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A Simulation Study of Some Logistic, Poisson, and Multiple Ridge Regression Estimators

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Abstract. This paper is a Monte Carlo simulation study of some logistic, Poisson, and multiple ridge regression estimators. This study proposes new ridge regression estimators using linear combinations of known ridge parameters, developed through grid search and methods that leverage mean squared error (MSE) values from prior simulations. The performance of each known and proposed estimators are then compared using MSE criterion. Results show that the proposed estimators performed better on many cases. Furthermore, each estimator was applied to secondary data and was compared based on their respective estimated coefficients.

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Key Words and Phrases: Ridge regression, ridge parameter, logistic regression, Poisson regression, multiple regression, Monte Carlo simulation, *stats* R package

1. Introduction

Regression analysis is a powerful tool in statistics, widely used to model relationships between variables. However, challenges such as multicollinearity and data-specific requirements can limit the effectiveness of traditional regression methods. In response to these challenges, ridge regression techniques have been developed to enhance model stability and accuracy [1, 2]. The general form of ridge regression applies an L2 penalty to the coefficient estimates, modifying the standard maximum likelihood (ML) estimation to control overfitting. This paper focuses on logistic ridge regression, Poisson ridge regression, and multiple ridge regression. Related studies are found in [3–8].

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Logistic Ridge Regression is applied in binary classification problems. The likelihood function is $p_i = 1/(1 + e^{-\mathbf{x}_i^T \boldsymbol{\beta}})$ and the ridge-regularized log-likelihood function is

$$J(\boldsymbol{\beta}) = -\sum_{i=1}^{n} [Y_i \log(p_i) + (1 - Y_i) \log(1 - p_i)] + k \sum_{j=1}^{p} \beta_j^2,$$

where p_i represents the predicted probabilities for the outcome Y, n is the number of observations, p is the number of independent variables, $\mathbf{Y}$ is the dependent $n \times 1$ vector, $\mathbf{X}$ is the independent $n \times p$ matrix, $\boldsymbol{\beta}$ is the $p \times 1$ coefficient vector, and k is the ridge parameter.

Poisson ridge regression is used to model count data. The likelihood function is

$$L(\boldsymbol{\beta}) = \prod_{i=1}^{n} \frac{e^{\mathbf{x}_i^T \boldsymbol{\beta} Y_i}}{Y_i!},$$

and the ridge-regularized log-likelihood function is

$$J(\boldsymbol{\beta}) = -\sum_{i=1}^{n} \left[Y_i \, \mathbf{x}_i^T \boldsymbol{\beta} - e^{\mathbf{x}_i^T \boldsymbol{\beta}} \right] + k \sum_{j=1}^{p} \beta_j^2.$$

The coefficients are estimated by solving

$$\hat{\boldsymbol{\beta}}_{\text{ridge}} = \arg\min_{\boldsymbol{\beta}} J(\boldsymbol{\beta}).$$

Multiple Ridge Regression applies the ridge penalty to multiple linear regression, which models the relationship between a continuous dependent variable and multiple predictors. The ridge estimator modifies the Ordinary Least Squares (OLS) estimator:

$$\hat{\boldsymbol{\beta}}_{\text{ridge}} = (X^T X + kI)^{-1} X^T Y$$

2. Methodology

This study investigates the performance of ridge-regularized estimators applied to logistic regression, Poisson regression, and multiple linear regression models in the presence of multicollinearity. The objective is to compare several known ridge parameter estimators and newly proposed ones based on their ability to minimize the mean squared error (MSE) of coefficient estimates. A Monte Carlo simulation framework is employed for the evaluation.

2.1. Theoretical Framework for Ridge Estimation

The ridge estimator for the regression coefficients $\boldsymbol{\beta}$ is

$$\hat{\boldsymbol{\beta}}_{\text{ridge}} = (X^T X + kI)^{-1} X^T Y$$

where $k > 0$ is the ridge parameter, I is the $p \times p$ identity matrix, and X is the matrix of predictors. This estimator is particularly useful in the presence of multicollinearity, where the OLS estimator becomes unstable.

To analyze the effect of ridge regularization, we express the linear model using spectral decomposition. Let the matrix X^TX have eigen-decomposition:

$$X^TX = D\Lambda D^T,$$

where D is an orthogonal matrix of eigenvectors and $\Lambda = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_p)$ contains the eigenvalues of X^TX. Defining $Z = XD$ and $\boldsymbol{\alpha} = D^T\boldsymbol{\beta}$, the model becomes:

$$Y = Z\boldsymbol{\alpha} + \varepsilon.$$

The ridge estimator of $\boldsymbol{\alpha}$ is:

$$\hat{\boldsymbol{\alpha}}(k) = (Z^TZ + kI)^{-1}Z^TY.$$

The mean squared error (MSE) of $\hat{\boldsymbol{\alpha}}(k)$ can be decomposed into variance and bias components and is given by:

$$\text{MSE}(\hat{\boldsymbol{\alpha}}(k)) = \sigma^2 \sum_{i=1}^{p} \left(\frac{\lambda_i}{\lambda_i + k}\right)^2 + \sum_{i=1}^{p} \left(\frac{k\alpha_i^2}{\lambda_i + k}\right)^2,$$

where σ^2 is the error variance, and α_i is the ith element of $\boldsymbol{\alpha}$. In practice, since σ^2 and $\boldsymbol{\alpha}$ are unknown, the estimated MSE used for comparison is:

$$\widehat{\text{MSE}}(\hat{\boldsymbol{\alpha}}(k)) = \hat{\sigma}^2 \sum_{i=1}^{p} \left(\frac{\lambda_i}{\lambda_i + k}\right)^2 + \sum_{i=1}^{p} \left(\frac{k\hat{\alpha}_i^2}{\lambda_i + k}\right)^2.$$

This framework is also extended to logistic and Poisson regression models via their respective iterative methods of approximations. For these models, the MSE is computed based on the penalized likelihood estimates of $\hat{\beta}$.

2.2. Ridge Parameter Estimators

The following are the known ridge parameters compared in this simulation study:

$$HK = k_1 = \hat{k}_{HK1} = \frac{\sigma^2}{a_{\max}^2}, \text{ where } \sigma^2 = \frac{\sum_{i=1}^{n}(y_i - \hat{\mu}_i)^2}{n-p-1}[1];$$

$$k_2 = \hat{k}_{HKM} = \frac{1}{a_{\max}^2}[2];$$

$$k_3 = \hat{k}_{GM} = \frac{\sigma^2}{\left(\prod_{i=1}^{p} a_i^2\right)^{1/p}}[9];$$

$$k_4 = \hat{k}_{MED} = \text{Median}(m_i^2), \text{ where } m_i = \frac{\sigma^2}{a_i^2}[10];$$

$$k_5 = \hat{k}_{SS} = \max(s_i), \text{ where } s_i = \frac{t_i\sigma^2}{(n-p)\sigma^2 + t_i a_i^2}[10];$$

$$k_6 = \hat{k}_{KM2} = \max\left(\frac{1}{m_i}\right)[11];$$

$$k_7 = \hat{k}_{KM4} = \left(\prod_{i=1}^{p}\frac{1}{m_i}\right)^{1/p}[11];$$

$$k_8 = \hat{k}_{KM5} = \text{median}\left(\frac{1}{m_i}\right)[11].$$

2.3. Proposed Estimators

For logistic and Poisson regression, new ridge estimators k_{9l} and k_{9p} are introduced as weighted linear combinations of the above estimators. The weights are computed based on the inverse of the mean MSE from preliminary simulations:

$$c_i = \frac{\frac{1}{\text{Mean MSE of } k_i}}{\sum_j \frac{1}{\text{Mean MSE of } k_j}}.$$

For multiple linear regression, the proposed estimator k_{9m} is determined using a grid search method. A sequence of k values from 0.01 to 10 is tested, and the value minimizing the MSE is selected as the optimal k, denoted k_{search}. Regression analysis is then performed on k_{search} values using the known ridge parameters and model dimensions (n and p) to generate a predictive formula for k_{9m}.

2.4. Simulation Procedure

The simulation design consists of the following steps:

1. **Parameter Setup.** The parameter ranges are defined as follows: the sample size n varies from 10 to 250; the number of predictors p ranges from 2 to 12; the correlation level ρ is between 0.70 and 0.99 inclusively; and the intercept β_0 is set to either 0 or 1. These parameter values are commonly used in simulation studies [3–5].

2. **Generate Multicollinear Predictors.** The matrix $X \in \mathbb{R}^{n\times p}$ is generated using the method of McDonald and Galarneau [12]:

$$x_{ij} = \sqrt{1-\rho^2}\cdot z_{ij} + \rho\cdot z_{i0}, \quad i = 1,\ldots,n, \quad j = 1,\ldots,p,$$

where $z_{ij} \sim \mathcal{N}(0,1)$ are standard normal random variables, inducing correlation among predictors.

3. **Generate Response Variable.** Set the true coefficients $\boldsymbol{\beta}$ such that $\sum \beta_i^2 = 1$. Generate the response vector Y appropriate for each regression model (logistic, Poisson, or linear) using X and $\boldsymbol{\beta}$.

4. **Estimate Coefficients and Compute MSE.** For each estimator, the regression method is applied then optimized through the `optim` function in R to compute $\hat{\boldsymbol{\beta}}$ and evaluate the MSE:
$$\text{MSE} = \frac{1}{p}(\boldsymbol{\beta} - \hat{\boldsymbol{\beta}})^T(\boldsymbol{\beta} - \hat{\boldsymbol{\beta}}).$$

5. **Repetition and Averaging.** Perform steps 2 to 4 for a total of 1,000 times. For each estimator, compute the average MSE across the 1,000 replications. The estimator with the lowest average MSE is considered the best estimator.

3. Results

In developing new ridge estimators, the following results were determined and are proposed based on their performance in their respective regression models:

$k_{9l} = 0.451 \cdot k_6 + 0.269 \cdot k_7 + 0.2797 \cdot k_8$ for logistic regression,

$k_{9p} = 0.290 \cdot k_2 + 0.169 \cdot k_6 + 0.283 \cdot k_7 + 0.253 \cdot k_8$ for Poisson regression, and

$k_{9m} = \hat{k}_{\text{search}} = 0.329 \cdot n + 0.464 \cdot p - 0.282 \cdot k_2, -0.176 \cdot k_6$ for multiple regression.

The following subsections discusses the results in tables for each regression model, Tables 1, 3 and 5 shows the simulation results where the proposed estimator has the least estimated MSE.

3.1. Logistic Ridge Regression Simulation Results

For logistic ridge regression, the widely used ML estimator, the known HK, k_2 to k_8 estimators and the proposed k_{9l} estimator are applied and compared in simulations. Table 1 shows some of the simulation results where the MSE values of each estimator are compared when $\beta_0 = 0$ and $\rho = 0.99$ for $p = 2, 3, 4, 8, 12$ with $n = 20p + 10, 30p, 40p, 60p,$ and $100p$. It is observed in the simulation that the new estimator k_{9l} has the least MSE in some cases. Specifically, when $\beta_0 = 0$, $\rho = 0.99$, $p = 2, 3, 4, 8, 12$ and when $n = 60, 80, 160, 170, 180, 240,$ and 250.

Table 1: Estimated MSEs for the Logistic Ridge Regression Simulation with $\beta_0 = 0$ and $\rho = 0.99$

		Estimators									
p	n	ML	HK	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_{9l}
2	60	0.658	0.569	0.370	0.533	0.416	0.517	0.217	0.219	0.214	**0.213**
2	80	0.452	0.403	0.277	0.386	0.316	0.380	0.161	0.166	0.161	**0.159**
3	180	6.817	5.097	2.355	2.601	0.812	1.747	0.076	0.078	0.073	**0.070**
4	160	13.74	10.81	5.144	4.518	1.244	3.088	0.099	0.096	0.087	**0.086**
8	170	42.43	35.62	17.41	10.89	3.43	9.19	0.129	0.129	0.113	**0.103**
8	240	27.98	23.67	11.98	7.215	2.87	7.63	0.088	0.157	0.127	**0.082**
12	250	53.22	46.255	23.671	13.07	5.048	14.199	0.105	0.180	0.140	**0.090**

To apply the logistic estimators to secondary data, we used the breast cancer data obtained from Taha (2022, Kaggle dataset). We set the dependent variable as the type of cancer diagnosis (0=Benign Cancer, 1 = Malignant Cancer) with $n = 180$ and $p = 3$. Three explanatory variables are included: radius mean (R), perimeter mean (P) and area mean (A), which represent specific average values of the cancer image features.

Table 2 shows the coefficient results of the bivariate correlation r to Diagnosis and the different logistic regression estimator's coefficients. It can be observed that only the estimated coefficients of k_6, k_7, k_8, and the new estimator k_{9l} has the same sign as that of the correlation coefficients of the independent variables to the dependent. On the other hand, the ML and HK estimates do not match the signs to that of the correlation coefficients, a clear indicator of the negative effect of multicollinearity present in the secondary data.

Table 2: Correlation Coefficients r and Estimated Logistic Ridge Coefficients of each Estimator

		Estimators									
Var	r	ML	HK	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_{9l}
R	0.71	-43.9	-43.8	-42.4	-40.7	-11.4	-28.1	3.02	2.99	2.94	3.02
P	0.73	38.03	37.9	36.7	35.1	9.7	24.6	0.004	0.024	0.048	0.007
A	0.71	11.32	11.3	11.07	10.8	5.2	8.8	0.003	0.021	0.043	0.007

3.2. Poisson Ridge Regression Simulation Results

Table 3: Estimated MSEs for the Poisson Ridge Regression Simulation with $\beta_o = 0$ and $p = 2$

		Estimators									
p	n	ML	HK	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_{9p}
0.85	15	0.102	0.077	0.032	0.074	0.072	0.070	0.054	0.034	0.034	**0.032**
0.85	20	0.064	0.054	0.023	0.045	0.040	0.050	0.028	0.022	0.020	**0.020**
0.90	15	0.142	0.091	0.031	0.065	0.062	0.087	0.054	0.035	0.033	**0.031**
0.90	20	0.076	0.061	0.025	0.044	0.041	0.060	0.030	0.026	0.022	**0.022**
0.95	10	2.392	0.714	0.107	0.484	0.129	0.352	0.122	0.067	0.076	**0.064**
0.96	10	4.892	0.474	0.084	0.282	0.113	0.241	0.134	0.075	0.087	**0.073**
0.96	20	0.123	0.080	0.025	0.077	0.074	0.081	0.030	0.024	0.021	**0.021**
0.97	10	2.256	0.594	0.114	0.413	0.147	0.336	0.129	0.075	0.085	**0.072**
0.98	10	2.785	0.454	0.079	0.294	0.111	0.246	0.131	0.074	0.085	**0.069**
0.99	20	0.468	0.151	0.025	0.109	0.063	0.123	0.026	0.019	0.018	**0.016**

In Poisson ridge regression, the ML estimator, the known HK, k_2 to k_8 estimators and the proposed k_{9p} estimator are applied and compared in simulations. Table 3 shows some of the simulation results where the MSE values of each estimator are compared when $\beta_0 = 0$, $\rho = 0.85$, 0.90, 0.95, 0.96, 0.97, 0.98, 0.99, $p = 2$, 3 with $n = 10$, 15, 20, 30, 50. It has been observed that the new Poisson ridge estimator (k_{9p}) has the least MSE in some cases. Specifically, when $\beta_0 = 0$, $p = 2$, 3 at the different levels of ρ and values of n.

Applying the Poisson Ridge estimators to the secondary data from the Botswana Demographic and Health Survey (Central Statistics Office, 1988). We set the number of children as the dependent variable, and year born and age as the independent variables . Table 4 shows the coefficient results of the bivariate correlation coefficients r to the number of children and the different Poisson regression estimator's coefficients. It is observed that k_3 to k_8 and the proposed k_{9p} estimated coefficients coincides with the signs in the correlation coefficients of the independent variables to the dependent. On the other hand, the ML and HK estimated coefficients does not match the sign of the correlation coefficient an indication of the negative effect of multicollinearity in the secondary data.

Table 4: Correlation Coefficients r and Estimated Poisson Ridge Coefficients of Estimators

		Estimators									
Var	r	ML	HK	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_{9p}
Year	-0.73	4.9	0.393	1.32	-0.53	-0.57	-0.57	-0.59	-0.55	-0.58	-0.57
Age	0.74	5.53	1.00	1.93	0.074	0.03	0.029	0.012	0.047	0.022	0.031

3.3. Multiple Ridge Regression Simulation Results

In the multiple ridge regression, Table 5 shows some results of the simulation where the MSE values of each estimator. Overall, it has been observed that the proposed ridge estimator k_{9m} has the least MSE in some cases. Specifically, when $\rho \geq 0.72$, $p \geq 3$ and $n \geq 10$.

Table 5: Estimated MSEs for the Multiple Ridge Regression Simulation with $p = 3$

		Estimators									
p	n	ML	HK	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_{9m}
0.72	10	0.083	0.064	0.039	0.047	0.437	0.049	0.026	0.037	0.032	**0.024**
0.72	20	0.032	0.029	0.023	0.027	0.024	0.027	0.018	0.024	0.023	**0.014**
0.72	100	0.007	0.007	0.006	0.004	0.005	0.006	0.006	0.007	0.007	**0.004**
0.85	10	0.157	0.106	0.051	0.066	0.060	0.084	0.026	0.041	0.034	**0.022**
0.85	20	0.057	0.047	0.031	0.031	0.032	0.045	0.021	0.033	0.029	**0.012**
0.85	150	0.006	0.006	0.006	0.005	0.005	0.006	0.005	0.006	0.006	**0.005**
0.90	10	0.220	0.134	0.058	0.072	0.063	0.102	0.022	0.036	0.030	**0.018**
0.90	20	0.083	0.064	0.037	0.040	0.041	0.063	0.022	0.037	0.033	**0.013**
0.90	30	0.049	0.041	0.027	0.026	0.027	0.041	0.019	0.030	0.028	**0.009**
0.95	10	0.472	0.276	0.103	0.128	0.082	0.161	0.019	0.030	0.024	**0.018**
0.95	20	0.161	0.105	0.047	0.056	0.056	0.098	0.020	0.037	0.029	**0.012**
0.95	30	0.109	0.080	0.042	0.046	0.048	0.079	0.022	0.038	0.033	**0.010**

Applying the estimators to secondary data obtained from the World Bank (2023), which provides economic and labor data for the Philippines from 2010–2021 as shown in Table 6. We set the dependent variable Unemployment as percent of total labor force regressed by the independent variables consumer price index or CPI (2010 base year), wholesale price index or WPI (2010 base year), total labor force or Labor (L), total population (P), and household (H) final consumption expenditure (annual percent growth) data from year 2010 to 2021.

Table 6: Some Philippine Indicator Values from World Bank (2023)

Year	CPI	WPI	Labor(L)	Population(P)	Household(H)	Unemployment
2010	100.0000	100.0000	38081598	38081598	3.589800	3.61
2011	104.7184	108.6630	39490704	96337913	5.552134	3.59
2012	107.8882	109.9084	40075145	98032317	6.798970	3.50
2013	110.6746	111.7949	40789298	99700107	5.819003	3.50
2014	114.6565	115.7143	42179845	101325201	5.784455	3.60
2015	115.4295	117.6191	42622144	103031365	6.444572	3.07
2016	116.8766	118.6996	43756699	104875266	7.149608	2.70
2017	120.2113	120.9341	42974771	106738501	5.957367	2.55
2018	126.5938	123.2875	43800369	108568836	5.765216	2.34
2019	129.6220	125.3114	45091808	110380804	5.866915	2.24
2020	132.7241	128.3150	42419079	112190977	-7.956529	2.52
2021	137.9364	132.2527	44857443	113880328	4.211487	3.40

Table 7 shows the bivariate correlation coefficients r of each independent variable to the Unemployment variable and the different multiple ridge estimators' coefficients. It can be noted that k_6 to k_8 and the proposed k_{9m} coefficients match the signs of the correlation coefficients. On the other hand, the OLS and HK estimated coefficients do not match all the signs of the correlation coefficients an indication of the presence of multicollinearity in the secondary data.

Table 7: Correlation Coefficients r and Estimated Ridge Regression Coefficients of Estimators

		Estimators									
Var	r	ML	HK	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_{9m}
CPI	-0.63	2.93	2.69	1.62	0.94	0.95	0.35	-0.10	-0.0006	-0.08	-0.095
WPI	-0.63	1.09	1.10	1.04	0.79	0.795	0.34	-0.01	-0.005	-0.08	-0.098
L	-0.69	-0.88	-0.93	-1.11	-1.06	-1.06	-0.74	-0.17	-0.37	-0.24	-0.195
P	-0.70	-3.79	-3.51	-2.18	-1.31	-1.32	-0.59	-0.15	-0.27	-0.19	-0.167
H	0.22	0.28	0.30	0.38	0.36	0.36	0.24	0.04	0.10	0.059	0.047

4. Conclusion

The method of ridge regression is useful in reducing the negative effects of multicollinearity by using a small bias in the form of a ridge parameter k. There have been many proposed ridge parameter, the best ridge parameter has yet been determined and no constant value of k is certain to give an estimator that is consistently better in terms of MSE than the ML or OLS.

In this study, we used Monte Carlo simulation with 1,000 replications in the R software. In all the simulation combinations considered, where the number of independent variables p, level of correlation ρ, intercept β_0, and sample sizes n are varied, the simulation results showed that overall the known and proposed ridge parameter estimators has lower MSE than that of the OLS, ML, and HK estimators. Furthermore, the proposed estimators performed best on some cases. Finally, the methods of the known and proposed estimators were applied to multicollinear data from secondary sources and found that the proposed estimators' coefficients match the signs of the corresponding correlation coefficient.

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