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Fekete–Szegő Inequalities for New Subclasses of Bi-Univalent Functions Defined by Sălăgean q-Differential Operator

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Abstract. In this paper, we introduce a new operator based on the Sălăgean q-differential approach to define a new class of analytic functions. Using this operator, we obtain estimates for the first two coefficients in the Taylor series, $|a_2|$ and $|a_3|$. A significant part of the study focuses on the Fekete–Szegő inequalities for the function classes $\mathcal{M}^{\zeta,m}_{\sigma,q,\Sigma}(\curlywedge,\kappa,\alpha)$ and $\mathcal{M}^{\zeta,m}_{\sigma,q,\Sigma}(\gamma,\curlywedge,\kappa)$. Through our analysis, we derive several important results, including some special cases that we present in this paper as Corollaries.

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1. Introduction

Let Λ denote the class of all analytic functions $\mathfrak{I}$ defined in the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$ and normalized by the conditions $\mathfrak{I}(0) = 0$ and $\mathfrak{I}'(0) = 1$. Each $\mathfrak{I} \in \Lambda$

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has a Taylor series expansion of the form:

$$\mathfrak{I}(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad z \in \mathbb{U}. \tag{1}$$

For every $\mathfrak{I} \in \mathcal{S}$, there exists an inverse map $\mathfrak{I}^{-1}$ satisfying the following conditions:

$$\mathfrak{I}^{-1}(\mathfrak{I}(z)) = z, \quad z \in \mathbb{U},$$

$$\mathfrak{I}(\mathfrak{I}^{-1}(\varpi)) = \varpi, \quad |\varpi| < r_0(\mathfrak{I}); \quad r_0(\mathfrak{I}) \geq \frac{1}{4}.$$

The inverse function is given by the series:

$$\mathfrak{I}^{-1}(\varpi) = \varpi - a_2\varpi^2 + (2a_2^2 - a_3)\varpi^3 - (5a_2^3 - 5a_2a_3 + a_4)\varpi^4 + \cdots . \tag{2}$$

Definition 1. *A single-valued complex function $\mathfrak{I}$ is said to be univalent in a simply connected domain D if it does not take the same value twice in D; that is, $\mathfrak{I}(z_1) \neq \mathfrak{I}(z_2)$ whenever $z_1 \neq z_2$, for all $z_1, z_2 \in D$.*

Definition 2. *A function $\mathfrak{I} \in \Lambda$ is said to be bi-univalent in $\mathbb{U}$ if both $\mathfrak{I}(z)$ and $\mathfrak{I}^{-1}(z)$ are univalent in $\mathbb{U}$.*

Let Σ denote the class of bi-univalent functions in $\mathbb{U}$ defined by (1). Examples of functions in Σ include:

$$\frac{z}{1-z}, \quad -\log(1-z), \quad \frac{1}{2}\log\left(\frac{1+z}{1-z}\right), \ldots .$$

It is worth noting that the familiar Koebe function is not a member of Σ because it maps the unit disk $\mathbb{U}$ univalently onto the entire complex plane except for the part of the negative real axis from $-\frac{1}{4}$ to $-\infty$.

The class $\mathcal{S}^*(\alpha)$ of starlike functions of order α in $\mathbb{U}$ has been extensively studied and is a subset of $\mathcal{S}$. By definition:

$$\mathcal{S}^*(\alpha) = \left\{\mathfrak{I} \in \mathcal{S} : \operatorname{Re}\left(\frac{\mathfrak{I}'(z)}{\mathfrak{I}(z)}\right) > \alpha, \ z \in \mathbb{U}, \ 0 \leq \alpha < 1\right\}. \tag{3}$$

Ezrohi [1] introduced the class $\mathcal{H}(\alpha)$, defined as:

$$\mathcal{H}(\alpha) = \left\{\mathfrak{I} \in \mathcal{S} : \operatorname{Re}\{\mathfrak{I}'(z)\} > \alpha, \ z \in \mathbb{U}, \ 0 \leq \alpha < 1\right\}. \tag{4}$$

Similarly, the class $\mathcal{K}(\alpha)$ was introduced by [2]:

$$\mathcal{K}(\alpha) = \left\{\mathcal{F} \in \mathcal{S} : \operatorname{Re}\left(1 + \frac{z\mathcal{F}''(z)}{\mathcal{F}'(z)}\right) > \alpha, \ z \in \mathbb{U}, \ 0 \leq \alpha < 1\right\}. \tag{5}$$

A function $\mathfrak{I} \in \Lambda$ belongs to the class $\mathcal{S}_\Sigma^*(\alpha)$ of strongly bi-starlike functions of order α $(0 < \alpha \leq 1)$ if:

$$|\arg\left(\frac{z\mathfrak{I}'(z)}{\mathfrak{I}(z)}\right)| < \frac{\alpha\pi}{2}, \quad z \in \mathbb{U},$$

$$|\arg\left(\frac{\varpi\mathcal{G}'(\varpi)}{\mathcal{G}(\varpi)}\right)| < \frac{\alpha\pi}{2}, \quad \varpi \in \mathbb{U},$$

where $\mathcal{G} = \mathfrak{I}^{-1}$.

Here, we revisit the q-difference operator, a fundamental tool in q-calculus that plays a key role in various fields such as hypergeometric series, quantum physics, and operator theory. The q-calculus framework, introduced by Jackson [3], has been extended to fractional q-calculus operators, as utilized by Kanas and Răducanu [4]. For more details, readers are referred to [3, 5–31]. Below, we outline key definitions and concepts, assuming $0 < q < 1$.

The Jackson q-derivative of a function $\mathfrak{I} \in \Lambda$ is defined as [3]:

$$D_q\mathfrak{I}(z) = \begin{cases} \frac{\mathfrak{I}(z)-\mathfrak{I}(qz)}{(1-q)z}, & z \neq 0, \\ \mathfrak{I}'(0), & z = 0, \end{cases} \tag{6}$$

with the second q-derivative given by:

$$D_q^2\mathfrak{I}(z) = D_q(D_q\mathfrak{I}(z)).$$

Using the above, $D_q\mathfrak{I}(z)$ can be expressed as:

$$D_q\mathfrak{I}(z) = 1 + \sum_{n=2}^{\infty}[n]_q a_n z^{n-1}, \tag{7}$$

where the q-basic number $[n]_q$ is defined as:

$$[n]_q = \frac{1-q^n}{1-q}.$$

As $q \to 1^-$, $[n]_q \to n$. For $h(z) = z^n$, the q-derivative becomes:

$$D_q h(z) = [n]_q z^{n-1}.$$

This result converges to the classical derivative $h'(z) = nz^{n-1}$ as $q \to 1^-$.

Recently, Govindaraj and Sivasubramanian [32] introduced the Sălăgean q-differential operator:

$$\mathcal{D}_q^0\mathfrak{I}(z) = \mathfrak{I}(z), \quad \mathcal{D}_q^1\mathfrak{I}(z) = z\mathcal{D}_q\mathfrak{I}(z),$$

$$\mathcal{D}_q^m\mathfrak{I}(z) = z\mathcal{D}_q^m\left(\mathcal{D}_q^{m-1}\mathfrak{I}(z)\right),$$

$$\mathcal{D}_q^m\mathfrak{I}(z) = z + \sum_{n=2}^{\infty}[n]_q^m a_n z^n, \quad m \in \mathbb{N}_0,\ z \in \mathbb{U}. \tag{8}$$

[33] Define the generalized operator:

$$\mathbb{D}^0\mathfrak{I}(z) = \mathcal{D}_q^m\mathfrak{I}(z),$$

$$\mathbb{D}_{\sigma,q}^{1,m}\Im(z) = (1-\sigma)\mathcal{D}_q^m\Im(z) + \sigma z\big(\mathcal{D}_q^m\Im(z)\big)',$$
$$= z + \sum_{n=2}^{\infty}[n]_q^m\big[1+(n-1)\sigma\big]a_n z^n, \tag{9}$$

$$\mathbb{D}_{\sigma,q}^{\zeta,m}\Im(z) = z + \sum_{n=2}^{\infty}[n]_q^m\big[1+(n-1)\sigma\big]^{\zeta}a_n z^n, \quad \sigma>0,\ \zeta\in\mathbb{N}_0. \tag{10}$$

As $q \to 1^-$, the operator reduces to:

$$\mathbb{D}_{\sigma}^{\zeta,m}\Im(z) = z + \sum_{n=2}^{\infty}n^m\big[1+(n-1)\sigma\big]^{\zeta}a_n z^n, \quad \sigma>0,\ m,\zeta\in\mathbb{N}_0. \tag{11}$$

Definition 3. *A function $\Im(z)$, as described in* (1)*, belongs to the class $\mathcal{M}_{\sigma,q,\Sigma}^{\zeta,m}(\curlywedge,\kappa,\alpha)$ if:*

$$|\arg\left(1+\frac{1}{\kappa}[(1-\curlywedge)(\frac{d}{dq}\mathbb{D}_{\sigma,q}^{\zeta,m}\Im(z)) + \curlywedge\frac{\mathbb{D}_{\sigma,q}^{\zeta,m}\Im(z)}{z}-1]\right)| < \frac{\alpha\pi}{2},$$

and

$$|\arg\left(1+\frac{1}{\kappa}[(1-\curlywedge)(\frac{d}{dq}\mathbb{D}_{\sigma,q}^{\zeta,m}\mathcal{G}(\varpi)) + \curlywedge\frac{\mathbb{D}_{\sigma,q}^{\zeta,m}\mathcal{G}(\varpi)}{z}-1]\right)| < \frac{\alpha\pi}{2},$$

where $0<\alpha\le 1,\ \curlywedge\ge 0,\ \kappa\ge 1, \sigma>0,\ m,\zeta\in\mathbb{N}_0\ z,\varpi\in\mathbb{U}$.

Definition 4. *A function $\Im(z)$, as described in* (1)*, belongs to the class $\mathcal{M}_{\sigma,q,\Sigma}^{\zeta,m}(\gamma,\curlywedge,\kappa)$ if:*

$$Re\left(1+\frac{1}{\kappa}[(1-\curlywedge)(\frac{d}{dq}\mathbb{D}_{\sigma,q}^{\zeta,m}\Im(z)) + \curlywedge\frac{\mathbb{D}_{\sigma,q}^{\zeta,m}\Im(z)}{z}-1]\right) > \gamma,$$

and

$$Re\left(1+\frac{1}{\kappa}[(1-\curlywedge)(\frac{d}{dq}\mathbb{D}_{\sigma,q}^{\zeta,m}\mathcal{G}(\varpi)) + \curlywedge\frac{\mathbb{D}_{\sigma,q}^{\zeta,m}\mathcal{G}(\varpi)}{z}-1]\right) > \gamma,$$

where $0\le\gamma<1,\ \curlywedge\ge 0,\ \kappa\ge 1, \sigma>0,\ m,\zeta\in\mathbb{N}_0\ z,\varpi\in\mathbb{U}$.

To prove our theorem, we will make use of the following lemma:

Lemma 1 ([34])**.** *If h belongs to the family $\mathcal{H}$, where $\mathcal{H}$ represents all analytic functions in $\mathbb{U}$ satisfying $Re(h(z))>0$ and $h(z)=1+h_1z+h_2z^2+\cdots$, then $|h_i|\le 2$ for each index i.*

Coefficients Bounds for Classes $\mathcal{M}^{\zeta,m}_{\sigma,q,\Sigma}(\leftthreetimes,\kappa,\alpha)$ and $\mathcal{M}^{\zeta,m}_{\sigma,q,\Sigma}(\gamma,\leftthreetimes,\kappa)$

Theorem 1. *Let $\Im(z)$ given by (1) be in the class $\mathcal{M}^{\zeta,m}_{\sigma,q,\Sigma}(\leftthreetimes,\kappa,\alpha)$, with* $0<\alpha\leq 1,\ \leftthreetimes\geq 0,\ \kappa\geq 1, \sigma>0,\ m,\zeta\in\mathbb{N}_0\ z,\varpi\in\mathbb{U}$. *Then*

$$|a_2|\leq \frac{8\alpha\kappa}{\sqrt{4\alpha\kappa\big[1+2\sigma\big]^{\zeta}((1-\leftthreetimes)[3]_q^{m+1}+\leftthreetimes[3]_q^{m})-(\alpha-1)\big[1+\sigma\big]^{2\zeta}((1-\leftthreetimes)[2]_q^{m+1}+\leftthreetimes[2]_q^{m})^2}},$$

and

$$|a_3|\leq \frac{2\alpha}{|\frac{\left[1+2\sigma\right]^{\zeta}}{\kappa}((1-\leftthreetimes)[3]_q^{m+1}+\leftthreetimes[3]_q^{m})|}+\frac{8\alpha^2\kappa^2}{|\big[1+\sigma\big]^{2\zeta}((1-\leftthreetimes)[2]_q^{m+1}+\leftthreetimes[2]_q^{m})^2|}.$$

Proof. To establish the theorem, the definition (3) is utilized in its equivalent forms:

$$1+\frac{1}{\kappa}[(1-\leftthreetimes)(\frac{d}{dq}\mathbb{D}^{\zeta,m}_{\sigma,q}\Im(z))+\leftthreetimes\frac{\mathbb{D}^{\zeta,m}_{\sigma,q}\Im(z)}{z}-1]=[v(z)]^{\alpha}, \tag{12}$$

$$1+\frac{1}{\kappa}[(1-\leftthreetimes)(\frac{d}{dq}\mathbb{D}^{\zeta,m}_{\sigma,q}\mathcal{G}(\varpi))+\leftthreetimes\frac{\mathbb{D}^{\zeta,m}_{\sigma,q}\mathcal{G}(\varpi)}{z}-1]=[c(\varpi)]^{\alpha}, \tag{13}$$

where $v(z)$ and $c(w)$ belong to the class H and satisfy the conditions defined in (1) . These functions can be expressed as:

$$v(z)=1+v_1z+v_2z^2+v_3z^3+\cdots, \tag{14}$$

$$c(\varpi)=1+c_1\varpi+c_2\varpi^2+c_3\varpi^3+\cdots. \tag{15}$$

By equating coefficients in the above equations, the following relations are obtained:

$$\frac{\big[1+\sigma\big]^{\zeta}}{\kappa}((1-\leftthreetimes)[2]_q^{m+1}+\leftthreetimes[2]_q^{m})a_2=\alpha v_1, \tag{16}$$

$$\frac{\big[1+2\sigma\big]^{\zeta}}{\kappa}((1-\leftthreetimes)[3]_q^{m+1}+\leftthreetimes[3]_q^{m})a_3=\alpha v_2+\frac{\alpha(\alpha-1)}{2}v_1^2, \tag{17}$$

$$-\frac{\big[1+\sigma\big]^{\zeta}}{\kappa}((1-\leftthreetimes)[2]_q^{m+1}+\leftthreetimes[2]_q^{m})a_2=\alpha c_1, \tag{18}$$

and

$$\frac{\big[1+2\sigma\big]^{\zeta}}{\kappa}((1-\leftthreetimes)[3]_q^{m+1}+\leftthreetimes[3]_q^{m})(2a_2^2-a_3)=\alpha c_2+\frac{\alpha(\alpha-1)}{2}c_1^2. \tag{19}$$

Using the equations (16),(18) , it follows that:

$$v_1=-c_1, \tag{20}$$

$$\frac{\left[1+\sigma\right]^{2\zeta}}{\kappa^2}((1-\curlywedge)[2]_q^{m+1}+\curlywedge[2]_q^m)^2a_2^2=\alpha^2(v_1^2+c_1^2). \tag{21}$$

From equations (17) and (19) , it can be concluded that:

$$4\alpha\kappa\left[1+2\sigma\right]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^m)a_2^2=2\alpha^2\kappa^2(v_2+c_2)+$$

$$(\alpha-1)\left[1+\sigma\right]^{2\zeta}((1-\curlywedge)[2]_q^{m+1}+\curlywedge[2]_q^m)^2a_2^2. \tag{22}$$

Consequently, we obtain:

$$a_2^2=\frac{2\alpha^2\kappa^2(v_2+c_2)}{4\alpha\kappa\left[1+2\sigma\right]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^m)-(\alpha-1)\left[1+\sigma\right]^{2\zeta}((1-\curlywedge)[2]_q^{m+1}+\curlywedge[2]_q^m)^2}, \tag{23}$$

and the upper bound for $|a_2|$ is determined as:

$$|a_2|\leq\frac{8\alpha\kappa}{\sqrt{4\alpha\kappa\left[1+2\sigma\right]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^m)-(\alpha-1)\left[1+\sigma\right]^{2\zeta}((1-\curlywedge)[2]_q^{m+1}+\curlywedge[2]_q^m)^2}}.$$

By using equations (17) , (19) and (21) we have:

$$a_3=\frac{\alpha(v_2-c_2)}{2\frac{\left[1+2\sigma\right]^{\zeta}}{\kappa}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^m)}+\frac{\alpha^2\kappa^2(v_1^2+c_1^2)}{\left[1+\sigma\right]^{2\zeta}((1-\curlywedge)[2]_q^{m+1}+\curlywedge[2]_q^m)^2}, \tag{24}$$

and the following upper bound is obtained:

$$|a_3|\leq\frac{2\alpha}{|\frac{\left[1+2\sigma\right]^{\zeta}}{\kappa}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^m)|}+\frac{8\alpha^2\kappa^2}{|\left[1+\sigma\right]^{2\zeta}((1-\curlywedge)[2]_q^{m+1}+\curlywedge[2]_q^m)^2|}.$$

This concludes the proof.

Theorem 2. *Let $\Im(z)$ given by (1) be in the class $\mathcal{M}_{\sigma,q,\Sigma}^{\zeta,m}(\gamma,\curlywedge,\kappa)$, where $0\leq\gamma<1$, $\curlywedge,\delta\geq 0$, $\kappa\geq 1,\sigma>0$, $m,\zeta\in\mathbb{N}_0$ $z,\varpi\in\mathbb{U}$. Then*

$$|a_2|\leq\sqrt{\frac{2\kappa(1-\gamma)}{|\left[1+2\sigma\right]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^m)|}}$$

and

$$|a_3|\leq\frac{8\kappa^2(1-\gamma)^2}{|\left[1+\sigma\right]^{2\zeta}((1-\curlywedge)[2]_q^{m+1}+\curlywedge[2]_q^m)^2|}+\frac{2\kappa(1-\gamma)}{|\left[1+2\sigma\right]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^m)|}$$

Proof. It can be inferred from definition (4) that there exist $v(z)$ and $c(\varpi) \in \mathcal{H}$ such that

$$1+\frac{1}{\kappa}[(1-\leftthreetimes)(\frac{d}{dq}\mathbb{D}_{\sigma,q}^{\zeta,m}\Im(z))+\leftthreetimes\frac{\mathbb{D}_{\sigma,q}^{\zeta,m}\Im(z)}{z}-1]=\gamma+(1-\gamma)v(z), \tag{25}$$

and

$$1+\frac{1}{\kappa}[(1-\leftthreetimes)(\frac{d}{dq}\mathbb{D}_{\sigma,q}^{\zeta,m}\mathcal{G}(\varpi))+\leftthreetimes\frac{\mathbb{D}_{\sigma,q}^{\zeta,m}\mathcal{G}(\varpi)}{z}-1]=\gamma+(1-\gamma)c(\varpi). \tag{26}$$

Equating coefficients in (25) and (26) , we obtain:

$$\frac{\left[1+\sigma\right]^{\zeta}}{\kappa}((1-\leftthreetimes)[2]_q^{m+1}+\leftthreetimes[2]_q^m)a_2=(1-\gamma)v_1, \tag{27}$$

$$\frac{\left[1+2\sigma\right]^{\zeta}}{\kappa}((1-\leftthreetimes)[3]_q^{m+1}+\leftthreetimes[3]_q^m)a_3=(1-\gamma)v_2, \tag{28}$$

$$-\frac{\left[1+\sigma\right]^{\zeta}}{\kappa}((1-\leftthreetimes)[2]_q^{m+1}+\leftthreetimes[2]_q^m)a_2=(1-\gamma)c_1, \tag{29}$$

and

$$\frac{\left[1+2\sigma\right]^{\zeta}}{\kappa}((1-\leftthreetimes)[3]_q^{m+1}+\leftthreetimes[3]_q^m)(2a_2^2-a_3)=(1-\gamma)c_2. \tag{30}$$

Utilizing equations (27) and (29) , we deduce the following:

$$v_1=-c_1, \tag{31}$$

and

$$\frac{\left[1+\sigma\right]^{2\zeta}}{\kappa^2}((1-\leftthreetimes)[2]_q^{m+1}+\leftthreetimes[2]_q^m)^2a_2^2=(1-\gamma)^2(v_1^2+c_1^2). \tag{32}$$

From equations (28) and (30) , it can be concluded that:

$$\frac{2\left[1+2\sigma\right]^{\zeta}}{\kappa}((1-\leftthreetimes)[3]_q^{m+1}+\leftthreetimes[3]_q^m)a_2^2=(1-\gamma)(v_2+c_2). \tag{33}$$

Consequently, we obtain:

$$a_2=\sqrt{\frac{\kappa(1-\gamma)(v_2+c_2)}{2\left[1+2\sigma\right]^{\zeta}((1-\leftthreetimes)[3]_q^{m+1}+\leftthreetimes[3]_q^m)}}. \tag{34}$$

This determines the upper bound for $|a_2|$:

$$|a_2|\leq\sqrt{\frac{2\kappa(1-\gamma)}{|\left[1+2\sigma\right]^{\zeta}((1-\leftthreetimes)[3]_q^{m+1}+\leftthreetimes[3]_q^m)|}} \tag{35}$$

Next, for the purpose of establishing the constraint on $|a_3|$, we subtract (28) and (30), using (32), we get:

$$\frac{2\big[1+2\sigma\big]^{\zeta}}{\kappa}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^{m})(a_3-a_2^2)=(1-\gamma)(v_2-c_2). \tag{36}$$

Alternatively, it can be expressed as:

$$a_3=a_2^2+\frac{\kappa(1-\gamma)(v_2-c_2)}{2\big[1+2\sigma\big]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^{m})}. \tag{37}$$

By using equation (31) in (32), we have:

$$a_3=\frac{\kappa^2(1-\gamma)^2(v_1^2+c_1^2)}{\big[1+\sigma\big]^{2\zeta}((1-\curlywedge)[2]_q^{m+1}+\curlywedge[2]_q^{m})^2}+\frac{\kappa(1-\gamma)(v_2-c_2)}{2\big[1+2\sigma\big]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^{m})} \tag{38}$$

We can establish the following upper bound for $|a_3|$:

$$|a_3|\leq\frac{8\kappa^2(1-\gamma)^2}{|\big[1+\sigma\big]^{2\zeta}((1-\curlywedge)[2]_q^{m+1}+\curlywedge[2]_q^{m})^2|}+\frac{2\kappa(1-\gamma)}{|\big[1+2\sigma\big]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^{m})|} \tag{39}$$

This completes the proof.

2. Corollaries and Consequences

By substituting $\curlywedge=1$ in Theorem (1) and Theorem (2), we arrive at the following corollaries, respectively:

Corollary 1. *Let* $\Im(z)$ *given by* (1) *be in the class* $\mathcal{M}^{\zeta,m}_{\sigma,q,\Sigma}(1,\kappa,\alpha)$*, with* $0<\alpha\leq 1,\ \kappa\geq 1,\sigma>0,\curlywedge=1,m,\zeta\in\mathbb{N}_0\ z,\varpi\in\mathbb{U}$*. Then*

$$|a_2|\leq\frac{8\alpha\kappa}{\sqrt{4\alpha\kappa\big[1+2\sigma\big]^{\zeta}([3]_q^{m})-(\alpha-1)\big[1+\sigma\big]^{2\zeta}([2]_q^{m})^2}}.$$

and

$$|a_3|\leq\frac{2\alpha}{|\frac{\left[1+2\sigma\right]^{\zeta}}{\kappa}([3]_q^{m})|}+\frac{8\alpha^2\kappa^2}{|\big[1+\sigma\big]^{2\zeta}([2]_q^{m})^2|}.$$

Corollary 2. *Let* $\Im(z)$ *given by* (1) *be in the class* $\mathcal{M}^{\zeta,m}_{\sigma,q,\Sigma}(\gamma,1,\kappa)$*, where* $0\leq\gamma<1,\ \kappa\geq 1,\sigma>0,\curlywedge=1,m,\zeta\in\mathbb{N}_0\ z,\varpi\in\mathbb{U}$*. Then*

$$|a_2|\leq\sqrt{\frac{2\kappa(1-\gamma)}{|\big[1+2\sigma\big]^{\zeta}([3]_q^{m})|}}$$

and

$$|a_3|\leq\frac{8\kappa^2(1-\gamma)^2}{|\big[1+\sigma\big]^{2\zeta}([2]_q^{m})^2|}+\frac{2\kappa(1-\gamma)}{|\big[1+2\sigma\big]^{\zeta}([3]_q^{m})|}$$

By substituting $\kappa = 1$ in Theorem (1) and Theorem (2), we arrive at the following corollaries, respectively:

Corollary 3. *Let* $\Im(z)$ *given by* (1) *be in the class* $\mathcal{M}_{\sigma,q,\Sigma}^{\zeta,m}(\curlywedge, 1, \alpha)$, *with* $0 < \alpha \leq 1$, $\curlywedge \geq 0$, $\kappa = 1, \sigma > 0$, $m, \zeta \in \mathbb{N}_0$ $z, \varpi \in \mathbb{U}$. *Then*

$$|a_2| \leq \frac{8\alpha}{\sqrt{4\alpha\big[1+2\sigma\big]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^m)-(\alpha-1)\big[1+\sigma\big]^{2\zeta}((1-\curlywedge)[2]_q^{m+1}+\curlywedge[2]_q^m)^2}}.$$

and

$$|a_3| \leq \frac{2\alpha}{|\big[1+2\sigma\big]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^m)|}+\frac{8\alpha^2}{|\big[1+\sigma\big]^{2\zeta}((1-\curlywedge)[2]_q^{m+1}+\curlywedge[2]_q^m)^2|}.$$

Corollary 4. *Let* $\Im(z)$ *given by* (1) *be in the class* $\mathcal{M}_{\sigma,q,\Sigma}^{\zeta,m}(\gamma, \curlywedge, 1)$, *where* $0 \leq \gamma < 1$, $\curlywedge \geq 0$, $\kappa = 1, \sigma > 0$, $m, \zeta \in \mathbb{N}_0$ $z, \varpi \in \mathbb{U}$. *Then*

$$|a_2| \leq \sqrt{\frac{2(1-\gamma)}{|\big[1+2\sigma\big]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^m)|}}$$

and

$$|a_3| \leq \frac{8(1-\gamma)^2}{|\big[1+\sigma\big]^{2\zeta}((1-\curlywedge)[2]_q^{m+1}+\curlywedge[2]_q^m)^2|}+\frac{2(1-\gamma)}{|\big[1+2\sigma\big]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^m)|}$$

By substituting $\alpha = 1$ and $\gamma = 0$ respectively in The previous corollaries , we arrive:

Corollary 5. *Let* $\Im(z)$ *given by* (1) *be in the class* $\mathcal{M}_{\sigma,q,\Sigma}^{\zeta,m}(\curlywedge, 1, 1)$, *with* $0 < \alpha \leq 1$, $\curlywedge \geq 0$, $\kappa = 1, \sigma > 0$, $m, \zeta \in \mathbb{N}_0$ $z, \varpi \in \mathbb{U}$. *Then*

$$|a_2| \leq \frac{8}{\sqrt{4\big[1+2\sigma\big]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^m)}}.$$

and

$$|a_3| \leq \frac{2}{|\big[1+2\sigma\big]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^m)|}+\frac{8}{|\big[1+\sigma\big]^{2\zeta}((1-\curlywedge)[2]_q^{m+1}+\curlywedge[2]_q^m)^2|}.$$

Corollary 6. *Let* $\Im(z)$ *given by* (1) *be in the class* $\mathcal{M}_{\sigma,q,\Sigma}^{\zeta,m}(0, \curlywedge, 1)$, *where* $0 \leq \gamma < 1$, $\curlywedge \geq 0$, $\kappa = 1, \sigma > 0$, $m, \zeta \in \mathbb{N}_0$ $z, \varpi \in \mathbb{U}$. *Then*

$$|a_2| \leq \sqrt{\frac{2}{|\big[1+2\sigma\big]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^m)|}}$$

and

$$|a_3| \leq \frac{8}{|\big[1+\sigma\big]^{2\zeta}((1-\curlywedge)[2]_q^{m+1}+\curlywedge[2]_q^m)^2|}+\frac{2}{|\big[1+2\sigma\big]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^m)|}$$

3. Fekete–Szegő Inequalities for the Functions in the Classes $\mathcal{M}^{\zeta,m}_{\sigma,q,\Sigma}(\leftthreetimes,\kappa,\alpha)$ and $\mathcal{M}^{\zeta,m}_{\sigma,q,\Sigma}(\gamma,\leftthreetimes,\kappa)$

In this section, the focus is on the Fekete–Szegő inequalities for the Functions in the Classes $\mathcal{M}^{\zeta,m}_{\sigma,q,\Sigma}(\leftthreetimes,\kappa,\alpha)$ and $\mathcal{M}^{\zeta,m}_{\sigma,q,\Sigma}(\gamma,\leftthreetimes,\kappa)$.

Theorem 3. *Let $\Im(z)$ given by (1) be in the class $\mathcal{M}^{\zeta,m}_{\sigma,q,\Sigma}(\leftthreetimes,\kappa,\alpha)$, with $0<\alpha\leq 1$, $\leftthreetimes\geq 0$, $\kappa\geq 1, \sigma>0$, $m,\zeta\in\mathbb{N}_0$ $z,\varpi\in\mathbb{U}$. Then*

$$|a_3-\Theta a_2^2|\leq\begin{cases}\dfrac{2\alpha}{|\frac{\left[1+2\sigma\right]^{\zeta}}{\kappa}\left((1-\leftthreetimes)[3]_q^{m+1}+\leftthreetimes[3]_q^{m}\right)|} & \text{for } |h(\Theta)|\leq\dfrac{1}{|\frac{\left[1+2\sigma\right]^{\zeta}}{\kappa}\left((1-\leftthreetimes)[3]_q^{m+1}+\leftthreetimes[3]_q^{m}\right)|}\\ 2\alpha|h(\Theta)| & \text{for } |h(\Theta)|\geq\dfrac{1}{|\frac{\left[1+2\sigma\right]^{\zeta}}{\kappa}\left((1-\leftthreetimes)[3]_q^{m+1}+\leftthreetimes[3]_q^{m}\right)|}\end{cases}\tag{40}$$

Proof. From equations (23) and (24), it is derived that:

$$a_3-\Theta a_2^2=\frac{\alpha(\upsilon_2-c_2)}{2\frac{\left[1+2\sigma\right]^{\zeta}}{\kappa}((1-\leftthreetimes)[3]_q^{m+1}+\leftthreetimes[3]_q^{m})}+(1-\Theta)a_2^2$$

Also,

$$\begin{aligned}a_3-\Theta a_2^2=&\frac{\alpha(\upsilon_2-c_2)}{2\frac{\left[1+2\sigma\right]^{\zeta}}{\kappa}\left((1-\leftthreetimes)[3]_q^{m+1}+\leftthreetimes[3]_q^{m}\right)}\\&+\frac{2(1-\Theta)\alpha^2\kappa^2(\upsilon_2+c_2)}{4\alpha\kappa\left[1+2\sigma\right]^{\zeta}\left((1-\leftthreetimes)[3]_q^{m+1}+\leftthreetimes[3]_q^{m}\right)-(\alpha-1)\left[1+\sigma\right]^{2\zeta}\left((1-\leftthreetimes)[2]_q^{m+1}+\leftthreetimes[2]_q^{m}\right)^2}\end{aligned}$$

Simplify to:

$$\begin{aligned}a_3-\Theta a_2^2=\alpha\Bigg[&\left(h(\Theta)+\frac{1}{\frac{\left[1+2\sigma\right]^{\zeta}}{\kappa}((1-\leftthreetimes)[3]_q^{m+1}+\leftthreetimes[3]_q^{m})}\right)\upsilon_2\\&+\left(h(\Theta)-\frac{1}{\frac{\left[1+2\sigma\right]^{\zeta}}{\kappa}((1-\leftthreetimes)[3]_q^{m+1}+\leftthreetimes[3]_q^{m})}\right)c_2\Bigg]\end{aligned}\tag{41}$$

where

$$h(\Theta)=\frac{2\alpha(1-\Theta)\kappa^2}{4\alpha\kappa\left[1+2\sigma\right]^{\zeta}\left((1-\leftthreetimes)[3]_q^{m+1}+\leftthreetimes[3]_q^{m}\right)-(\alpha-1)\left[1+\sigma\right]^{2\zeta}\left((1-\leftthreetimes)[2]_q^{m+1}+\leftthreetimes[2]_q^{m}\right)^2}\tag{42}$$

Theorem 4. *Let $\Im(z)$ given by (1) be in the class $\mathcal{M}^{\zeta,m}_{\sigma,q,\Sigma}(\gamma,\curlywedge,\kappa)$, where $0 \le \gamma < 1,\ \curlywedge,\delta \ge 0,\ \kappa \ge 1, \sigma > 0,\ m,\zeta \in \mathbb{N}_0\ z,\varpi \in \mathbb{U}$. Then*

$$|a_3 - \vartheta a_2^2| \le \begin{cases} \frac{2\kappa(1-\gamma)}{|2\left[1+2\sigma\right]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^m)|} & \text{for } |h(\vartheta)| \le \frac{1}{|2\left[1+2\sigma\right]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^m)|} \\ 2\kappa(1-\gamma)|h(\vartheta)| & \text{for } |h(\vartheta)| \ge \frac{1}{|2\left[1+2\sigma\right]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^m)|} \end{cases} \tag{43}$$

Proof. From equations (37) and (33), it is derived that:

$$a_3 - \vartheta a_2^2 = \frac{\kappa(1-\gamma)(v_2 - c_2)}{2\left[1+2\sigma\right]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^m)} + (1-\vartheta)a_2^2$$

Also,

$$a_3 - \vartheta a_2^2 = \frac{\kappa(1-\gamma)(v_2 - c_2)}{2\left[1+2\sigma\right]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^m)} + \frac{\kappa(1-\vartheta)(1-\gamma)(v_2 + c_2)}{2\left[1+2\sigma\right]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^m)}$$

Simplify to:

$$a_3 - \vartheta a_2^2 = \kappa(1-\gamma)\left[\left(h(\vartheta) + \frac{1}{2\left[1+2\sigma\right]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^m)}\right)v_2 + \left(h(\vartheta) - \frac{1}{2\left[1+2\sigma\right]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^m)}\right)c_2\right] \tag{44}$$

where

$$h(\vartheta) = \frac{(1-\vartheta)}{2\left[1+2\sigma\right]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^m)} \tag{45}$$

4. Corollaries and Consequences

By substituting $\curlywedge = 1$ in Theorem (1) and Theorem (2), we arrive at the following corollaries, respectively:

Corollary 7. *Let $\Im(z)$ given by (1) be in the class $\mathcal{M}^{\zeta,m}_{\sigma,q,\Sigma}(1,\kappa,\alpha)$, with $0 < \alpha \le 1,\ \curlywedge \ge 0,\ \kappa \ge 1, \sigma > 0,\ m,\zeta \in \mathbb{N}_0\ z,\varpi \in \mathbb{U}$. Then*

$$|a_3-\Theta a_2^2| \leq \begin{cases} \frac{2\kappa\alpha}{|\left[1+2\sigma\right]^{\zeta}\left([3]_q^m\right)|} & \textit{for } |h(\Theta)| \leq \frac{1}{|\frac{\left[1+2\sigma\right]^{\zeta}}{\kappa}\left([3]_q^m\right)|} \\ 2\alpha|\frac{2\alpha(1-\Theta)\kappa^2}{4\alpha\kappa\left[1+2\sigma\right]^{\zeta}\left([3]_q^m\right)-(\alpha-1)\left[1+\sigma\right]^{2\zeta}\left([2]_q^m\right)^2}| & \textit{for } |h(\Theta)| \geq \frac{1}{|\frac{\left[1+2\sigma\right]^{\zeta}}{\kappa}\left([3]_q^m\right)|} \end{cases} \tag{46}$$

where

$$h(\Theta) = \frac{2\alpha(1-\Theta)\kappa^2}{4\alpha\kappa\left[1+2\sigma\right]^{\zeta}\left([3]_q^m\right)-(\alpha-1)\left[1+\sigma\right]^{2\zeta}\left([2]_q^m\right)^2} \tag{47}$$

Corollary 8. *Let $\Im(z)$ given by (1) be in the class $\mathcal{M}_{\sigma,q,\Sigma}^{\zeta,m}(\gamma,1,\kappa)$, where* $0 \leq \gamma < 1,\ \curlywedge,\delta \geq 0,\ \kappa \geq 1, \sigma > 0,\ m,\zeta \in \mathbb{N}_0\ z,\varpi \in \mathbb{U}$. *Then*

$$|a_3-\vartheta a_2^2| \leq \begin{cases} \frac{2\kappa(1-\gamma)}{|2\left[1+2\sigma\right]^{\zeta}([3]_q^m)|} & \textit{for } |\frac{(1-\vartheta)}{2\left[1+2\sigma\right]^{\zeta}([3]_q^m)}| \leq \frac{1}{|2\left[1+2\sigma\right]^{\zeta}([3]_q^m)|} \\ 2\kappa(1-\gamma)|\frac{(1-\vartheta)}{2\left[1+2\sigma\right]^{\zeta}([3]_q^m)}| & \textit{for } |\frac{(1-\vartheta)}{2\left[1+2\sigma\right]^{\zeta}([3]_q^m)}| \geq \frac{1}{|2\left[1+2\sigma\right]^{\zeta}([3]_q^m)|} \end{cases} \tag{48}$$

By substituting $\kappa = 1$ in Theorem (1) and Theorem (2), we arrive at the following corollaries, respectively:

Corollary 9. *Let $\Im(z)$ given by (1) be in the class $\mathcal{M}_{\sigma,q,\Sigma}^{\zeta,m}(\curlywedge,1,\alpha)$, with* $0 < \alpha \leq 1,\ \curlywedge \geq 0,\ \sigma > 0,\ m,\zeta \in \mathbb{N}_0\ z,\varpi \in \mathbb{U}$. *Then*

$$|a_3-\Theta a_2^2| \leq \begin{cases} \frac{2\alpha}{|\left[1+2\sigma\right]^{\zeta}\left((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^m\right)|} & \textit{for } |h(\Theta)| \leq \frac{1}{|\left[1+2\sigma\right]^{\zeta}\left((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^m\right)|} \\ 2\alpha|h(\Theta)| & \textit{for } |h(\Theta)| \geq \frac{1}{|\left[1+2\sigma\right]^{\zeta}\left((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^m\right)|} \end{cases} \tag{49}$$

where

$$h(\Theta) = \frac{2\alpha(1-\Theta)}{4\alpha\left[1+2\sigma\right]^{\zeta}\left((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^m\right)-(\alpha-1)\left[1+\sigma\right]^{2\zeta}\left((1-\curlywedge)[2]_q^{m+1}+\curlywedge[2]_q^m\right)^2} \tag{50}$$

Corollary 10. *Let $\Im(z)$ given by (1) be in the class $\mathcal{M}_{\sigma,q,\Sigma}^{\zeta,m}(\gamma,\curlywedge,1)$, where* $0 \leq \gamma < 1,\ \curlywedge,\delta \geq 0,\ \sigma > 0,\ m,\zeta \in \mathbb{N}_0\ z,\varpi \in \mathbb{U}$. *Then*

$$|a_3-\vartheta a_2^2| \le \begin{cases} \frac{2(1-\gamma)}{|2\left[1+2\sigma\right]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^{m})|} & \textit{for } |h(\vartheta)| \le \frac{1}{|2\left[1+2\sigma\right]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^{m})|} \\ 2(1-\gamma)|h(\vartheta)| & \textit{for } |h(\vartheta)| \ge \frac{1}{|2\left[1+2\sigma\right]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^{m})|} \end{cases} \tag{51}$$

where

$$h(\vartheta)=\frac{(1-\vartheta)}{2\left[1+2\sigma\right]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^{m})} \tag{52}$$

By substituting $\alpha=1$ and $\gamma=0$ respectively in The previous corollaries , we arrive:

Corollary 11. *Let* $\Im(z)$ *given by* (1) *be in the class* $\mathcal{M}^{\zeta,m}_{\sigma,q,\Sigma}(\curlywedge,1,1)$*, with* $0<\alpha\le 1,\ \curlywedge\ge 0,\ \sigma>0,\ m,\zeta\in\mathbb{N}_0\ z,\varpi\in\mathbb{U}$*. Then*

$$|a_3-\Theta a_2^2| \le \begin{cases} \frac{2}{|\left[1+2\sigma\right]^{\zeta}\left((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^{m}\right)|} & \textit{for } |h(\Theta)| \le \frac{1}{|\left[1+2\sigma\right]^{\zeta}\left((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^{m}\right)|} \\ 2|h(\Theta)| & \textit{for } |h(\Theta)| \ge \frac{1}{|\left[1+2\sigma\right]^{\zeta}\left((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^{m}\right)|} \end{cases} \tag{53}$$

where

$$h(\Theta)=\frac{2(1-\Theta)}{4\left[1+2\sigma\right]^{\zeta}\left((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^{m}\right)} \tag{54}$$

Corollary 12. *Let* $\Im(z)$ *given by* (1) *be in the class* $\mathcal{M}^{\zeta,m}_{\sigma,q,\Sigma}(0,\curlywedge,1)$*, where* $0\le\gamma< 1,\ \curlywedge,\delta\ge 0,\ \sigma>0,\ m,\zeta\in\mathbb{N}_0\ z,\varpi\in\mathbb{U}$*. Then*

$$|a_3-\vartheta a_2^2| \le \begin{cases} \frac{1}{|\left[1+2\sigma\right]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^{m})|} & \textit{for } |h(\vartheta)| \le \frac{1}{|2\left[1+2\sigma\right]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^{m})|} \\ 2|h(\vartheta)| & \textit{for } |h(\vartheta)| \ge \frac{1}{|2\left[1+2\sigma\right]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^{m})|} \end{cases} \tag{55}$$

where

$$h(\vartheta)=\frac{(1-\vartheta)}{2\left[1+2\sigma\right]^{\zeta}((1-\curlywedge)[3]_q^{m+1}+\curlywedge[3]_q^{m})} \tag{56}$$

5. Conclusions

In this paper, we introduced a new operator based on the Salagean q-differential approach to define a new class of analytic functions. We provided estimates for the Maclaurin coefficients $|a_2|$ and $|a_3|$, and addressed the Fekete–Szegő problems. Additionally, by specializing the parameters $\mathcal{M}^{\zeta,m}_{\sigma,q,\Sigma}(\leftthreetimes,\kappa,\alpha)$ and $\mathcal{M}^{\zeta,m}_{\sigma,q,\Sigma}(\gamma,\leftthreetimes,\kappa)$, we hope this study will inspire other researchers to extend this family to harmonic functions and symmetric q-calculus. Our approach can also be adapted to incorporate the symmetric q-sine and q-cosine domains as alternatives to the current domain.

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