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Twice order slip on the flows of fractionalized MHD viscoelastic fluid

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Abstract. The objective of this article is to investigate the effect of twice order slip on the MHD flow of fractionalized Maxwell fluid through a permeable medium produced by oscillatory movement of an infinite bottom plate. The governing equations are developed by fractional calculus approach. The exact analytical results for velocity field and related shear stress are calculated using Laplace transforms and presented in terms of generalized M-function satisfying all imposed initial and boundary conditions. The flow results for fractionalized Maxwell, traditional Maxwell and Newtonian fluid with and without slips, in the presence and absence of magnetic and porous effects are derived as the limiting cases. The impact of fractional parameter, slip coefficients, magnetic force and porosity parameter over the velocity field and shear stress are discussed and analyzed through graphical illustrations. The outcomes demonstrate that the speed comparing to streams with slip condition is lower than that for stream with non-slip conditions, and the speed with second-slip condition is lower than that with first-order slip condition.

2010 Mathematics Subject Classifications: 76A05, 76A10

Key Words and Phrases: Twice order slip, MHD Maxwell fluid, fractional derivative, unsteady flow, **M**-function, velocity field, shear stress, Laplace transforms

1. Introduction

The viscoelastic non-Newtonian fluids has got immense importance in research and development of biomedical and chemical industries like metallurgy, plastic, polymer, oil and food industries. Such type fluids include blood, slurries, polymer solutions, cerebrospinal fluid, granular materials, emulsions, gel, suspensions, exotic lubricants, colloidal solutions, composites, earth's mantle, elastomers, drilling mud, clay coatings, pulps, endobronchial secretions, oils and greases. The stream attributes of non-Newtonian viscoelastic fluids are quite different than that of the Newtonian fluid due to complex rheological behavior, and having both elastic and viscous properties [29, 30]. The inadequacy of Naiver-Stoke

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equations to describe all the properties of viscoelasticity encourages the researchers to produce the equations of non-Newtonian viscoelastic fluid from constitutive relation. The obtained equations are higher order and complex differential equations in comparison with the Naiver-Stoke governing equations of the Newtonian fluids due to nonlinear association between shear rate bend and shear stress which depend upon time, shear rate and viscoelastic properties. These characteristics give raise to many industrially originated flow phenomena that are predictable and can be modified by Mathematical or Physical models, classified as integral, differential or rate type model. Thus, many constitutive relations of these fluids are proposed [23]. However, the flow of some polymer melts cannot be described by differential type model due to their incapability of predicting the stress relaxation. Maxwell fluid model described by physical characteristics equivalent to spring and a damper in series, is one of the rate type viscoelastic non-Newtonian fluid model that is widely used to describe the reaction of polymeric fluids taking relaxation phenomena into consideration but in a shear flow it never depicts the connection between shear stress and shear rate[4, 5]. Initially Maxwell fluid model developed to deal with viscoelastic air response [19].

The viscoelastic fluid flow induced by sinusoidal oscillatory motion of the flat plate is the fundamental problem in theoretical, industrial and engineering fields and it additionally happens in biological studies like acoustic gushing around oscillatory object, quasi-periodic flow of blood in cardiovascular system, fluctuating unsteady boundary layer flow and flow in vibrating media. In 1886, Stokes [25] solved the problem for viscous fluid flow induced by rotatory oscillating infinite rod. The viscous fluid flow due to oscillatory shearing movement of an infinite flat plate is termed as stokes second problem and Couette flow if two parallel walls bound the fluid. The close form transient solution to the viscous flow because an oscillatory plate was firstly presented by Fenton [12]. Erdogan [10] determined the two beginning solutions to viscous flow produced by oscillatory movement of the plate and concluded that steady-state streams are set up with same recurrence as the boundary velocity. Yin [27] investigated some similarities and differences between classical and fractional Maxwell model and determined the exact solutions in time and frequency domains showing that fractional model can describe the real viscoelastic fluids better than classical linear model[3, 6, 14, 26]. Gomez-Aguilar [13] solved nonlinear fractional partial differential equation using homotopy perturbation transform method and presented a rapidly converging solution in series form.

Exact solution for stokes second problem and the solutions for Maxwell fluid due to oscillating plate were obtained by Fetecau [8], he presented the results as a sum of steady state and transient part where the transients disappear for large times, while the time required to reach the steady state for cosine oscillations of the plate is smaller than that for the sine oscillations. Khan [28] investigated dual nature solution of fluid flow in porous medium and observed no appreciable effect of slip on dimensionless velocity in upper branch solution. Agkul [1] obtained uniformly convergent solutions of a class of variable order fractional order differential equations by reproducing kernel method and proved its applicability and validity by comparing with other numerical and exact solutions he also checked the accuracy and efficiency of reproducing kernel method finding approximate solution of viscoelastic fluid model. Zheng [17] presented the exact analytical solutions in terms of generalized G and R functions for fractionalized Maxwell fluid flow because of oscillating and regularly quickening plate and examined that velocity is the function of frequency of oscillating plate, increasing for cosine and decreasing for sine oscillations.

The effect of slip on non-Newtonian fluid flow is the core issue in technological applications since in polymer rheological flow the polymer melts exhibit tangible macroscopic wall slip which is lack of continuity in the velocity field in solid-fluid interface usually caused by wall surface texture, material rheology of fluids, low viscosity layers, thin lubricating layers, resin-rich formation, large velocity gradients, dismantling of network structure, adhesion loss are the presents of oil at the wall. The boundary slip is significant in non-natural heart valves polishing, hysteresis, rarefied fluid flow and multiple inter face flow. The fluid velocity is linearly proportional to shear stress at the wall. Mooney [20] quantified the slip during the movement of a Newtonian fluid inside a capillary viscometer dependent on a hydrodynamical theory and concluded that fluid velocity is directly related to normal stress at the enclosure. Beavers [7] introduced a slip stream condition at the enclosure. Before the occurrence of slip, the magnitude of shear stress tends to some critical value, termed as the slip yield stress and fluid velocity is related to normal stress at the enclosure [9]. For dual slip wall effects, the slip initiates first at internal divider from there on at the external enclosure, and it has three stream routines, the no slip, slip just at inward divider and slip at the two enclosures. The effects of slip on oscillating fractionalized Maxwell fluid are observed by Jamil [15] and concluded that oscillation frequency and amplitude of shear stress have inverse relation with slip parameter. Bhatti [16] considered unsteady Stokes flow through porous channel with periodic suction and injection with slip conditions and analyzed the impact of slip on axial and radial velocity for different values of slip parameters on various cross sections of the channel from which the fluid is passed.

In recent years, Fang [11] considered the second order slip and calculated velocity in closed analytical form for viscous fluid over a continuously stretching sheet with heat transfer and combined effect of two slips which greatly influence the fluid flow and shear stress. Sharma [22] developed the twice order velocity slip flow model and observed decreasing behavior of velocity and skin friction for increasing second slip parameter. Liu [18] examined the effects of second order slip on the fractional Maxwell MHD fluid using Riemann-Liouville fractional operator and commented that velocity with slip condition is lower than that with no slip condition and velocity with second slip is lower than first slip velocity. Ganesh and Qasem [21] considered velocity and thermal slips of second order with entropy generation and showed that higher efficiency of thermal fluidic system can be obtained by increasing the slip. Twice order slip in fractional Maxwell fluid using Caputo operator is numerically analyzed by Aman [24] by comparing the results through Tzou's and Stechfest's algorithms, and resulted that slip parameters reduced the flow velocity and shear stress factors.

The objective of this study is twofold, firstly to establish new exact solutions for fractionalized MHD Maxwell fluid in a porous medium due to oscillating plate. Secondly, it is to investigate the effects of twice order slip due to oscillating phenomena of the bottom plate. More specifically, our goal is to calculate the exact analytical solutions for velocity field and shear stress corresponding to the motion of fractionalized Maxwell fluid due to sine and cosine oscillations of the plate. Initially the plate is at rest at time $t = 0^+$, the plate start to oscillate with the velocity $U\sin(\omega t)$ or $UH(t)\cos(\omega t)$ in its own plane, where U is the oscillation amplitude, $H(t)$ is the Heaviside unit step function and ω is the oscillation frequency. The general solutions are obtained using discrete Laplace transforms and represented in terms of generalized **M**-function satisfying all imposed initial and boundary conditions. As the special cases, the general solutions are particularized to give similar solutions for the fractionalized Maxwell, ordinary Maxwell and Newtonian fluid with first order slip or no slip and existence and nonexistence of magnetic or porous effect by varying fractional parameter $\alpha \longrightarrow 1$ and zero relaxation time. Finally, the results are discussed and graphically analyzed for different values of the parameters of interest.

2. Formation of the problem

Consider an incompressible fractionalized MHD Maxwell fluid possessing over an boundlessly expanded plate that is arranged perpendicularly to the y-axis in the (x, z) plane. At first, the fluid is at rest and at the instant $t = 0^+$, an oscillating velocity $U\sin(\omega t)$ or $UH(t)\cos(\omega t)$ is impulsively applied in its possessed plane. Because of the shear, the fluid over the plate starts to move. Its velocity and shear stress is of the form

$$\mathbf{V} = u(y,t)\widehat{i}, \qquad \mathbf{S} = S(y,t), \tag{1}$$

where $\widehat{i}$ is the unit vector in the direction of x. The meaningful equation of Maxwell fluid in the absence of body forces and pressure gradient and with magnetic effect in porous medium are given by

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial u(y,t)}{\partial t} = \nu\frac{\partial^2 u(y,t)}{\partial y^2} - \frac{\sigma B_0^2}{\rho}\left(1+\lambda\frac{\partial}{\partial t}\right)u(y,t) - \frac{\nu\phi}{\kappa}u(y,t); \quad y,\, t > 0, \tag{2}$$

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\tau(y,t) = \mu\frac{\partial u(y,t)}{\partial y}, \tag{3}$$

where $\tau(y,t) = S_{xy}(y,t)$ are the non-zero shear stresses, $\nu = \mu/\rho$ is the kinematic viscosity of the fluid μ is the dynamic viscosity, ρ is the density of the fluid, ϕ is the porosity and κ

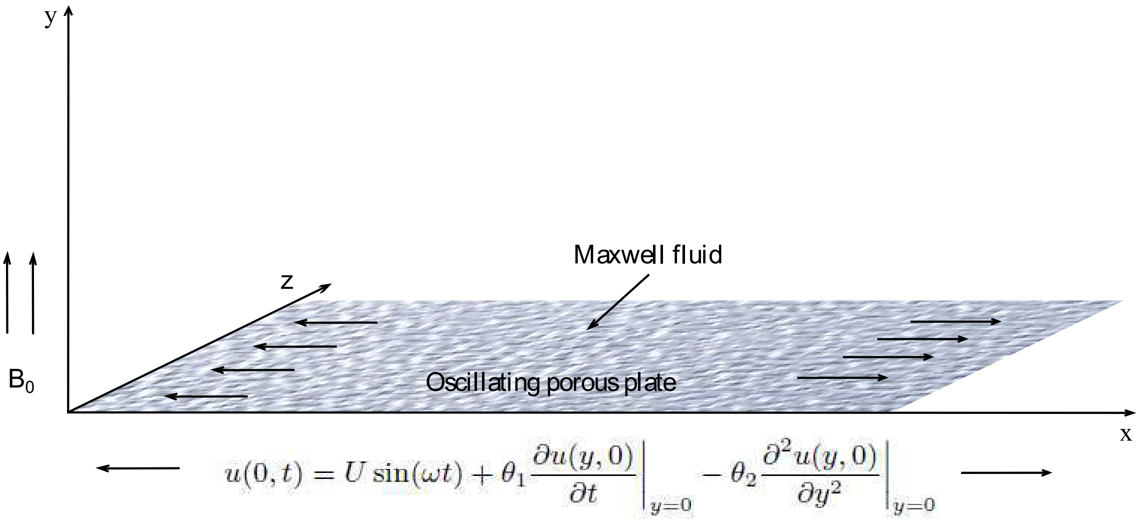

Figure 1: Geometry of the problem

is the permeability of the porous medium, B_0 is the magnitude of applied magnetic field and σ is the electrically conductively of fluid.

The governing equations corresponding to an incompressible fractionalized MHD Maxwell fluid in porous medium, performing the same motion are

$$\left(1+\lambda^{\alpha} D_t^{\alpha}\right)\frac{\partial u(y,t)}{\partial t} = \nu\frac{\partial^2 u(y,t)}{\partial y^2} - M\left(1+\lambda^{\alpha} D_t^{\alpha}\right)u(y,t) - \Psi u(y,t), \tag{4}$$

$$\left(1+\lambda^{\alpha} D_t^{\alpha}\right)\tau(y,t) = \mu\frac{\partial u(y,t)}{\partial y}, \tag{5}$$

where $M = \sigma B_0^2/\rho$, $\Psi = \frac{\nu\phi}{\kappa}$, α is fractional parameter and the fractional operator D_t^{α} named Caputo is defined by [13]

$$D_t^{\alpha} f(t) = \begin{cases} \frac{1}{\Gamma(1-\alpha)}\int_0^t \frac{f'(\tau)}{(t-\tau)^{\alpha}}\,d\tau, & 0<\alpha<1; \\ \frac{df(t)}{dt} & \alpha = 1, \end{cases} \tag{6}$$

and $\Gamma(\cdot)$ is the Gamma function. The fitting initial and boundary limitations are

$$u(y,0) = \frac{\partial u(y,0)}{\partial t} = 0; \quad \tau(y,0) = 0, \quad y>0, \tag{7}$$

$$u(0,t) = U\sin(\omega t) + \theta_1 \frac{\partial u(y,0)}{\partial t}\bigg|_{y=0} - \theta_2 \frac{\partial^2 u(y,0)}{\partial y^2}\bigg|_{y=0}; \quad t \geq 0, \tag{8}$$

$$u(0,t) = UH(t)\cos(\omega t) + \theta_1 \frac{\partial u(y,0)}{\partial t}\bigg|_{y=0} - \theta_2 \frac{\partial^2 u(y,0)}{\partial y^2}\bigg|_{y=0}; \quad t \geq 0, \tag{9}$$

where $H(t)$ is the Heaviside function. Moreover, the natural conditions

$$u(y,t),\ \frac{\partial u(y,t)}{\partial y} \to 0 \text{ as } y \to \infty \quad \text{and} \quad t > 0, \tag{10}$$

have to be also satisfied. The system of fractional partial differential equation (4) and (5), with fitting initial and boundary limitations (7) − (10), will be tackled by Laplace transforms. So as to maintain a strategic distance from extensive calculations of buildups and contour integrals, the discrete inverse Laplace transform method is applied.

The proposed method yield the results in more compact, simplified and generalized form. Participation of each factor can easily be observed in the solutions, whereas the solutions obtained by above method can directly be reduced into the all of its subclass fluids in the presence or absence of porous, magnetic, and slip parameters. Obtained results completely agree with Jamil [15] for first slip, while comparing the effects of twice order slip with exiting literature[18, 21, 24] for other nonlinear motions of the plate, similar behavior for oscillatory motion of the plate is observed. The solutions show that the velocity comparing to flow with slip condition is lower than that with no-slip conditions, and the velocity with twice order slip condition is lower than that with first order slip condition.

3. Solution of the problem

3.1. Calculation of the velocity field

Application of the Laplace transforms to Eq. (4) and using the initial condition $(7)_{1,2}$, we obtained

$$\frac{\partial^2 \overline{u}(y,q)}{\partial y^2} - \frac{(q+M)(1+\lambda^\alpha q^\alpha)+\Psi}{\nu}\overline{u}(y,q) = 0, \tag{11}$$

subject to boundary conditions for sine oscillations

$$\overline{u}(0,t) = U\frac{\omega}{q^2+\omega^2} + \theta_1 \frac{\partial \overline{u}(y,0)}{\partial y}\bigg|_{y=0} - \theta_2 \frac{\partial^2 \overline{u}(y,0)}{\partial y^2}\bigg|_{y=0}, \tag{12}$$

and natural conditions

$$\overline{u}(y,q),\ \frac{\partial \overline{u}(y,q)}{\partial y} \to 0 \text{ as } y \to \infty, \tag{13}$$

where $\overline{u}(y,q)$is the image function of $u(y,t)$ and q is the transform parameter. Solving Eqs.(11) and (12), utilizing conditions (13), we get

$$\overline{u}(y,q) = \frac{U\omega}{(q^2+\omega^2)\left\{1+\theta_1\left[\frac{(q+M)(1+\lambda^\alpha q^\alpha)+\Psi}{\nu}\right]^{\frac{1}{2}}+\theta_2\left[\frac{(q+M)(1+\lambda^\alpha q^\alpha)+\Psi}{\nu}\right]\right\}}$$

$$\times \exp\left\{-\left[\frac{(q+M)(1+\lambda^\alpha q^\alpha)+\Psi}{\nu}\right]^{\frac{1}{2}} y\right\}. \tag{14}$$

To acquire $u(y,t) = L^{-1}\{\overline{u}(y,q)\}$ and to evade the prolix computations of residuals and contours integrals, we utilize the inverse laplace transform method. However, for a proper presentation of the velocity field, the series form representation of Eq. (14) is

$$\overline{u}(y,q) = \frac{U\omega}{q^2+\omega^2} + U\omega\sum_{i=1}^{\infty}(-1)^i\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j\sum_{l=0}^{\infty}\frac{(-\Psi)^l}{l!}\sum_{m=0}^{\infty}\frac{(-M)^m}{m!}\sum_{n=0}^{\infty}(-\omega^2)^n\lambda^{\alpha(\frac{i+j}{2}-l)}$$

$$\times\sum_{p=0}^{\infty}\frac{\Gamma(j-i)\Gamma(l-\frac{i+j}{2})\Gamma(m-\frac{i+j}{2}+l)\Gamma(p-\frac{i+j}{2}+l)(-\lambda^\alpha)^{-p}}{p!\Gamma(-i)\Gamma(-\frac{i+j}{2})\Gamma(-\frac{i+j}{2}+l)\Gamma(-\frac{i+j}{2}+l)}\frac{1}{q^{\alpha p-(1+\alpha)(\frac{i+j}{2}-l)+m+2n+2}}$$

$$+U\omega\sum_{i=0}^{\infty}(-1)^i\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j\sum_{k=1}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k\sum_{l=0}^{\infty}\frac{(-\Psi)^l}{l!}\sum_{m=0}^{\infty}\frac{(-M)^m}{m!}\sum_{n=0}^{\infty}(-\omega^2)^n\lambda^{\alpha(\frac{i+j+k}{2}-l)}$$

$$\times\sum_{p=0}^{\infty}\frac{\Gamma(j-i)\Gamma(l-\frac{i+j+k}{2})\Gamma(m-\frac{i+j+k}{2}+l)\Gamma(p-\frac{i+j+k}{2}+l)(-\lambda^\alpha)^{-p}}{p!\Gamma(-i)\Gamma(-\frac{i+j+k}{2})\Gamma(-\frac{i+j+k}{2}+l)\Gamma(-\frac{i+j+k}{2}+l)}\frac{1}{q^{\alpha p-(1+\alpha)(\frac{i+j+k}{2}-l)+m+2n+2}}. \tag{15}$$

Applying the discrete inverse Laplace transform, we have

$$u(y,t) = U\sin(\omega t) + U\omega\sum_{i=1}^{\infty}(-1)^i\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j\sum_{l=0}^{\infty}\frac{(-\Psi)^l}{l!}\sum_{m=0}^{\infty}\frac{(-M)^m}{m!}\sum_{n=0}^{\infty}(-\omega^2)^n$$

$$\times\lambda^{\alpha(\frac{i+j}{2}-l)}t^{-(1+\alpha)(\frac{i+j}{2}-l)+m+2n+1}$$

$$\times\sum_{p=0}^{\infty}\frac{(-\frac{t^{\alpha}}{\lambda^{\alpha}})^{p}\Gamma(j-i)\Gamma(l-\frac{i+j}{2})\Gamma(m-\frac{i+j}{2}+l)\Gamma(p-\frac{i+j}{2}+l)}{p!\Gamma(-i)\Gamma(-\frac{i+j}{2})\Gamma(-\frac{i+j}{2}+l)\Gamma(-\frac{i+j}{2}+l)\Gamma(\alpha p-(1+\alpha)(\frac{i+j}{2}-l)+m+2n+2)}$$

$$+U\omega\sum_{i=0}^{\infty}(-1)^{i}\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^{j}\sum_{k=1}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^{k}\sum_{l=0}^{\infty}\frac{(-\Psi)^{l}}{l!}\sum_{m=0}^{\infty}\frac{(-M)^{m}}{m!}\sum_{n=0}^{\infty}(-\omega^{2})^{n}$$

$$\times\lambda^{\alpha(\frac{i+j+k}{2}-l)}t^{-(1+\alpha)(\frac{i+j+k}{2}-l)+m+2n+1}$$

$$\times\sum_{p=0}^{\infty}\frac{(-\frac{t^{\alpha}}{\lambda^{\alpha}})^{p}\Gamma(j-i)\Gamma(l-\frac{i+j+k}{2})\Gamma(m-\frac{i+j+k}{2}+l)\Gamma(p-\frac{i+j+k}{2}+l)}{p!\Gamma(-i)\Gamma(-\frac{i+j+k}{2})\Gamma(-\frac{i+j+k}{2}+l)\Gamma(-\frac{i+j+k}{2}+l)\Gamma\left(\alpha p-(1+\alpha)(\frac{i+j+k}{2}-l)+m+2n+2\right)}.\quad(16)$$

Rewriting the above velocity expression in terms of generalized **M**-function corresponding to sine oscillations

$$u_s(y,t)=U\sin(\omega t)+U\omega\sum_{i=1}^{\infty}(-1)^{i}\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^{j}\sum_{l=0}^{\infty}\frac{(-\Psi)^{l}}{l!}\sum_{m=0}^{\infty}\frac{(-M)^{m}}{m!}\sum_{n=0}^{\infty}(-\omega^{2})^{n}$$

$$\times\lambda^{\alpha(\frac{i+j}{2}-l)}\mathbf{M}^{1,4}_{4,6}\left[\frac{t^{\alpha}}{\lambda^{\alpha}}\middle|\begin{matrix}(1+i-j,0),\left(1+\frac{i+j}{2}-l,0\right),\left(1+\frac{i+j}{2}-l-m,0\right),\left(1+\frac{i+j}{2}-l,1\right)\\(0,1),(1+i,0),\left(1+\frac{i+j}{2},0\right),\left(1+\frac{i+j}{2}-l,0\right),\left(1+\frac{i+j}{2}-l,0\right),\left((1+\alpha)\left(\frac{i+j}{2}-l\right)-m-2n-1,\alpha\right)\end{matrix}\right]$$

$$+U\omega\sum_{i=0}^{\infty}(-1)^{i}\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^{j}\sum_{k=1}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^{k}\sum_{l=0}^{\infty}\frac{(-\Psi)^{l}}{l!}\sum_{m=0}^{\infty}\frac{(-M)^{m}}{m!}\sum_{n=0}^{\infty}(-\omega^{2})^{n}$$

$$\times\lambda^{\alpha(\frac{i+j+k}{2}-l)}\mathbf{M}^{1,4}_{4,6}\left[\frac{t^{\alpha}}{\lambda^{\alpha}}\middle|\begin{matrix}(1+i-j,0),\left(1+\frac{i+j+k}{2}-l,0\right),\left(1+\frac{i+j+k}{2}-l-m,0\right),\left(1+\frac{i+j+k}{2}-l,1\right)\\(0,1),(1+i,0),\left(1+\frac{i+j+k}{2},0\right),\left(1+\frac{i+j+k}{2}-l,0\right),\left(1+\frac{i+j+k}{2}-l,0\right),\left((1+\alpha)\left(\frac{i+j+k}{2}-l\right)-m-2n-1,\alpha\right)\end{matrix}\right],\quad(17)$$

where the new generalized **M**-function together property of the Fox H-function [2] is defined by

$$\mathbf{M}^{1,k}_{k,n+1}\left[z\middle|\begin{matrix}(1-a_1,A_1),\ldots(1-a_k,A_k)\\(0,1),(1-b_1,B_1),\ldots(1-b_n,B_n)\end{matrix}\right]=t^{b_n-1}H^{1,k}_{k,n+1}\left[z\middle|\begin{matrix}(1-a_1,A_1),\ldots(1-a_k,A_k)\\(0,1),(1-b_1,B_1),\ldots(1-b_n,B_n)\end{matrix}\right]$$

$$= t^{b_n - 1} \sum_{p=0}^{\infty} \frac{(-z)^p \prod_{j=1}^{k} \Gamma(a_j + A_j p)}{p! \prod_{j=1}^{n} \Gamma(b_j + B_j p)}. \tag{18}$$

Similarly, the velocity field for cosine oscillations is

$$u_c(y,t) = UH(t)\cos(\omega t) + UH(t) \sum_{i=1}^{\infty} (-1)^i \sum_{j=0}^{\infty} \frac{1}{j!} \left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j} \left(-\frac{\theta_2}{\nu}\right)^j \sum_{l=0}^{\infty} \frac{(-\Psi)^l}{l!} \sum_{m=0}^{\infty} \frac{(-M)^m}{m!}$$

$$\times \sum_{n=0}^{\infty} (-\omega^2)^n \lambda^{\alpha(\frac{i+j}{2}-l)} \mathbf{M}_{4,6}^{1,4}\left[\frac{t^\alpha}{\lambda^\alpha}\middle|\begin{matrix}(1+i-j,0),\left(1+\frac{i+j}{2}-l,0\right),\left(1+\frac{i+j}{2}-l-m,0\right),\left(1+\frac{i+j}{2}-l,1\right)\\(0,1),(1+i,0),\left(1+\frac{i+j}{2},0\right),\left(1+\frac{i+j}{2}-l,0\right),\left(1+\frac{i+j}{2}-l,0\right),\left((1+\alpha)\left(\frac{i+j}{2}-l\right)-m-2n,\alpha\right)\end{matrix}\right]$$

$$+UH(t) \sum_{i=0}^{\infty} (-1)^i \sum_{j=0}^{\infty} \frac{1}{j!} \left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j} \left(-\frac{\theta_2}{\nu}\right)^j \sum_{k=1}^{\infty} \frac{1}{k!} \left(-\frac{y}{\sqrt{\nu}}\right)^k \sum_{l=0}^{\infty} \frac{(-\Psi)^l}{l!} \sum_{m=0}^{\infty} \frac{(-M)^m}{m!} \sum_{n=0}^{\infty} (-\omega^2)^n$$

$$\times \lambda^{\alpha(\frac{i+j+k}{2}-l)} \mathbf{M}_{4,6}^{1,4}\left[\frac{t^\alpha}{\lambda^\alpha}\middle|\begin{matrix}(1+i-j,0),\left(1+\frac{i+j+k}{2}-l,0\right),\left(1+\frac{i+j+k}{2}-l-m,0\right),\left(1+\frac{i+j+k}{2}-l,1\right)\\(0,1),(1+i,0),\left(1+\frac{i+j+k}{2},0\right),\left(1+\frac{i+j+k}{2}-l,0\right),\left(1+\frac{i+j+k}{2}-l,0\right),\left((1+\alpha)\left(\frac{i+j+k}{2}-l\right)-m-2n,\alpha\right)\end{matrix}\right]. \tag{19}$$

3.2. Calculation of the shear stress

Applying the Laplace transform, principle of sequential fractional derivative to Eq. (5) and using the initial conditions $(7)_3$, we get

$$\overline{\tau}(y,q) = \frac{\mu}{1+\lambda^\alpha q^\alpha} \frac{\partial \overline{u}(y,q)}{\partial y}, \tag{20}$$

where $\overline{\tau}(y,q)$ is the Laplace transform of $\tau(y,t)$. Using Eq. (14) in the above expression, we have

$$\overline{\tau}(y,q) = -\frac{U\mu\omega\left[(q+M)(1+\lambda^\alpha q^\alpha)+\Psi\right]^{\frac{1}{2}}}{\sqrt{\nu}(q^2+\omega^2)(1+\lambda^\alpha q^\alpha)\left\{1+\theta_1\left[\frac{(q+M)(1+\lambda^\alpha q^\alpha)+\Psi}{\nu}\right]^{\frac{1}{2}}+\theta_2\left[\frac{(q+M)(1+\lambda^\alpha q^\alpha)+\Psi}{\nu}\right]\right\}}$$

$$\times \exp\left\{-\left[\frac{(q+M)(1+\lambda^\alpha q^\alpha)+\Psi}{\nu}\right]^{\frac{1}{2}} y\right\}. \tag{21}$$

The series form of the above expression is

$$\overline{\tau}(y,q)=-\frac{U\mu\omega}{\sqrt{\nu}}\sum_{i=0}^{\infty}(-1)^{i}\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^{j}\sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^{k}\sum_{l=0}^{\infty}\frac{(-\Psi)^{l}}{l!}\sum_{m=0}^{\infty}\frac{(-M)^{m}}{m!}\sum_{n=0}^{\infty}(-\omega^2)^{n}$$

$$\times\lambda^{\alpha(\frac{i+j+k-1}{2}-l)}\sum_{p=0}^{\infty}\frac{\Gamma(j-i)\Gamma(l-\frac{i+j+k+1}{2})\Gamma(m-\frac{i+j+k+1}{2}+l)\Gamma(p-\frac{i+j+k-1}{2}+l)\,(-\lambda^{\alpha})^{-p}}{p!\Gamma(-i)\Gamma(-\frac{i+j+k+1}{2})\Gamma(-\frac{i+j+k+1}{2}+l)\Gamma(-\frac{i+j+k-1}{2}+l)}$$

$$\times\frac{1}{q^{\alpha p-(1+\alpha)\left(\frac{i+j+k}{2}-l\right)-\frac{1}{2}(1-\alpha)+m+2n+2}}. \tag{22}$$

Application of discrete inverse Laplace transform yield the following result,

$$\overline{\tau}(y,q)=-\frac{U\mu\omega}{\sqrt{\nu}}\sum_{i=0}^{\infty}(-1)^{i}\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^{j}\sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^{k}\sum_{l=0}^{\infty}\frac{(-\Psi)^{l}}{l!}\sum_{m=0}^{\infty}\frac{(-M)^{m}}{m!}$$

$$\times\sum_{n=0}^{\infty}(-\omega^2)^{n}\lambda^{\alpha(\frac{i+j+k-1}{2}-l)}t^{-(1+\alpha)\left(\frac{i+j+k}{2}-l\right)-\frac{1}{2}(1-\alpha)+m+2n+1}$$

$$\times\sum_{p=0}^{\infty}\frac{(-\frac{t^{\alpha}}{\lambda^{\alpha}})^{p}\Gamma(j-i)\Gamma(l-\frac{i+j+k+1}{2})\Gamma(m-\frac{i+j+k+1}{2}+l)}{p!\Gamma(-i)\Gamma(-\frac{i+j+k+1}{2})\Gamma(-\frac{i+j+k+1}{2}+l)\Gamma(-\frac{i+j+k}{2}+l)}$$

$$\times\frac{\Gamma(p-\frac{i+j+k-1}{2}+l)}{\Gamma(\alpha p-(1+\alpha)(\frac{i+j+k}{2}-l)-\frac{1}{2}(1-\alpha)+m+2n+2)}. \tag{23}$$

Generalized **M**-function representation of the above expression is

$$\tau_s(y,t)=-\frac{U\mu\omega}{\sqrt{\nu}}\sum_{i=0}^{\infty}(-1)^{i}\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^{j}$$

$$\times\sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^{k}\sum_{l=0}^{\infty}\frac{(-\Psi)^{l}}{l!}\sum_{m=0}^{\infty}\frac{(-M)^{m}}{m!}\sum_{n=0}^{\infty}(-\omega^2)^{n}\lambda^{\alpha(\frac{i+j+k-1}{2}-l)}$$

$$\times \mathbf{M}_{4,6}^{1,4}\left[\frac{t^{\alpha}}{\lambda^{\alpha}}\left|\begin{matrix}(1+i-j,0),\left(1+\frac{i+j+k+1}{2}-l,0\right),\left(1+\frac{i+j+k+1}{2}-l-m,0\right),\left(1+\frac{i+j+k-1}{2}-l,1\right)\\(0,1),(1+i,0),\left(1+\frac{i+j+k+1}{2},0\right),\left(1+\frac{i+j+k+1}{2}-l,0\right),\left(1+\frac{i+j+k-1}{2}-l,0\right),\left((1+\alpha)\left(\frac{i+j+k}{2}-l\right)+\frac{1}{2}(1-\alpha)-m-2n-1,\alpha\right)\end{matrix}\right.\right]. \quad (24)$$

Similarly, shear stress for cosine oscillations is

$$\tau_c(y,t) = -\frac{UH(t)\mu}{\sqrt{\nu}}\sum_{i=0}^{\infty}(-1)^i\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j$$

$$\times\sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k\sum_{l=0}^{\infty}\frac{(-\Psi)^l}{l!}\sum_{m=0}^{\infty}\frac{(-M)^m}{m!}\sum_{n=0}^{\infty}(-\omega^2)^n\lambda^{\alpha\left(\frac{i+j+k-1}{2}-l\right)}$$

$$\times \mathbf{M}_{4,6}^{1,4}\left[\frac{t^{\alpha}}{\lambda^{\alpha}}\left|\begin{matrix}(1+i-j,0),\left(1+\frac{i+j+k+1}{2}-l,0\right),\left(1+\frac{i+j+k+1}{2}-l-m,0\right),\left(1+\frac{i+j+k-1}{2}-l,1\right)\\(0,1),(1+i,0),\left(1+\frac{i+j+k+1}{2},0\right),\left(1+\frac{i+j+k+1}{2}-l,0\right),\left(1+\frac{i+j+k-1}{2}-l,0\right),\left((1+\alpha)\left(\frac{i+j+k}{2}-l\right)+\frac{1}{2}(1-\alpha)-m-2n,\alpha\right)\end{matrix}\right.\right]. \quad (25)$$

4. Special cases

4.1. Ordinary MHD Maxwell fluid in porous medium

When $\alpha \longrightarrow 1$, the Eq. (17), (19), (24), (25) reduces to the solutions for usual Maxwell fluid.

$$u_s(y,t) = U\sin(\omega t) + U\omega\sum_{i=1}^{\infty}(-1)^i\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j\sum_{l=0}^{\infty}\frac{(-\Psi)^l}{l!}\sum_{m=0}^{\infty}\frac{(-M)^m}{m!}\sum_{n=0}^{\infty}(-\omega^2)^n$$

$$\times\lambda^{\left(\frac{i+j}{2}-l\right)}\mathbf{M}_{4,6}^{1,4}\left[\frac{t}{\lambda}\left|\begin{matrix}(1+i-j,0),\left(1+\frac{i+j}{2}-l,0\right),\left(1+\frac{i+j}{2}-l-m,0\right),\left(1+\frac{i+j}{2}-l,1\right)\\(0,1),(1+i,0),\left(1+\frac{i+j}{2},0\right),\left(1+\frac{i+j}{2}-l,0\right),\left(1+\frac{i+j}{2}-l,0\right),(i+j-2l-m-2n-1,1)\end{matrix}\right.\right]$$

$$+U\omega\sum_{i=0}^{\infty}(-1)^i\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j\sum_{k=1}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k\sum_{l=0}^{\infty}\frac{(-\Psi)^l}{l!}\sum_{m=0}^{\infty}\frac{(-M)^m}{m!}\sum_{n=0}^{\infty}(-\omega^2)^n$$

$$\times\lambda^{\left(\frac{i+j+k}{2}-l\right)}\mathbf{M}_{4,6}^{1,4}\left[\frac{t}{\lambda}\left|\begin{matrix}(1+i-j,0),\left(1+\frac{i+j+k}{2}-l,0\right),\left(1+\frac{i+j+k}{2}-l-m,0\right),\left(1+\frac{i+j+k}{2}-l,1\right)\\(0,1),(1+i,0),\left(1+\frac{i+j+k}{2},0\right),\left(1+\frac{i+j+k}{2}-l,0\right),\left(1+\frac{i+j+k}{2}-l,0\right),(i+j+k-2l-m-2n-1,1)\end{matrix}\right.\right], \quad (26)$$

$$u_c(y,t) = UH(t)\cos(\omega t) + UH(t)\sum_{i=1}^{\infty}(-1)^i \sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j \sum_{l=0}^{\infty}\frac{(-\Psi)^l}{l!}\sum_{m=0}^{\infty}\frac{(-M)^m}{m!}$$

$$\times \sum_{n=0}^{\infty}(-\omega^2)^n \lambda^{\left(\frac{i+j}{2}-l\right)} \mathbf{M}^{1,4}_{4,6}\left[\frac{t}{\lambda}\Bigg|^{(1+i-j,0),\left(1+\frac{i+j}{2}-l,0\right),\left(1+\frac{i+j}{2}-l-m,0\right),\left(1+\frac{i+j}{2}-l,1\right)}_{(0,1),(1+i,0),\left(1+\frac{i+j}{2},0\right),\left(1+\frac{i+j}{2}-l,0\right),\left(1+\frac{i+j}{2}-l,0\right),(i+j-2l-m-2n,1)}\right]$$

$$+UH(t)\sum_{i=0}^{\infty}(-1)^i \sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j \sum_{k=1}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k \sum_{l=0}^{\infty}\frac{(-\Psi)^l}{l!}\sum_{m=0}^{\infty}\frac{(-M)^m}{m!}$$

$$\times \sum_{n=0}^{\infty}(-\omega^2)^n \lambda^{\left(\frac{i+j+k}{2}-l\right)} \mathbf{M}^{1,4}_{4,6}\left[\frac{t}{\lambda}\Bigg|^{(1+i-j,0),\left(1+\frac{i+j+k}{2}-l,0\right),\left(1+\frac{i+j+k}{2}-l-m,0\right),\left(1+\frac{i+j+k}{2}-l,1\right)}_{(0,1),(1+i,0),\left(1+\frac{i+j+k}{2},0\right),\left(1+\frac{i+j+k}{2}-l,0\right),\left(1+\frac{i+j+k}{2}-l,0\right),(i+j+k-2l-m-2n,1)}\right], \quad (27)$$

and the corresponding shear stresses are

$$\tau_s(y,t) = -\frac{U\mu\omega}{\sqrt{\nu}}\sum_{i=0}^{\infty}(-1)^i \sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j$$

$$\times \sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k \sum_{l=0}^{\infty}\frac{(-\Psi)^l}{l!}\sum_{m=0}^{\infty}\frac{(-M)^m}{m!}\sum_{n=0}^{\infty}(-\omega^2)^n \lambda^{\left(\frac{i+j+k-1}{2}-l\right)}$$

$$\times \mathbf{M}^{1,4}_{4,6}\left[\frac{t}{\lambda}\Bigg|^{(1+i-j,0),\left(1+\frac{i+j+k+1}{2}-l,0\right),\left(1+\frac{i+j+k+1}{2}-l-m,0\right),\left(1+\frac{i+j+k-1}{2}-l,1\right)}_{(0,1),(1+i,0),\left(1+\frac{i+j+k+1}{2},0\right),\left(1+\frac{i+j+k+1}{2}-l,0\right),\left(1+\frac{i+j+k-1}{2}-l,0\right),(i+j+k-2l-m-2n-1,1)}\right]. \quad (28)$$

$$\tau_c(y,t) = -\frac{UH(t)\mu}{\sqrt{\nu}}\sum_{i=0}^{\infty}(-1)^i \sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j$$

$$\times \sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k \sum_{l=0}^{\infty}\frac{(-\Psi)^l}{l!}\sum_{m=0}^{\infty}\frac{(-M)^m}{m!}\sum_{n=0}^{\infty}(-\omega^2)^n \lambda^{\left(\frac{i+j+k-1}{2}-l\right)}$$

$$\times \mathbf{M}^{1,4}_{4,6}\left[\frac{t}{\lambda}\Bigg|^{(1+i-j,0),\left(1+\frac{i+j+k+1}{2}-l,0\right),\left(1+\frac{i+j+k+1}{2}-l-m,0\right),\left(1+\frac{i+j+k-1}{2}-l,1\right)}_{(0,1),(1+i,0),\left(1+\frac{i+j+k+1}{2},0\right),\left(1+\frac{i+j+k+1}{2}-l,0\right),\left(1+\frac{i+j+k-1}{2}-l,0\right),(i+j+k-2l-m-2n,1)}\right]. \quad (29)$$

4.2. Fractionalized Maxwell fluid in porous medium

When $M \to 0$, the Eq. (17), (19), (24), (25) reduces to

$$u_s(y,t) = U\sin(\omega t) + U\omega \sum_{i=1}^{\infty}(-1)^i \sum_{j=0}^{\infty} \frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j} \left(-\frac{\theta_2}{\nu}\right)^j \sum_{l=0}^{\infty} \frac{(-\Psi)^l}{l!} \sum_{n=0}^{\infty}(-\omega^2)^n \lambda^{\alpha(\frac{i+j}{2}-l)}$$

$$\times \mathbf{M}_{3,5}^{1,3}\left[\frac{t^\alpha}{\lambda^\alpha}\Bigg|_{(0,1),(1+i,0),\left(1+\frac{i+j}{2},0\right),\left(1+\frac{i+j}{2}-l,0\right),\left((1+\alpha)\left(\frac{i+j}{2}-l\right)-2n-1,\alpha\right)}^{(1+i-j,0),\left(1+\frac{i+j}{2}-l,0\right),\left(1+\frac{i+j}{2}-l,1\right)}\right]$$

$$+U\omega \sum_{i=0}^{\infty}(-1)^i \sum_{j=0}^{\infty} \frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j} \left(-\frac{\theta_2}{\nu}\right)^j \sum_{k=1}^{\infty} \frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k \sum_{l=0}^{\infty} \frac{(-\Psi)^l}{l!} \sum_{n=0}^{\infty}(-\omega^2)^n \lambda^{\alpha(\frac{i+j+k}{2}-l)}$$

$$\times \mathbf{M}_{3,5}^{1,3}\left[\frac{t^\alpha}{\lambda^\alpha}\Bigg|_{(0,1),(1+i,0),\left(1+\frac{i+j+k}{2},0\right),\left(1+\frac{i+j+k}{2}-l,0\right),\left((1+\alpha)\left(\frac{i+j+k}{2}-l\right)-2n-1,\alpha\right)}^{(1+i-j,0),\left(1+\frac{i+j+k}{2}-l,0\right),\left(1+\frac{i+j+k}{2}-l,1\right)}\right], \tag{30}$$

$$u_c(y,t) = UH(t)\cos(\omega t) + UH(t) \sum_{i=1}^{\infty}(-1)^i \sum_{j=0}^{\infty} \frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j} \left(-\frac{\theta_2}{\nu}\right)^j \sum_{l=0}^{\infty} \frac{(-\Psi)^l}{l!} \sum_{n=0}^{\infty}(-\omega^2)^n$$

$$\times \lambda^{\alpha(\frac{i+j}{2}-l)} \mathbf{M}_{3,5}^{1,3}\left[\frac{t^\alpha}{\lambda^\alpha}\Bigg|_{(0,1),(1+i,0),\left(1+\frac{i+j}{2},0\right),\left(1+\frac{i+j}{2}-l,0\right),\left((1+\alpha)\left(\frac{i+j}{2}-l\right)-2n,\alpha\right)}^{(1+i-j,0),\left(1+\frac{i+j}{2}-l,0\right),\left(1+\frac{i+j}{2}-l,1\right)}\right]$$

$$+UH(t) \sum_{i=0}^{\infty}(-1)^i \sum_{j=0}^{\infty} \frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j} \left(-\frac{\theta_2}{\nu}\right)^j \sum_{k=1}^{\infty} \frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k \sum_{l=0}^{\infty} \frac{(-\Psi)^l}{l!} \sum_{n=0}^{\infty}(-\omega^2)^n$$

$$\times \lambda^{\alpha(\frac{i+j+k}{2}-l)} \mathbf{M}_{3,5}^{1,3}\left[\frac{t^\alpha}{\lambda^\alpha}\Bigg|_{(0,1),(1+i,0),\left(1+\frac{i+j+k}{2},0\right),\left(1+\frac{i+j+k}{2}-l,0\right),\left((1+\alpha)\left(\frac{i+j+k}{2}-l\right)-2n,\alpha\right)}^{(1+i-j,0),\left(1+\frac{i+j+k}{2}-l,0\right),\left(1+\frac{i+j+k}{2}-l,1\right)}\right], \tag{31}$$

and the associated shear stresses are

$$\tau_s(y,t) = -\frac{U\mu\omega}{\sqrt{\nu}} \sum_{i=0}^{\infty}(-1)^i \sum_{j=0}^{\infty} \frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j} \left(-\frac{\theta_2}{\nu}\right)^j \sum_{k=0}^{\infty} \frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k \sum_{l=0}^{\infty} \frac{(-\Psi)^l}{l!} \sum_{n=0}^{\infty}(-\omega^2)^n$$

$$\times\lambda^{\alpha(\frac{i+j+k-1}{2}-l)}\mathbf{M}^{1,3}_{3,5}\left[\frac{t^\alpha}{\lambda^\alpha}\Bigg|^{(1+i-j,0),\left(1+\frac{i+j+k+1}{2}-l,0\right),\left(1+\frac{i+j+k-1}{2}-l,1\right)}_{(0,1),(1+i,0),\left(1+\frac{i+j+k+1}{2},0\right),\left(1+\frac{i+j+k-1}{2}-l,0\right),\left((1+\alpha)\left(\frac{i+j+k}{2}-l\right)+\frac{1}{2}(1-\alpha)-2n-1,\alpha\right)}\right], \quad (32)$$

$$\tau_c(y,t)=-\frac{UH(t)\mu}{\sqrt{\nu}}\sum_{i=0}^{\infty}(-1)^i\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j\sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k\sum_{l=0}^{\infty}\frac{(-\Psi)^l}{l!}\sum_{n=0}^{\infty}(-\omega^2)^n$$

$$\times\lambda^{\alpha(\frac{i+j+k-1}{2}-l)}\mathbf{M}^{1,3}_{3,5}\left[\frac{t^\alpha}{\lambda^\alpha}\Bigg|^{(1+i-j,0),\left(1+\frac{i+j+k+1}{2}-l,0\right),\left(1+\frac{i+j+k-1}{2}-l,1\right)}_{(0,1),(1+i,0),\left(1+\frac{i+j+k+1}{2},0\right),\left(1+\frac{i+j+k-1}{2}-l,0\right),\left((1+\alpha)\left(\frac{i+j+k}{2}-l\right)+\frac{1}{2}(1-\alpha)-2n,\alpha\right)}\right]. \quad (33)$$

4.3. Fractionalized MHD Maxwell fluid without porous effect

Putting $\Psi\to 0$, the Eq. (17), (19), (24), (25) simplified into

$$u_s(y,t)=U\sin(\omega t)+U\omega\sum_{i=1}^{\infty}(-1)^i\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j\sum_{m=0}^{\infty}\frac{(-M)^m}{m!}\sum_{n=0}^{\infty}(-\omega^2)^n$$

$$\times\lambda^{\alpha(\frac{i+j}{2})}\mathbf{M}^{1,3}_{3,5}\left[\frac{t^\alpha}{\lambda^\alpha}\Bigg|^{(1+i-j,0),\left(1+\frac{i+j}{2}-m,0\right),\left(1+\frac{i+j}{2},1\right)}_{(0,1),(1+i,0),\left(1+\frac{i+j}{2},0\right),\left(1+\frac{i+j}{2},0\right),\left((1+\alpha)\left(\frac{i+j}{2}\right)-m-2n-1,\alpha\right)}\right]$$

$$+U\omega\sum_{i=0}^{\infty}(-1)^i\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j\sum_{k=1}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k\sum_{m=0}^{\infty}\frac{(-M)^m}{m!}\sum_{n=0}^{\infty}(-\omega^2)^n\lambda^{\alpha(\frac{i+j+k}{2})}$$

$$\times\mathbf{M}^{1,3}_{3,5}\left[\frac{t^\alpha}{\lambda^\alpha}\Bigg|^{(1+i-j,0),\left(1+\frac{i+j+k}{2}-m,0\right),\left(1+\frac{i+j+k}{2},1\right)}_{(0,1),(1+i,0),\left(1+\frac{i+j+k}{2},0\right),\left(1+\frac{i+j+k}{2},0\right),\left((1+\alpha)\left(\frac{i+j+k}{2}\right)-m-2n-1,\alpha\right)}\right], \quad (34)$$

$$u_c(y,t)=UH(t)\cos(\omega t)+UH(t)\sum_{i=1}^{\infty}(-1)^i\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j\sum_{m=0}^{\infty}\frac{(-M)^m}{m!}\sum_{n=0}^{\infty}(-\omega^2)^n$$

$$\times\lambda^{\alpha(\frac{i+j}{2})}\mathbf{M}^{1,3}_{3,5}\left[\frac{t^\alpha}{\lambda^\alpha}\Bigg|^{(1+i-j,0),\left(1+\frac{i+j}{2}-m,0\right),\left(1+\frac{i+j}{2},1\right)}_{(0,1),(1+i,0),\left(1+\frac{i+j}{2},0\right),\left(1+\frac{i+j}{2},0\right),\left((1+\alpha)\left(\frac{i+j}{2}\right)-m-2n,\alpha\right)}\right]$$

$$+UH(t)\sum_{i=0}^{\infty}(-1)^i\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j\sum_{k=1}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k\sum_{m=0}^{\infty}\frac{(-M)^m}{m!}\sum_{n=0}^{\infty}(-\omega^2)^n\lambda^{\alpha(\frac{i+j+k}{2})}$$

$$\times\mathbf{M}_{3,5}^{1,3}\left[\frac{t^\alpha}{\lambda^\alpha}\Bigg|_{(0,1),(1+i,0),\left(1+\frac{i+j+k}{2},0\right),\left(1+\frac{i+j+k}{2},0\right),\left((1+\alpha)\left(\frac{i+j+k}{2}\right)-m-2n-1,\alpha\right)}^{(1+i-j,0),\left(1+\frac{i+j+k}{2}-m,0\right),\left(1+\frac{i+j+k}{2},1\right)}\right], \quad (35)$$

and

$$\tau_s(y,t)=-\frac{U\mu\omega}{\sqrt{\nu}}\sum_{i=0}^{\infty}(-1)^i\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j\sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k\sum_{m=0}^{\infty}\frac{(-M)^m}{m!}\sum_{n=0}^{\infty}(-\omega^2)^n$$

$$\times\lambda^{\alpha(\frac{i+j+k-1}{2})}\mathbf{M}_{3,5}^{1,3}\left[\frac{t^\alpha}{\lambda^\alpha}\Bigg|_{(0,1),(1+i,0),\left(1+\frac{i+j+k+1}{2},0\right),\left(1+\frac{i+j+k-1}{2},0\right),\left((1+\alpha)\left(\frac{i+j+k}{2}\right)+\frac{1}{2}(1-\alpha)-m-2n-1,\alpha\right)}^{(1+i-j,0),\left(1+\frac{i+j+k+1}{2}-m,0\right),\left(1+\frac{i+j+k-1}{2},1\right)}\right], (36)$$

$$\tau_c(y,t)=-\frac{UH(t)\mu}{\sqrt{\nu}}\sum_{i=0}^{\infty}(-1)^i\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j\sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k\sum_{m=0}^{\infty}\frac{(-M)^m}{m!}\sum_{n=0}^{\infty}(-\omega^2)^n$$

$$\times\lambda^{\alpha(\frac{i+j+k-1}{2})}\mathbf{M}_{3,5}^{1,3}\left[\frac{t^\alpha}{\lambda^\alpha}\Bigg|_{(0,1),(1+i,0),\left(1+\frac{i+j+k+1}{2},0\right),\left(1+\frac{i+j+k-1}{2},0\right),\left((1+\alpha)\left(\frac{i+j+k}{2}\right)+\frac{1}{2}(1-\alpha)-m-2n,\alpha\right)}^{(1+i-j,0),\left(1+\frac{i+j+k+1}{2}-m,0\right),\left(1+\frac{i+j+k-1}{2},1\right)}\right]. (37)$$

4.4. Fractionalized Maxwell fluid without MHD & porous effects

Making $M \to 0$ and $\Psi = 0$, Eq. (17), (19), (24) and (25) yield the following expressions

$$u_s(y,t)=U\sin(\omega t)+U\omega\sum_{i=1}^{\infty}(-1)^i\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j\sum_{n=0}^{\infty}(-\omega^2)^n\lambda^{\alpha(\frac{i+j}{2})}$$

$$\times\mathbf{M}_{2,4}^{1,2}\left[\frac{t^\alpha}{\lambda^\alpha}\Bigg|_{(0,1),(1+i,0),\left(1+\frac{i+j}{2},0\right),\left((1+\alpha)\left(\frac{i+j}{2}\right)-2n-1,\alpha\right)}^{(1+i-j,0),\left(1+\frac{i+j}{2},1\right)}\right]$$

$$+U\omega\sum_{i=0}^{\infty}(-1)^i\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j\sum_{k=1}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k\sum_{n=0}^{\infty}(-\omega^2)^n\lambda^{\alpha\left(\frac{i+j+k}{2}\right)}$$

$$\times\mathbf{M}^{1,2}_{2,4}\left[\frac{t^\alpha}{\lambda^\alpha}\middle|^{(1+i-j,0),\left(1+\frac{i+j+k}{2},1\right)}_{(0,1),(1+i,0),\left(1+\frac{i+j+k}{2},0\right),\left((1+\alpha)\left(\frac{i+j+k}{2}\right)-2n-1,\alpha\right)}\right], \quad (38)$$

$$u_c(y,t)=UH(t)\cos(\omega t)+UH(t)\sum_{i=1}^{\infty}(-1)^i\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j\sum_{n=0}^{\infty}(-\omega^2)^n\lambda^{\alpha\left(\frac{i+j}{2}\right)}$$

$$\times\mathbf{M}^{1,2}_{2,4}\left[\frac{t^\alpha}{\lambda^\alpha}\middle|^{(1+i-j,0),\left(1+\frac{i+j}{2},1\right)}_{(0,1),(1+i,0),\left(1+\frac{i+j}{2},0\right),\left((1+\alpha)\left(\frac{i+j}{2}\right)-2n-1,\alpha\right)}\right]$$

$$+UH(t)\sum_{i=0}^{\infty}(-1)^i\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j\sum_{k=1}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k\sum_{n=0}^{\infty}(-\omega^2)^n\lambda^{\alpha\left(\frac{i+j+k}{2}\right)}$$

$$\times\mathbf{M}^{1,2}_{2,4}\left[\frac{t^\alpha}{\lambda^\alpha}\middle|^{(1+i-j,0),\left(1+\frac{i+j+k}{2},1\right)}_{(0,1),(1+i,0),\left(1+\frac{i+j+k}{2},0\right),\left((1+\alpha)\left(\frac{i+j+k}{2}\right)-2n,\alpha\right)}\right], \quad (39)$$

$$\tau_s(y,t)=-\frac{U\mu\omega}{\sqrt{\nu}}\sum_{i=0}^{\infty}(-1)^i\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j\sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k\sum_{n=0}^{\infty}(-\omega^2)^n\lambda^{\alpha\left(\frac{i+j+k-1}{2}\right)}$$

$$\times\mathbf{M}^{1,2}_{2,4}\left[\frac{t^\alpha}{\lambda^\alpha}\middle|^{(1+i-j,0),\left(1+\frac{i+j+k-1}{2},1\right)}_{(0,1),(1+i,0),\left(1+\frac{i+j+k-1}{2},0\right),\left((1+\alpha)\left(\frac{i+j+k}{2}\right)+\frac{1}{2}(1-\alpha)-2n-1,\alpha\right)}\right], \quad (40)$$

$$\tau_c(y,t)=-\frac{UH(t)\mu}{\sqrt{\nu}}\sum_{i=0}^{\infty}(-1)^i\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j\sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k\sum_{n=0}^{\infty}(-\omega^2)^n\lambda^{\alpha\left(\frac{i+j+k-1}{2}\right)}$$

$$\times\mathbf{M}^{1,2}_{2,4}\left[\frac{t^\alpha}{\lambda^\alpha}\middle|^{(1+i-j,0),\left(1+\frac{i+j+k-1}{2},1\right)}_{(0,1),(1+i,0),\left(1+\frac{i+j+k-1}{2},0\right),\left((1+\alpha)\left(\frac{i+j+k}{2}\right)+\frac{1}{2}(1-\alpha)-2n,\alpha\right)}\right]. \quad (41)$$

4.5. Fractionalized MHD Maxwell fluid in porous medium with first order slip only

Making $\theta_2 \to 0$ in Eq. (17), (19), (24) and (25), we ge the required solution as

$$u_s(y,t) = U\sin(\omega t) + U\omega \sum_{i=1}^{\infty}\left(-\frac{\theta_1}{\sqrt{\nu}}\right)^i \sum_{l=0}^{\infty}\frac{(-\Psi)^l}{l!}\sum_{m=0}^{\infty}\frac{(-M)^m}{m!}\sum_{n=0}^{\infty}(-\omega^2)^n \lambda^{\alpha(\frac{i}{2}-l)}$$

$$\times \mathbf{M}^{1,3}_{3,5}\left[\frac{t^\alpha}{\lambda^\alpha}\middle|^{(1+\frac{i}{2}-l,0),(1+\frac{i}{2}-l-m,0),(1+\frac{i}{2}-l,1)}_{(0,1),(1+\frac{i}{2},0),(1+\frac{i}{2}-l,0),(1+\frac{i}{2}-l,0),((1+\alpha)(\frac{i}{2}-l)-m-2n-1,\alpha)}\right]$$

$$+U\omega \sum_{i=0}^{\infty}\left(-\frac{\theta_1}{\sqrt{\nu}}\right)^i \sum_{k=1}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k \sum_{l=0}^{\infty}\frac{(-\Psi)^l}{l!}\sum_{m=0}^{\infty}\frac{(-M)^m}{m!}\sum_{n=0}^{\infty}(-\omega^2)^n \lambda^{\alpha(\frac{i+k}{2}-l)}$$

$$\times \mathbf{M}^{1,3}_{3,5}\left[\frac{t^\alpha}{\lambda^\alpha}\middle|^{(1+\frac{i+k}{2}-l,0),(1+\frac{i+k}{2}-l-m,0),(1+\frac{i+k}{2}-l,1)}_{(0,1),(1+\frac{i+k}{2},0),(1+\frac{i+k}{2}-l,0),(1+\frac{i+k}{2}-l,0),((1+\alpha)(\frac{i+k}{2}-l)-m-2n-1,\alpha)}\right], \tag{42}$$

$$u_c(y,t) = UH(t)\cos(\omega t) + UH(t) \sum_{i=1}^{\infty}\left(-\frac{\theta_1}{\sqrt{\nu}}\right)^i \sum_{l=0}^{\infty}\frac{(-\Psi)^l}{l!}\sum_{m=0}^{\infty}\frac{(-M)^m}{m!}\sum_{n=0}^{\infty}(-\omega^2)^n \lambda^{\alpha(\frac{i}{2}-l)}$$

$$\times \mathbf{M}^{1,3}_{3,5}\left[\frac{t^\alpha}{\lambda^\alpha}\middle|^{(1+\frac{i}{2}-l,0),(1+\frac{i}{2}-l-m,0),(1+\frac{i}{2}-l,1)}_{(0,1),(1+\frac{i}{2},0),(1+\frac{i}{2}-l,0),(1+\frac{i}{2}-l,0),((1+\alpha)(\frac{i}{2}-l)-m-2n,\alpha)}\right]$$

$$+UH(t) \sum_{i=0}^{\infty}\left(-\frac{\theta_1}{\sqrt{\nu}}\right)^i \sum_{k=1}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k \sum_{l=0}^{\infty}\frac{(-\Psi)^l}{l!}\sum_{m=0}^{\infty}\frac{(-M)^m}{m!}\sum_{n=0}^{\infty}(-\omega^2)^n \lambda^{\alpha(\frac{i+k}{2}-l)}$$

$$\times \mathbf{M}^{1,3}_{3,5}\left[\frac{t^\alpha}{\lambda^\alpha}\middle|^{(1+\frac{i+k}{2}-l,0),(1+\frac{i+k}{2}-l-m,0),(1+\frac{i+k}{2}-l,1)}_{(0,1),(1+\frac{i+k}{2},0),(1+\frac{i+k}{2}-l,0),(1+\frac{i+k}{2}-l,0),((1+\alpha)(\frac{i+k}{2}-l)-m-2n,\alpha)}\right], \tag{43}$$

and

$$\tau_s(y,t) = -\frac{U\mu\omega}{\sqrt{\nu}} \sum_{i=0}^{\infty}\left(-\frac{\theta_1}{\sqrt{\nu}}\right)^i \sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k \sum_{l=0}^{\infty}\frac{(-\Psi)^l}{l!}\sum_{m=0}^{\infty}\frac{(-M)^m}{m!}\sum_{n=0}^{\infty}(-\omega^2)^n \lambda^{\alpha(\frac{i+k-1}{2}-l)}$$

$$\times \mathbf{M}_{3,5}^{1,3}\left[\frac{t^{\alpha}}{\lambda^{\alpha}}\Bigg|_{(0,1),\left(1+\frac{i+k+1}{2},0\right),\left(1+\frac{i+k+1}{2}-l,0\right),\left(1+\frac{i+k-1}{2}-l,0\right),\left((1+\alpha)\left(\frac{i+k}{2}-l\right)+\frac{1}{2}(1-\alpha)-m-2n-1,\alpha\right)}^{\left(1+\frac{i+k+1}{2}-l,0\right),\left(1+\frac{i+k+1}{2}-l-m,0\right),\left(1+\frac{i+k-1}{2}-l,1\right)}\right], (44)$$

$$\tau_c(y,t) = -\frac{UH(t)\mu}{\sqrt{\nu}}\sum_{i=0}^{\infty}\left(-\frac{\theta_1}{\sqrt{\nu}}\right)^i\sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k\sum_{l=0}^{\infty}\frac{(-\Psi)^l}{l!}\sum_{m=0}^{\infty}\frac{(-M)^m}{m!}\sum_{n=0}^{\infty}(-\omega^2)^n\lambda^{\alpha\left(\frac{i+k-1}{2}-l\right)}$$

$$\times \mathbf{M}_{3,5}^{1,3}\left[\frac{t^{\alpha}}{\lambda^{\alpha}}\Bigg|_{(0,1),\left(1+\frac{i+k+1}{2},0\right),\left(1+\frac{i+k+1}{2}-l,0\right),\left(1+\frac{i+k-1}{2}-l,0\right),\left((1+\alpha)\left(\frac{i+k}{2}-l\right)+\frac{1}{2}(1-\alpha)-m-2n,\alpha\right)}^{\left(1+\frac{i+k+1}{2}-l,0\right),\left(1+\frac{i+k+1}{2}-l-m,0\right),\left(1+\frac{i+k-1}{2}-l,1\right)}\right]. (45)$$

4.6. Fractionalized MHD Maxwell fluid in porous medium with no slip

The general solutions for no slip condition is obtained by considering $\theta_1 \to 0$ and $\theta_2 \to 0$ into Eq. (17), (19), (24) and (25), provide

$$u_s(y,t) = U\sin(\omega t) + U\omega\sum_{k=1}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k\sum_{l=0}^{\infty}\frac{(-\Psi)^l}{l!}\sum_{m=0}^{\infty}\frac{(-M)^m}{m!}\sum_{n=0}^{\infty}(-\omega^2)^n\lambda^{\alpha\left(\frac{k}{2}-l\right)}$$

$$\times \mathbf{M}_{3,5}^{1,3}\left[\frac{t^{\alpha}}{\lambda^{\alpha}}\Bigg|_{(0,1),\left(1+\frac{k}{2},0\right),\left(1+\frac{k}{2}-l,0\right),\left(1+\frac{k}{2}-l,0\right),\left((1+\alpha)\left(\frac{k}{2}-l\right)-m-2n-1,\alpha\right)}^{\left(1+\frac{k}{2}-l,0\right),\left(1+\frac{k}{2}-l-m,0\right),\left(1+\frac{k}{2}-l,1\right)}\right], \quad (46)$$

$$u_c(y,t) = UH(t)\cos(\omega t) + UH(t)\sum_{k=1}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k\sum_{l=0}^{\infty}\frac{(-\Psi)^l}{l!}\sum_{m=0}^{\infty}\frac{(-M)^m}{m!}\sum_{n=0}^{\infty}(-\omega^2)^n\lambda^{\alpha\left(\frac{k}{2}-l\right)}$$

$$\times \mathbf{M}_{3,5}^{1,3}\left[\frac{t^{\alpha}}{\lambda^{\alpha}}\Bigg|_{(0,1),\left(1+\frac{k}{2},0\right),\left(1+\frac{k}{2}-l,0\right),\left(1+\frac{k}{2}-l,0\right),\left((1+\alpha)\left(\frac{k}{2}-l\right)-m-2n,\alpha\right)}^{\left(1+\frac{k}{2}-l,0\right),\left(1+\frac{k}{2}-l-m,0\right),\left(1+\frac{k}{2}-l,1\right)}\right], \quad (47)$$

$$\tau_s(y,t) = -\frac{U\mu\omega}{\sqrt{\nu}}\sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k\sum_{l=0}^{\infty}\frac{(-\Psi)^l}{l!}\sum_{m=0}^{\infty}\frac{(-M)^m}{m!}\sum_{n=0}^{\infty}(-\omega^2)^n\lambda^{\alpha\left(\frac{k-1}{2}-l\right)}$$

$$\times \mathbf{M}_{3,5}^{1,3}\left[\frac{t^{\alpha}}{\lambda^{\alpha}}\Bigg|_{(0,1),\left(1+\frac{k+1}{2},0\right),\left(1+\frac{k+1}{2}-l,0\right),\left(1+\frac{k-1}{2}-l,0\right),\left((1+\alpha)\left(\frac{k}{2}-l\right)+\frac{1}{2}(1-\alpha)-m-2n-1,\alpha\right)}^{\left(1+\frac{k+1}{2}-l,0\right),\left(1+\frac{k+1}{2}-l-m,0\right),\left(1+\frac{k-1}{2}-l,1\right)}\right], \quad (48)$$

$$\tau_c(y,t)=-\frac{UH(t)\mu}{\sqrt{\nu}}\sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k\sum_{l=0}^{\infty}\frac{(-\Psi)^l}{l!}\sum_{m=0}^{\infty}\frac{(-M)^m}{m!}\sum_{n=0}^{\infty}(-\omega^2)^n\lambda^{\alpha\left(\frac{k-1}{2}-l\right)}$$

$$\times \mathbf{M}_{3,5}^{1,3}\left[\frac{t^{\alpha}}{\lambda^{\alpha}}\Bigg|_{(0,1),\left(1+\frac{k+1}{2},0\right),\left(1+\frac{k+1}{2}-l,0\right),\left(1+\frac{k-1}{2}-l,0\right),\left((1+\alpha)\left(\frac{k}{2}-l\right)+\frac{1}{2}(1-\alpha)-m-2n,\alpha\right)}^{\left(1+\frac{k+1}{2}-l,0\right),\left(1+\frac{k+1}{2}-l-m,0\right),\left(1+\frac{k-1}{2}-l,1\right)}\right]. \quad (49)$$

4.7. MHD Newtonian fluid in porous medium with twice order slip

Making $\lambda \to 0$ into equation (14) and(21), the recalculated velocity and shear stresses are

$$u_s(y,t)=U\sin(\omega t)+U\omega\sum_{i=1}^{\infty}(-1)^i\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j\sum_{l=0}^{\infty}\frac{(-\Psi)^l}{l!}\sum_{n=0}^{\infty}(-\omega^2)^n$$

$$\times \mathbf{M}_{3,5}^{1,3}\left[Mt\Bigg|_{(0,1),(1+i,0),\left(1+\frac{i+j}{2},0\right),\left(1+\frac{i+j}{2}-l,0\right),\left(\frac{i+j}{2}-l-2n-1,1\right)}^{(1+i-j,0),\left(1+\frac{i+j}{2}-l,0\right),\left(1+\frac{i+j}{2}-l,1\right)}\right]$$

$$+U\omega\sum_{i=0}^{\infty}(-1)^i\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j\sum_{k=1}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k\sum_{l=0}^{\infty}\frac{(-\Psi)^l}{l!}\sum_{n=0}^{\infty}(-\omega^2)^n$$

$$\times \mathbf{M}_{3,5}^{1,3}\left[Mt\Bigg|_{(0,1),(1+i,0),\left(1+\frac{i+j+k}{2},0\right),\left(1+\frac{i+j+k}{2}-l,0\right),\left(\frac{i+j+k}{2}-l-2n-1,1\right)}^{(1+i-j,0),\left(1+\frac{i+j+k}{2}-l,0\right),\left(1+\frac{i+j+k}{2}-l,1\right)}\right], \quad (50)$$

$$\tau_s(y,t)=-\frac{U\mu\omega}{\sqrt{\nu}}\sum_{i=0}^{\infty}(-1)^i\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j\sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k\sum_{l=0}^{\infty}\frac{(-\Psi)^l}{l!}\sum_{n=0}^{\infty}(-\omega^2)^n$$

$$\times \mathbf{M}_{3,5}^{1,3}\left[Mt\Bigg|_{(0,1),(1+i,0),\left(1+\frac{i+j+k+1}{2},0\right),\left(1+\frac{i+j+k+1}{2}-l,0\right),\left(\frac{i+j+k+1}{2}-l-2n-1,1\right)}^{(1+i-j,0),\left(1+\frac{i+j+k+1}{2}-l,0\right),\left(1+\frac{i+j+k+1}{2}-l,1\right)}\right]. \quad (51)$$

For cosine oscillations, we have

$$u_c(y,t) = UH(t)\cos(\omega t) + UH(t)\sum_{i=1}^{\infty}(-1)^i \sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j \sum_{l=0}^{\infty}\frac{(-\Psi)^l}{l!}\sum_{n=0}^{\infty}(-\omega^2)^n$$

$$\times \mathbf{M}_{3,5}^{1,3}\left[Mt\Big|^{(1+i-j,0),\left(1+\frac{i+j}{2}-l,0\right),\left(1+\frac{i+j}{2}-l,1\right)}_{(0,1),(1+i,0),\left(1+\frac{i+j}{2},0\right),\left(1+\frac{i+j}{2}-l,0\right),\left(\frac{i+j}{2}-l-2n,1\right)}\right]$$

$$+UH(t)\sum_{i=0}^{\infty}(-1)^i \sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j \sum_{k=1}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k \sum_{l=0}^{\infty}\frac{(-\Psi)^l}{l!}\sum_{n=0}^{\infty}(-\omega^2)^n$$

$$\times \mathbf{M}_{3,5}^{1,3}\left[Mt\Big|^{(1+i-j,0),\left(1+\frac{i+j+k}{2}-l,0\right),\left(1+\frac{i+j+k}{2}-l,1\right)}_{(0,1),(1+i,0),\left(1+\frac{i+j+k}{2},0\right),\left(1+\frac{i+j+k}{2}-l,0\right),\left(\frac{i+j+k}{2}-l-2n,1\right)}\right], \tag{52}$$

$$\tau_c(y,t) = -\frac{UH(t)\mu}{\sqrt{\nu}}\sum_{i=0}^{\infty}(-1)^i \sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j \sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k \sum_{l=0}^{\infty}\frac{(-\Psi)^l}{l!}\sum_{n=0}^{\infty}(-\omega^2)^n$$

$$\times \mathbf{M}_{3,5}^{1,3}\left[Mt\Big|^{(1+i-j,0),\left(1+\frac{i+j+k+1}{2}-l,0\right),\left(1+\frac{i+j+k+1}{2}-l,1\right)}_{(0,1),(1+i,0),\left(1+\frac{i+j+k+1}{2},0\right),\left(1+\frac{i+j+k+1}{2}-l,0\right),\left(\frac{i+j+k+1}{2}-l-2n,1\right)}\right]. \tag{53}$$

4.8. MHD Newtonian fluid without porous effect

Putting $\Psi \to 0$ in the Eqs. (50), (51), (52), and (53) yields the results

$$u_s(y,t) = U\sin(\omega t) + U\omega\sum_{i=1}^{\infty}(-1)^i \sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j \sum_{n=0}^{\infty}(-\omega^2)^n$$

$$\times \mathbf{M}_{2,4}^{1,2}\left[Mt\Big|^{(1+i-j,0),\left(1+\frac{i+j}{2},1\right)}_{(0,1),(1+i,0),\left(1+\frac{i+j}{2},0\right),\left(\frac{i+j}{2}-2n-1,1\right)}\right]$$

$$+U\omega\sum_{i=0}^{\infty}(-1)^{i}\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^{j}\sum_{k=1}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^{k}\sum_{n=0}^{\infty}(-\omega^2)^{n}$$

$$\times\mathbf{M}_{2,4}^{1,2}\left[Mt\Big|_{(0,1),(1+i,0),\left(1+\frac{i+j+k}{2},0\right),\left(\frac{i+j+k}{2}-2n-1,1\right)}^{(1+i-j,0),\left(1+\frac{i+j+k}{2},1\right)}\right], \tag{54}$$

$$u_c(y,t)=UH(t)\cos(\omega t)+UH(t)\sum_{i=1}^{\infty}(-1)^{i}\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^{j}\sum_{n=0}^{\infty}(-\omega^2)^{n}$$

$$\times\mathbf{M}_{2,4}^{1,2}\left[Mt\Big|_{(0,1),(1+i,0),\left(1+\frac{i+j}{2},0\right),\left(\frac{i+j}{2}-2n,1\right)}^{(1+i-j,0),\left(1+\frac{i+j}{2},1\right)}\right]$$

$$+UH(t)\sum_{i=0}^{\infty}(-1)^{i}\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^{j}\sum_{k=1}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^{k}\sum_{n=0}^{\infty}(-\omega^2)^{n}$$

$$\times\mathbf{M}_{2,4}^{1,2}\left[Mt\Big|_{(0,1),(1+i,0),\left(1+\frac{i+j+k}{2},0\right),\left(\frac{i+j+k}{2}-2n,1\right)}^{(1+i-j,0),\left(1+\frac{i+j+k}{2},1\right)}\right], \tag{55}$$

$$\tau_s(y,t)=-\frac{U\mu\omega}{\sqrt{\nu}}\sum_{i=0}^{\infty}(-1)^{i}\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^{j}\sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^{k}\sum_{n=0}^{\infty}(-\omega^2)^{n}$$

$$\times\mathbf{M}_{2,4}^{1,2}\left[Mt\Big|_{(0,1),(1+i,0),\left(1+\frac{i+j+k+1}{2},0\right)\left(\frac{i+j+k+1}{2}-2n-1,1\right)}^{(1+i-j,0),\left(1+\frac{i+j+k+1}{2},1\right)}\right], \tag{56}$$

$$\tau_c(y,t)=-\frac{UH(t)\mu}{\sqrt{\nu}}\sum_{i=0}^{\infty}(-1)^{i}\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^{j}\sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^{k}\sum_{n=0}^{\infty}(-\omega^2)^{n}$$

$$\times\mathbf{M}_{2,4}^{1,2}\left[Mt\Big|_{(0,1),(1+i,0),\left(1+\frac{i+j+k+1}{2},0\right)\left(\frac{i+j+k+1}{2}-2n,1\right)}^{(1+i-j,0),\left(1+\frac{i+j+k+1}{2},1\right)}\right]. \tag{57}$$

4.9. Newtonian fluid in porous medium without MHD

Putting $M \to 0$ in the Eqs. (50), (51), (52), and (53), we get

$$u_s(y,t) = U\sin(\omega t) + U\omega\sum_{i=1}^{\infty}(-1)^i\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j\sum_{n=0}^{\infty}(-\omega^2)^n$$

$$\times\mathbf{M}_{2,4}^{1,2}\left[\Psi t\middle|_{(0,1),(1+i,0),\left(1+\frac{i+j}{2},0\right),\left(\frac{i+j}{2}-2n-1,1\right)}^{(1+i-j,0),\left(1+\frac{i+j}{2},1\right)}\right]$$

$$+U\omega\sum_{i=0}^{\infty}(-1)^i\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j\sum_{k=1}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k\sum_{n=0}^{\infty}(-\omega^2)^n$$

$$\times\mathbf{M}_{2,4}^{1,2}\left[\Psi t\middle|_{(0,1),(1+i,0),\left(1+\frac{i+j+k}{2},0\right),\left(\frac{i+j+k}{2}-2n-1,1\right)}^{(1+i-j,0),\left(1+\frac{i+j+k}{2},1\right)}\right], \tag{58}$$

$$u_c(y,t) = UH(t)\cos(\omega t) + UH(t)\sum_{i=1}^{\infty}(-1)^i\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j\sum_{n=0}^{\infty}(-\omega^2)^n$$

$$\times\mathbf{M}_{2,4}^{1,2}\left[\Psi t\middle|_{(0,1),(1+i,0),\left(1+\frac{i+j}{2},0\right),\left(\frac{i+j}{2}-2n,1\right)}^{(1+i-j,0),\left(1+\frac{i+j}{2},1\right)}\right]$$

$$+UH(t)\sum_{i=0}^{\infty}(-1)^i\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j\sum_{k=1}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k\sum_{n=0}^{\infty}(-\omega^2)^n$$

$$\times\mathbf{M}_{2,4}^{1,2}\left[\Psi t\middle|_{(0,1),(1+i,0),\left(1+\frac{i+j+k}{2},0\right),\left(\frac{i+j+k}{2}-2n,1\right)}^{(1+i-j,0),\left(1+\frac{i+j+k}{2},1\right)}\right], \tag{59}$$

and

$$\tau_s(y,t) = -\frac{U\mu\omega}{\sqrt{\nu}}\sum_{i=0}^{\infty}(-1)^i\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j\sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k\sum_{n=0}^{\infty}(-\omega^2)^n$$

$$\times \mathbf{M}^{1,2}_{2,4}\left[\Psi t\Big|^{(1+i-j,0),\left(1+\frac{i+j+k+1}{2},1\right)}_{(0,1),(1+i,0),\left(1+\frac{i+j+k+1}{2},0\right)\left(\frac{i+j+k+1}{2}-2n-1,1\right)}\right], \tag{60}$$

$$\tau_c(y,t) = -\frac{UH(t)\mu}{\sqrt{\nu}}\sum_{i=0}^{\infty}(-1)^i\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^{i-j}\left(-\frac{\theta_2}{\nu}\right)^j\sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k\sum_{n=0}^{\infty}(-\omega^2)^n$$

$$\times \mathbf{M}^{1,2}_{2,4}\left[\Psi t\Big|^{(1+i-j,0),\left(1+\frac{i+j+k+1}{2},1\right)}_{(0,1),(1+i,0),\left(1+\frac{i+j+k+1}{2},0\right),\left(\frac{i+j+k+1}{2}-2n,1\right)}\right]. \tag{61}$$

4.10. Newtonian fluid without MHD & porous effects

Putting $\lambda \to 0$, $M \to 0$ and $\Psi \to 0$ into Eqs. (14) and (21) and proceed to calculate the velocity fields and shear stresses, yields the results

$$u_s(y,t) = U\sin(\omega t) + U\omega\sum_{i=1}^{\infty}\left(-\frac{\theta_1}{\sqrt{\nu}}\right)^i\sum_{n=0}^{\infty}(-\omega^2)^n\mathbf{M}^{1,1}_{1,3}\left[\frac{\theta_2}{\theta_1\sqrt{\nu t}}\Big|^{(1+i,1)}_{(0,1),(1+i,0),\left(\frac{i}{2}-2n-1,-\frac{1}{2}\right)}\right]$$

$$+U\omega\sum_{i=0}^{\infty}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^i\sum_{k=1}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k\sum_{n=0}^{\infty}(-\omega^2)^n\mathbf{M}^{1,1}_{1,3}\left[\frac{\theta_2}{\theta_1\sqrt{\nu t}}\Big|^{(1+i,1)}_{(0,1),(1+i,0),\left(\frac{i+k}{2}-2n-1,-\frac{1}{2}\right)}\right], \tag{62}$$

$$u_c(y,t) = UH(t)\cos(\omega t) + UH(t)\sum_{i=1}^{\infty}\left(-\frac{\theta_1}{\sqrt{\nu}}\right)^i\sum_{n=0}^{\infty}(-\omega^2)^n\mathbf{M}^{1,1}_{1,3}\left[\frac{\theta_2}{\theta_1\sqrt{\nu t}}\Big|^{(1+i,1)}_{(0,1),(1+i,0),\left(\frac{i}{2}-2n,-\frac{1}{2}\right)}\right]$$

$$+UH(t)\sum_{i=0}^{\infty}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^i\sum_{k=1}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k\sum_{n=0}^{\infty}(-\omega^2)^n\mathbf{M}^{1,1}_{1,3}\left[\frac{\theta_2}{\theta_1\sqrt{\nu t}}\Big|^{(1+i,1)}_{(0,1),(1+i,0),\left(\frac{i+k}{2}-2n,-\frac{1}{2}\right)}\right], \tag{63}$$

and

$$\tau_s(y,t) = -\frac{U\mu\omega}{\sqrt{\nu}}\sum_{i=0}^{\infty}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^i\sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k\sum_{n=0}^{\infty}(-\omega^2)^n\mathbf{M}^{1,1}_{1,3}\left[\frac{\theta_2}{\theta_1\sqrt{\nu t}}\Big|^{(1+i,1)}_{(0,1),(1+i,0)\left(\frac{i+k+1}{2}-2n-1,-\frac{1}{2}\right)}\right], \tag{64}$$

$$\tau_c(y,t) = -\frac{UH(t)\mu}{\sqrt{\nu}}\sum_{i=0}^{\infty}\left(\frac{\theta_1}{\sqrt{\nu}}\right)^i\sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{y}{\sqrt{\nu}}\right)^k\sum_{n=0}^{\infty}(-\omega^2)^n\mathbf{M}^{1,1}_{1,3}\left[\frac{\theta_2}{\theta_1\sqrt{\nu t}}\Big|^{(1+i,1)}_{(0,1),(1+i,0)\left(\frac{i+k+1}{2}-2n,-\frac{1}{2}\right)}\right]. \tag{65}$$

5. Numerical results and discussion

The objective of this study is to acquire the exact analytical solutions of velocity fields $u(y,t)$ and the shear stresses $\tau(y,t)$ for the unsteady flow of incompressible fractionalized MHD Maxwell fluid under the effect of twice order slip between porous plate and the fluid in consideration of magnetic field normal to the plate. The relative velocity field between fluid and the plate is considered to be proportional to velocity gradients at the plate. The results are acquired by Laplace transform and presented in the form of generalized **M**-function satisfying the all applied conditions.

In order to have a brief investigation of different parameters over the flow field, we discuss some special cases of the general solution by particularizing $M, \psi, \theta_1, \theta_2$ to be 0 and α to be 1, it reduce solutions corresponding to the flow of ordinary Maxwell in the presence and absence of second slip, first slip, magnetic and porous affects. Solutions for Newtonian fluid with twice order slip are easily obtained by vanishing the Maxwell parameter λ.

For better understanding of the physical attributes of solutions acquired in Sections 3 and 4 of the paper, we represent some plots of velocity field and concerned shear stress in Figs. 2-15 for which it can be noted that motion of fluid effect is oscillating for diverse values of time.

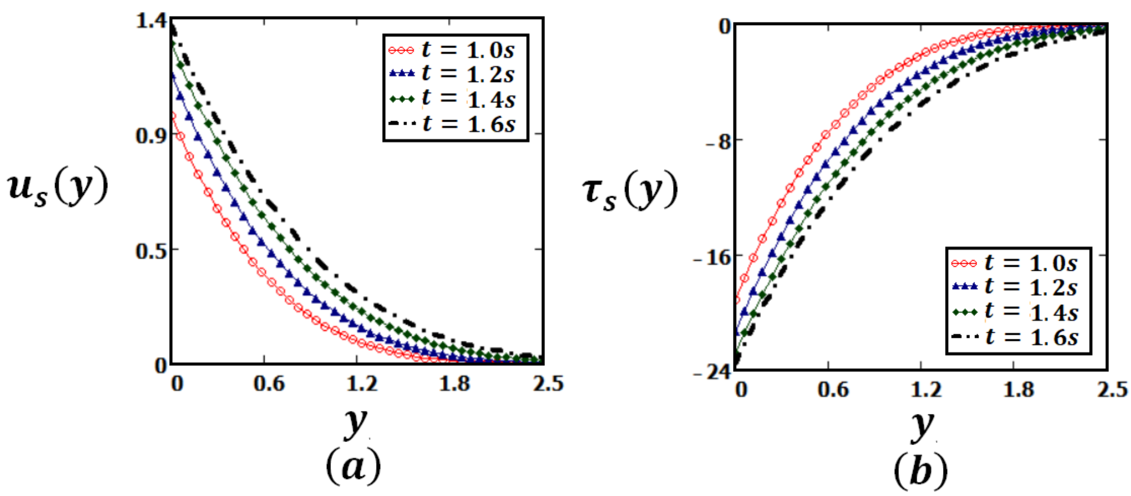

Figure 2: Profiles for fractionalized Maxwell fluid given by Eqs. (17) and (24), for $U = 2$, $\nu = 2.108$, $\mu = 33$, $\lambda = 2$, $M = 0.5$, $\Psi = 2$, $\alpha = 0.5$, $\omega = 1$, $\theta_1 = 0.2$, $\theta_2 = 0.3$ and different values of t.

Fig. 2 indicates that velocity and the shear stress are numerically increasing functions of time. Velocity of fluid near the plate is high while it tends to zero for the fluid particles much above the plate as mentioned in the natural condition (10).

In Fig. 3 oscillatory behavior can be observed with respect to time while by moving higher above the plate, both the flow characteristics decrease.

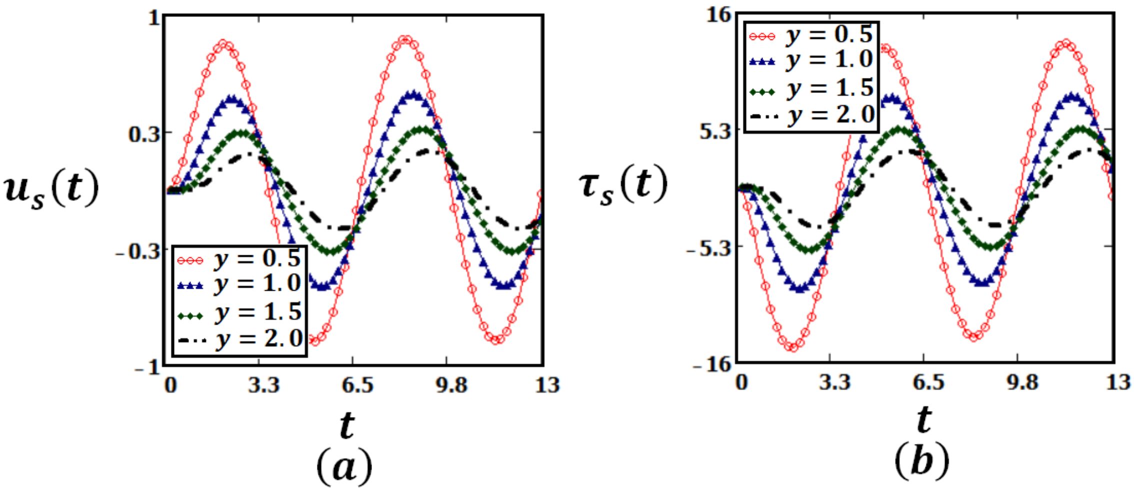

Figure 3: Profiles for fractionalized Maxwell fluid given by Eqs. (17) and (24), for $U = 2$, $\nu = 2.108$, $\mu = 33$, $\lambda = 2$, $M = 0.5$, $\Psi = 2$, $\alpha = 0.5$, $\omega = 1$, $\theta_1 = 0.2$, $\theta_2 = 0.3$ and different values of y.

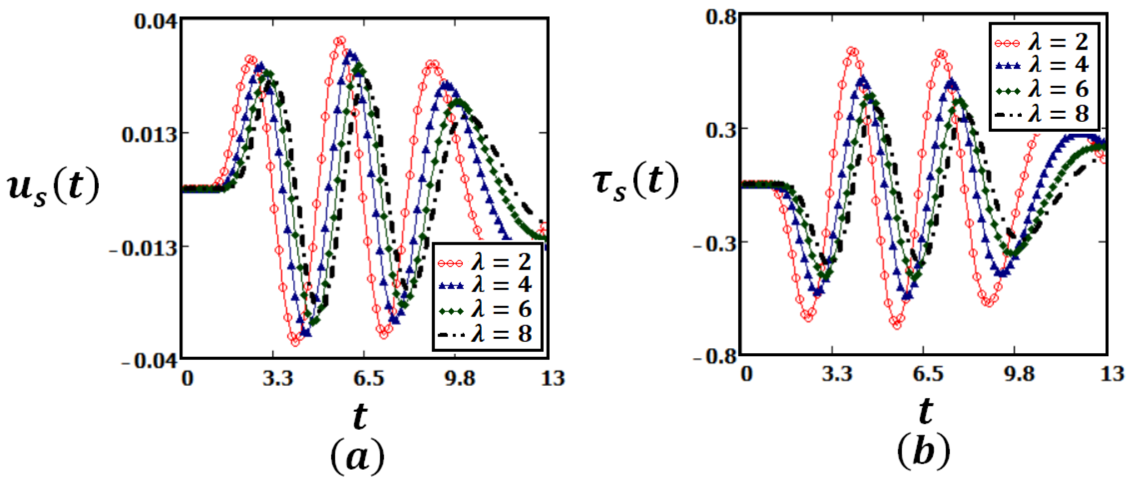

Figure 4: Profiles for fractionalized Maxwell fluid given by Eqs. (17) and (24), for $U = 2$, $\nu = 2.108$, $\mu = 33$, $M = 0.2$, $\Psi = 3$, $\alpha = 0.5$, $\omega = 2$, $\theta_1 = 0.2$, $\theta_2 = 0.3$, $y = 3$ and different values of λ.

Fig. 4 shows agreement with relaxation time, by increasing the λ values, response of fluid motion gets late and slower, the effect of relaxation time values is numerically smaller to both velocity and shear stress.

As expected, Figs. 5-6 clearly reflect the drawbacks of enhancing the coefficients of magnetic and porous parameters M and ψ, a similar effects can be seen in depictions of velocity and shear stress profiles.

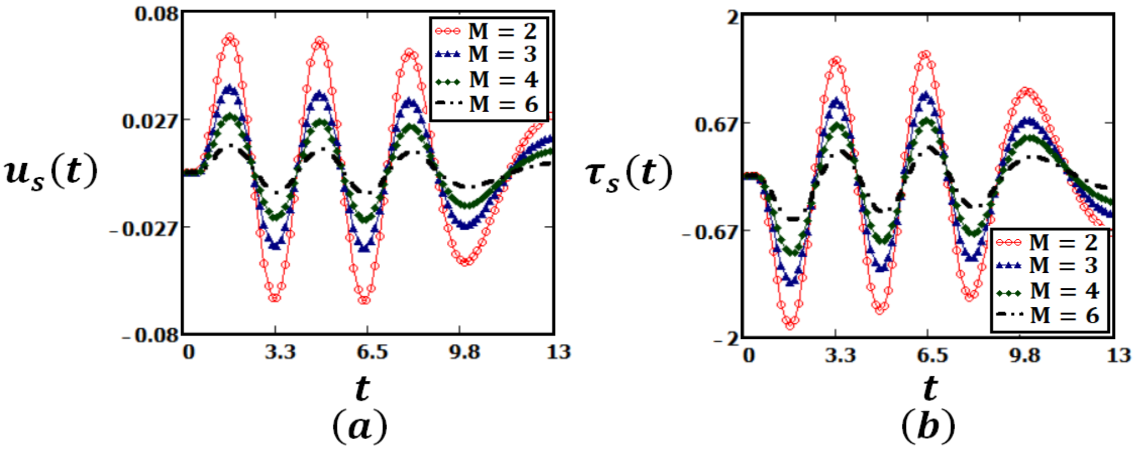

Figure 5: Profiles for fractionalized Maxwell fluid given by Eqs. (17) and (24), for $U = 2$, $\nu = 2.108$, $\mu = 33$, $\lambda = 2$, $\Psi = 3$, $\alpha = 0.5$, $\omega = 2$, $\theta_1 = 0.2$, $\theta_2 = 0.3$, $y = 1.5$ and different values of M.

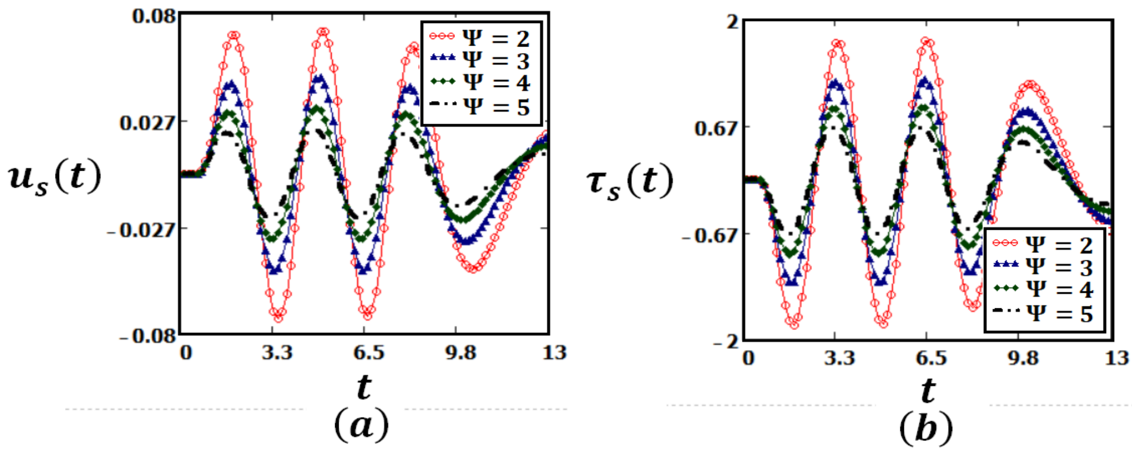

Figure 6: Profiles for fractionalized Maxwell fluid given by Eqs. (17) and (24), for $U = 2$, $\nu = 2.108$, $\mu = 33$, $\lambda = 0.5$, $M = 0.5$, $\alpha = 0.5$, $\omega = 2$, $\theta_1 = 0.2$, $\theta_2 = 0.3$, $y = 2.5$ and different values of Ψ.

Higher values of magnetic and porous parameter decrease the velocity of fluid as well as shear stress profiles, whereas the smaller values of these parameter increase the amplitude velocity and shear stress profiles.

Fig. 7 shows that increasing the kinematic viscosity ν of the fluid, both the flow characteristics goes up. It indicates that kinematic viscosity of the fluid has important role in controlling the flow characteristics.

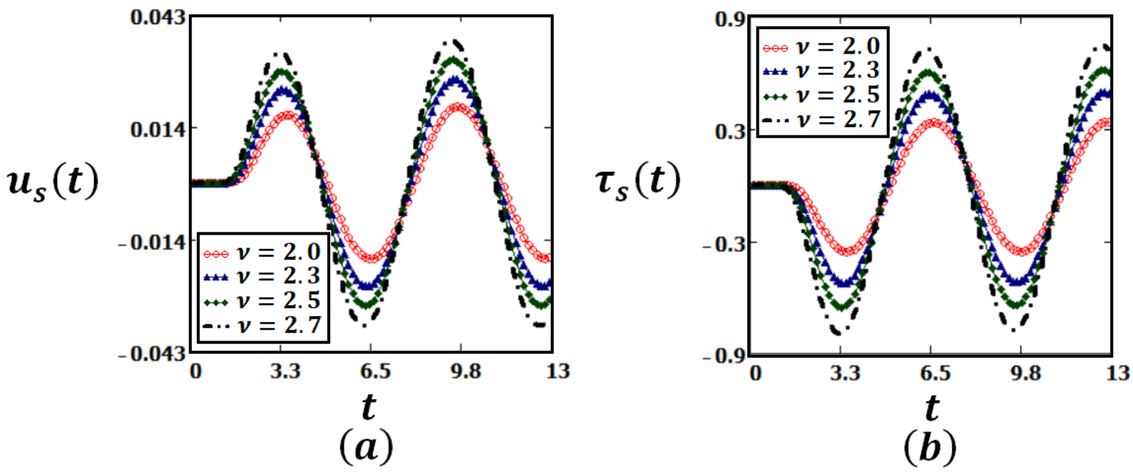

Figure 7: Profiles for fractionalized Maxwell fluid given by Eqs. (17) and (24), for $U = 2$, $\rho = 15.655$, $\lambda = 2$, $M = 0.2$, $\Psi = 3$, $\alpha = 0.5$, $\omega = 1$, $\theta_1 = 0.2$, $\theta_2 = 0.3$, $t = 3.5$ and different values of ν.

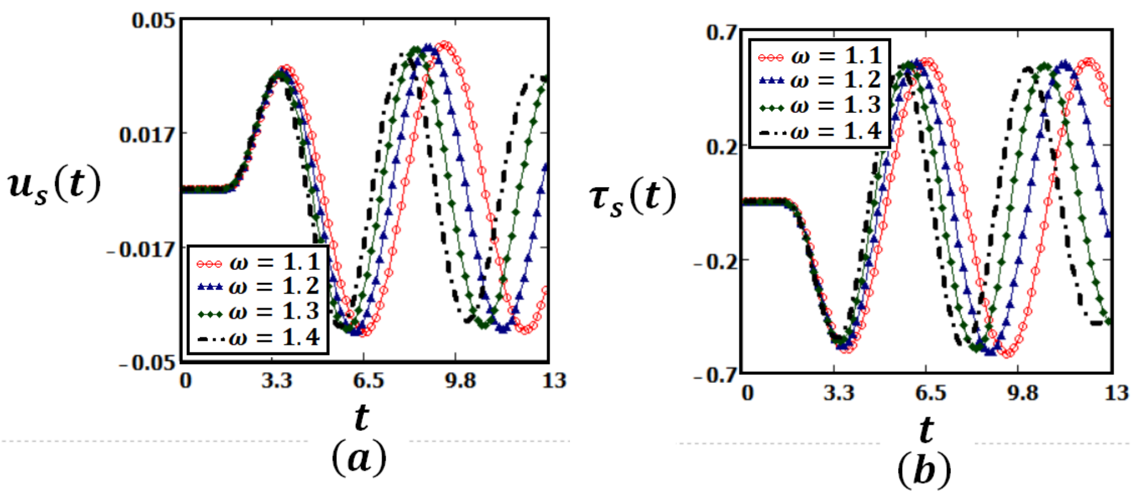

Figure 8: Profiles for fractionalized Maxwell fluid given by Eqs. (17) and (24), for $U = 2$, $\nu = 2.108$, $\mu = 33$, $\lambda = 6$, $M = 0.2$, $\Psi = 3$, $\alpha = 0.5$, $\theta_1 = 0.2$, $\theta_2 = 0.3$, $y = 3$ and different values of ω.

Fig. 8 indicates that by raising the oscillating amplitude of the bottom plate, flow speed and the tension between fluid and the plate can be decreased. It can be concluded that frequency of oscillation of the plate is directly proportional to the velocity field and shear stress of the fluid.

The effects of fractional parameter over the flow characters are depicted in Fig. 9, which favor the importance of fractionalized form of governing equations of the fluid in such a way that higher the α values, more rapidly the velocity and shear stress gets high magnitude.

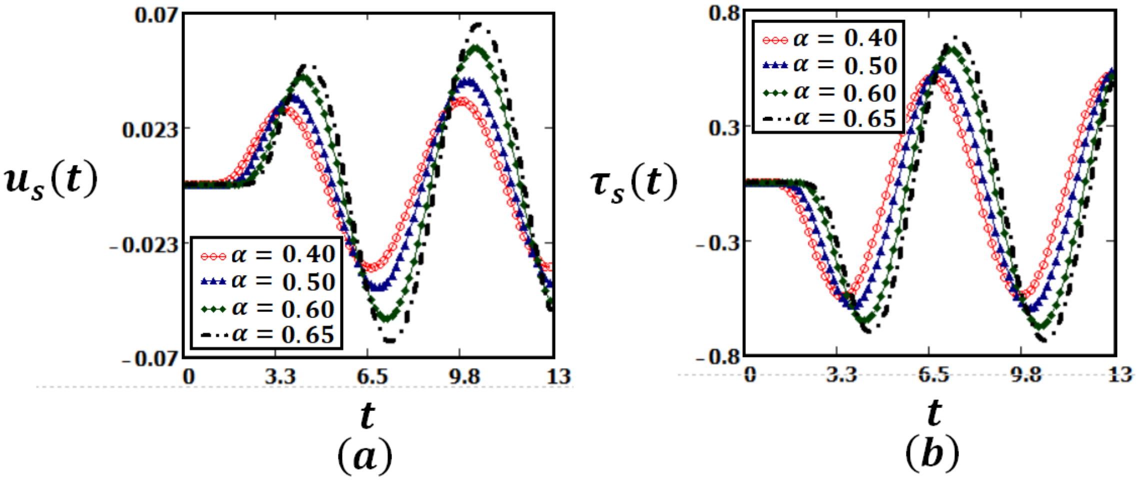

Figure 9: Profiles for fractionalized Maxwell fluid given by by Eqs. (17) and (24), for $U = 2, \quad \nu = 2.108, \ \mu = 33, \ \lambda = 6, \quad M = 0.2, \ \Psi = 3, \ \omega = 1, \ \theta_1 = 0.2, \ \theta_2 = 0.3, \ \ y = 3s$ and different values of α.

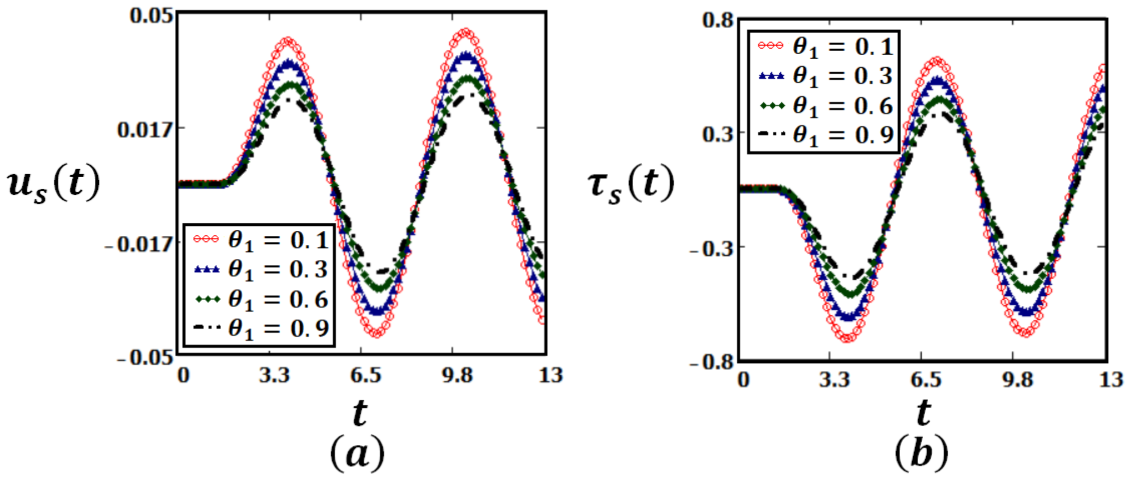

Figure 10: Profiles for fractionalized Maxwell fluid given by Eqs. (17) and (24), for $U = 2, \quad \nu = 2.108, \ \mu = 33, \ \lambda = 5, \quad M = 0.5, \ \Psi = 2, \ \alpha = 0.5, \quad \omega = 1, \ \theta_2 = 0.3, \ \ y = 3$ and different values of θ_1.

The first order slip effects over the general solution is depicted in Figs. 10 which indicates the inverse proportion between coefficients of first order slip and velocity field and shear stress, both characters slows down by increasing the first order slip.

Twice order slip effects over the general solution are presented in Fig. 11 which indicates that velocity and shear stress decrease by increasing the twice order slip.

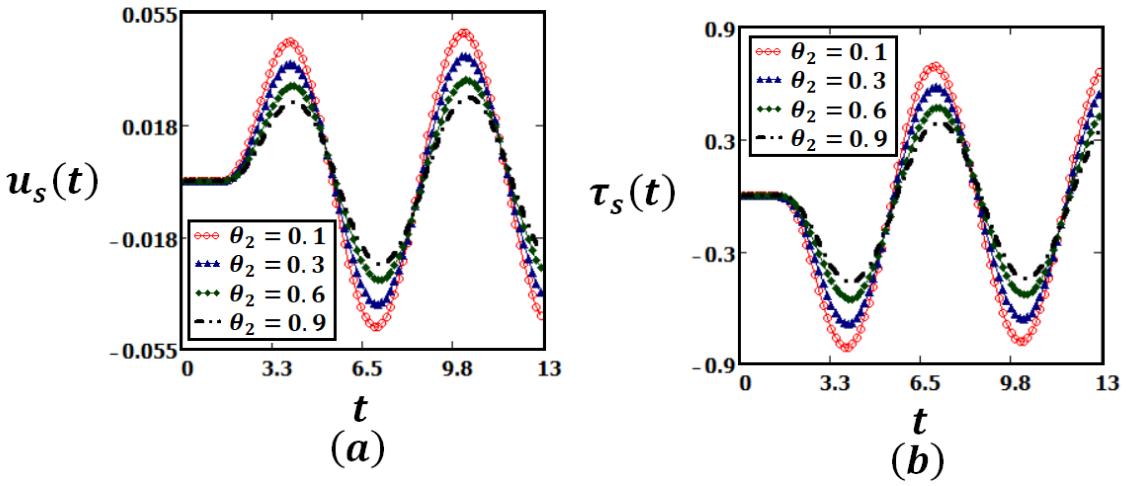

Figure 11: Profiles for fractionalized Maxwell fluid given by Eqs. (17) and (24), for $U = 2$, $\nu = 2.108$, $\mu = 33$, $\lambda = 5$, $M = 0.5$, $\Psi = 2$, $\alpha = 0.5$, $\omega = 1$, $\theta_1 = 0.1$, $y = 3$ and different values of θ_2.

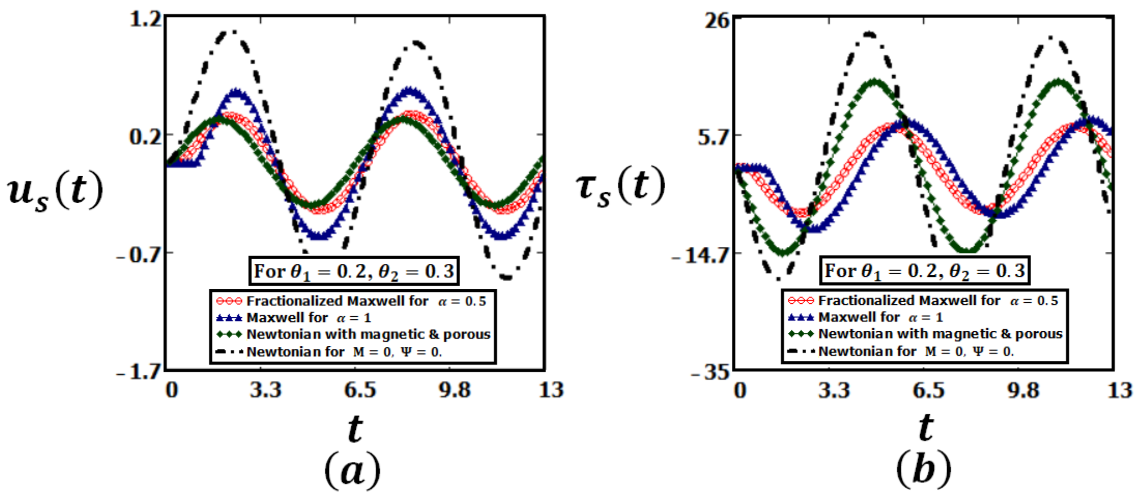

Figure 12: Profiles for fractionalized Maxwell fluid given by Eqs. (17) and (24), Maxwell fluid with MHD & porous given by Eqs. (26) and (28), Newtonian fluid with MHD & porous given by Eqs. (50) and (51) and Newtonian fluid given by Eqs. (62) and (64) for $U = 2$, $\nu = 2.108$, $\mu = 33$, $\lambda = 2$, $M = 0.2$, $\Psi = 3$, $\alpha = 0.5$, $\omega = 1$, $\theta_1 = 0.2$, $\theta_3 = 0.3$ and $y = 1$.

Fractionalized Maxwell, ordinary Maxwell, Newtonian fluid models and Newtonian fluid without porous and magnetic effects along with twice and/or first slip or no slip are depicted in Figs. 12-15 and as expected, the flow velocity and shear stress of Newtonian fluid without porous and magnetic effect is at peak in comparison with other mentioned fluid types in all Figs. 12-15.

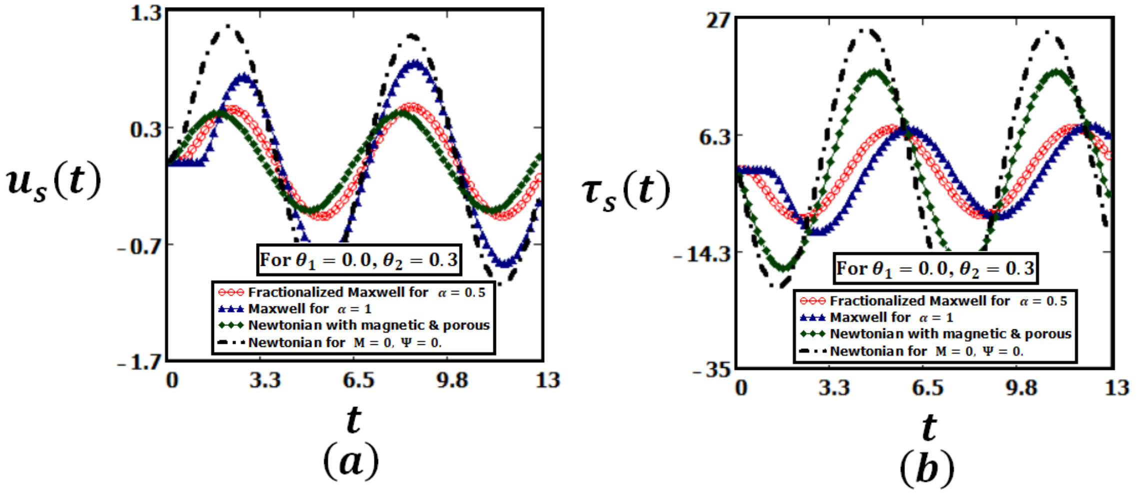

Figure 13: Profiles for fractionalized Maxwell fluid given by Eqs. (17) and (24), Maxwell fluid with MHD & porous given by Eqs. (26) and (28), Newtonian fluid with MHD & porous given by Eqs. (50) and (51) and Newtonian fluid given by Eqs. (62) and (64) for $U = 2$, $\nu = 2.108$, $\mu = 33$, $\lambda = 2$, $M = 0.2$, $\Psi = 3$, $\alpha = 0.5$, $\omega = 1$, $\theta_1 = 0.0$, $\theta_2 = 0.3$ and $y = 1$.

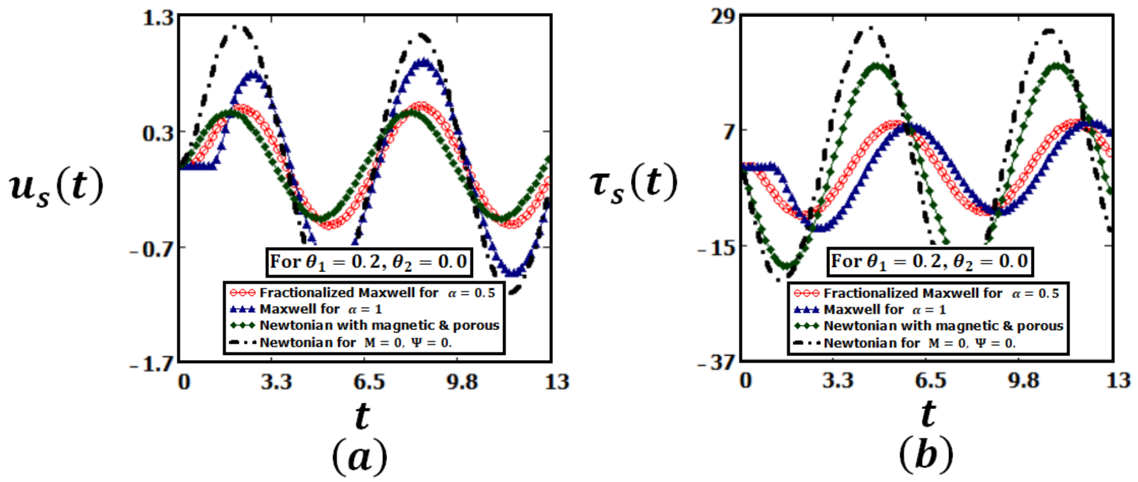

Figure 14: Profiles for fractionalized Maxwell fluid given by Eqs. (17) and (24), Maxwell fluid with MHD & porous given by Eqs. (26) and (28), Newtonian fluid with MHD & porous given by Eqs. (50) and (51) and Newtonian fluid given by Eqs. (62) and (64) for $U = 2$, $\nu = 2.108$, $\mu = 33$, $\lambda = 2$, $M = 0.2$, $\Psi = 3$, $\alpha = 0.5$, $\omega = 1$, $\theta_1 = 0.2$, $\theta_2 = 0.0$ and $y = 1$.

With combined effect of both the first and twice order slips, Newtonian fluid without magnetic and porous parameters has maximum velocity and shear stress and Newtonian fluid has minimum as compared to other three types presented as shown in Figs. 12.

Fig. 13 is based on assumption that if the first order slip does not occur, than the flow with twice order slips has a larger magnitude in comparison to those of Figs. 12.

Comparing the magnitudes of velocity and shear stress of the flow with and without twice order slips, smaller magnitude with twice order slip is observed as it clear from Figs. 13-15. In other words twice order slip offers more resistance to the flow of fluid than the first order slip. The units of the material constants in all figures are SI units and all graphs are made by using Mathcad software.

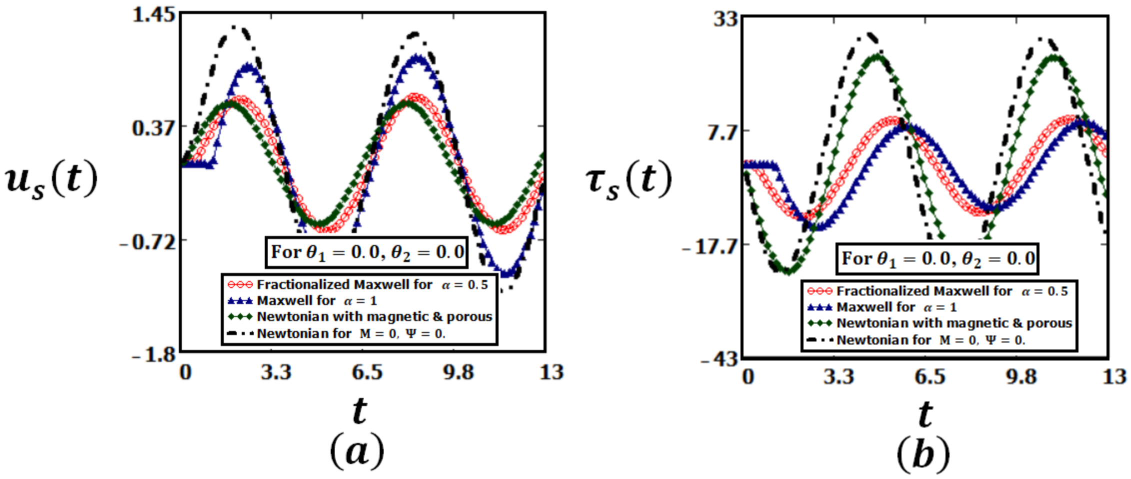

Figure 15: Profiles for fractionalized Maxwell fluid given by Eqs. (17) and (24), Maxwell fluid with MHD & porous given by Eqs. (26) and (28), Newtonian fluid with MHD & porous given by Eqs. (50) and (51) and Newtonian fluid given by Eqs. (62) and (64) for $U = 2$, $\nu = 2.108$, $\mu = 33$, $\lambda = 2$, $M = 0.2$, $\Psi = 3$, $\alpha = 0.5$, $\omega = 1$, $\theta_1 = 0.0$, $\theta_2 = 0.0$ and $y = 1$.

6. Concluding remarks

The effects of twice slip between the oscillating plate and the fluid are more intense than that of first order slip only, while both the slips play similar role of decaying the flow values numerically, specifically the flow values are higher for all four types of fluids without slips. To the best of author's knowledge, rare attempts have been made to examine the effects of twice order slip, thus the future work will be to investigate the effects of twice order slip for different class of fluid with a variety of wall slip boundary conditions.

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