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Almost Contra-$(\tau_1, \tau_2)p$-continuity for Functions

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Abstract. This paper introduces a new class of functions between bitopological spaces, namely almost contra-$(\tau_1, \tau_2)p$-continuous functions. Moreover, some characterizations and several properties concerning almost contra-$(\tau_1, \tau_2)p$-continuous functions are established.

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1. Introduction

In 1966, Dontchev [1] introduced the concepts of contra-continuity and strong S-closedness in topological spaces. Moreover, Dontchev [1] obtained very interesting and important results concerning contra-continuity, compactness, S-closedness and strong S-closedness. In 1999, Dontchev et al. [2] defined a new class of functions called regular set-connected functions. Furthermore, Dontchev and Noiri [3] introduced and studied the concept of RC-continuity between topological spaces which is weaker than contra-continuity. In [4], the present authors introduced and investigated a new class of functions called contra-super-continuous functions which lies between classes of RC-continuous functions and contra-continuous functions. In 2002, Jafari and Noiri [5] introduced a new class of function called contra-precontinuous functions which is weaker than contra-continuous functions and studied several basic properties of contra-precontinuous functions. In particular, Jafari and Noiri [5] defined contra-preclosed graphs and investigated relations between contra-precontinuity and contra-preclosed graphs. In 2004, Ekici [6] introduced and studied a new class of functions called almost contra-precontinuous functions which generalize classes of regular set-connected functions [2], contra-precontinuous functions [5], contra-continuous functions [1], almost s-continuous functions [7] and perfectly continuous functions [8]. Ekici [6] obtained basic properties and preservation theorems of almost

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contra-precontinuous functions and relationships between almost contra-precontinuity and P-regular graphs. Noiri and Jafari [9] obtained the further characterizations and properties of almost contra-precontinuous functions and showed that (s, p)-continuity due to Jafari [10] is equivalent to almost contra-precontinuity. In 2007, Al-Omari and Noorani [11] introduced the concept of almost contra ω-continuous functions via the notion of ω-open sets and investigated several characterizations of contra ω-continuous functions and almost contra ω-continuous functions. On the other hand, the present authors introduced and studied the notions of of (τ_1, τ_2)-continuous functions [12], almost (τ_1, τ_2)-continuous functions [13], weakly (τ_1, τ_2)-continuous functions [14], quasi $\theta(\tau_1, \tau_2)$-continuous functions [15], $\delta(\tau_1, \tau_2)$-continuous functions [16], almost quasi (τ_1, τ_2)-continuous functions [17], weakly quasi (τ_1, τ_2)-continuous functions [18], faintly (τ_1, τ_2)-continuous functions [19], almost nearly (τ_1, τ_2)-continuous functions, almost weakly (τ_1, τ_2)-continuous functions [20] and almost contra-(Λ, sp)-continuous functions [21]. In this paper, we introduce the concept of almost contra-$(\tau_1, \tau_2)p$-continuous functions. We also investigate some characterizations of almost contra-$(\tau_1, \tau_2)p$-continuous functions.

2. Preliminaries

Throughout the present paper, spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or simply X and Y) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a bitopological space (X, τ_1, τ_2). The closure of A and the interior of A with respect to τ_i are denoted by τ_i-Cl(A) and τ_i-Int(A), respectively, for $i = 1, 2$. A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-*closed* [22] if $A = \tau_1\text{-Cl}(\tau_2\text{-Cl}(A))$. The complement of a $\tau_1\tau_2$-closed set is called $\tau_1\tau_2$-*open*. The intersection of all $\tau_1\tau_2$-closed sets of X containing A is called the $\tau_1\tau_2$-*closure* [22] of A and is denoted by $\tau_1\tau_2$-Cl(A). The union of all $\tau_1\tau_2$-open sets of X contained in A is called the $\tau_1\tau_2$-*interior* [22] of A and is denoted by $\tau_1\tau_2$-Int(A).

Lemma 1. [22] *Let A and B be subsets of a bitopological space (X, τ_1, τ_2). For the $\tau_1\tau_2$-closure, the following properties hold:*

(1) $A \subseteq \tau_1\tau_2\text{-Cl}(A)$ and $\tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Cl}(A)) = \tau_1\tau_2\text{-Cl}(A)$.

(2) If $A \subseteq B$, then $\tau_1\tau_2\text{-Cl}(A) \subseteq \tau_1\tau_2\text{-Cl}(B)$.

(3) $\tau_1\tau_2\text{-Cl}(A)$ is $\tau_1\tau_2$-closed.

(4) A is $\tau_1\tau_2$-closed if and only if $A = \tau_1\tau_2\text{-Cl}(A)$.

(5) $\tau_1\tau_2\text{-Cl}(X - A) = X - \tau_1\tau_2\text{-Int}(A)$.

A subset A of a bitopological space (X, τ_1, τ_2) is called $(\tau_1, \tau_2)r$-*open* [23] (resp. $(\tau_1, \tau_2)s$-*open* [24], $(\tau_1, \tau_2)p$-*open* [24], $(\tau_1, \tau_2)\beta$-*open* [24]) if $A = \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A))$ (resp. $A \subseteq \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A))$, $A \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A))$, $A \subseteq \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A)))$). The complement of a $(\tau_1, \tau_2)r$-open (resp. $(\tau_1, \tau_2)s$-open, $(\tau_1, \tau_2)p$-open, $(\tau_1, \tau_2)\beta$-open) set is said to be $(\tau_1, \tau_2)r$-*closed* (resp. $(\tau_1, \tau_2)s$-*closed*, $(\tau_1, \tau_2)p$-*closed*, $(\tau_1, \tau_2)\beta$-*closed*). A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\alpha(\tau_1, \tau_2)$-*open* [25] if $A \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A)))$. The complement of an $\alpha(\tau_1, \tau_2)$-open set is said to be $\alpha(\tau_1, \tau_2)$-*closed*. Let A be a subset of a bitopological space (X, τ_1, τ_2). The intersection of all $(\tau_1, \tau_2)p$-closed (resp. $(\tau_1, \tau_2)s$-closed, $\alpha(\tau_1, \tau_2)$-closed) sets of X containing A is called the $(\tau_1, \tau_2)p$-*closure* [26] (resp. $(\tau_1, \tau_2)s$-*closure* [24], $\alpha(\tau_1, \tau_2)$-*closure* [27]) of A and is denoted by $(\tau_1, \tau_2)\text{-pCl}(A)$ (resp. $(\tau_1, \tau_2)\text{-sCl}(A)$, $\alpha(\tau_1, \tau_2)\text{-Cl}(A)$). The union of all $(\tau_1, \tau_2)p$-open (resp. $(\tau_1, \tau_2)s$-open, $\alpha(\tau_1, \tau_2)$-open) sets of X contained in A is called the $(\tau_1, \tau_2)p$-*interior* [26] (resp. $(\tau_1, \tau_2)s$-*interior* [24], $\alpha(\tau_1, \tau_2)$-*interior* [27]) of A and is denoted by $(\tau_1, \tau_2)\text{-pInt}(A)$ (resp. $(\tau_1, \tau_2)\text{-sInt}(A)$, $\alpha(\tau_1, \tau_2)\text{-Int}(A)$).

Lemma 2. *For a subset A of a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) $(\tau_1, \tau_2)\text{-}pCl(A) = \tau_1\tau_2\text{-}Cl(\tau_1\tau_2\text{-}Int(A)) \cup A$ [26];

(2) $(\tau_1, \tau_2)\text{-}pInt(A) = \tau_1\tau_2\text{-}Int(\tau_1\tau_2\text{-}Cl(A)) \cap A$ [20];

(3) $(\tau_1, \tau_2)\text{-}sCl(A) = \tau_1\tau_2\text{-}Int(\tau_1\tau_2\text{-}Cl(A)) \cup A$ [24];

(4) $(\tau_1, \tau_2)\text{-}sInt(A) = \tau_1\tau_2\text{-}Cl(\tau_1\tau_2\text{-}Int(A)) \cap A$ [28].

Let A be a subset of a bitopological space (X, τ_1, τ_2). A point $x \in X$ is called a $s(\tau_1, \tau_2)\theta$-*cluster point* of A if $\tau_1\tau_2\text{-Cl}(U) \cap A \neq \emptyset$ for every $(\tau_1, \tau_2)s$-open set U containing x. The set of all $s(\tau_1, \tau_2)\theta$-cluster points of A is called the $s(\tau_1, \tau_2)\theta$-*closure* of A and is denoted by $s(\tau_1, \tau_2)\theta\text{-Cl}(A)$. A subset A of a bitopological space (X, τ_1, τ_2) is called $s(\tau_1, \tau_2)\theta$-*closed* if $s(\tau_1, \tau_2)\theta\text{-Cl}(A) = A$. The complement of a $s(\tau_1, \tau_2)\theta$-closed set is said to be $s(\tau_1, \tau_2)\theta$-*open*. The union of all $s(\tau_1, \tau_2)\theta$-open sets of X contained in A is called the $s(\tau_1, \tau_2)\theta$-*interior* of A and is denoted by $s(\tau_1, \tau_2)\theta\text{-Int}(A)$.

3. Almost contra-$(\tau_1, \tau_2)p$-continuous functions

In this section, we introduce the concept of almost contra-$(\tau_1, \tau_2)p$-continuous functions. Moreover, some characterizations of almost contra-$(\tau_1, \tau_2)p$-continuous functions are discussed.

Definition 1. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be almost contra-$(\tau_1, \tau_2)p$-continuous if for each $x \in X$ and for each $(\sigma_1, \sigma_2)r$-closed set F of Y containing $f(x)$, there exists a $(\tau_1, \tau_2)p$-open set U of X containing x such that $f(U) \subseteq F$.*

Theorem 1. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) f is almost contra-$(\tau_1, \tau_2)p$-continuous;

(2) $f^{-1}(F)$ is $(\tau_1, \tau_2)p$-open in X for every $(\sigma_1, \sigma_2)r$-closed set F of Y;

(3) $f^{-1}(V)$ is $(\tau_1, \tau_2)p$-closed in X for every $(\sigma_1, \sigma_2)r$-open set V of Y;

(4) $f^{-1}(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))$ *is* $(\tau_1, \tau_2)p$*-closed in* X *for every* $\sigma_1\sigma_2$*-open set* V *of* Y*;*

(5) $f^{-1}(\sigma_1\sigma_2\text{-}Cl(\sigma_1\sigma_2\text{-}Int(F)))$ *is* $(\tau_1, \tau_2)p$*-open in* X *for every* $\sigma_1\sigma_2$*-closed set* F *of* Y*.*

Proof. (1) ⇒ (2): Let F be any $(\sigma_1, \sigma_2)r$-closed set of Y and $x \in f^{-1}(F)$. Then, $f(x) \in F$. By (1), there exists a $(\tau_1, \tau_2)p$-open set U of X containing x such that $f(U) \subseteq F$. Thus, $x \in U \subseteq f^{-1}(F)$ and hence $x \in (\tau_1, \tau_2)\text{-pInt}(f^{-1}(F))$. This implies that

$$f^{-1}(F) \subseteq (\tau_1, \tau_2)\text{-pInt}(f^{-1}(F)).$$

Therefore, $f^{-1}(F)$ is $(\tau_1, \tau_2)p$-open in X.

(2) ⇒ (3): The proof is obvious.

(3) ⇒ (4): Let V be any $\sigma_1\sigma_2$-open set of Y. Then, we have $\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))$ is $(\sigma_1, \sigma_2)r$-open in Y. Thus by (3), $f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$ is $(\tau_1, \tau_2)p$-closed in X.

(4) ⇒ (5): Let F be any $\sigma_1\sigma_2$-closed set of Y. Then, $Y - F$ is $\sigma_1\sigma_2$-open in Y. By (4), we have $f^{-1}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(Y - F))) = Y - f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(F)))$ is $(\tau_1, \tau_2)p$-closed in X. Thus, $f^{-1}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(F)))$ is $(\tau_1, \tau_2)p$-open in X.

(5) ⇒ (1): Let F be any $(\sigma_1, \sigma_2)r$-closed set of Y containing $f(x)$. Since F is $\sigma_1\sigma_2$-closed in Y and by (5), $f^{-1}(F)$ is $(\tau_1, \tau_2)p$-open in X. Let $U = f^{-1}(F)$. Then, U is a $(\tau_1, \tau_2)p$-open set U of X containing x such that $f(U) \subseteq F$. This shows that f is almost contra-$(\tau_1, \tau_2)p$-continuous.

Definition 2. *A function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is said to be* $(\tau_1, \tau_2)p$*-open if* $f(U)$ *is* $(\sigma_1, \sigma_2)p$*-open in* Y *for every* $(\tau_1, \tau_2)p$*-open set* U *of* X*.*

Theorem 2. *If* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is a* $(\tau_1, \tau_2)p$*-open surjection and*

$$g : (Y, \sigma_1, \sigma_2) \to (Z, \rho_1, \rho_2)$$

is a function such that $g \circ f : (X, \tau_1, \tau_2) \to (Z, \rho_1, \rho_2)$ *is almost contra-*$(\tau_1, \tau_2)p$*-continuous, then* g *is almost contra-*$(\sigma_1, \sigma_2)p$*-continuous.*

Proof. Let F be any $(\rho_1, \rho_2)r$-closed set of Z. Since $g \circ f$ is almost contra-$(\tau_1, \tau_2)p$-continuous, by Theorem 1 we have $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is $(\tau_1, \tau_2)p$-open in X. Since f is $(\tau_1, \tau_2)p$-open surjective, $f(f^{-1}(g^{-1}(F))) = g^{-1}(F)$ is $(\sigma_1, \sigma_2)p$-open in Y. Thus, g is almost contra-$(\sigma_1, \sigma_2)p$-continuous.

Definition 3. *A function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is said to be* $(\tau_1, \tau_2)p$*-closed if* $f(K)$ *is* $(\sigma_1, \sigma_2)p$*-closed in* Y *for every* $(\tau_1, \tau_2)p$*-closed set* K *of* X*.*

Theorem 3. *If* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is a* $(\tau_1, \tau_2)p$*-closed surjection and*

$$g : (Y, \sigma_1, \sigma_2) \to (Z, \rho_1, \rho_2)$$

is a function such that $g \circ f : (X, \tau_1, \tau_2) \to (Z, \rho_1, \rho_2)$ *is almost contra-*$(\tau_1, \tau_2)p$*-continuous, then* g *is almost contra-*$(\sigma_1, \sigma_2)p$*-continuous.*

Proof. The proof is similar to that of Theorem 2.

Definition 4. *A bitopological space (X, τ_1, τ_2) is said to be weakly $\tau_1\tau_2$-Hausdorff if each element of X is an intersection of $(\tau_1, \tau_2)r$-closed sets.*

Definition 5. *A bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)p$-T_1 if for each pair of distinct points x and y of X, there exist $(\tau_1, \tau_2)p$-open sets U and V containing x and y, respectively, such that $y \notin U$ and $x \notin V$.*

Theorem 4. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is an almost contra-$(\tau_1, \tau_2)p$-continuous injection and (Y, σ_1, σ_2) is weakly $\sigma_1\sigma_2$-Hausdorff, then (X, τ_1, τ_2) is $(\tau_1, \tau_2)p$-T_1.*

Proof. Suppose that (Y, σ_1, σ_2) is weakly $\sigma_1\sigma_2$-Hausdorff. For any distinct points x and y in X, there exist $(\sigma_1, \sigma_2)r$-closed sets H and K of Y such that $f(x) \in H$, $f(y) \notin H$, $f(y) \in K$ and $f(x) \notin K$. Since f is almost contra-$(\tau_1, \tau_2)p$-continuous, $f^{-1}(H)$ and $f^{-1}(K)$ are $(\tau_1, \tau_2)p$-open sets of X such that $x \in f^{-1}(H)$, $y \notin f^{-1}(H)$, $y \in f^{-1}(K)$ and $x \notin f^{-1}(K)$. This shows that (X, τ_1, τ_2) is $(\tau_1, \tau_2)p$-T_1.

Recall that a bitopological space (X, τ_1, τ_2) is said to be $\tau_1\tau_2$-*connected* [22] if X cannot be written as the union of two nonempty disjoint $\tau_1\tau_2$-open sets.

Definition 6. *A bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)p$-connected if X cannot be written as the union of two nonempty disjoint $(\tau_1, \tau_2)p$-open sets.*

Theorem 5. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is an almost contra-$(\tau_1, \tau_2)p$-continuous surjection and (X, τ_1, τ_2) is $(\tau_1, \tau_2)p$-connected, then (Y, σ_1, σ_2) is $\sigma_1\sigma_2$-connected.*

Proof. Suppose that (Y, σ_1, σ_2) is not $\sigma_1\sigma_2$-connected. Then, there exist nonempty $\sigma_1\sigma_2$-open sets V and W such that $Y = V \cup W$. Therefore, V and W are $\sigma_1\sigma_2$-clopen in Y. Since f is almost contra-$(\tau_1, \tau_2)p$-continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are $(\tau_1, \tau_2)p$-open in X. Furthermore, $f^{-1}(V)$ and $f^{-1}(W)$ are nonempty disjoint and $X = f^{-1}(V) \cup f^{-1}(W)$. This shows that (X, τ_1, τ_2) is not $(\tau_1, \tau_2)p$-connected. This is a contradiction. Thus, (Y, σ_1, σ_2) is $\sigma_1\sigma_2$-connected.

Definition 7. *A bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)p$-compact if every $(\tau_1, \tau_2)p$-open cover of X has a finite subcover.*

Definition 8. *A bitopological space (X, τ_1, τ_2) is said to be S-$\tau_1\tau_2$-closed if every $(\tau_1, \tau_2)r$-closed cover of X has a finite subcover.*

Theorem 6. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is an almost contra-$(\tau_1, \tau_2)p$-continuous surjection and (X, τ_1, τ_2) is $(\tau_1, \tau_2)p$-compact, then (Y, σ_1, σ_2) is S-$\sigma_1\sigma_2$-closed.*

Proof. Let $\{V_\gamma \mid \gamma \in \nabla\}$ be any $(\sigma_1, \sigma_2)r$-closed cover of Y. Since f is almost contra-$(\tau_1, \tau_2)p$-continuous, we have $\{f^{-1}(V_\gamma) \mid \gamma \in \nabla\}$ is a $(\tau_1, \tau_2)p$-open cover of X and therefore there exists a finite subset ∇_0 of ∇ such that $X = \cup\{f^{-1}(V_\gamma) \mid \gamma \in \nabla_0\}$. Thus, we have $Y = \cup\{V_\gamma \mid \gamma \in \nabla_0\}$ and hence (Y, σ_1, σ_2) is S-$\sigma_1\sigma_2$-closed.

Definition 9. *A bitopological space (X, τ_1, τ_2) is said to be $\mathcal{P}$-$\tau_1\tau_2$-closed if every $(\tau_1, \tau_2)p$-closed cover of X has a finite subcover.*

Definition 10. *A bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)r$-compact if every $(\tau_1, \tau_2)r$-open cover of X has a finite subcover.*

Theorem 7. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is an almost contra-$(\tau_1, \tau_2)p$-continuous surjection and (X, τ_1, τ_2) is $\mathcal{P}$-$\tau_1\tau_2$-closed, then (Y, σ_1, σ_2) is $(\sigma_1, \sigma_2)r$-compact.*

Proof. Let $\{V_\gamma \mid \gamma \in \nabla\}$ be any $(\sigma_1, \sigma_2)r$-open cover of Y. Since f is almost contra-$(\tau_1, \tau_2)p$-continuous, we have $\{f^{-1}(V_\gamma) \mid \gamma \in \nabla\}$ is a $(\tau_1, \tau_2)p$-closed cover of X. Since (X, τ_1, τ_2) is $\mathcal{P}$-$\tau_1\tau_2$-closed, there exists a finite subset ∇_0 of ∇ such that

$$X = \cup\{f^{-1}(V_\gamma) \mid \gamma \in \nabla_0\}.$$

Thus, we have $Y = \cup\{V_\gamma \mid \gamma \in \nabla_0\}$ and hence (Y, σ_1, σ_2) is $(\sigma_1, \sigma_2)r$-compact.

Let A be a subset of a bitopological space (X, τ_1, τ_2). A point $x \in X$ is called a $s(\tau_1, \tau_2)\theta$-*cluster point* of A if $\tau_1\tau_2$-Cl$(U) \cap A \neq \emptyset$ for every $(\tau_1, \tau_2)s$-open set U containing x. The set of all $s(\tau_1, \tau_2)\theta$-cluster points of A is called the $s(\tau_1, \tau_2)\theta$-*closure* of A and is denoted by $s(\tau_1, \tau_2)\theta$-Cl(A). A subset A of a bitopological space (X, τ_1, τ_2) is called $s(\tau_1, \tau_2)\theta$-*closed* if $s(\tau_1, \tau_2)\theta$-Cl$(A) = A$. The complement of a $s(\tau_1, \tau_2)\theta$-closed set is said to be $s(\tau_1, \tau_2)\theta$-*open*. The union of all $s(\tau_1, \tau_2)\theta$-open sets of X contained in A is called the $s(\tau_1, \tau_2)\theta$-*interior* of A and is denoted by $s(\tau_1, \tau_2)\theta$-Int(A).

Definition 11. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be $p(\tau_1, \tau_2)s$-continuous if for each $x \in X$ and for each $(\sigma_1, \sigma_2)s$-open set V of Y containing $f(x)$, there exists a $(\tau_1, \tau_2)p$-open set U of X containing x such that $f(U) \subseteq \sigma_1\sigma_2$-Cl$(V)$.*

Theorem 8. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) f is $p(\tau_1, \tau_2)s$-continuous;

(2) f is almost contra-$(\tau_1, \tau_2)p$-continuous;

(3) $f^{-1}(V)$ is $(\tau_1, \tau_2)p$-open in X for each $s(\sigma_1, \sigma_2)\theta$-open set V of Y;

(4) $f^{-1}(F)$ is $(\tau_1, \tau_2)p$-closed in X for each $s(\sigma_1, \sigma_2)\theta$-closed set F of Y.

Proof. (1) $\Rightarrow$ (2): Let F be any $(\sigma_1, \sigma_2)r$-closed set of Y and $x \in f^{-1}(F)$. Then, $f(x) \in F$ and F is $(\sigma_1, \sigma_2)s$-open. Since f is $p(\tau_1, \tau_2)s$-continuous, there exists a $(\tau_1, \tau_2)p$-open set U of X containing x such that $f(U) \subseteq \sigma_1\sigma_2$-Cl$(F) = F$. Therefore, we have $x \in U \subseteq f^{-1}(F)$ which implies that $x \in (\tau_1, \tau_2)$-pInt$(f^{-1}(F))$. Thus,

$$f^{-1}(F) \subseteq (\tau_1, \tau_2)\text{-pInt}(f^{-1}(F))$$

and hence $f^{-1}(F) = (\tau_1, \tau_2)$-pInt$(f^{-1}(F))$. This shows that $f^{-1}(F)$ is $(\tau_1, \tau_2)p$-open in X. It follows from Theorem 1 that f is almost contra-$(\tau_1, \tau_2)p$-continuous.

(2) $\Rightarrow$ (3): This follows from the fact that every $s(\sigma_1, \sigma_2)\theta$-open set is the union of $(\sigma_1, \sigma_2)r$-closed sets.

(3) $\Leftrightarrow$ (4): This is obvious.

(4) $\Rightarrow$ (1): Let $x \in X$ and V be any $(\sigma_1, \sigma_2)s$-open set of Y containing $f(x)$. Since $\sigma_1\sigma_2$-Cl(V) is $(\sigma_1, \sigma_2)r$-closed, we have $\sigma_1\sigma_2$-Cl(V) is $s(\sigma_1, \sigma_2)\theta$-open. Thus by (4), $f^{-1}(\sigma_1\sigma_2\text{-Cl}(V))$ is $(\tau_1, \tau_2)p$-open in X. Now, put $U = f^{-1}(\sigma_1\sigma_2\text{-Cl}(V))$. Then, U is a $(\tau_1, \tau_2)p$-open set of X containing x and $f(U) \subseteq \sigma_1\sigma_2$-Cl$(V)$. This shows that f is $p(\tau_1, \tau_2)s$-continuous.

Theorem 9. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) f is almost contra-$(\tau_1, \tau_2)p$-continuous;

(2) $f((\tau_1, \tau_2)\text{-}p\mathit{Cl}(A)) \subseteq s(\sigma_1, \sigma_2)\theta\text{-}\mathit{Cl}(f(A))$ for every subset A of X;

(3) $(\tau_1, \tau_2)\text{-}p\mathit{Cl}(f^{-1}(B)) \subseteq f^{-1}(s(\sigma_1, \sigma_2)\theta\text{-}\mathit{Cl}(B))$ for every subset B of Y.

Proof. (1) $\Rightarrow$ (2): Let A be any subset of X. Let $x \in (\tau_1, \tau_2)$-pCl(A) and V be any $(\sigma_1, \sigma_2)s$-open set of Y containing $f(x)$. Since f is almost contra-$(\tau_1, \tau_2)p$-continuous, by Theorem 8 there exists a $(\tau_1, \tau_2)p$-open set U of X containing x such that

$$f(U) \subseteq \sigma_1\sigma_2\text{-Cl}(V).$$

Since $x \in (\tau_1, \tau_2)$-pCl(A), we have $U \cap A \neq \emptyset$ and hence

$$\emptyset \neq f(U) \cap f(A) \subseteq \sigma_1\sigma_2\text{-Cl}(V) \cap f(A).$$

Therefore, $f(x) \in s(\sigma_1, \sigma_2)\theta$-Cl$(f(A))$. Thus, $f((\tau_1, \tau_2)\text{-pCl}(A)) \subseteq s(\sigma_1, \sigma_2)\theta\text{-Cl}(f(A))$.

(2) $\Rightarrow$ (3): Let B be any subset of Y. By (2), we have

$$\begin{aligned} f((\tau_1, \tau_2)\text{-pCl}(f^{-1}(B))) &\subseteq s(\sigma_1, \sigma_2)\theta\text{-Cl}(f(f^{-1}(B))) \\ &\subseteq s(\sigma_1, \sigma_2)\theta\text{-Cl}(B) \end{aligned}$$

and hence $(\tau_1, \tau_2)\text{-pCl}(f^{-1}(B)) \subseteq f^{-1}(s(\sigma_1, \sigma_2)\theta\text{-Cl}(B))$.

(3) $\Rightarrow$ (1): Let V be any $(\sigma_1, \sigma_2)s$-open set of Y containing $f(x)$. Since

$$\sigma_1\sigma_2\text{-Cl}(V) \cap (Y - \sigma_1\sigma_2\text{-Cl}(V)) = \emptyset,$$

we have $f(x) \notin s(\sigma_1, \sigma_2)\theta\text{-Cl}(Y - \sigma_1\sigma_2\text{-Cl}(V))$ and hence

$$x \notin f^{-1}(s(\sigma_1, \sigma_2)\theta\text{-Cl}(Y - \sigma_1\sigma_2\text{-Cl}(V))).$$

By (3), $x \notin (\tau_1, \tau_2)\text{-pCl}(f^{-1}(Y - \sigma_1\sigma_2\text{-Cl}(V)))$. There exists a $(\tau_1, \tau_2)p$-open set U of X containing x such that $U \cap f^{-1}(Y - \sigma_1\sigma_2\text{-Cl}(V)) = \emptyset$; hence $f(U) \cap (Y - \sigma_1\sigma_2\text{-Cl}(V)) = \emptyset$. This shows that $f(U) \subseteq \sigma_1\sigma_2$-Cl$(V)$. Thus by Theorem 8, f is almost contra-$(\tau_1, \tau_2)p$-continuous.

Theorem 10. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) f is almost contra-$(\tau_1, \tau_2)p$-continuous;

(2) $f^{-1}(\sigma_1\sigma_2\text{-}Cl(V))$ is $(\tau_1, \tau_2)p$-open in X for every $(\sigma_1, \sigma_2)\beta$-open set V of Y;

(3) $f^{-1}(\sigma_1\sigma_2\text{-}Cl(V))$ is $(\tau_1, \tau_2)p$-open in X for every $(\sigma_1, \sigma_2)s$-open set V of Y;

(4) $f^{-1}(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))$ is $(\tau_1, \tau_2)p$-closed in X for every $(\sigma_1, \sigma_2)p$-open set V of Y.

Proof. (1) ⇒ (2): Let V be any $(\sigma_1, \sigma_2)\beta$-open set of Y. Then, $\sigma_1\sigma_2\text{-Cl}(V)$ is $(\sigma_1, \sigma_2)r$-closed in Y, by Theorem 1 we have $f^{-1}(\sigma_1\sigma_2\text{-Cl}(V))$ is $(\tau_1, \tau_2)p$-open in X.

(2) ⇒ (3): This is obvious since every $(\sigma_1, \sigma_2)s$-open set is $(\sigma_1, \sigma_2)\beta$-open.

(3) ⇒ (4): Let V be any $(\sigma_1, \sigma_2)p$-open set of Y. Then, $Y - \sigma_1\sigma_2\text{-Cl}(V)$ is $(\sigma_1, \sigma_2)r$-closed and hence $Y - \sigma_1\sigma_2\text{-Cl}(V)$ is $(\sigma_1, \sigma_2)s$-open. Thus by (3),

$$f^{-1}(\sigma_1\sigma_2\text{-Cl}(Y - \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))))$$

is $(\tau_1, \tau_2)p$-open in X. Since

$$\begin{aligned} X - f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))) &= f^{-1}(Y - \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))) \\ &= f^{-1}(\sigma_1\sigma_2\text{-Cl}(Y - \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))), \end{aligned}$$

we have $f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$ is $(\tau_1, \tau_2)p$-closed in X.

(4) ⇒ (1): Let V be any $(\sigma_1, \sigma_2)r$-open set of Y. Then, V is $(\sigma_1, \sigma_2)p$-open in Y. By (4), we have $f^{-1}(V) = f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$ is $(\tau_1, \tau_2)p$-closed in X. It follows from Theorem 1 that f is almost contra-$(\tau_1, \tau_2)p$-continuous.

Lemma 3. *For a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) $\alpha(\tau_1, \tau_2)\text{-}Cl(V) = \tau_1\tau_2\text{-}Cl(V)$ for every $(\tau_1, \tau_2)\beta$-open set V of X;

(2) $(\tau_1, \tau_2)\text{-}pCl(V) = \tau_1\tau_2\text{-}Cl(V)$ for every $(\tau_1, \tau_2)s$-open set V of X;

(3) $(\tau_1, \tau_2)\text{-}sCl(V) = \tau_1\tau_2\text{-}Int(\tau_1\tau_2\text{-}Cl(V))$ for every $(\tau_1, \tau_2)p$-open set V of X.

Corollary 1. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) f is almost contra-$(\tau_1, \tau_2)p$-continuous;

(2) $f^{-1}(\alpha(\sigma_1, \sigma_2)\text{-}Cl(V))$ is $(\tau_1, \tau_2)p$-open in X for every $(\sigma_1, \sigma_2)\beta$-open set V of Y;

(3) $f^{-1}((\sigma_1, \sigma_2)\text{-}pCl(V))$ is $(\tau_1, \tau_2)p$-open in X for every $(\sigma_1, \sigma_2)s$-open set V of Y;

(4) $f^{-1}((\sigma_1, \sigma_2)\text{-}sCl(V))$ is $(\tau_1, \tau_2)p$-closed in X for every $(\sigma_1, \sigma_2)p$-open set V of Y.

Proof. This is an immediate consequence of Theorem 10 and Lemma 3.

Definition 12. [20] *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be almost weakly (τ_1, τ_2)-continuous if for each $x \in X$ and each $\sigma_1\sigma_2$-open set V of Y containing $f(x)$,*

$$x \in \tau_1\tau_2\text{-}Int(\tau_1\tau_2\text{-}Cl(f^{-1}(\sigma_1\sigma_2\text{-}Cl(V)))).$$

Lemma 4. [20] *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) f is almost weakly (τ_1, τ_2)-continuous;

(2) $f^{-1}(V) \subseteq \tau_1\tau_2\text{-}Int(\tau_1\tau_2\text{-}Cl(f^{-1}(\sigma_1\sigma_2\text{-}Cl(V))))$ for every $\sigma_1\sigma_2$-open set V of Y;

(3) $\tau_1\tau_2\text{-}Cl(\tau_1\tau_2\text{-}Int(f^{-1}(V))) \subseteq f^{-1}(\sigma_1\sigma_2\text{-}Cl(V))$ for every $\sigma_1\sigma_2$-open set V of Y;

(4) $(\tau_1, \tau_2)\text{-}pCl(f^{-1}(V)) \subseteq f^{-1}(\sigma_1\sigma_2\text{-}Cl(V))$ for every $\sigma_1\sigma_2$-open set V of Y;

(5) $f^{-1}(V) \subseteq (\tau_1, \tau_2)\text{-}pInt(f^{-1}(\sigma_1\sigma_2\text{-}Cl(V)))$ for every $\sigma_1\sigma_2$-open set V of Y;

(6) for each $x \in X$ and each $\sigma_1\sigma_2$-open set V of Y containing $f(x)$, there exists a $(\tau_1, \tau_2)p$-open set U of X containing x such that $f(U) \subseteq \sigma_1\sigma_2\text{-}Cl(V)$.

Theorem 11. *If a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is almost contra-$(\tau_1, \tau_2)p$-continuous, then f is almost weakly (τ_1, τ_2)-continuous.*

Proof. Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$. Then, $\sigma_1\sigma_2\text{-Cl}(V)$ is a $(\sigma_1, \sigma_2)r$-closed set of Y containing $f(x)$. Since f is almost contra-$(\tau_1, \tau_2)p$-continuous, there exists a $(\tau_1, \tau_2)p$-open set U of X containing x such that $f(U) \subseteq \sigma_1\sigma_2\text{-Cl}(V)$. By Lemma 4, f is almost weakly (τ_1, τ_2)-continuous.

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References

[1] J. Dontchev. Contra-continuous functions and strongly S-closed spaces. *International Journal of Mathematics and Mathematical Sciences*, 19:303–310, 1966.

[2] J. Dontchev, M. Ganster, and I. Reilly. More on almost s-continuity. *Indian Journal of Mathematics*, 41:139–146, 1999.

[3] J. Dontchev and T. Noiri. Contra-semicontinuous functions. *Mathematica Pannonica*, 10:159–168, 1999.

[4] S. Jafari and T. Noiri. Contra-super-continuous functions. *Annales Universitatis Scientiarum Budapestinensis de Rolando Eötvös, Sectio Mathematica*, 42:27–34, 1999.

[5] S. Jafari and T. Noiri. On contra-precontinuous functions. *Bulletin of the Malaysian Mathematical Sciences Society*, 25:115–128, 2002.

[6] E. Ekici. Almost contra-precontinuous functions. *Bulletin of the Malaysian Mathematical Sciences Society*, 27:53–65, 2004.

[7] T. Noiri, B. Ahmad, and M. Khan. Almost s-continuous functions. *Kyungpook Mathematical Journal*, 35:311–322, 1995.

[8] T. Noiri. Super-continuity and some strong forms of continuity. *Indian Journal of Pure and Applied Mathematics*, 15:241–250, 1984.

[9] T. Noiri and S. Jafari. Some properties of almost contra-precontinuous functions. *Bulletin of the Malaysian Mathematical Sciences Society*, 28:107–116, 2005.

[10] S. Jafari. On semi-pre-irresolute functions. *Far East Journal of Mathematical Sciences*, 6:1003–1010, 1998.

[11] A. Al-Omari and M. S. M. Noorani. Contra ω-continuous and almost contra ω-continuous functions. *International Journal of Mathematics and Mathematical Sciences*, 2007:40469, 2007.

[12] C. Boonpok and N. Srisarakham. (τ_1, τ_2)-continuity for functions. *Asia Pacific Journal of Mathematics*, 11:21, 2024.

[13] C. Boonpok and P. Pue-on. Characterizations of almost (τ_1, τ_2)-continuous functions. *International Journal of Analysis and Applications*, 22:33, 2024.

[14] C. Boonpok and C. Klanarong. On weakly (τ_1, τ_2)-continuous functions. *European Journal of Pure and Applied Mathematics*, 17(1):416–425, 2024.

[15] N. Srisarakham, S. Sompong, and C. Boonpok. Quasi $\theta(\tau_1, \tau_2)$-continuous functions. *European Journal of Pure and Applied Mathematics*, 18(1):5722, 2025.

[16] C. Prachanpol, C. Boonpok, and C. Viriyapong. $\delta(\tau_1, \tau_2)$-continuous functions. *European Journal of Pure and Applied Mathematics*, 17(4):3730–3742, 2024.

[17] B. Kong-ied, S. Sompong, and C. Boonpok. Almost quasi (τ_1, τ_2)-continuous functions. *Asia Pacific Journal of Mathematics*, 11:64, 2024.

[18] M. Chiangpradit, S. Sompong, and C. Boonpok. Weakly quasi (τ_1, τ_2)-continuous functions. *International Journal of Analysis and Applications*, 22:125, 2024.

[19] N. Srisarakham, A. Sama-Ae, and C. Boonpok. Characterizations of faintly (τ_1, τ_2)-continuous functions. *European Journal of Pure and Applied Mathematics*, 17(4):2753–2762, 2024.

[20] J. Khampakdee, S. Sompong, and C. Boonpok. Almost weakly (τ_1, τ_2)-continuous functions. *European Journal of Pure and Applied Mathematics*, 18(1):5721, 2025.

[21] C. Boonpok and J. Khampakdee. Upper and lower almost contra-(Λ, sp)-continuity. *European Journal of Pure and Applied Mathematics*, 16(1):156–168, 2023.

[22] C. Boonpok, C. Viriyapong, and M. Thongmoon. On upper and lower (τ_1, τ_2)-precontinuous multifunctions. *Journal of Mathematics and Computer Science*, 18:282–293, 2018.

[23] C. Viriyapong and C. Boonpok. $(\tau_1, \tau_2)\alpha$-continuity for multifunctions. *Journal of Mathematics*, 2020:6285763, 2020.

[24] C. Boonpok. $(\tau_1, \tau_2)\delta$-semicontinuous multifunctions. *Heliyon*, 6:e05367, 2020.

[25] N. Viriyapong, S. Sompong, and C. Boonpok. (τ_1, τ_2)-extremal disconnectedness in

bitopological spaces. *International Journal of Mathematics and Computer Science*, 19(3):855–860, 2024.

[26] N. Viriyapong, S. Sompong, and C. Boonpok. Upper and lower s-$(\tau_1, \tau_2)p$-continuous multifunctions. *European Journal of Pure and Applied Mathematics*, 17(3):2210–2220, 2024.

[27] C. Viriyapong, S. Sompong, and C. Boonpok. Upper and lower slight $\alpha(\tau_1, \tau_2)$-continuity. *European Journal of Pure and Applied Mathematics*, 17(3):2142–2154, 2024.

[28] P. Pue-on, S. Sompong, and C. Boonpok. Almost quasi (τ_1, τ_2)-continuity for multifunctions. *International Journal of Analysis and Applications*, 22:97, 2024.