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Marangoni Convection in Hybrid Nanofluid Flow over a Disk

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Abstract. In this work Marangoni convection problem is studied for hybrid nanofluids. Both positive and negative Marangoni parameter cases are studied. The magnetohydrodynamic (MHD) flow of heat transfer over a stretching disk with porous suction and injection effects is examined. The problem is first converted from partial differential equations (PDEs) into ordinary differential equations (ODEs) by similarity transformation and then solved numerically by using the bvp4c solver in MATLAB. Results are computed and discussed for embedded parameters. A comparative analysis with existing literature is also provided presenting the skin friction and Nusselt number results in a table format.

2020 Mathematics Subject Classifications: 76A05, 76A10, 76U05, 76S05, 76W05, 26A33, 35R11, 35A22

Key Words and Phrases: Marangoni Convection, Hybrid Nanofluid, Permeable Disk, MWCNT and ZnO_2

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1. Introduction

With the use of contemporary materials and methods, the study of fluid dynamics has advanced significantly, opening the door for creative technological and engineering applications. Because of their improved thermal characteristics, hybrid nanofluids which are created by suspending nanoparticles of several materials inside a base fluid have become a fascinating field of study. The behavior of hybrid nanofluid flow over a permeable disk in the presence of a magnetic field has exciting opportunities in industrial and biomedical applications. The application of a magnetic field influences the fluid dynamics by inducing magnetohydrodynamic (MHD) effects, which can be utilized to control and optimize the flow characteristics. Additionally, Marangoni convection, driven by surface tension gradients typically caused by temperature or concentration differences at the fluid interface, plays a crucial role in the stability and efficiency of fluid flow systems. The interplay between Marangoni convection and hybrid nanofluid flow under magnetic influence over

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a permeable surface introduces complex but beneficial phenomena that can enhance heat transfer and fluid mixing processes. This study aims to explore the intricate dynamics of hybrid nanofluid flow under the combined influence of a magnetic field and Marangoni convection over a permeable disk. Understanding these interactions is essential for advancing the design and optimization of systems where precise heat and mass transfer control is critical, such as in cooling technologies, microfluidic devices, and biomedical engineering. Sethy et al. [1] studied the Cattaneo-Christov heat flux-based Darcy-Forchheimer hybrid nanofluid flow with Marangoni convection above a porous disk. They concluded that the presence of Marangoni convection significantly enhances the system's heat transfer rate and fluid flow stability. Mohanty et al. [2] investigated thermo-solutal Marangoni convective Darcy-Forchheimer bio-hybrid nanofluid flow over a permeable disk with activation energy. Their findings highlighted the critical role of activation energy in controlling the thermal and solutal Marangoni convection, improving the efficiency of heat and mass transfer. Khan et al. [3] studied drilling nanoliquids with clay nanoparticles using fractional fractal model. Mohanty et al. [4] investigated thermosolutal Marangoni stagnation point GO–$\mathrm{MoS_2}$/water hybrid nanofluid over a stretching sheet with an inclined magnetic field. Their results demonstrated that the inclined magnetic field improves the fluid flow and heat transfer characteristics, making the system more efficient. Abu Bakar et al. [5] analyzed the unsteady flow of hybrid nanofluid over a permeable shrinking inclined rotating disk with radiation and velocity slip effects. They concluded that the inclusion of radiation and velocity slip effects leads to a more precise control of fluid flow and thermal management in dynamic systems. Chaurasiya et al. [6] investigated the impact of magnetic induction on the flow of self-rewetting power-law fluid over a disk surface: Onset of Marangoni convection. Their study revealed that magnetic induction can delay or accelerate the onset of Marangoni convection, which is crucial for applications requiring precise thermal control. Abbas et al. [7] analyzed the bioconvective flow of tangent hyperbolic hybrid nanofluid through different geometries with temperature and concentration-dependent heat source. They concluded that the bioconvection phenomenon enhances the heat and mass transfer efficiency, particularly in systems with variable thermal properties. Abdullah et al. [8] analyzed the effect of MHD on Marangoni boundary layer of hybrid nanofluid flow past a permeable stretching surface. Their findings indicated that the MHD effects significantly modify the boundary layer characteristics, enhancing the overall heat transfer performance. Abbas et al. [9] investigated thermal Marangoni convection in two-phase quadratic convective flow of dusty MHD trihybrid nanofluid with a non-linear heat source. Their conclusions echoed the previous findings, emphasizing the positive impact of non-linear heat sources on heat transfer efficiency. Ullah et al. [10] analyzed the thin film flow of ternary hybrid nanofluid over a rotating disk under the influence of a magnetic field due to nonlinear convection. They concluded that nonlinear convection plays a crucial role in optimizing the thin film flow and heat transfer rates in rotating systems. Abbas et al. [11] discussed the numerical analysis of Marangoni convective flow of gyrotactic microorganisms in dusty Jeffrey hybrid nanofluid over a Riga plate with Soret and Dufour effects. Their study highlighted the importance of these effects in enhancing heat and mass transfer in biological and engineering applications. Batool et al. [12] discussed the significance of interfacial nanolayer and mixed convection in radiative Casson hybrid nanofluid flow by a permeable rotating cone. They concluded that the interfacial nanolayer plays a pivotal role in enhancing heat transfer and flow stability. Jiang et al. [13] studied mixed convection heat transfer and entropy generation of MHD hybrid nanofluid in a cubic porous cavity with a wavy wall and rotating cylinders. They found that the wavy wall and rotating cylinders significantly enhance the heat transfer and reduce entropy generation, making the system more efficient. Abbas et al. [14] analyzed the numerical simulation of magneto thermal Marangoni convective flow of dusty Sutterby hybrid nanofluid with variable thermal conductivity. They concluded that variable thermal conductivity significantly affects the flow and heat transfer characteristics, offering better control over the thermal management processes. Alhushaybari et al. [15] studied the impact of a non-linear heat source and magnetic field on the Carreau nanofluid Marangoni convective flow–a numerical investigation. Their findings indicated that the non-linear heat source and magnetic field jointly enhance heat transfer efficiency and flow stability. Rehman et al. [16] analyzed the impact of Marangoni convection on carbon nanotube blood-based hybrid nanofluid with thermal radiation, viscous dissipation, and couple stress. Their analytical study showed that these factors significantly improve the heat transfer and fluid flow properties, which is beneficial for biomedical applications. Abu Bakar et al. [17] expressed the effects of homogeneous–heterogeneous reactions and hybrid nanofluid on Bödewadt flow over a permeable stretching/shrinking rotating disk with radiation. They concluded that these reactions and radiation effects optimize the flow and thermal properties, enhancing the system's efficiency. Mohanty et al. [18] studied the impact of the interfacial nanolayer on Marangoni convective Darcy-Forchheimer hybrid nanofluid flow over an infinite porous disk with Cattaneo-Christov heat flux. Their results demonstrated that the interfacial nanolayer significantly enhances heat transfer and flow stability. Jiang et al. [19] discussed mixed convection heat transfer and entropy generation of MHD hybrid nanofluid in a cubic porous cavity with a wavy wall and rotating cylinders. They concluded that this configuration significantly optimizes the heat transfer and minimizes entropy generation, leading to more efficient thermal management systems. Hybrid nanofluids, combining nanoparticles like Multi-Walled Carbon Nanotubes (MWCNT) and Zinc Peroxide (ZnO_2), are investigated for their enhanced thermal and rheological properties, particularly on a rotating disk subject to MHD effects, radiation, and boundary conditions of suction and injection. These hybrid nanofluids leverage the excellent thermal conductivity and mechanical strength of MWCNT and the thermal stability of ZnO_2, potentially resulting in superior heat transfer capabilities. The study aims to understand the influence of MHD on flow and thermal properties, evaluate the impact of radiative heat transfer, and assess how suction and injection affect nanoparticle distribution and heat transfer performance. This research is crucial for optimizing thermal management systems, improving energy efficiency, and enhancing performance in various industrial applications. Hybrid nanofluids, combining nanoparticles such as MWCNT and ZnO_2, have gained significant attention for their enhanced thermal and rheological properties, particularly when applied to rotating disk systems under the influence of MHD effects, radiation, and boundary conditions like suction and injection. Hemmat et al. [20] investigated the rheological behavior, economic performance, and developed a model for MWCNT-ZnO (30:70)/10W40 hybrid nanofluid using response surface methodology, highlighting its potential for optimized thermal management. Upadhya et al. [21] compared nanofluids containing Single-Walled Carbon Nanotubes (SWCNT) with MWCNT and Fe_3O_4 across a spinning disk with suspended joule heating and non-linear thermal radiation, using multi-linear optimization to demonstrate superior thermal properties. Memon et al. [22] focused on enhancing the stability of binary nanofluids comprising Al_2O_3, ZnO, and TiO_2, which is critical for consistent heat transfer performance. Gupta et al. [23] studied the heat transfer of MHD flow in hybrid nanofluids (SWCNT-MWCNT/$C_3H_8O_2$) over a permeable surface using the Cattaneo–Christov model, providing insights into improved heat transfer mechanisms. Madiwal and Naduvinamani [24] examined the heat and mass transfer of Casson hybrid nanofluid (MoS_2+ ZnO) based on engine oil over a stretched wall with chemical reactions and thermo-diffusion effects, underscoring the potential for enhanced thermal management in industrial applications [25–28]. These studies collectively emphasize the importance of hybrid nanofluids in optimizing heat transfer systems, improving energy efficiency, and enhancing the performance of various industrial processes, offering a comprehensive understanding of their behavior under different conditions and configurations.

2. Mathematical modeling and assumption

Let us consider two-dimensional boundary layer flow of a hybrid nanofluid of Cu-ZnO_2/MWCNT suspended in water, flowing over a permeable disk. Utilizing cylindrical coordinates (r, α, z), where the z-axis represents vertical direction and the r-axis signifies horizontal direction, the flow maintains symmetry to the $z = 0$ plane and exhibits axis symmetry around the z-axis, with $\partial/\partial\alpha = 0$ for all variables. This flow is propelled by surface tension resulting from the surface temperature gradient, known as the Marangoni effect. The disk, subject to stretching or shrinking, resides within a porous medium, where no slip between nanoparticles and water is anticipated, given their thermal equilibrium. We presume the disk's surface temperature as $T_w(r)$, while the ambient fluid temperature remains T_∞. Furthermore, we consider a constant mass flux velocity, denoted as w_0, where $w_0 < 0$ signifies suction and $w_0 > 0$ indicates injection. From ref [25], we have incorporated MHD for momentum equation here for this study.

Governing the following equations

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$u\,\frac{\partial u}{\partial r} + w\,\frac{\partial u}{\partial z} = \frac{\mu_{hnf}}{\rho_{hnf}}\left(\frac{\partial^2 u}{\partial z^2} - \frac{u}{k}\right) - \frac{\sigma_{hnf}}{\rho_{hnf}}\,B_0^2 u \tag{2}$$

$$u\,\frac{\partial T}{\partial r} + w\,\frac{\partial T}{\partial z} = \frac{k_{hnf}}{(\rho\, c_p)_{hnf}}\frac{\partial^2 T}{\partial z^2} \tag{3}$$

With the boundary conditions from Ref [25]

$$\mu_{hnf}\,\frac{\partial u}{\partial z} = \lambda\,\frac{\partial \sigma}{\partial r}, \quad w = w_0, \quad T = T_w(r) = T_\infty + T_0\left(\frac{r}{R}\right)^2 \quad \text{at} \quad z = 0 \tag{4}$$

$$u \to 0, \quad T \to T_\infty \quad \text{as} \quad z \to \infty \tag{5}$$

The velocity components are represented by u (in the r-axis direction) and w (in the z-axis direction), while T denotes the fluid temperature and T_0 stands for the characteristic temperature at the disk surface. R represents the characteristic radius of the disk. The parameter w_0 is determined as $w_0 = -S\sqrt{\Omega\,\theta_f}$, where S denotes the constant mass flux parameter, with $S > 0$ for suction and $S < 0$ for injection, respectively. λ signifies the constant stretching/shrinking parameter, with $\lambda > 0$ for a stretching disk, $\lambda < 0$ for a shrinking disk, and $\lambda = 0$ for a static disk. The surface tension σ is assumed to vary linearly with temperature:

$$\sigma = \sigma_0 - \gamma_T\,(T - T_\infty) \tag{6}$$

Here, $\gamma_T = -\left(\partial\sigma/\partial T\right)_{T=T_\infty}$, where $\gamma_T > 0$ represents the temperature surface tension coefficient, and $\sigma_0 > 0$. The interfacial surface tension gradient influences the flow as $\partial\sigma/\partial r = \partial\sigma/\partial T \cdot \partial T/\partial r$ due to the temperature gradient.

Similarity variables from Ref [25]

$$u = r\,\Omega\,f(\eta), \quad w = \sqrt{\Omega\,v_f}\,h(\eta), \quad T_w(r) = T_\infty + T_0\left(\frac{r}{R}\right)^2, \quad \eta = z\,\sqrt{\Omega\,v_f} \tag{7}$$

By transforming into odes by using above similarity variables we get from Ref [20]

$$2f + h' \tag{8}$$

$$\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f}\,(f'' - Pf) - f^2 - hf' - \frac{\sigma_{hnf}/\sigma_f}{\rho_{hnf}/\rho_f}\,Mf = 0 \tag{9}$$

$$\frac{1}{Pr}\,\frac{k_{hnf}/k_f}{(\rho\,c_p)_{hnf}/(\rho\,c_p)_f}\,\theta'' - (2f\theta + h\theta') = 0 \tag{10}$$

Subject to conditions from Ref [25]

$$f'(0) = 2\frac{\mu_f}{\mu_{hnf}}\,Ma\,\lambda, \quad h(0) = -S, \quad \theta(0) = 1 \tag{11}$$

$$f(\eta) \to 0, \quad \theta(\eta) \to 0 \quad \text{as} \quad \eta \to \infty \tag{12}$$

where $P = \nu_f/k\,\Omega$ is the porosity parameter with k as porous medium permeability $Pr = (\mu\,c_p)_f/k_f$ is Prandtl number and $M = (\sigma_f B_0^2)/(\rho_f\,\Omega)$ is the magnetic parameter and $Ma = (\sqrt{\nu_f/\Omega}\,T_0\gamma_T)/\Omega\,\mu_f$ is the Marangoni Parameter. The kinematic viscosity, dynamic viscosity, density, thermal conductivity, and heat capacity with fixed pressure are symbolized as ν, μ, ρ, k, and c_p, respectively, while heat capacity is denoted as $(\rho\, c_p)$. The subscripts hnf, f, 1, and 2 refer to the hybrid nanofluid, base fluid, the nanoparticle of alumina, and the nanoparticle of copper, respectively. The volume fractions for alumina and copper are ϕ_1 and ϕ_2, accordingly.

The Local Nusselt number is $Nu_x = -r\,(k_{hnf}/k_f(T_0 - T_\infty))\,(\partial T/\partial z)_{z\to 0}$, it can be simply expressed as $Re^{-1/2}\,Nu_x = -(k_{hnf}/k_f)\theta'(0)$ where $Re = (\Omega\, r^2)/\nu_f$ is the local Reynolds number.

Table 1: Value of nanoparticles (Water, ZnO_2, MWCNT)

Physical characteristics	ZnO_2	MWCNT	Water
$c_p\ (J/kg\,K)$	514	711	4179
$\rho\ (kg/m^3)$	5606	2100	997.1
$k\ (W/m\,K)$	23.4	3000	0.613
$\sigma\ (S/m)$	1.587	10^{-12}	1.9×10^{-4}

Table 2: The effective characteristics of hybrid nano liquids associated with ZnO_2 and MWCNT.

Characteristics	Correlation
Density	$\rho_{hnf} = \phi_{\mathrm{MWCNT}}\,\rho_{\mathrm{ZnO_2}}\left[(1-\phi_{\mathrm{ZnO_2}})\,\rho_f + \phi_{\mathrm{ZnO_2}}\,\rho_{\mathrm{ZnO_2}}\right](1-\phi_{\mathrm{MWCNT}})$
Effective viscosity	$\mu_{hnf} = \frac{\mu_f}{(1-\phi_{\mathrm{ZnO_2}})^{2.5}\,(1-\phi_{\mathrm{MWCNT}})^{2.5}}$
Heat capacitance	$(\rho\, c_p)_{hnf} = (1-\phi_{\mathrm{MWCNT}})\left[(1-\phi_{\mathrm{ZnO_2}})\,(\rho\, c_p)_f + \phi_{\mathrm{ZnO_2}}\,(\rho\, c_p)_{\mathrm{ZnO_2}}\right] + \phi_{\mathrm{MWCNT}}\,(\rho\, c_p)_{\mathrm{MWCNT}}$
Thermal diffusivity	$\alpha_{hnf} = \frac{k_f}{(\rho\, c_p)_{hnf}}$
Effective thermal conductivity	$\frac{k_{hnf}}{k_{bf}} = \left[\frac{(k_{\mathrm{ZnO_2}}+2k_{bf}-2\phi_{\mathrm{MWCNT}}(k_{bf}-k_{\mathrm{MWCNT}}))}{(k_{\mathrm{MWCNT}}+2k_{bf}+\phi_{\mathrm{MWCNT}}(k_{bf}-k_{\mathrm{MWCNT}}))}\right]$ where $\frac{k_{bf}}{k_f} = \left[\frac{(k_{\mathrm{ZnO_2}}+2k_f-2\phi_{\mathrm{ZnO_2}}(k_f-k_{\mathrm{ZnO_2}}))}{(k_1+2k_f+\phi_{\mathrm{ZnO_2}}(k_f-k_{\mathrm{ZnO_2}}))}\right]$

3. Solution method and linearisation of ODE

BVP4C, short for Boundary Value Problem Solver for Ordinary Differential Equations (ODEs) in MATLAB, is a powerful tool for solving boundary value problems (BVPs) numerically. It's particularly useful for solving differential equations subject to boundary conditions. In the context of your specific problem, investigating the effect of a magnetic field on hybrid nanofluid flow over a permeable disk under Marangoni convection, BVP4C could prove instrumental. Start by formulating the governing equations for the problem. These typically involve the Navier-Stokes equations for fluid flow, coupled with equations describing heat transfer, possibly incorporating the effects of magnetic fields, thermal conductivity, and Marangoni convection. Specify the boundary conditions for the problem. These conditions describe the behavior of the solution at the edges of the domain. For example, at the surface of the disk, you might have conditions related to the flow velocity, temperature, and magnetic field strength. Convert the governing equations into dimensionless form to simplify the problem and reduce the number of parameters. This step often involves introducing dimensionless variables and parameters. Discretize the domain and equations. This involves dividing the domain into a grid or mesh and approximating the derivatives in the equations using finite differences, finite elements, or other numerical methods. Set up the BVP4C solver in MATLAB. This involves specifying the differential equations, boundary conditions, and any additional parameters. BVP4C uses a collocation method to solve BVPs, where the solution is approximated as a polynomial that satisfies the differential equations at a set of collocation points within each subinterval of the domain. Once everything is set up, use the BVP4C solver to solve the boundary value problem numerically. The solver iteratively adjusts the polynomial coefficients to satisfy the boundary conditions and minimize the residual error in the equations. After obtaining the solution, analyze and visualize the results to gain insights into the behavior of the system. This may involve plotting velocity profiles, temperature distributions, magnetic field lines, and other relevant quantities. By leveraging BVP4C, you can explore the complex interplay between magnetic fields, Marangoni convection, and hybrid nanofluid flow over a permeable disk, providing valuable insights into this intriguing phenomenon.

$$
\begin{aligned}
&r_1 = f,\, r_2 = f',\, r_3 = f'' = r_2',\, r_4 = \theta,\, r_5 = \theta',\, r_6 = \theta'' \\
&r_2' = \frac{\mathrm{Pr}_1\left(\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f}\right) + r_1^2 + h\, r_2 + \frac{\sigma_{hnf}/\sigma_f}{\rho_{hnf}/\rho_f} M\, r_1}{\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f}}, \\
&r_5' = \left(\frac{2\, r_1\, r_4 + h\, r_5}{\frac{\rho c_{p_{hnf}}}{\rho c_{p_f}} \frac{1}{\mathrm{Pr}}}\right).
\end{aligned}
$$

4. Results and discussion

This study examines how hybrid nanofluids with MHD effects transfer heat over stretching disks with porous suction and injection, considering both positive and negative Marangoni parameters. By converting the partial differential equations to ordinary differential equations and using the bvp4c solver, we analyze skin friction and the Nusselt number for various water concentrations and Marangoni effects, using nanoparticles like water, ZnO_2, and MWCNT. We provide detailed results on velocity and temperature profiles under different suction and injection conditions. Additionally, we compare our results with existing studies, presenting findings in a table. This research helps understand heat transfer in MHD nanofluids over stretching disks, offering valuable insights for engineering applications.

Figure 1 depicts the influence of magnetic and Marangoni parameters on the dimensionless radial velocity of nanofluids flowing over an impermeable stretching disk. The decrease in velocity profile observed in Figure 1 for nanofluids, including hybrid nanofluids with nanoparticles such as water, ZnO_2, and MWCNT, due to changes in Marangoni parameters, can be attributed to several physical mechanisms: Marangoni effect arises from surface tension gradients within the nanofluid. Different nanoparticles can alter these gradients due to variations in their surface properties. For instance, certain nanoparticles like ZnO_2 or MWCNTs can modify the surface characteristics of the fluid, leading to changes in surface tension gradients. Temperature gradients can also cause the Marangoni effect along the fluid surface. When temperature gradients exist, it induce thermocapillary flow,

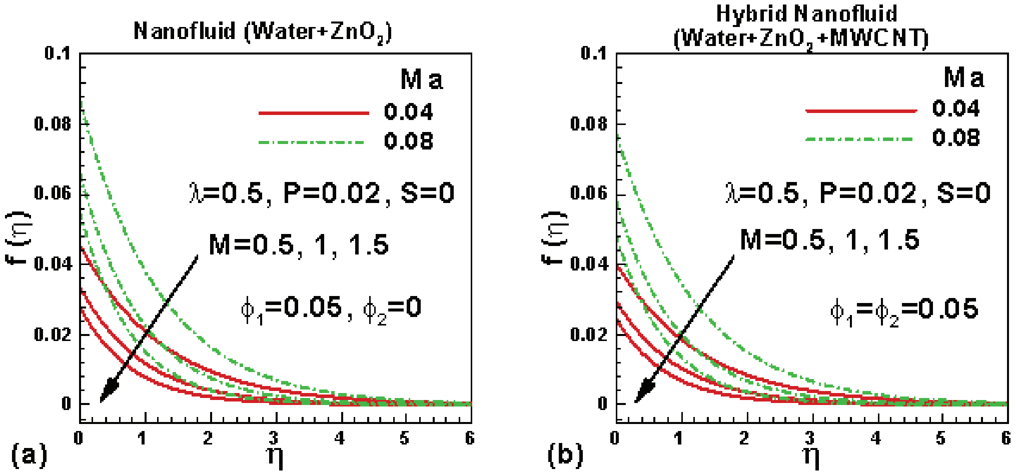

Figure 1: Effects of magnetic and Marangoni parameters on dimensionless radial velocity of nanofluids over an impermeable stretching disk.

which can influence the flow dynamics of the nanofluid. The presence of nanoparticles, especially those with high thermal conductivity like MWCNTs, can affect temperature distribution in the fluid, thereby altering the thermocapillary flow and Marangoni effect. Nanoparticles dispersed in the nanofluid can interact with the fluid interface, affecting its surface properties and the Marangoni effect. For example, nanoparticles may adsorb onto the fluid interface, modifying surface tension gradients and consequently the flow behavior. The addition of nanoparticles can increase the viscosity of the nanofluid, which can dampen fluid motion and lead to a decrease in velocity profile. This effect is particularly significant in the presence of Marangoni parameters, as viscosity influences the fluid's response to surface tension gradients. In hybrid nanofluids containing multiple types of nanoparticles, interactions between different nanoparticles and their combined effects on fluid properties can further influence the Marangoni effect. These interactions can be complex and depend on factors such as nanoparticle concentration, size, and surface chemistry. Overall, the decrease in velocity profile for nanofluids, including hybrid nanofluids, in Figure 1 due to changes in Marangoni parameters involves a combination of surface tension gradients, thermocapillary flow, interfacial interactions, increased viscosity, and interactions between different nanoparticles. These mechanisms collectively influence the flow dynamics of the nanofluid over the impermeable stretching disk.

Figure 2 illustrates how changes in magnetic and Marangoni parameters affect the dimensionless radial velocity of nanofluids over an impermeable stretching disk. In Figure 2, the decrease in velocity profile for both nanofluids and hybrid nanofluids containing nanoparticles like water, ZnO$_2$, and MWCNTs, due to changes in Marangoni parameters over a permeable stretching disk with injection, can be attributed to various physical mechanisms: Marangoni effect arises from surface tension gradients within the nanofluid. The presence of nanoparticles alters these gradients due to changes in surface properties. For instance, different nanoparticles like ZnO$_2$ and MWCNTs may modify the fluid's surface tension, affecting the Marangoni effect. Temperature gradients along the fluid surface in-

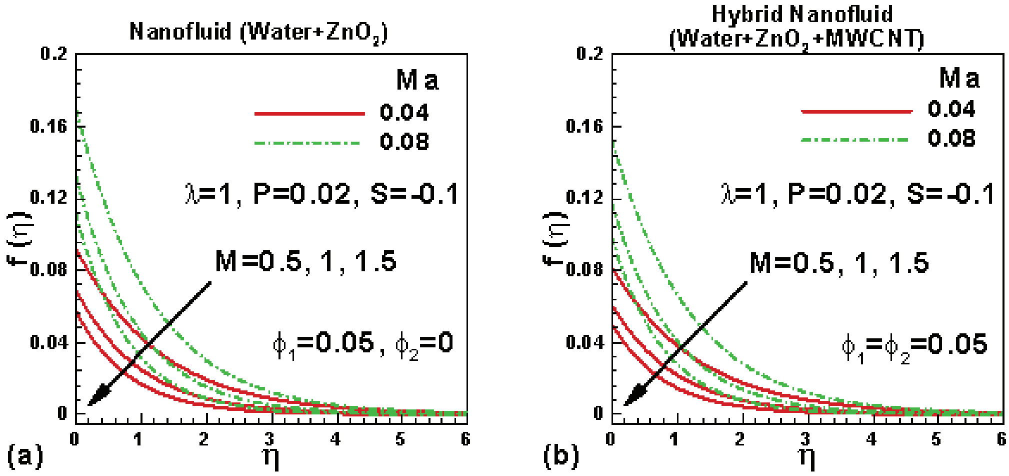

Figure 2: Effects of magnetic and Marangoni parameters on dimensionless radial velocity of nanofluids over a permeable stretching disk with injection.

duce thermocapillary flow, contributing to the Marangoni effect. Nanoparticles, especially those with high thermal conductivity like MWCNTs, can influence temperature distribution in the fluid, altering thermocapillary flow and consequently the Marangoni effect. Nanoparticles dispersed in the nanofluid interact with the fluid interface, modifying its surface properties and the Marangoni effect. For example, nanoparticles may adsorb onto the fluid interface, changing surface tension gradients and flow behavior. Nanoparticles increase the viscosity of the nanofluid, damping fluid motion and leading to a decrease in velocity profile. This effect is significant in the presence of Marangoni parameters, as viscosity influences the fluid's response to surface tension gradients. Injection at the permeable stretching disk modifies the boundary condition, affecting the flow dynamics near the disk. Injection can alter the flow pattern and velocity distribution, potentially contributing to the decrease in velocity profile. In hybrid nanofluids, interactions between different nanoparticles and their combined effects on fluid properties influence the Marangoni effect. These interactions depend on factors such as nanoparticle concentration, size, and surface chemistry, and can further contribute to the observed decrease in velocity profile. Overall, the decrease in velocity profile for nanofluids and hybrid nanofluids in Figure 3, due to changes in Marangoni parameters over a permeable stretching disk with injection, involves a combination of surface tension gradients, thermocapillary flow, interfacial interactions, increased viscosity, the influence of injection boundary conditions, and interactions between different nanoparticles. These mechanisms collectively influence the flow dynamics of the nanofluid over the permeable stretching disk.

Figure 3 illustrates the impact of magnetic and Marangoni parameters on the dimensionless axial velocity of nanofluids flowing over an impermeable stretching disk. The increase in axial velocity profile for both nano and hybrid nanofluids containing nanoparticles like water, ZnO_2, and MWCNTs, due to positive values of Marangoni parameters over an impermeable stretching disk, as depicted in Figure 3, can be attributed to several physical mechanisms: Positive Marangoni parameters induce surface tension gradients

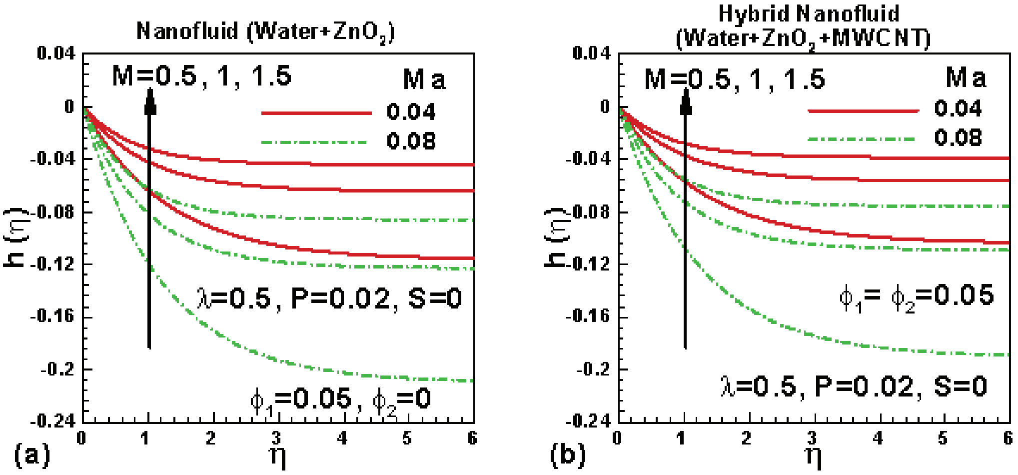

Figure 3: Effects of magnetic and Marangoni parameters on dimensionless axial velocity of nanofluids over an impermeable stretching disk.

that enhance thermocapillary flow along the fluid surface. This increased flow can lead to higher velocities in the axial direction, resulting in an increase in axial velocity profile along the stretching disk. The presence of nanoparticles in the nanofluid, such as ZnO_2 and MWCNTs, can reduce the viscosity of the fluid and enhance its mobility. As a result, the fluid can flow more easily over the stretching disk, leading to an increase in axial velocity profile. Positive Marangoni parameters can reduce the thickness of the boundary layer near the fluid-solid interface. This reduction in boundary layer thickness allows for more efficient fluid flow and leads to higher velocities in the axial direction. The influence of magnetic parameters, combined with positive Marangoni parameters, can further enhance fluid motion and mixing. This enhanced motion contributes to an increase in axial velocity profile along the stretching disk. In hybrid nanofluids, interactions between different nanoparticles may also play a role. The combined effects of nanoparticles, such as their concentration, size, and surface chemistry, can influence fluid properties and contribute to the observed increase in axial velocity profile. Overall, the increase in axial velocity profile for nano and hybrid nanofluids over an impermeable stretching disk with positive values of Marangoni parameters, as depicted in Figure 6, involves a combination of enhanced thermocapillary flow, improved fluid mobility, reduction of boundary layer thickness, interaction with magnetic parameters, and effects specific to hybrid nanofluids. These mechanisms collectively lead to the observed increase in axial velocity profile.

Figure 4 indicates that how alterations in magnetic and Marangoni parameters impact the dimensionless temperature of nanofluids over an impermeable stretching disk. The increase in temperature profile for both nano and hybrid nanofluids containing nanoparticles like water, ZnO_2, and MWCNTs, due to positive values of Marangoni parameters over an impermeable stretching disk, as depicted in Figure 4, can be attributed to several physical mechanisms: Positive Marangoni parameters induce surface tension gradients that enhance thermocapillary flow along the fluid surface. This increased flow can lead to higher rates of convective heat transfer, resulting in an increase in temperature profile

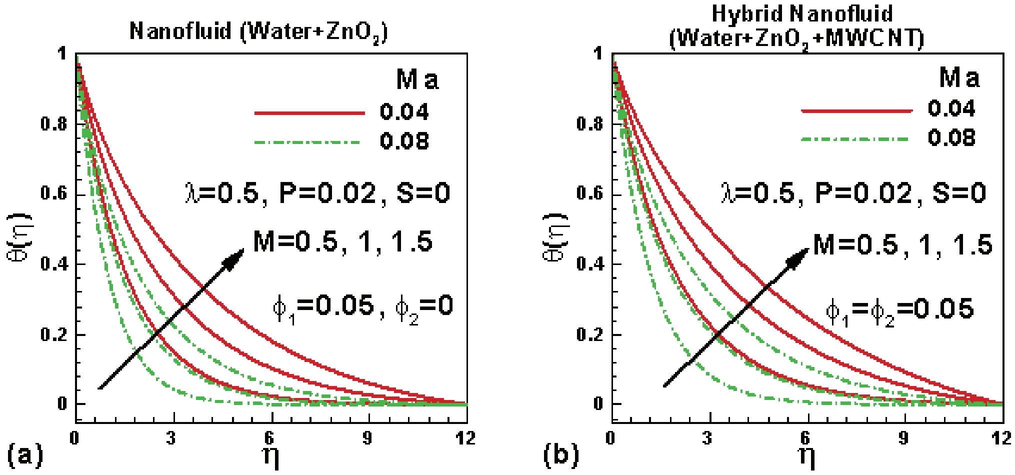

Figure 4: Effects of magnetic and Marangoni parameters on dimensionless temperature of nanofluids over an impermeable stretching disk.

along the stretching disk. The presence of nanoparticles in the nanofluid, such as ZnO_2 and MWCNTs, can enhance heat transfer properties due to their high thermal conductivity. As the fluid flows over the stretching disk, the nanoparticles facilitate efficient heat dissipation, leading to an increase in temperature profile. Positive Marangoni parameters can reduce the thickness of the thermal boundary layer near the fluid-solid interface. This reduction in boundary layer thickness allows for more efficient heat transfer from the stretching disk to the fluid, resulting in an increase in temperature profile. The influence of magnetic parameters, combined with positive Marangoni parameters, can further enhance fluid motion and mixing. This enhanced motion facilitates heat transfer within the fluid, contributing to the increase in temperature profile along the stretching disk. In hybrid nanofluids, interactions between different nanoparticles may also play a role. The combined effects of nanoparticles, such as their concentration, size, and surface chemistry, can enhance heat transfer properties and contribute to the observed increase in temperature profile. Overall, the increase in temperature profile for nano and hybrid nanofluids over an impermeable stretching disk with positive values of Marangoni parameters, as depicted in Figure 4, involves a combination of enhanced thermocapillary flow, improved heat dissipation, reduction of thermal boundary layer thickness, interaction with magnetic parameters, and effects specific to hybrid nanofluids. These mechanisms collectively lead to the observed increase in temperature profile.

Figure 5 illustrates the influence of magnetic and Marangoni parameters on the dimensionless temperature of nanofluids over a permeable stretching disk with injection. The increase in temperature profile for both nano and hybrid nanofluids containing nanoparticles like water, ZnO_2, and MWCNTs, due to positive values of Marangoni parameters over a permeable stretching disk with injection, as depicted in Figure 5, can be attributed to several physical mechanisms: Positive Marangoni parameters induce surface tension gradients that enhance thermocapillary flow along the fluid surface. This increased flow can lead to higher rates of convective heat transfer, resulting in an increase in temper-

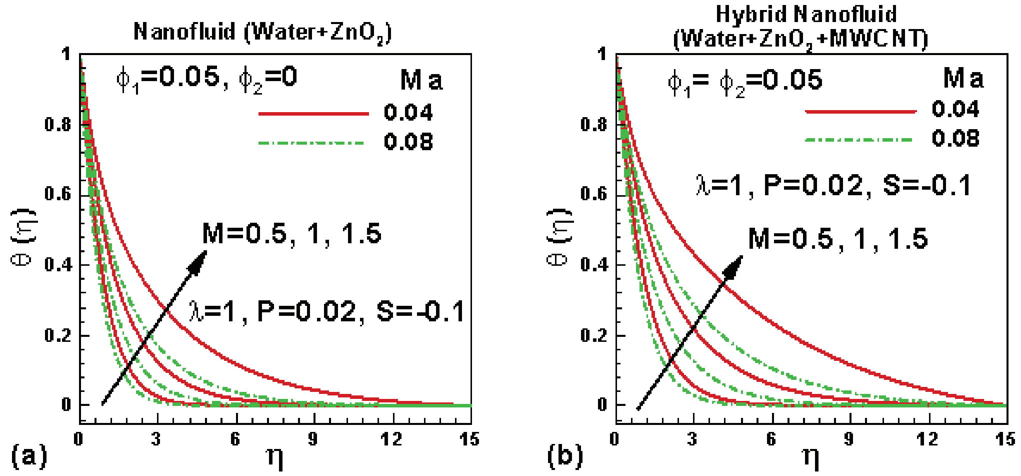

Figure 5: Effects of magnetic and Marangoni parameters on dimensionless temperature of nanofluids over a permeable stretching disk with injection.

ature profile along the stretching disk with injection. The presence of nanoparticles in the nanofluid, such as ZnO_2 and MWCNTs, can enhance heat transfer properties due to their high thermal conductivity. As the fluid flows over the stretching disk with injection, the nanoparticles facilitate efficient heat dissipation, leading to an increase in temperature profile. Injection at the permeable stretching disk promotes fluid mixing, which can lead to higher velocities and enhanced convective heat transfer within the fluid. This increased mixing contributes to the observed increase in temperature profile. The influence of magnetic parameters, combined with positive Marangoni parameters, can further enhance fluid motion and mixing. This enhanced motion facilitates heat transfer within the fluid, contributing to the increase in temperature profile along the stretching disk with injection. In hybrid nanofluids, interactions between different nanoparticles may also play a role. The combined effects of nanoparticles, such as their concentration, size, and surface chemistry, can enhance heat transfer properties and contribute to the observed increase in temperature profile. Overall, the increase in temperature profile for nano and hybrid nanofluids over a permeable stretching disk with injection, due to positive values of Marangoni parameters as depicted in Figure 5, involves a combination of enhanced thermocapillary flow, improved heat dissipation, injection-induced mixing, interaction with magnetic parameters, and effects specific to hybrid nanofluids. These mechanisms collectively lead to the observed increase in temperature profile.

Table 1 provides the Values of nanoparticles (Water, ZnO2, MWCNT). In Table 2 the effective characteristics of hybrid nano liquids associated with ZnO2 and MWCNT are provided. Table 3 provides a comprehensive comparison of Skin friction and Nusselt number for various selected values of water concentration and Marangoni effects. It highlights the numerical values of skin friction and Nusselt number for different combinations of water concentration and Marangoni effects, offering insights into the influence of these parameters on the heat transfer characteristics. This comparative analysis aids in understanding the variations in skin friction and Nusselt number across different scenarios, contributing

Table 3: Comparison of $f''(0)$ and $-\theta'(0)$ for selected values of ϕ_{Cu} and Ma when $Pr = 7$, $\phi_{MWCNT}, \lambda = 1, S = 0, P = 0, Ma = 0.1$

ϕ_{Cu}	$f''(0)$			$-\theta'(0)$		
	Lin and Zheng [26]	Wahid et al. [25]	Present results	Lin and Zheng [26]	Wahid et al. [25]	Present results
0	0.307593	0.307357	0.307429	2.42527	2.436961	2.430326
1.5	0.292523	0.292312	0.292377	2.30518	2.310947	2.306619
3	0.279196	0.279012	0.279068	2.19574	2.196472	2.194103
4.5	0.267258	0.267093	0.267143	2.0946	2.091621	2.09073
Ma						
0.05	0.165779	0.165917	0.165848	1.62999	1.633861	1.63193
0.15	0.345094	0.345125	0.345110	2.35221	2.356451	2.35433
0.25	0.485152	0.485157	0.485151	2.78771	2.793882	2.79080
0.35	0.607148	0.607148	0.607148	3.12053	3.125485	3.12301
0.45	0.717893	0.717879	0.717891	3.39309	3.398591	3.39584

to the advancement of knowledge in thermal fluid dynamics.

5. Conclusions

This study explores the heat transfer behavior of MHD hybrid nanofluids over-stretching disks with porous suction and injection effects, considering both positive and negative Marangoni parameters. We transform the governing partial differential equations into ordinary differential equations and analyze them using the bvp4c solver. The investigation focuses on the skin friction and Nusselt number for various water concentrations and Marangoni effects, incorporating nanoparticles of water, ZnO_2, and MWCNT. We provide detailed results on the radial and axial velocity profiles, as well as temperature profiles, under different suction and injection conditions, including negative Marangoni parameters. Additionally, we compare our findings with existing literature for skin friction and Nusselt number and found them with an excellent agreement. This comprehensive approach enhances the understanding of heat transfer dynamics in MHD nanofluids over-stretching disks with porous media effects, offering valuable insights for thermal fluid dynamics and engineering applications across various fields.

Future Recommendations

In future this work can be extended to other non-Newtonian fluids with various effects. This problem can be also extended to another physical configuration. Different types of nanoparticles and base fluids can be used in future studies.

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