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Graphical Invariants for some Transformed Networks

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Abstract. A topological index is a numerical character associated with a graph that is invariant under graph isomorphism and describe the graphs topology. There are several graph operations that may be used to change it into a new structure, such as constructing a stellation, bounded dual, complement, subdivided, line graph, minor, dual, and medial. In this paper, we construct transformed networks from the concealled non-kekulean benzenoid hydrocarbon structure by applying stellation and bounded dual operations, further we study their degree based topological properties by appropriately labeling the graph. Degree based topological indices are playing significant role among other types of indices in chemical, pharmaceutical and bio-informatics industry, since they corelate the structure with its physicochemical properties.

2020 Mathematics Subject Classifications: 05C08, 05C92, 37F20

Key Words and Phrases: Topological Indices; Concealled Non-Kekulean Benezenoid Hydrocarbon; Sum Connectivity Index; General Sum Connectivity Index; Atomic Bond Connectivity Index; Geometric Arithmetic Index

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1. Introduction

The subject of mathematics known as graph theory deals with network of points connected by lines. Graph theory began as a fun way to solve math problems, but it has now evolved into a major field of mathematics with applications in chemistry, operations

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research, social sciences, and computer science. An important use of (connected and undirected) graphs is the representation of an atomic structure by a graph where the vertices represent atoms and the edges indicate bonding, which is explored in the field of chemical graph theory. These indices are playing vital role to study various networks since they help researchers in QSPR (Quantitative Structure Property Relationship) and QSAR (Quantitative Structure Activity Relationship) study.

Alikhani et al. calculated the atom-bond connectivity index of some families of dendrimers [3]. Babujee et al. worked on topological indices and new graph structures [4]. Farahani worked on a new version of Zagreb index of circumcoronene series of benzenoid and calculated some connectivity indices of different classes of graphs [6], [7], [8], [9]. Hayat et al. calculated some degree-based topological indices of certain nanotubes and networks [11], [14]. Few networks are discussed in [16], [13], [12], [42], [29], [15], [23]. Ma et al. studied the energy and operations of graphs [28]. Randic also calculated the benzenoid rings resonance energies and local aromaticity of benzenoid hydrocarbons [31]. Saleem worked on retractions and homomorphisms on some operations of graphs [33]. Siddiqui et al. worked on Zagreb indices and Zagreb polynomials of some nanostar dendrimers [34]. Yu et al. defined indices through M-polynomial [37].

There are thousands of topological indices developed over the decades in the field of chemical graph theory. Since different indices deal with different structure properties, others give better estimation. The results of our study are novel, motivated by the application of indices in the advanced material technology, to form new materials and study its various properties. If d_θ and d_ψ are the degrees of the vertices θ and ψ, respectively in K and $\theta\psi \in E(K)$ then following are the formulae of different degree based topological indices which are computed in this paper. The General Randic Index was defined as [30]

$$R_\alpha(K) = \sum_{\theta\psi \in E(K)} (d_\theta d_\psi)^\alpha \tag{1}$$

The Sum Connectivity Index was defined as [40]

$$\chi(K) = \sum_{\theta\psi \in E(K)} (d_\theta + d_\psi)^{-1/2} \tag{2}$$

The General Sum Connectivity Index was defined by Zhou [41]

$$\chi_\alpha(K) = \sum_{\theta\psi \in E(K)} (d_\theta + d_\psi)^\alpha \tag{3}$$

Ranjini in 2013, stated the Redefined First, Second and Third Zagreb Indices [32]

$$ReZG_1(K) = \sum_{\theta\psi \in E(K)} \left(\frac{d_\theta + d_\psi}{d_\theta d_\psi}\right) \tag{4}$$

$$ReZG_2(K) = \sum_{\theta\psi \in E(K)} \left(\frac{d_\theta d_\psi}{d_\theta + d_\psi}\right) \tag{5}$$

$$ReZG_3(K) = \sum_{\theta\psi \in E(K)} (d_\theta d_\psi)(d_\theta + d_\psi) \tag{6}$$

Whereas in 2010 the Atomic Bond Connectivity Index was defined as [5]

$$ABC(K) = \sum_{\theta\psi \in E(K)} \sqrt{\frac{d_\theta + d_\psi - 2}{d_\theta d_\psi}} \tag{7}$$

Furtula stated Geometric Arithmetic Index as [10]

$$GA(K) = \sum_{\theta\psi \in E(K)} \frac{2\sqrt{d_\theta d_\psi}}{d_\theta + d_\psi} \tag{8}$$

In 2015, the General Version of Harmonic Index was defined [36]

$$H_k(K) = \sum_{\theta\psi \in E(K)} \left(\frac{2}{d_\theta + d_\psi}\right)^k \tag{9}$$

2. Results and Discussion

To understand the concept of stellation and bounded dual, see the following Figures. The Figure 1, is of single benzene ring. Similarly, Figure 2, shows stellation on benzene ring and Figure 3, shows stellation plus bounded dual on the benzene rings of concealled non-kekulean benezenoid hydrocarbon, respectively.

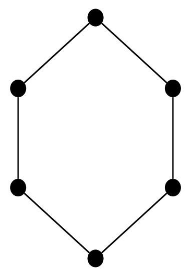

Figure 1: Single Ring of Concealled Non-Kekulean Benzenoid Hydrocarbon.

Let G_1 be the simple and undirected molecular graph when stellation operation is applied on concealled non-kekulean benzenoid hydrocarbon and Figure 4 shows the stellation network for $n = 6$. Six different types of edges of graph, G_1 for $n \geq 4$ and their count are given in Table 1.
Following are some results of topological indices for G_1.

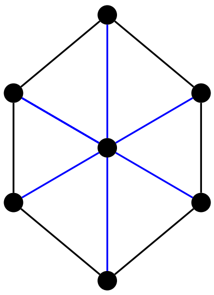

Figure 2: Stellation is Blue. Single Ring of Concealled Non-Kekulean Benzenoid Hydrocarbon.

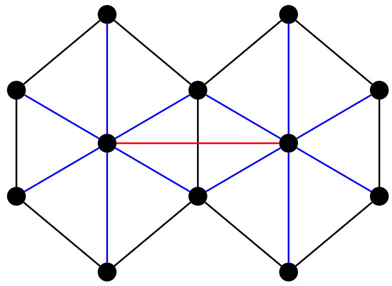

Figure 3: Stellation is Blue. Bounded Dual is Red. Both Operations are on Couple of Benzene Ring.

Table 1: Types and number of edges

Types of Edges	Number of Edges
(3 , 3)	8
(3 , 5)	$12+4n$
(3 , 6)	$14+2n$
(5 , 6)	$24+6n$
(6 , 6)	$35n$-40
(5 , 5)	2

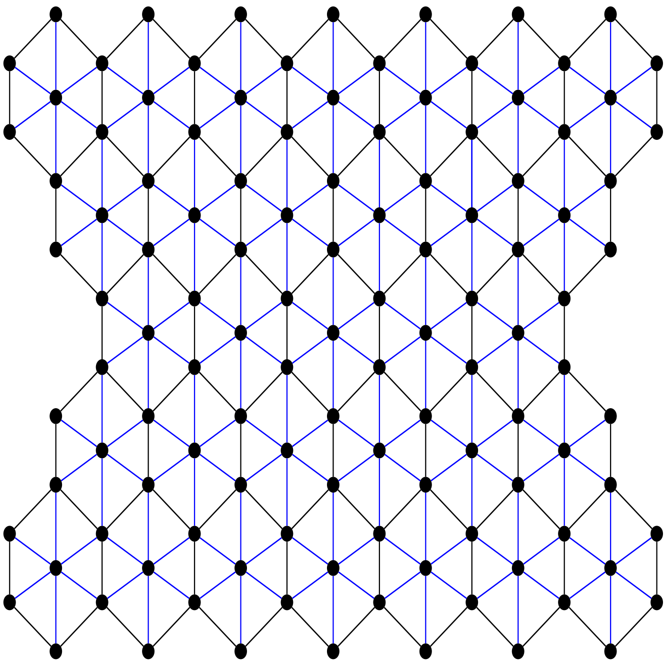

Figure 4: Stellation Operation on Concealled Non-Kekulean Benzenoid Hydrocarbon for $n = 6$.

Theorem 2.1

For G_1, the General Randic Index, Sum Connectivity Index, General Sum Connectivity Index are as follows, respectively.

$$\begin{aligned}
i) \quad R_\alpha(G_1) &= 2\big[(4)3^{2\alpha} + (2)3^{1+\alpha}5^\alpha + (7)2^\alpha 3^{2\alpha} + 2^{2+\alpha}3^{1+\alpha}5^\alpha - (5)2^{2(1+\alpha)}3^{2\alpha} \\
&\quad +5^{2\alpha}\big] + n\big[(4)3^\alpha 5^\alpha + 2^{1+\alpha}3^{2\alpha} + 2^{1+\alpha}3^{1+\alpha}5^\alpha + (35)2^{2\alpha}3^{2\alpha}\big] \\
ii) \quad \chi(G_1) &= \frac{4\sqrt{6}+9\sqrt{2}+14-20\sqrt{3}}{3} + \frac{24}{\sqrt{11}} + \sqrt{\frac{2}{5}} + n\Bigg(\sqrt{2} + \frac{2}{3} + \frac{6}{\sqrt{11}} \\
&\quad + \frac{35}{2\sqrt{3}}\Bigg) \\
iii) \quad \chi_\alpha(G_1) &= 2\big[2^{2+\alpha}3^\alpha + (3)2^{1+3\alpha} + (7)3^{2\alpha} + (12)11^\alpha - (5)2^{2(1+\alpha)}3^\alpha + 2^\alpha 5^\alpha\big] \\
&\quad +n\big[2^{2+3\alpha} + (2)3^{2\alpha} + (6)11^\alpha + (35)2^{2\alpha}3^\alpha\big]
\end{aligned}$$

Proof

i) According to Equation (1)

$$R_\alpha(G_1) = \sum_{\theta\psi \in E(G_1)} (d_\theta d_\psi)^\alpha$$

By using the information given in Table 1.

$$\begin{aligned}
R_\alpha(G_1) &= (8)(3\times 3)^\alpha + (12+4n)(3\times 5)^\alpha + (14+2n)(3\times 6)^\alpha + (24+6n)(5\times 6)^\alpha \\
&\quad +(35n-40)(6\times 6)^\alpha + (2)(5\times 5)^\alpha \\
&= 2\big[(4)3^{2\alpha} + (2)3^{1+\alpha}5^\alpha + (7)2^\alpha 3^{2\alpha} + 2^{2+\alpha}3^{1+\alpha}5^\alpha - (5)2^{2(1+\alpha)}3^{2\alpha} + 5^{2\alpha}\big] \\
&\quad +n\big[(4)3^\alpha 5^\alpha + 2^{1+\alpha}3^{2\alpha} + 2^{1+\alpha}3^{1+\alpha}5^\alpha + (35)2^{2\alpha}3^{2\alpha}\big]
\end{aligned}$$

ii) According to Equation (2)

$$\chi(G_1) = \sum_{\theta\psi \in E(G_1)} (d_\theta + d_\psi)^{-1/2}$$

By using the information given in Table 1.

$$\begin{aligned}
\chi(G_1) &= (8)(3+3)^{-1/2} + (12+4n)(3+5)^{-1/2} + (14+2n)(3+6)^{-1/2} + (24+6n)(5 \\
&\quad +6)^{-1/2} + (35n-40)(6+6)^{-1/2} + (2)(5+5)^{-1/2} \\
&= \frac{4\sqrt{6}+9\sqrt{2}+14-20\sqrt{3}}{3} + \frac{24}{\sqrt{11}} + \sqrt{\frac{2}{5}} + n\left(\sqrt{2} + \frac{2}{3} + \frac{6}{\sqrt{11}} + \frac{35}{2\sqrt{3}}\right)
\end{aligned}$$

iii) According to Equation (3)

$$\chi_\alpha(G_1) = \sum_{\theta\psi \in E(G_1)} (d_\theta + d_\psi)^\alpha$$

By using the information given in Table 1.

$$\begin{aligned} \chi_\alpha(G_1) &= (8)(3+3)^\alpha + (12+4n)(3+5)^\alpha + (14+2n)(3+6)^\alpha + (24+6n)(5+6)^\alpha \\ &\quad +(35n-40)(6+6)^\alpha + (2)(5+5)^\alpha \\ &= 2\big[2^{2+\alpha}3^\alpha + (3)2^{1+3\alpha} + (7)3^{2\alpha} + (12)11^\alpha - (5)2^{2(1+\alpha)}3^\alpha + 2^\alpha 5^\alpha\big] + n\big[2^{2+3\alpha} \\ &\quad +(2)3^{2\alpha} + (6)11^\alpha + (35)2^{2\alpha}3^\alpha\big] \end{aligned}$$

Theorem 2.2

For G_1, the 1st Zagreb Index, 2nd Zagreb Index and 3rd Zagreb Index are as follows, respectively.

$$\begin{aligned} &i) & ReZG_1(G_1) &= 15 + 17n \\ &ii) & ReZG_2(G_1) &= \frac{285 + 2923n}{22} \\ &iii) & ReZG_3(G_1) &= 17904n - 4720 \end{aligned}$$

Proof

i) According to Equation (4)

$$ReZG_1(G_1) = \sum_{\theta\psi \in E(G_1)} \left(\frac{d_\theta + d_\psi}{d_\theta d_\psi}\right)$$

By using the information given in Table 1.

$$\begin{aligned} ReZG_1(G_1) &= (8)\left(\frac{3+3}{3\times 3}\right) + (12+4n)\left(\frac{3+5}{3\times 5}\right) + (14+2n)\left(\frac{3+6}{3\times 6}\right) \\ &\quad +(24+6n)\left(\frac{5+6}{5\times 6}\right) + (35n-40)\left(\frac{6+6}{6\times 6}\right) + (2)\left(\frac{5+5}{5\times 5}\right) \\ &= 15 + 17n \end{aligned}$$

ii) According to Equation (5)

$$ReZG_2(G_1) = \sum_{\theta\psi \in E(G_1)} \left(\frac{d_\theta d_\psi}{d_\theta + d_\psi}\right)$$

By using the information given in Table 1.

$$\begin{aligned}
ReZG_{2(G_1)} &= (8)\left(\frac{3\times 3}{3+3}\right)+(12+4n)\left(\frac{3\times 5}{3+5}\right)+(14+2n)\left(\frac{3\times 6}{3+6}\right)+(24+6n)\left(\frac{5\times 6}{5+6}\right)\\
&\quad +(35n-40)\left(\frac{6\times 6}{6+6}\right)+(2)\left(\frac{5\times 5}{5+5}\right)\\
&= \frac{285+2923n}{22}
\end{aligned}$$

iii) According to Equation (6)

$$ReZG_3(G_1) = \sum_{\theta\psi\in E(G_1)} (d_\theta d_\psi)(d_\theta + d_\psi)$$

By using the information given in Table 1.

$$\begin{aligned}
ReZG_3(G_1) &= (8)(3\times 3)(3+3)+(12+4n)(3\times 5)(3+5)+(14+2n)(3\times 6)(3+6)+(24\\
&\quad +6n)(5\times 6)(5+6)+(35n-40)(6\times 6)(6+6)+(2)(5\times 5)(5+5)\\
&= 17904n-4720
\end{aligned}$$

Theorem 2.3

For G_1, the Atomic Bond Connectivity, Geometric Arithmetic Index and General Version of Harmonic Index are as follows, respectively.

$$\begin{aligned}
i)\quad ABC(G_1) &= \frac{1}{\sqrt{2}}\left[2\left(\frac{40\sqrt{2}+35\sqrt{7}-64\sqrt{5}+36\sqrt{15}+12}{15}\right)\right.\\
&\quad \left.+n\left(\frac{199+18\sqrt{3}+2\sqrt{35}}{3\sqrt{5}}\right)\right]\\
ii)\quad GA(G_1) &= \frac{99\big(\sqrt{15}-10\big)+4\sqrt{2}\big(77+36\sqrt{15}\big)}{33}\\
&\quad +n\left[\frac{11(105+4\sqrt{2})+3\sqrt{15}(11+12\sqrt{2})}{33}\right]\\
iii)\quad H_k(G_1) &= \frac{2^3}{3^k}\left(\frac{2^k-5}{2^k}\right)+(3)2^{2(1-k)}+\frac{(7)2^{1+k}}{3^{2k}}+\frac{(3)2^{3+k}}{11^k}+\frac{2}{5^k}\\
&\quad +n\left(2^{2(1-k)}+\frac{2^{1+k}}{3^{2k}}+\frac{(3)2^{1+k}}{11^k}+\frac{35}{2^k3^k}\right)
\end{aligned}$$

Proof

i) According to Equation (7)

$$ABC(G_1) = \sum_{\theta\psi \in E(G_1)} \sqrt{\frac{d_\theta + d_\psi - 2}{d_\theta d_\psi}}$$

By using the information given in Table 1.

$$\begin{aligned} ABC(G_1) &= (8)\sqrt{\frac{3+3-2}{3\times 3}} + (12+4n)\sqrt{\frac{3+5-2}{3\times 5}} + (14+2n)\sqrt{\frac{3+6-2}{3\times 6}} \\ &\quad +(24+6n)\sqrt{\frac{5+6-2}{5\times 6}} + (35n-40)\sqrt{\frac{6+6-2}{6\times 6}} + (2)\sqrt{\frac{5+5-2}{5\times 5}} \\ &= \frac{1}{\sqrt{2}}\left[2\left(\frac{40\sqrt{2}+35\sqrt{7}-64\sqrt{5}+36\sqrt{15}+12}{15}\right) + n\left(\frac{199+18\sqrt{3}+2\sqrt{35}}{3\sqrt{5}}\right)\right] \end{aligned}$$

ii) According to Equation (8)

$$GA(G_1) = \sum_{\theta\psi \in E(G_1)} \frac{2\sqrt{d_\theta d_\psi}}{d_\theta + d_\psi}$$

By using the information given in Table 1.

$$\begin{aligned} GA(G_1) &= (8)\frac{2\sqrt{3\times 3}}{3+3} + (12+4n)\frac{2\sqrt{3\times 5}}{3+5} + (14+2n)\frac{2\sqrt{3\times 6}}{3+6} + (24+6n)\frac{2\sqrt{5\times 6}}{5+6} \\ &\quad +(35n-40)\frac{2\sqrt{6\times 6}}{6+6} + (2)\frac{2\sqrt{5\times 5}}{5+5} \\ &= \frac{99(\sqrt{15}-10)+4\sqrt{2}(77+36\sqrt{15})}{33} + n\left[\frac{11(105+4\sqrt{2})+3\sqrt{15}(11+12\sqrt{2})}{33}\right] \end{aligned}$$

iii) According to Equation (9)

$$H_k(G_1) = \sum_{\theta\psi \in E(G_1)} \left(\frac{2}{d_\theta + d_\psi}\right)^k$$

By using the information given in Table 1.

$$H_k(G_1) = (8)\left(\frac{2}{3+3}\right)^k + (12+4n)\left(\frac{2}{3+5}\right)^k + (14+2n)\left(\frac{2}{3+6}\right)^k$$

$$+(24+6n)\left(\frac{2}{5+6}\right)^k + (35n-40)\left(\frac{2}{6+6}\right)^k + (2)\left(\frac{2}{5+5}\right)^k$$
$$= \frac{2^3}{3^k}\left(\frac{2^k-5}{2^k}\right) + (3)2^{2(1-k)} + \frac{(7)2^{1+k}}{3^{2k}} + \frac{(3)2^{3+k}}{11^k} + \frac{2}{5^k}$$
$$+n\left(2^{2(1-k)} + \frac{2^{1+k}}{3^{2k}} + \frac{(3)2^{1+k}}{11^k} + \frac{35}{2^k3^k}\right)$$

Now, we are going to calculate topological indices when both the bounded dual and stellation operations are applied on the graph of concealled non-kekulean benzenoid hydrocarbon, say G_2. Let G_2 be the simple and undirected transformed network and following are twenty types of edges for G_2, $n \geq 4$. These types of edges and their count are given in Table 2 and Figure 5 shows the stellation and bounded dual on single structure for $n = 7$.

Table 2: Types and number of edges

Types of Edges	Number of Edges
(3 , 3)	8
(3 , 5)	$4n$+12
(5 , 6)	8+2n
(8 , 3)	12
(6 , 8)	4
(8 , 5)	8
(10 , 10)	2n
(3 , 10)	2+2n
(10 , 6)	6+6n
(10 , 8)	8
(5 , 10)	4+4n
(6 , 6)	11n-16
(5 , 5)	2
(6 , 12)	18n-42
(10 , 12)	4n-4
(5 , 11)	4
(6 , 11)	8
(12 , 11)	6
(10 , 11)	4
(12 , 12)	7n-22

Theorem 2.4

For G_2, the General Randic Index, Sum Connectivity Index, General Sum Connectivity Index are as follows, respectively.

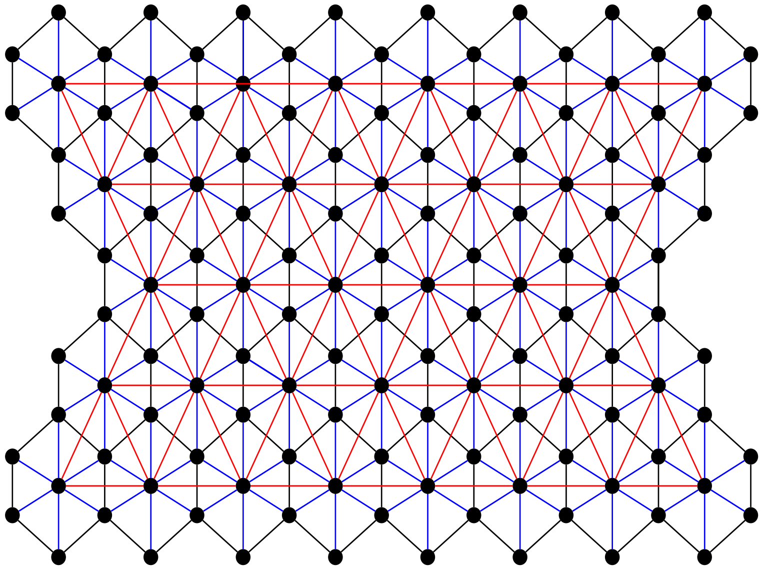

Figure 5: Stellation and Bounded Dual on the Same Structure of Concealled Non-Kekulean Benzenoid Hydrocarbon for $n = 7$.

$$\begin{aligned}
i) \quad R_\alpha(G_2) &= 3^{2\alpha}8 + 3^{1+\alpha}5^\alpha 4 + 2^{3+\alpha}3^\alpha 5^\alpha + 2^{2+3\alpha}3^{1+\alpha} + 2^{2(1+2\alpha)}3^\alpha \\
&+ 2^{3(1+\alpha)}5^\alpha + 2^{1+\alpha}3^\alpha 5^\alpha + 2^{1+2\alpha}3^{1+\alpha}5^\alpha + 2^{3+4\alpha}5^\alpha + 2^{2+\alpha}5^{2\alpha} \\
&- 2^{2(2+\alpha)}3^{2\alpha} + 5^{2\alpha}2 - 2^{1+3\alpha}3^{1+2\alpha}7 - 2^{2+3\alpha}3^\alpha 5^\alpha + 5^\alpha 11^\alpha 4 \\
&+ 2^{3+\alpha}3^\alpha 11^\alpha + 2^{1+2\alpha}3^{1+\alpha}11^\alpha + 2^{2+\alpha}5^\alpha 11^\alpha - 2^{1+4\alpha}3^{2\alpha}11 \\
&+ n[3^\alpha 5^\alpha 4 + 2^{1+\alpha}5^\alpha 3^\alpha + 2^{1+2\alpha}5^{2\alpha} + 2^{1+\alpha}3^\alpha 5^\alpha + 2^{1+2\alpha}3^{1+\alpha}5^\alpha \\
&+ 2^{2+\alpha}5^{2\alpha} + 2^{2\alpha}3^{2\alpha}11 + 2^{1+3\alpha}3^{2(1+\alpha)} + 2^{2+3\alpha}3^\alpha 5^\alpha + 3^{2\alpha}2^{4\alpha}7] \\
ii) \quad \chi(G_2) &= \frac{4}{\sqrt{3}}\left[\sqrt{2} + \sqrt{\frac{2}{3}} + \sqrt{\frac{1}{5}} - 2 + \sqrt{\frac{1}{7}} - \frac{11}{4\sqrt{2}}\right] + \frac{1}{\sqrt{11}}\left[20 + \sqrt{2}\right] \\
&+ \frac{10}{\sqrt{13}} + 2\sqrt{\frac{2}{7}} + \sqrt{\frac{2}{5}} + \frac{8}{\sqrt{17}} - 4\sqrt{2} + \frac{5}{2} + n\left[\frac{2}{\sqrt{11}}\left(1 + \sqrt{2}\right)\right. \\
&\left. + \frac{1}{\sqrt{5}}\left(\frac{\sqrt{3}+4}{\sqrt{3}}\right) + \frac{1}{2\sqrt{3}}\left(\frac{11\sqrt{2}+7}{\sqrt{2}}\right) + 4\sqrt{2} + \frac{2}{\sqrt{13}} + \frac{3}{2}\right] \\
iii) \quad \chi_\alpha(G_2) &= 2^{3+\alpha}3^\alpha + 2^{2+3\alpha}3 + 11^\alpha 8 + 11^\alpha 12 + 2^{2+\alpha}7^\alpha + 13^\alpha 8 + 13^\alpha 2 \\
&+ 2^{1+4\alpha}3 + 2^{3+\alpha}3^{2\alpha} + 5^\alpha 3^\alpha 4 - 2^{2(2+\alpha)}3^\alpha + 2^{1+\alpha}5^\alpha - 3^{1+2\alpha}2^{1+\alpha}7 \\
&- 2^{2+\alpha}11^\alpha + 2^{2(1+2\alpha)} + 17^\alpha 8 + 23^\alpha 6 + 7^\alpha 3^\alpha 4 - 2^{1+3\alpha}3^\alpha 11 \\
&+ n\big[2^{2+3\alpha} + 11^\alpha 2 + 2^{1+2\alpha}5^\alpha + 13^\alpha 2 + 2^{1+4\alpha}3 + 5^\alpha 3^\alpha 4 \\
&+ 2^{2\alpha}3^\alpha 11 + 3^{2(1+\alpha)}2^{1+\alpha} + 2^{2+\alpha}11^\alpha + 2^{3\alpha}3^\alpha 7\big].
\end{aligned}$$

Proof

i) According to Equation (1)

$$R_\alpha(G_2) = \sum_{\theta\psi\in E(G_2)} (d_\theta d_\psi)^\alpha$$

By using types of edges given in Table 2, we get.

$$\begin{aligned}
R_\alpha(G_2) &= (8)(3\times 3)^\alpha + (12+4n)(3\times 5)^\alpha + (8+2n)(5\times 6)^\alpha + (12)(8\times 3)^\alpha + (4)(6\times 8)^\alpha \\
&\quad +(8)(8\times 5)^\alpha + (2n)(10\times 10)^\alpha + (2+2n)(3\times 10)^\alpha + (6+6n)(10\times 6)^\alpha \\
&\quad +(8)(10\times 8)^\alpha + (4+4n)(5\times 10)^\alpha + (11n-16)(6\times 6)^\alpha + (2)(5\times 5)^\alpha \\
&\quad +(18n-42)(6\times 12)^\alpha + (4n-4)(10\times 12)^\alpha + (4)(5\times 11)^\alpha + (8)(6\times 11)^\alpha \\
&\quad +(6)(12\times 11)^\alpha + (4)(10\times 11)^\alpha + (7n-22)(12\times 12)^\alpha \\
&= 3^{2\alpha}8 + 3^{1+\alpha}5^\alpha 4 + 2^{3+\alpha}3^\alpha 5^\alpha + 2^{2+3\alpha}3^{1+\alpha} + 2^{2(1+2\alpha)}3^\alpha + 2^{3(1+\alpha)}5^\alpha + 2^{1+\alpha}3^\alpha 5^\alpha \\
&\quad +2^{1+2\alpha}3^{1+\alpha}5^\alpha + 2^{3+4\alpha}5^\alpha + 2^{2+\alpha}5^{2\alpha} - 2^{2(2+\alpha)}3^{2\alpha} + 5^{2\alpha}2 - 2^{1+3\alpha}3^{1+2\alpha}7 \\
&\quad -2^{2+3\alpha}3^\alpha 5^\alpha + 5^\alpha 11^\alpha 4 + 2^{3+\alpha}3^\alpha 11^\alpha + 2^{1+2\alpha}3^{1+\alpha}11^\alpha + 2^{2+\alpha}5^\alpha 11^\alpha \\
&\quad -2^{1+4\alpha}3^{2\alpha}11 + n[3^\alpha 5^\alpha 4 + 2^{1+\alpha}5^\alpha 3^\alpha + 2^{1+2\alpha}5^{2\alpha} + 2^{1+\alpha}3^\alpha 5^\alpha + 2^{1+2\alpha}3^{1+\alpha}5^\alpha \\
&\quad +2^{2+\alpha}5^{2\alpha} + 2^{2\alpha}3^{2\alpha}11 + 2^{1+3\alpha}3^{2(1+\alpha)} + 2^{2+3\alpha}3^\alpha 5^\alpha + 3^{2\alpha}2^{4\alpha}7].
\end{aligned}$$

ii) According to Equation (2)

$$\chi(G_2) = \sum_{\theta\psi\in E(G_2)} (d_\theta + d_\psi)^{-1/2}$$

By using types of edges given in Table 2, we get.

$$\begin{aligned}
\chi(G_2) &= (8)(3+3)^{-1/2} + (4n+12)(3+5)^{-1/2} + (8+2n)(5+6)^{-1/2} + (12)(8+3)^{-1/2} \\
&\quad +(4)(6+8)^{-1/2} + (8)(8+5)^{-1/2} + (2n)(10+10)^{-1/2} + (2+2n)(3+10)^{-1/2} \\
&\quad +(6+6n)(10+6)^{-1/2} + (8)(10+8)^{-1/2} + (4+4n)(5+10)^{-1/2} + (11n-16) \\
&\quad \times(6+6)^{-1/2} + (2)(5+5)^{-1/2} + (18n-42)(6+12)^{-1/2} + (4n-4)(10+12)^{-1/2} \\
&\quad +(4)(5+11)^{-1/2} + (8)(6+11)^{-1/2} + (6)(12+11)^{-1/2} + (4)(10+11)^{-1/2} \\
&\quad +(7n-22)(12+12)^{-1/2} \\
&= \frac{4}{\sqrt{3}}\left[\sqrt{2} + \sqrt{\frac{2}{3}} + \sqrt{\frac{1}{5}} - 2 + \sqrt{\frac{1}{7}} - \frac{11}{4\sqrt{2}}\right] + \frac{1}{\sqrt{11}}\left[20+\sqrt{2}\right] + \frac{10}{\sqrt{13}} + 2\sqrt{\frac{2}{7}} \\
&\quad +\sqrt{\frac{2}{5}} + \frac{8}{\sqrt{17}} - 4\sqrt{2} + \frac{5}{2} + n\left[\frac{2}{\sqrt{11}}\left(1+\sqrt{2}\right) + \frac{1}{\sqrt{5}}\left(1+\frac{4}{\sqrt{3}}\right) + \frac{1}{2\sqrt{3}}\left(11\right.\right. \\
&\quad \left.\left.+\frac{7}{\sqrt{2}}\right) + 4\sqrt{2} + \frac{2}{\sqrt{13}} + \frac{3}{2}\right].
\end{aligned}$$

iii) According to Equation (3)

$$\chi_\alpha(G_2) = \sum_{\theta\psi\in E(G_2)} (d_\theta + d_\psi)^\alpha$$

By using types of edges given in Table 2, we get.

$$\begin{aligned}
\chi_\alpha(G_2) &= (8)(3+3)^\alpha + (4n+12)(3+5)^\alpha + (8+2n)(5+6)^\alpha + (12)(8+3)^\alpha + (4)(6 \\
&\quad +8)^\alpha + (8)(8+5)^\alpha + (2n)(10+10)^\alpha + (2+2n)(3+10)^\alpha + (6+6n)(10+6)^\alpha \\
&\quad +(8)(10+8)^\alpha + (4+4n)(5+10)^\alpha + (11n-16)(6+6)^\alpha + (2)(5+5)^\alpha + (18n \\
&\quad -42)(6+12)^\alpha + (4n-4)(10+12)^\alpha + (4)(5+11)^\alpha + (8)(6+11)^\alpha + (6)(12 \\
&\quad +11)^\alpha + (4)(10+11)^\alpha + (7n-22)(12+12)^\alpha \\
&= 2^{3+\alpha}3^\alpha + 2^{2+3\alpha}3 + 11^\alpha 8 + 11^\alpha 12 + 2^{2+\alpha}7^\alpha + 13^\alpha 8 + 13^\alpha 2 + 2^{1+4\alpha}3 + 2^{3+\alpha}3^{2\alpha} \\
&\quad +5^\alpha 3^\alpha 4 - 2^{2(2+\alpha)}3^\alpha + 2^{1+\alpha}5^\alpha - 3^{1+2\alpha}2^{1+\alpha}7 - 2^{2+\alpha}11^\alpha + 2^{2(1+2\alpha)} + 17^\alpha 8 \\
&\quad +23^\alpha 6 + 7^\alpha 3^\alpha 4 - 2^{1+3\alpha}3^\alpha 11 + n\big[2^{2+3\alpha} + 11^\alpha 2 + 2^{1+2\alpha}5^\alpha + 13^\alpha 2 + 2^{1+4\alpha}3 \\
&\quad +5^\alpha 3^\alpha 4 + 2^{2\alpha}3^\alpha 11 + 3^{2(1+\alpha)}2^{1+\alpha} + 2^{2+\alpha}11^\alpha + 2^{3\alpha}3^\alpha 7\big].
\end{aligned}$$

Theorem 2.5

For G_2, the 1st Zagreb Index, 2nd Zagreb Index and 3rd Zagreb Index are as follows, respectively.

$$\begin{aligned}
i) \quad ReZG_1(G_2) &= 15 + 17n \\
ii) \quad ReZG_2(G_2) &= 232.22n - 67.79 \\
iii) \quad ReZG_3(G_2) &= 77512n - 71896.
\end{aligned}$$

Proof

i) According to Equation (4)

$$ReZG_1(G_2) = \sum_{\theta\psi\in E(G_2)} \left(\frac{d_\theta + d_\psi}{d_\theta d_\psi}\right)$$

By using types of edges given in Table 2, we get.

$$\begin{aligned}
ReZG_1(G_2) &= (8)\left(\frac{3+3}{3\times 3}\right) + (4n+12)\left(\frac{3+5}{3\times 5}\right) + (8+2n)\left(\frac{5+6}{5\times 6}\right) + (12)\left(\frac{8+3}{8\times 3}\right) \\
&\quad +(4)\left(\frac{6+8}{6\times 8}\right) + (8)\left(\frac{8+5}{8\times 5}\right) + (2n)\left(\frac{10+10}{10\times 10}\right) + (2+2n)\left(\frac{3+10}{3\times 10}\right)
\end{aligned}$$

$$
\begin{aligned}
&+(6+6n)\left(\frac{10+6}{10\times 6}\right)+(8)\left(\frac{10+8}{10\times 8}\right)+(4+4n)\left(\frac{5+10}{5\times 10}\right)+(11n-16)\left(\frac{6+6}{6\times 6}\right)\\
&+(2)\left(\frac{5+5}{5\times 5}\right)+(18n-42)\left(\frac{6+12}{6\times 12}\right)+(4n-4)\left(\frac{10+12}{10\times 12}\right)+(4)\left(\frac{5+11}{5\times 11}\right)\\
&+(8)\left(\frac{6+11}{6\times 11}\right)+(6)\left(\frac{12+11}{12\times 11}\right)+(4)\left(\frac{10+11}{10\times 11}\right)+(7n-22)\left(\frac{12+12}{12\times 12}\right)\\
= \; & \frac{14850}{990}+n\left(\frac{510}{30}\right)\\
= \; & 15+17n.
\end{aligned}
$$

ii) According to Equation (5)

$$
ReZG_2(G_2) = \sum_{\theta\psi\in E(G_2)}\left(\frac{d_\theta d_\psi}{d_\theta+d_\psi}\right)
$$

By using types of edges given in Table 2, we get.

$$
\begin{aligned}
ReZG_2(G_2) = \; & (8)\left(\frac{3\times 3}{3+3}\right)+(4n+12)\left(\frac{3\times 5}{3+5}\right)+(8+2n)\left(\frac{5\times 6}{5+6}\right)+(12)\left(\frac{8\times 3}{8+3}\right)\\
&+(4)\left(\frac{6\times 8}{6+8}\right)+(8)\left(\frac{8\times 5}{8+5}\right)+(2n)\left(\frac{10\times 10}{10+10}\right)+(2+2n)\left(\frac{3\times 10}{3+10}\right)+(6\\
&+6n)\left(\frac{10\times 6}{10+6}\right)+(8)\left(\frac{10\times 8}{10+8}\right)+(4+4n)\left(\frac{5\times 10}{5+10}\right)+(11n-16)\left(\frac{6\times 6}{6+6}\right)\\
&+(2)\left(\frac{5\times 5}{5+5}\right)+(18n-42)\left(\frac{6\times 12}{6+12}\right)+(4n-4)\left(\frac{10\times 12}{10+12}\right)+(4)\left(\frac{5\times 11}{5+11}\right)\\
&+(8)\left(\frac{6\times 11}{6+11}\right)+(6)\left(\frac{12\times 11}{12+11}\right)+(4)\left(\frac{10\times 11}{10+11}\right)+(7n-22)\left(\frac{12\times 12}{12+12}\right)\\
= \; & -67.79+232.22n\\
= \; & 232.22n-67.79.
\end{aligned}
$$

iii) According to Equation (6)

$$
ReZG_3(G_2) = \sum_{\theta\psi\in E(G_2)}(d_\theta d_\psi)(d_\theta+d_\psi)
$$

By using types of edges given in Table 2, we get.

$$
\begin{aligned}
ReZG_3(G_2) = \; & (8)(3\times 3)(3+3)+(4n+12)(3\times 5)(3+5)+(8+2n)(5\times 6)(5+6)\\
&+(12)(8\times 3)(8+3)+(4)(6\times 8)(6+8)+(8)(8\times 5)(8+5)+(2n)(10
\end{aligned}
$$

$$
\begin{aligned}
& \times 10)(10+10)+(2+2n)(3\times 10)(3+10)+(6+6n)(10\times 6)(10+6) \\
& +(8)(8\times 10)(8+10)+(4+4n)(5\times 10)(5+10)+(11n-16)(6\times 6)(6 \\
& +6)+(2)(5\times 5)(5+5)+(18n-42)(6\times 12)(6+12)+(4n-4)(10 \\
& \times 12)(10+12)+(4)(5\times 11)(5+11)+(8)(6\times 11)(6+11)+(6)(12 \\
& \times 11)(12+11)+(4)(10\times 11)(10+11)+(7n-22)(12\times 12)(12+12) \\
= & -71896+77512n \\
= & \; 77512n-71896.
\end{aligned}
$$

Theorem 2.6

For G_2, the Atomic Bond Connectivity, Geometric Arithmetic Index and General Version of Harmonic Index are as follows, respectively.

$$
\begin{aligned}
i)\quad ABC(G_2) \;=\; & \frac{2(11-4\sqrt{10})}{3}+\frac{8\sqrt{3}(3+\sqrt{2})+2\sqrt{11}(2\sqrt{3}+1)+2(12+7\sqrt{5}+3\sqrt{7})}{\sqrt{30}} \\
& +\frac{2(444+12\sqrt{13}-55\sqrt{11})}{30\sqrt{2}}+\frac{\sqrt{7}(8+3\sqrt{10})+4(\sqrt{19}+10)}{\sqrt{110}} \\
& +n\Bigg[\frac{2(4\sqrt{3}+3+\sqrt{11}+3\sqrt{7})}{\sqrt{30}}+\frac{2(33+2\sqrt{13})}{5\sqrt{2}}+\frac{\sqrt{2}(22\sqrt{5}+7\sqrt{11})}{12} \\
& +\frac{4}{\sqrt{6}}\Bigg].
\end{aligned}
$$

$$
\begin{aligned}
ii)\quad GA(G_2) \;=\; & 3\sqrt{15}-28-28\sqrt{2}+\frac{8\sqrt{6}(2\sqrt{5}+6-\sqrt{5})}{11}+\frac{8(6\sqrt{3}+\sqrt{110})}{21} \\
& +\frac{4\sqrt{10}(8+\sqrt{3})}{13}+\frac{\sqrt{5}(3\sqrt{3}+\sqrt{11})}{2}+\frac{8(4\sqrt{5}+3\sqrt{2})}{9}+\frac{16\sqrt{66}}{17} \\
& +\frac{24\sqrt{33}}{23}+n\left[\sqrt{15}+20+12\sqrt{2}+\frac{12\sqrt{30}}{11}+\frac{4\sqrt{30}}{13}+\frac{9\sqrt{15}+16\sqrt{2}}{6}\right].
\end{aligned}
$$

$$
\begin{aligned}
iii)\quad H_k(G_2) \;=\; & \frac{2^{2+k}5}{11^k}+\frac{2^{1+k}5}{13^k}+\frac{2^{2(1+k)}-5^k16+2^{1+k}3^k}{2^k3^k5^k}+\frac{2^{1+2k}-7^k11}{2^{k-1}3^k7^k} \\
& +\frac{2^{1+k}3+5}{2^{3k-1}}+\frac{2(3^k4-17)}{3^{2k}}+\frac{4}{7^k}-\frac{4}{11^k}+\frac{2^{3+k}}{17^k}+\frac{2^{1+k}3}{23^k} \\
& +n\Bigg[\frac{3^k2+2^{2(1+k)}+5^k11}{2^k3^k5^k}+\frac{2^{1+k}+3}{2^{3k-1}}+\frac{4(2^{k-1}+1)}{11^k} \\
& +\frac{2^{2k+1}3+3^{k-1}7}{3^{2k-1}2^{2k}}+\frac{2^{1+k}}{13^k}\Bigg].
\end{aligned}
$$

Proof

i) According to Equation (7)

$$ABC(G_2) = \sum_{\theta\psi \in E(G_2)} \sqrt{\frac{d_\theta + d_\psi - 2}{d_\theta d_\psi}}$$

By using types of edges given in Table 2, we get.

$$\begin{aligned}
ABC(G_2) &= (8)\sqrt{\frac{3+3-2}{3\times 3}} + (4n+12)\sqrt{\frac{3+5-2}{3\times 5}} + (8+2n)\sqrt{\frac{5+6-2}{5\times 6}} + (12)\sqrt{\frac{8+3-2}{8\times 3}} \\
&+(4)\sqrt{\frac{6+8-2}{6\times 8}} + (8)\sqrt{\frac{8+5-2}{8\times 5}} + (2n)\sqrt{\frac{10+10-2}{10\times 10}} + (2+2n)\sqrt{\frac{3+10-2}{3\times 10}} \\
&+(6+6n)\sqrt{\frac{10+6-2}{10\times 6}} + (8)\sqrt{\frac{10+8-2}{10\times 8}} + (4+4n)\sqrt{\frac{5+10-2}{5\times 10}} \\
&+(11n-16)\sqrt{\frac{6+6-2}{6\times 6}} + (2)\sqrt{\frac{5+5-2}{5\times 5}} + (18n-42)\sqrt{\frac{6+12-2}{6\times 12}} \\
&+(4n-4))\sqrt{\frac{10+12-2}{10\times 12}} + (4)\sqrt{\frac{5+11-2}{5\times 11}} + (8)\sqrt{\frac{6+11-2}{6\times 11}} + (6)\sqrt{\frac{12+11-2}{12\times 11}} \\
&+(4)\sqrt{\frac{10+11-2}{10\times 11}} + (7n-22)\sqrt{\frac{12+12-2}{12\times 12}} \\
&= \frac{2(11-4\sqrt{10})}{3} + \frac{8\sqrt{3}(3+\sqrt{2}) + 2\sqrt{11}(2\sqrt{3}+1) + 2(12+7\sqrt{5}+3\sqrt{7})}{\sqrt{30}} \\
&+\frac{2(444+12\sqrt{13}-55\sqrt{11})}{30\sqrt{2}} + \frac{\sqrt{7}(8+3\sqrt{10}) + 4(\sqrt{19}+10)}{\sqrt{110}} \\
&+n\left[\frac{2(4\sqrt{3}+3+\sqrt{11}+3\sqrt{7})}{\sqrt{30}} + \frac{2(33+2\sqrt{13})}{5\sqrt{2}} + \frac{\sqrt{2}(22\sqrt{5}+7\sqrt{11})}{12} + \frac{4}{\sqrt{6}}\right].
\end{aligned}$$

ii) According to Equation (8)

$$GA(G_2) = \sum_{\theta\psi \in E(G_2)} \frac{2\sqrt{d_\theta d_\psi}}{d_\theta + d_\psi}$$

By using types of edges given in Table 2, we get.

$$\begin{aligned}
GA(G_2) &= (8)\left(\frac{2\sqrt{3\times 3}}{3+3}\right) + (4n+12)\left(\frac{2\sqrt{3\times 5}}{3+5}\right) + (8+2n)\left(\frac{2\sqrt{5\times 6}}{5+6}\right) + (12)\left(\frac{2\sqrt{8\times 3}}{8+3}\right) \\
&+(4)\left(\frac{2\sqrt{8\times 6}}{8+6}\right) + (8)\left(\frac{2\sqrt{8\times 5}}{8+5}\right) + (2n)\left(\frac{2\sqrt{10\times 10}}{10+10}\right) + (2+2n)\left(\frac{2\sqrt{3\times 10}}{3+10}\right) \\
&+(6+6n)\left(\frac{2\sqrt{10\times 6}}{10+6}\right) + (8)\left(\frac{2\sqrt{10\times 8}}{10+8}\right) + (4+4n)\left(\frac{2\sqrt{5\times 10}}{5+10}\right)
\end{aligned}$$

$$
\begin{aligned}
&+(11n-16)\left(\frac{2\sqrt{6\times 6}}{6+6}\right)+(2)\left(\frac{2\sqrt{5\times 5}}{5+5}\right)+(18n-42)\left(\frac{2\sqrt{6\times 12}}{6+12}\right)\\
&+(4n-4)\left(\frac{2\sqrt{10\times 12}}{10+12}\right)+(4)\left(\frac{2\sqrt{5\times 11}}{5+11}\right)+(8)\left(\frac{2\sqrt{6\times 11}}{6+11}\right)+(6)\left(\frac{2\sqrt{12\times 11}}{12+11}\right)\\
&+(4)\left(\frac{2\sqrt{10\times 11}}{10+11}\right)+(7n-22)\left(\frac{2\sqrt{12\times 12}}{12+12}\right)\\
=\;& 3\sqrt{15}-28-28\sqrt{2}+\frac{8\sqrt{6}(2\sqrt{5}+6-\sqrt{5})}{11}+\frac{8(6\sqrt{3}+\sqrt{110})}{21}+\frac{4\sqrt{10}(8+\sqrt{3})}{13}\\
&+\frac{\sqrt{5}(3\sqrt{3}+\sqrt{11})}{2}+\frac{8(4\sqrt{5}+3\sqrt{2})}{9}+\frac{16\sqrt{66}}{17}+\frac{24\sqrt{33}}{23}+n\Bigg[\sqrt{15}+20+12\sqrt{2}\\
&+\frac{12\sqrt{30}}{11}+\frac{4\sqrt{30}}{13}+\frac{9\sqrt{15}+16\sqrt{2}}{6}\Bigg].
\end{aligned}
$$

iii) According to Equation (9)

$$H_k(G_2)=\sum_{\theta\psi\in E(G_2)}\left(\frac{2}{d_\theta+d_\psi}\right)^k$$

By using types of edges given in Table 2, we get.

$$
\begin{aligned}
H_k(G_2) =\;& (8)\left(\frac{2}{3+3}\right)^k+(4n+12)\left(\frac{2}{3+5}\right)^k+(8+2n)\left(\frac{2}{5+6}\right)^k+(12)\left(\frac{2}{8+3}\right)^k\\
&+(4)\left(\frac{2}{6+8}\right)^k+(8)\left(\frac{2}{8+5}\right)^k+(2n)\left(\frac{2}{10+10}\right)^k+(2+2n)\left(\frac{2}{3+10}\right)^k\\
&+(6+6n)\left(\frac{2}{10+6}\right)^k+(8)\left(\frac{2}{10+8}\right)^k+(4+4n)\left(\frac{2}{5+10}\right)^k+(11n-16)\left(\frac{2}{6+6}\right)^k\\
&+(2)\left(\frac{2}{5+5}\right)^k+(18n-42)\left(\frac{2}{6+12}\right)^k+(4n-4)\left(\frac{2}{10+12}\right)^k+(4)\left(\frac{2}{5+11}\right)^k\\
&+(8)\left(\frac{2}{6+11}\right)^k+(6)\left(\frac{2}{12+11}\right)^k+(4)\left(\frac{2}{10+11}\right)^k+(7n-22)\left(\frac{2}{12+12}\right)^k\\
=\;& \frac{2^{2+k}5}{11^k}+\frac{2^{1+k}5}{13^k}+\frac{2^{2(1+k)}-5^k16+2^{1+k}3^k}{2^k3^k5^k}+\frac{2^{1+2k}-7^k11}{2^{k-1}3^k7^k}+\frac{2^{1+k}3+5}{2^{3k-1}}\\
&+\frac{2(3^k4-17)}{3^{2k}}+\frac{4}{7^k}-\frac{4}{11^k}+\frac{2^{3+k}}{17^k}+\frac{2^{1+k}3}{23^k}+n\Bigg[\frac{3^k2+2^{2(1+k)}+5^k11}{2^k3^k5^k}\\
&+\frac{2^{1+k}+3}{2^{3k-1}}+\frac{4(2^{k-1}+1)}{11^k}+\frac{2^{2k+1}3+3^{k-1}7}{3^{2k-1}2^{2k}}+\frac{2^{1+k}}{13^k}\Bigg].
\end{aligned}
$$

3. Conclusions

In this paper, certain degree based topological indices [1, 2, 18–21, 25–27], namely; general Randic index, sum connectivity index, general sum connectivity index, first, second and third Zagreb indices, atomic bond connectivity index, geometric arithmetic index, general version of harmonic index are computed for transformed structures by applying stellation and bounded dual operations, derived networks are named G_1 and G_2. Analytically closed formulae of above mentioned topological indices for these networks are stated. The results provide a basis to understand a deep topology for G_1 and G_2 graphs. The compelling factor of these topological indices is that they are structure invariant. In future, the interested reader is encouraged to design some new architectures/networks and study their topological properties through these numerical descriptors [17, 22, 24, 35, 38, 39]. One can also examine distance based topological indices for the transformed structures considered in this article.

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