_2}, \text{ and hence } \pi_\epsilon \text{ is a supra } \epsilon\text{-closed function.}$$

Theorem 18. *Let $\pi_\epsilon : (\lambda_1, \nu_1) \rightarrow (\lambda_2, \nu_2)$ be a bijective function with ϑ_2 as an associated STS with ν_2, then π_ϵ is supra ϵ-open function if and only if it is supra ϵ-closed.*

Proof. " $\Rightarrow$ " *Let $R \in \nu_1^c$, then $R^c \in \nu_1$. Since π_ϵ is supra bijective ϵ-open function,*

$$[\pi_\epsilon(R)]^c = \pi_\epsilon(R^c) \in SO_\epsilon(\lambda_1).$$

It follows that,

$$\pi_\epsilon(R) \in SO_\epsilon(\lambda_2).$$

Therefore, π_ϵ is a supra ϵ-closed function.
" $\Leftarrow$ " *It is followed by a comparable argument.*

Proposition 1. *Let $\pi_\epsilon : (\lambda_1, \nu_1) \rightarrow (\lambda_2, \nu_2)$ be a bijective function with ϑ_2 as an associated STS with ν_2, then the next assertions are equivalent:*

(1) *π_ϵ is a supra ϵ-open function.*

(2) *π_ϵ is a supra ϵ-closed function.*

(3) *π_ϵ^{-1} is a supra ϵ-cts function.*

Proof.

(1) $\Rightarrow$ (2) *Direct from Theorem 18.*

(2) $\Rightarrow$ (3) *Let $Z \in \nu_1^c$. Since π_ϵ is supra bijective function and given (2), $(\pi_\epsilon^{-1})^{-1}(Z) = \pi_\epsilon(Z) \in SC_\epsilon(\lambda_2)$. Hence, π_ϵ^{-1} is a supra ϵ-cts function.*

(3) $\Rightarrow$ (1) *Let $S \in \nu_1$. Then, $\pi_\epsilon(S) = (\pi_\epsilon^{-1})^{-1}(S) \in SO_\epsilon(\lambda_2)$, given (3). Therefore, π_ϵ is a supra ϵ-open function.*

Theorem 19. *Let $\pi_\epsilon : (\lambda_1, \nu_1) \to (\lambda_2, \nu_2)$ and $\psi_\epsilon : (\lambda_2, \nu_2) \to (\lambda_3, \nu_3)$ be two functions with $\vartheta_1, \vartheta_2, \vartheta_3$ associated STSs with ν_1, ν_2, ν_3, respectively. Then*

(1) *If $\psi_\epsilon \circ \pi_\epsilon$ is a supra ϵ-open function and π_ϵ is a surjective cts function, then ψ_ϵ is a supra ϵ-open function.*

(2) *If $\psi_\epsilon \circ \pi_\epsilon$ is an open function and ψ_ϵ is an injective supra ϵ-cts function, then π_ϵ is a supra ϵ-open function.*

(3) *If $\psi_\epsilon \circ \pi_\epsilon$ is a supra open function and ψ_ϵ is an injective supra ϵ^*-cts function, then π_ϵ is a supra ϵ-open function.*

(4) *If $\psi_\epsilon \circ \pi_\epsilon$ is a supra ϵ-open and ψ_ϵ is an injective supra ϵ-irresolute function, then π_ϵ is a supra ϵ-open function.*

Proof.

(1) *Let $F \in \nu_2$. Since π_ϵ is a cts function, $\pi_\epsilon^{-1}(F) \in \nu_1$. Given $\psi_\epsilon \circ \pi_\epsilon$ is a supra ϵ-open function and π_ϵ is a surjective function, then $(\psi_\epsilon \circ \pi_\epsilon)[\pi_\epsilon^{-1}(F)] = \psi_\epsilon[\pi_\epsilon(\pi_\epsilon^{-1}(F))] = \psi_\epsilon(F) \in SO_\epsilon(\lambda_3)$. Therefore, ψ_ϵ is a supra ϵ-open function.*

(2) *Let $F \in \nu_1$. Since $\psi_\epsilon \circ \pi_\epsilon$ is an open function, $(\psi_\epsilon \circ \pi_\epsilon)(F) \in \nu_3$. Given ψ_ϵ is an injective supra ϵ-cts function, then $\psi_\epsilon^{-1}[\psi_\epsilon \circ \pi_\epsilon(F)] = (\psi_\epsilon^{-1} \circ \psi_\epsilon)(\pi_\epsilon(F)) = \pi_\epsilon(F) \in SO_\epsilon(\lambda_2)$. Therefore, π_ϵ is a supra ϵ-open function.function.*

(3) *Let $F \in \nu_1$. Since $\psi_\epsilon \circ \pi_\epsilon$ is a supra open function, $(\psi_\epsilon \circ \pi_\epsilon)(F) \in \vartheta_3$. Given ψ_ϵ is an injective supra ϵ^*-cts function, then $\psi_\epsilon^{-1}[\psi_\epsilon \circ \pi_\epsilon(F)] = (\psi_\epsilon^{-1} \circ \psi_\epsilon)(\pi_\epsilon(F)) = \pi_\epsilon(F) \in SO_\epsilon(\lambda_2)$. Therefore, π_ϵ is a supra ϵ-open function.*

(4) *Let $F \in \nu_1$. Since $\psi_\epsilon \circ \pi_\epsilon$ is a supra ϵ-open function, $(\psi_\epsilon \circ \pi_\epsilon)(F) \in SO_\epsilon(\lambda_3)$. Given ψ_ϵ is an injective supra ϵ-irresolute function, then $\psi_\epsilon^{-1}[\psi_\epsilon \circ \pi_\epsilon(F)] = (\psi_\epsilon^{-1} \circ \psi_\epsilon)(\pi_\epsilon(F)) = \pi_\epsilon(F) \in SO_\epsilon(\lambda_2)$. Therefore, π_ϵ is a supra ϵ-open function.*

Definition 13. *A bijective function $\pi_\epsilon : (\lambda_1, \nu_1) \to (\lambda_2, \nu_2)$ with ϑ_1, ϑ_2 associated STSs with ν_1, ν_2 , respectively, is said to be supra ϵ-homeomorphism if it is supra ϵ-cts and supra ϵ-open.*

Theorem 20. *For a bijective supra ϵ-cts function $\pi_\epsilon : (\lambda_1, \nu_1) \rightarrow (\lambda_2, \nu_2)$ with ϑ_1, ϑ_2 associated STSs with ν_1, ν_2 , respectively. The statements that follow are interchangeable:*

(1) π_ϵ *is supra ϵ-homeomorphism.*

(2) ψ_{sd}^{-1} *is supra ϵ-cts.*

(3) π_ϵ *is supra ϵ-closed.*

Proof. *It is instantly evident from Definition 13 and Theorem 18.*

Theorem 21. *A bijective function $\pi_\epsilon : (\lambda_1, \nu_1) \rightarrow (\lambda_2, \nu_2)$ with ν_1, ν_2 associated STSs with ϑ_1, ϑ_2, respectively, is an supra ϵ-homeomorphism in the event that one of the subsequent conditions is fulfilled:*

(1) $\pi_\epsilon(cl^s_\epsilon(H)) \subseteq cl(\pi_\epsilon(H))$ *and* $cl^s_\epsilon(\pi_\epsilon(H)) \subseteq \pi_\epsilon(cl(H))$, $\forall\ (H) \subseteq \lambda_1$.

(2) $\pi_\epsilon(int(H)) \subseteq int^s_\epsilon(\pi_\epsilon(H))$, $\forall\ (H) \subseteq \lambda_1$ *and* $\psi_{sd}^{-1}(int(H)) \subseteq int^s_\epsilon(\psi_{sd}^{-1}(Z, \Theta_2))$, $\forall\ H \subseteq \lambda_2$.

Proof. If the first condition is fulfilled, then $\pi_\epsilon(cl^s_\epsilon(H)) \subseteq cl(\pi_\epsilon(H))$, implies π_ϵ is supra ϵ-cts, given Theorem 7 (4). Moreover, $cl^s_\epsilon(\pi_\epsilon(H)) \subseteq \pi_\epsilon(cl(H))$, implies π_ϵ is supra ϵ-closed, given Proposition 17. Consequently, π_ϵ is supra ϵ-homeomorphism, in line with Theorem 20.
If the first condition is fulfilled, then by a similar way π_ϵ is supra ϵ-homeomorphism, in line with Definition 13, Theorem 7 (5) and Theorem 16.

5. Conclusion

Abd El-latif et al. introduced the utilization of supra ϵ-open sets to supra topological spaces[43]. Then, using their previously established category of supra open sets, they investigated new types of operators known as supra ϵ-interior (closure, accumulation, exterior, and boundary, respectively). We introduce and explore new forms of supra continuity in this work. The concept of supra ϵ-continuous functions was developed by us, building on the previous sorts of weaker forms of such conceptions. Moreover, we provide a diagram to analyze the connections between our new class and earlier supra continuity concepts. Additionally, we presented the notions of supra ϵ-irresolute functions and supra ϵ-cts functions and thoroughly described their key characteristics. Furthermore, we propose novel approaches for supra functions, which we refer to as supra ϵ-open functions, supra ϵ-closed functions, and supra ϵ-homeomorphism functions. We also discuss the main features of each notions.
From the particular methods described in this article, additional research on the theoretical elements of these generalized concepts could be carried out by looking at the following subjects: Utilizing these methods in supra soft ideal topological spaces [30]. In addition, we investigate certain topological characteristics, such as supra (connectedness, separation axioms, and compactness), that are motivated by specific approaches discussed in this work.

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