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A Study on (c,d) $\mathcal{IF}-\mathcal{Q}$ Uniform $\mathtt{Ir}^*$ Centred Structure Compactification

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Abstract. Compactification is one of the novel extensions in topological space. Nets and filters are used to study the detailed characterization of compactness and convergence in topological spaces. The major framework of this article delves into (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure compactification. An innovative space that integrates a (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform topological space with (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ space. It explains about the irreducibility in (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform topological space. Also combines with (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform centred system that deals the intersection of open sets. This study also involves (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure filters and (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure nets which explains a detailed analysis on sequences and its convergence in (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform topological space.

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Key Words and Phrases: (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ structure space, (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure space, (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure filter and (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure net

1. Introduction

L. Zadeh in 1965, [1] had proposed a set that deals with vagueness, imprecision called Fuzzy set from universal set X and [2] give a detailed explanation on this fuzzy set. Fuzzy set explores the characterisation using parameter, linguistic variables etc. Each elements in fuzzy set is represented as membership values from the set X to $[0,1]$. It had various applications in image processing, decision making, fuzzy logics and fuzzy inference systems etc. K. Atanassov in 1986, [3] enhances a unique concept called intuitionistic fuzzy sets. It ensures the both membership and non-membership values in intuitionistic fuzzy sets.
Mathematical analysis explores the concepts of limits, contuinity, open sets, closed sets, compactness are discussed in Real numbers. After that, various mathematicians convey his ideas and explores his conception to different geometrical space is represented as topology.

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Topological space is other wise expressed as rubber- sheet geometry, it characterizes the shapes and its deformations. Connectedness, compactness and continuity are major three C's in topological space. Among these, compactness ensures about the open coverings in a give topological spaces. This also leads to a higher dimension topological space called large inductive dimension, short inductive covering dimensions, embeddings, manifolds, functions spaces, and also it leads to a another dimensional concept as algebraic topology. C. L. Chang in 1968, [4], defines fuzzy topological spaces. It enhances a various ideas on structures and spaces also its properties. Later, intuitionistic fuzzy topological space was defined by D. Coker in 1997 [5]. In both fuzzy topological space and intuitionistic fuzzy topological space leads to an concepts for connectedness, compactness etc.
B. Hutton in 1975 and 1977 [6, 7] notion on uniformities and normalities in fuzzy topological space. Among all, the main aim in this article is to explores a new essence called (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure compactification using (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure filters and (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structue nets. In this concept, various results regarding compactifications and its properties are explored. The relationship between nets and filters in topological spaces is to understand the compact spaces and its characteristics. The (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure compactification is motivated by a need to bridge certain gaps in the theory of compactifications of uniform spaces, particularly those involving irregular and quasi-uniform structures. Traditional compactifications, such as Stone–Čech and Samuel compactifications, are largely constructed under assumptions of regularity, symmetry, or completeness. However, many natural and important spaces in both topology and analysis, especially quasi-uniform spaces and structures arising in generalized function theory, lack these properties. The extension of (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure compactification more flexible treatment of quasi-uniformities that do not necessarily satisfy classical uniform conditions, enabling a broader class of spaces to be compactified meaningfully. Also the broader field of topology, this research positions itself at the intersection of compactification theory, uniform space theory, and generalized convergence structures. It contributes to the ongoing effort to generalize classical results to more flexible, non-standard settings, which are increasingly relevant in modern applications such as rigid motions in theoretical physics, other higher dimension topological spaces, and the study of generalized metric spaces. An filters can associate with nets and vice versa to analyze convergence and compactness. Among that the relationship between nets, filters and compactifications are also associated with each other. Filters are used to analyse the compact spaces and construct compactifications. Likewise, the nets are tedious to ensure the convergence in compactified space.

2. Literature Review

E. Narmada and et. al [8] had framed a C structure to define an compactification in intuitionistic fuzzy topological spaces. The article, explores an detailed analysis of C structure spaces using T^C filters for compactification. In 2014, G. K. Revathi et. al [9] had researched a new approach on Wallman-type compactification via intuitionistic fuzzy rough centred texture spaces. Also in 2015, G. K. Revathi et. al [10] express an novel idea of compactification via semigroup and intuitionistic fuzzy convergence topological spaces. Later on that, in 2015, Ridvan Sahin developed his ideas to soft set. Soft sets deals with paramaters. The author had define a compactification and its properties on soft sets. In 2019, Ceren Sultan ELIMALI et. al [11] defined an FAN-GOTTESMAN compactifications and stone spaces along with properties are discussed. Likewise [12]the topological group of transformations explain about the pointwise convergence topology and admissible group topology with the structure equivariant compactifications. Also [13] introduced the category of stable compactifications and Raney extensions of proximity frames in duality spaces. In this articles, the author defines an comparitive analysis on other type of compactifications. Also they explore his ideas in clopen sets, ultrafilters, non-convergent spaces, etc. An application related to compactifications [14–16] are used in both the theoretical approaches like lie algebra, bounded operators, etc,. and also applied in different fields like physics, robotics, etc.

3. Motivation and Contribution of the Study

The major motivation behind the (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform topological space is an major extension of intuitionsitic fuzzy topological space. A (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform topological space is a highly generalized mathematical structure that blends concepts from uniform topology, intuitionistic fuzzy sets, and quasi-uniformity. It is designed to model uncertainity in topological spaces. This space has led to the numerous concepts in topological space is functors, morphisms etc. Also in (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ structure space deals with the irreducibility in topological spaces. This leads to Zariski topology, spectral space and Jac-spectral space etc., These are the higher dimension topological space combined with algebraic geometry and commutative algebra. Here, this motivate for novel research ideas which can be incorporated to various domains. Likewise, Centred systems deals only with the collection of open sets in the given topological space. So, here the reserach work incorporates the different spaces in (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform topological space. Based on the literature survey in Section-2, the authors introduced the novel idea called (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}$ topological space. Many researchers depicted and applied compactifications in robotics, physics, lie groups, etc., which leads authors to study (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure compactification. To increase the readers interest, the authors introduced (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure filters and (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure nets which supported the study of (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure compactification.

4. Proposed Structure of the Paper

In this article, an idea of Compactification on (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure space is defined. The below flowchart shows the process of compactification is executed via (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred Filters and (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform Nets.

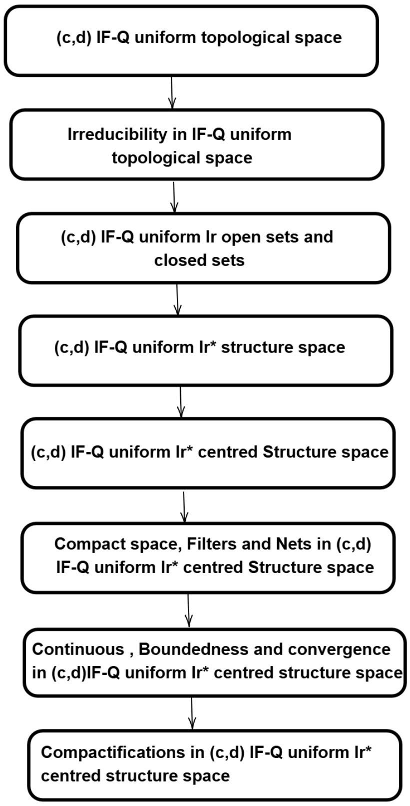

Figure 1: A Study on (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure compactification

5. Preliminaries

Here IF denotes the intuitionistic fuzzy set on X and throughout the article, and the universe of discourse X is a non-empty set.

Definition 1. [3] Let X be a universal set and an IF set A in X is defined as $A = \{\langle x, \mu_A(x), \nu_{A(x)}\rangle : x \in X\}$ where $\mu_A(x) : X \to [0,1]$ and $\nu_A(x) : X \to [0,1]$ are the membership and non-membership functions respectively for every $x \in X$, with the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition 2. [5] An IF topology τ on X is a collection of intuitionistic fuzzy sets, the following assertion are to be hold:

(i) $0_\sim$,$1_\sim \in \tau$,

(ii) Each $A_i \in \tau$, then $\cap_{i=1}^{n} A_i \in \tau$.

(iii) For all $A_i \in \tau$ then $\cup_i \in JA \in \tau$.

The pair (X, τ) is referred to as IF topological space on X

Definition 3. [17] Suppose Ξ_X represents the collection of IF mappings, $g : IF(X) \to IF(X)$ then the following are:

(i) $g(0_\sim)$ =$0_\sim$

(ii) $A \subseteq g(A)$, $\forall A \in$ IF(X)

(iii) $g(\cup A_i)$= $\cup g(A_i), \forall A_i \in IF(X)$, i $\in$ J

For $g \in \Xi_X$, the function $g^{-1}(A)$= $\cap\{B : g(\bar{B}) \subseteq \bar{A}\} \in \Xi_X$, given for all $A \in IF(X)$, $g \sqcap g^{'}(A)$=$\cap\{g(A_1) \cup g^{'}(A_2) : A_1 \cup A_2 = A\}$, $(g \circ g^{'})$ (A)=$g(g^{'}(A))$.

Definition 4. [17] Let υ: $\Xi_X \to I \times I$ be an IF mapping. Then υ is characterized as an IF quasi uniformity on X, if it fulfills the circumstances:

(i) $\upsilon(g_1 \sqcap g_2) \supseteq \upsilon(g_1) \cap \upsilon(g_2)$ for g_1, $g_2 \in \Xi_X$

(ii) $g \in \Xi_X$ we have $\cup\{\upsilon(g_1) : g_1 \circ g_1 \subseteq g\} \supseteq \upsilon(g)$

(iii) $g_1 \supseteq g$ then $\upsilon(g_1) \supseteq \upsilon(g)$

(iv) $g \in \Xi_X$ then $\upsilon(f)$=$1_\sim$.

A pair (X, υ) is claimed as $\mathcal{IF} - \mathcal{Q}$ uniform space.

Definition 5. [17] A pair (X, υ) be an $\mathcal{IF} - \mathcal{Q}$ uniform space. Let $c \in (0, 1]$ =I_0 and $d \in [0, 1)$ =I_1 with $c + d \leq 1$ and $A \in IF(X)$.
$(c,d)IFQI_\upsilon(A) = \cup\{B : f(B) \subseteq A$, some of $f \in \Xi(X)$ and $\upsilon(f) > (c,d)\}$

Definition 6. [17] Consider (X, υ) be an $\mathcal{IF} - \mathcal{Q}$ uniform space The mapping τ_υ is defined by: $IF(X) \to I \times I$ is defined by
$\tau_\upsilon(A) = \cup\{(c,d) : (c,d)\ IFQI_\upsilon(A)=A,\ c \in I_0,\ d \in I_1$ with $c + d \leq 1\}$.Hence, a pair (X, τ_υ) is called $\mathcal{IF} - \mathcal{Q}$ uniform topological space. The elements of (X, τ_υ) is called *(c,d)* $\mathcal{IF} - \mathcal{Q}$ uniform open sets and its complement is *(c,d)* $\mathcal{IF} - \mathcal{Q}$ uniform closed sets.

Example 1. Assume $X = \{w,y\}$.
$A = \left\langle x, \left(\frac{w}{0.3}, \frac{y}{0.5}\right), \left(\frac{w}{0.4}, \frac{y}{0.1}\right)\right\rangle$, $B = \left\langle x, \left(\frac{w}{0.1}, \frac{y}{0.2}\right), \left(\frac{w}{0.5}, \frac{y}{0.6}\right)\right\rangle$ and $C = \left\langle x, \left(\frac{w}{0.3}, \frac{y}{0.2}\right), \left(\frac{w}{0.4}, \frac{y}{0.3}\right)\right\rangle$ be any three IF sets on X and let E is a non-void IF set.

Let Ξ_X: $IF(X) \to IF(X)$ be an IF mapping. Let g_1, g_2, g_3 and $g_4 \in \Xi_X$ be defined as:

$$g_1(E) = \begin{cases} 0_\sim, & ifE = 0_\sim \\ 1_\sim, & otherwise \end{cases}$$

$$g_2(E) = \begin{cases} 0_\sim, & ifE = 0_\sim \\ A, & ifE \subseteq A \\ 1_\sim, & otherwise. \end{cases}$$

$$g_3(E) = \begin{cases} 0_\sim, & ifE = 0_\sim \\ B, & ifE \subseteq B \\ 1_\sim, & otherwise. \end{cases}$$

$$g_4(E) = \begin{cases} 0_\sim, & ifE = 0_\sim \\ C, & ifE \subseteq C \\ 1_\sim, & otherwise. \end{cases}$$

$$\upsilon(g) = \begin{cases} (1,0), & ifg = g_1 \\ (2/7,5/8), & ifg = g_2 \\ (3/6,1/5), & ifg = g_3 \\ (5/9,9/11), & ifg = g_4 \\ (4/7,1/9), & ifg = g_2 \sqcap g_3 \\ (5/7,8/9), & ifg = g_2 \sqcap g_4 \\ (5/9,6/7), & ifg = g_3 \sqcap g_4 \\ (0,1), & otherwise \end{cases}$$

Clearly, (X, υ) is an $\mathcal{IF} - \mathcal{Q}$ uniform space, for c=0.02 and d=0.05. define intuitionistic fuzzy mapping, τ_υ: $IF(X) \to I \times I$ as

$$\tau_\upsilon(E) = \begin{cases} (0,1), & ifE = 0_\sim \\ (4/5,2/7), & ifE = A \\ (2/5,3/7), & ifE = B \\ (1/8,2/7), & ifE = C \\ (1,0), & otherwise \end{cases}$$

Hence τ_υ=$\{A, B, C, 0_\sim, 1_\sim\}$. Clearly (X, τ_υ) is an $\mathcal{IF} - \mathcal{Q}$ uniform topological space.

Definition 7. [17] A pair (X, τ_υ) be an $\mathcal{IF} - \mathcal{Q}$ uniform topological space and A be an IF set. The $\mathcal{IF} - \mathcal{Q}$ uniform interior of A is then described as $(c,d)IFQint_\upsilon(A)$=$\cup\{B : B \subseteq A$ and B is a (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform open set where $c \in I_0$, $d \in I_1$, and c+$d \leq 1\}$.

Definition 8. [17] Let (X, τ_v) be an $\mathcal{IF} - \mathcal{Q}$ uniform topological space and A be a IF set. Then the $\mathcal{IF} - \mathcal{Q}$ uniform closure of A is expressed as $(c,d)IFQcl_v(A){=}\cap\{B : B \supseteq A$ and B is an (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform closed set where $c \in I_0,\ d \in I_1$ with $c{+}d \leq 1\}$

6. Compactification in (c,d) $\mathcal{IF} - \mathcal{Q}$ Uniform $\mathtt{Ir}^*$-structure space

Throughout the article the word "bounded" is represented as "bdd".

6.1. Irreducibility and (c,d) $\mathcal{IF} - \mathcal{Q}$ Uniform $\mathtt{Ir}^*$ Centred Structure space in $\mathcal{IF} - \mathcal{Q}$ Uniform Topological space

Definition 9. Let (X, τ_v) represents an $\mathcal{IF} - \mathcal{Q}$ uniform topological space. A, B and C $\in IF(X)$. An IF set A is said to be irreducible iff $A \subseteq B \cup C$, such that $A \subseteq B$ or $A \subseteq C$ with $B \neq 1_\sim$ and $C \neq 1_\sim$

Definition 10. A pair (X, τ_v) be $\mathcal{IF} - \mathcal{Q}$ uniform topological space and $A, B,$ C be any (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform open sets is said be to irreducible, iff $A \subseteq B \cup C$ such that $A \subseteq B$ or $A \subseteq C$. Then A is said to be (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform irreducible open set. The complement of (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform irreducible open sets is said to be (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform irreducible closed set.

Example 2. Let $X = \{g,h\}$ be a non empty set, $A = \left\langle x, \left(\frac{g}{0.1}, \frac{h}{0.2}\right), \left(\frac{g}{0.3}, \frac{h}{0.5}\right)\right\rangle$ $B = \left\langle x, \left(\frac{g}{0.2}, \frac{h}{0.3}\right), \left(\frac{g}{0.2}, \frac{h}{0.2}\right)\right\rangle, C = \left\langle x, \left(\frac{g}{0.4}, \frac{h}{0.4}\right), \left(\frac{h}{0.1}, \frac{h}{0.1}\right)\right\rangle$ and $D = \left\langle x, \left(\frac{g}{0.8}, \frac{h}{0.1}\right), \left(\frac{g}{0.03}, \frac{h}{0.01}\right)\right\rangle$ be any four IF sets on X and let E is a non-empty IF set on X. Consider the IF mapping, Ξ_X: $IF(X) \to IF(X)$ $g_1, g_2,$ $g_3,$ g_4 and $g_5 \in \Xi_X$ be defined as:

$$g_1(E) = \begin{cases} 0_\sim, & if E = 0_\sim \\ 1_\sim, & otherwise. \end{cases}$$

$$g_2(E) = \begin{cases} 0_\sim, & if E = 0_\sim \\ A, & if E \subseteq A \\ 1_\sim, & otherwise. \end{cases}$$

$$g_3(E) = \begin{cases} 0_\sim, & if E = 0_\sim \\ B, & if E \subseteq B \\ 1_\sim, & otherwise. \end{cases}$$

$$g_4(E) = \begin{cases} 0_\sim, & if E = 0_\sim \\ C, & if E \subseteq C \\ 1_\sim, & otherwise. \end{cases}$$

$$g_5(\mathcal{E}) = \begin{cases} 0_\sim, & if E = 0_\sim \\ D, & if E \subseteq D \\ 1_\sim, & otherwise. \end{cases}$$

$$\upsilon(g) = \begin{cases} (1,0), & if g = g_1 \\ (2/7,5/8), & if g = g_2 \\ (3/6,1/5), & if g = g_3 \\ (3/7,1/6), & if g = g_4 \\ (3/8,1/8), & if g = g_5 \\ (4/9,1/9), & if g = g_2 \sqcap g_3 \\ (5/7,5/9), & if g = g_2 \sqcap g_4 \\ (6/7,6/9), & if g = g_2 \sqcap g_5 \\ (3/7,8/9), & if g = g_3 \sqcap g_4 \\ (4/9,1/10), & if g = g_3 \sqcap g_5 \\ (1/6,4/7), & if g = g_4 \sqcap g_5 \\ (0,1), & otherwise \end{cases}$$

Clearly, (X, υ) is an $\mathcal{IF} - \mathcal{Q}$ uniform space, for c=0.08 and d=0.03. define an IF mapping, τ_υ: $IF(X) \to I \times I$ as

$$\tau_\upsilon(E) = \begin{cases} (0,1), & if E = 0_\sim \\ (2/5,3/7), & if E = A \\ (1/8,2/7), & if E = B \\ (1/3,3/8), & if E = C \\ (2/5,3/5), & if E = D \\ (1,0), & otherwise \end{cases}$$

Then τ_υ=$\{A, B, C, D, 0_\sim, 1_\sim\}$ be an $\mathcal{IF} - \mathcal{Q}$ uniform topological space. The elements of τ_υ are called (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform open sets and the complements are (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform closed sets. Clearly (X, τ_υ) is an $\mathcal{IF} - \mathcal{Q}$ uniform topological space. Let A, B and C are (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform irreducible open sets. From the Definition 6.2, the collection $\mathtt{Ir}$= $\{A, B, C, 0_\sim\}$ are (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform irreducible open sets.

Remark 1. From the above Example 6.3, Clearly $\mathtt{Ir}^*$= $\mathtt{Ir} \cup \{1_\sim\}$. Then $\mathtt{Ir}^*$= $\{A, B, C, 0_\sim\} \cup$ $1_\sim$ is said to be (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ irreducible open sets.

Definition 11. Let (X, τ_υ) be an $\mathcal{IF} - \mathcal{Q}$ uniform topological space on X. Then (X, τ_υ) is said to be (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ structure space, then the corresponding requirements are to be fulfilled:

(i) $0_\sim, 1_\sim \in \mathtt{Ir}^*$.

(ii) If $\{A_i : i \in I, \forall A_i \in \mathtt{Ir}^*\}$ where $\cup_{i\in I} A_i \in \mathtt{Ir}^*$.

(iii) If $A, B \in \mathtt{Ir}^*$, then $A \cap B \in \mathtt{Ir}^*$.

Every member of (X, τ_v) is said to be (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ structure space. The elements of (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ structure space is (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ structure open sets. The complements of (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ structure open set is (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ structure closed sets.

Definition 12. Let (X,τ_v) be $\mathcal{IF} - \mathcal{Q}$ uniform topological space. Then (X,τ_v) is said to be (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ structure hausdorff space, iff for every $x_1, x_2 \in X$ and $x_1 \neq x_2$ implies that there exists $G_1= \langle x, \mu_{G_1}, \nu_{G_1}\rangle$, $G_2= \langle x, \mu_{G_2}, \nu_{G_2}\rangle \in \tau_v$ with $\mu_{G_1}(x_1)= 1_\sim$, $\nu_{G_1}(x_1)= 0_\sim$, $\mu_{G_2}(x_2)= 1_\sim$, $\nu_{G_2}(x_2)= 0_\sim$ and $G_1 \cap G_2= 0_\sim$

Definition 13. Let (X, τ_v) is said to be (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ structure hausdorff space. The collection $P = \{A_i\}_{i\in\delta}$ of all (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ structure open sets of (X, τ_v) referred to as (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure system, if for any finite collection of elements in $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ structure space such that $\cap_{i=1}^{n} A_i \neq 0_\sim$.

Definition 14. Consider, the sets $P_X= \{P_i : i \in \delta\}$ where $P_i^{'}s$ are (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure systems in (X, τ_v) which are also called as (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure points. Then the family τ_P is said to be an (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure, if it satisfies the following conditions:

(i) $\emptyset, P_X \in \tau_{\mathcal{P}}$

(ii) $\cup_{i\in J}\ \tau_P$ is in τ_P.(arbitrary union)

(iii) $\cap_{i=1}^{n}\ \tau_P$ is in τ_P.(finite intersection)

The pair (P_X, τ_P) is called (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure space. Each members of (P_X, τ_P) are called (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure open set. The complement of an (c,d) $\mathcal{IF} - \mathcal{Q}$ $\mathtt{Ir}^*$ centred structure uniform open set is (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure closed set.

Definition 15. Let (P_X,τ_P) be (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure space and $A \subseteq P_X$. Then (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure closure and (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure interior of A is defined as
$\mathtt{Ir}^*\ C_p\ \mathrm{Cl}(A)=\cap\{B : B$ is an (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure closed set and $A \subseteq B\}$
$\mathtt{Ir}^*\ C_p\ \mathrm{Int}(A)=\cap\{B : B$ is an (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure closed set and $A \supseteq B\}$

Definition 16. Let (P_X,τ_P) be the (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure space. A collection $\{A_i : i \in \delta\}$ of (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure open sets in a (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure space (P_X, τ_P) is called a (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure open cover of (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred subset B of P_X, if $B \subseteq \cup\ \{A_i : i \in \delta\}$.

Definition 17. Let (P_X,τ_P) be the (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure space is said to be a (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure compact, if for every (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ open cover of P_X possess a finite subcover.

Definition 18. A pairs (P_X, τ_P) and (P_Y, τ_P^*) be any two (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure space. Then f: (P_X, τ_P) $\to$ (P_Y, τ_P^*) is an (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure continuous function, if $f^{-1}(V)$ is an (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure open set in ($\mathcal{P}_X$, τ_P) for each (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$centred open set V in (P_Y, τ_P^*).

Definition 19. Let (P_X, τ_P) be a (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure space and let $P \in P_X$. Then a (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure subset $N \subseteq P_X$ is said to be (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ structure neighborhood, if there exists a (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ structure open sets G such that $P \in G \subseteq N$.

Definition 20. Let (P_X, τ_P) be a (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure space. A subset A of P_X is said to be a (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$centred structure dense, if $\texttt{Ir}^*C_P$ cl(A)=P_X.

Definition 21. A pair (P_X, τ_P) be a (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure space. Then a non-empty family $\mathbf{F}$ of subsets of P_X is called (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure filter on P_X, iff it satisfies the following conditions:

(i) $\emptyset \in \mathbf{F}$

(ii) Assume $F \in \mathbf{F}$ and $F \subseteq H$, then $H \in \mathbf{F}$.

(iii) Consider $F_1, F_2 \in \mathbf{F}$, then $F_1 \cap F_2 \in \mathbf{F}$

Definition 22. Consider (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure net in an (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure space (P_X, τ_P) is a function from a directed set Δ to P_X. It is denoted as $\{P_\zeta\}_{\zeta\in\varsigma}$

6.2. Nets, Filters and Convergence in (c,d) $\mathcal{IF}-\mathcal{Q}$ Uniform $\texttt{Ir}^*$ Centred Structure space

Notation: Here, throughout this article the notation $\oslash$ is used for eventually and $\ominus$ is used for frequently

Definition 23. Let $\{\ P_\zeta\ \}_{\ \zeta\in\varsigma}$ be a (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure net in an (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure space P_X and let G be a (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure subset of P_X. Then the (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure net is expressed as

(i) in G iff $\{\mathcal{P}_\zeta\} \in G$, $\forall\zeta \in \varsigma$.

(ii) $\oslash \in G$ iff there is an existence of $\vartheta \in \varsigma \forall\alpha \in \varsigma\ \alpha \geq \vartheta$, $\{P_\zeta\} \in G$

(iii) $\ominus \in G$, iff for all $\vartheta \in \varsigma$, there is an existence of $\zeta \in \varsigma, \zeta \geq \vartheta$ and $\{P_\zeta\} \in G$

Definition 24. Let $\{P_\zeta\}$ is a (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure net in the (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure P_X and P is a (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure element of P_X. An (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure net converges towards P iff for every (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure neighborhood U of P, such that $\{P_\zeta\}$ $\oslash$ in U.

Definition 25. A pair (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure element P_1 of P_X is said to be a (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure accumulation point or cluster point of a (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure net iff for every (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure neighborhood U of P_1, so that (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure net is $\ominus$ in U.

Definition 26. Let (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure net $\{P\}$ in a set P_X is said to be (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure universal or (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure ultranet, if for every (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure subset A of P_X, either $\{P\}$ is $\oslash$ in A or $\{P\}$ is $\oslash$ in $P_X - A$.

Definition 27. A collection ϕ represents (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure continuous function on an (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure space P_X. A (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure net $\{P_i\}$ in P_X will be called as (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure ϕ net, converges to each f in the mapping ϕ.

Definition 28. A mapping ϕ contains a collection of bdd real valued continuous function on P_X. Here, P_X is a (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure compact set, for every ϕ net has an (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure cluster point in P_X.

Definition 29. Let P_X be an (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure space, $C^*(P_X)=$ $\{f_\zeta : \zeta \in \varsigma\}$ be the collection of all bdd real-valued continuous function on P_X. Regarding a $C^*(P_X)$ net $\{P_i\}$ and $F_{\{P_i\}}= \{U : U$ is a (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure open set in P_X and $\{P_i\}$ is $\oslash$ in $U\}$. By knowing that, $F_{\{P_i\}}$ is an (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure open filter and some $f_\zeta \in C^*(P_X)$, any $\epsilon > 0, (f_\zeta)^{-1}((r_\zeta - \varepsilon, r_\zeta + \varepsilon)) \in F_{\{P_i\}}$ where $r_\zeta= \lim \{f_\zeta(P_i)\}$ $F_{\{P_i\}}$ is the (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure open filter in P_X induced by $\{P_i\}$.

Definition 30. Let F is a (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred strcture filter on P_X. Let $\varsigma_F= \{(P,F) : P \in F \in F\}$. So ς_F is expressed as the relation $(P_1,F_1) \leq (P_2,F_2)$ iff $F_2 \subset F_1$ and the function $M : \varsigma_F \to P_X$ expressed as $M(P,F_1) = P$ is an (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure Net in P_X. It is called as (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred net based on F.

Definition 31. The pair (c,d) be an $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure filter F_P converges to P in P_X, if the (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure net converges to P with respect to F_P.

Definition 32. Let P be a (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure open filter on P_X and $\{P_i\}$ be an (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred net related to P, and $I=\{U : U$ is a (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure open in P_X also $\{P_i\}$ is $\oslash$ in $U\}$. Then $I = P$

Remark 2. Every $C^*(P_X)$ net $\{P_i\}$ in P_X and also $\{w_k^{P_i}\}$ be the (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred Net based on the (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure open filter $F_{\{P_i\}}$ induced by $\{P_i\}$.

(i) $\{w_k^{P_i}\}$ is distinctly established as $F_{\{P_i\}}$ and $F_{\{P_i\}}= F_{\{P_j\}}$ iff $\{w_k^{P_i}\}= \{w_k^{P_j}\}$.

(ii) $F_{\{P_i\}}$=$F_{\{w_k^{P_i}\}}$ $= \{O : O$ is (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure open set in P_X and $\{w_k^{P_i}\}$ is eventually in O $\}$

(iii) $\{w_k^{P_i}\}$ is a $C^*(\mathcal{P}_X)$ net along with lim $\{f_\zeta(w_k^{P_i})\}$= lim $\{f_\zeta(P_i)\}$ $\forall$ f_ζ in $C^*(P_X)$.

(iv) requirements are to be hold:

  (a) $\{w_k^{P_i}\}$ convergent to P

  (b) Here, converged to P with respect to $\{P_i\}$.

  (c) $F_{\{P_i\}}$ converges to P.

Let Y_P= $\{\{w_k^{P_i}\}^* : \{P_i\}$ is an $C^*(P_X)$ net does not converge to P_X, $\{w_k^{P_i}\}$ is (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure Net based on (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure filter $F_{\{P_i\}}\}$, P_X^*= $P_X \cup Y_P$, the disjoint union of P_X and Y_P. For each (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure open set $U \subset P_X$ define $U^* \subseteq P_X^*$ and the set U^*= $U \cup \{\{w_k^{P_i}\}^* : \{w_k^{P_i}\}^*\} \in Y_P$ and $\{w_k^{P_i}\}$ is $\oslash$ in $U\}$. It is apparent that if $U \subset V$, then $U^* \subset V^*$.

Proposition 1. Let U and V be any two (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure open sets in P_X, then $(U \cap V)^*$= $U^* \cap V^*$.

Proof. Let $P_2 \in (U \cap V)^* \cap Y_P$, then P_2= $\{w_k^{P_i}\}^*$ and $\{w_k^{P_i}\}$ is $\oslash$ in $U \cap V$. This indicates that $\{w_k^{P_i}\}$ is $\oslash \in U$,V. Here, $\{w_k^{P_i}\}^* \in U^* \cap V^*$. If $P_2 \in (U^* \cap V^*) \cap Y_P$, then P_2= $\{w_k^{P_i}\}^*$ and $\{w_k^{P_i}\}$ is $\oslash$ is both in U and V. So $\{w_k^{P_i}\}$ is $\oslash \in U \cap V$. Then $P_2 \in (U \cap V)^*$

Proposition 2. Assume B= $\{U^*$: U be an (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure open set in P_X $\}$. Then B is an (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure base for an (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\texttt{Ir}^*$ centred structure on P_X^*, if

(i) $P_X^* = \{U^* : U^* \in B\}$

(ii) Each U^*, $V^* \in B$ with $P_2 \in U^* \cap V^*$ some of the W^*= $(U^* \cap V^*) \in B$, $P_2 \in W^* \subset U^* \cap V^*$

Proof.

(i) $P_X^* = \{U^* : U^* \in B\}$, let $P_2 \in Y_P$, then $\mathcal{P}_2 = \{w_k^{P_i}\}^*$. For any $f_\zeta \in C^*(P_X)$, let $r_\zeta =$ lim $\{f_\zeta(w_k^{P_i})$, then $\{w_k^{P_i}\}$ is $\oslash$ in $f_\alpha^{-1}((r_\zeta - \varepsilon, r_\zeta + \varepsilon))$, for any $\varepsilon > 0$, that is $\{w_k^{P_i}\}^*$ is in $f_\zeta^{-1}((r_\zeta - \varepsilon, r_\zeta + \varepsilon))$ for all $\upsilon > 0$, accordingly $Y_P \subset \cup\{U^* : U^* \in B\}$ consequently $P_X^* \subset \cup\{U^* : U^* \in B\}$. For $\{U^* : U^* \in B\} \subset P_X^*$ is explained.

(ii) if $P_2 \in U^* \cap V^*$, for any U^* and V^* in B, since $(U \cap V)^*$ is in B and $(U \cap V)^* = U^* \cap V^*$, thus $P_2 \in (U \cap V)^* \subset U^* \cap V^*$.

Remark 3. Consider the P_X^* with (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure induced by the (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure base B. For all f_ζ in $C^*(P_X)$, defined f_ζ^*: $P_X^* \to R$ define that $f_\zeta^*(P_1) = f_\zeta(P_1)$ if $P_1 \in P_X$, $f_\zeta^*(\{w_k^{P_i}\}^*) =$ lim $\{f_\zeta(w_k^{P_i})\}$ for any $\{w_k^{P_i}\}^*$ in Y_P and it is stated as f_ζ^* is clearly specified that it is bdd real valued function on P_X^*.

Proposition 3. For Any f_ζ in $C^*(P_X)$, f_ζ^* is a bounded real valued continuous function on P_X^*.

Proof. It is enough to proof the continuity, f_ζ^* at any P_3 in P_X^*, let $t_\zeta = f_\zeta^*(P_3)$. It will be shown that for any $\varepsilon > 0$, there is an (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure open set $U^* \in B$ such that $P_3 \in U^* \subset (f_\zeta^*)^{-1}((t_\zeta - \varepsilon, t_\zeta + \varepsilon))$. Let $\mathsf{U} = f_\zeta^{-1}((t_\zeta - \varepsilon/2, t_\zeta + \varepsilon/2))$. If $P_3 \in P_X$, since $f_\zeta(P_3) = f_\zeta^*(P_3) = t_\zeta$, thus $P_3 \in f_\zeta^{-1}((t_\zeta - \varepsilon/2, t_\zeta + \varepsilon/2)) \subset (f_\zeta^{-1}((t_\zeta - \varepsilon/2, t_\zeta + \varepsilon/2)))^*$. If $P_3 \in Y_P$, then $P_3 = \{w_k^{P_i}\}^*$. Since $t_\zeta = f_\zeta^*(P_3) =$ lim $\{f_\zeta(w_k^{P_i})$, so $\{w_k^{P_i}\}$ is $\oslash$ in $f_\zeta^{-1}((t_\zeta - \varepsilon/2, t_\zeta + \varepsilon/2))$. that is $P_3 = \{w_k^{P_i}\}^* \in (f_\zeta^{-1}((t_\zeta - \varepsilon/2, t_\zeta + \varepsilon/2)))^*$. Finally show that, $(f_\zeta^{-1}((t_\zeta - \varepsilon/2, t_\zeta + \varepsilon/2)))^* \subset (f_\zeta^*)^{-1}((t_\zeta - \varepsilon, t_\zeta + \varepsilon))$. If P_1 is in $P_X \cap (f_\zeta^{-1}((t_\zeta - \varepsilon/2, t_\zeta + \varepsilon/2)))^*$ then $P_1 \in f_\zeta^{-1}((t_\zeta - \varepsilon/2, t_\zeta + \varepsilon/2))$ that is $f_\zeta^*(P_1) = f_\zeta(P_1) \in (t_\zeta - \varepsilon, t_\zeta + \varepsilon)$. So, $P_1 \in (f_\zeta^*)^{-1}((t_\zeta - \varepsilon, t_\zeta + \varepsilon))$. If $P_2 \in (f_\zeta^{-1}((t_\zeta - \varepsilon/2, t_\zeta + \varepsilon/2)))^* \cap Y_P$, then $P_2 = \{w_k^{P_i}\}^*$ and $\{w_k^{P_i}\}$ is $\oslash$ in $(f_\zeta)^{-1}((t_\zeta - \varepsilon/2, t_\zeta + \varepsilon/2))$ thus $f_\zeta^*(P_2) =$ lim $\{f_\zeta(w_k^{P_i})\} \in [t_\zeta - \varepsilon/2, t_\zeta + \varepsilon/2] \subset (t_\zeta - \varepsilon, t_\zeta + \varepsilon)$ that is $P_2 \in (f_\zeta^*)^{-1}((t_\zeta - \varepsilon, t_\zeta + \varepsilon))$.

Proposition 4. Let $K : P_X \to P_X^*$ be defined by $K(P_1) = P_1$, then K is an (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure continuous mapping from P_X into P_X^*.

Proof. For any (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure open set $U^* \in B$, $K^{-1}(U^*) = U$ is a (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure open set in P_X, so K is an (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure continuous mapping on P_X.

Proposition 5. For any P_2 in $P_X^* - P_X$ with $P_2 = \{w_k^{P_i}\}^*$, $\{k(w_k^{P_i})\}$ converges to $P_2 = \{(w_k^{P_i})\}^*$.

Proof. Let U^* be any (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure open set in B containing P_2 then $\{(w_k^{P_i}\}$ and $\oslash \in U$ in P_X. This implies that $\{k(w_k^{P_i})\}$ is $\oslash$ in U^*, thus $\{K(w_k^{P_i})\}$ it is converged to $P_2 = \{w_k^{P_i}\}^*$.

Proposition 6. $K(P_X)$ is an (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure dense in P^*_X

Proof. For any P_2 in $P^*_X - P_X$. $P_2 = \{w^{P_i}_k\}^*$. By the above Proposition 5, implies that $\{K(w^{P_i}_i)\}$ converges to P_2=$\{w^{P_i}_k\}^*$. Thus $\mathtt{Ir}^*$ C_p Cl($K(P_X)$)=P^*_X.

Remark 4. Here, C=$\{f^*_\zeta : f_\zeta \in \delta\}$ represent $\{f^*_\zeta : f_\zeta \in C^*(P_X)\}$. Each C net P_i in P^*_X and $E = \{O : O$ is an (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure open in P^*_X and $\{P_i\}$ is $\oslash$ in $O\}$.
$L = \{U : U$ is an (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure open in P_X and $U^* \in E\}$.

Proposition 7. For a C net $\{P_i\}$, in P^*_X. Let r_ζ= lim $\{f^*_\zeta(P_i)\}$. All $f^*_\zeta \in C$. Then for arbitrary $\varepsilon > 0$, $(f^*_\zeta)^{-1}((r_\zeta - \varepsilon, r_\zeta + \varepsilon)) \subset (f^{-1}_\zeta((r_\zeta - \varepsilon, r_\zeta + \varepsilon)))^*$.

Proof. Consider, $P_3 \in (f^*_\zeta)^{-1}((r_\zeta - \varepsilon, r_\zeta + \varepsilon))$, then $f^*_\zeta(P_3) \in (r_\zeta - \varepsilon, r_\zeta + \varepsilon)$. If P_3 =$K(P_1) = P_1$,$\forall$ $P \in P_X$. Given that, $f_\zeta(P_1)$= $f^*_\zeta(P_3)$, so P_1 is in $(f_\zeta)^{-1}((r_\zeta - \varepsilon, r_\zeta + \varepsilon))^*$. If P_3= $\{w^{P_i}_k\}^*$ in Y_P, then lim $\{f_\zeta(w^{P_i}_k)\}$= $f^*_\zeta(P_3) \in (r_\zeta - \varepsilon, r_\zeta + \varepsilon)$. This implies that $\{w^{P_i}_k\}$ is $\oslash$ in $(f_\zeta)^{-1}$ $((r_\zeta - \varepsilon, r_\zeta + \varepsilon))$ thus $\{w^{P_i}_k\}$ is in $(f_\zeta)^{-1}(r_\zeta - \varepsilon, r_\zeta + \varepsilon)^*$.

Corollary 1. For a C net $\{P_i\}$ in P^*_X. Let r_α=lim $\{f^*_\zeta(P_i)\}$ for every $f^*_\zeta \in C$. Then for arbitrary $\varepsilon > 0$, $(f^*_\zeta)^{-1}((r_\zeta - \varepsilon, r_\zeta + \varepsilon)) \in E$ and $(f_\zeta)^{-1}((r_\zeta - \varepsilon, r_\zeta + \varepsilon)) \in L$.

Proof. Here, $(f^*_\zeta)^{-1}$ $((r_\zeta - \varepsilon, r_\zeta + \varepsilon)) \in E$. By the above Proposition 7, $\{P_i\}$ is $\oslash$ in $(f_\zeta)^{-1}((r_\zeta - \varepsilon, r_\zeta + \varepsilon))^*$, thus $(f_\zeta)^{-1}((r_\zeta - \varepsilon, r_\zeta + \varepsilon)) \in L$ -

Proposition 8. E and L are (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure open filter on P^*_X and P_X respectively.

Proof. Building on the proof of Proposition 1 and Corollary 1 , it is evident that $\mathcal{E}$ is an (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure filter on P^*_X. By Corollary 1 $L \neq \emptyset$. If U, V are (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred strcuture open sets in L, then U^* and $V^* \in E$. Since $(U \cap V)^*$= $U^* \cap V^*$ and $U^* \cap V^* \in E$ thus $U \cap V \in L$. If W is an (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure open set both $W \supset O$ and $W^* \supset \emptyset^*$. Hence it is implies that $W^* \in E$ and $W \in L$.

Proposition 9. The C net $\{P_i\}$ converges with respect to P^*_X.

Proof. Let $\{w_k\}$ be the (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ basis on centred structure net L. Since for any $\alpha \in \delta$ and $\varepsilon > 0$, $(f^{-1}_\zeta)((r_\zeta - \varepsilon, r_\zeta + \varepsilon)) \in L$, where r_ζ=lim $\{f^*_\zeta P_i\}$. So, $f_\zeta(w_k)$ converges to r_ζ $\forall$ $\zeta \in \delta$. i.e $\{w_k\}$ is a $C^*(P_X)$ net. Since (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure open filter $F_{\{w_k\}}$ formed by the $C^*(P_X)$ net $\{w_k\}$ is exactly in L. So, if $\{w^{P_k}_k\}$ is the (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure net according to $F_{\{w_k\}}$, then $\{w_k\}$= $\{w^{P_k}_k\}$.
Case 1: If $\{w_k\}$ converges to an (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure point P in $\mathcal{P}_X$. Let U^* be an (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure open set B contains $K(P)$, then $\mathcal{P} \in U$.Here, U is an (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure open set in P_X. Since $\{w_k\}$ converges to P, by considering Definition 31, U in L and therefore U^* is in E. It states that $\{P_i\}$ converged to $K(P)$ in P^*_X.

Case 2: If $\{w_k\}$ diverges to $\{P_X\}$ then $\{w_k^*\}= \{w_k^{P_k}\}^*$ is in $Y_{\mathcal{P}}$. For all U^* is in B containing $\{w_k^{P_k}\}$ is $\oslash$ in U in P_X then by Definition 32 implies that $\mathcal{U}$ in L and therefore U^* is in $\mathcal{E}$. Thus $\{P_i\}$ converges to $\{w_k^{P_k}\}^*= \{w_k\}^* \in P_X^*$.

Proposition 10. (P_X^*, K) is an (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure compactification of P_X.

Proof. A family of C is said to be bdd real-valued continuous functions on $\mathcal{P}_X^*$ and for all C net $\{P_i\}$ converges to P_X^*. From the Definition 28, P_X^* is an (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure compact space. Here, the Proposition 6, implies that (P_X^*, K) is an (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure compactification of P_X.

Proposition 11. Let $C(P_X^*)$ be the collection of all real valued continuous functions on P_X^*. Then $C(P_X^*)= C= \{f_\zeta^* : f_\zeta \in C^*(P_X)\}$

Proof. Let $g \in C(P_X^*)$. Since $\mathcal{P}_X^*$ is an (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure compact, so $g \circ K \in C(P_X^*)$. By Proposition 5 and 6 along with based on the continuity of g, it follows that $(g \circ K)^*(\{w_k^{\mathcal{P}_i}\}) = \lim \{(g \circ K)(w_i^{P_i})\}= \lim \{g(K(w_k^{P_i}))\}=g(\lim \{K(w_K^{P_i})\})$ $=g(\{w_k^{P_i}\}^*)$ $\forall$ $\{w_k^{P_i}\} \in Y_P$ and $(g \circ K)^*$ $(K(P))=$ $(g \circ K)^*$ $(\mathcal{P}) = g(K(P))$ $\forall$ $P \in P_X$. It is stated that, $C(P_X^*) \subset C= \{f_\zeta^* : f_\zeta \in C^*(P_X)\}$.

7. Results and Discussions

The (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ Centred structure compactification is an unique approach of compactification methods compared to others. Here, it was studied with the irreducibility, centred systems, compact spaces, Nets and filters etc., Also with convergence using boundedness and continous functions. While comparing to other compactifications as mentioned in Section-2 Literature survey. Fan-Gottesmann compactification deals with clopen open sets with filters and nets alongs with convergence not using the boundedness. Also, some of the compactifications make a way to conceptual ideas of framing from non-compact spaces to compact spaces and others. Apart from all these, (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ Centred Structure compactification is unique one which is being centred system and irreducible sets.

8. Conclusion

This research work will lead to a fine understanding about (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure space and its properties. It enhances the theoretical approaches to various contexts in compactifications. The inter-relations between (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure nets and (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure filters provide a robust theoretical framework for understanding (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure compactness in various types of spaces. Likewise, (c,d) $\mathcal{IF} - \mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure nets provides a way to generalize sequences and convergence in (c,d)

$\mathcal{IF}-\mathcal{Q}$ uniform topological space while (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure filters discussed a framework for convergence and compactness. It will lead to applications of compactifications on various fields. Future framework of (c,d) $\mathcal{IF}-\mathcal{Q}$ uniform $\mathtt{Ir}^*$ centred structure compactification can be explored into category theory, fixed point theory along with metric spaces.

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