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Soft Hyper GR-Algebra

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Abstract. In this paper, we apply the notion of soft sets to the theory of hyper GR-algebra. Also, we introduce the concept of soft hyper GR-algebras and some properties of soft hyper GR-ideals.

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Key Words and Phrases: hyper GR-algebra, soft hyper GR-algebra, soft set

1. Introduction

During the Congress of Scandinavian Mathematics in 1954, Marty [9] introduced the concept of hyperstructure theory (also known as multialgebra) and defined groups based on the concept of hyperoperation, which is a generalization of a binary operation in algebra, and did an analysis on the application of its properties to groups. The notion of hyper GR-algebra was first initiated by Indangan and Petalcorin [3] in 2016. From then, some studies have been developed to establish some of its properties.

On the other hand, the concept of soft sets was initiated by Molodtsov [10] in 1999 as a new mathematical tool for dealing with uncertainties. It is free from difficulties that have troubled the usual theoretical approaches. Since then, there are various studies on soft sets. In 2003, Maji [8] proposed some basic operations on soft sets. Moreover, Ali [2] in 2009 revised some of these operations and Alcantud [1] in 2015 extended some of the theories on soft sets.

In this paper we apply the soft set theory to hyper GR-algebras.

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2. Preliminaries

An algebra of type (2,0) is an algebra with a binary operation and a constant element.

Definition 1. [**3**] Let H be a nonempty set and $\circledast$ be a hyperoperation on H. Then $(H;\circledast,0)$ is a called a hyper GR-algebra if it satisfies the following conditions, for all $x,y,z \in H$:

(i) (HGR1) $(x \circledast z) \circledast (y \circledast z) \ll x \circledast y$;

(ii) (HGR2) $(x \circledast y) \circledast z = (x \circledast z) \circledast y$;

(iii) (HGR3) $x \ll x$;

(iv) (HGR4) $0 \circledast (0 \circledast x) \ll x$, $x \neq 0$; and

(v) (HGR5) $(x \circledast y) \circledast z \ll y \circledast z$.

Example 1. [**3**] Let $H = \{0,1,2\}$ with hyperoperation $\circledast$ defined by the Cayley table below.

$\circledast$	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{0,1,2\}$	$\{0,1\}$	$\{0,1\}$
2	$\{0,2\}$	$\{0,1,2\}$	$\{0,2\}$

By routine calculation, we see that $(H;\circledast,0)$ is a hyper GR-algebra.

Example 2. [**3**] Let $H = \mathbb{Z}$, where $\mathbb{Z}$ is the set of integers such that for all $x,y,\in H$, $x \circledast y = \{0,x,y\}$. Then H is a hyper GR-algebra.

Definition 2. [**3**] A hyper GR-algebra H is *faithful* if for all $A,B \subseteq H$, $0 \in A \circledast B$ implies $A \ll B$.

Example 3. The hyper GR-algebra $H = \{0,1,2\}$ in Example 1 is faithful.

Definition 3. [4] Let H_1 and H_2 be hyper GR-algebras where $\circledast_1$ and $\circledast_2$ are the hyperoperations of H_1 and H_2, respectively,and $f : H_1 \to H_2$ be function. Then f is called a *hyper GR-algebra hyper homomorphism* if

(i) $f(0_1) = 0_2$; and

ii) $f(x \circledast_1 y) = f(x) \circledast_2 f(y)$.

A hyper homomorphism f is a *hyper monorphism* if f is one-to-one and f is a *hyper epimorphism* if f is onto; f is called a *hyper isomorphism* if f is a hyper monorphism and hyper epimorphism (denoted by $\cong_H$).

Definition 4. [3] Let H be a hyper GR-algebra and S be a subset of H containing 0. If S is a hyper GR-algebra with respect to the hyperoperation $\circledast$ on H, then we say that H is a *hyper subGR-algebra* of H.

Lemma 1. [4] Let $f : H \to Y$ be homomorphism of hyper GR-algebras. If S is a hyper subGR-algebra of H, then $f(S)$ is a hyper subGR-algebra.

Theorem 1. [3] (Hyper SubGR-algebra Criterion)
Let H be a hyper GR-algebra and S be a nonempty subset of H. Then S is a *hyper subGR-algebra* of H if and only if $x \circledast y \subseteq S$, for all $x, y \in S$.

Theorem 2. [3] If $\{I_i | i \in \Lambda\}$ is a nonempty collection of hyper GR-ideals of a hyper GR-algebra H, then so is $\bigcap\limits_{i\in\Lambda} I_i$.

Definition 5. [6] Let U be an initial universal set and E a set of all possible parameters under consideration. If $A \subset E$, then a *soft set* (F, A) *over* U is defined to be the set of ordered pairs

$$(F, A) = \{(x, f_A(x)) : x \in E, f_A(x) \in P(U)\},$$

where $f_A : E \to P(U)$ such that $f_A(x) = \varnothing$ if $x \notin A$. The function f_A is called the *approximation function of the soft set* (F, A). The subscript A in the notion f_A indicates that f_A is the approximate function of (F, A). In what follows, let $S(U)$ denote the set of all soft sets over U by Cagman et al. [7].

Definition 6. [5] Let (F, A) and (G, B) be two soft sets over U. The intersection of (F, A) and (G, B) is defined to be the soft set (H, C) satisfying the following conditions:

(i) $C = A \cap B \neq \varnothing$

(ii) $H(e) = F(e) \cap G(e)$, for all $e \in C$.

In this case , we write $(F, A) \,\widetilde{\cap}\, (G, B) = (H, C)$.

Definition 7. [5] Let (F, A) and (G, B) be two soft sets over a common universe U. Then the union of (F, A) and (G, B) is defined to be a soft set (H, C) satisfying the following conditions:

(i) $C = A \cup B$;

(ii) for all $e \in C$,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A \setminus B \\ G(e), & \text{if } e \in B \setminus A \\ F(e) \cup G(e), & \text{if } e \in A \cap B. \end{cases}$$

In this case, we write $(F, A) \,\widetilde{\cup}\, (G, B) = (H, C)$.

Definition 8. [**5**] If (F, A) and (G, B) are two sets over U, then "(F, A) and (G, B)" denoted by $(F, A) \mathbin{\widetilde{\wedge}} (G, B)$ is defined by $(F, A) \mathbin{\widetilde{\wedge}} (G, B) = (H, A \times B)$ where $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$ for all $(\alpha, \beta) \in A \times B$.

Definition 9. [**5**] For two soft sets (F, A) and (G, B) over U, then "(F, A) or (G, B) " denoted by $(F, A) \mathbin{\widetilde{\vee}} (G, B)$ is defined by $(F, A) \mathbin{\widetilde{\vee}} (G, B) = (H, A \times B)$ where $H(\alpha, \beta) = F(\alpha) \cup G(\beta)$ for all $(\alpha, \beta) \in A \times B$.

Definition 10. [**5**] For two soft sets (F, A) and (G, B) over U, we say that (F, A) is a soft subset of (G, B), denoted by $(F, A) \mathrel{\widetilde{\subset}} (G, B)$, if it satisfies:

(i) $A \subseteq B$

(ii) For every $\epsilon \in A$, $F(\epsilon) = G(\epsilon)$.

Definition 11. [**6**] Let $(F, A) \in S(U)$ and $\tau \subseteq U$. Then the *τ-exclusive set* of (F, A) is defined to be the set $e((F, A), \tau) = \{x \in A : f_A(x) \subseteq \tau\}$.

From Definition 11, we have the following properties [**6**]:

1. $e((F, A), U) = A$,
2. $f_A(x) = \cap\{\tau \subseteq U : x \in e((F, A), \tau)\}, \forall x \in A$, and
3. $\tau_1 \subseteq \tau_2$ implies $e((F, A), \tau_1) \subseteq e((F, A), \tau_2), \forall \tau_1, \tau_2 \subseteq U$.

3. Soft Hyper GR-Algebra

Let H be a hyper GR-algebra, A a nonempty set, and $\mathring{R}$ an arbitrary binary relation between an element of A and an element of $P(H)$, that is, $\mathring{R} \subseteq A \times P(H)$. A set-valued function $F : A \to P(H)$ can be defined as $F(a) = \bigcup B$ where $B \subset H$ and $a\mathring{R}B$, for all $a \in A$. Then (F, A) is then a soft set over H.

Definition 12. Let (F, A) be a soft set over a hyper GR-algebra H. Then (F, A) is called a *soft hyper GR-algebra* over H if $F(a) = \bigcup\limits_{B \subset H, a\mathring{R}B} B$ is a hyper GR-algebra of H, for all $a \in A$.

Example 4. Consider $H = \{0, 1, 2, 3\}$ defined by the Cayley table below.

$\circledast$	0	1	2	3
0	{0,1}	{0,1}	{0,1}	{0,1}
1	{1}	{0,1}	{0,1}	{0,1}
2	{0,2}	{0,2}	{0,1,2}	{0,1,2}
3	{3}	{0,1,3}	{0,1,3}	{0,1,3}.

By Definition 1 $(H; \circledast, 0)$ is a hyper GR-algebra. We will verify if $(H; \circledast, 0)$ is a soft hyper GR-algebra. Let $A = H$ and define a relation $\mathring{R}$ such that $a\mathring{R}B$ if and only if $B = a^n$, where $B \subset H$ and $a \in A$, let $F : A \to P(H)$ be a set-valued function defined as follows: $F(a) = \bigcup_{B \subset H, aRB \Leftrightarrow B = a^n} B$, for all $a \in A$, where $a^n = ((((a \circledast a) \circledast a) \circledast a) \circledast ... \circledast a)$. Then $F(0) = \{0, 1\}$, $F(1) = \{0, 1\}$, $F(2) = \{0, 1, 2\}$ and $F(3) = \{0, 1, 3\}$. Note that S is nonempty. Let $S = F(0) = F(1) = \{0, 1\}$. Then $0 \circledast 0 = 0 \circledast 1 = 1 \circledast 1 = \{0, 1\} \in S$ and $1 \circledast 0 = \{1\} \in S$. Thus, by Theorem 1, $F(0) = F(1)$ is a hyper subGR-algebra. Similarly, $F(2)$ and $F(3)$ are hyper subGR-algebras. Hence, $F(a)$ is a hyper GR-algebra over H, for all $a \in A$. Therefore, (F, A) is a soft hyper GR-algebra.

Example 5. Consider the same hyper GR-algebra $H = \{0, 1, 2, 3\}$ in Example 4. Let $A = \{a, b\}$ and $\mathring{R} = \{(a, \{0, 1\}), (a, \{0, 1, 2\}), (b, \{0, 1\}), (b, \{0, 1, 3\})\}$. Then

$$F(a) = \bigcup_{B \subset H, a\mathring{R}B} B = \{0, 1, 2\}$$

and

$$F(b) = \bigcup_{B \subset H, b\mathring{R}B} B = \{0, 1, 3\}$$

which are both hyper subGR-algebra of H. Hence, (F, A) is a soft hyper GR-algebra with respect to $\mathring{R}$.

Example 6. Consider the same hyper GR-algebra $H = \{0, 1, 2, 3\}$ in Example 4. Let $A = \{a, b\}$ and $\mathring{R} = \{(a, \{1, 3\}), (a, \{0\}), (b, \{0\}), (b, \{2\})\}$. Then

$$F(a) = \bigcup_{B \subset H, a\mathring{R}B} B = \{0, 1, 3\}$$

which is a hyper subGR-algebra. Now,

$$F(b) = \bigcup_{B \subset H, a\mathring{R}B} B = \{0, 2\}.$$

However, $0 \circledast 0 = \{0, 1\} \nsubseteq F(b)$. Thus, $F(b)$ is not a hyper subGR-algebra. Hence, (F, A) is not a soft hyper GR-algebra.

Theorem 3. Let (F, A) be a soft hyper GR-algebra over H. If $B \subseteq A$, then (F, B) is a soft hyper GR-algebra over H.

Proof: Since (F, A) is a soft hyper GR-algebra it follows that $F(a)$ is a hyper GR-algebra for all $a \in A$. Since $B \subseteq A$, $F(a)$ is a hyper GR-algebra over H for all $a \in B$. Hence, (F, B) is a soft hyper GR-algebra over H. □

Theorem 4. Let (F, A) and (G, B) be two soft hyper GR-algebras over H. If $A \cap B \neq \varnothing$, then the intersection $(F, A) \,\widetilde{\cap}\, (G, B)$ is a soft hyper GR-algebra over H.

Proof: Using Definition 6, we can write $(F, A)\widetilde{\cap}(G, B) = (D, C)$, where $A \cap B = C \neq \varnothing$ and $D(x) = F(x) \cap G(x)$ for all $x \in G$. Note that $D : C \to P(H)$ is a mapping since the intersection of two hyper GR-algebra is a hyper GR-algebra, thus, (D, C) is a soft set over H. Since (F, A) and (G, B) are soft hyper GR-algebras over H, it follows that $D(x) = F(x)$ or $D(x) = G(x)$ for all $x \in C$. Thus, (D, C) is a soft hyper GR-algebra over H. Hence, $(D, C) = (F, A) \widetilde{\cap} (G, B)$ is a soft hyper GR-algebra over H. □

Theorem 5. Let (F, A) and (G, B) be two soft hyper GR-algebras over H. If $A \cap B = \varnothing$, then the union $(F, A) \widetilde{\cup} (G, B)$ is a soft hyper GR-algebra over H.

Proof: Using Definition 7, we can write $(F, A) \widetilde{\cup} (G, B) = (J, C)$, where $C = A \cup B$, and for all $x \in C$,

$$J(x) = \begin{cases} F(x), & \text{if } x \in A \setminus B \\ G(x), & \text{if } x \in B \setminus A \\ F(x) \cup G(x), & \text{if } x \in A \cap B. \end{cases}$$

Since $A \cap B = \varnothing$, this implies that either $x \in A \setminus B$ or $x \in B \setminus A$ for all $x \in C$. If $x \in A \setminus B$, $J(x) = F(x)$. Thus, (J, C) is a soft hyper GR-algebra over H. If $x \in B \setminus A$, $J(x) = G(x)$. Thus, (J, C) is a soft hyper GR-algebra over H. Hence, $(J, C) = (F, A) \widetilde{\cup} (G, B)$ is a soft hyper GR-algebra over H. □

Theorem 6. If (F, A) and (G, B) are soft hyper GR-algebras over H. Then $(F, A)\widetilde{\wedge}(G, B)$ is a soft hyper GR-algebra over H.

Proof: By Definition 8, $(F, A) \widetilde{\wedge} (G, B) = (J, A \times B)$. Since $F(x)$ and $G(y)$ are hyper GR-algebras of H, it follows that the intersection $(F \cap G)(x, y)$, is also a hyper subGR-algebra of H. Hence, $J(x, y)$ is a hyper subGR-algebra of H for all $(x, y) \in A \times B$, and so $(F, A) \widetilde{\wedge} (G, B) = (J, A \times B)$ is a soft hyper GR-algebra over H. □

Definition 13. A soft hyper GR-algebra (F, A) over H is said to be *trivial* (respectively, *whole*) if $F(a) = \{0\}$ (respectively, $F(a) = X$) for all $a \in A$.

Example 7. Let $H = \{0, 1\}$. Define $\circledast$ as shown in the table below.

$\circledast$	0	1
0	$\{0\}$	$\{0\}$
1	$\{0\}$	$\{0\}$.

By routine calculations, $(H; \circledast, 0)$ is a hyper GR-algebra. Let $A = H$ and let $F : A \to P(H)$ be the set-valued function defined as follows:

$$F(a) = \bigcup_{B \subset H, a\mathring{R}B \Leftrightarrow 0 \circledast x \ll B} B.$$

Then, $F(0) = F(1) = F(2) = \{0\}$. Since $\{0\}$ is a hyper subGR-algebra of H, (F, A) is a trivial soft hyper GR-algebra of H.

Example 8. Let $H = \{0, 1, 2\}$. Define $\circledast$ as shown in the table below:

$\circledast$	0	1	2
0	{0}	{0}	{0}
1	{0,1,2}	{0,1}	{0,1}
2	{0,2}	{0,1,2}	{0,1,2}.

By routine calculations, $(H; \circledast, 0)$ is a hyper GR-algebra. Let $A = H$ and let $F : A \to P(H)$ be the set-valued function defined as follows:

$$F(a) = \bigcup_{B \subset H, a\mathring{R}B \Leftrightarrow x \circledast 0 \ll B} B,$$

where $B \subset H$ and define a relation $\mathring{R}$ such that $a\mathring{R}B$ if and only if $x \circledast 0 \ll B$ and $a \in A$. Then $F(0) = F(1) = F(2) = \{0, 1, 2\} = H$. Since H is a hyper GR-algebra of H, (F, A) is whole soft hyper GR-algebra of H.

Lemma 2. Let $f : H \to Y$ be a homomorphism of hyper GR-algebras. If (F, A) is a soft hyper GR-algebra over H, then $(f(F), A)$ is a soft hyper GR-algebra over Y.

Proof: Let $a \in A$. Since $F(a)$ is a hyper subGR-algebra on H and f is a homomorphism, it follows that $f(F)(a) = f(F(a))$ is a hyper subGR-algebra on Y by Lemma 1. Hence, $(f(F), A)$ is a soft hyper GR-algebra on Y. □

Theorem 7. Let $f : H \to Y$ be a homomorphism of hyper GR-algebras and let (F, A) be a soft hyper GR-algebra over H.

(i) $(f(F), A)$ is trivial soft hyper Gr-algebra over Y if and only if $F(x) \subseteq \ker f$ for all $x \in A$.

(ii) If f is onto and (F, A) is whole, then $(f(F), A)$ is a whole soft hyper GR-algebra over Y.

(iii) If $(f(F), A)$ is whole and f is one-to-one, then f is onto and (F, A) is a whole soft hyper GR-algebra over Y.

(iv) If f is bijective, then (F, A) is whole over H if and only if $(f(F), A)$ is whole over H.

Proof: (i) Suppose $F(a) \subseteq kerf$ for all $a \in A$. Then $f(F)(a) = f(F(a)) = \{0_Y\}$ for all $a \in A$. Hence, $(f(F), A)$ is a trivial soft hyper GR-algebra over Y by Definition 13 and Lemma 2. Conversely, suppose that $(f(F), A)$ is a trivial soft hyper GR-algebra over Y. Then $f(F)(a) = f(F(a)) = \{0_Y\}$ for all $a \in A$. This means that $F(a) \subseteq \ker f$, for all $a \in A$.
(ii) Assume that f is onto and (F, A) is whole. Then $F(a) = H$ for all $a \in A$, and so $f(F)(a) = f(F(a)) = H$ for all $a \in A$. It follows from Definition 13 and Lemma 2 that $(f(F), A)$ is a whole soft hyper GR-algebra.

(iii) Suppose $(f(F), A)$ is whole. Then $f(F)(a) = f(F(a)) = H$ for all $a \in A$. Thus, $f(H) = Y$ since $F(a) \subseteq H$ for all $a \in A$. Hence, f is onto. Now, let $a \in A$ and $z \in H$. Since $f(z) \in Y = f(F(a))$, there exists $x \in F(a)$ such that $f(x) = f(z)$. Since f is one-to-one, it follows that $x = z \in F(a)$. Therefore, $F(a) = H$ implying that (F, A) is whole.
(iv) The proof follows from (ii) and (iii). □

Definition 14. Let (F, A) and (G, C) be two soft hyper GR-algebras over H. Then (F, A) is called a *soft hyper subGR-algebra of* (G, C), written as $(F, A) \tilde{<} (G, C)$, if it satisfies the following:

(i) $A \subseteq C$

(ii) $F(a)$ is a hyper subGR-algebra of $G(a)$ for all $a \in A$.

Example 9. Consider the hyper GR-algebra in Example 8. For $C = H$, let $G : C \to P(H)$ be the set-valued function defined by

$$G(a) = \bigcup_{B \subset H, a\mathring{R}B \Leftrightarrow B \ll a} B,$$

for all $a \in C$. Then

$$G(0) = \bigcup_{B \subset H, 0\mathring{R}B \Leftrightarrow B \ll 0} B = \{0\},$$

$$G(1) = \bigcup_{B \subset H, 1\mathring{R}B \Leftrightarrow B \ll 1} B = \{0, 1, 2\}$$

and

$$G(2) = \bigcup_{B \subset H, 2\mathring{R}B \Leftrightarrow B \ll 2} B = \{0, 1, 2\}.$$

Let $A = \{0, 1, 2\}$ and $F : A \to P(H)$ be the set-valued function defined by

$$F(a) = \bigcup_{B \subset H, a\mathring{R}B \Leftrightarrow B = a^n} B,$$

where $a^n = ((((a \circledast a) \circledast a) \circledast a) \circledast ... \circledast a)$ for all $a \in A$. Then

$$F(0) = \bigcup_{B \subset H, 0\mathring{R}B \Leftrightarrow B = 0^n} B = \{0\},$$

$$F(1) = \bigcup_{B \subset H, 1\mathring{R}B \Leftrightarrow B = 1^n} B = \{0, 1\}$$

and

$$F(2) = \bigcup_{B \subset H, 2\mathring{R}B \Leftrightarrow B = 2^n} B = \{0, 1, 2\}.$$

Hence, $F(0), F(1)$ and $F(2)$ are soft hyper subGR-algebras on $G(0), G(1)$ and $G(2)$, respectively. Therefore, $(F, A) \tilde{<} (G, C)$.

Theorem 8. Let (F, A) and (G, A) be two soft hyper GR-algebras over H. Then $(F, A) \mathrel{\widetilde{<}} (G, A)$ if and only if $F(a) \subseteq G(a)$ for all $a \in A$.

Proof: Let $(F, A) \mathrel{\widetilde{<}} (G, A)$. By Definition 14, $F(a)$ is a hyper subGR-algebra of $G(a)$ for all $a \in A$. This implies that $F(a) \subseteq G(a)$ for all $a \in A$. Conversely, let $F(a) \subseteq G(a)$ for all $a \in A$. Since (F, A) and (G, A) are soft hyper GR-algebras over H and $F(a) \subseteq G(a)$ for all $a \in A$, it follows that $F(a)$ is a hyper subGR-algebra of $G(a)$ for all $a \in A$. Hence, $(F, A) \mathrel{\widetilde{<}} (G, A)$. □

Theorem 9. Let (F, A) be soft hyper GR-algebra over H and let (G_1, C_1) and (G_2, C_2) be two soft hyper subGR-algebras of (F, A). Then

(i) $(G_1, C_1) \mathbin{\widetilde{\cap}} (G_2, C_2) \mathrel{\widetilde{<}} (F, A)$

(ii) $C_1 \cap C_2 = \varnothing \implies (G_1, C_1) \mathbin{\widetilde{\cup}} (G_2, C_2) \mathrel{\widetilde{<}} (F, A)$.

Proof: (i) By Definition 6, we can write $(G_1, C_1)\widetilde{\cap}(G_2, C_2) = (G, C)$, where $C = C_1 \cap C_2$ and $G(a) = G_1(a) \cap G_2(a)$ for all $a \in C$. Since $G_1(a), G_2(a) \subseteq F(a)$, $G_1(a) \cap G_2(a) \subseteq F(a)$ for all $a \in C$. Thus, $G(a) = G_1(a) \cap G_2(a)$ is a hyper subGR-algebra of $F(a)$ for all $a \in C$.
(ii) Assume that $C_1 \cap C_2 = \varnothing$. By Definition 7, we can write $(G_1, C_1) \mathbin{\widetilde{\cup}} (G_2, C_2) = (G, C)$, where $C = C_1 \cup C_2$ and

$$G(x) = \begin{cases} G_1(a), & \text{if } a \in C_1 \setminus C_2 \\ G_2(a), & \text{if } a \in C_2 \setminus C_1 \\ G_1(a) \cup G_2(a), & \text{if } a \in C_1 \cap C_2 \end{cases}$$

for all $a \in C$. By the hypothesis, $C_1, C_2 \subseteq A$. This implies that $C = C_1 \cup C_2 \subseteq A$. Also, $G_i(a)$ is a hyper subGR-algebra of $F(a)$ for all $a \in C_i, i = 1, 2$. Since $C_1 \cap C_2 = \varnothing$, by Theorem 5, $G(a)$ is a hyper subGR-algebra of $F(a)$ for all $a \in C$. Therefore, $(G_1, C_1) \mathbin{\widetilde{\cup}} (G_2, C_2) \mathrel{\widetilde{<}} (F, A)$. □

Definition 15. Let (F, A) be a soft set over hyper GR-algebra H. A soft set (G, I) over H is called a *soft hyper GR-ideal* of (F, A), written as $(G, M) \mathrel{\widetilde{\diamond}} (F, A)$ if the following are satisfied:

(i) $I \subset A$ with $I \neq \varnothing$

(ii) for all $a \in I$, $G(a)$ is a hyper GR-ideal on $F(a)$.

Example 10. Consider the same hyper GR-algebra $H = \{0, 1, 2, 3\}$ in Example 4. Let $A = H$ and $I = \{0, 1\}$. Suppose $F : A \to P(H)$ is defined by $F(a) = \bigcup\limits_{b \subset H, a\mathring{R}B} B$ with $\mathring{R} = \{(0, \{1\}), (0, \{0, 2\}), (1, \{0, 2\}), (1, \{0, 1, 3\})\}$. Then

$$F(0) = \bigcup_{B \subset H, 0\mathring{R}B} B = \{0, 1, 2\}$$

and

$$F(1) = \bigcup_{B \subset H, 1\mathring{R}B} B = \{0, 1, 2, 3\}.$$

Define $G(a) = \bigcup_{B \subset H, a\mathring{R}B \Leftrightarrow B = a^n} B$ for some $n \in \mathbb{N}$. Then $G(0) = \{0, 1\}, G(1) = \{0, 1\}, G(2) = \{0, 1, 2\}$ and $G(3) = \{0, 1, 3\}$.

a	b	a ⊛ b	≪ I	$a \in I$
0	0	{0,1}	✓	✓
0	1	{0,1}	✓	✓
1	0	{1}	✓	✓
2	0	{0,2}	×	
2	1	{0,2}	×	

Table 3.1

a	b	a ⊛ b	$\ll G(1) = \{0, 1\}$	$a \in G(1) = \{0, 1\}$
0	0	{0,1}	✓	✓
1	0	{1}	✓	✓
1	1	{0,1}	✓	✓.
2	0	{0,2}	×	
2	1	{0,2}	×	
3	0	{3}	×	
3	1	{0,1,3}	×	

Table 3.2

Note that $G(0)$ is a hyper GR-ideal of $F(0)$ (see Table 3.1) and $G(1)$ is a hyper GR-ideal of $F(1)$ is a hyper GR-ideal (see Table 3.2). Hence, for all $a \in I$, $G(a)$ is a hyper GR-ideal of $F(a)$. Therefore, (G, I) is a soft hyper GR-ideal of (F, A).

Theorem 10. Let (F, A) be a soft hyper GR-algebra over H. Suppose (G, M_1) and (J, M_2) are two soft hyper GR-ideals of (F, A) such that $M_1 \cap M_2 \neq \varnothing$. Then $(G, M_1) \,\widetilde{\cap}\, (J, M_2)$ is a soft hyper GR-ideal of (F, A).

Proof: Suppose (G, M_1) and (J, M_2) are two soft hyper GR-ideals such that $M_1 \cap M_2 \neq \varnothing$. Take $M = M_1 \cap M_2$. Clearly, $M \subseteq A$ and by hypothesis $M \neq \varnothing$. By Theorem 2,M is a hyper GR-ideal. Let $m \in M$. Then this implies that $m \in M_1$ and $m \in M_2$. Also, $G(m) \diamond F(m)$ and $J(m) \diamond F(m)$. Thus, $[G(m) \cap J(m)] \diamond F(m)$. Hence, $(G, M_1) \,\widetilde{\cap}\, (J, M_2)$ is a soft hyper GR-ideal of (F, A). □

If $M = M_1 = M_2$, then we have the following Corollary.

Corollary 1. Let (F, A) be a soft hyper GR-algebra over H. For any soft sets (G, M) and (J, M) over H, we have $(G, M) \,\widetilde{\diamond}\, (F, A) \implies (G, M) \,\widetilde{\cap}\, (J, M) \,\widetilde{\diamond}\, (F, A)$.

Theorem 11. Let (F, A) be a soft hyper GR-algebra over H. For any soft sets (G, I) and (J, K) with $I \cap K = \varnothing$, we have

$$(G, I) \,\widetilde{\diamond}\, (F, A), (J, K) \,\widetilde{\diamond}\, (F, A) \implies (G, I) \,\widetilde{\cup}\, (J, K) \,\widetilde{\diamond}\, (F, A).$$

Proof: Using Definition 7, we can write $(G, I) \,\widetilde{\cup}\, (J, K) = (R, U)$, where $U = I \cup K$, and for all $x \in U$,

$$R(x) = \begin{cases} G(x), & \text{if } x \in I \setminus K \\ J(x), & \text{if } x \in K \setminus I \\ G(x) \cup J(x), & \text{if } x \in I \cap K. \end{cases}$$

Now, $I \cap K = \varnothing$ implies either $x \in I \setminus K$ or $x \in K \setminus I$ for all $x \in U$. If $x \in I \setminus K$, then $R(x) = G(x) \diamond F(x)$. If $x \in K \setminus I$, then $R(x) = J(x) \diamond F(x)$. Thus, $R(x) \diamond F(x)$ for all $x \in U$. Hence, $(G, I) \,\widetilde{\cup}\, (J, K) = (R, U) \,\widetilde{\diamond}\, (F, A)$. □

Theorem 12. Let (F, A) be a soft hyper GR-algebra over H. If (G, M) and (J, K) are soft hyper GR-ideals of (F, A), then $(G, M) \,\widetilde{\wedge}\, (J, K)$ is a soft hyper GR-ideal of (F, A).

Proof: Using Definition 8, we can write $(G, M)\widetilde{\wedge}(J, K) = (R, M \times K)$, where $R(x, y) = G(x) \cap J(y)$ for all $(x, y) \in M \times K$. By Theorem 10,

$$\begin{aligned} G(x) \diamond F(x), J(y) \diamond F(x) &\implies G(x) \cap J(y) \diamond F(x) \\ &\implies R(x, y) \diamond F(x) \text{ for all } (x, y) \in M \times K. \end{aligned}$$

Therefore, $(G, M) \,\widetilde{\wedge}\, (J, K) = (R, M \times K)$ is a soft hyper GR-ideal of (F, A). □

Definition 16. Let S be a hyper subGR-algebra on H. A subset M of H is a *hyper GR-commutative ideal* of H related to S denoted by $M \diamond_{hgrc} S$ if it satisfies the following:

(i) $0 \in M$;

(ii) $(x \circledast y) \circledast z \subseteq M$ and $z \in M$ imply that $x \circledast (y \circledast (y \circledast x)) \subseteq M$ for all $x, y \in S$.

Example 11. Consider the same hyper GR-algebra $H = \{0, 1, 2, 3\}$ in Example 4. Then $S = \{0, 1, 3\}$ is a hyper subGR-algebra of H and $M = \{0, 1, 2\}$ is a hyper GR-commutative ideal of S.

Definition 17. Let (F, A) be a soft set over a hyper GR-algebra H. A soft set (G, M) over H is called a *soft hyper GR-commutative ideal* of (F, A) denoted by $(G, M) \,\widetilde{\diamond}_{hgrc}\, (F, A)$, if the following are satisfied:

(i) $M \subset A$ with $M \neq \varnothing$;

(ii) for all $a \in M$, $G(a) \diamond_{hgrc} F(a)$.

Example 12. Consider the same hyper GR-algebra $H = \{0, 1, 2, 3\}$ in Example 4. Let $M = \{0, 1, 2\}$ and (F, A) be a soft set over H, where $A = H$. Let $F : A \to P(H)$ be defined by

$$F(a) = \bigcup_{B \subset H, a \mathring{R} B} B$$

with $\mathring{R} = \{(0,0),(0,\{1,2\}),(1,\{0,1\}),(1,\{1\}),(2,\{0,1\}),(2,\{0,3\})\}$. Then

$$F(0) = \bigcup_{B \subset H, 0\mathring{R}B} B = \{0,1,2\}$$
$$F(1) = \bigcup_{B \subset H, 1\mathring{R}B} B = \{0,1\}$$
$$\text{and } F(2) = \bigcup_{B \subset H, 2\mathring{R}B} B = \{0,1,3\}.$$

Also, let $G : A \to P(H)$ be a set-valued function defined by $G(a) = \bigcup_{B \subset H, a\mathring{R}B \Leftrightarrow B = a^n} B$, where $a^n = ((((a \circledast a) \circledast a) \circledast a) \circledast \ldots \circledast a)$. Then $G(0) = G(1) = \{0,1\}, G(2) = \{0,1,2\}$ and $G(3) = \{0,1,3\}$. Then $G(0) = \{0,1\} \diamond_{hgrc} F(0) = \{0,1,2\}, G(1) = \{0,1\} \diamond_{hgrc} F(1) = \{0,1\}$ since they are equal, and $G(2) = \{0,1,2\} \diamond_{hgrc} F(2) = \{0,1,3\}$. Hence, for all $a \in M, G(a) \,\widetilde{\diamond}_{hgrc}\, F(a)$.

Theorem 13. Let S be a hyper subGR-algebra of a hyper GR-algebra H. If $I_1 \diamond_{hgrc} S$ and $I_2 \diamond_{hgrc} S$, then $I_1 \cap I_2 \diamond_{hgrc} S$.

Proof: Since $0 \in I_1$ and $0 \in I_2$, $0 \in I_1 \cap I_2$. Suppose $x, y \in S$ and $(x \circledast y) \circledast z \subseteq M = I_1 \cap I_2$ with $z \in I_1 \cap I_2$. Since $I_1 \diamond_{hgrc} S$ and $I_2 \diamond_{hgrc} S$, it follows that $(x_1 \circledast y_1) \circledast z_1 \subseteq M$ and $(x_2 \circledast y_2) \circledast z_2 \subseteq M$ implying that $x_1 \circledast (y_1 \circledast (y_1 \circledast x_1)) \subseteq M$ and $x_2 \circledast (y_2 \circledast (y_2 \circledast x_2))$, respectively, for all $x_1, x_2, y_1, y_2 \in S$ and for all $z_1, z_2 \in M$. Let $x \subseteq I_1$, $y \subseteq I_2$ and $z \subseteq M$. Consider that $x_1, x_2, y_1, y_2 \subseteq S$ and for all $z_1, z_2 \subseteq M$ such that $x = x_1 \cap x_2$, $y = y_1 \cap y_2$ and $z = z_1 \cap z_2$. Then

$$\begin{aligned}(x \circledast y) \circledast z &= [(x_1 \cap x_2) \circledast (y_1 \cap y_2)] \circledast (z_1 \cap z_2)\\ &= [(x_1 \circledast y_1) \cap (x_2 \circledast y_2)] \circledast (z_1 \cap z_2)\\ &= [(x_1 \circledast y_1) \circledast (z_1 \cap z_2)] \cap [(x_2 \circledast y_2) \circledast (z_1 \cap z_2)]\\ &= \left[(x_1' \circledast y_1') \circledast z_1'\right] \cap \left[(x_2' \circledast y_2') \circledast z_2'\right]\\ &\subseteq I_1 \cap I_2 = M.\end{aligned}$$

By Definition 16, $x \circledast (y \circledast (y \circledast x)) \subseteq M$. Hence, $M = I_1 \cap I_2 \diamond_{hgrc} S$. □

Theorem 14. Let (F, A) be a soft hyper GR-algebra over H. For any soft sets (G_1, M_1) and (G_2, M_2) over H, where $M_1 \cap M_2 \neq \varnothing$ we have

$$(G_1, M_1) \,\widetilde{\diamond}_{hgrc}\, (F, A), (G_2, M_2) \,\widetilde{\diamond}_{hgrc}\, (F, A) \implies (G_1, M_1) \,\widetilde{\cap}\, (G_2, M_2) \,\widetilde{\diamond}_{hgrc}\, (F, A).$$

Proof: By Definition 6, we write $(G_1, M_1) \,\widetilde{\cap}\, (G_2, M_2) = (G, M)$ where $M = M_1 \cap M_2$ and $G(a) = G_1(a) \cap G_2(a)$ for all $a \in M$. Clearly, $M \subseteq A$. By Theorem 13, $G_1(a) \cap G_2(a) \diamond_{hgrc} F(a)$. Hence, $(G_1, M_1) \,\widetilde{\cap}\, (G_2, M_2) = (G, M) \,\widetilde{\diamond}_{hgrc}\, (F, A)$. □

If $M = M_1 = M_2$, then we have the following Corollary.

Corollary 2. Let (F, A) be a soft hyper GR-algebra over H. For any soft sets (G, M) and (J, M) over H, we have

$$(G, M) \mathrel{\widetilde{\diamond}_{hgrc}} (F, A), (J, M) \mathrel{\widetilde{\diamond}_{hgrc}} (F, A) \implies (G, M) \mathbin{\widetilde{\cap}} (J, M) \mathrel{\widetilde{\diamond}_{hgrc}} (F, A).$$

Theorem 15. Let (F, A) be a soft hyper GR-algebra over H. For any soft sets (G, M) and (J, N), with $M \cap N = \varnothing$, we have

$$(G, M) \mathrel{\widetilde{\diamond}_{hgrc}} (F, A), (J, N) \mathrel{\widetilde{\diamond}_{hgrc}} (F, A) \implies (G, M) \mathbin{\widetilde{\cup}} (J, N) \mathrel{\widetilde{\diamond}_{hgrc}} (F, A).$$

Proof: Using Definition 7, we can write $(G, M) \mathbin{\widetilde{\cup}} (J, N) = (R, U)$, where $U = M \cup N$, and for all $x \in U$,

$$R(x) = \begin{cases} G(x), & \text{if } x \in M \setminus N \\ J(x), & \text{if } x \in N \setminus M \\ G(x) \cup J(x), & \text{if } x \in M \cap N. \end{cases}$$

Since $M \cap N = \varnothing$, it implies that either $x \in M \setminus N$ or $x \in N \setminus M$, for all $x \in U$. If $x \in M \setminus N$, $R(x) = G(x) \diamond_{hgrc} F(x)$. If $x \in N \setminus M$, $R(x) = J(x) \diamond_{hgrc} F(x)$. Thus, $R(x) \diamond_{hgrc} F(x)$ for all $x \in U$. Hence, $(G, M) \mathbin{\widetilde{\cup}} (J, N) = (R, U) \mathrel{\widetilde{\diamond}_{hgrc}} (F, A)$. □

Definition 18. Let E be a hyper GR-algebra. Given a hyper subGR-algebra A of E, let $(F, A) \in S(U)$. Then (F, A) is called a *union-soft hyper GR-algebra* over U if f_A satisfies

$$f_A(x \circledast y) \subseteq f_A(x) \cup f_A(y), \forall x, y \in A.$$

Example 13. Consider the hyper GR-algebra $H = \{0, 1, 2, 3\}$ defined in Example 4. Let τ_1, τ_2, τ_3 be subsets of H such that $\tau_1 \subseteq \tau_2 \subseteq \tau_3$. Define a soft set (F, A) as follows:

$$(F, A) = \{(0, \tau_1), (1, \tau_1), (2, \tau_2), (3, \tau_3)\}.$$

By routine calculations, (F, A) is a union-soft hyper GR-algebra.

Example 14. Consider the hyper GR-algebra $H = \{0, 1, 2, 3\}$ defined in Example 4. Let $\tau_1, \tau_2, \tau_3, \tau_4$ be subsets of H such that $\tau_1 \subsetneq \tau_2 \subsetneq \tau_3 \subsetneq \tau_4$. Define a soft set (F, A) as follows:

$$(F, A) = \{(0, \tau_1), (1, \tau_2),)(2, \tau_3), (3, \tau_4)\}.$$

By routine calculations, (F, A) is not a union-soft hyper GR-algebra since $f_A(0 \circledast 0) = \tau_2 \nsubseteq f_A(0) \cup f_A(0) = \tau_1$.

Theorem 16. Let E be a hyper GR-algebra. Given a hyper subGR-algebra A of E, let $(F, A) \in S(U)$. Then (F, A) is a union-soft hyper GR-algebra over U if and only if the nonempty τ-exclusive set of (F, A) is a hyper subGR-algebra of A for all $\tau \subseteq U$.

Proof: Assume (F, A) is the union-soft hyper GR-algebra over U. Let $\tau \subseteq U$ and $x, y \in e((F, A); \tau)$. Then $f_A(x) \subseteq \tau$ and $f_A(y) \subseteq \tau$. It follows from the definition that $f_A(x \circledast y) \subseteq f_A(x) \cup f_A(y) \subseteq \tau$. Hence, $x \circledast y \subseteq e((F, A); \tau)$ and so $e((F, A); \tau)$ is a hyper subGR-algebra. Conversely, suppose that the nonempty τ-exclusive set of (F, A) is a hyper subGR-algebra of A for all $\tau \subseteq U$. Let $x, y \in A$ such that $f_A(x) = \tau_1$ and $f_A(y) = \tau_2$. Take $\tau = \tau_1 \cup \tau_2$. Then $x, y \in e((F, A); \tau)$ and so $x \circledast y \subseteq e((F, A); \tau)$. Thus, $f_A(x \circledast y) \subseteq \tau = \tau_1 \cup \tau_2 = f_A(x) \cup f_A(y)$. Hence, (F, A) is a union-soft hyper GR-algebra. □

Theorem 17. Let E be a hyper GR-algebra. Given a hyper subGR-algebra A of E, let $(F, A) \in S(U)$. Suppose $(F, A)^* \in S(U)$ with approximation function f_A^* defined by $f_A^* : E \to P(U)$,

$$x \longmapsto \begin{cases} f_A(x), & \text{if } x \in e((F, A); \tau) \\ U, & \text{otherwise.} \end{cases}$$

If (F, A) is a union-soft hyper GR-algebra over U, then so is $(F, A)^*$.

Proof: Since (F, A) is a union-soft hyper GR-algebra over U, it follows from Theorem 16 that $e((F, A); \tau)$ is a hyper subGR-algebra of A for all $\tau \subseteq U$. Let $x, y \in A$. If $x, y \in e((F, A); \tau)$, then $x \circledast y \subseteq e((F, A); \tau)$ and so $f_A^*(x \circledast y) = f_A(x \circledast y) \subseteq f_A(x) \cup f_A(y) = f_A^*(x) \cup f_A^*(y)$. $x \notin e((F, A); \tau)$ or $y \notin e((F, A); \tau)$, then $f_A^*(x) = U$ or $f_A^*(y) = U$. Hence, $f_A(x \circledast y) \subset U = f_A^*(x) \cup f_A^*(y)$. Therefore, $(F, A)^*$ is a union-soft hyper GR-algebra over U. □

Definition 19. Let E be a hyper GR-algebra. Given a hyper subGR-algebra A of E, let $(F, A) \in S(U)$. Then (F, A) is called a *union-soft hyper GR-ideal* over U if $f_A(x)$ satisfies the following:

(i) $f_A(0) \subseteq f_A(x), \forall x \in A$

(ii) $f_A(x) \subseteq f_A(x \circledast y) \cup f_A(y), \forall x, y \in A$.

Example 15. Consider the hyper GR-algebra $H = \{0, 1, 2, 3\}$ defined in Example 4. Let $\tau_1, \tau_2, \tau_3, \tau_4$ be subsets of H such that $\tau_1 \subseteq \tau_2 \subseteq \tau_3 \subseteq \tau_4$. Define a soft set (F, A) as follows:

$$(F, A) = \{(0, \tau_1), (1, \tau_2), (3, \tau_3), (4, \tau_4)\}.$$

By routine calculations, (F, A) is a union-soft hyper GR-ideal.

Example 16. Consider the $H = \{0, 1, 2\}$ defined in the Cayley table below.

$\circledast$	0	1	2
0	{0}	{0}	{0}
1	{0,1}	{0,1}	{0,1}
2	{0,2}	{0,1}	{0,1,2}.

By routine calculations, $(H; \circledast, 0)$ is hyper GR-algebra. Let τ_1, τ_2, τ_3 be subsets of H such that $\tau_1 \subsetneq \tau_2 \subsetneq \tau_3$. Define a soft set (F, A) as follows:

$$(F, A) = \{(0, \tau_1), (1, \tau_2), (2, \tau_3)\}.$$

By routine calculations, (F, A) is not a union-soft hyper GR-ideal since $F(2) = \tau_3 \nsubseteq F(2 \circledast 1) \cup F(1) = \tau_2$.

Theorem 18. Let E be a hyper GR-algebra. Given a hyper subGR-algebra A of E, suppose $(F, A) \subseteq S(U)$. If (F, A) is a union-soft hyper GR-ideal over U, then for all $x, y \in A$, $f_A(x) \subseteq [f_A(x \circledast y) \cap f_A(x)] \cup [f_A(x \circledast y) \cap f_A(y)] \cup [f_A(x) \cap f_A(y)] \cup f_A(y)$.

Proof: Note that $f_A(x) \subseteq f_A(x) \cup f_A(y)$ and $f_A(x) \subseteq f_A(x \circledast y) \cup f_A(y)$. This implies that $f_A(x) = [f_A(x \circledast y) \cup f_A(y)] \cap [f_A(x) \cup f_A(y)]$. Hence, $f_A(x) \subseteq [f_A(x \circledast y) \cap f_A(x)] \cup [f_A(x \circledast y) \cap f_A(y)] \cup [f_A(x) \cap f_A(y)] \cup f_A(y)$. $\square$

Theorem 19. Let E be a hyper GR-algebra. Given a hyper subGR-algebra A of E, suppose $(F, A) \subseteq S(U)$. If (F, A) is a union-soft hyper GR-ideal over U, then the nonempty τ-exclusive set of (F, A) is an ideal of A for all $\tau \subseteq U$.

Proof: Let (F, A) be a union-soft hyper GR-ideal over U. Let $\tau \subset U$ such that $e((F, A); \tau) \neq \varnothing$. Then for some $x \in A$, $f_A(x) \subseteq \tau$. It follows from Definition 19 (i) that $f_A(0) \subseteq f_A(x) \subseteq \tau$. Thus, $0 \in e((F, A); \tau)$. Now let $x, y \in A$ such that $x \circledast y \subseteq e((F, A); \tau)$ and $y \subseteq e((F, A); \tau)$. Hence, $f_A(x \circledast y) \subseteq \tau$ and $f_A(y) \subseteq \tau$. By Definition 19(ii) $f_A(x) \subseteq f_A(x \circledast y) \cup f_A(y) \subseteq \tau$. Hence, $x \in e((F, A); \tau)$ and so $e((F, A); \tau)$ is an ideal of A. $\square$

4. Conclusion

In this paper, the notion of soft hyper GR-algebra is presented. Some of its properties and characterization are also presented such as the soft hyper subGR-algebra, the soft hyper GR-ideal, the soft hyper GR-commutative ideal and the union-soft hyper GR-ideal.

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