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Integrated Sustainable Inventory and Remanufacturing Optimization in a Circular Economy: A Two-Echelon Supply Chain Approach Under Carbon Regulations

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Abstract. This study presents an integrated sustainable inventory and remanufacturing model within a two-echelon supply chain framework, comprising a manufacturer and a retailer. The model captures the impact of carbon emissions regulations through both carbon tax and cap-and-trade mechanisms, considering the dual nature of demand: price- and time-dependent for manufacturing and circularity-index-dependent for remanufacturing. Unlike conventional multi-echelon models, this study isolates the core dyadic relationship between the manufacturer and retailer to streamline decision-making and emphasize emission and circularity optimization. Using analytical optimization and numerical simulations in MATLAB R2024b, the study demonstrates that the cap-and-trade mechanism offers slightly higher profitability while maintaining environmental targets. Key contributions include the simultaneous integration of deteriorating items, dual demand functions and policy-driven emissions constraints in a supply chain context.

2020 Mathematics Subject Classifications: 90B05, 90B50

Key Words and Phrases: Supply Chain, Circularity index, EPQ, Carbon tax & cap & trade, Total profit, Two-echelon

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1. Introduction

In today's competitive business environment, the collaboration between retailers and manufacturers is a critical component of effective supply chain management. This partnership enables manufacturers to utilize retailers market insights while ensuring a steady supply of products, fostering mutual growth and operational efficiency. Such collaborations are vital in addressing challenges like fluctuating consumer preferences, rapid innovation and intense competition. Modern production systems often face imperfections due to machine breakdowns, human errors, and variability in raw materials. Addressing these challenges requires adaptive strategies, including quality improvement programs and flexible production schedules, to ensure efficiency and product quality. Additionally, product deterioration, particularly in perishable goods and high-tech items, underscores the need for effective inventory management and innovative solutions to minimize losses and maintain customer satisfaction. This paper explores the retailer-manufacturer collaboration, with a specific focus on the remanufacturing process under carbon-indicator-driven demand. It also examines the role of carbon emissions in supply chain management, highlighting the importance of sustainable practices. By addressing these interconnected elements, the study aims to provide actionable insights for enhancing supply chain resilience and sustainability.

Inventory system mathematical modeling is one of the many academic domains where CE concepts have been used due to their significance for environmental sustainability. Despite being relatively new in this field of study, CE applications have gained more interest recently since Rabta's (2020) [1] groundbreaking work was published. Rabta (2020) [2] used the economic order quantity (EOQ) model theory, initially put out by Harris (1913)[3], to build an economic order quantity extension for a product with a CE indication. Carbon emissions indicator is only an index that ranges from 0 to 1, indicating how much a product's manufacturing and consumption adhere to the CE's tenets. It directly affects the product's unit gross profit as well as the rate of demand for it. The integration of sustainability and efficiency in supply chain systems has become increasingly significant in addressing contemporary challenges such as environmental impact, product quality, and resource optimization. Inventory systems, in particular, are at the forefront of these efforts, where the interplay between deteriorating items, circular economy (CE) principles, and carbon emissions policies necessitates innovative approaches. Rabta (2020) [1] introduced a pivotal model that emphasized the dual optimization of order quantities and CE indicators, paving the way for exploring critical facets such as item deterioration, quality assurance, supply chain integration, and environmental regulations.

An illustrative example can be seen in the management of pharmaceutical products, which often have strict expiration dates and require controlled environments to maintain efficacy. Deteriorating items like these pose challenges for inventory control, necessitating advanced strategies for monitoring quality, minimizing waste, and ensuring timely distribution. Moreover, recycling and reprocessing expired pharmaceuticals align with CE principles, reducing environmental impact and conserving resources. These processes must be balanced with stringent quality standards and compliance with regulatory frameworks.

Simultaneously, effective supply chain integration is vital for leveraging core competencies and achieving operational coherence among diverse stakeholders. As businesses increasingly outsource non-core activities, fostering synergy within the supply chain becomes imperative to ensure seamless functionality. Additionally, growing global awareness of environmental sustainability has intensified the need for compliance with carbon emissions policies, including carbon taxes and cap-and-trade mechanisms.

The objective of this article is to create a CE-based integrated sustainable inventory model for a two-tier supply chain that focuses on deteriorating and subpar products. By examining various carbon emissions policies, the study seeks to optimize inventory replenishment strategies and CE indicators while addressing key research questions:

- How can inventory levels of deteriorating items be optimized to reduce waste and ensure quality?
- What degree of circularity should be incorporated into inventory systems for maximum sustainability?
- Which carbon emissions policy is most effective for balancing profitability and environmental responsibility?
- How do different demand rates and profit structures impact the overall efficiency of sustainable inventory models?

1.1. Orientation of the Manuscript

The structure of the paper is as follows: Section 1 provides the introduction. Section 2 reviews the existing literature relevant to the proposed model. In Section 3, the notations and assumptions utilized in the study are introduced. Section 4 outlines the problem description of the developed model. The mathematical formulation and corresponding solution are presented in Section 5. Section 6 discusses the variations and extensions of the proposed model. Section 7 describes the solution methodology and algorithm adopted. Section 8 presents numerical examples, implemented using MATLAB R2024b. Section 9 includes a sensitivity analysis and graphical representation of the results. Managerial insights derived from the model are discussed in Section 10. Finally, Section 11 concludes the study with key observations and practical implications.

2. Literature Review

The proposed supply chain inventory system faces challenges related to the circularity (CE) indicator of items, their potential for deterioration and the presence of substandard-quality items. Additionally, the system operates under varying carbon emissions regulations. As a result, inventory models that address carbon emissions policy, item deterioration, imperfect quality and CE indicators are the primary focus of the research study.

2.1. Deteriorating Items in EPQ Models:

Deterioration is a significant factor affecting inventory decisions. The incorporation of deterioration rates into EPQ models has been widely studied. Goyal and Giri (2001) [4] analyzed replenishment policies for deteriorating items, laying the foundation for future research. Later, Abad (2003) [5] integrated deterioration with variable production rates in an EPQ context. In recent years, Sarkar et al. (2020) [6] introduced an EPQ model addressing perishability and environmental sustainability simultaneously. Dey et al. (2022) [7] optimized the production schedule in conjunction with investments for automated inspection and green technology integration in a manufacturing-remanufacturing system for assembled goods. In Sindhuja and Arathi (2023) [8], a preservative-based inventory model for depreciating products with quality demand is investigated. This model incorporates circularity index and price-sensitive demand dynamics. Employing constant deterioration methods aims to enhance manufacturer profits through a new approach to defective items-based inventory models, ultimately aiming for cost-effectiveness in future manufacturing endeavors.

2.2. Circular Economy and EPQ Models:

Rabta (2020) [1] introduced an EOQ model integrating a product's circularity indicator to study its impact on inventory replenishment, modeling demand, and profit using various functional forms and optimizing for profit with circularity and order quantity as decision variables. Rabta (2020) [2] economic order quantity model has been executed for multi-echelon supply chains and manufacturing systems. Incorporating CE principles into EPQ models enhances resource efficiency and promotes sustainability. Kazancoglu et al. (2020) [9] extended the EPQ framework to include remanufacturing and recycling activities, highlighting the potential for reduced environmental impact. Singh et al. (2021) [10] introduced a hybrid EPQ model combining CE principles with traditional inventory management, demonstrating cost and waste reductions. John and Mishra (2023a) [11] and Khan et al. (2023) [12] created EPQ extensions. John and Mishra included emissions, carbon cap & trade policy and investment in green technology, while Khan et al. took production, setup and storage of carbon emissions into account. Wani and Mishra (2022) [13] and Thomas and Mishra (2022) [14] expanded two-echelon supply chain models, emphasizing sustainability investments and carbon emissions. John and Mishra (2023b) [15] developed a three-echelon model for the textile industry, incorporating emissions, green technologies, and textile waste.

2.3. Imperfect Quality in EPQ Models:

Imperfect production is a crucial aspect influencing inventory and production decisions. Porteus (1986) [16] was among the first to study quality control within EPQ systems. Salameh and Jaber's (2000) [17] inventory model has garnered significant attention for its applicability across various production systems. To find the ideal order amount, Goyal and Cardenas-Barron (2002) [18] suggested a less complicated approach. Chang (2004) [19] extended the model to a fuzzy environment with fuzzy demand rates and quality fractions. Wee et al. (2007) [20] introduced an executed allowing shortages with full backordering, while Khan et al. (2011) [21] accounted for Type I and Type II errors in the screening process, addressing the original model's assumption of perfect screening. In order to compare the performance of a three-tier supply chain with a Stackelberg game theory approach for commodities of imperfect quality, Sana (2011) [22] constructed the model. More recently, Khan and Qianli (2017) [23] integrated quality improvement into EPQ models for sustainable supply chains. In related work, Sana et al. (2022) [24] proposed an EPQ framework considering imperfect quality under carbon regulations, offering insights into balancing quality and sustainability. Salameh and Jaber's (2000) [17] model was expanded by Gautam et al. (2022) [25] by adding reworking, price- and green-dependent demand, and energy consumption. Additional noteworthy extensions include the use of high-tech products (Ruidas et al., 2023b [26]), trade credit with two levels (Tiwari et al., 2022 [27]), expanding products (Sebatjane & Adetunji, 2019, 2020, 2024 [28–30]) and sophisticated algorithms such as outer approximated performance with the use of penalty and equality relaxation (Gharaei et al., 2019 [29]).

2.4. Carbon Emissions in EPQ Models:

Environmental regulations have increasingly influenced EPQ-based research. Benjaafar et al. (2013) [31] explored the impact of carbon emissions policies on production systems, emphasizing the importance of regulatory compliance. Studies by Govindan and Hasanagic (2018) [32] further explored CE integration into EPQ models under carbon constraints. Chen et al. (2020) [33] developed a green EPQ model incorporating carbon taxes, demonstrating its effectiveness in achieving sustainability targets. The influence of the reduction of emission techniques and take-back laws on manufacturing enterprises' reproduction and carbon tax was examined by Ding et al. (2020) [34]. Yu et al. (2020) [35] developed an inventory model for degradable products while accounting for carbon policy investments in preservation technologies. They examined the best ordering choices that retailers would make in the constraints of the tax and cap & trade. By controlling product stocks and delivery, Wangsa et al. (2022) [36] were able to optimize expenditures associated with purchasing, inspection, food waste, packing, cold storage, transportation, and carbon emissions. In order to address a sustainable and traceable fish closed-loop supply chain, Andi Purnomo et al. (2022) [37] created a mathematical model that took carbon emissions from transportation, production, and warehousing into account. The model sought to minimize overall costs across a number of time periods, taking into account production, inventory, transportation, traceability, and emission costs. It did this by using mixed-integer linear programming. In a multi-echelon dairy supply chain, Vanany et al. (2023) [38] concentrated on reducing expenses, carbon emissions, and food waste. An method to multi-objective mixed-integer linear programming was utilized to optimize a system that involved farmers, distributors, retailers, and a processing plant. Dey et al. (2023) [39] examined three distinct policy types within their model to reduce carbon emissions and demonstrated that constrained carbon regulation was the most effective approach. While previous literature surveys have made advancements in carbon emission-based inventory models and strategies for cost reduction through emission control, this paper takes a slightly different approach by incorporating the remanufacturing method. This introduces a novel component with greater potential for cost reduction compared to conventional methods.

2.5. Sustainability and Environmental Considerations in EPQ Models:

Recent advancements in Economic Production Quantity (EPQ) models have increasingly focused on integrating sustainability and real-world complexities into supply chain management. Karim and Nakade (2022) [40] reviewed EPQ frameworks, emphasizing the incorporation of carbon emissions and product recycling. Sana (2022) [41] developed a two-echelon supply chain model to optimize inventory strategies under structural constraints, while Sana (2023) [42] explored the impact of greenhouse gas emission costs on pricing and lot-sizing in imperfect production systems. Sivashankari et al. (2024) [43] analyzed how advertising and price-dependent demand influence production and pricing in a sustainable system with imperfect quality. Barman et al. (2024) [44] studied pricing policies in dual-channel supply chains, integrating green investment and sales efforts with a revenue-sharing contract. Suvetha et al. (2024) [45] identified emerging trends, including additive manufacturing and fuzzy logic applications, in EPQ research from 2000 to 2022. Salas-Navarro et al. (2020) [46] developed an EPQ model that considers probabilistic demand and defective items, while Salas-Navarro et al. (2020) [47] proposed a three-echelon model factoring in marketing efforts on demand. Shaikh et al. (2018) [48] offered solutions for EPQ models with exponentially deteriorating items under partial trade credit policies. These studies collectively highlight the shift toward more sustainable, adaptable EPQ models addressing environmental impact, demand variability and financial constraints.

2.6. Research Gap:

Despite increasing research on sustainable supply chains, several critical gaps remain:

(i) **Supply Chain Structure:** Most studies consider multi-echelon supply chains that include distributors and other intermediaries (e.g., Dey et al., (2023) [39]; Ding et al., (2020) [34]). In contrast, this study focuses on a simplified two-echelon structure involving only a manufacturer and a retailer. This allows for direct coordination and streamlined decision-making, enhancing analytical tractability and managerial relevance.

(ii) **Demand Modeling:** Prior research in inventory and production systems primarily incorporates price-dependent or time-dependent demand models (e.g., Abad, (2003) [5]; Chen et al., (2020) [33]). However, circularity index-based demand, which reflects consumer preference for environmentally sustainable products, has not been integrated into such models. This study fills this gap by incorporating both traditional and circularity-driven demand functions.

(iii) **Carbon Emission Regulations:** Most existing works examine only one type of carbon policy—either carbon tax or cap-and-trade (e.g., Benjaafar et al., (2013) [31]; Sana & Chaudhuri, (2022) [24]). The current study addresses this limitation by analyzing and comparing both policies within a unified framework, offering greater insight into regulatory trade-offs.

(iv) **Quality Imperfections and Deterioration:** Studies have addressed production imperfections (e.g., Salameh & Jaber, (2000) [17]; Chang, (2004) [19]) and product deterioration (e.g., Goyal & Giri, (2001) [4]) separately. However, these two factors often coexist in real-world scenarios. This work integrates both deterioration and imperfect quality simultaneously, enhancing the model's practical applicability to sustainable supply chains.

2.7. Contribution of the Study:

To address these gaps, this study:

(i) Develops a two-echelon sustainable supply chain model that excludes distributors, enhancing decision-making efficiency.

(ii) Integrates dual-demand structures, where manufacturing is influenced by price and time, while remanufacturing depends on circularity index demand.

(iii) Analyzes and compares multiple carbon emissions regulations, extending prior research that focused on singular policies.

(iv) Incorporates imperfect quality and deterioration factors, optimizing inventory replenishment in remanufacturing systems.

This study introduces a sustainable production-inventory model for a two-echelon manufacturer-retailer supply chain with remanufacturing under various carbon emissions policies. The findings reveal that cap & trade policies yield the highest profitability, despite a moderate circularity index. Compared to previous studies (Dey et al., (2023) [39]; Ding et al., (2020) [34]; Rabta, (2020) [1]), this model enhances decision making by integrating dual demand functions, multiple carbon regulations, and quality deterioration factors. These insights contribute to the growing body of research on sustainable supply chains, helping policymakers and companies develop environmentally conscious and cost-effective inventory strategies.

3. Notations and Assumptions

In this manuscript, the following notations, acronyms and presumptions are used.

3.1. Notations

Table 1 & 2, describe the notation for all parameters used in this manuscript.

3.2. Assumptions

The model formulation is based on the following assumptions:

(i) The supply chain consists of a manufacturer and a retailer. The inventory system manages a single type of deteriorating product with a circularity index (ϵ), ranging from 0 to 1.

(ii) Inventory levels decline over time due to deterioration at a constant rate θ, both at the manufacturer and the retailer levels.

(iii) The manufacturing process is imperfect, resulting in a fixed fraction $\mathscr{Y}$ of defective units. These are detected through 100% quality inspection performed by the retailer.

(iv) All imperfect items are collected and remanufactured in batches by the manufacturer after screening.

(v) The demand rate for remanufactured products depends on the circularity index, expressed as $\mathscr{D}(\epsilon) = \mathscr{D}_0 + \mathscr{A}\epsilon$, while the unit gross profit is defined as $\mathscr{G}\mathscr{P}(\epsilon) = \mathscr{G}_0 + \mathscr{B}\epsilon$, in line with Rabta (2020) [1].

(vi) The manufacturing and remanufacturing rates, i.e., $\mathscr{P}_{\mathscr{M}}\&\mathscr{P}_{\mathscr{R}}$ exceed the maximum demand rate $\mathscr{D}_0 + \mathscr{A}$ and the screening rate i.e., $\mathscr{X}$ is assumed to be significantly higher than the demand rate to ensure immediate processing.

(vii) Both high-quality items and remanufactured products contribute to the total gross profit, reflecting realistic cost recovery mechanisms.

(viii) Carbon emissions arise from both manufacturing and remanufacturing activities and are regulated under two policy mechanisms: carbon tax and cap-and-trade.

(ix) All system parameters are deterministic and constant over time. Shortages are not permitted in this model.

(x) Classical optimization techniques are employed to determine optimal production cycle and circularity level. The Hessian matrix is used to confirm the concavity and local optimality of the objective function.

4. Problem Description

This study investigates a sustainable inventory and remanufacturing problem in a two-echelon supply chain consisting of a manufacturer and a retailer, operating under carbon emissions regulations. The manufacturer produces new items that deteriorate over time and are subject to imperfect quality, while the retailer handles quality inspection and facilitates the return of defective units for remanufacturing. The demand for newly manufactured products is dependent on both selling price and time, whereas the demand for remanufactured products is driven by a circularity index, which reflects the product's environmental performance. The supply chain operates under either a carbon tax or a cap-and-trade policy and the objective is to determine optimal production and remanufacturing cycle times that maximize total profit while satisfying environmental constraints. The model also considers the effects of deterioration, carbon emissions, imperfect quality, and circular economy dynamics on inventory decisions. A mathematical framework is developed to analyze the system behavior under both regulatory regimes, and numerical validation is conducted using MATLAB to assess the impact of key parameters on supply chain performance. Figure 1, shows the interaction between the manufacturer and retailer, supporting sustainable and cost-effective inventory and remanufacturing decisions under carbon regulations.

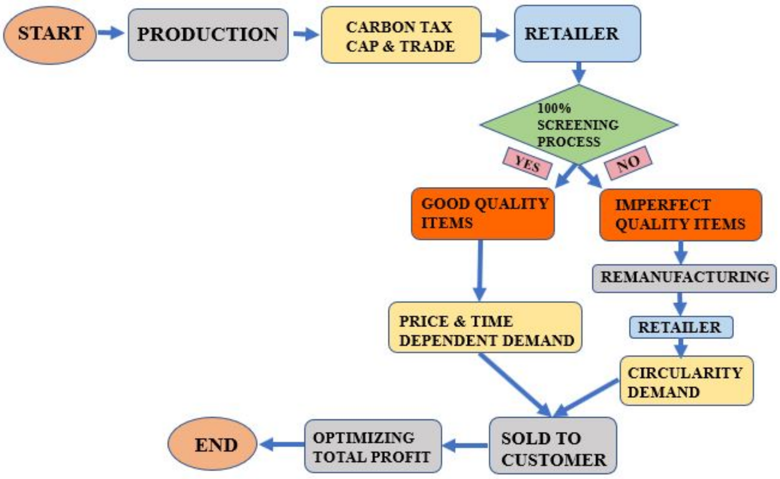

Figure 1: Flowchart of the proposed model

5. Mathematical Model with Solution

This section details the mathematical model of the proposed framework, while Figures 2 to 6 illustrate the overall configuration of the inventory system.

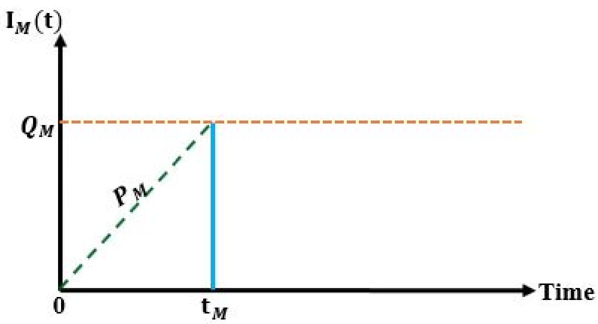

Figure 2: Inventory system of manufacturer

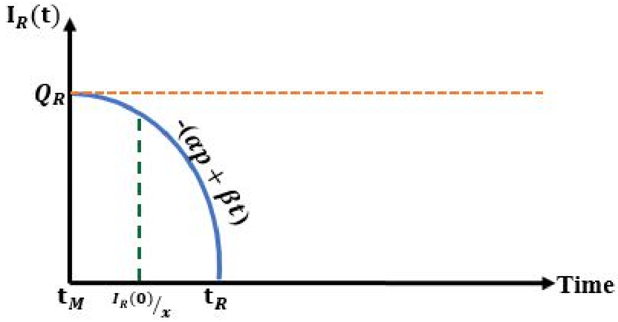

Figure 3: Inventory system of retailer

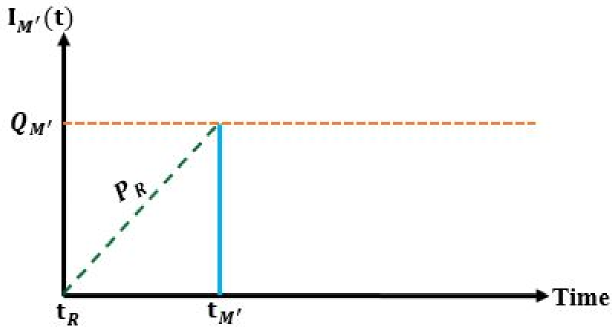

Figure 4: Inventory system of re-manufacturer

5.1. Case:1 Manufacturing inventory model for deteriorating items with price & time dependent demand

5.1.1. Total cost and emission function of the manufacturer:

As shown in Figure 2, the inventory levels at any time during the period $(0,t_{\mathscr{M}})$ are governed by the following differential equations:

$$\frac{d\mathscr{I}_{\mathscr{M}}(t)}{dt}+\theta\mathscr{I}_{\mathscr{M}}(t)=\mathscr{P}_{\mathscr{M}}, \quad 0\leq t\leq t_{\mathscr{M}}; \tag{1}$$

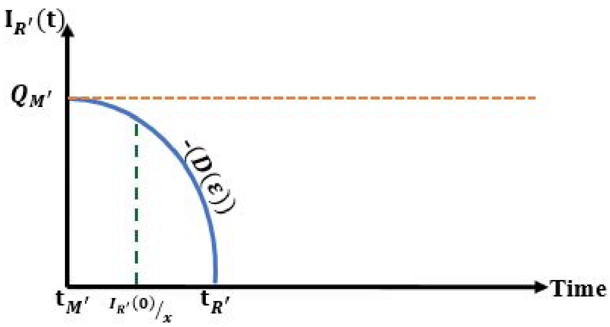

Figure 5: Inventory system of retailer in re-manufacturing process

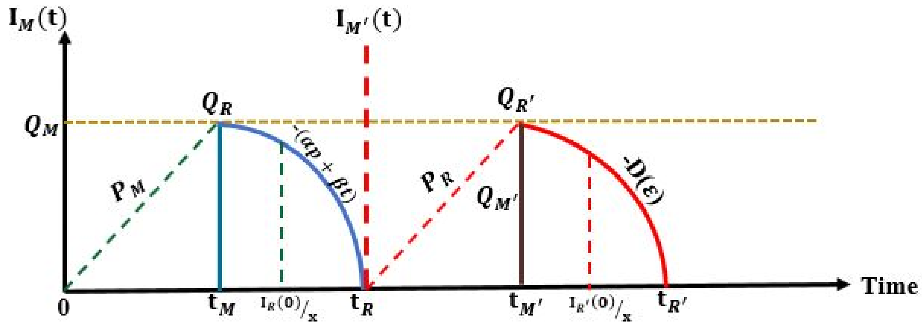

Figure 6: Inventory system of the model

With the given initial and boundary conditions, $\mathscr{I}_{\mathscr{M}}(0) = 0$ at $t = 0$, the manufacturer's stock level at any given time t can be determined as follows:

$$\mathscr{I}_{\mathscr{M}}(t) = \frac{\mathscr{P}_{\mathscr{M}}}{\theta}[1 - e^{-\theta t}]. \tag{2}$$

The quantity delivered by the manufacturer to the retailer, represented as $\mathscr{Q}_{\mathscr{M}}$, can be obtained by utilizing the boundary condition $\mathscr{I}_{\mathscr{M}}(t_{\mathscr{M}}) = \mathscr{Q}_{\mathscr{M}}$ at $t = t_{\mathscr{M}}$. By incorporating this condition, the preceding equations can be solved as follows:

$$\mathscr{Q}_{\mathscr{M}} = \mathscr{I}_{\mathscr{M}}(t_{\mathscr{M}}) = \frac{\mathscr{P}_{\mathscr{M}}}{\theta}[1 - e^{-\theta t_{\mathscr{M}}}]. \tag{3}$$

As illustrated in Figure 2, the manufacturer produces $\mathscr{Q}_{\mathscr{M}}$ goods during the manufacturing period $t_{\mathscr{M}}$ and starts a current manufacturing cycle every T time units. Fixed setup cost of $\mathscr{S}_{\mathscr{M}}$ incurred at the start of each cycle, the setup cost per unit time for the manufacturer is,

$$\mathscr{S}\mathscr{C}_{\mathscr{M}} = \frac{\mathscr{S}_{\mathscr{M}}}{T}. \tag{4}$$

In this study, products are stored until their demand is met, extending up to time T. During the interval from 0 to $t_{\mathscr{M}}$, the holding cost is calculated for all items produced and retained until demand is satisfied. This cost is determined based on the fixed holding cost $\mathscr{H}_{\mathscr{M}}$, the inventory level $\mathscr{I}_{\mathscr{M}}(t)$ and the demand $D(t)$.

$$\mathscr{H}\mathscr{C}_{\mathscr{M}} = \frac{\mathscr{H}_{\mathscr{M}}}{T}\left[\ \int_0^{t_{\mathscr{M}}} \mathscr{I}_{\mathscr{M}}(t)dt\ \right],$$

$$\mathscr{H}\mathscr{C}_{\mathscr{M}} = \frac{\mathscr{P}_{\mathscr{M}}\mathscr{H}_{\mathscr{M}}}{\theta^2 T}\left[\ \theta t_{\mathscr{M}} + e^{-\theta t_{\mathscr{M}}} - 1\ \right]. \tag{5}$$

The deteriorated inventory of the manufacturer per cycle is given by $\mathscr{P}_{\mathscr{M}} t_{\mathscr{M}}$)-$\mathscr{Q}_{\mathscr{M}}$,

$$\mathscr{P}_{\mathscr{M}} t_{\mathscr{M}} - \mathscr{Q}_{\mathscr{M}} = \mathscr{P}_{\mathscr{M}} t_{\mathscr{M}} - \frac{\mathscr{P}_{\mathscr{M}}}{\theta}[1 - e^{-\theta t_{\mathscr{M}}}],$$

The manufacturer's deterioration cost per unit time, $\mathscr{D}\mathscr{C}_{\mathscr{M}}$, is determined using the depreciated inventory per cycle, the deterioration cost per unit ($\mathscr{D}_{\mathscr{M}}$) and the cycle of replenishment duration T can be used to compute as,

$$\mathscr{D}\mathscr{C}_{\mathscr{M}} = \frac{\mathscr{D}_{\mathscr{M}}}{T}\left[\ \mathscr{P}_{\mathscr{M}} t_{\mathscr{M}} - \tfrac{\mathscr{P}_{\mathscr{M}}}{\theta}[1 - e^{-\theta t_{\mathscr{M}}}]\ \right]. \tag{6}$$

The overall cost to the manufacturer includes ordering, degradation, and holding charges. Therefore, by summing equations (4), (5) and (6), the total cost per unit time ($\mathscr{T}\mathscr{C}_{\mathscr{M}}$) can be obtained as follows:

$$\mathscr{T}\mathscr{C}_{\mathscr{M}} = \left[\begin{array}{l} \frac{\mathscr{S}_{\mathscr{M}}}{T} + \left[\ \frac{\mathscr{P}_{\mathscr{M}}\mathscr{H}_{\mathscr{M}}}{\theta^2 T}\left[\ \theta t_{\mathscr{M}} + e^{-\theta t_{\mathscr{M}}} - 1\ \right]\ \right] \\ + \left[\ \frac{\mathscr{D}_{\mathscr{M}}}{T}\left[\ \mathscr{P}_{\mathscr{M}} t_{\mathscr{M}} - \frac{\mathscr{P}_{\mathscr{M}}}{\theta}[1 - e^{-\theta t_{\mathscr{M}}}]\ \right]\ \right] \end{array}\right]. \tag{7}$$

The manufacturer's inventory holding, production setup and depreciation processes generate carbon emissions amounting to $\hat{\mathscr{S}}_{\mathscr{M}}$ units per cycle, $\hat{\mathscr{H}}_{\mathscr{M}}$ units per unit time and $\hat{\mathscr{D}}_{\mathscr{M}}$ per units time. Consequently, the manufacturer's overall carbon emissions per time unit, $\mathscr{T}\mathscr{E}_{\mathscr{M}}$ are calculated as follows:

$$\mathscr{T}\mathscr{E}_{\mathscr{M}} = \left[\begin{array}{l} \frac{\hat{\mathscr{S}}_{\mathscr{M}}}{T} + \left[\ \frac{\mathscr{P}_{\mathscr{M}}\hat{\mathscr{H}}_{\mathscr{M}}}{\theta^2 T}\left[\ \theta t_{\mathscr{M}} + e^{-\theta t_{\mathscr{M}}} - 1\ \right]\ \right] \\ + \left[\ \frac{\hat{\mathscr{D}}_{\mathscr{M}}}{T}\left[\ \mathscr{P}_{\mathscr{M}} t_{\mathscr{M}} - \frac{\mathscr{P}_{\mathscr{M}}}{\theta}[1 - e^{-\theta t_{\mathscr{M}}}]\ \right]\ \right] \end{array}\right]. \tag{8}$$

5.1.2. Retailer's overall manufacturing process costs and emissions

In Figure 3, the manufacturer receives an order of $\mathscr{Q}_{\mathscr{M}}$ products to the store at the beginning of each cycle, with a fraction $\mathscr{Y}$ of these items being of lower quality. The retailer performs a 100% screening of the lot at a rate of $\mathscr{X}$ after receiving the order in order to separate the good from the bad. The lower-quality items $\mathscr{Y}\mathscr{Q}_{\mathscr{R}}$ are separated from the lot and kept as a single batch at time $t = \frac{\mathscr{I}_{\mathscr{R}}(0)}{\mathscr{X}}$ after the screening process is finished. Since each cycle extends for $t_{\mathscr{R}}$ time units and the retailer is obligated to pay a fixed order cost of $\mathscr{O}_{\mathscr{R}}$ at the beginning of each cycle, the ordering cost per unit time,

$$\mathscr{O}\mathscr{C}_{\mathscr{R}} = \frac{\mathscr{O}_{\mathscr{R}}}{t_{\mathscr{R}}} \tag{9}$$

Assuming that the quantity of high-quality items received by the retailer is at least equal to the demand during the screening period, the following constraint is enforced on the inventory system of the retailer, as derived from Lee and Kim (2014).

$$(1 - \mathscr{Y})\mathscr{I}_{\mathscr{R}}(0) \geq \ \mathscr{D}(t)\frac{\mathscr{I}_{\mathscr{R}}(0)}{\mathscr{X}}, \tag{10}$$

Because demand and deterioration cause the retailer's inventory to deplete at rates of $\mathscr{D}(t)$ and θ, respectively, and because lower-quality items are not used to satisfy the demand $\mathscr{D}(t)$ for high-quality items, the following differential equation governs the retailer's inventory level over time.

$$\frac{d\mathscr{I}_{\mathscr{R}}(t)}{dt} + \theta \mathscr{I}_{\mathscr{R}}(t) = -(1-\mathscr{Y})\mathscr{D}(t), \quad 0 \le t \le t_{\mathscr{R}}; \tag{11}$$

The retailer's inventory system is divided into two periods, the non-screening period, which runs from $t = \frac{\mathscr{I}_{\mathscr{R}}(0)}{\mathscr{X}}$ to $t = t_{\mathscr{R}}$, and the screening period, which runs from $t = 0$ to $t = \frac{\mathscr{I}_{\mathscr{R}}(0)}{\mathscr{X}}$. Taking into account the boundary constraint $\mathscr{I}_{\mathscr{R}}(t_{\mathscr{R}}) = 0$, the following differential equations depict the variations in the retailer's inventory level over time during the screening and non-screening periods, respectively.

$$\mathscr{I}_{\mathscr{R}}(t) = \frac{1}{\theta^2}\left[\ (\alpha\theta\mathscr{P} - \beta)e^{-\theta t} - \alpha\theta\mathscr{P} - \beta\theta t + \beta\ \right], \quad 0 \le t \le \frac{\mathscr{I}_{\mathscr{R}}(0)}{\mathscr{X}}; \tag{12}$$

$$\mathscr{I}_{\mathscr{R}}(t) = \frac{(1-\mathscr{Y})}{\theta^2}\begin{bmatrix} e^{\theta(t_{\mathscr{R}}-t)}[\alpha\theta\mathscr{P} + \beta\theta t_{\mathscr{R}} - \beta] \\ -[\alpha\theta\mathscr{P} + \beta\theta t - \beta] \end{bmatrix}, \quad \frac{\mathscr{I}_{\mathscr{R}}(0)}{\mathscr{X}} \le t \le t_{\mathscr{R}}; \tag{13}$$

Similarly, the buyer's ordered quantity is provided as follows:

$$\mathscr{Q}_{\mathscr{R}} = \mathscr{I}_{\mathscr{R}}(0) = \alpha\mathscr{P}t_{\mathscr{R}} + \beta t_{\mathscr{R}}^2. \tag{14}$$

The retail replenishment cycle duration ($t_{\mathscr{R}}$) is divided by the annual cost of keeping one unit of inventory in storage ($\mathscr{H}_{\mathscr{R}}$), and the average inventory level per cycle is multiplied by the retailer to determine the holding cost per unit time ($\mathscr{HC}_{\mathscr{R}}$). Therefore, the holding cost is as follows:

$$\mathscr{HC}_{\mathscr{R}} = \frac{\mathscr{H}_{\mathscr{R}}}{t_{\mathscr{R}}}\left[\ \int_0^{\frac{\mathscr{I}_{\mathscr{R}}(0)}{\mathscr{X}}} \mathscr{I}_{\mathscr{R}}(t)dt + \int_{\frac{\mathscr{I}_{\mathscr{R}}(0)}{\mathscr{X}}}^{t_{\mathscr{R}}} \mathscr{I}_{\mathscr{R}}(t)dt\ \right],$$

$$\mathscr{HC}_{\mathscr{R}} = \frac{\mathscr{H}_{\mathscr{R}}}{t_{\mathscr{R}}}\begin{bmatrix} \frac{1}{2\theta\mathscr{X}^2}\left[\ (-\alpha\beta\theta + 2\beta)(\alpha^2\mathscr{P}^2t_{\mathscr{R}}^2 + \beta^2 t_{\mathscr{R}}^4 + 2\alpha\beta\mathscr{P}t_{\mathscr{R}}^3)\ \right] \\ +\frac{(1-\mathscr{Y})(\alpha\mathscr{P}+\beta t_{\mathscr{R}})}{2\mathscr{X}}\begin{bmatrix} 2t_{\mathscr{R}}(\mathscr{X}t_{\mathscr{R}} - \alpha\mathscr{P}t_{\mathscr{R}} - \beta t_{\mathscr{R}}^2) \\ -\frac{1}{\mathscr{X}^2}(\mathscr{X}t_{\mathscr{R}} - (\alpha\mathscr{P}t_{\mathscr{R}} + \beta t_{\mathscr{R}}^2))^2 \end{bmatrix} \end{bmatrix}. \tag{15}$$

In the retailer's cycle $\mathscr{D}(t)t_{\mathscr{R}}$, the order quantity $\mathscr{Q}_{\mathscr{R}}$ lowers the demand, which is the amount of inventory that depreciates per cycle. The amount of deteriorating inventory that the merchant has per cycle can be roughly estimated using the yields of Tiwari et al. (2018).

$$\mathscr{Q}_{\mathscr{R}} - \mathscr{D}(t)t_{\mathscr{R}} = \beta t_{\mathscr{R}}(t_{\mathscr{R}} - t),$$

The retailer's deterioration cost per unit time $\mathscr{DC}_{\mathscr{R}}$ is determined as follows, considering the deteriorated inventory per cycle, the deterioration cost per unit $\mathscr{D}_{\mathscr{R}}$ and the replenishment cycle time $t_{\mathscr{R}}$.

$$\mathscr{DC}_{\mathscr{R}} = \mathscr{D}_{\mathscr{R}}\beta(t_{\mathscr{R}} - t). \tag{16}$$

If the retailer completes a screening procedure (i.e., screens all $\mathscr{Q}_{\mathscr{R}}$ units) and the cost to screen each unit is $\mathscr{Z}_{\mathscr{R}}$, then the screening costs per unit time $\mathscr{Z}\mathscr{C}_{\mathscr{R}}$ is calculated as follows.

$$\mathscr{Z}\mathscr{C}_{\mathscr{R}} = \frac{\mathscr{Z}_{\mathscr{R}}\mathscr{Q}_{\mathscr{R}}}{t_{\mathscr{R}}} = \mathscr{Z}_{\mathscr{R}}[\alpha\mathscr{P} + \beta t_{\mathscr{R}}]. \tag{17}$$

The retailer's total cost function includes ordering, holding, deterioration, and screening costs. Thus, the retailer's total cost per unit time $\mathscr{T}\mathscr{C}_{\mathscr{R}}$ is found by adding equations $(9), (15), (16), and (17)$, as shown below.

$$\mathscr{T}\mathscr{C}_{\mathscr{R}} = \left[\begin{array}{c} \frac{\mathscr{H}_{\mathscr{R}}}{t_{\mathscr{R}}}\left[\begin{array}{l} \frac{1}{2\theta\mathscr{X}^2}\left[\; (-\alpha\beta\theta + 2\beta)(\alpha^2\mathscr{P}^2t_{\mathscr{R}}^2 + \beta^2t_{\mathscr{R}}^4 + 2\alpha\beta\mathscr{P}t_{\mathscr{R}}^3)\;\right] \\ +\frac{(1-\mathscr{Y})(\alpha\mathscr{P}+\beta t_{\mathscr{R}})}{2\mathscr{X}}\left[\begin{array}{c} 2t_{\mathscr{R}}(\mathscr{X}t_{\mathscr{R}} - \alpha\mathscr{P}t_{\mathscr{R}} - \beta t_{\mathscr{R}}^2) \\ -\frac{1}{\mathscr{X}^2}(\mathscr{X}t_{\mathscr{R}} - (\alpha\mathscr{P}t_{\mathscr{R}} + \beta t_{\mathscr{R}}^2))^2 \end{array}\right] \end{array}\right] \\ +\mathscr{D}_{\mathscr{R}}\beta(t_{\mathscr{R}} - t) + \mathscr{Z}_{\mathscr{R}}[\alpha\mathscr{P} + \beta t_{\mathscr{R}}] + \frac{\mathscr{O}_{\mathscr{R}}}{t_{\mathscr{R}}} \end{array}\right]. \tag{18}$$

At the retail facility, various inventory management activities contribute to carbon dioxide emissions. Specifically, these activities generate $\hat{\mathscr{O}}_{\mathscr{R}}$ emissions units per cycle, $\hat{\mathscr{H}}_{\mathscr{R}}$ emissions units per unit per unit time from holding and $\hat{\mathscr{D}}_{\mathscr{R}}$ emissions units per unit per unit time from deterioration. As a result, the retailer's total carbon emissions per unit time $\mathscr{T}\mathscr{E}_{\mathscr{R}}$ are calculated using the following formula:

$$\mathscr{T}\mathscr{E}_{\mathscr{R}} = \left[\begin{array}{c} \frac{\hat{\mathscr{H}}_{\mathscr{R}}}{t_{\mathscr{R}}}\left[\begin{array}{l} \frac{1}{2\theta\mathscr{X}^2}\left[\; (-\alpha\beta\theta + 2\beta)(\alpha^2\mathscr{P}^2t_{\mathscr{R}}^2 + \beta^2t_{\mathscr{R}}^4 + 2\alpha\beta\mathscr{P}t_{\mathscr{R}}^3)\;\right] \\ +\frac{(1-\mathscr{Y})(\alpha\mathscr{P}+\beta t_{\mathscr{R}})}{2\mathscr{X}}\left[\begin{array}{c} 2t_{\mathscr{R}}(\mathscr{X}t_{\mathscr{R}} - \alpha\mathscr{P}t_{\mathscr{R}} - \beta t_{\mathscr{R}}^2) \\ -\frac{1}{\mathscr{X}^2}(\mathscr{X}t_{\mathscr{R}} - (\alpha\mathscr{P}t_{\mathscr{R}} + \beta t_{\mathscr{R}}^2))^2 \end{array}\right] \end{array}\right] \\ +\hat{\mathscr{D}}_{\mathscr{R}}\beta(t_{\mathscr{R}} - t) + \frac{\hat{\mathscr{O}}_{\mathscr{R}}}{t_{\mathscr{R}}} \end{array}\right]. \tag{19}$$

$\mathscr{G}\mathscr{P}(t)$ is the unit gross profit, which is determined by subtracting the acquisition cost of the items from the selling price. $\mathscr{D}(t)$ indicates the demand rate. The average total supply chain profit, therefore, is $\mathscr{D}(t)\mathscr{G}\mathscr{P}(t) - (\mathscr{T}\mathscr{C}_{\mathscr{M}} + \mathscr{T}\mathscr{C}_{\mathscr{R}})$. Thus, the following formula is used to calculate the total supply chain profit per unit time $\mathscr{T}\mathscr{P}_{\mathscr{S}\mathscr{C}}$:

$$\mathscr{T}\mathscr{P}_{\mathscr{S}\mathscr{C}} = \left[\begin{array}{c} \mathscr{D}(t)\mathscr{G}\mathscr{P}(t) - \left[\begin{array}{l} \frac{\mathscr{S}_{\mathscr{M}}}{T} + \left[\; \frac{\mathscr{P}_{\mathscr{M}}\mathscr{H}_{\mathscr{M}}}{\theta^2 T}\left[\; \theta t_{\mathscr{M}} + e^{-\theta t_{\mathscr{M}}} - 1\;\right]\;\right] \\ +\left[\; \frac{\mathscr{D}_{\mathscr{M}}}{T}\left[\; \mathscr{P}_{\mathscr{M}}t_{\mathscr{M}} - \frac{\mathscr{P}_{\mathscr{M}}}{\theta}[1 - e^{-\theta t_{\mathscr{M}}}]\;\right]\;\right] \end{array}\right] \\ -\left[\begin{array}{c} \frac{\mathscr{H}_{\mathscr{R}}}{t_{\mathscr{R}}}\left[\begin{array}{l} \frac{1}{2\theta\mathscr{X}^2}\left[\; (-\alpha\beta\theta + 2\beta)(\alpha^2\mathscr{P}^2t_{\mathscr{R}}^2 + \beta^2t_{\mathscr{R}}^4 + 2\alpha\beta\mathscr{P}t_{\mathscr{R}}^3)\;\right] \\ +\frac{(1-\mathscr{Y})(\alpha\mathscr{P}+\beta t_{\mathscr{R}})}{2\mathscr{X}}\left[\begin{array}{c} 2t_{\mathscr{R}}(\mathscr{X}t_{\mathscr{R}} - \alpha\mathscr{P}t_{\mathscr{R}} - \beta t_{\mathscr{R}}^2) \\ -\frac{1}{\mathscr{X}^2}(\mathscr{X}t_{\mathscr{R}} - (\alpha\mathscr{P}t_{\mathscr{R}} + \beta t_{\mathscr{R}}^2))^2 \end{array}\right] \end{array}\right] \\ +\mathscr{D}_{\mathscr{R}}\beta(t_{\mathscr{R}} - t) + \mathscr{Z}_{\mathscr{R}}[\alpha\mathscr{P} + \beta t_{\mathscr{R}}] + \frac{\mathscr{O}_{\mathscr{R}}}{t_{\mathscr{R}}} \end{array}\right] \end{array}\right]. \tag{20}$$

The supply chain's total carbon emissions originate from both the retailer's and the manufacturer's facilities. Therefore, the overall carbon emissions per unit time in the supply chain are determined as follows:

$$\mathscr{TE}_{\mathscr{SC}} = \left[\begin{array}{c} \left[\begin{array}{c} \frac{\hat{\mathscr{H}_{\mathscr{R}}}}{t_{\mathscr{R}}} \left[\begin{array}{c} \frac{1}{2\theta\mathscr{X}^2} \left[\ (-\alpha\beta\theta + 2\beta)(\alpha^2\mathscr{P}^2 t_{\mathscr{R}}^2 + \beta^2 t_{\mathscr{R}}^4 + 2\alpha\beta\mathscr{P} t_{\mathscr{R}}^3) \ \right] \\ + \frac{(1-\mathscr{Y})(\alpha\mathscr{P}+\beta t_{\mathscr{R}})}{2\mathscr{X}} \left[\begin{array}{c} 2t_{\mathscr{R}}(\mathscr{X} t_{\mathscr{R}} - \alpha\mathscr{P} t_{\mathscr{R}} - \beta t_{\mathscr{R}}^2) \\ -\frac{1}{\mathscr{X}^2}(\mathscr{X} t_{\mathscr{R}} - (\alpha\mathscr{P} t_{\mathscr{R}} + \beta t_{\mathscr{R}}^2))^2 \end{array} \right] \end{array} \right] \\ + \hat{\mathscr{D}}_{\mathscr{R}}\beta(t_{\mathscr{R}} - t) + \frac{\hat{\mathscr{O}_{\mathscr{R}}}}{t_{\mathscr{R}}} \end{array} \right] \\ + \left[\begin{array}{c} \frac{\hat{\mathscr{S}_{\mathscr{M}}}}{T} + \left[\ \frac{\mathscr{P}_{\mathscr{M}}\hat{\mathscr{H}_{\mathscr{M}}}}{\theta^2 T} \left[\ \theta t_{\mathscr{M}} + e^{-\theta t_{\mathscr{M}}} - 1 \ \right] \right] \\ + \left[\ \frac{\hat{\mathscr{D}_{\mathscr{M}}}}{T} \left[\ \mathscr{P}_{\mathscr{M}} t_{\mathscr{M}} - \frac{\mathscr{P}_{\mathscr{M}}}{\theta}[1 - e^{-\theta t_{\mathscr{M}}}] \ \right] \right] \end{array} \right] \end{array} \right]. \tag{21}$$

5.2. Case:2 Remanufacturing inventory model for deteriorating items with circularity index demand

5.2.1. Remanufacturer's total cost and emission functions:

The production process begins with a constant rate $\mathscr{P}_{\mathscr{M}} > \mathscr{D}$, producing defective items at a rate of $\mathscr{Y}$, resulting in $\theta_1 = \mathscr{Y}\mathscr{P}_{\mathscr{M}}$. Remanufacturing starts at a rate $\mathscr{P}_{\mathscr{R}}$ to convert defective items into perfect ones, with scrap generated at a rate of $\mathscr{Z}$, leading to $\theta_2 = \mathscr{Z}\mathscr{P}_{\mathscr{R}}$. The perfect items are then prepared for retail sale within time $t_{\mathscr{R}'}$. The on-hand inventory of perfect items after remanufacturing is $\mathscr{I}_{\mathscr{M}'}$. Inventory levels between $(t_{\mathscr{R}},t_{\mathscr{M}'})$ are described by the following differential equations,

$$\frac{d\mathscr{I}_{\mathscr{M}'}(t)}{dt} + \theta\mathscr{I}_{\mathscr{M}'}(t) = \mathscr{P}_{\mathscr{R}}, \quad t_{\mathscr{R}} \leq t \leq t_{\mathscr{M}'}; \tag{22}$$

With the boundary condition $\mathscr{I}_{\mathscr{M}'}(t) = \mathscr{Y}\mathscr{P}_{\mathscr{M}}$ at $t = t_{\mathscr{R}}$, the remanufacturer's stock level at any given time t can be expressed as,

$$\mathscr{I}_{\mathscr{M}'}(t) = e^{-\theta t} \left[\ \frac{{\mathscr{P}_{\mathscr{R}}}^2 e^{\theta t} - \mathscr{Y}\mathscr{P}_{\mathscr{M}}\theta^2}{\theta\mathscr{P}_{\mathscr{R}}} \ \right] \tag{23}$$

The remanufacturer's delivery quantity to the retailer $\mathscr{Q}_{\mathscr{M}'}$ is obtained by applying the initial condition $\mathscr{I}_{\mathscr{M}'}(t_{\mathscr{M}'}) = \mathscr{Q}'_{\mathscr{M}}$ at $t = t_{\mathscr{M}'}$. The equations can be solved as follows,

$$\mathscr{Q}_{\mathscr{M}'}(t) = e^{-\theta t_{\mathscr{M}'}} \left[\ \frac{{\mathscr{P}_{\mathscr{R}}}^2 e^{\theta t_{\mathscr{M}'}} - \mathscr{Y}\mathscr{P}_{\mathscr{M}}\theta^2}{\theta\mathscr{P}_{\mathscr{R}}} \ \right] \tag{24}$$

As depicted in Figure 4, the remanufacturer produces $\mathscr{Q}'_{\mathscr{M}}$ goods during the manufacturing period $t_{\mathscr{M}'}$ and begins a manufacturing cycle every T time units. The fixed setup cost $\mathscr{S}_{\mathscr{M}'}$ is incurred at the beginning of every cycle, and the manufacturer's setup cost per time unit is calculated as follows,

$$\mathscr{SC}_{\mathscr{M}'} = \frac{\mathscr{S}_{\mathscr{M}'}}{T}. \tag{25}$$

Products are stored until their demand is met, extending up to time T. From $t_{\mathscr{R}}$ to $t_{\mathscr{M}'}$, the holding costs include all items produced and retained until demand is satisfied. This cost is calculated based on the fixed holding cost $\mathscr{H}_{\mathscr{M}'}$, the inventory level $\mathscr{I}_{\mathscr{M}'}(t)$ and the product demand $D(\epsilon)$.

$$\mathscr{H}\mathscr{C}_{\mathscr{M}'} = \frac{\mathscr{H}_{\mathscr{M}'}}{T}\left[\ \int_{t_{\mathscr{R}}}^{t_{\mathscr{M}'}} \mathscr{I}_{\mathscr{M}'}(t)dt\ \right],$$

$$\mathscr{H}\mathscr{C}_{\mathscr{M}'} = \frac{\mathscr{H}_{\mathscr{M}'}}{T}\left[\ \frac{\mathscr{P}_{\mathscr{R}}}{\theta}[t_{\mathscr{M}'} - t_{\mathscr{R}}] + \frac{\mathscr{Y}\mathscr{P}_{\mathscr{M}}}{\mathscr{P}_{\mathscr{R}}} e^{-\theta(t_{\mathscr{M}'} - t_{\mathscr{R}})}\ \right] \tag{26}$$

The remanufacturer's deteriorated inventory per cycle is determined by subtracting the quantity of goods delivered to the retailer $\mathscr{Q}'_{\mathscr{M}}$ from the remanufacturer's production quantity $\mathscr{P}_{\mathscr{R}} t_{\mathscr{M}'}$.

$$\mathscr{P}_{\mathscr{R}} t_{\mathscr{M}'} - \mathscr{Q}_{\mathscr{M}'} = \mathscr{P}_{\mathscr{R}} t_{\mathscr{M}'} - e^{-\theta t_{\mathscr{M}'}}\left[\ \frac{{\mathscr{P}_{\mathscr{R}}}^2 e^{\theta t_{\mathscr{M}'}} - \mathscr{Y}\mathscr{P}_{\mathscr{M}}\theta^2}{\theta \mathscr{P}_{\mathscr{R}}}\ \right]$$

Deterioration cost per unit time for the remanufacturer the amount of deteriorated inventory every cycle, the deterioration cost per unit $\mathscr{D}_{\mathscr{M}'}$ and the duration of the replenishment cycle T can be used to determine $\mathscr{D}\mathscr{C}_{\mathscr{M}'}$ as follows:

$$\mathscr{D}\mathscr{C}_{\mathscr{M}'} = \frac{\mathscr{D}_{\mathscr{M}'}}{T}\left[\ \mathscr{P}_{\mathscr{R}} t_{\mathscr{M}'} - e^{-\theta t_{\mathscr{M}'}}\left[\ \frac{{\mathscr{P}_{\mathscr{R}}}^2 e^{\theta t_{\mathscr{M}'}} - \mathscr{Y}\mathscr{P}_{\mathscr{M}}\theta^2}{\theta \mathscr{P}_{\mathscr{R}}}\ \right]\ \right] \tag{27}$$

The remanufacturer's total cost includes setup, holding, and deteriorating costs. Therefore, summing equations (23), (24), and (25) gives the remanufacturer's total cost per unit time ($\mathscr{T}\mathscr{C}_{\mathscr{M}'}$) as follows,

$$\mathscr{T}\mathscr{C}_{\mathscr{M}'} = \left[\begin{array}{l} \frac{\mathscr{S}_{\mathscr{M}'}}{T} + \frac{\mathscr{H}_{\mathscr{M}'}}{T}\left[\ \frac{\mathscr{P}_{\mathscr{R}}}{\theta}[t_{\mathscr{M}'} - t_{\mathscr{R}}] + \frac{\mathscr{Y}\mathscr{P}_{\mathscr{M}}}{\mathscr{P}_{\mathscr{R}}} e^{-\theta(t_{\mathscr{M}'} - t_{\mathscr{R}})}\ \right] \\ + \frac{\mathscr{D}_{\mathscr{M}'}}{T}\left[\ \mathscr{P}_{\mathscr{R}} t_{\mathscr{M}'} - e^{-\theta t_{\mathscr{M}'}}\left[\ \frac{{\mathscr{P}_{\mathscr{R}}}^2 e^{\theta t_{\mathscr{M}'}} - \mathscr{Y}\mathscr{P}_{\mathscr{M}}\theta^2}{\theta \mathscr{P}_{\mathscr{R}}}\ \right]\ \right] \end{array}\right]. \tag{28}$$

Carbon emissions are $\hat{\mathscr{S}}_{\mathscr{M}'}$ units per cycle, $\hat{\mathscr{H}}_{\mathscr{M}'}$ units per unit time, and $\hat{\mathscr{D}}_{\mathscr{M}'}$ units per unit time, respectively, from the remanufacturer's production setup, deterioration and inventory holding activities. $\mathscr{T}\mathscr{E}_{\mathscr{M}'}$, the total carbon emissions per unit of time, is determined by

$$\mathscr{T}\mathscr{E}_{\mathscr{M}'} = \left[\begin{array}{l} \frac{\hat{\mathscr{S}}_{\mathscr{M}'}}{T} + \frac{\hat{\mathscr{H}}_{\mathscr{M}'}}{T}\left[\ \frac{\mathscr{P}_{\mathscr{R}}}{\theta}[t_{\mathscr{M}'} - t_{\mathscr{R}}] + \frac{\mathscr{Y}\mathscr{P}_{\mathscr{M}}}{\mathscr{P}_{\mathscr{R}}} e^{-\theta(t_{\mathscr{M}'} - t_{\mathscr{R}})}\ \right] \\ + \frac{\hat{\mathscr{D}}_{\mathscr{M}'}}{T}\left[\ \mathscr{P}_{\mathscr{R}} t_{\mathscr{M}'} - e^{-\theta t_{\mathscr{M}'}}\left[\ \frac{{\mathscr{P}_{\mathscr{R}}}^2 e^{\theta t_{\mathscr{M}'}} - \mathscr{Y}\mathscr{P}_{\mathscr{M}}\theta^2}{\theta \mathscr{P}_{\mathscr{R}}}\ \right]\ \right] \end{array}\right]. \tag{29}$$

5.2.2. Retailer's total cost and emissions function of remanufacturing process

According to Figure 5, the retailer receives $\mathscr{Q}_{\mathscr{M}'}$ items at the start of each cycle, with a fraction $\mathscr{Z}$ being of imperfect quality. The retailer separates the high-quality items from the lower-quality ones by performing a 100% screening at a pace of $\mathscr{X}$. The lower-quality items $\mathscr{Z}\mathscr{Q}_{\mathscr{R}'}$ are salvaged in a single batch at $t = \frac{\mathscr{I}_{\mathscr{R}'}(0)}{\mathscr{X}}$ at the conclusion of the screening procedure.

At the beginning of each cycle, which lasts $t_{\mathscr{R}'}$ time units, the retailer incurs a fixed order cost $\mathscr{O}_{\mathscr{R}'}$. The following formula is used to determine the ordering cost per unit of time:

$$\mathscr{O}\mathscr{C}_{\mathscr{R}'} = \frac{\mathscr{O}_{\mathscr{R}'}}{t_{\mathscr{R}'}} \tag{30}$$

The following restriction, derived from Lee and Kim (2014) [16], applies to the retailer's inventory system when the quantity of high-quality products received is sufficient to meet demand during the screening period:

$$(1-\mathscr{Z})\mathscr{I}_{\mathscr{R}'}(0) \geq \ \mathscr{D}(\epsilon)\frac{\mathscr{I}_{\mathscr{R}'}(0)}{\mathscr{X}}, \tag{31}$$

Assuming poorer quality items are excluded from fulfilling the demand rate $\mathscr{D}(\epsilon)$ for good quality goods, and considering inventory depletion due to demand $\mathscr{D}(\epsilon)$ and deterioration at rate θ, the retailer's inventory changes over time are governed by the following differential equation,

$$\frac{d\mathscr{I}_{\mathscr{R}'}(t)}{dt} + \theta\mathscr{I}_{\mathscr{R}'}(t) = -\left[\ (1-\mathscr{Z})\mathscr{D}(\epsilon) + \mathscr{Z}\mathscr{D}(\epsilon)\ \right], \quad 0 \leq t \leq t_{\mathscr{R}'}; \tag{32}$$

The two periods in the retailer's inventory system are the screening period [$t = 0$ to $t = \frac{\mathscr{I}_{\mathscr{R}'}(0)}{\mathscr{X}}$] and the and, according to Tiwari et al. (2018) [36], the non-screening period [$t = \frac{\mathscr{I}_{\mathscr{R}'}(0)}{\mathscr{X}}$ to $t = t_{\mathscr{R}'}$]. Inventory changes throughout these periods are described by the following differential equations with the boundary condition $\mathscr{I}_{\mathscr{R}}(t_{\mathscr{R}'}) = 0$.

$$\mathscr{I}_{\mathscr{R}'}(t) = \frac{\mathscr{D}(\epsilon)}{\theta}\left[\ e^{-\theta t} - 1\ \right], \qquad 0 \leq t \leq \frac{\mathscr{I}_{\mathscr{R}'}(0)}{\mathscr{X}}; \tag{33}$$

$$\mathscr{I}_{\mathscr{R}'}(t) = \frac{(1-\mathscr{Z})\mathscr{D}(\epsilon)}{\theta}\left[\ e^{\theta(t_{\mathscr{R}'}-t)} - 1\ \right], \qquad \frac{\mathscr{I}_{\mathscr{R}'}(0)}{\mathscr{X}} \leq t \leq t_{\mathscr{R}'}; \tag{34}$$

The buyer's purchase quantity is determined by

$$\mathscr{Q}_{\mathscr{R}'} = \mathscr{I}_{\mathscr{R}'}(0) = \frac{\mathscr{D}(\epsilon)}{\theta}\left[\ e^{\theta t_{\mathscr{R}'}} - 1\ \right]. \tag{35}$$

The retailer's holding cost per unit time $\mathscr{H}\mathscr{C}_{\mathscr{R}'}$ is determined by dividing the length of the replenishment cycle $t_{\mathscr{R}}$ by the average stock level per cycle and the yearly holding cost per unit $\mathscr{H}_{\mathscr{R}}$.

$$\mathscr{H}\mathscr{C}_{\mathscr{R}'} = \frac{\mathscr{H}_{\mathscr{R}'}}{t_{\mathscr{R}'}}\left[\ \int_0^{\frac{\mathscr{I}_{\mathscr{R}'}(0)}{\mathscr{X}}} \mathscr{I}_{\mathscr{R}'}(t)dt + \int_{\frac{\mathscr{I}_{\mathscr{R}'}(0)}{\mathscr{X}}}^{t_{\mathscr{R}'}} \mathscr{I}_{\mathscr{R}'}(t)dt\ \right],$$

$$\mathscr{HC}_{\mathscr{R}'} = \frac{\mathscr{H}_{\mathscr{R}'}}{t_{\mathscr{R}'}} \left[\frac{(1-\mathscr{Z})\mathscr{D}(\epsilon)t_{\mathscr{R}'}^2}{2\mathscr{X}^2}[\mathscr{X}^2 - (\mathscr{D}(\epsilon))^2] - \frac{(\mathscr{D}(\epsilon))^2 t_{\mathscr{R}'}}{2\theta\mathscr{X}^2}[\theta\mathscr{D}(\epsilon)t_{\mathscr{R}'} + 2\mathscr{X}] \right]. \tag{36}$$

The retailer's deteriorated inventory per cycle is the difference between the order quantity $\mathscr{Q}_{\mathscr{R}'}$ and the demand during the period $\mathscr{D}(\epsilon)t_{\mathscr{R}'}$. As per this quantity can be approximated as follows,

$$\mathscr{Q}_{\mathscr{R}'} - \mathscr{D}(\epsilon)t_{\mathscr{R}'} = \theta\mathscr{D}(\epsilon)t_{\mathscr{R}'}^2,$$

The retailer's deterioration cost per unit of time $\mathscr{DC}_{\mathscr{R}'}$ is calculated by considering the deteriorated inventory each cycle, the deterioration cost per unit $\mathscr{D}_{\mathscr{R}'}$ and the replenishing cycle time $t_{\mathscr{R}'}$.

$$\mathscr{DC}_{\mathscr{R}'} = \mathscr{D}_{\mathscr{R}'}\theta\mathscr{D}(\epsilon)t_{\mathscr{R}'}. \tag{37}$$

Since the retailer performs 100% screening of the entire lot $\mathscr{Q}_{\mathscr{R}'}$ at a cost of $\mathscr{Z}_{\mathscr{R}'}$ per unit, the retailer's screening cost per unit time $\mathscr{ZC}_{\mathscr{R}'}$ is calculated as follows,

$$\mathscr{ZC}_{\mathscr{R}'} = \frac{\mathscr{Z}_{\mathscr{R}'}\mathscr{Q}_{\mathscr{R}'}}{t_{\mathscr{R}'}} = \theta\mathscr{D}(\epsilon)\mathscr{Z}_{\mathscr{R}'}t_{\mathscr{R}'}. \tag{38}$$

$$\mathscr{DPC}_{\mathscr{R}'} = \theta\mathscr{Z}\mathscr{P}_{\mathscr{R}}. \tag{39}$$

The retailer's total cost function includes ordering, holding, deteriorating, screening, and disposal costs. Therefore, summing equations (28), (34), (35), (36) and (37) gives the retailer's total cost per unit time $\mathscr{TC}_{\mathscr{R}'}$ as follows,

$$\mathscr{TC}_{\mathscr{R}'} = \left[\begin{array}{c} \frac{\mathscr{H}_{\mathscr{R}'}}{t_{\mathscr{R}'}} \left[\frac{(1-\mathscr{Z})\mathscr{D}(\epsilon)t_{\mathscr{R}'}^2}{2\mathscr{X}^2}[\mathscr{X}^2 - (\mathscr{D}(\epsilon))^2] - \frac{(\mathscr{D}(\epsilon))^2 t_{\mathscr{R}'}}{2\theta\mathscr{X}^2}[\theta\mathscr{D}(\epsilon)t_{\mathscr{R}'} + 2\mathscr{X}] \right] \\ + \frac{\mathscr{O}_{\mathscr{R}'}}{t_{\mathscr{R}'}} + \mathscr{D}_{\mathscr{R}'}\theta\mathscr{D}(\epsilon)t_{\mathscr{R}'} + \theta\mathscr{D}(\epsilon)\mathscr{Z}_{\mathscr{R}'}t_{\mathscr{R}'} + \theta\mathscr{Z}\mathscr{P}_{\mathscr{R}} \end{array} \right]. \tag{40}$$

Carbon emissions are produced by the retail facility's inventory management operations, such as ordering, holding, and deterioration. These actions are specifically related to $\hat{\mathscr{O}}_{\mathscr{R}'}$. Each cycle's emissions, $\hat{\mathscr{H}}_{\mathscr{R}'}$ and $\hat{\mathscr{D}}_{\mathscr{R}'}$ units of emissions per time units, respectively. Consequently, $\mathscr{TE}_{\mathscr{R}'}$, the retailer's total emissions per units of time,

$$\mathscr{TE}_{\mathscr{R}'} = \left[\frac{\hat{\mathscr{H}}_{\mathscr{R}'}}{t_{\mathscr{R}'}} \left[\begin{array}{c} \frac{(1-\mathscr{Z})\mathscr{D}(\epsilon)t_{\mathscr{R}'}^2}{2\mathscr{X}^2}[\mathscr{X}^2 - (\mathscr{D}(\epsilon))^2] \\ - \frac{(\mathscr{D}(\epsilon))^2 t_{\mathscr{R}'}}{2\theta\mathscr{X}^2}[\theta\mathscr{D}(\epsilon)t_{\mathscr{R}'} + 2\mathscr{X}] \end{array} \right] + \frac{\hat{\mathscr{O}}_{\mathscr{R}'}}{t_{\mathscr{R}'}} + \hat{\mathscr{D}}_{\mathscr{R}'}\theta\mathscr{D}(\epsilon)t_{\mathscr{R}'} \right]. \tag{41}$$

$\mathscr{GP}(\epsilon)$ is the unit gross profit, which is achieved by subtracting the acquisition cost from the selling price. $\mathscr{D}(\epsilon)$ is the demand rate. $\mathscr{D}(\epsilon)\mathscr{GP}(\epsilon)$ less the sum of $\mathscr{TC}_{\mathscr{M}'}$ and $\mathscr{TC}_{\mathscr{R}'}$)

yields the average total supply chain profit. Therefore, $\mathscr{T}\mathscr{P}_{\mathscr{S}\mathscr{C}'}$, the total supply chain profit per unit time, is

$$\mathscr{T}\mathscr{P}_{\mathscr{S}\mathscr{C}'} = \begin{bmatrix} \mathscr{D}(\epsilon)\mathscr{G}\mathscr{P}(\epsilon) - \begin{bmatrix} \frac{\mathscr{H}_{\mathscr{R}'}}{t_{\mathscr{R}'}} \begin{bmatrix} \frac{(1-\mathscr{Z})\mathscr{D}(\epsilon)t^2_{\mathscr{R}'}}{2\mathscr{X}^2}[\mathscr{X}^2 - (\mathscr{D}(\epsilon))^2] \\ -\frac{(\mathscr{D}(\epsilon))^2 t_{\mathscr{R}'}}{2\theta\mathscr{X}^2}[\theta\mathscr{D}(\epsilon)t_{\mathscr{R}'} + 2\mathscr{X}] \end{bmatrix} \\ +\frac{\mathscr{O}_{\mathscr{R}'}}{t_{\mathscr{R}'}} + \mathscr{D}_{\mathscr{R}'}\theta\mathscr{D}(\epsilon)t_{\mathscr{R}'} \\ +\theta\mathscr{D}(\epsilon)\mathscr{Z}_{\mathscr{R}'}t_{\mathscr{R}'} + \theta\mathscr{Z}\mathscr{P}_{\mathscr{R}} \end{bmatrix} \\ - \begin{bmatrix} \frac{\mathscr{S}_{\mathscr{M}'}}{T} + \frac{\mathscr{H}_{\mathscr{M}'}}{T} \begin{bmatrix} \frac{\mathscr{P}_{\mathscr{R}}}{\theta}[t_{\mathscr{M}'} - t_{\mathscr{R}}] + \frac{\mathscr{Y}\mathscr{P}_{\mathscr{M}}}{\mathscr{P}_{\mathscr{R}}} e^{-\theta(t_{\mathscr{M}'} - t_{\mathscr{R}})} \end{bmatrix} \\ +\frac{\mathscr{D}_{\mathscr{M}'}}{T} \begin{bmatrix} \mathscr{P}_{\mathscr{R}}t_{\mathscr{M}'} - e^{-\theta t_{\mathscr{M}'}} \begin{bmatrix} \frac{\mathscr{P}_{\mathscr{R}}{}^2 e^{\theta t_{\mathscr{M}'}} - \mathscr{Y}\mathscr{P}_{\mathscr{M}}\theta^2}{\theta\mathscr{P}_{\mathscr{R}}} \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix}. \tag{42}$$

The total carbon emissions in the supply chain originate from both the retailer's and the manufacturer's resources. Consequently, the overall carbon emissions per unit time for the supply chain are calculated as follows:

$$\mathscr{T}\mathscr{E}_{\mathscr{S}\mathscr{C}'} = \begin{bmatrix} \begin{bmatrix} \frac{\hat{\mathscr{H}}_{\mathscr{R}'}}{t_{\mathscr{R}'}} \begin{bmatrix} \frac{(1-\mathscr{Z})\mathscr{D}(\epsilon)t^2_{\mathscr{R}'}}{2\mathscr{X}^2}[\mathscr{X}^2 - (\mathscr{D}(\epsilon))^2] \\ -\frac{(\mathscr{D}(\epsilon))^2 t_{\mathscr{R}'}}{2\theta\mathscr{X}^2}[\theta\mathscr{D}(\epsilon)t_{\mathscr{R}'} + 2\mathscr{X}] \end{bmatrix} + \frac{\hat{\mathscr{O}}_{\mathscr{R}'}}{t_{\mathscr{R}'}} + \hat{\mathscr{D}}_{\mathscr{R}'}\theta\mathscr{D}(\epsilon)t_{\mathscr{R}'} \end{bmatrix} \\ + \begin{bmatrix} \frac{\hat{\mathscr{S}}_{\mathscr{M}'}}{T} + \frac{\hat{\mathscr{H}}_{\mathscr{M}'}}{T} \begin{bmatrix} \frac{\mathscr{P}_{\mathscr{R}}}{\theta}[t_{\mathscr{M}'} - t_{\mathscr{R}}] + \frac{\mathscr{Y}\mathscr{P}_{\mathscr{M}}}{\mathscr{P}_{\mathscr{R}}} e^{-\theta(t_{\mathscr{M}'} - t_{\mathscr{R}})} \end{bmatrix} \\ +\frac{\hat{\mathscr{D}}_{\mathscr{M}'}}{T} \begin{bmatrix} \mathscr{P}_{\mathscr{R}}t_{\mathscr{M}'} - e^{-\theta t_{\mathscr{M}'}} \begin{bmatrix} \frac{\mathscr{P}_{\mathscr{R}}{}^2 e^{\theta t_{\mathscr{M}'}} - \mathscr{Y}\mathscr{P}_{\mathscr{M}}\theta^2}{\theta\mathscr{P}_{\mathscr{R}}} \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix}. \tag{43}$$

6. Carbon emissions policies and solution approaches

This section examines two carbon emissions policies: carbon tax and cap-and-trade regulations. Each strategy in the study has two mathematical formulations and additional analysis is done using various combinations of the unit profit and demand functions.

6.1. Mathematical models and solution methods for carbon emissions policies

Carbon tax and cap & trade emissions policies are commonly used by governments to motivate firms to reduce their carbon emissions. Both of these policies are used to assess the suggested supply chain model, and mathematical formulations are created for situations in which the unit profit and demand functions are both linear. Due to the complexity of these formulations, analytical solutions are not feasible, so heuristic algorithms are suggested as solution methods.

6.1.1. Carbon tax regulation

The supply chain must pay an extra tax under a carbon tax policy, which is determined by the quantity of carbon emissions generated. The monetary tax levied per unit of carbon emissions per unit of time is denoted by τ. The following is a mathematical formulation of the problem:

Total profit of manufacturing process:

$$
\mathscr{TP}_{\mathscr{SC}1}(t_{\mathscr{R}}, t_{\mathscr{M}}, T) = \left[\begin{array}{c} \mathscr{D}(t)\mathscr{GP}(t) \\ -\left[\begin{array}{c} \frac{\mathscr{S}_{\mathscr{M}}+\tau\hat{\mathscr{S}}_{\mathscr{M}}}{T} \\ +\left[\frac{\mathscr{P}_{\mathscr{M}}(\mathscr{H}_{\mathscr{M}}+\tau\hat{\mathscr{H}}_{\mathscr{M}})}{\theta^2 T}\left[\theta t_{\mathscr{M}} + e^{-\theta t_{\mathscr{M}}} - 1\right]\right] \\ +\left[\frac{(\mathscr{D}_{\mathscr{M}}+\tau\hat{\mathscr{D}}_{\mathscr{M}})}{T}\left[\mathscr{P}_{\mathscr{M}} t_{\mathscr{M}} - \frac{\mathscr{P}_{\mathscr{M}}}{\theta}[1-e^{-\theta t_{\mathscr{M}}}]\right]\right]\end{array}\right] \\ -\left[\begin{array}{c} \frac{(\mathscr{H}_{\mathscr{R}}+\tau\hat{\mathscr{H}}_{\mathscr{R}})}{t_{\mathscr{R}}}\left[\begin{array}{c} \frac{1}{2\theta\mathscr{X}^2}\left[\begin{array}{c}(-\alpha\beta\theta+2\beta)(\alpha^2\mathscr{P}^2t_{\mathscr{R}}^2 \\ +\beta^2 t_{\mathscr{R}}^4 + 2\alpha\beta\mathscr{P}t_{\mathscr{R}}^3)\end{array}\right] \\ +\frac{(1-\mathscr{Y})(\alpha\mathscr{P}+\beta t_{\mathscr{R}})}{2\mathscr{X}}\left[\begin{array}{c} 2t_{\mathscr{R}}(\mathscr{X}t_{\mathscr{R}} \\ -\alpha\mathscr{P}t_{\mathscr{R}} - \beta t_{\mathscr{R}}^2) \\ -\frac{1}{\mathscr{X}^2}(\mathscr{X}t_{\mathscr{R}} \\ -(\alpha\mathscr{P}t_{\mathscr{R}}+\beta t_{\mathscr{R}}^2))^2\end{array}\right]\end{array}\right] \\ +(\mathscr{D}_{\mathscr{R}}+\tau\hat{\mathscr{D}}_{\mathscr{R}})\beta(t_{\mathscr{R}}-t) + \mathscr{Z}_{\mathscr{R}}[\alpha\mathscr{P}+\beta t_{\mathscr{R}}] + \frac{(\mathscr{O}_{\mathscr{R}}+\tau\hat{\mathscr{O}}_{\mathscr{R}})}{t_{\mathscr{R}}}\end{array}\right]\end{array}\right]. \tag{44}
$$

Total profit of remanufacturing process:

$$
\mathscr{TP}_{\mathscr{SC}'2}(t_{\mathscr{R}'}, t_{\mathscr{M}'}, \epsilon) = \left[\begin{array}{c} \mathscr{D}(\epsilon)\mathscr{GP}(\epsilon) \\ -\left[\begin{array}{c} \frac{(\mathscr{H}_{\mathscr{R}'}+\tau\hat{\mathscr{H}}_{\mathscr{R}'})}{t_{\mathscr{R}'}}\left[\begin{array}{c}\frac{(1-\mathscr{Z})\mathscr{D}(\epsilon)t_{\mathscr{R}'}^2}{2\mathscr{X}^2}[\mathscr{X}^2-(\mathscr{D}(\epsilon))^2] \\ -\frac{(\mathscr{D}(\epsilon))^2 t_{\mathscr{R}'}}{2\theta\mathscr{X}^2}[\theta\mathscr{D}(\epsilon)t_{\mathscr{R}'}+2\mathscr{X}]\end{array}\right] \\ +\frac{(\mathscr{O}_{\mathscr{R}'}+\tau\hat{\mathscr{O}}_{\mathscr{R}'})}{t_{\mathscr{R}'}} + (\mathscr{D}_{\mathscr{R}'}+\tau\hat{\mathscr{D}}_{\mathscr{R}'})\theta\mathscr{D}(\epsilon)t_{\mathscr{R}'} \\ +\theta\mathscr{D}(\epsilon)\mathscr{Z}_{\mathscr{R}'}t_{\mathscr{R}'} + \theta\mathscr{Z}\mathscr{P}_{\mathscr{R}}\end{array}\right] \\ -\left[\begin{array}{c} \frac{(\mathscr{S}_{\mathscr{M}'}+\tau\hat{\mathscr{S}}_{\mathscr{M}'})}{T} \\ +\frac{(\mathscr{H}_{\mathscr{M}'}+\tau\hat{\mathscr{H}}_{\mathscr{M}'})}{T}\left[\begin{array}{c}\frac{\mathscr{P}_{\mathscr{R}}}{\theta}[t_{\mathscr{M}'}-t_{\mathscr{R}}] \\ +\frac{\mathscr{Y}\mathscr{P}_{\mathscr{M}}}{\mathscr{P}_{\mathscr{R}}}e^{-\theta(t_{\mathscr{M}'}-t_{\mathscr{R}})}\end{array}\right] \\ +\frac{(\mathscr{D}_{\mathscr{M}'}+\tau\hat{\mathscr{D}}_{\mathscr{M}'})}{T}\left[\begin{array}{c}\mathscr{P}_{\mathscr{R}}t_{\mathscr{M}'} \\ -e^{-\theta t_{\mathscr{M}'}}\left[\frac{\mathscr{P}_{\mathscr{R}}{}^2 e^{\theta t_{\mathscr{M}'}}-\mathscr{Y}\mathscr{P}_{\mathscr{M}}\theta^2}{\theta\mathscr{P}_{\mathscr{R}}}\right]\end{array}\right]\end{array}\right]\end{array}\right]. \tag{45}
$$

$\mathscr{D}(\epsilon) = \mathscr{D}_0 + \mathscr{A}\epsilon, \qquad \mathscr{GP}(\epsilon) = \mathscr{G}_0 + \mathscr{B}\epsilon.$

6.1.2. Carbon cap & trade regulation

In a carbon cap & trade regime, the supply chain is subject to a carbon emissions cap, and excess emissions credits can be bought and sold through an emissions trading market. The supply chain can sell excess credits at market value if emissions are below the cap. On the other hand, going over the cap requires buying more credits at market value, which adds to the expenses. Let δ be the market price for purchasing and disposing of emissions credits, and let ρ be the emissions cap per unit of time. The following is a mathematical formulation of the problem:

Total profit of manufacturing process:

$$\mathscr{TP}_{\mathscr{SC}3}(t_{\mathscr{R}},t_{\mathscr{M}},T)=\left[\begin{array}{c}\mathscr{D}(t)\mathscr{G}\mathscr{P}(t)+\rho\delta\\ -\left[\begin{array}{c}\frac{\mathscr{S}_{\mathscr{M}}+\delta\hat{\mathscr{S}}_{\mathscr{M}}}{T}\\ +\left[\frac{\mathscr{P}_{\mathscr{M}}(\mathscr{H}_{\mathscr{M}}+\delta\hat{\mathscr{H}}_{\mathscr{M}})}{\theta^2T}\left[\theta t_{\mathscr{M}}+e^{-\theta t_{\mathscr{M}}}-1\right]\right]\\ +\left[\frac{(\mathscr{D}_{\mathscr{M}}+\delta\hat{\mathscr{D}}_{\mathscr{M}})}{T}\left[\mathscr{P}_{\mathscr{M}}t_{\mathscr{M}}-\frac{\mathscr{P}_{\mathscr{M}}}{\theta}[1-e^{-\theta t_{\mathscr{M}}}]\right]\right]\end{array}\right]\\ -\left[\begin{array}{c}\frac{(\mathscr{H}_{\mathscr{R}}+\delta\hat{\mathscr{H}}_{\mathscr{R}})}{t_{\mathscr{R}}}\left[\begin{array}{c}\frac{1}{2\theta\mathscr{X}^2}\left[\begin{array}{c}(-\alpha\beta\theta+2\beta)(\alpha^2\mathscr{P}^2t_{\mathscr{R}}^2\\ +\beta^2t_{\mathscr{R}}^4+2\alpha\beta\mathscr{P}t_{\mathscr{R}}^3)\end{array}\right]\\ +\frac{(1-\mathscr{Y})(\alpha\mathscr{P}+\beta t_{\mathscr{R}})}{2\mathscr{X}}\left[\begin{array}{c}2t_{\mathscr{R}}(\mathscr{X}t_{\mathscr{R}}\\ -\alpha\mathscr{P}t_{\mathscr{R}}-\beta t_{\mathscr{R}}^2)\\ -\frac{1}{\mathscr{X}^2}(\mathscr{X}t_{\mathscr{R}}\\ -(\alpha\mathscr{P}t_{\mathscr{R}}+\beta t_{\mathscr{R}}^2))^2\end{array}\right]\end{array}\right]\\ +(\mathscr{D}_{\mathscr{R}}+\delta\hat{\mathscr{D}}_{\mathscr{R}})\beta(t_{\mathscr{R}}-T)+\mathscr{Z}_{\mathscr{R}}[\alpha\mathscr{P}+\beta t_{\mathscr{R}}]+\frac{(\mathscr{O}_{\mathscr{R}}+\delta\hat{\mathscr{O}}_{\mathscr{R}})}{t_{\mathscr{R}}}\end{array}\right]\end{array}\right]. \tag{46}$$

Total profit of remanufacturing process:

$$\mathscr{TP}_{\mathscr{SC}'4}(t_{\mathscr{R}'},t_{\mathscr{M}'},\epsilon)=\left[\begin{array}{c}\mathscr{D}(\epsilon)\mathscr{G}\mathscr{P}(\epsilon)+\rho\delta\\ -\left[\begin{array}{c}\frac{(\mathscr{H}_{\mathscr{R}'}+\delta\hat{\mathscr{H}}_{\mathscr{R}'})}{t_{\mathscr{R}'}}\left[\begin{array}{c}\frac{(1-\mathscr{Z})\mathscr{D}(\epsilon)t_{\mathscr{R}'}^2}{2\mathscr{X}^2}[\mathscr{X}^2-(\mathscr{D}(\epsilon))^2]\\ -\frac{(\mathscr{D}(\epsilon))^2t_{\mathscr{R}'}}{2\theta\mathscr{X}^2}[\theta\mathscr{D}(\epsilon)t_{\mathscr{R}'}+2\mathscr{X}]\end{array}\right]\\ +\frac{(\mathscr{O}_{\mathscr{R}'}+\delta\hat{\mathscr{O}}_{\mathscr{R}'})}{t_{\mathscr{R}'}}+(\mathscr{D}_{\mathscr{R}'}+\delta\hat{\mathscr{D}}_{\mathscr{R}'})\theta\mathscr{D}(\epsilon)t_{\mathscr{R}'}\\ +\theta\mathscr{D}(\epsilon)\mathscr{Z}_{\mathscr{R}'}t_{\mathscr{R}'}+\theta\mathscr{Z}\mathscr{P}_{\mathscr{R}}\end{array}\right]\\ -\left[\begin{array}{c}\frac{(\mathscr{S}_{\mathscr{M}'}+\delta\hat{\mathscr{S}}_{\mathscr{M}'})}{T}\\ +\frac{(\mathscr{H}_{\mathscr{M}'}+\delta\hat{\mathscr{H}}_{\mathscr{M}'})}{T}\left[\begin{array}{c}\frac{\mathscr{P}_{\mathscr{R}}}{\theta}[t_{\mathscr{M}'}-t_{\mathscr{R}}]\\ +\frac{\mathscr{Y}\mathscr{P}_{\mathscr{M}}}{\mathscr{P}_{\mathscr{R}}}e^{-\theta(t_{\mathscr{M}'}-t_{\mathscr{R}})}\end{array}\right]\\ +\frac{(\mathscr{D}_{\mathscr{M}'}+\delta\hat{\mathscr{D}}_{\mathscr{M}'})}{T}\left[\begin{array}{c}\mathscr{P}_{\mathscr{R}}t_{\mathscr{M}'}\\ -e^{-\theta t_{\mathscr{M}'}}\left[\frac{{\mathscr{P}_{\mathscr{R}}}^2e^{\theta t_{\mathscr{M}'}}-\mathscr{Y}\mathscr{P}_{\mathscr{M}}\theta^2}{\theta\mathscr{P}_{\mathscr{R}}}\right]\end{array}\right]\end{array}\right]\end{array}\right]. \tag{47}$$

$\mathscr{D}(\epsilon)=\mathscr{D}_0+\mathscr{A}\epsilon, \qquad \mathscr{G}\mathscr{P}(\epsilon)=\mathscr{G}_0+\mathscr{B}\epsilon.$

7. Solution Methodology

To determine the optimal values in the proposed two-echelon supply chain model, we employed classical optimization techniques suitable for nonlinear objective functions. These methods rely on iterative procedures that use gradient and Hessian information to navigate the solution space. Approaches such as the steepest descent and conjugate gradient methods follow the direction of the negative gradient to reach a local optimum, while Newton's method utilizes second-order derivatives via the Hessian matrix to accelerate convergence. Given the highly nonlinear nature of the objective function—which incorporates cost, deterioration, circularity and carbon emissions—numerical optimization was performed using a structured algorithm. Since classical methods generally converge to local optima, we evaluated the Hessian matrix to assess concavity. Where the Hessian is negative definite, we confirm the existence of a local maximum. To enhance confidence in the results, multiple initial values were used to explore the solution space and reduce the risk of convergence to suboptimal points. This approach ensures robustness in deriving economically and environmentally optimal decisions under the model's constraints.

7.1. Solution Algorithm

This section presents a solution algorithm designed to obtain the optimal results for the proposed study. Figure 7 provides a visual depiction of the computational steps involved in the model.

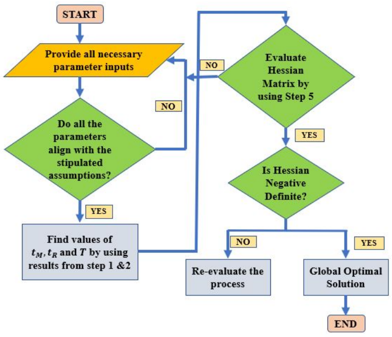

Figure 7: Algorithm of the proposed model

Step 1: Consider the following parameters values: $\mathscr{O}_{\mathscr{R}}, \hat{\mathscr{O}}_{\mathscr{R}}, \mathscr{H}_{\mathscr{R}}, \hat{\mathscr{H}}_{\mathscr{R}}$, $\mathscr{D}_{\mathscr{R}}, \hat{\mathscr{D}}_{\mathscr{R}}, \mathscr{S}_{\mathscr{M}}, \mathscr{S}_{\mathscr{M}}, \mathscr{H}_{\mathscr{M}}, \hat{\mathscr{H}}_{\mathscr{M}}, \mathscr{D}_{\mathscr{M}}, \mathscr{P}_{\mathscr{M}}, \tau, \rho$, and δ.

Step 2: From the equation (44), to find $\frac{\partial TP}{\partial t_{\mathscr{R}}}, \frac{\partial TP}{\partial t_{\mathscr{M}}}$ and $\frac{\partial TP}{\partial T}$. Detailed solutions are given.

Step 3: To determine the values of $t_{\mathscr{R}}$, $t_{\mathscr{M}}$, and T using Step 2, utilize the values from Step 1.

Step 4: Substituting, $t_{\mathscr{R}}$, $t_{\mathscr{M}}$ and T in equations (44) and you will get the values of total profit.

Step 5: Evaluate distinct prominent minor of the Hessian matrix,

$$\begin{bmatrix} \frac{\partial^2(TP)}{\partial t_{\mathscr{M}}^2} & \frac{\partial^2(TP)}{\partial t_{\mathscr{M}} \partial t_{\mathscr{R}}} & \frac{\partial^2(TP)}{\partial t_{\mathscr{M}} \partial T} \\ \frac{\partial^2(TP)}{\partial t_{\mathscr{M}} \partial t_{\mathscr{R}}} & \frac{\partial^2(TP)}{\partial t_{\mathscr{R}}^2} & \frac{\partial^2(TP)}{\partial t_{\mathscr{R}} \partial T} \\ \frac{\partial^2(TP)}{\partial t_{\mathscr{M}} \partial T} & \frac{\partial^2(TP)}{\partial t_{\mathscr{R}} \partial T} & \frac{\partial^2(TP)}{\partial T^2} \end{bmatrix}$$

at the point $t_{\mathscr{M}}$, $t_{\mathscr{R}}$ and T.

Step 6: This provides the global optimal solution if the hessian matrix is negative definite at the points ($t_{\mathscr{M}}$, $t_{\mathscr{R}}$, and T).

Step 7: Determine TP ($t_{\mathscr{M}}$, $t_{\mathscr{R}}$ and T), the system's optimal profit.

8. Numerical Analysis

This section provides numerical examples to demonstrate the applicability and validate the findings of the study in real-world scenarios. The effectiveness of green technology investment, outsourcing, deterioration, circularity index and flexibility in the manufacturing and remanufacturing process are discussed using numerical data. The models at best fit of Rabta et al. (2020) [1], Dey et al. (2022) [7] and Sebatjane et al. (2024) [30] are used to determine the parametric values for various parameters. MATLAB R2024b was used on a Windows 10 PC with 16 GB of RAM and 128 GB of SSD to determine the best values for the choice criteria.

The manufacturing and remanufacturing process (with and without circularity index) are the two types of models we looked at because the deterioration rate is random. We determined the overall profit for each model by applying the profit formula found in the Sebatjane [28]. The best results with green technology investment are shown in the following sections, which also examine a number of unusual circumstances.

8.1. Numerical Assessment for Manufacturing and Remanufacturing Process

In this portion, we explore the best outcomes for this investigation concerning price & time dependent and circularity index demand in manufacturing & remanufacturing process. The parameter values are associated with the retailer's fixed screening cost ($\mathscr{Z}_{\mathscr{R}}$&$\mathscr{Z}'_{\mathscr{R}}$)= 0.2 $\$/unit$, retailer's fixed ordering cost ($\mathscr{O}_{\mathscr{R}}$&$\mathscr{O}'_{\mathscr{R}}$)= 50 ($\$/unit$), retailer's fixed deteriorating cost ($\mathscr{D}_{\mathscr{R}}$&$\mathscr{D}'_{\mathscr{R}}$)= 0.75 ($\$/unit/yr$), retailer's fixed holding cost ($\mathscr{H}_{\mathscr{R}}$&$\mathscr{H}'_{\mathscr{R}}$)= 0.075 ($\$/unit/yr$), retailer's carbon emissions as a result of the expense of ordering merchandise ($\hat{\mathscr{O}}_{\mathscr{R}}$&$\hat{\mathscr{O}}_{\mathscr{R}'}$)= 15 ($lb$ of Co_2), retailer's carbon emissions as a result of the expense of keeping inventory ($\hat{\mathscr{H}}_{\mathscr{R}}$&$\hat{\mathscr{H}}_{\mathscr{R}'}$)= 0.0481 ($lb$ of $Co_2/unit/yr$), retailer's carbon emissions as a result of declining inventory prices ($\hat{\mathscr{D}}_{\mathscr{R}}$&$\hat{\mathscr{D}}_{\mathscr{R}'}$)= 0.15 ($lb$ of $Co_2/unit/yr$), manufacturer's fixed setup cost ($\mathscr{S}_{\mathscr{M}}$&$\mathscr{S}'_{\mathscr{M}}$)= 2,500 ($\$/unit$), manufacturer's fixed holding cost ($\mathscr{H}_{\mathscr{M}}$&$\mathscr{H}'_{\mathscr{M}}$)= 0.05 ($\$/unit/yr$), manufacturer's fixed deteriorating cost ($\mathscr{D}_{\mathscr{M}}$&$\mathscr{D}'_{\mathscr{M}}$)= 0.375 ($\$/unit/yr$),Carbon emissions from the producer as a result of inventory setup expenses ($\hat{\mathscr{S}}_{\mathscr{M}}$&$\hat{\mathscr{S}}_{\mathscr{M}'}$)= 500 ($lb$ of Co_2), manufacturer's carbon emissions as a result of the expense of keeping inventory ($\hat{\mathscr{H}}_{\mathscr{M}}$&$\hat{\mathscr{H}}_{\mathscr{M}'}$)= 0.01875 ($lb$ of Co_2), manufacturer's carbon emission due to inventory deteriorating cost ($\hat{\mathscr{D}}_{\mathscr{M}}$&$\hat{\mathscr{D}}_{\mathscr{M}'}$)= 0.0927 ($lb$ of Co_2), fixed deterioration rate (θ)= 0.25, fraction of poorer items in manufacturing process ($\mathscr{Y}$)= 0.5, fixed screening rate ($\mathscr{X}$)= 1,75,000 ($\$/units/yr$), production rate in manufacturing period ($\mathscr{P}_{\mathscr{M}}$)= 1,23,000 ($units/yr$), production rate in remanufacturing period ($\mathscr{P}_{\mathscr{R}}$)= 1,00,000 ($units/yr$), demand rate($D_0$)= 65,200 ($units/yr$), gross profit ($g_o$)= 2($\$/unit$), demand rate including circularity index ($\mathscr{A}$)= 9,800 ($units/yr$), gross profit including circularity index ($\mathscr{B}$)= 0.145 ($\$/unit$), carbon emissions tax (τ)= 1.25 ($\$$ $/lb$ of Co_2), carbon emissions cap (ρ)= 8000 (lb of Co_2 $/yr$), market price of carbon emissions (δ)= 2.5 ($\$$ $/lb$ of Co_2).

The principal minors of the Hessian matrix at the optimal decision variable values are as follows: $| H_{11} |= -3.8235 < 0$, $| H_{22} |= 1.1673 > 0$ and $| H_{33} |= -2.0131 < 0$. When all primary minors have alternating signs, the generated profit reaches its global maximum, indicating that the revenue function is negative definite at the optimal decision variable values.

Table 3, presents the optimal results of the production system under two different carbon emission policies: carbon tax and carbon cap-and-trade, based on the specified parameter values. The table evaluates the impact of these policies on key decision variables and total profit in both manufacturing and remanufacturing processes. Given the influence of carbon regulations on production efficiency, we determine the optimal values for cycle times and total profit under each policy. The results indicate that the cap-and-trade policy leads to higher profitability in both processes compared to a carbon tax.

Table 3, shows that under the carbon cap & trade policy, the total profit reaches $\$92,393$ per cycle for the manufacturing process and $\$99,058$ per cycle for the remanufacturing process. In comparison, under the carbon tax policy, the total profit is slightly lower, at $\$90,271$ and $\$97,692$, respectively. This suggests that a cap & trade system provides more economic benefits by allowing flexibility in managing emissions. The optimal cycle times for manufacturing and remanufacturing are also affected by the policy choice. In the manufacturing process, the cycle time ($t_{\mathscr{M}}$) is slightly higher under cap & trade (0.1392) than under a carbon tax (0.1304), while the remanufacturing cycle time ($t_{\mathscr{R}}$) is lower under cap & trade (1.2931 vs. 1.5498 under tax). Similarly, for the remanufacturing process, the manufacturing cycle time is 0.1405 under cap & trade and 0.1472 under carbon tax, while the remanufacturing cycle time remains slightly lower under cap & trade (1.2169 vs. 1.2324). Overall, the results indicate that cap-and-trade enhances profitability by 2.35% in manufacturing and 1.40% in remanufacturing compared to a carbon tax. Furthermore, the total cycle time (T) remains relatively stable across policies, ensuring that production efficiency is not significantly disrupted.

9. Sensitivity Analysis

In this part, a sensitivity analysis of the parameters is reviewed. Through individually modifying each parameter within a range of 50%, while maintaining the other parameters constant, we conduct a comprehensive sensitivity analysis.

9.1. *Sensitivity Analysis for Case 1:*

Tables 4 and 5 present the results of the sensitivity analysis in manufacturing process.

9.1.1. *Sensitivity Analysis for Case 1:*

Tables 6, 7 and 8 present the results of the sensitivity analysis in remanufacturing process.

9.2. *Sensitivity Analysis for Case 2:*

Tables 9, 10 and 11 present the results of the sensitivity analysis in manufacturing process.

9.2.1. *Sensitivity Analysis for Case 2:*

Tables 12, 13 and 14 present the results of the sensitivity analysis in remanufacturing process.

9.3. Sensitivity analysis of demand related parameters:

Demand plays a critical role in both manufacturing and remanufacturing systems, significantly impacting overall supply chain profitability. This study examines the influence of demand on total profit, highlighting the importance of strategic decision-making for industry managers. By effectively managing demand through optimal pricing strategies, managers can enhance revenue and profitability. This study considers two distinct types of demand: (i) price- and time-dependent demand, which is associated with the manufacturing process and (ii) circularity index-based demand, which pertains to the remanufacturing process. Demand varies across different parameters, ranging from minimum to maximum values, except for $\mathscr{D}_0$ and $\mathscr{G}_0$. The findings indicate that an increase in demand for these parameters significantly enhances revenue generation. Sensitivity analysis of demand-related parameters reveals that emission-related parameters contribute to a profit increase of approximately 50% to 69%, whereas other parameters result in a profit increase of approximately 25% to 45%. These scaling parameters, particularly those associated with demand-driven remanufacturing, exhibit high sensitivity. Figures 8 to 12 provide a visual representation of total profit and its correlation with demand-related parameters.

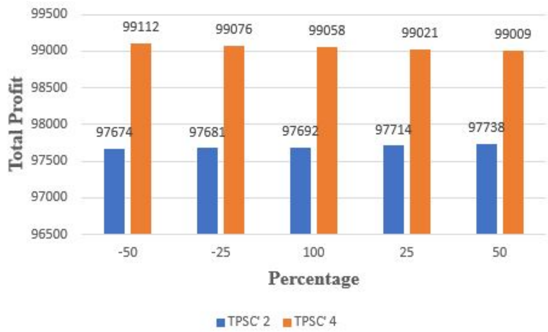

Figure 8: Comparison of the overall profit in parameters $\mathscr{D}_0$

9.4. Sensitivity analysis of cost related parameters:

The cost function is essential for optimizing manufacturing and remanufacturing processes by minimizing expenses and maximizing profitability. Key cost components include

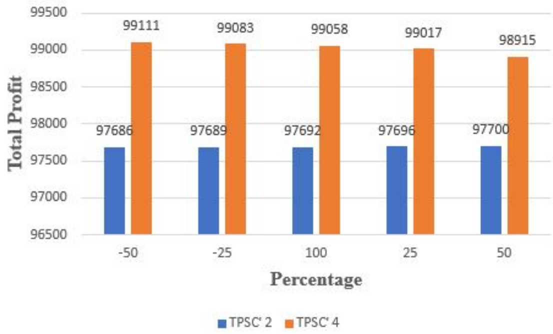

Figure 9: Comparison of the overall profit in parameters $\mathscr{G}_0$

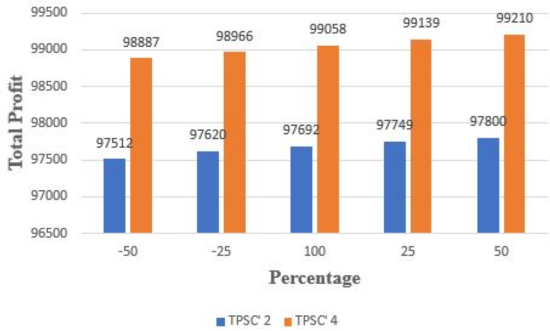

Figure 10: Comparison of the overall profit in parameters $\mathscr{A}$

ordering cost, setup cost, holding cost, production cost, screening cost, and deteriorating cost, all of which influence overall operational efficiency. In manufacturing, it helps manage raw material procurement, labor, and energy consumption, while in remanufacturing, it assesses refurbishment, recycling, and reverse logistics costs. An efficient cost function aids in resource allocation, waste reduction, and pricing strategies, enhancing supply chain performance. Moreover, it supports sustainable practices by balancing economic and environmental objectives in industrial systems. The total profit of the remanufacturing process varies between 97,337 and 99,990, influenced by fluctuations in the parameters $\mathscr{O}_{\mathscr{R}}$, $\hat{\mathscr{O}}_{\mathscr{R}}$, $\hat{\mathscr{H}}_{\mathscr{R}}$, $\mathscr{H}_{\mathscr{M}}$ and $\hat{\mathscr{H}}_{\mathscr{M}}$ within a range of -50% to +50%. Additionally, the setup

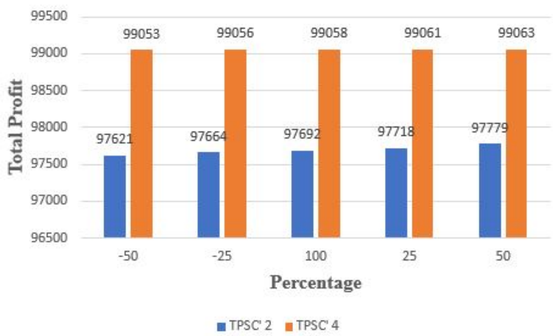

Figure 11: Comparison of the overall profit in parameters $\mathscr{B}$

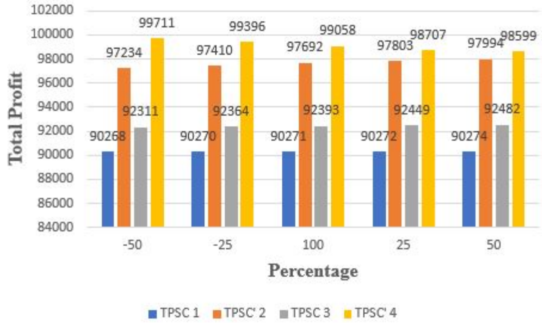

Figure 12: Comparison of the overall profit in parameters θ

cost parameters $\mathscr{S}_{\mathscr{M}}$ and $\hat{\mathscr{S}}_{\mathscr{M}}$ exhibit an increasing trend from top to bottom. Figures 13 to 24 provide a visual representation of total profit variations in response to changes in cost parameters.

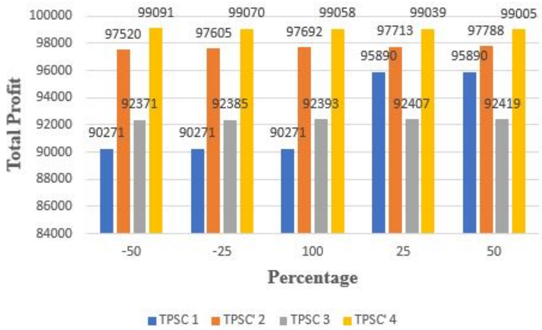

Figure 13: Comparison of the overall profit in parameters $\hat{\mathscr{D}}_{\mathscr{M}}$

9.5. Sensitivity analysis of emissions related parameters:

In this study, two environmental policies carbon tax and carbon cap & trade are considered to assess their impact on manufacturing and remanufacturing systems. The carbon tax directly penalizes emissions, encouraging firms to adopt cleaner technologies and optimize resource utilization. Meanwhile, the cap-and-trade system sets an emission limit while allowing firms to trade allowances, promoting cost-effective emissions reduction. Both policies play a crucial role in enhancing sustainability, reducing environmental impact, and improving long-term profitability by balancing economic and ecological objectives in industrial operations. The parameters ρ and δ exhibit the highest recorded values among all considered parameters, with values of 99,199 and 99,671, respectively. Similarly, the carbon tax parameter τ demonstrates a high value of 97,797. Figures 25 to 29 provide a visual representation of total profit variations corresponding to these parameters.

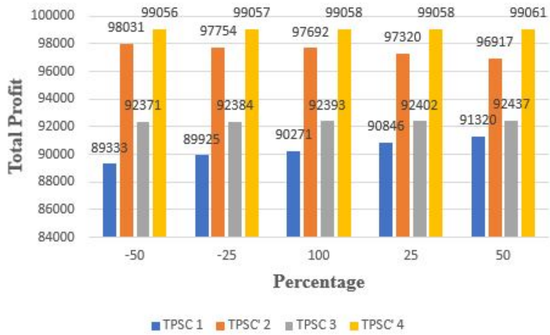

Figure 14: Comparison of the overall profit in parameters $\mathscr{D}_{\mathscr{M}}$

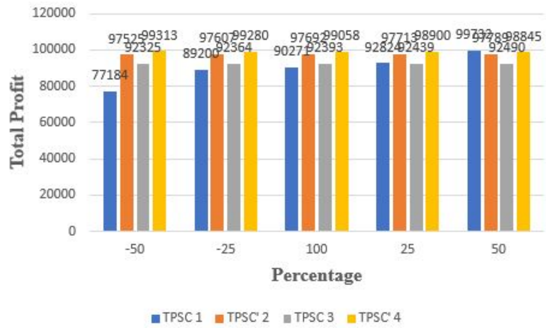

Figure 15: Comparison of the overall profit in parameters $\hat{\mathscr{D}}_{\mathscr{R}}$

10. Managerial Insights

This study makes meaningful theoretical and practical contributions, providing important guidance for businesses and policymakers. The findings underscore the significance of managing inventory efficiently while addressing carbon emissions, item deterioration, and fluctuating demand patterns.

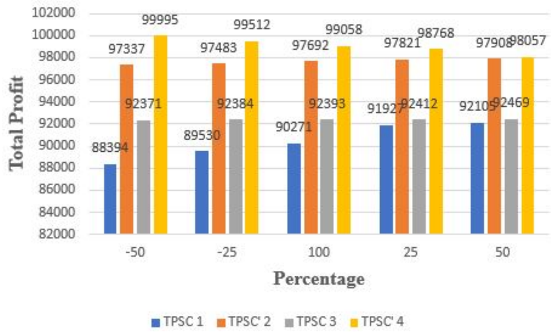

Figure 16: Comparison of the overall profit in parameters $\mathscr{D}_{\mathscr{R}}$

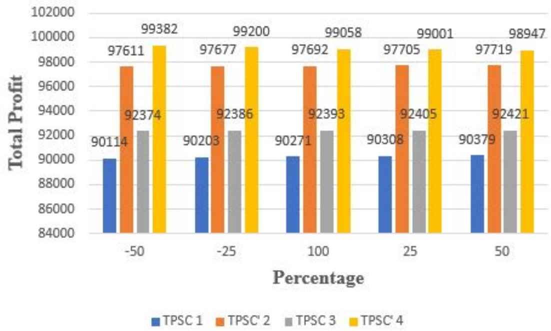

Figure 17: Comparison of the overall profit in parameters $\hat{\mathscr{H}}_{\mathscr{M}}$

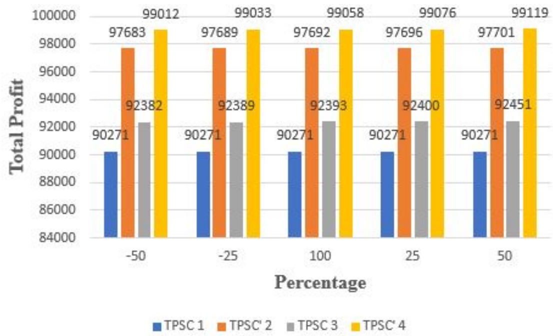

Figure 18: Comparison of the overall profit in parameters $\mathscr{H}_{\mathscr{M}}$

10.1. Theoretical Implementation

The findings of this study provide valuable implications for supply chain managers aiming to balance economic performance with environmental compliance. Coordinated decision-making between the manufacturer and retailer, as modeled in the two-echelon framework, leads to more efficient inventory and remanufacturing strategies under carbon regulations. The model demonstrates that cap-and-trade policies offer greater flexibility and slightly higher profitability than carbon taxation, suggesting that firms can benefit from strategic participation in carbon credit markets. Furthermore, the use of a circularity

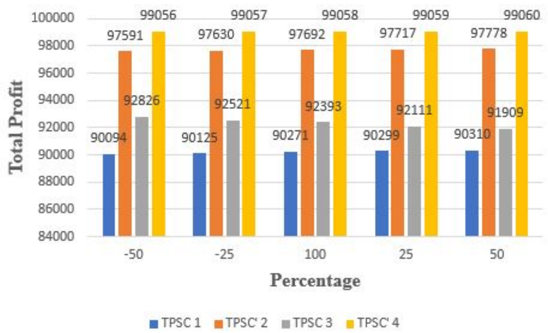

Figure 19: Comparison of the overall profit in parameters $\mathscr{H}_{\mathscr{R}}$

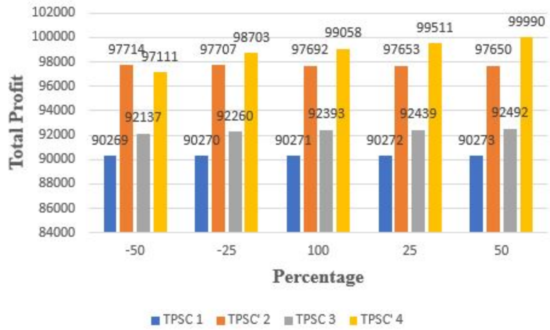

Figure 20: Comparison of the overall profit in parameters $\hat{\mathscr{O}}_{\mathscr{R}}$

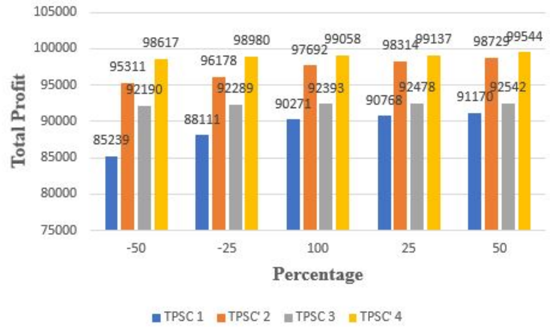

Figure 21: Comparison of the overall profit in parameters $\mathscr{O}_{\mathscr{R}}$

index as a demand driver emphasizes the operational and market advantages of promoting product circularity, particularly in remanufacturing contexts. Effective quality inspection at the retail level supports value recovery from imperfect items, aligning with circular economy goals. The model also highlights the importance of minimizing deterioration losses through optimized cycle times, which is especially relevant for semi-durable goods. Finally, the integration of analytical tools such as MATLAB enables managers to simulate real-world scenarios and optimize decisions across cost, environmental, and service objectives. These insights are particularly useful for small and mid-sized enterprises that

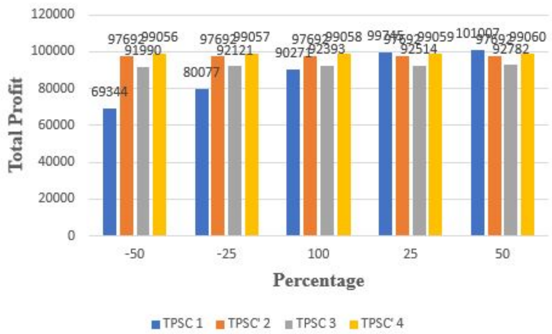

Figure 22: Comparison of the overall profit in parameters $\hat{\mathscr{S}}_{\mathscr{M}}$

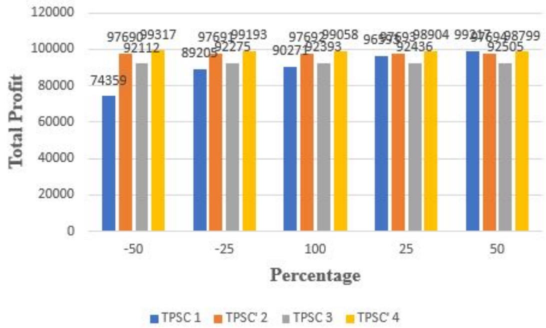

Figure 23: Comparison of the overall profit in parameters $\mathscr{S}_{\mathscr{M}}$

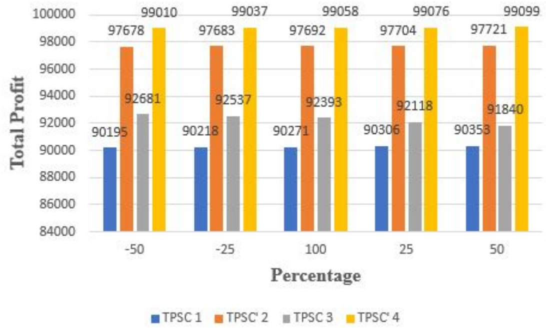

Figure 24: Comparison of the overall profit in parameters $\mathscr{X}_{\mathscr{R}}$

operate in simplified supply chain structures yet seek to improve sustainability outcomes.

10.2. Practical Implementation

The proposed two-echelon sustainable supply chain model has strong practical relevance across several industrial sectors where product deterioration, remanufacturing potential, and environmental constraints intersect. The dual-demand framework and integration of carbon policies provide a strategic decision-making tool for real-world operations. Key application domains include:

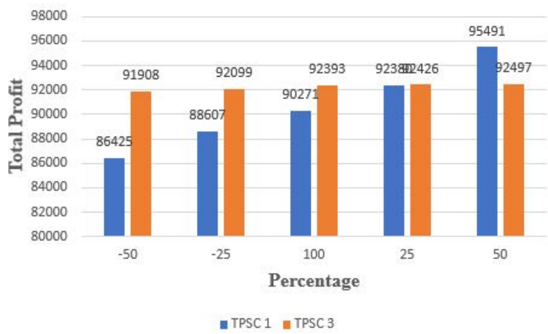

Figure 25: Comparison of the overall profit in parameters $\mathscr{P}_{\mathscr{M}}$

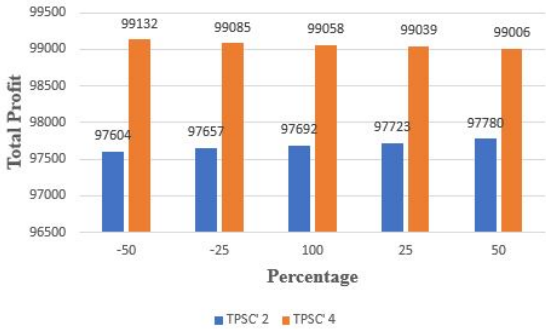

Figure 26: Comparison of the overall profit in parameters $\mathscr{P}_{\mathscr{R}}$

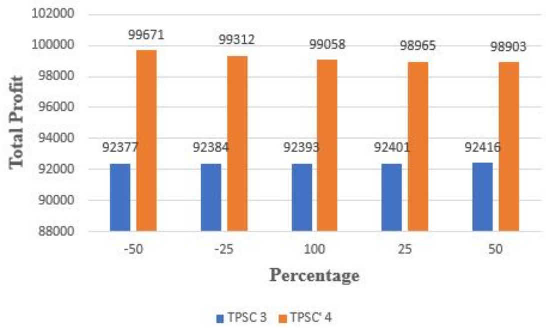

Figure 27: Comparison of the overall profit in parameters ρ

(i) **Electronics and Consumer Appliances:** Manufacturers of high-value consumer goods—such as smartphones, laptops, and televisions—routinely manage defective units through repair, refurbishing, or resale channels. Remanufacturing practices in this sector align with circular economy goals and are influenced by extended producer responsibility (EPR) policies. The model aids in determining optimal production and remanufacturing cycles by considering deterioration of unsold items, emission costs, and profitability. Notable examples include companies like Apple and Dell, which have established certified refurbishing programs under sustainability commitments.

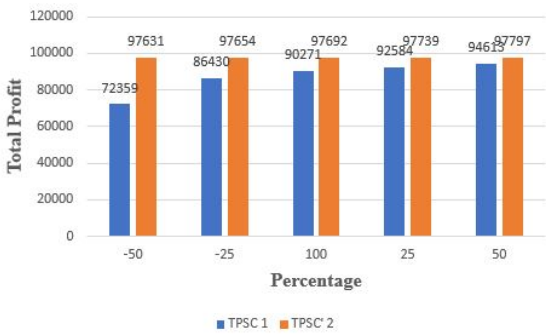

Figure 28: Comparison of the overall profit in parameters τ

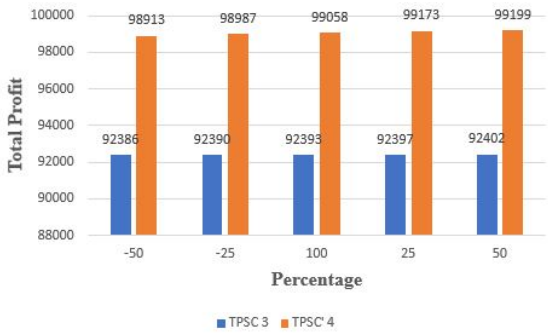

Figure 29: Comparison of the overall profit in parameters δ

(ii) **Automotive Components and Heavy Machinery:** Remanufacturing is well-established in automotive supply chains, particularly for engines, gearboxes, alternators, and turbochargers. Organizations such as Bosch, Caterpillar, and Cummins manage large-scale remanufacturing operations that reduce raw material consumption and emissions. The proposed model is suitable for coordinating replenishment and remanufacturing decisions for deteriorating parts, optimizing profit under emissions caps or taxes. The circularity index can quantify the reused material content or lifecycle extension of remanufactured components.

(iii) **Medical Equipment and Diagnostic Devices:** Hospitals and manufacturers frequently remanufacture or refurbish medical imaging equipment (e.g., MRI, CT scanners) and surgical tools due to their high cost and regulatory lifespan. The model is particularly applicable in optimizing inventory management for returned or used medical devices. It supports decisions based on equipment deterioration, environmental compliance, and cost-effectiveness, in accordance with healthcare regulatory frameworks (e.g., FDA or EU MDR guidelines).

(iv) **Battery Recycling and Energy Storage Systems:** In the context of electric vehicles (EVs) and renewable energy, remanufacturing and recycling of lithium-ion batteries have become critical. Companies like Redwood Materials and Tesla engage in battery recovery to extract valuable materials such as lithium, cobalt, and nickel. The proposed model facilitates optimization of reverse logistics, refurbishment cycles, and emission controls in remanufacturing processes, with circularity index reflecting the proportion of recovered energy materials.

(v) **Industrial Equipment and Construction Tools :** Industries involved in mining, construction, or manufacturing often refurbish tools and machinery to extend service life and reduce replacement costs. The model can be applied to plan maintenance and remanufacturing cycles for deteriorating mechanical parts under fluctuating demand and carbon regulations. Companies like Komatsu and Volvo Construction Equipment employ similar practices under sustainability initiatives.

10.3. Policy & Industry Recommendations

This study offers key recommendations for promoting sustainable production and circular economy practices. Policymakers are encouraged to enhance cap-and-trade systems and include circularity metrics in reporting standards. Industry practitioners should align operations with environmental regulations, improve quality inspections, and manage deterioration through forecasting tools. Using decision-support systems like MATLAB can further support data-driven, low-carbon supply chain strategies that balance profitability with sustainability.

11. Conclusion

This paper presents a comprehensive two-echelon supply chain model that integrates sustainable inventory and remanufacturing decisions under carbon regulation. Unlike traditional models that focus solely on production or inventory, this framework emphasizes the interaction between the manufacturer and retailer, optimizing operations based on deteriorating items, imperfect quality, and environmentally influenced demand. Circularity index demand introduces a novel dimension in remanufacturing, reflecting real-world shifts toward circular economy practices. Our findings indicate that cap-and-trade policies are slightly more effective in balancing profitability and environmental goals compared to carbon tax. The sensitivity analysis highlights the importance of circularity-based demand parameters and carbon cost variations. This model is best suited for semi-durable goods, such as electronics or medical devices, rather than highly perishable items. Future extensions may include stochastic elements, traceability technologies and multiple product types to further enhance realism and applicability.

Future studies can extend this work by incorporating uncertainty in demand, carbon credit pricing, and return rates, reflecting more realistic supply chain environments. The model may be expanded to include stochastic deterioration, multiple product types, and capacity constraints in remanufacturing. Additionally, integrating blockchain or IoT-based traceability mechanisms could enhance circularity tracking and compliance in closed-loop systems. Exploring decentralized decision-making frameworks and contractual coordination between supply chain tiers may also provide deeper insights into collaborative sustainability strategies. Finally, empirical validation using industry-specific data would further strengthen the practical applicability of the model across diverse sectors.

While the proposed model offers valuable insights into sustainable inventory and remanufacturing planning within a two-echelon supply chain, certain limitations must be acknowledged. The model assumes deterministic parameters, including demand rates, deterioration and return quantities, which may not fully capture real-world uncertainties. Additionally, the system considers perfect quality screening and instantaneous remanufacturing, whereas practical operations may involve delays and classification errors. The current model also excludes capacity constraints, lead times and stochastic emissions, which can influence supply chain performance in practice. Addressing these limitations in future studies by incorporating uncertainty modeling, stochastic demand, dynamic carbon pricing and real-time remanufacturing logistics would enhance the model's robustness and applicability.

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References

[1] B. Rabta. Optimizing circular economy indicators in inventory systems: a multi-objective optimization approach. *Sustainability*, 14(3):1395, 2022.

[2] B. Rabta. An economic order quantity inventory model for a product with a circular economy indicator. *Computers & Industrial Engineering*, 140:106215, 2020.

[3] F. Harris. How many parts to make at once. *Factory, The Magazine of Management*, 10:135–136, 1913.

[4] S. K. Goyal and B. C. Giri. Recent trends in modeling of deteriorating inventory. *European Journal of Operational Research*, 134(1):1–16, 2001.

[5] P. L. Abad. Optimal pricing and lot-sizing under conditions of perishability and partial backordering. *Management Science*, 42(8):1093–1104, 1996.

[6] B. Sarkar and B. Ganguly. EPQ with imperfect production and environmental regulations. *Mathematical and Computer Modelling*, 170:55–73, 2020.

[7] B. K. Dey, J. Park, and H. Seok. Carbon emission and waste reduction of a manufacturing-remanufacturing system using green technology and automated inspection. *RAIRO - Operations Research*, 56(4):2801–2831, 2022.

[8] S. Sindhuja and P. Arathi. An inventory model for deteriorating products under preservation technology with time-dependent quality demand. *Cogent Engineering*, 10(1):2166068, 2023.

[9] Y. Kazancoglu, I. Kazancoglu, and M. Sagnak. A new holistic conceptual framework for the implementation of the circular economy in closed-loop supply chains. *International Journal of Production Research*, 59(22):6765–6787, 2021.

[10] S. Singh and H. Kaur. Integrating circular economy principles into the sustainable supply chain management: evidence from indian dairy industry. *International Journal of Business and Globalisation*, 31(3):363–381, 2022.

[11] E. P. John and U. Mishra. Sustainable circular economy production system with emission control in LED bulb companies. *Environmental Science and Pollution Research*, 30(22):59963–59990, 2023. 2023a.

[12] M. A. A. Khan, L. E. Cárdenas-Barrón, G. Treviño-Garza, and A. Céspedes-Mota. Optimal circular economy index policy in a production system with carbon emissions. *Expert Systems with Applications*, 212:118684, 2023.

[13] N. A. Wani and U. Mishra. An integrated circular economic model with controllable carbon emission and deterioration from an apple orchard. *Journal of Cleaner Production*, 374:133945, 2022.

[14] A. Thomas and U. Mishra. A sustainable circular economic supply chain system with waste minimization using 3D printing and emissions reduction in plastic reforming industry. *Journal of Cleaner Production*, 345:131128, 2022.

[15] E. P. John and U. Mishra. A sustainable three-layer circular economic model with controllable waste, emission and wastewater from the textile and fashion industry. *Journal of Cleaner Production*, 388:135965, 2023. 2023b.

[16] E. L. Porteus. Optimal lot sizing, process quality improvement and setup cost reduction. *Operations Research*, 34(1):137–144, 1986.

[17] M. K. Salameh and M. Y. Jaber. Economic production quantity model for items with imperfect quality. *International Journal of Production Economics*, 64(1-3):59–64, 2000.

[18] S. K. Goyal and L. E. Cárdenas-Barrón. Note on: economic production quantity model for items with imperfect quality—a practical approach. *International Journal of Production Economics*, 77(1):85–87, 2002.

[19] H. C. Chang. An application of fuzzy sets theory to the EOQ model with imperfect quality items. *Computers & Operations Research*, 31(12):2079–2092, 2004.

[20] H. M. Wee, J. Yu, and M. C. Chen. Optimal inventory model for items with imperfect quality and shortage backordering. *Omega*, 35(1):7–11, 2007.

[21] M. Khan, M. Y. Jaber, and M. Bonney. An economic order quantity (EOQ) for items with imperfect quality and inspection errors. *International Journal of Production Economics*, 133(1):113–118, 2011.

[22] S. S. Sana. A production-inventory model of imperfect quality products in a three-layer supply chain. *Decision Support Systems*, 50(2):539–547, 2011.

[23] S. A. R. Khan and D. Qianli. Impact of green supply chain management practices on firms' performance: an empirical study from the perspective of pakistan. *Environmental Science and Pollution Research*, 24(20):16829–16844, 2017.

[24] S. S. Sana and K. S. Chaudhuri. EPQ model under carbon emissions regulations. *Journal of Industrial and Management Optimization*, 18(1):211–230, 2022.

[25] P. Gautam, S. Maheshwari, and C. K. Jaggi. Sustainable production inventory model with greening degree and dual determinants of defective items. *Journal of Cleaner Production*, 367:132962, 2022.

[26] S. Ruidas, M. R. Seikh, and P. K. Nayak. A production inventory model for high-tech products involving two production runs and a product variation. *Journal of Industrial and Management Optimization*, 19(3):2178–2205, 2023. 2023b.

[27] S. Tiwari, L. E. Cárdenas-Barrón, A. I. Malik, and C. K. Jaggi. Retailer's credit and inventory decisions for imperfect quality and deteriorating items under two-level trade credit. *Computers & Operations Research*, 138:105617, 2022.

[28] M. Sebatjane and O. Adetunji. Economic order quantity model for growing items with imperfect quality. *Operations Research Perspectives*, 6:100131, 2019.

[29] A. Gharaei, M. Karimi, and S. A. H. Shekarabi. An integrated multi-product, multi-

buyer supply chain under penalty, green, and quality control policies and a vendor managed inventory with consignment stock agreement: the outer approximation with equality relaxation and augmented penalty algorithm. *Applied Mathematical Modelling*, 69:223–254, 2019.

[30] M. Sebatjane and O. Adetunji. A four-echelon supply chain inventory model for growing items with imperfect quality and errors in quality inspection. *Annals of Operations Research*, 335(1):327–359, 2024.

[31] S. Benjaafar, Y. Li, and M. Daskin. Carbon footprint and the management of supply chains: insights from simple models. *IEEE Transactions on Automation Science and Engineering*, 10(1):99–116, 2013.

[32] K. Govindan and M. Hasanagic. A systematic review on drivers, barriers, and practices towards circular economy: a supply chain perspective. *International Journal of Production Research*, 56(1-2):278–311, 2018.

[33] J. Chen, X. Li, and Y. Wang. Green EPQ model with carbon tax and shortage backlogging. *Journal of Cleaner Production*, 257:120451, 2020.

[34] J. Ding, W. Chen, and W. Wang. Production and carbon emission reduction decisions for remanufacturing firms under carbon tax and take-back legislation. *Computers & Industrial Engineering*, 143:106419, 2020.

[35] C. Yu, Z. Qu, T. W. Archibald, and Z. Luan. An inventory model of a deteriorating product considering carbon emissions. *Computers & Industrial Engineering*, 148:106694, 2020.

[36] I. D. Wangsa, I. Vanany, and N. Siswanto. An optimization model for fresh-food electronic commerce supply chain with carbon emissions and food waste. *Journal of Industrial and Production Engineering*, 40(1):1–21, 2023.

[37] M. R. Andi Purnomo, I. D. Wangsa, N. Rizky, W. A. Jauhari, and I. Zahria. A multi-echelon fish closed-loop supply chain network problem with carbon emission and traceability. *Expert Systems with Applications*, 210:118316, 2022.

[38] I. Vanany, I. D. Wangsa, and N. A. Jeremi. A multi-objective mixed integer linear model for sustainable dairy supply chain with food waste and environmental pollutants. *Process Integration and Optimization for Sustainability*, 8(3):723–740, 2024.

[39] B. K. Dey, A. Datta, and B. Sarkar. Effectiveness of carbon policies and multi-period delay in payments in a global supply chain under remanufacturing consideration. *Journal of Cleaner Production*, 402:136761, 2023.

[40] R. Karim and K. Nakade. A literature review on the sustainable EPQ model, focusing on carbon emissions and product recycling. *Logistics*, 6(3):55, 2022.

[41] S. S. Sana. A structural mathematical model on two echelon supply chain system. *Annals of Operations Research*, 315(2):1997–2025, 2022.

[42] S. S. Sana. The effects of greenhouse gas costs on optimal pricing and production lot size in an imperfect production system. *RAIRO - Operations Research*, 57(4):2209–2230, 2023.

[43] C. K. Sivashankari, S. Sathishkumar, S. S. Sana, and V. Choudri. Sustainable production for imperfect production system with advertisement and Bertrand's price-dependent demand. *Central European Journal of Operations Research*, 32(3):827–857,

2024.

[44] A. Barman, R. Das, P. K. De, and S. S. Sana. Pricing policies for a competitive dual-channel supply chain with green investment and sales effort under revenue-sharing contract. *RAIRO - Operations Research*, 58(5):3985–4012, 2024.

[45] R. Suvetha, K. Rangarajan, and P. Rajadurai. Recent trends of economic production quantity models (2000–2022): a literature review. *International Journal of Mathematics in Operational Research*, 20(4):377–409, 2021.

[46] K. Salas-Navarro, J. Acevedo-Chedid, W. F. Flórez, H. Ospina-Mateus, S. S. Sana, and L. E. Cárdenas-Barrón. An EPQ inventory model considering an imperfect production system with probabilistic demand and collaborative approach. *Journal of Advances in Management Research*, 17(2):282–304, 2020.

[47] K. Salas-Navarro, J. Acevedo-Chedid, W. F. Flórez, H. Ospina-Mateus, L. E. Cárdenas-Barrón, and S. S. Sana. A collaborative EPQ inventory model for a three-echelon supply chain with multiple products considering the effect of marketing effort on demand. *Journal of Industrial and Management Optimization*, 16(4):1613–1633, 2020.

[48] A. A. Shaikh, L. E. Cárdenas-Barrón, and S. Tiwari. Closed-form solutions for the EPQ-based inventory model for exponentially deteriorating items under retailer partial trade credit policy in supply chain. *International Journal of Applied and Computational Mathematics*, 4(1):24, 2018.

Appendix

Table 1: Notations of the proposed model.

Input Parameters	
θ	Deterioration rate of items
$\mathscr{Y}$ & $\mathscr{Z}$	The proportion of inferior products in the retailer's order amount while it is being manufactured and remanufactured
$\mathscr{X}$	Screening rate at the retailer (units / yr)
$\mathscr{Z}_{\mathscr{R}}$ & $\mathscr{Z}_{\mathscr{R}'}$	Retailer's fixed screening cost in manufacturing & re-manufacturing period ($ / unit)
$\mathscr{O}_{\mathscr{R}}$ & $\mathscr{O}_{\mathscr{R}'}$	Retailer's fixed ordering cost in manufacturing & re-manufacturing period ($)
$\mathscr{D}_{\mathscr{R}}$ & $\mathscr{D}_{\mathscr{R}'}$	Retailer's fixed deteriorating cost in manufacturing & re-manufacturing period ($ / unit)
$\mathscr{H}_{\mathscr{R}}$ & $\mathscr{H}_{\mathscr{R}'}$	Retailer's fixed holding cost in manufacturing & re-manufacturing period ($ / unit)
$\mathscr{TC}_{\mathscr{R}}$ & $\mathscr{TC}_{\mathscr{R}'}$	Retailer's total cost in manufacturing & re-manufacturing period ($ / unit)
$\mathscr{TE}_{\mathscr{R}}$ & $\mathscr{TE}_{\mathscr{R}'}$	Retailer's total carbon emission cost in manufacturing & re-manufacturing period ($ / unit)
$\hat{\mathscr{O}}_{\mathscr{R}}$ & $\hat{\mathscr{O}}_{\mathscr{R}'}$	Carbon emissions from inventory ordering by retailers (lb of CO_2)
$\hat{\mathscr{D}}_{\mathscr{R}}$ & $\hat{\mathscr{D}}_{\mathscr{R}'}$	Carbon emissions from the retailer's declining inventory (lb of CO_2 /units / yr)
$\hat{\mathscr{H}}_{\mathscr{R}}$ & $\hat{\mathscr{H}}_{\mathscr{R}'}$	Carbon emissions of retailers inventory holdings (lb of CO_2 /units / yr)
$\mathscr{P}_{\mathscr{M}}$ & $\mathscr{P}_{\mathscr{R}}$	Production rate in Manufacturing and Re-manufacturing period (units / yr)
$\mathscr{S}_{\mathscr{M}}$ & $\mathscr{S}_{\mathscr{M}'}$	Manufacturer's fixed setup cost in manufacturing & re-manufacturing period ($)
$\mathscr{D}_{\mathscr{M}}$ & $\mathscr{D}_{\mathscr{M}'}$	Manufacturer's fixed deteriorating cost in manufacturing & re-manufacturing period ($ / unit)
$\mathscr{H}_{\mathscr{M}}$ & $\mathscr{H}_{\mathscr{M}'}$	Manufacturer's fixed holding cost in manufacturing & re-manufacturing period ($ / unit)
$\mathscr{TC}_{\mathscr{M}}$ & $\mathscr{TC}_{\mathscr{M}'}$	Manufacturer's total cost in manufacturing & re-manufacturing period ($ / unit)
$\mathscr{TE}_{\mathscr{M}}$ & $\mathscr{TE}_{\mathscr{M}'}$	Manufacturer's total carbon emission cost in manufacturing & re-manufacturing period ($ / unit)
$\hat{\mathscr{S}}_{\mathscr{M}}$ & $\hat{\mathscr{S}}_{\mathscr{M}'}$	Carbon emissions from the manufacturer's production setup (lb of CO_2)
$\hat{\mathscr{D}}_{\mathscr{M}}$ & $\hat{\mathscr{D}}_{\mathscr{M}'}$	Carbon emissions from the manufacturer's declining inventory (lb of CO_2 /units / yr)
$\hat{\mathscr{H}}_{\mathscr{M}}$ & $\hat{\mathscr{H}}_{\mathscr{M}'}$	Manufacturer's carbon emission from inventory holding (lb of CO_2 /units / yr)

Table 2: Notations of the proposed model.

Input Parameters	
$\mathscr{D}_0$	Rate of base demand without implementing the circularity index(units / yr)
$\mathscr{G}_0$	Gross profit per base unit less the circularity index factor ($ / unit)
$\mathscr{A}$	Demand function sensitivity to the circularity index (units / yr)
$\mathscr{B}$	Unit gross profit function sensitivity to the circularity index ($ /unit)
ρ	Carbon emissions cap (lb of CO_2 / yr)
τ	Carbon tax on emissions ($/ lb of CO_2)
δ	Market price for carbon emissions credits ($ / lb of CO_2)
Explicit Decision Variables	
$t_{\mathscr{R}}\&t_{\mathscr{R}'}$	Retailer's cycle time in manufacturing and re-manufacturing period (yr)
$t_{\mathscr{M}}\&t_{\mathscr{M}'}$	Manufacturer's cycle time in manufacturing and re-manufacturing period (yr)
ξ	Circularity index ranging from 0 to 1
Implicit Decision Variables	
T	Inventory cycle time
$\mathscr{Q}_{\mathscr{R}}\&\mathscr{Q}'_{\mathscr{R}}$	Retailer's order quantity in manufacturing and re-manufacturing period (units)
$\mathscr{Q}_{\mathscr{M}}\&\mathscr{Q}'_{\mathscr{M}}$	Deliveries made by the manufacturer to the retailer during the production and remanufacturing phases. (units)
Functions	
$\mathscr{I}_{\mathscr{R}}(t)\&\mathscr{I}'_{\mathscr{R}}(t)$	Inventory levels of retailers in manufacturing and remanufacturing at any given time t (units)
$\mathscr{G}\mathscr{P}(\xi)$	Gross profit per unit as a function of circularity ($ /unit)
$\mathscr{I}_{\mathscr{M}}(t)\&\mathscr{I}'_{\mathscr{M}}(t)$	Inventory levels of manufacturers in production and remanufacturing processes at any given time t (units)
$\mathscr{D}(\xi)$	Demand function for remanufactured items; defined as $\mathscr{D}_0 + \mathscr{A}\xi$ (units / yr)
$\mathscr{T}\mathscr{P}_{\mathscr{S}\mathscr{C}}$	Total profit of the supply chain ($ / yr)
$\mathscr{T}\mathscr{E}_{\mathscr{S}\mathscr{C}}$	Total carbon emissions from the supply chain (lb of CO_2/ yr)
Abbreviations	
AGI	Annual Green technology Investment
AHC	Annual Holding Cost
TP	Total Profit
APC	Annual Purchasing Cost
TCE	Total Carbon Emission
ASC	Annual Setup Cost
NAR	Net Annual Revenue

Table 3: Optimum results under carbon tax and carbon cap and trade policy.

	Manufacturing Process		Remanufacturing Process	
Decision Variables	Carbon tax	Carbon cap & trade	Carbon tax	Carbon cap & trade
$t_{\mathscr{M}}$	0.1304	0.1392	0.1472	0.1405
$t_{\mathscr{R}}$	1.5498	1.2931	1.2324	1.2169
T	2.0283	2.0157	2.0217	2.0043
Total Profit ($\$/cycle$)	90,271	92,393	97,692	99,058

Table 4: Manufacturing process concerning the parameters $\mathscr{O}_{\mathscr{R}}, \hat{\mathscr{O}}_{\mathscr{R}}, \mathscr{H}_{\mathscr{R}}, \hat{\mathscr{H}}_{\mathscr{R}}, \mathscr{D}_{\mathscr{R}}, \hat{\mathscr{D}}_{\mathscr{R}}, \mathscr{S}_{\mathscr{M}}, \hat{\mathscr{S}}_{\mathscr{M}}$ and $\mathscr{H}_{\mathscr{M}}$ under carbon tax policy.

Parameter	Percentage	$t_{\mathscr{M}}$	$t_{\mathscr{R}}$	T	$\mathscr{T}\mathscr{P}_{\mathscr{S}\mathscr{C}1}$
5* $\mathscr{O}_{\mathscr{R}}$	-50%	0.1127	1.4302	2.1996	85,239
	-25%	0.1291	1.4999	2.1001	88,111
	100%	0.1304	1.5498	2.0283	90,271
	+25%	0.1405	1.6027	2.0037	90,768
	+50%	0.1493	1.6831	1.9380	91,170
5* $\hat{\mathscr{O}}_{\mathscr{R}}$	-50%	0.1302	1.5496	2.0281	90,269
	-25%	0.1303	1.5497	2.0282	90,270
	100%	0.1304	1.5498	2.0283	90,271
	+25%	0.1305	1.5499	2.0284	90,272
	+50%	0.1306	1.5500	2.0285	90,273
5* $\mathscr{H}_{\mathscr{R}}$	-50%	0.1304	1.2317	1.9371	90,094
	-25%	0.1304	1.4052	2.0035	90,125
	100%	0.1304	1.5498	2.0283	90,271
	+25%	0.1304	1.5931	2.0394	90,299
	+50%	0.1304	1.6021	2.0501	90,310
5* $\hat{\mathscr{H}}_{\mathscr{R}}$	-50%	0.0972	1.5488	2.0281	76,313
	-25%	0.1089	1.5493	2.0282	82,106
	100%	0.1304	1.5498	2.0283	90,271
	+25%	0.1315	1.5504	2.0284	91,534
	+50%	0.1377	1.5511	2.0285	92,079
5* $\mathscr{D}_{\mathscr{R}}$	-50%	0.2947	1.7832	2.1988	88,394
	-25%	0.2039	1.6549	2.1220	89,530
	100%	0.1304	1.5498	2.0283	90,271
	+25%	0.1211	1.4311	2.0030	91,927
	+50%	0.1021	1.3927	1.9900	92,105
5* $\hat{\mathscr{D}}_{\mathscr{R}}$	-50%	0.1101	1.3271	2.9314	77,184
	-25%	0.1203	1.4750	2.5177	89,200
	100%	0.1304	1.5498	2.0283	90,271
	+25%	0.1409	1.6111	1.9117	92,824
	+50%	0.1507	1.7004	1.2349	99,732
5* $\mathscr{S}_{\mathscr{M}}$	-50%	0.0751	1.5494	2.0095	74,359
	-25%	0.0922	1.5496	2.0139	89,205
	100%	0.1304	1.5498	2.0283	90,271
	+25%	0.1718	1.5500	2.0314	96,593
	+50%	0.2349	1.5502	2.0406	99,217
5* $\hat{\mathscr{S}}_{\mathscr{M}}$	-50%	0.1302	0.7632	1.0937	69,344
	-25%	0.1303	0.9315	1.8754	80,077
	100%	0.1304	1.5498	2.0283	90,271
	+25%	0.1305	1.5901	2.0976	99,745
	+50%	0.1306	1.6537	2.1085	1,01,007
5* $\mathscr{H}_{\mathscr{M}}$	-50%	0.1304	1.5498	2.0283	90,271
	-25%	0.1304	1.5498	2.0283	90,271
	100%	0.1304	1.5498	2.0283	90,271
	+25%	0.1304	1.5498	2.0283	90,271
	+50%	0.1304	1.5498	2.0283	90,271

Table 5: Manufacturing process concerning the parameters $\hat{\mathscr{H}}_{\mathscr{M}}, \mathscr{D}_{\mathscr{M}}, \hat{\mathscr{D}}_{\mathscr{M}}, \theta, \mathscr{Z}_{\mathscr{R}}, Y, X, \mathscr{P}_{\mathscr{M}}$ and τ under carbon tax policy.

Parameter	Percentage	$t_{\mathscr{M}}$	$t_{\mathscr{R}}$	T	$\mathscr{T}\mathscr{P}_{\mathscr{S}\mathscr{C}1}$
5* $\hat{\mathscr{H}}_{\mathscr{M}}$	-50%	0.1291	1.3287	1.9317	90,114
	-25%	0.1297	1.4155	1.9924	90,203
	100%	0.1304	1.5498	2.0283	90,271
	+25%	0.1307	1.6022	2.0954	90,308
	+50%	0.1312	1.6384	2.1036	90,379
5* $\mathscr{D}_{\mathscr{M}}$	-50%	0.1283	1.4322	2.0134	89,333
	-25%	0.1291	1.4905	2.0199	89,925
	100%	0.1304	1.5498	2.0283	90,271
	+25%	0.1311	1.5807	2.0307	90,846
	+50%	0.1357	1.5994	2.0358	91,320
5* $\hat{\mathscr{D}}_{\mathscr{M}}$	-50%	0.1304	1.5498	2.0283	90,271
	-25%	0.1304	1.5498	2.0283	90,271
	100%	0.1304	1.5498	2.0283	90,271
	+25%	0.1307	1.5531	2.1377	95,890
	+50%	0.1307	1.5531	2.1377	95,890
5* θ	-50%	0.1304	1.5498	2.0281	90,268
	-25%	0.1304	1.5498	2.0282	90,270
	100%	0.1304	1.5498	2.0283	90,271
	+25%	0.1304	1.5498	2.0284	90,272
	+50%	0.1304	1.5498	2.0285	90,274
5* $\mathscr{Z}_{\mathscr{R}}$	-50%	0.1452	1.5271	2.0179	90,195
	-25%	0.1375	1.5335	2.0211	90,218
	100%	0.1304	1.5498	2.0283	90,271
	+25%	0.1286	1.5584	2.0314	90,306
	+50%	0.1208	1.5679	2.0396	90,353
5* Y	-50%	0.1304	1.5498	2.0283	90,271
	-25%	0.1304	1.5498	2.0283	90,271
	100%	0.1304	1.5498	2.0283	90,271
	+25%	0.1304	1.5498	2.0283	90,271
	+50%	0.1304	1.5498	2.0283	90,271
5* X	-50%	0.1302	1.5496	2.0281	90,269
	-25%	0.1303	1.5497	2.0282	90,270
	100%	0.1304	1.5498	2.0283	90,271
	+25%	0.1305	1.5499	2.0284	90,272
	+50%	0.1306	1.5500	2.0285	90,273
5* $\mathscr{P}_{\mathscr{M}}$	-50%	0.1304	1.5431	2.0032	86,425
	-25%	0.1304	1.5477	2.0157	88,607
	100%	0.1304	1.5498	2.0283	90,271
	+25%	0.1306	1.5539	2.0311	92,380
	+50%	0.1306	1.5590	2.0399	95,491
5* τ	-50%	0.1175	1.7901	2.1189	81,394
	-25%	0.1293	1.6324	2.0547	86,736
	100%	0.1304	1.5498	2.0283	90,271
	+25%	0.1389	1.4730	2.0025	93,115
	+50%	0.1472	1.3956	1.9621	99,218

Table 6: Remanufacturing process concerning the parameters $\mathscr{O}_{\mathscr{R}'}, \hat{\mathscr{O}}_{\mathscr{R}'}, \mathscr{H}_{\mathscr{R}'}, \hat{\mathscr{H}}_{\mathscr{R}'}, \mathscr{D}_{\mathscr{R}'}, \hat{\mathscr{D}}_{\mathscr{R}'}, \mathscr{S}_{\mathscr{M}'}, \hat{\mathscr{S}}_{\mathscr{M}'}$ and $\mathscr{H}_{\mathscr{M}'}$ under carbon tax policy.

Parameter	Percentage	$t_{\mathscr{M}'}$	$t_{\mathscr{R}'}$	T	$\mathscr{T}\mathscr{P}_{\mathscr{S}\mathscr{C}'2}$
5* $\mathscr{O}_{\mathscr{R}'}$	-50%	0.1230	1.2322	2.0193	95,311
	-25%	0.1317	1.2323	2.0205	96,178
	100%	0.1472	1.2324	2.0217	97,692
	+25%	0.1511	1.2325	2.0224	98,314
	+50%	0.1590	1.2326	2.0236	98,729
5* $\hat{\mathscr{O}}_{\mathscr{R}'}$	-50%	0.1734	1.2321	2.0535	97,714
	-25%	0.1502	1.2323	2.0411	97,707
	100%	0.1472	1.2324	2.0217	97,692
	+25%	0.1236	1.2325	2.0010	97,653
	+50%	0.1098	1.2325	1.9832	97,650
5* $\mathscr{H}_{\mathscr{R}'}$	-50%	0.1221	1.2301	2.0204	97,591
	-25%	0.1305	1.2319	2.0211	97,630
	100%	0.1472	1.2324	2.0217	97,692
	+25%	0.1509	1.2328	2.0223	97,717
	+50%	0.1637	1.2335	2.0229	97,778
5* $\hat{\mathscr{H}}_{\mathscr{R}'}$	-50%	0.1531	1.2917	2.0534	98,337
	-25%	0.1509	1.2697	2.0319	97,920
	100%	0.1472	1.2324	2.0217	97,692
	+25%	0.1458	1.2016	2.0092	97,219
	+50%	0.1420	1.1995	1.9971	97,034
5* $\mathscr{D}_{\mathscr{R}'}$	-50%	0.1294	1.1936	2.0138	97,337
	-25%	0.1313	1.2172	2.0190	97,483
	100%	0.1472	1.2324	2.0217	97,692
	+25%	0.1509	1.2506	2.0254	97,821
	+50%	0.1581	1.2781	2.0279	97,908
5* $\hat{\mathscr{D}}_{\mathscr{R}'}$	-50%	0.1301	1.2101	2.0134	97,525
	-25%	0.1396	1.2297	2.0182	97,607
	100%	0.1472	1.2324	2.0217	97,692
	+25%	0.1499	1.2358	2.0256	97,713
	+50%	0.1584	1.2373	2.0290	97,789
5* $\mathscr{S}_{\mathscr{M}'}$	-50%	0.1470	1.2320	2.0215	97,690
	-25%	0.1471	1.2322	2.0216	97,691
	100%	0.1472	1.2324	2.0217	97,692
	+25%	0.1473	1.2326	2.0218	97,693
	+50%	0.1474	1.2328	2.0219	97,694
5* $\hat{\mathscr{S}}_{\mathscr{M}'}$	-50%	0.1472	1.2324	2.0217	97,692
	-25%	0.1472	1.2324	2.0217	97,692
	100%	0.1472	1.2324	2.0217	97,692
	+25%	0.1472	1.2324	2.0217	97,692
	+50%	0.1472	1.2324	2.0217	97,692
5* $\mathscr{H}_{\mathscr{M}'}$	-50%	0.1593	1.1938	2.0211	97,683
	-25%	0.1511	1.2102	2.0214	97,689
	100%	0.1472	1.2324	2.0217	97,692
	+25%	0.1439	1.2575	2.0221	97,696
	+50%	0.1397	1.2781	2.0225	97,701

Table 7: Remanufacturing process concerning the parameters $\hat{\mathscr{H}}_{\mathscr{M}'}, \mathscr{D}_{\mathscr{M}'}, \hat{\mathscr{D}}_{\mathscr{M}'}, \theta, \mathscr{Z}_{\mathscr{R}'}, \mathscr{P}_{\mathscr{R}}, \mathscr{D}_0, \mathscr{A}$ and g_0 under carbon tax policy.

Parameter	Percentage	$t_{\mathscr{M}'}$	$t_{\mathscr{R}'}$	T	$\mathscr{T}\mathscr{P}_{\mathscr{S}\mathscr{C}'2}$
5* $\hat{\mathscr{H}}_{\mathscr{M}'}$	-50%	0.1298	1.2311	2.0183	97,611
	-25%	0.1354	1.2319	2.0201	97,677
	100%	0.1472	1.2324	2.0217	97,692
	+25%	0.1581	1.2332	2.0265	97,705
	+50%	0.1606	1.2338	2.0290	97,719
5* $\mathscr{D}_{\mathscr{M}'}$	-50%	0.1938	1.3711	2.0591	98,031
	-25%	0.1765	1.3097	2.0428	97,754
	100%	0.1472	1.2324	2.0217	97,692
	+25%	0.1281	1.2009	2.0053	97,320
	+50%	0.1054	1.1976	1.1926	96,917
5* $\hat{\mathscr{D}}_{\mathscr{M}'}$	-50%	0.1359	1.1983	2.0191	97,520
	-25%	0.1413	1.2052	2.0209	97,605
	100%	0.1472	1.2324	2.0217	97,692
	+25%	0.1508	1.2461	2.0228	97,713
	+50%	0.1544	1.2599	2.0236	97,788
5* θ	-50%	0.1470	1.2320	1.7281	97,234
	-25%	0.1471	1.2322	1.9901	97,410
	100%	0.1472	1.2324	2.0217	97,692
	+25%	0.1473	1.2326	2.0493	97,803
	+50%	0.1474	1.2328	2.0668	97,994
5* $\mathscr{Z}_{\mathscr{R}'}$	-50%	0.1423	1.2322	2.0203	97,678
	-25%	0.1456	1.2323	2.0211	97,683
	100%	0.1472	1.2324	2.0217	97,692
	+25%	0.1497	1.2325	2.0224	97,704
	+50%	0.1505	1.2326	2.0238	97,721
5* $\mathscr{P}_{\mathscr{R}}$	-50%	0.1334	1.2322	2.0180	97,604
	-25%	0.1390	1.2323	2.0204	97,657
	100%	0.1472	1.2324	2.0217	97,692
	+25%	0.1499	1.2325	2.0249	97,723
	+50%	0.1523	1.2326	2.0271	97,780
5* $\mathscr{D}_0$	-50%	0.1453	1.2310	2.0211	97,674
	-25%	0.1461	1.2317	2.0215	97,681
	100%	0.1472	1.2324	2.0217	97,692
	+25%	0.1485	1.2339	2.0220	97,714
	+50%	0.1496	1.2358	2.0224	97,738
5* $\mathscr{A}$	-50%	0.1359	1.2301	2.0215	97,512
	-25%	0.1423	1.2311	2.0216	97,620
	100%	0.1472	1.2324	2.0217	97,692
	+25%	0.1519	1.2347	2.0218	97,749
	+50%	0.1587	1.2356	2.0219	97,800
5* g_0	-50%	0.1468	1.2320	2.0211	97,686
	-25%	0.1470	1.2322	2.0214	97,689
	100%	0.1472	1.2324	2.0217	97,692
	+25%	0.1474	1.2326	2.0220	97,696
	+50%	0.1476	1.2328	2.0223	97,700

Table 8: Remanufacturing process concerning the parameters $\mathscr{B}$ and τ under carbon tax policy.

Parameter	Percentage	$t_{\mathscr{M}'}$	$t_{\mathscr{R}'}$	T	$\mathscr{T}\mathscr{P}_{\mathscr{S}\mathscr{C}'2}$
5* $\mathscr{B}$	-50%	0.1329	1.8921	2.0212	97,621
	-25%	0.1390	1.5512	2.0215	97,664
	100%	0.1472	1.2324	2.0217	97,692
	+25%	0.1501	1.0732	2.0220	97,718
	+50%	0.1584	0.9321	2.0226	97,779
5* τ	-50%	0.1465	1.2109	2.0198	97,631
	-25%	0.1468	1.2272	2.0209	97,654
	100%	0.1472	1.2324	2.0217	97,692
	+25%	0.1479	1.2390	2.0224	97,739
	+50%	0.1488	1.2416	2.0238	97,797

Table 9: Manufacturing process concerning the parameters $\mathscr{O}_{\mathscr{R}}, \hat{\mathscr{O}}_{\mathscr{R}}, \mathscr{H}_{\mathscr{R}}, \hat{\mathscr{H}}_{\mathscr{R}}, \mathscr{D}_{\mathscr{R}}, \hat{\mathscr{D}}_{\mathscr{R}}, \mathscr{S}_{\mathscr{M}}, \hat{\mathscr{S}}_{\mathscr{M}}$ and $\mathscr{H}_{\mathscr{M}}$ under carbon cap and trade policy.

Parameter	Percentage	$t_{\mathscr{M}}$	$t_{\mathscr{R}}$	T	$\mathscr{T}\mathscr{P}_{\mathscr{S}\mathscr{C}3}$
5* $\mathscr{O}_{\mathscr{R}}$	-50%	0.1390	1.2720	1.0908	92,190
	-25%	0.1391	1.2809	1.0999	92,289
	100%	0.1392	1.2931	2.0157	92,393
	+25%	0.1393	1.3019	2.0194	92,478
	+50%	0.1394	1.3679	2.0236	92,542
5* $\hat{\mathscr{O}}_{\mathscr{R}}$	-50%	0.1277	1.2291	2.0122	92,137
	-25%	0.1329	1.2635	2.0139	92,260
	100%	0.1392	1.2931	2.0157	92,393
	+25%	0.1401	1.3107	2.0171	92,439
	+50%	0.1473	1.3776	2.0190	92,492
5* $\mathscr{H}_{\mathscr{R}}$	-50%	0.1713	1.3509	2.1057	92,826
	-25%	0.1586	1.3182	2.0685	92,521
	100%	0.1392	1.2931	2.0157	92,393
	+25%	0.1100	1.2830	1.9801	92,111
	+50%	0.0935	1.2497	1.8054	91,909
5* $\hat{\mathscr{H}}_{\mathscr{R}}$	-50%	0.1156	1.2317	1.8507	91,973
	-25%	0.1209	1.2677	1.9912	92,011
	100%	0.1392	1.2931	2.0157	92,393
	+25%	0.1470	1.3351	2.0391	92,467
	+50%	0.1514	1.3602	2.0509	92,616
5* $\mathscr{D}_{\mathscr{R}}$	-50%	0.1387	1.2922	2.0141	92,371
	-25%	0.1388	1.2927	2.0150	92,384
	100%	0.1392	1.2931	2.0157	92,393
	+25%	0.1396	1.2938	2.0166	92,412
	+50%	0.1405	1.2949	2.0174	92,469
5* $\hat{\mathscr{D}}_{\mathscr{R}}$	-50%	0.1373	1.2888	2.0111	92,325
	-25%	0.1384	1.2911	2.0135	92,364
	100%	0.1392	1.2931	2.0157	92,393
	+25%	0.1405	1.2960	2.0178	92,439
	+50%	0.1438	1.2993	2.0213	92,490
5* $\mathscr{S}_{\mathscr{M}}$	-50%	0.1279	1.2900	2.0011	92,112
	-25%	0.1303	1.2919	2.0099	92,275
	100%	0.1392	1.2931	2.0157	92,393
	+25%	0.1486	1.2964	2.0192	92,436
	+50%	0.1590	1.2995	2.0278	92,505
5* $\hat{\mathscr{S}}_{\mathscr{M}}$	-50%	0.1376	1.2726	2.0062	91,990
	-25%	0.1384	1.2860	2.0109	92,121
	100%	0.1392	1.2931	2.0157	92,393
	+25%	0.1409	1.3015	2.0236	92,514
	+50%	0.1427	1.3742	2.0390	92,782
5* $\mathscr{H}_{\mathscr{M}}$	-50%	0.1390	1.2921	2.0157	92,382
	-25%	0.1391	1.2927	2.0157	92,389
	100%	0.1392	1.2931	2.0157	92,393
	+25%	0.1393	1.2938	2.0157	92,400
	+50%	0.1394	1.2949	2.0157	92,451

Table 10: Manufacturing process concerning the parameters $\hat{\mathscr{H}}_{\mathscr{M}}, \mathscr{D}_{\mathscr{M}}, \hat{\mathscr{D}}_{\mathscr{M}}, \theta, \mathscr{Z}_{\mathscr{R}}, Y, X, \mathscr{P}_{\mathscr{M}}, \rho$ and δ under carbon cap and trade policy.

Parameter	Percentage	$t_{\mathscr{M}}$	$t_{\mathscr{R}}$	T	$\mathscr{TP}_{\mathscr{SC}3}$
5* $\hat{\mathscr{H}}_{\mathscr{M}}$	-50%	0.1300	1.2929	2.0152	92,374
	-25%	0.1347	1.2930	2.0155	92,386
	100%	0.1392	1.2931	2.0157	92,393
	+25%	0.1405	1.2932	2.0160	92,405
	+50%	0.1469	1.2933	2.0166	92,421
5* $\mathscr{D}_{\mathscr{M}}$	-50%	0.1247	1.2929	2.0132	92,371
	-25%	0.1309	1.2929	2.0141	92,384
	100%	0.1392	1.2931	2.0157	92,393
	+25%	0.1437	1.2934	2.0169	92,402
	+50%	0.1490	1.2935	2.0178	92,437
5* $\hat{\mathscr{D}}_{\mathscr{M}}$	-50%	0.1373	1.2903	2.0120	92,371
	-25%	0.1380	1.2915	2.0139	92,385
	100%	0.1392	1.2931	2.0157	92,393
	+25%	0.1409	1.2949	2.0171	92,407
	+50%	0.1427	1.2964	2.0194	92,419
5* θ	-50%	0.1607	1.3674	2.0139	92,311
	-25%	0.1538	1.3199	2.0144	92,364
	100%	0.1392	1.2931	2.0157	92,393
	+25%	0.1284	1.2290	2.0163	92,449
	+50%	0.1105	1.2068	2.0178	92,482
5* $\mathscr{Z}_{\mathscr{R}}$	-50%	0.1691	1.2380	2.0133	92,681
	-25%	0.1583	1.2611	2.0144	92,537
	100%	0.1392	1.2931	2.0157	92,393
	+25%	0.1194	1.3277	2.0169	92,118
	+50%	0.0917	1.3609	2.0176	91,840
5* Y	-50%	0.1390	1.2939	2.0155	92,371
	-25%	0.1391	1.2935	2.0156	92,384
	100%	0.1392	1.2931	2.0157	92,393
	+25%	0.1393	1.2927	2.0158	92,416
	+50%	0.1394	1.2918	2.0159	92,439
5* X	-50%	0.1379	1.2714	2.1511	93,610
	-25%	0.1386	1.2859	2.0805	92,901
	100%	0.1392	1.2931	2.0157	92,393
	+25%	0.1401	1.3017	2.0003	91,717
	+50%	0.1435	1.3290	1.9452	91,022
5* $\mathscr{P}_{\mathscr{M}}$	-50%	0.1317	1.3217	2.0111	91,908
	-25%	0.1351	1.3108	2.0139	92,099
	100%	0.1392	1.2931	2.0157	92,393
	+25%	0.1415	1.2629	2.0182	92,426
	+50%	0.1478	1.2394	2.0208	92,497

Table 11: Manufacturing process concerning the parameters $\rho, \alpha, \beta, t, b$ and δ under carbon cap and trade policy.

Parameter	Percentage	$t_{\mathscr{M}}$	$t_{\mathscr{R}}$	T	$\mathscr{T}\mathscr{P}_{\mathscr{S}\mathscr{C}3}$
5* ρ	-50%	0.1391	1.2912	2.0132	92,377
	-25%	0.1391	1.2926	2.0145	92,384
	100%	0.1392	1.2931	2.0157	92,393
	+25%	0.1394	1.2938	2.0163	92,401
	+50%	0.1396	1.2947	2.0177	92,416
5* δ	-50%	0.1390	1.2929	2.0157	92,386
	-25%	0.1391	1.2930	2.0157	92,390
	100%	0.1392	1.2931	2.0157	92,393
	+25%	0.1393	1.2932	2.0157	92,397
	+50%	0.1394	1.2933	2.0157	92,402

Table 12: Remanufacturing process concerning the parameters $\mathscr{O}_{\mathscr{R}'}, \hat{\mathscr{O}}_{\mathscr{R}'}, \mathscr{H}_{\mathscr{R}'}, \hat{\mathscr{H}}_{\mathscr{R}'}, \mathscr{D}_{\mathscr{R}'}, \hat{\mathscr{D}}_{\mathscr{R}'}, \mathscr{S}_{\mathscr{M}'}, \hat{\mathscr{S}}_{\mathscr{M}'}$ and $\mathscr{H}_{\mathscr{M}'}$ under carbon cap and trade policy.

Parameter	Percentage	$t_{\mathscr{M}'}$	$t_{\mathscr{R}'}$	T	$\mathscr{T}\mathscr{P}_{\mathscr{S}\mathscr{C}'4}$
5* $\mathscr{O}_{\mathscr{R}'}$	-50%	0.1381	1.2026	1.1739	98,617
	-25%	0.1393	1.2104	1.1907	98,980
	100%	0.1405	1.2169	2.0043	99,058
	+25%	0.1429	1.2192	2.0110	99,137
	+50%	0.1450	1.2211	2.0155	99,544
5* $\hat{\mathscr{O}}_{\mathscr{R}'}$	-50%	0.1332	1.2074	1.9901	97,111
	-25%	0.1379	1.2111	1.9987	98,703
	100%	0.1405	1.2169	2.0043	99,058
	+25%	0.1458	1.2205	2.0101	99,511
	+50%	0.1490	1.2923	2.0456	99,990
5* $\mathscr{H}_{\mathscr{R}'}$	-50%	0.1403	1.2167	2.0041	99,056
	-25%	0.1404	1.2168	2.0042	99,057
	100%	0.1405	1.2169	2.0043	99,058
	+25%	0.1406	1.2170	2.0044	99,059
	+50%	0.1407	1.2171	2.0045	99,060
5* $\hat{\mathscr{H}}_{\mathscr{R}'}$	-50%	0.1400	1.2168	2.0041	99,032
	-25%	0.1403	1.2169	2.0042	99,046
	100%	0.1405	1.2169	2.0043	99,058
	+25%	0.1408	1.2170	2.0044	99,065
	+50%	0.1411	1.2170	2.0045	99,077
5* $\mathscr{D}_{\mathscr{R}'}$	-50%	0.1891	1.4444	2.9600	99,995
	-25%	0.1633	1.3805	2.4802	99,512
	100%	0.1405	1.2169	2.0043	99,058
	+25%	0.1299	1.1573	1.8999	98,768
	+50%	0.9767	1.1057	1.5077	98,057
5* $\hat{\mathscr{D}}_{\mathscr{R}'}$	-50%	0.1913	1.2430	2.0043	99,313
	-25%	0.1639	1.2327	2.0043	99,280
	100%	0.1405	1.2169	2.0043	99,058
	+25%	0.1210	1.2102	2.0043	98,900
	+50%	0.9084	1.2006	2.0043	98,845
5* $\mathscr{S}_{\mathscr{M}'}$	-50%	0.1492	1.2155	2.0041	99,317
	-25%	0.1451	1.2162	2.0042	99,193
	100%	0.1405	1.2169	2.0043	99,058
	+25%	0.1383	1.2177	2.0044	98,904
	+50%	0.1317	1.2184	2.0045	98,799
5* $\hat{\mathscr{S}}_{\mathscr{M}'}$	-50%	0.1405	1.2167	2.0042	99,056
	-25%	0.1405	1.2168	2.0042	99,057
	100%	0.1405	1.2169	2.0043	99,058
	+25%	0.1405	1.2170	2.0043	99,059
	+50%	0.1405	1.2171	2.0044	99,060
5* $\mathscr{H}_{\mathscr{M}'}$	-50%	0.1400	1.2165	2.0037	99,012
	-25%	0.1403	1.2167	2.0041	99,033
	100%	0.1405	1.2169	2.0043	99,058
	+25%	0.1407	1.2172	2.0045	99,076
	+50%	0.1409	1.2175	2.0047	99,119

Table 13: Remanufacturing process concerning the parameters $\hat{\mathscr{H}_{\mathscr{M}'}}, \mathscr{D}_{\mathscr{M}'}, \hat{\mathscr{D}_{\mathscr{M}'}}, \theta, \mathscr{Z}_{\mathscr{R}'}, \mathscr{P}_{\mathscr{R}}, \mathscr{D}_0, \mathscr{A}$ and g_0 under carbon cap and trade policy.

5* $\hat{\mathscr{H}_{\mathscr{M}'}}$	-50%	0.1511	1.2190	2.0088	99,382
	-25%	0.1470	1.2178	2.0061	99,200
	100%	0.1405	1.2169	2.0043	99,058
	+25%	0.1383	1.2151	2.0029	99,001
	+50%	0.1327	1.2137	2.0017	98,947
5* $\mathscr{D}_{\mathscr{M}'}$	-50%	0.1403	1.2169	2.0043	99,056
	-25%	0.1404	1.2169	2.0043	99,057
	100%	0.1405	1.2169	2.0043	99,058
	+25%	0.1406	1.2169	2.0043	99,058
	+50%	0.1407	1.2170	2.0044	99,061
5* $\hat{\mathscr{D}_{\mathscr{M}'}}$	-50%	0.1404	1.2167	2.0039	99,091
	-25%	0.1404	1.2168	2.0041	99,070
	100%	0.1405	1.2169	2.0043	99,058
	+25%	0.1405	1.2170	2.0045	99,039
	+50%	0.1405	1.2171	2.0047	99,005
5* θ	-50%	0.1490	1.2169	2.0000	99,711
	-25%	0.1451	1.2169	2.0027	99,396
	100%	0.1405	1.2169	2.0043	99,058
	+25%	0.1379	1.2169	2.0069	98,707
	+50%	0.1336	1.2169	2.0085	98,599
5* $\mathscr{Z}_{\mathscr{R}'}$	-50%	0.1400	1.2111	2.0043	99,010
	-25%	0.1403	1.2147	2.0043	99,037
	100%	0.1405	1.2169	2.0043	99,058
	+25%	0.1407	1.2183	2.0043	99,076
	+50%	0.1409	1.2205	2.0043	99,099
5* $\mathscr{P}_{\mathscr{R}}$	-50%	0.1478	1.2244	2.0099	99,132
	-25%	0.1439	1.2205	2.0061	99,085
	100%	0.1405	1.2169	2.0043	99,058
	+25%	0.1381	1.2116	2.0027	99,039
	+50%	0.1347	1.2078	2.0004	99,006
5* $\mathscr{D}_0$	-50%	0.1496	1.2318	2.0123	99,112
	-25%	0.1449	1.2271	2.0095	99,076
	100%	0.1405	1.2169	2.0043	99,058
	+25%	0.1371	1.2085	2.0007	99,021
	+50%	0.1348	1.1734	1.9711	99,009
5* $\mathscr{A}$	-50%	0.1390	1.2099	1.9045	98,887
	-25%	0.1397	1.2138	1.9650	98,966
	100%	0.1405	1.2169	2.0043	99,058
	+25%	0.1414	1.2190	2.0081	99,139
	+50%	0.1462	1.2237	2.0194	99,210
5* g_0	-50%	0.1473	1.2192	2.0071	99,111
	-25%	0.1451	1.2176	2.0058	99,083
	100%	0.1405	1.2169	2.0043	99,058
	+25%	0.1377	1.2137	2.0027	99,017
	+50%	0.1328	1.2102	2.0005	98,915

Table 14: Remanufacturing process concerning the parameters $\mathscr{B}, \rho$ and δ under carbon cap and trade policy.

Parameter	Percentage	$t_{\mathscr{M}'}$	$t_{\mathscr{R}'}$	T	$\mathscr{T}\mathscr{P}_{\mathscr{S}\mathscr{C}'4}$
5* $\mathscr{B}$	-50%	0.1401	1.2163	2.0037	99,053
	-25%	0.1403	1.2166	2.0040	99,056
	100%	0.1405	1.2169	2.0043	99,058
	+25%	0.1407	1.2172	2.0046	99,061
	+50%	0.1409	1.2175	2.0049	99,063
5* ρ	-50%	0.1476	1.2193	2.0158	99,671
	-25%	0.1431	1.2174	2.0102	99,312
	100%	0.1405	1.2169	2.0043	99,058
	+25%	0.1383	1.2152	2.0009	98,965
	+50%	0.1329	1.2131	1.9904	98,903
5* δ	-50%	0.1391	1.2087	2.0038	98,913
	-25%	0.1397	1.2112	2.0040	98,987
	100%	0.1405	1.2169	2.0043	99,058
	+25%	0.1413	1.2191	2.0045	99,173
	+50%	0.1426	1.2234	2.0048	99,199