On $I_{s^g}$-Continuous Functions in Ideal Topological Spaces

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Abstract. By using $I_{s^g}$-closed sets due to Khan and Hamza [5], we introduce the notion of $I_{s^g}$-continuous functions in ideal topological spaces. We obtain several properties of $I_{s^g}$-continuity and the relationship between this function and other related functions.

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1. Introduction

Khan and Hamza [5] introduced and investigated the notion of $I_{s^g}$-closed sets in ideal topological spaces as a generalization of $I_g$-closed sets due to Dontchev et al. [2]. In this paper, by using $I_{s^g}$-closed sets we introduce $I_{s^g}$-continuous functions, strongly $I_{s^g}$-continuous functions and weakly $I_{s^g}$-continuous functions. It turns out that weak $I_{s^g}$-continuity is weaker than weak $I$-continuity defined by Ackgoz et al. [1]. We obtain several properties of $I_{s^g}$-continuity and the relationship between this function and other related functions.

2. Preliminaries

Let $(X, \tau)$ be a topological space with no separation properties assumed. For a subset $A$ of a topological space $(X, \tau)$, $cl(A)$ and $Int(A)$ denote the closure and interior of $A$ in $(X, \tau)$, respectively. An ideal $I$ on a set $X$ is a non-empty collection of subsets of $X$ which satisfies the following properties:

(1) $A \in I$ and $B \subset A$ implies $B \in I$,

(2) $A \in I$ and $B \in I$ implies $A \cup B \in I$.

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An ideal topological space is a topological space \((X, \tau, I)\) with an ideal \(I\) on \(X\) and is denoted by \((X, \tau, I)\). For a subset \(A \subseteq X\), \(A^*(I, \tau) = \{x \in X : A \cap U \notin I\ \text{for every } U \in \tau(x)\}\), where, \(\tau(x) = \{U \in \tau : x \in U\}\), is called the local function of \(A\) with respect to \(I\) and \(\tau\) \cite{4, 6}. We simply write \(A^*\) or \(A^*_X\) instead of \(A^*(I, \tau)\) and \(B^*_A\) for \(B^*(I_A, \tau_A)\) in case there is no chance for confusion. For every ideal topological space \((X, \tau, I)\), there exists a topology \(\tau^*(I)\), finer than \(\tau\), generated by the base \(\beta(I, \tau) = \{U - J : U \in \tau \land J \in I\}\). It is known in \cite{4} that \(\beta(I, \tau)\) is not necessarily a topology. When there is no ambiguity, \(\tau^*(I)\) is denoted by \(\tau^*\). Recall that \(A\) is said to be \((\tau^*)\)-dense in itself (resp. \((\tau^*)\)-perfect) if \(A \subseteq A^*\) (resp. \(A^* \subseteq A\), \(A = A^*\)). For a subset \(A\) of \(X\), \(\text{cl}^*(A)\) and \(\text{Int}^*(A)\) will, respectively, denote the closure and interior of \(A\) in \((X, \tau^*)\). A subset \(A\) of \(X\) is said to be semi-open \cite{7} if there exists an open set \(U\) in \(X\) such that \(U \subseteq A \subseteq \text{cl}(U)\). The complement of a semi-open set is said to be semi-closed. A subset \(A\) is said to be semi-regular if \(A\) is semi-open and semi-closed. A subset \(A\) of \(X\) is said to be generalized closed \cite{8} (briefly, \(g\)-closed) if \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \(X\). The complement of a \(g\)-closed set is said to be \(g\)-open. A space \(X\) is called a \(T_{1/2}\)-space \cite{3} if every \(g\)-closed set in \(X\) is closed. Recall that if \((X, \tau, I)\) is an ideal topological space and \(A\) is a subset of \(X\), then \((A, \tau_A, I_A)\) is an ideal topological space, where \(\tau_A\) is the relative topology on \(A\) and \(I_A = \{A \cap J : J \in I\}\).

3. \(I_{s^*g}\)-Closed Sets

The notion of \(I_{s^*g}\)-closed sets was defined by Khan and Hamza \cite{5}. In this section we will obtain further properties of \(I_{s^*g}\)-closed sets in ideal topological spaces.

**Definition 1.** A subset \(A\) of a space \((X, \tau, I)\) is said to be \(I_{s^*g}\)-closed \cite{5} if \(A^* \subseteq U\) whenever \(A \subseteq U\) and \(U\) is semi-open in \(X\). The complement of an \(I_{s^*g}\)-closed set is said to be \(I_{s^*g}\)-open, equivalently if \(F \subseteq \text{Int}^*(A)\) whenever \(F \subseteq A\) for every semi-closed set \(F\) in \(X\).

**Lemma 1.** Every open set is \(I_{s^*g}\)-open.

**Lemma 2** (\cite{27, 7}). Let \((X, \tau, I)\) be an ideal topological space and \(B \subseteq A \subseteq X\). Then \(B^*(I_A, \tau_A) = B^*(I, \tau) \cap A\).

**Lemma 3.** If \(U\) is open and \(A\) is \(I_{s^*g}\)-open, then \(U \cap A\) is \(I_{s^*g}\)-open.

**Proof.** We prove that \(X - (U \cap A)\) is \(I_{s^*g}\)-closed. Let \(X - (U \cap A) \subseteq G\) where \(G\) is semi-open in \(X\). This implies \((X - U) \cup (X - A) \subseteq G\). Since \((X - A) \subseteq G\) and \((X - A)\) is \(I_{s^*g}\)-closed in \(X\), therefore \((X - A)^* \subseteq G\). Moreover \(X - U\) is closed and contained in \(G\), therefore, \((X - U)^* \subseteq \text{cl}(X - U) \subseteq G\). Hence \((X -(U \cap A))^* = ((X - U) \cup (X - A))^* = (X - U)^* \cup (X - A)^* \subseteq G\). This proves that \(U \cap A\) is \(I_{s^*g}\)-open.

**Theorem 1.** Let \((X, \tau, I)\) be an ideal topological space and \(B \subseteq A \subseteq X\). If \(B\) is an \(I_{s^*g}\)-closed set relative to \(A\), where \(A\) is open and \(I_{s^*g}\)-closed in \(X\), then \(B\) is \(I_{s^*g}\)-closed in \(X\).

**Proof.** Let \(B \subseteq G\), where \(G\) is semi-open in \(X\). Then \(B \subseteq A \cap G\) and \(A \cap G\) is semi-open in \(X\) and hence in \(A\). Therefore \(B_A^* \subseteq A \cap G\). It follows from Lemma 2 that \(A \cap B_A^* \subseteq A \cap G\) or
A \subset G \cup (X - B_\chi^*)$. By Theorem 2.3 of [4], $B_\chi^*$ is closed in $X$ and $G \cup (X - B_\chi^*)$ is semi-open in $X$. Since $A$ is $I_{s^g}$-closed in $X, A_\chi^* \subset G \cup (X - B_\chi^*)$ and hence $B_\chi^* = B_\chi^* \cap A_\chi^* \subset B^* \cap [G \cup (X - B_\chi^*)] \subset G$. Therefore, we obtain $B_\chi^* \subset G$. This proves that $B$ is $I_{s^g}$-closed in $X$.

**Theorem 2.** Let $A$ be a semi-open set in a space $(X, \tau, I)$ and $B \subset A \subset X$. If $B$ is $I_{s^g}$-closed in $X$, then $B$ is $I_{s^g}$-closed relative to $A$.

**Proof.** Let $B \subset U$ where $U$ is semi-open in $A$. Then there exists a semi-open set $V$ in $X$ such that $U = A \cap V$. Thus $B \subset A \cap V$. Now $B \subset V$ implies that $B_\chi^* \subset V$. It follows that $A \cap B_\chi^* \subset A \cap V$. By Lemma 2, $B_\chi^* \subset A \cap V = U$. This proves that $B$ is a $I_{s^g}$-closed relative to $A$.

**Corollary 1.** Let $B \subset A \subset X$ and $A$ be open and $I_{s^g}$-closed in $(X, \tau, I)$. Then $B$ is $I_{s^g}$-closed relative to $A$ if and only if $B$ is $I_{s^g}$-closed in $X$.

**Theorem 3.** If $B$ is a subset of a space $(X, \tau, I)$ such that $A \subset B \subset A^*$ and $A$ is $I_{s^g}$-closed in $X$, then $B$ is also $I_{s^g}$-closed in $X$.

**Proof.** Let $G$ be a semi-open set in $X$ containing $B$, then $A \subset G$. Since $A$ is $I_{s^g}$-closed, therefore $A^* \subset G$ and hence $B^* \subset (A^*)^* \subset A^* \subset G$. This implies that $B$ is $I_{s^g}$-closed in $X$.

**Theorem 4.** Let $B \subset A \subset X$ and suppose that $B$ is $I_{s^g}$-open in $X$ and $A$ is a semi-regular set in $X$. Then $B$ is $I_{s^g}$-open relative to $A$.

**Proof.** We prove that $A - B$ is $I_{s^g}$-closed relative to $A$. Let $U \in SO(A)$ such that $(A - B) \subset U$. Now $(A - B) \subset (X - B) \subset U \cup (X - A)$, where $U \cup (X - A) \in SO(X)$ because $A \in SR(X)$. Since $X - B$ is $I_{s^g}$-closed in $X$, therefore $(X - B)_\chi^* \subset U \cup (X - A)$ or $(X - B)_\chi^* \cap A \subset (U \cup (X - A)) \cap A \subset U$. By Lemma 2, $(A - B)_\chi^* = (A - B)_\chi^* \cap A \subset (X - B)_\chi^* \cap A \subset U$ and hence $(A - B)_\chi^* \subset U$. This proves that $B$ is $I_{s^g}$-open relative to $A$.

**Theorem 5.** Let $B \subset A \subset X$. $B$ is $I_{s^g}$-open in $A$ and $A$ is open in $X$ then $B$ is $I_{s^g}$-open in $X$.

**Proof.** Let $F$ be a semi-closed subset of $B$ in $X$. Since $A$ is open, therefore $F \in SC(A)$. Since $B$ is $I_{s^g}$-open in $A$, therefore $F \subset Int^*_A(B) = A \cap Int^*_X(B) \subset Int^*_X(B)$. This proves that $B$ is $I_{s^g}$-open in $X$.

4. $I_{s^g}$-Continuous Functions

**Definition 2.** A function $f : (X, \tau, I) \rightarrow (Y, \Omega, J)$ is said to be weakly $I$-continuous [1] if for each $x \in X$ and each open set $V$ in $Y$ containing $f(x)$, there exists an open set $U$ containing $x$ such that $f(U) \subset cl^* (V)$.

**Definition 3.** A function $f : (X, \tau, I) \rightarrow (Y, \Omega)$ is said to be $I_{s^g}$-continuous if for every $U \in \Omega$, $f^{-1}(U)$ is $I_{s^g}$-open in $(X, \tau_X, I)$. 
Remark 1. Every continuous function is $I_{s,g}$-continuous and the converse need not be true as seen from Example 2 (below).

Definition 4. A function $f : (X, \tau) \rightarrow (Y, \Omega, J)$ is said to be strongly $I_{s,g}$-continuous if for every $I_{s,g}$-open set $U$ in $Y$, $f^{-1}(U)$ is open in $X$.

Remark 2. Every strongly $I_{s,g}$-continuous function is continuous but the converse is not true in general.

Example 1. Let $X = \{a, b, c, d\}$ with $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, c, d\}\}$. Let $Y = \{a, b, c, d\}$ with $\Omega = \{\phi, Y, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ and $J = \{\phi, \{a\}\}$. Let $f : (X, \tau) \rightarrow (Y, \Omega, J)$ be defined by $f(a) = b, f(b) = a, f(c) = a$ and $f(d) = d$. Then $f$ is continuous. Let $U = \{a, c\}$ then $U$ is $I_{s,g}$-open in $Y$ but $f^{-1}(U) = \{a, c\}$ is not open in $X$. Hence $f$ is not strongly $I_{s,g}$-continuous.

Definition 5. A function $f : (X, \tau, I) \rightarrow (Y, \Omega, J)$ is said to be weakly $I_{s,g}$-continuous if for each $x \in X$ and each open set $V$ in $Y$ containing $f(x)$, there exists an $I_{s,g}$-open set $U$ containing $x$ such that $f(U) \subset cl^g(V)$.

Remark 3. (1) Every weakly $I$-continuous function is weakly $I_{s,g}$-continuous but the converse is not true in general.

(2) Every $I_{s,g}$-continuous function is weakly $I_{s,g}$-continuous.

By the above definitions, for a function $f : (X, \tau, I) \rightarrow (Y, \Omega, J)$ we obtain the following implications:

$$
\begin{align*}
\text{strong } I_{s,g}\text{-continuity} & \Rightarrow \text{ continuity } \Rightarrow I_{s,g}\text{-continuity} \\
& \downarrow \\
\text{Weak } I\text{-continuity} & \Rightarrow \text{ weak } I_{s,g}\text{-continuity}
\end{align*}
$$

Remark 4. $I_{s,g}$-continuity and weak $I$-continuity are independent of each other.

Example 2. Let $X = Y = \{a, b, c, d\}$ and $\tau = \Omega = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ with $I = \{\phi, \{a\}, \{b\}, \{a, b\}\}$. Define $f : (X, \tau, I) \rightarrow (Y, \Omega, I)$ by $f(a) = a, f(b) = c, f(c) = b$ and $f(d) = b$. Then $f$ is $I_{s,g}$-continuous but not weak $I$-continuous. Since for $c \in X$, $f(c) = b$ and an open set $V = \{a, b\}$ containing $f(c)$, the only open set containing $c$ is $U = X$ and $f(U) \notin cl^g(V) = \{a, b\}$.

Example 3. Let $X = \{a, b, c, d\}$, $\tau = \Omega = \{\phi, X, \{a\}, \{b\}, \{a, b\}\} and I = \{\phi, \{a\}, \{b\}, \{a, b\}\}$. Let $Y = \{1, 2, 3, 4\}$, $\Omega = \{\phi, \{1, 2\}, Y\}$ and $J = \{\phi, \{3\}, \{4\}, \{3, 4\}\}$. Define $f : (X, \tau, I) \rightarrow (Y, \Omega, I)$ by $f(a) = 1, f(b) = 3, f(c) = 2$ and $f(d) = 4$. $f$ is weak $I$-continuous but not $I_{s,g}$-continuous. Since $V = \{1, 2\}$ is open in $Y$ but $f^{-1}(V) = \{a, c\}$ is not $I_{s,g}$-open in $X$.

Theorem 6. Let $f : (X, \tau, I) \rightarrow (Y, \Omega)$ be a function. Then, the following statements are equivalent:

(1) $f$ is $I_{s,g}$-continuous.

(2) The inverse image of each closed set in $Y$ is $I_{s,g}$-closed in $X$. 
Definition 7. Let $N$ be a subset of a space $(X, \tau, I)$ and $x \in X$. Then $N$ is called an $I_s^g$-open neighborhood of $x$ if there exists an $I_s^g$-open set $U$ containing $x$ such that $U \subset N$.

Theorem 7. Let $(X, \tau, I)$ be $T$-dense. Then, for a function $f : (X, \tau, I) \to (Y, \Omega)$ the following statements are equivalent:

1. $f$ is $I_s^g$-continuous.
2. For each $x \in X$ and each open set $V$ in $Y$ with $f(x) \in V$, there exists an $I_s^g$-open set $U$ containing $x$ such that $f(U) \subset V$.
3. For each $x \in X$ and each open set $V$ in $Y$ with $f(x) \in V$, $f^{-1}(V)$ is an $I_s^g$-open neighborhood of $x$.

Proof. (1) $\Rightarrow$ (2) Let $x \in X$ and let $V$ be an open set in $Y$ such that $f(x) \in V$. Since $f$ is $I_s^g$-continuous, $f^{-1}(V)$ is $I_s^g$-open in $X$. By putting $U = f^{-1}(V)$, we have $x \in U$ and $f(U) \subset V$.

(2) $\Rightarrow$ (3) Let $V$ be an open set in $Y$ and let $f(x) \in V$. Then by (2), there exists an $I_s^g$-open set $U$ containing $x$ such that $f(U) \subset V$. So $x \in U \subset f^{-1}(V)$. Hence $f^{-1}(V)$ is an $I_s^g$-open neighborhood of $x$.

(3) $\Rightarrow$ (1) Let $V$ be an open set in $Y$ and let $f(x) \in V$. Then by (3), $f^{-1}(V)$ is an $I_s^g$-neighborhood of $x$. Thus for each $x \in f^{-1}(V)$, there exists an $I_s^g$-open set $U_x$ containing $x$ such that $x \in U_x \subset f^{-1}(V)$. Hence $f^{-1}(V) = \bigcup_{x \in f^{-1}(V)} U_x$ and so by Theorem 2.12 [5], $f^{-1}(V)$ is $I_s^g$-open in $X$.

Theorem 8. A function $f : (X, \tau, J) \to (Y, \Omega, J)$ is strongly $I_s^g$-continuous if and only if the inverse image of every $I_s^g$-closed set in $Y$ is closed in $X$.

Theorem 9. (1) Let $f : (X, \tau) \to (Y, \Omega, J)$ be strongly $I_s^g$-continuous and $h : (Y, \Omega, J) \to (Z, \sigma)$ be $I_s^g$-continuous, then $h \circ f$ is continuous.

(2) Let $f : (X, \tau, I) \to (Y, \Omega)$ be $I_s^g$-continuous and $g : (Y, \Omega) \to (Z, \sigma)$ be continuous, then $g \circ f : (X, \tau, I) \to (Z, \sigma)$ is $I_s^g$-continuous.

Theorem 10. Let $f : (X, \tau, I) \to (Y, \Omega)$ be $I_s^g$-continuous and $U \in RO(X)$. Then the restriction $f | U : (U, \tau | U, I | U) \to (Y, \Omega)$ is $I_s^g$-continuous.

Proof. Let $V$ be any open set of $(Y, \tau_Y)$. Since $f$ is $I_s^g$-continuous, $f^{-1}(V)$ is $I_s^g$-open in $X$. By Theorem 2.14 of [5], $f^{-1}(V) \cap U$ is $I_s^g$-open in $X$. Thus by Theorem 4, $(f | U)^{-1}(V) = f^{-1}(V) \cap U$ is $I_s^g$-open in $U$ because $U$ is regular-open in $X$. This proves that $f | U : (U, \tau | U, I | U) \to (Y, \tau_Y)$ is $I_s^g$-continuous.
**Theorem 11.** Let \( f : (X, \tau, I) \to (Y, \Omega, J) \) be a function and \( \{U_\alpha : \alpha \in \nabla\} \) be an open cover of a \( T \)-dense space \( X \). If the restriction \( f | U_\alpha \) is \( I_{s'g} \)-continuous for each \( \alpha \in \nabla \), then \( f \) is \( I_{s'g} \)-continuous.

**Proof.** Suppose \( F \) is an arbitrary open set in \((Y, \Omega, J)\). Then for each \( \alpha \in \nabla \), we have \((f | U_\alpha)^{-1}(V) = f^{-1}(V) \cap U_\alpha\). Because \( f | U_\alpha \) is \( I_{s'g} \)-continuous, therefore, \( f^{-1}(V) \cap U_\alpha \) is \( I_{s'g} \)-open in \( X \) for each \( \alpha \in \nabla \). Since for each \( \alpha \in \nabla \), \( U_\alpha \) is open in \( X \), by Theorem 5, \( f^{-1}(V) \cap U_\alpha \) is \( I_{s'g} \)-open in \( X \). Now since \( X \) is \( T \)-dense, by [Theorem 2.12 5], \( \bigcup_{\alpha \in \nabla} f^{-1}(V) \cap U_\alpha = f^{-1}(V) \) is \( I_{s'g} \)-open in \( X \). This implies \( f \) is \( I_{s'g} \)-continuous.

**Theorem 12.** If \((X, \tau, I)\) is a \( T \)-dense space and \( f : (X, \tau, I) \to (Y, \Omega, I) \) is \( I_{s'g} \)-continuous, then graph function \( g : X \to X \times Y \), defined by \( g(x) = (x, f(x)) \) for each \( x \in X \), is \( I_{s'g} \)-continuous.

**Proof.** Let \( x \in X \) and \( W \) be any open set in \( X \times Y \) containing \( g(x) = (x, f(x)) \). Then there exists a basic open set \( U \times V \) such that \( g(x) \subset U \times V \subset W \). Since \( f \) is \( I_{s'g} \)-continuous, there exists an \( I_{s'g} \)-open set \( U_1 \) in \( X \) containing \( x \) such that \( f(U_1) \subset V \). By Lemma 3, \( U_1 \cap U \) is \( I_{s'g} \)-open in \( X \) and we have \( x \in U_1 \cap U \subset U \) and \( g(U_1 \cap U) \subset U \times V \subset W \). Since \( X \) is \( T \)-dense, therefore by Theorem 7, \( g \) is \( I_{s'g} \)-continuous.

**Theorem 13.** A function \( f : (X, \tau, I) \to (Y, \Omega, I) \) is \( I_{s'g} \)-continuous if the graph function \( g : X \to X \times Y \) is \( I_{s'g} \)-continuous.

**Proof.** Let \( V \) be an open set in \( Y \) containing \( f(x) \). Then \( X \times V \) is an open set in \( X \times Y \) and by the \( I_{s'g} \)-continuity of \( g \), there exists an \( I_{s'g} \)-open set \( U \) in \( X \) containing \( x \) such that \( g(U) \subset X \times V \). Therefore, we obtain \( f(U) \subset V \). This shows that \( f \) is \( I_{s'g} \)-continuous.

**Theorem 14.** Let \( \{X_\alpha : \alpha \in \nabla\} \) be any family of topological spaces. If \( f : (X, \tau, I) \to \Pi_{\alpha \in \nabla} X_\alpha \) is an \( I_{s'g} \)-continuous function, then \( P_\alpha \circ f : X \to X_\alpha \) is \( I_{s'g} \)-continuous for each \( \alpha \in \nabla \), where \( P_\alpha \) is the projection of \( \Pi X_\alpha \) onto \( X_\alpha \).

**Proof.** We will consider a fixed \( \alpha_0 \in \nabla \). Let \( G_{\alpha_0} \) be an open set of \( X_{\alpha_0} \). Then \( (P_{\alpha_0})^{-1}(G_{\alpha_0}) \) is open in \( \Pi X_\alpha \). Since \( f \) is \( I_{s'g} \)-continuous, \( f^{-1}((P_{\alpha_0})^{-1}(G_{\alpha_0})) = (P_{\alpha_0} \circ f)^{-1}(G_{\alpha_0}) \) is \( I_{s'g} \)-open in \( X \). Thus \( P_\alpha \circ f \) is \( I_{s'g} \)-continuous.

**Corollary 2.** For any bijective function \( f : (X, \tau) \to (Y, \Omega, J) \), the following are equivalent:

1. \( f^{-1} : (Y, \Omega, J) \to (X, \tau) \) is \( I_{s'g} \)-continuous.
2. \( f(U) \) is \( I_{s'g} \)-open in \( Y \) for every open set \( U \) in \( X \).
3. \( f(U) \) is \( I_{s'g} \)-closed in \( Y \) for every closed set \( U \) in \( X \).

**Proof.** It is trivial.

**Definition 8.** An ideal topological space \((X, \tau, I)\) is an RI-space \([1]\), if for each \( x \in X \) and each open neighbourhood \( V \) of \( x \), there exists an open neighbourhood \( U \) of \( x \) such that \( x \in U \subset cl^*(U) \subset V \).
Theorem 15. Let \((Y, \Omega, J)\) be an RI-space and \((X, \tau, I)\) be a \(T\)-dense space. Then 
\(f : (X, \tau, I) \to (Y, \Omega, J)\) is weak \(I_{s,g}\)-continuous if and only if \(f\) is \(I_{s,g}\)-continuous.

Proof. The sufficiency is clear.

Necessity. Let \(x \in X\) and \(V\) be an open set of \(Y\) containing \(f(x)\). Since \(Y\) is an RI-space, there exists an open set \(W\) of \(Y\) such that \(f(x) \in W \subset \text{cl}^*(W) \subset V\). Since \(f\) is weakly \(I_{s,g}\)-continuous, there exists an \(I_{s,g}\)-open set \(U\) such that \(x \in U\) and \(f(U) \subset \text{cl}^*(W)\). Hence we obtain that \(f(U) \subset \text{cl}^*(W) \subset V\). By Theorem 8, \(f\) is \(I_{s,g}\)-continuous.

References


