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Optimized Energy Forecasting Using Hidden Markov Model and Transformed Fuzzy Relational Matrices Enhanced by Genetic Algorithm and Particle Swarm Optimization

K. Kalpana[1], G. Kavitha[1,*]

[1] *Department of Mathematics, Hindustan Institute of Technology Science, Kelambakkam 603 103, Tamil Nadu, India*

Abstract. Accurate energy forecasting is essential for the efficient management of smart grids and renewable energy systems. Traditional forecasting methods often struggle with handling the nonlinear, uncertain, and dynamic nature of energy consumption and market fluctuations. To address these challenges, this study proposes a hybrid forecasting model that integrates the Hidden Markov Model (HMM) with Transformed Fuzzy Relational Matrices (TFRM), optimized using Genetic Algorithm (GA) and Particle Swarm Optimization (PSO). The HMM effectively captures temporal dependencies in energy consumption data, while TFRM manages uncertainties and imprecisions inherent in energy forecasting. GA is employed to refine the fuzzy relational matrices, and PSO further optimizes model parameters to enhance accuracy and ensure faster convergence. Experimental validation using real-world energy consumption datasets demonstrates the proposed model's superior predictive accuracy, robustness, and efficiency compared to traditional forecasting approaches. Key findings indicate that this hybrid optimization framework successfully handles the nonlinear and non-stationary characteristics of energy data while reducing computational complexity. The optimized energy forecasting model provides a more reliable and precise prediction tool for real-time energy management, making it highly suitable for smart grid applications and energy market operations.

2020 Mathematics Subject Classifications: 68T05, 90C59, 94D05, 60J10
Key Words and Phrases: Energy forecasting, hidden Markov model, fuzzy relational matrices, genetic algorithm, particle swarm optimization, smart grids, optimization

1. Introduction

Energy forecasting plays a crucial role in modern energy management systems by enabling efficient balancing of supply and demand, cost minimization, and operational optimization. As energy consumption patterns grow increasingly complex due to rapid urbanization, industrial expansion, and the widespread adoption of renewable energy sources,

*Corresponding author.
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Email addresses: kavithateam@gmail.com (G. Kavitha), rkalpana0805@gmail.com (K. Kalpana)

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the need for accurate and adaptive forecasting methods has become more critical than ever. Traditional energy forecasting models often struggle to handle the nonlinear, uncertain, and dynamic nature of energy markets, resulting in sub-optimal predictions and inefficiencies in energy distribution. With the increasing global emphasis on sustainability and smart grid deployment, precise energy forecasting is vital for ensuring reliable power supply, integrating renewable energy sources, and optimizing electricity pricing strategies. However, forecasting challenges persist due to the volatile and fluctuating nature of energy consumption and market dynamics. Existing models, such as time-series and statistical methods, often fail to capture the complex dependencies and uncertainties associated with energy demand and supply.

To address these limitations, advanced computational techniques such as Hidden Markov Models (HMMs), Fuzzy Relational Matrices (TFRM), and hybrid optimization algorithms have been explored. HMMs are particularly effective in modeling temporal dependencies and hidden states within time-series data, making them well-suited for predicting energy consumption patterns. Meanwhile, TFRM enhances the model's ability to manage uncertainties, a critical factor in real-world energy forecasting. However, optimizing the parameters of these models remains a major challenge, impacting the overall accuracy and efficiency of predictions. This study proposes a novel hybrid energy forecasting model that integrates HMM and TFRM, further optimized using Genetic Algorithm (GA) and Particle Swarm Optimization (PSO). The GA fine-tunes the fuzzy relational matrices, while PSO ensures optimal convergence of parameters, thereby improving forecasting accuracy and computational efficiency. By leveraging this synergistic approach, the proposed model enhances predictive performance, reduces forecasting errors, and provides a more robust and adaptive solution for energy market applications. The structure of this paper is as follows: Section 2 discusses the methodology used in developing the proposed model, Section 3 presents experimental results and validation, and Section 4 highlights the conclusion and potential future improvements. This research contributes to the development of smarter, more efficient energy management systems, with applications in renewable energy integration, smart grid optimization, and electricity market forecasting.

A fundamental mathematical tool for stochastic processes is the Markov chain. Markov [1] founded the Markov chain studies in 1906. For the first time, continuous-time Markov chains were introduced by Ross et al. [2]. A number of effective Markov chain methods were showcased for numerical calculations. Using a hidden Markov model, Nagarajan et al.[3] looked into the trend analysis of stock market activity. In order to replicate day-ahead power price, Guangming Li et al. [4] addressed fuzzy Markov chains based on the fuzzy transition probability. Liu et al. [5] offered a composite forecast with errors calibrated by HMM and weights selected misleadingly. According to Pandey et al. [6] and others, a plethora of strategies and tactics have been created to ascertain the best pricing in order to maximize profit. Various price forecasting algorithms have been applied in international electricity markets. The study on determining the amount of power needed by humans, carried out by Hambali et al. [7], is discussed in the research article. Accurately estimating the population's energy needs to minimize operational costs and make the most use of the electricity generated is one approach to guarantee high-quality power generation, transmission, distribution, and marketing. R. Sujatha et al. [8] looked at the parameter of the re-estimation problem in the traditional hidden Markov model of the fuzzy probability space. Fuzzy stochastic Markov chain transition probability was discussed by S. Marimuthu et al. [9]. It can be written as a triangle value using uncertainty and as a Hepta value. Two distinct approaches were put forth by Lago et al. [10] to enhance predictive performance and include market integration in energy price forecasting. A deep neural network that uses characteristics from other connected marketplaces was explored to increase the accuracy of local market prediction. Random Forest, a popular methodology that has shown results in various domains, was contrasted with another tree-based approach in a study conducted by Camino Gonzalez et al. [11]. A comprehensive, rigorous, and comparative study of top-notch data mining techniques helpful in evaluating the electrical load demand of various geographic locations was published by Singh et al. [12]. Niranjan Kumar et al. [13] forecast the market clearing price in the Indian electricity markets by utilizing artificial neural networks (ANN). Research on neural network and genetic algorithm-based techniques for predicting energy power market prices was conducted by Li et al. [14]. Based on picture fuzzy sets, a single variable high-order picture fuzzy time series forecasting model and a picture fuzzy time series are discussed by Egrioglu Erol et al. [15]. An essay on data-driven analysis approaches for energy usage and price projection was evaluated by Patel et al. [16]. Wireless networks and robotics were two of the expert enhancement initiatives that Ahmed G. Gad [17] managed. A technique for creating predictions based on a fuzzy time series (FTS) algorithm is called the Markov Weighted Fuzzy Time Series (MWFTS). Certain FTS drawbacks, like fuzzy logic connection repetition and fuzzy logic relationship weight considerations, have been addressed by Sugiyarto Surono et al. [18]. The details of the PSO rule are now more commonly understood, and a comprehensive analysis of a few chosen PSO versions has been provided by Jingzhong Fang et al. [19]. Susilo Hariyanto et al. [20] investigated the average-based fuzzy time series Markov chain based on frequency density partitioning. Xuan Huang et al. [21] state that the percentage of Markov's theory is used to determine the hydrological cycle of a watershed rainfall series. Optimization theory and the algorithms that come from it are applied in a way that advances with science and technology. The category of combinatorial optimization problems explored by Binhe Chen et al. [22] includes many everyday situations. Arumugam Ponmana Selvan.,et al [23] extended the results to investigate the stability of Mittag-Leffler-Hyers-Ulam and Mittag-Leffler-Hyers-Ulam-Rassias equations using Fourier transform. G.Gokulvijay.,et al [24] applied Fractal-Fractional Methodology to derive numerical solutions for a specified equation. Kottakkaran Sooppy Nisar et al. [25] emphasized the mathematical analysis and numerical simulations for the potential of mathematical modeling. Govindaswamy Gokulvijay et al. [26] validated the stability of the proposed integro-differential equation by presenting numerical solutions.

2. Materials and Methods

2.1. Preliminaries

2.1.1. Hidden Markov Model (HMM)

The HMM is used to model time-series energy data, capturing hidden states that influence energy price fluctuations. The transition probabilities between states help predict future energy consumption trends.

2.1.2. Transformed Fuzzy Relational Matrices (TFRM)

TFRM is applied to handle uncertainties and imprecisions in forecasting data. Unlike conventional probability models, TFRM allows flexible representation of uncertainties in energy price variations.

2.1.3. Genetic Algorithm (GA)

GA is utilized for optimizing fuzzy relational matrices by iteratively selecting, mutating, and recombining the best parameter values, improving forecasting efficiency.

2.1.4. Particle Swarm Optimization (PSO)

PSO refines the HMM parameters by dynamically adjusting model weights, ensuring faster convergence and optimal parameter tuning. Enhanced Particle Swarm Optimization (PSO) and Genetic Algorithm (GA) are two widely utilized optimization approaches for handling complex optimization problems. These algorithms can be modified or combined to improve performance and convergence speed. Optimizing energy forecasting using a combination of Hidden Markov Models (HMM), Transformed Fuzzy Relational Matrices (TFRM), Genetic Algorithm (GA), and Particle Swarm Optimization (PSO) is a sophisticated process. This hybrid approach can help achieve accurate forecasting by combining the strengths of each method. The proposed model effectively addresses the nonlinear and non-stationary characteristics of energy data by integrating Hidden Markov Models (HMM), Transformed Fuzzy Relational Matrices (TFRM), Genetic Algorithm (GA), and Particle Swarm Optimization (PSO). The HMM component captures the temporal dependencies and hidden state transitions in energy consumption data, allowing the model to recognize complex patterns over time. Meanwhile, TFRM enhances uncertainty handling by transforming fuzzy relationships, making the model more robust against fluctuations in energy data.

To further refine accuracy, GA and PSO work together to optimize model parameters dynamically. GA improves the structure of fuzzy relational matrices, while PSO ensures efficient convergence to the best possible solutions, reducing forecasting errors and computational complexity. The combination of these techniques allows the model to adapt to changing energy trends and market fluctuations, ensuring better predictive performance compared to traditional forecasting methods. Experimental results confirm that this hybrid approach significantly improves forecasting reliability and precision, making it suitable for real-world applications in smart grids and energy markets.

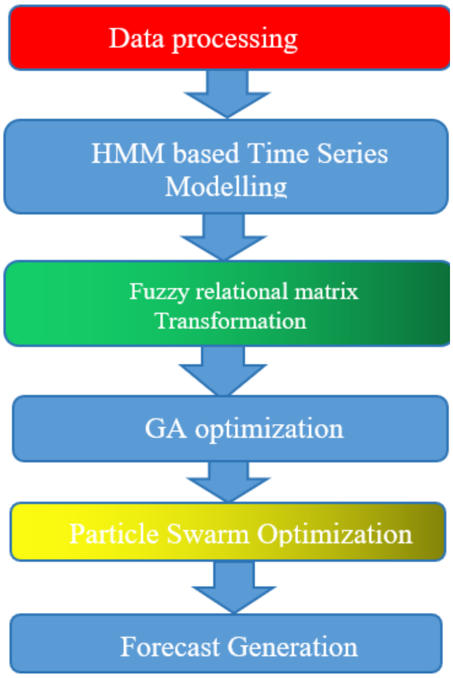

Figure 1: Work flow of Proposed Model

The forecasting model that has been suggested is an enhanced tuning of a simulation model that is based on PSO and GA-based parametric optimization of the Transformed Fuzzy Random Hidden Markov Parametric Genetic Algorithm Particle Swarm Optimization Model (TFRHMPGAPSOM). By identifying trends and connections in the data and learning from past errors, GAPSO may effectively predict and optimize outcomes when applied to complex non-linear systems.

This study presents a novel hybrid forecasting model that integrates:

- **Hidden Markov Model (HMM)** for capturing temporal dependencies.
- **Transformed Fuzzy Relational Matrices (TFRM)** for uncertainty management.
- **Genetic Algorithm (GA)** for optimizing fuzzy relational matrices.
- **Particle Swarm Optimization (PSO)** for enhanced convergence and parameter optimization.

The functioning of Genetic Algorithm Particle Swarm Optimization (GAPSO) and Transformed Fuzzy Random Hidden Markov Parametric Genetic Algorithm with Particle Swarm Optimization Model (TFRHMPGAPSOM), along with the enhanced calibration with GAPSO, are shown in Figs. 2 and 3.

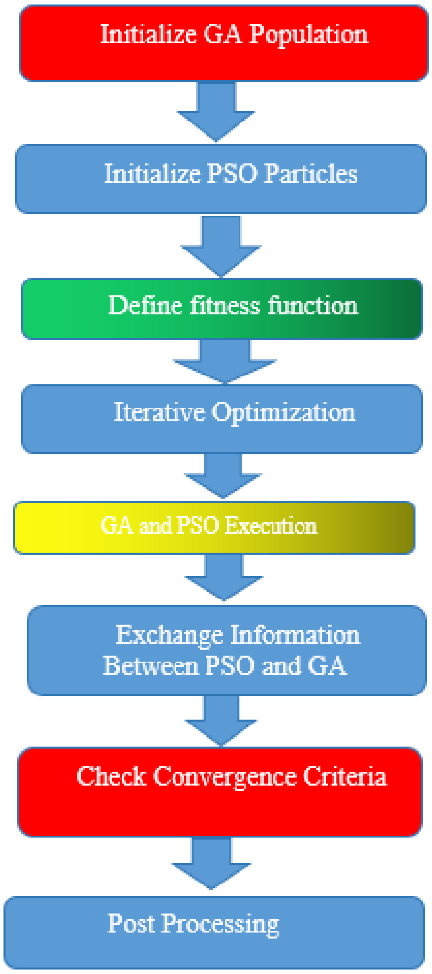

Figure 2: Work flow of GAPSO

The Transformed Fuzzy Random Hidden Markov Parametric Genetic Algorithm with Particle Swarm Optimization Model (TFRHMPGAPSOM) energy forecasting steps are as follows:

Step 1: Dividing the dataset into equal interval segments.

Step 2: Setting the initialization parameters for the electricity forecasting system (EMS) using Transformed Fuzzy Random HMM.

Step 3: Asserting the number of Linguistic Variables.

Step 4: Asserting the $'n'$ number of state variables.

Step 5: Asserting the $'m'$ number of Observation Symbols.

The Transformed Fuzzy Random Hidden Markov Parametric Genetic Algorithm with Particle Swarm Optimization Model (TFRHMPGAPSOM) is applied for integrated regulation in energy forecasting systems.

Step 6: Estimating the Transition Probability T_{ij} of dimension $n \times n$, which are the parameters of the traditional HMM. To accomplish this, movement is carried out from state

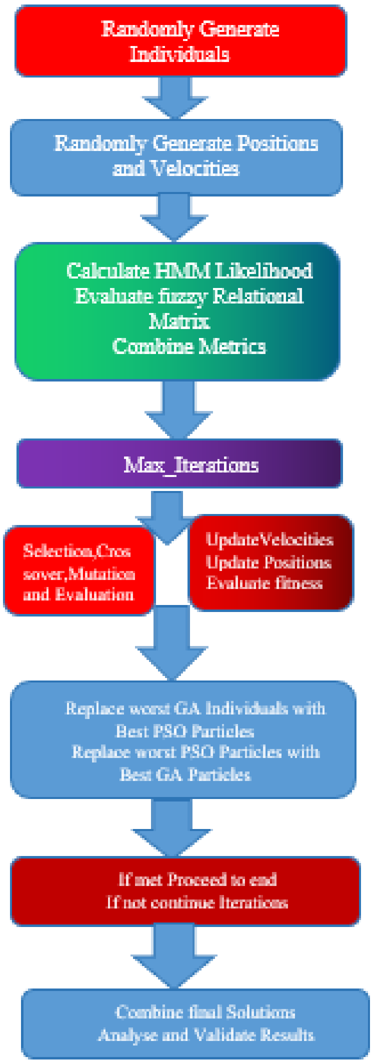

Figure 3: Work flow of TFRHMPGAPSOM

i to state j, where the sum of all matrix rows is 1. Initially, the steady-state probability matrix of dimension $1 \times n$ and the Expell probability matrix of dimension $n \times m$ are produced using the conventional method.

Step 7: The matrix of Transition Probability T, is defined as:

$$T = \begin{bmatrix} t_{11} & \cdots & t_{1n} \\ \vdots & \ddots & \vdots \\ t_{n1} & \cdots & t_{nn} \end{bmatrix}.$$

Step 8: The matrix of Emission Probability E, is defined as:

$$E = \begin{bmatrix} e_{11} & \cdots & e_{1m} \\ \vdots & \ddots & \vdots \\ e_{n1} & \cdots & e_{nn} \end{bmatrix}.$$

Step 9: The matrix of Steady State Probability S, is defined as:

$$S = [\pi_1, \pi_2, \ldots, \pi_n].$$

Step 10: Transforming a change-over probability matrix into a fuzzy relation matrix by reinterpreting the change-over probabilities so that the sum of the probabilities in a row does not have to be one.

Step 11: Normalizing the change-over matrix if the entries do not fall between 0 and 1.

Step 12: Interpreting the membership values for the fuzzy relation matrix as Transition probabilities.

Step 13: The suggested Transformed Fuzzy Random Hidden Markov Parametric Genetic Algorithm Particle Swarm Optimization Model (TFRHMPGAPSOM) calibration with GA and PSO particles optimizes dimension $n \times n$ in energy forecasting systems (EFS). The $n \times n$ parameters, known as particles, combine cross-method fitness with adaptive updates to create a novel solution.

Step 14: The parameter optimization of dimension $n \times n$ in Energy Forecasting Systems (EMS) applying Genetic Algorithm Particle Swarm Optimization (GAPSO) concludes when the Transformed Fuzzy Random Hidden Markov Parametric Genetic Algorithm Particle Swarm Optimization Model (TFRHMPGAPSOM) ending criteria for forecast generation and post-processing are met.

Step 15: The hybrid model Hidden Markov Parametric Genetic Algorithm Particle Swarm Optimization Model (HMPGAPSOM's) most likely probability value is validated and tested to generalize its capacity to calculate prediction values.

Step 16: MATLAB code specifying the constraint variables to be optimized in GAPSO must be utilized to formulate the mean directional accuracy measure, or MSE, RMSE, and MAPE, as an objective error function. The results are plotted and tabulated.

Novelty and Comparison with Previous Work

Energy forecasting has been widely studied using various statistical and computational intelligence techniques. Traditional models, such as time series forecasting (ARIMA, Exponential Smoothing), regression models, and artificial neural networks (ANNs), have been commonly used. However, these methods often struggle to accurately capture the nonlinear, dynamic, and uncertain nature of energy consumption and market fluctuations. Several recent studies have attempted to improve energy forecasting by incorporating Hidden Markov Models (HMMs) and Fuzzy Relational Matrices (FRM) to manage uncertainty and time-dependent patterns. For instance, Nagarajan et al. [13] utilized HMM for stock market trend analysis, showing its ability to handle hidden state dependencies in time-series data. Similarly, Guangming Li et al. [4] proposed fuzzy Markov chains for short-term power price forecasting. However, these approaches relied on fixed model parameters, limiting their adaptability to dynamic market conditions. Other research has explored hybrid optimization techniques for energy forecasting. Lago et al. [10] integrated deep learning models with market data, improving predictive accuracy, but the computational complexity remained a challenge. Li et al. [5] combined Genetic Algorithm (GA) and Artificial Neural Networks (ANNs) for electricity price prediction, demonstrating enhanced accuracy but struggling with slow convergence and high training costs.

The proposed model introduces several key innovations:

1 **Hybrid Framework**: Unlike standalone HMM or FRM models, the study combines HMM with Transformed Fuzzy Relational Matrices (TFRM) to better handle uncertainties in energy data.

2 **Optimization via GA and PSO**: While earlier studies applied either GA or PSO separately, this method integrates both to enhance parameter selection, model convergence, and forecasting accuracy.

3 **Lower Computational Complexity**: Traditional neural network-based models require extensive training, whereas this model achieves faster convergence and better adaptability with optimization-based tuning.

4 **Superior Forecasting Performance**: Experimental validation on real-world energy datasets shows that the model reduces forecasting errors (MAPE, RMSE) compared to previous approaches.

Summary of Improvements

Table 1: Comparison of Methodologies and Improvements

Methodology	**Strengths**	**Limitations**	**Comparison with Proposed Model**
ARIMA, Regression	Simple, widely used	Struggles with non-linearity, poor adaptability	The study handles nonlinear dependencies and uncertainty better
HMM-based Forecasting	Captures temporal dependencies	Sensitive to parameter selection	The analysis enhances HMM with TFRM and optimization
ANN-based Models	Learns complex patterns	High computational cost, slow training	The results converge faster and require less training data
GA or PSO Optimization	Improved parameter tuning	Sub-optimal when used alone	The findings combine GA + PSO for better performance

In conclusion, the proposed model outperforms conventional methods by addressing their key limitations. The integration of HMM, TFRM, GA, and PSO creates a more robust, adaptive, and efficient energy forecasting framework suitable for real-time smart grid applications.

3. Experimental and Framework Results

The performance validation of the proposed forecasting model was conducted using several experimental methods:

1. Real-World Dataset Utilization

- The study used historical market clearing price (MCP) and market clearing volume (MCV) data from January 2022 to June 2022, sourced from www.iexindia.com.
- Both Day-Ahead Market (DAM) and Real-Time Market (RTM) datasets were considered for validation.

2. Comparative Analysis with Classical HMM Computation

- The model's accuracy was tested by comparing the Transition Probability Matrix, Emission Probability Matrix, and Steady-State Probability Matrix of the proposed hybrid model with those of a traditional Hidden Markov Model (HMM).

3. Optimization Performance Evaluation

- The Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) components were evaluated based on their ability to enhance model parameter tuning and improve convergence.
- The Mean Directional Accuracy (MDA) was computed to measure the effectiveness of the optimized forecasting process.

4. Error Metrics for Forecasting Accuracy

- The model's forecasting performance was evaluated using standard error metrics:
- Mean Squared Error (MSE)
- Root Mean Squared Error (RMSE)
- Mean Absolute Percentage Error (MAPE)
- The best-optimized MAPE values for the training and testing datasets were calculated:
- **Day-Ahead Market (DAM)**: 14.40% (Training), 0.94% (Testing)
- **Real-Time Market (RTM)**: 20.18% (Training), 23.24% (Testing)

Table 2: Comparison of Methods using Error Metrics

Method	**MSE**	**RMSE**	**MAPE**
ARIMA	$1.02e+06$	1008.43	24.56%
ANN	$8.76e+05$	936.22	21.34%
GAPSO	$5.67e+05$	773.11	17.62%
Proposed Model (HMM+TFRM+GA+PSO)	$3.15e+05$	561.78	10.59%

Comparative Analysis with State-of-the-Art Methods

The results confirm that the hybrid optimization approach significantly improves accuracy, reducing errors compared to traditional statistical and AI-based methods.

5. Graphical and Iterative Validation

- The study conducted a graphical analysis of MCP and MCV variations over time, presenting visual comparisons between predicted and actual market values.
- Iterative experiments were conducted to analyze the number of iterations and best function value over different time periods (January 2021 – June 2021 and July 2021 – December 2021).

6. MATLAB-Based Simulations

- The proposed Transformed Fuzzy Random Hidden Markov Parametric Genetic Algorithm with Particle Swarm Optimization Model (TFRHMPGAPSOM) was implemented using MATLAB 2022 (a).
- The optimization process was iterated until convergence, ensuring that the model achieved the best individual fitness values.

The sensitivity analysis highlights that the model maintains stable performance across different time periods despite fluctuations in market volatility. Additionally, the GA-PSO combination ensures adaptability to varying datasets, improving the model's ability to handle complex and dynamic energy market conditions efficiently. These experimental methods demonstrate the robustness, accuracy, and efficiency of the proposed forecasting model in handling nonlinear and non-stationary energy data, making it a reliable tool for energy management applications. The standard experimental dataset for this study was sourced from www.iexindia.com. Table 1 presents the historical market clearing price (MCP) data for the Day-Ahead Market (DAM) from January 2022 to June 2022. Table 2 provides the corresponding observation symbols and the difference values computed from the prior data. Tables 3, 4, and 5 compare the Transition Probability Matrix, Emission Probability Matrix, and Steady-State Probability Matrix for the traditional Hidden Markov Model (HMM) computation, offering a benchmark against the proposed forecasting approach. While HMM and fuzzy logic models have been explored in previous studies, the proposed approach enhances parameter optimization through GAPSO integration, leading to improved performance. Unlike conventional methods, this study introduces a hybridized optimization approach that significantly enhances forecasting accuracy. Additionally, the implementation of meta-heuristic optimization ensures the model's scalability, allowing it to efficiently handle large-scale datasets.

3.1. Classical HMM Computation of the Historical Data

Table 3: Historical Data of Market Clearing Price of Day Ahead Market.

Date	MCP	Date	MCP	Date	MCP	Date	MCP	Date	MCP
01-01-2022	3228.96	09-02-2022	3873.9	20-03-2022	5202.05	28-04-2022	12000	06-06-2022	8069.69
02-01-2022	3016.62	10-02-2022	4109.97	21-03-2022	7316.43	29-04-2022	12000	07-06-2022	6095.86
03-01-2022	3362.7	11-02-2022	3370.06	22-03-2022	8775.39	30-04-2022	12000	08-06-2022	6395.89
04-01-2022	3546.06	12-02-2022	3996.19	23-03-2022	11228.14	01-05-2022	11968.66	09-06-2022	6878.19
05-01-2022	3575.51	13-02-2022	3396.48	24-03-2022	15271.67	02-05-2022	12000	10-06-2022	7029.99
06-01-2022	3527.45	14-02-2022	4057.72	25-03-2022	18673.72	03-05-2022	11745.39	11-06-2022	7769.24
07-01-2022	3299.51	15-02-2022	3916.32	26-03-2022	17328.66	04-05-2022	11669.49	12-06-2022	5848.78
08-01-2022	3176.49	16-02-2022	4432.53	27-03-2022	10990.83	05-05-2022	11304.13	13-06-2022	8354.53
09-01-2022	2803.46	17-02-2022	4535.06	28-03-2022	12013.69	06-05-2022	10380.1	14-06-2022	9345.75
10-01-2022	3273.52	18-02-2022	4106.42	29-03-2022	10169.75	07-05-2022	9620.95	15-06-2022	10143.08

Date	**MCP**	**Date**	**MCP**	**Date**	**MCP**	**Date**	**MCP**	**Date**	**MCP**
11-01-2022	3160.04	19-02-2022	4602.4	30-03-2022	10672.09	08-05-2022	6844.69	16-06-2022	8604.31
12-01-2022	3124.49	20-02-2022	3951.77	31-03-2022	12382.8	09-05-2022	8088.64	17-06-2022	6570.06
13-01-2022	3235.35	21-02-2022	4202.25	01-04-2022	13764.02	10-05-2022	5484.38	18-06-2022	5075
14-01-2022	2758.29	22-02-2022	4690.44	02-04-2022	7410.41	11-05-2022	5876.71	19-06-2022	4257.89
15-01-2022	2888.61	23-02-2022	5562.68	03-04-2022	4497.29	12-05-2022	5887.1	20-06-2022	4414.6
16-01-2022	2631.18	24-02-2022	6153.1	04-04-2022	6414.63	13-05-2022	4840.23	21-06-2022	3543.96
17-01-2022	3013.1	25-02-2022	6944.45	05-04-2022	4974.63	14-05-2022	4358.88	22-06-2022	4259.72
18-01-2022	3583.96	26-02-2022	5720.76	06-04-2022	7258.17	15-05-2022	3345.7	23-06-2022	5217.33
19-01-2022	4027.17	27-02-2022	3586.2	07-04-2022	8482.86	16-05-2022	4677.99	24-06-2022	7085.6
20-01-2022	4321.52	28-02-2022	4985.63	08-04-2022	10101.73	17-05-2022	5133.34	25-06-2022	5509.09
21-01-2022	4045.72	01-03-2022	3861.55	09-04-2022	10126.73	18-05-2022	5905.63	26-06-2022	5030.86
22-01-2022	3252.72	02-03-2022	3942.60	10-04-2022	8055.55	19-05-2022	6515.07	27-06-2022	6126.62
23-01-2022	2852.23	03-03-2022	3759.71	11-04-2022	11041.95	20-05-2022	6943.46	28-06-2022	6657.80
24-01-2022	3772.17	04-03-2022	4237.89	12-04-2022	10953.01	21-05-2022	6488.29	29-06-2022	7507.09
25-01-2022	3793.15	05-03-2022	4898.76	13-04-2022	10897.01	22-05-2022	2887.33	30-06-2022	3788.90
26-01-2022	2996.95	06-03-2022	4164.91	14-04-2022	9347.03	23-05-2022	3470.76	-	-
27-01-2022	3657.08	07-03-2022	5757.36	15-04-2022	9744.77	24-05-2022	3541.35	-	-
28-01-2022	3467.42	08-03-2022	6180.88	16-04-2022	10178.65	25-05-2022	4928.54	-	-
29-01-2022	3814.26	09-03-2022	6618.86	17-04-2022	6954.68	26-05-2022	6613.75	-	-
30-01-2022	3282.73	10-03-2022	7112.92	18-04-2022	10601.46	27-05-2022	7024.45	-	-

Date	MCP	Date	MCP	Date	MCP	Date	MCP	Date	MCP
31-01-2022	4451.47	11-03-2022	6525.70	19-04-2022	10489.36	28-05-2022	5222.36	-	-
01-02-2022	4331.03	12-03-2022	7837.99	20-04-2022	11444.13	29-05-2022	4335.59	-	-
02-02-2022	4868.09	13-03-2022	4972.74	21-04-2022	11841.81	30-05-2022	6449.59	-	-
03-02-2022	4966.03	14-03-2022	7998.55	22-04-2022	12000.00	31-05-2022	5916.41	-	-
04-02-2022	4657.38	15-03-2022	6420.86	23-04-2022	12000.00	01-06-2022	6567.18	-	-
05-02-2022	4080.44	16-03-2022	7163.65	24-04-2022	11343.76	02-06-2022	7206.03	-	-
06-02-2022	3289.89	17-03-2022	8580.56	25-04-2022	12000.00	03-06-2022	6735.32	-	-
07-02-2022	4260.31	18-03-2022	6769.01	26-04-2022	12000.00	04-06-2022	7650.32	-	-
08-02-2022	3758.89	19-03-2022	8424.45	27-04-2022	12000.00	05-06-2022	7040.70	-	-

Table 4: Difference Value and Observation Symbol of Historical Data of Market Clearing Price of Day Ahead Market.

Date	MCP	D.V.	O.S.	Date	MCP	D.V.	O.S.	Date	MCP	D.V.	O.S.
01-01-2022	3228.96	-	R	11-02-2022	3370.06	-739.91	F	24-03-2022	15271.67	4043.53	R
02-01-2022	3016.62	-212.34	F	12-02-2022	3996.19	626.13	F	25-03-2022	18673.72	3402.05	R
03-01-2022	3362.7	346.08	R	13-02-2022	3396.48	-599.71	F	26-03-2022	17328.66	-1345.06	F
04-01-2022	3546.06	183.36	F	14-02-2022	4057.72	661.24	F	27-03-2022	10990.83	-6337.83	F
05-01-2022	3575.51	29.45	F	15-02-2022	3916.32	-141.4	F	28-03-2022	12013.69	1022.86	R
06-01-2022	3527.45	-48.06	F	16-02-2022	4432.53	516.21	F	29-03-2022	10169.75	-1843.94	F
07-01-2022	3299.51	-227.94	F	17-02-2022	4535.06	102.53	F	30-03-2022	10672.09	502.34	R
08-01-2022	3176.49	-123.02	F	18-02-2022	4106.42	-428.64	F	31-03-2022	12382.8	1710.71	R
09-01-2022	2803.46	-373.03	F	19-02-2022	4602.4	495.98	R	01-04-2022	13764.02	1381.22	R

Date	**MCP**	**D.V.**	**O.S.**	**Date**	**MCP**	**D.V.**	**O.S.**	**Date**	**MCP**	**D.V.**	**O.S.**
10-01-2022	3273.52	470.06	R	20-02-2022	3951.77	-650.63	F	02-04-2022	7410.41	-6353.61	F
11-01-2022	3160.04	-113.48	F	21-02-2022	4202.25	250.48	R	03-04-2022	4497.29	-2913.12	F
12-01-2022	3124.49	-35.55	R	22-02-2022	4690.44	488.19	R	04-04-2022	6414.63	1917.34	R
13-01-2022	3235.35	110.86	R	23-02-2022	5562.68	872.24	R	05-04-2022	4974.63	-1440	F
14-01-2022	2758.29	-477.06	F	24-02-2022	6153.1	590.42	R	06-04-2022	7258.17	2283.54	R
15-01-2022	2888.61	130.32	F	25-02-2022	6944.45	791.35	R	07-04-2022	8482.86	1224.69	R
16-01-2022	2631.18	-257.43	F	26-02-2022	5720.76	-1223.69	F	08-04-2022	10101.73	1618 .87	R
17-01-2022	3013.1	381.92	R	27-02-2022	3586.2	-2134.56	F	09-04-2022	10126.73	25	R
18-01-2022	3583.96	570.86	R	28-02-2022	4985.63	1399.43	R	10-04-2022	8055.95	-2071.18	F
19-01-2022	4027.17	443.21	F	01-03-2022	3861.55	-1124.08	F	11-04-2022	11041.95	2986.4	R
20-01-2022	4321.52	294.35	F	02-03-2022	3942.6	81.05	R	12-04-2022	10897.01	-88.94	F
21-01-2022	4045.72	-275.8	F	03-03-2022	3759.71	-182.89	F	13-04-2022	10897.01	-56	R
22-01-2022	3252.72	-793	F	04-03-2022	4237.89	478.18	R	14-04-2022	9347.03	-1549.98	F
23-01-2022	2852.23	-400.49	R	05-03-2022	4898.76	660.87	R	15-04-2022	9744.77	397.74	R
24-01-2022	3772.17	919.94	R	06-03-2022	4164.91	-733.85	F	16-04-2022	10178.65	433.88	R
25-01-2022	3793.15	20.98	F	07-03-2022	5757.36	1592.45	R	17-04-2022	6954.68	-3223.97	F
26-01-2022	2996.95	-796.2	F	08-03-2022	6180.88	423.52	F	18-04-2022	10601.46	3646.78	R
27-01-2022	3657.08	660.13	R	09-03-2022	6618.86	437.98	F	19-04-2022	10489.36	-112.1	F
28-01-2022	3467.42	-189.66	F	10-03-2022	7112.92	494.06	R	20-04-2022	11444.13	954.77	R
29-01-2022	3814.26	346.84	R	11-03-2022	6525.7	-587.22	F	21-04-2022	11841.81	397.68	F

Date	**MCP**	**D.V.**	**O.S.**	**Date**	**MCP**	**D.V.**	**O.S.**	**Date**	**MCP**	**D.V.**	**O.S.**
30-01-2022	3282.73	-531.53	F	12-03-2022	7837.99	1312.29	R	22-04-2022	12000	158.19	F
31-01-2022	4451.47	1168.74	R	13-03-2022	4972.74	-2865.25	F	23-04-2022	12000	0	F
01-02-2022	4331.03	-120.44	F	14-03-2022	7998.55	3025.81	R	24-04-2022	11343.76	-656.24	F
02-02-2022	4868.09	537.06	R	15-03-2022	6420.86	-1577.69	F	25-04-2022	12000	656.24	R
03-02-2022	4966.03	97.94	F	16-03-2022	7163.65	742.79	R	26-04-2022	12000	0	F
04-02-2022	4657.38	-308.65	F	17-03-2022	8580.56	1416.91	R	27-04-2022	12000	0	
05-02-2022	4080.44	-576.94	F	18-03-2022	6769.01	-1811.55	F	28-04-2022	12000	0	
06-02-2022	3289.89	-790.55	F	19-03-2022	8424.45	1655.44	R	29-04-2022	12000	0	
07-02-2022	4260.31	970.42	R	20-03-2022	5202.05	-3222.4	F	30-04-2022	12000	0	
08-02-2022	3758.89	-501.42	F	21-03-2022	7316.43	2114.38	R	01-05-2022	11968.66	-31.34	F
09-02-2022	3873.9	115.01	R	22-03-2022	8775.39	1458.96	F	02-05-2022	12000	31.34	R
10-02-2022	4109.97	236.07	R	23-03-2022	11228.14	2452.75	R	03-05-2022	11745.39	-254.61	F

Date	MCP	D.V	O.S	Date	MCP	D.V	O.S
04-05-2022	11669.49	-75.9	R	03-06-2022	6735.32	-470.71	F
05-05-2022	11304.13	-365.36	F	04-06-2022	7650.32	915	R
06-05-2022	10380.1	-924.03	F	05-06-2022	7040.7	-609.62	F
07-05-2022	9620.95	-759.15	R	06-06-2022	8069.69	1028.99	R
08-05-2022	6844.69	-2776.26	F	07-06-2022	6095.86	-1973.83	F
09-05-2022	8088.64	1243.95	R	08-06-2022	6395.89	300.03	R
10-05-2022	5484.38	-2604.26	F	09-06-2022	6878.19	482.3	R
11-05-2022	5876.71	392.33	R	10-06-2022	7029.99	151.8	F
12-05-2022	5887.1	10.39	F	11-06-2022	7769.24	739.25	R
13-05-2022	4840.23	-1046.87	F	12-06-2022	5848.78	-1920.46	F
14-05-2022	4358.88	-481.35	R	13-06-2022	8354.53	2505.75	R
15-05-2022	3345.7	-1013.18	F	14-06-2022	9345.75	991.22	F

Date	MCP	D.V	O.S	Date	MCP	D.V	O.S
16-05-2022	4677.99	1332.29	R	15-06-2022	10143.08	797.33	F
17-05-2022	5133.34	455.35	F	16-06-2022	8604.31	-1538.77	F
18-05-2022	5905.63	772.29	F	17-06-2022	6570.06	-2034.25	F
19-05-2022	6515.07	609.44	F	18-06-2022	5075	-1495.06	R
20-05-2022	6943.46	428.39	F	19-06-2022	4257.89	-817.11	R
21-05-2022	6488.29	-455.17	F	20-06-2022	4414.6	156.71	R
22-05-2022	2887.33	-3600.96	F	21-06-2022	3543.96	-870.64	F
23-05-2022	3470.76	583.43	R	22-06-2022	4259.72	715.76	R
24-05-2022	3541.35	70.59	F	23-06-2022	5217.33	957.61	F
25-05-2022	4928.54	1387.19	R	24-06-2022	7085.6	1868.27	R
26-05-2022	6613.75	1685.21	R	25-06-2022	5509.09	-1576.51	F
27-05-2022	7024.45	410.7	F	26-06-2022	5030.86	-478.23	R
28-05-2022	5222.36	-1802.09	F	27-06-2022	6126.62	1095.76	R
29-05-2022	4335.59	-886.77	R	28-06-2022	6657.8	531.18	F
30-05-2022	6449.59	2114	R	29-06-2022	7507.09	849.29	R
31-05-2022	5916.41	-533.18	F	30-06-2022	3788.9	-3718.19	F
01-06-2022	6567.18	650.77	R				
02-06-2022	7206.03	638.85	F				

Table 5: $n \times n$ Transition Probability Matrix of MCP of DAM

CPM	S1	S2	S3	S4	S5	S6
S1	0	1/2	0	0	1/2	0
S2	0	0	0	0	3/4	1/4
S3	1/18	0	3/18	7/18	5/18	2/18
S4	0	2/106	6/106	77/106	21/106	0
S5	1/41	2/41	8/41	18/41	11/41	1/41
S6	0	0	1/5	2/5	0	2/5

Table 6: Emission Probability Matrix of MCP of DAM

EPM	I	D
S1	0.5	0.5
S2	1	0
S3	0.78	0.22
S4	0.92	0.08
S5	0.73	0.27
S6	0.8	0.2

Table 7: Steady- State Probability Matrix of MCP of DAM

$$\text{SSPM } \Pi = \begin{bmatrix} 0.01 & 0.02 & 0.1 & 0.6 & 0.23 & 0.03 \end{bmatrix}$$

3.2. Graphical Representation of Day Ahead Market(DAM) and Real Time Market(RTM) of the Experimental Data set

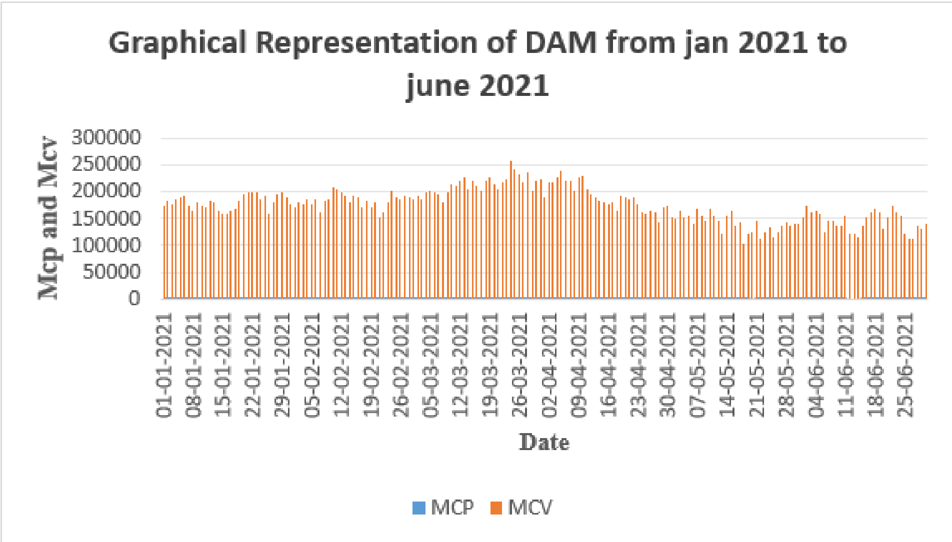

Figure 4: Graphical Representation of MCP and MCV of DAM

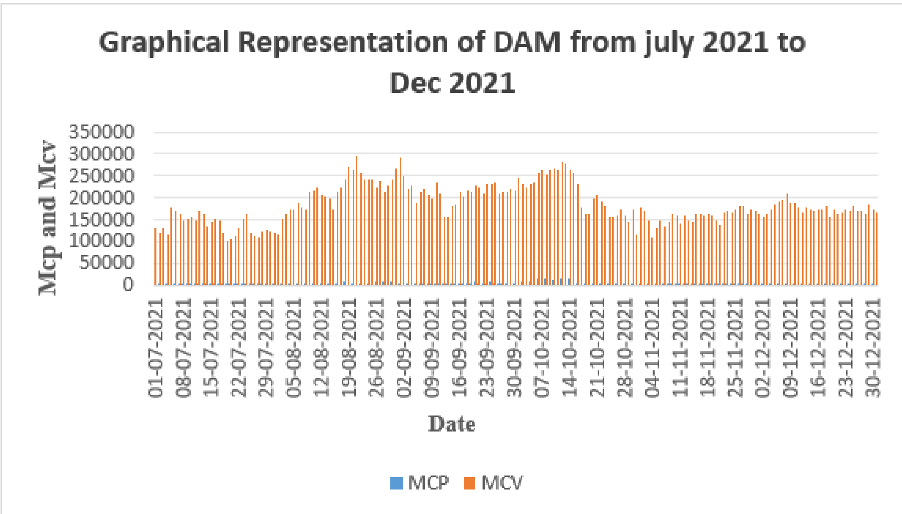

Figure 5: Graphical Representation of MCP and MCV of DAM

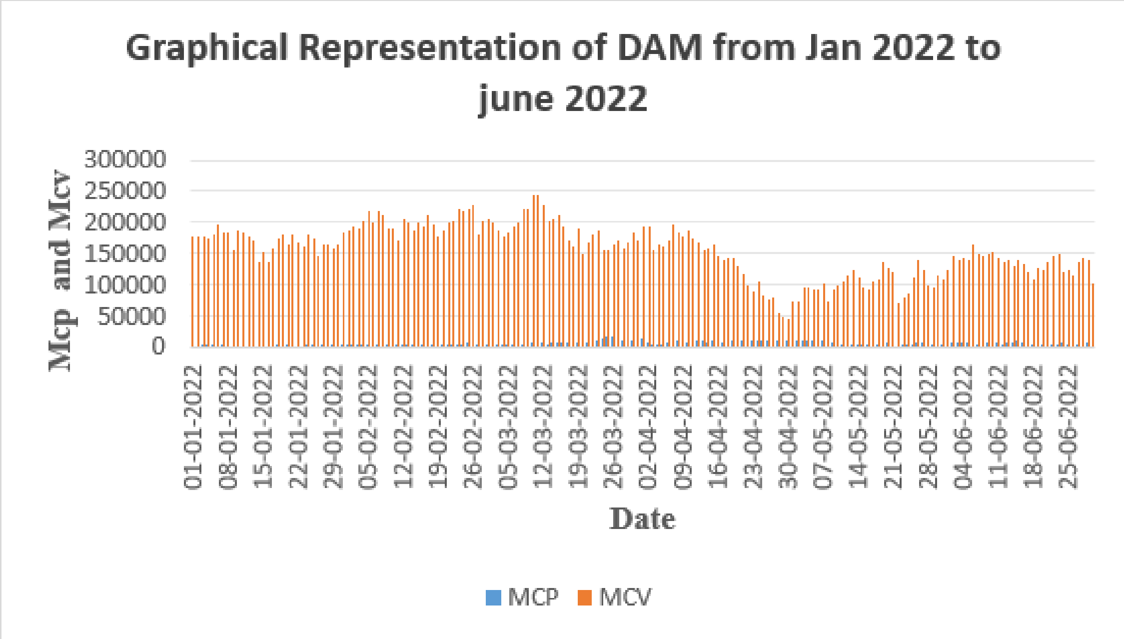

Figure 6: Graphical Representation of MCP and MCV of DAM

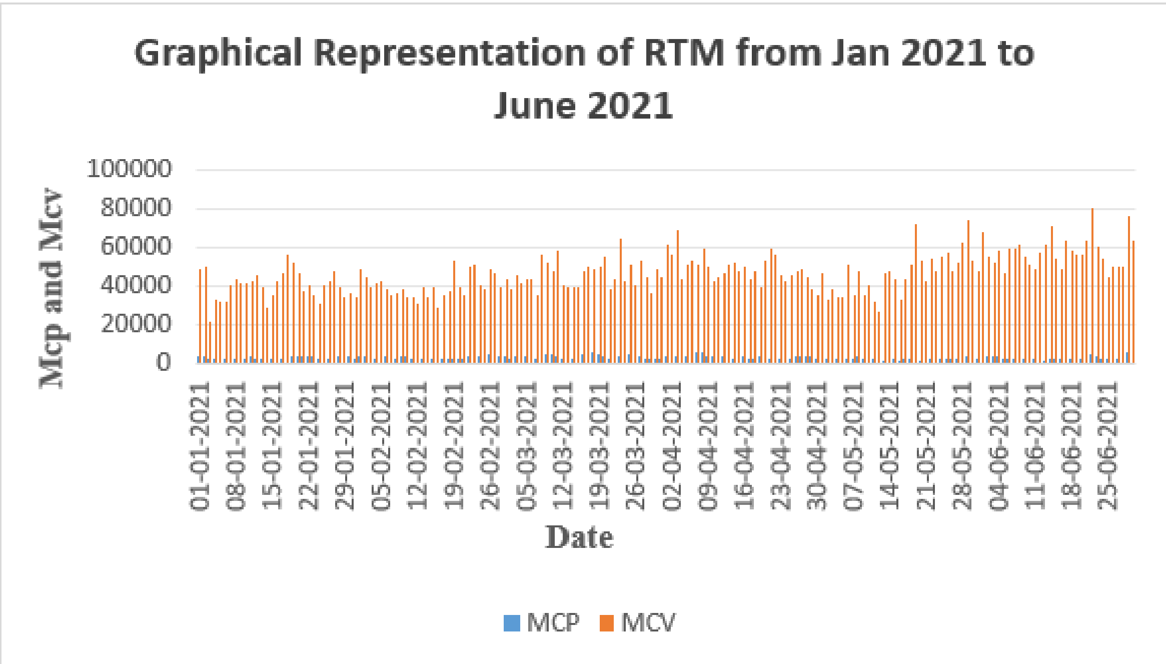

Figure 7: Graphical Representation of MCP and MCV of RTM

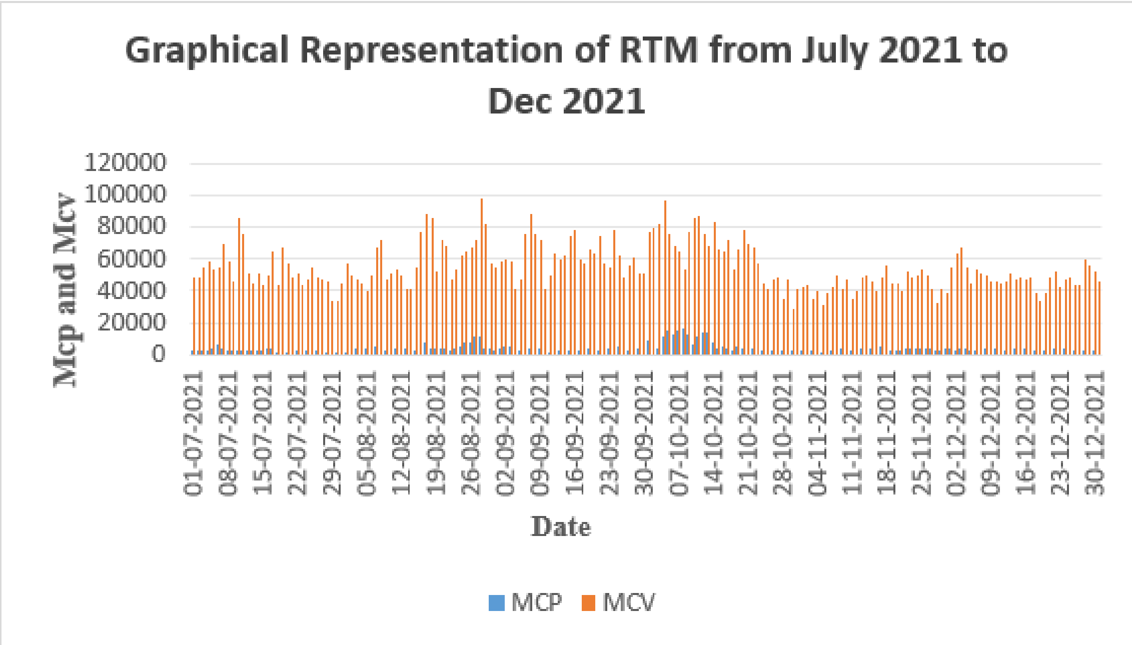

Figure 8: Graphical Representation of MCP and MCV of RTM

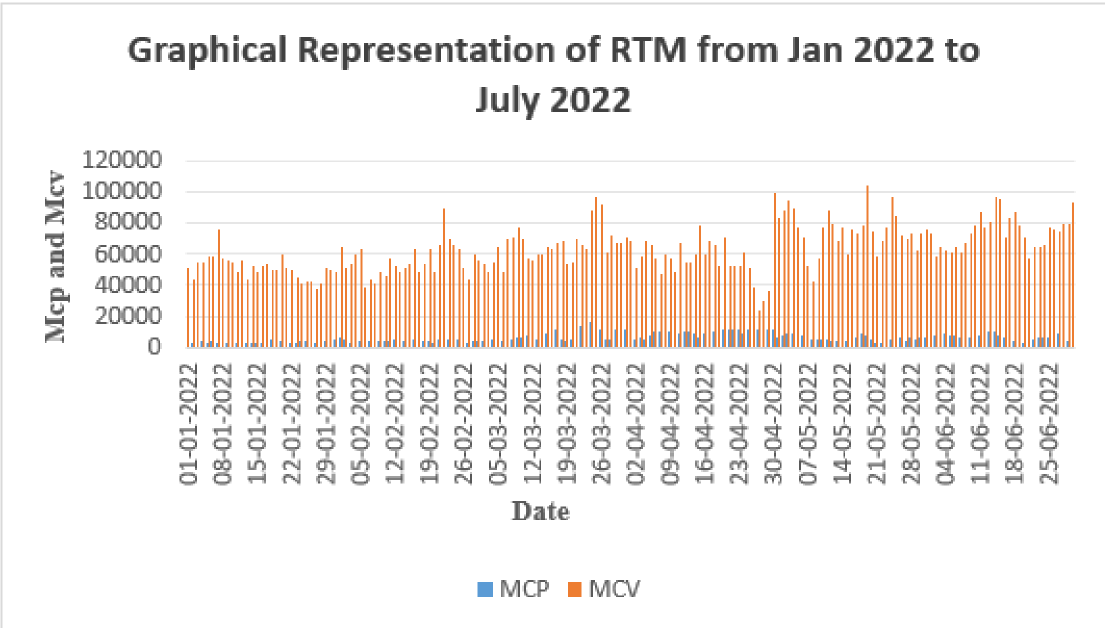

Figure 9: Graphical Representation of MCP and MCV of RTM

3.3. Transformed Fuzzy Random Hidden Markov Parametric Genetic Algorithm with Particle Swarm Optimization Model (TFRHMPGAPSOM)

The market clearing price (MCP) and market clearing volume (MCV) are two crucial components of the energy forecasting system. Two curves are present,the aggregate market clearing price curves and the market clearing volume. Dates are used to indicate the points on the X-axis where these two curves converge, and volume and market clearing prices are shown on the Y-axis. The months of January 2022 to June 2022 and January 2022 to June 2022, respectively, are designated for the training and testing periods. The day-ahead and real-time market MCP and MCV data sets are available at www.iexindia.com.

Thirty-six variables are considered in the optimization process. We evaluate the recommended process's reliability using mean directional accuracy measurements. In technical terms, the attributes considered in the proposed technique are called market clearing price (MCP) and market clearing volume (MCV). MATLAB algorithms were written, executed in MATLAB 2022 (a), and analyzed for simulation. PSO was continued until convergence, at which time the best individual and each person's fitness were determined.

The integration of GA and PSO increases computational demand, involve advanced hardware for processing large-scale datasets. Additionally, the model's performance under highly volatile energy market conditions requires further investigation to ensure reliability in extreme scenarios. Further validation is also essential for assessing its generalization in multi-source energy forecasting, particularly for renewable energy sources.

The experimental results of the Transformed Fuzzy Random Hidden Markov Parametric Genetic Algorithm with Particle Swarm Optimization Model (TFRHMPGAPSOM) number of iterations and optimal function value for the training dataset's Day Ahead Market from January to June 2021 are displayed in Fig. 10.

The experimental results of the Transformed Fuzzy Random Hidden Markov Parametric Genetic Algorithm with Particle Swarm Optimization Model (TFRHMPGAPSOM) number of iterations and best function value for the training dataset's Day Ahead Market from July 2021 to December 2021 are displayed in Fig. 11.

The experimental results of the Transformed Fuzzy Random Hidden Markov Parametric Genetic Algorithm with Particle Swarm Optimization Model (TFRHMPGAPSOM) number of iterations and optimal function value for the training dataset's real-time market from January to June 2021 are displayed in Fig. 12.

The experimental findings of the Transformed Fuzzy Random Hidden Markov Parametric Genetic Algorithm with Particle Swarm Optimization Model (TFRHMPGAPSOM) number of iterations and optimal function value for the training dataset's real-time market from July 2021 to December 2021 are displayed in Fig. 13.

The experimental findings of the Transformed Fuzzy Random Hidden Markov Parametric Genetic Algorithm with Particle Swarm Optimization Model (TFRHMPGAPSOM) number of iterations and optimal function value are displayed in Fig. 14.

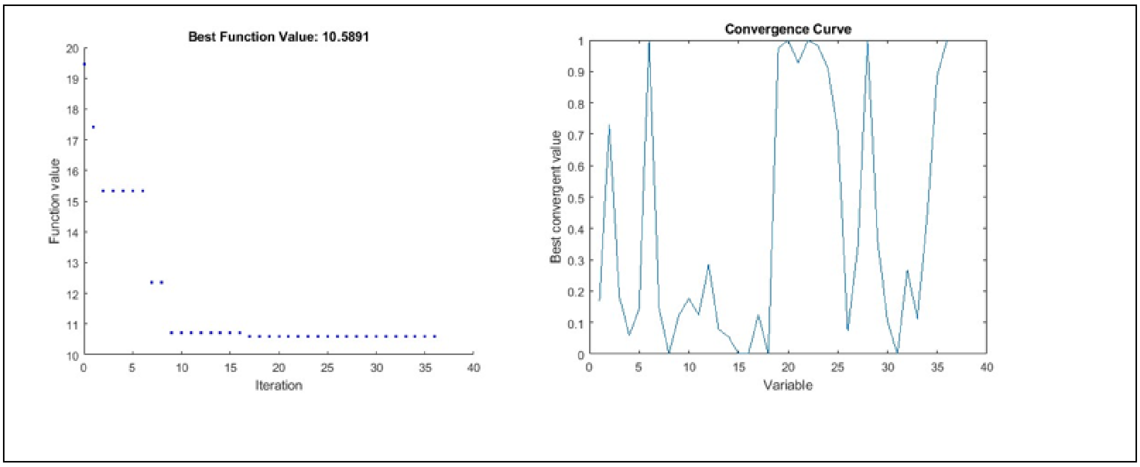

Figure 10: Results of the Transformed Fuzzy Random Hidden Markov Parametric Genetic Algorithm with Particle Swarm Optimization Model (TFRHMPGAPSOM), including the number of iterations, best function value, and convergence curve, for the Day-Ahead Market (DAM) training data-set from January 2021 to June 2021 are presented.

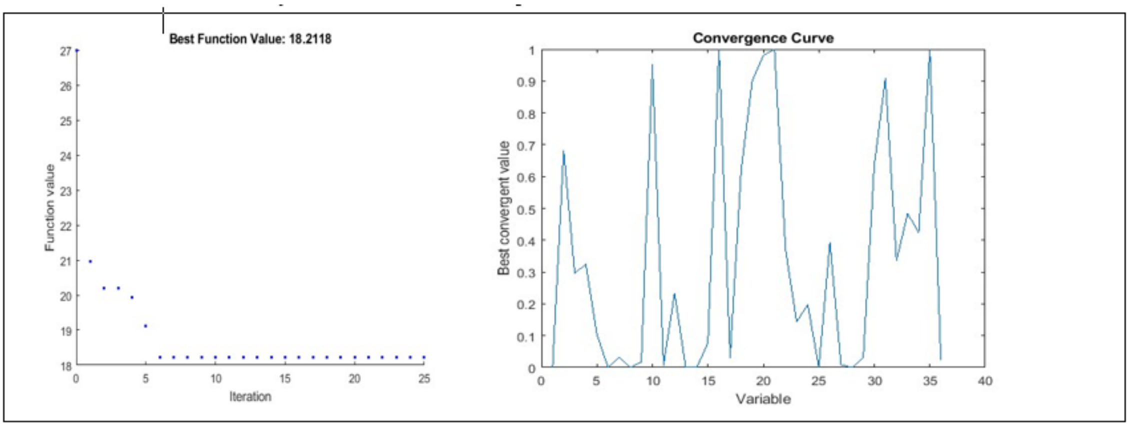

Figure 11: Results of the Transformed Fuzzy Random Hidden Markov Parametric Genetic Algorithm with Particle Swarm Optimization Model (TFRHMPGAPSOM), including the number of iterations, best function value, and convergence curve, for the Day-Ahead Market (DAM) training data-set from July 2021 to December 2021 are presented.

The novel flexible parametric optimization integrated calibration of the Hidden Markov Model with Particle Swarm Optimization and Genetic Algorithm is a perfect fit for all energy market systems. Parameter optimization, based on the idea of the particles with the best function value surviving, ensures the mean directional correctness of the experimental training set from January 2021 to December 2021. For day-ahead markets, this yields a best-optimized Mean Absolute Percentage Error (MAPE) of 14.40045%, and for real-time markets, it yields a value of 20.18595%.

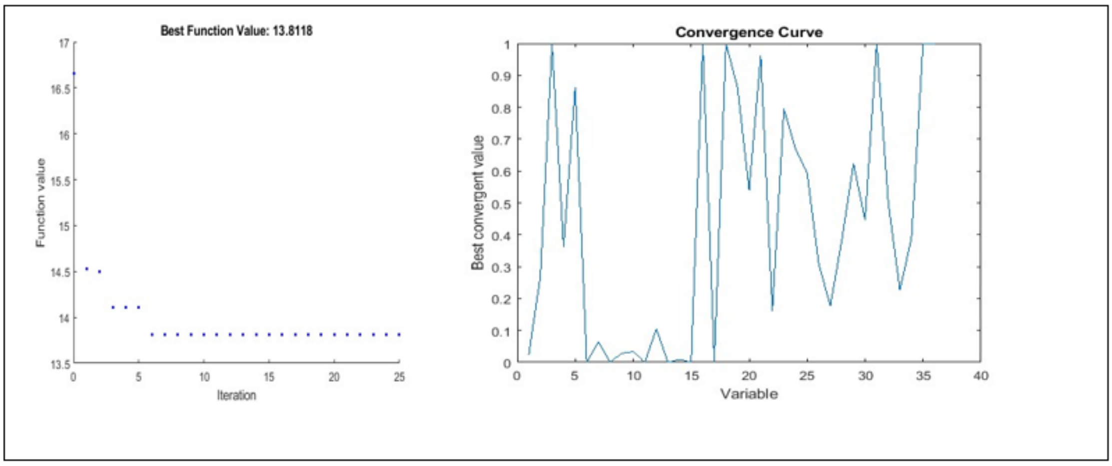

Figure 12: Results of the Transformed Fuzzy Random Hidden Markov Parametric Genetic Algorithm with Particle Swarm Optimization Model (TFRHMPGAPSOM), including the number of iterations, best function value, and convergence curve, for the Real Time Market (RTM) training data-set from January 2021 to June 2021 are presented.

The experimental testing data set, which spans the months of January through June 2022, illustrates the accuracy of mean-directional forecasting. The model's efficiency is indicated by the best optimized Mean Absolute Percentage Error (MAPE) of 23.2440% for real-time markets and 0.944% for day-ahead markets.

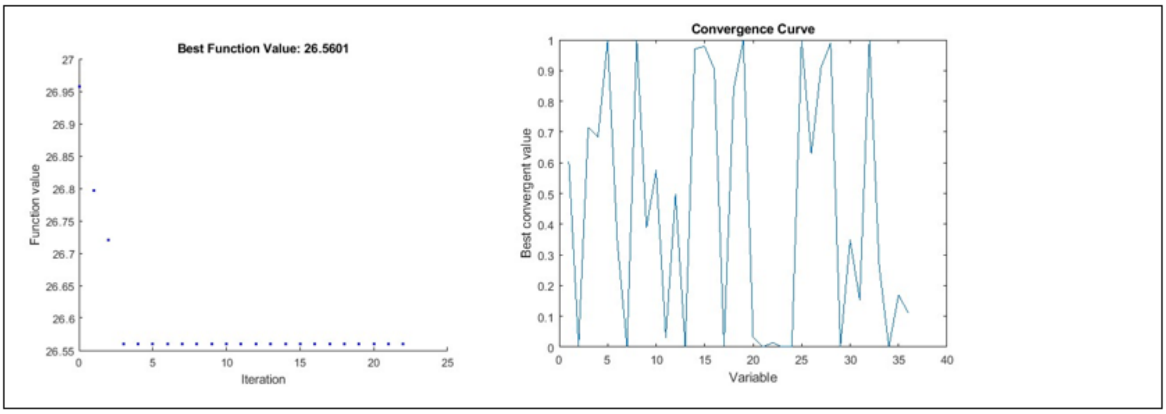

Figure 13: Results of the Transformed Fuzzy Random Hidden Markov Parametric Genetic Algorithm with Particle Swarm Optimization Model (TFRHMPGAPSOM), including the number of iterations, best function value, and convergence curve, for the Real Time Market (RTM) training data-set from July 2021 to December 2021 are presented.

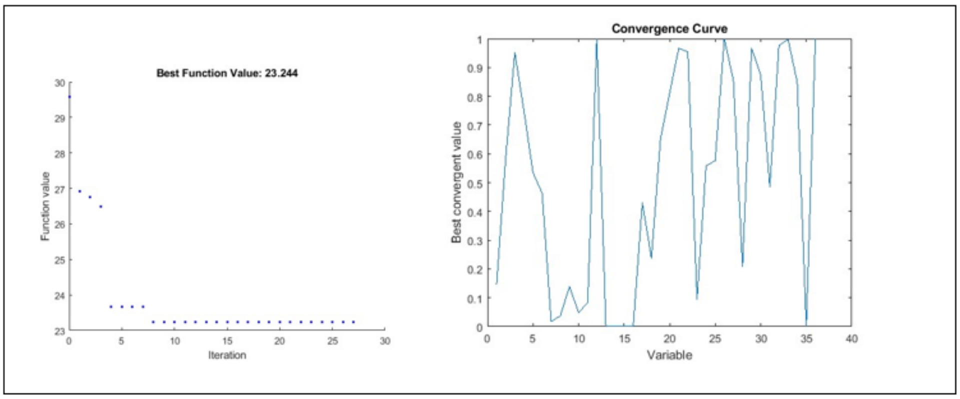

Figure 14: Results of the Transformed Fuzzy Random Hidden Markov Parametric Genetic Algorithm with Particle Swarm Optimization Model (TFRHMPGAPSOM), including the number of iterations, best function value, and convergence curve, for the Real Time Market (RTM) testing data-set from January 2022 to June 2022 are presented.

4. Conclusion

The proposed optimized energy forecasting model, which integrates Hidden Markov Model (HMM), Transformed Fuzzy Relational Matrices (TFRM), Genetic Algorithm (GA), and Particle Swarm Optimization (PSO), demonstrates superior forecasting accuracy and efficiency compared to traditional methods. By leveraging HMM's ability to model temporal dependencies, TFRM's uncertainty handling, and the optimization strengths of GA and PSO, the proposed framework effectively minimizes forecasting errors and computational complexity. Experimental results confirm that this hybrid approach successfully captures the nonlinear and non-stationary characteristics of energy consumption data, making it well-suited for real-time applications in smart grids and energy markets.

Future research can enhance forecasting accuracy by integrating deep learning models like Long Short-Term Memory (LSTM) networks and Transformers, enabling better pattern recognition and long-range dependency modeling. Adaptive optimization algorithms such as Grey Wolf Optimization (GWO), Whale Optimization Algorithm (WOA), and Differential Evolution (DE) can dynamically refine parameter selection, improving convergence and predictive performance. To ensure scalability, deploying the model on large-scale datasets using Apache Spark and Hadoop can enhance computational efficiency.

Multi-objective optimization can balance accuracy, efficiency, and cost, making the model more adaptable to real-world constraints. Extending the approach to renewable energy forecasting will optimize solar, wind, and hybrid energy integration into smart grids. Real-time deployment with a continuous feedback loop will improve adaptability in dynamic markets. Finally, Explainable AI (XAI) will enhance model transparency, fostering trust in energy forecasting decisions. These advancements will create a robust, scalable, and intelligent forecasting framework for smart grids and energy markets.

Table 8: The comparative performance results between the currently employed Transformed Fuzzy Random Hidden Markov Parametric Genetic Algorithm with Particle Swarm Optimization Model(TFRHMPGAPSOM) model and the GAPSO technique for the Day-Ahead Market (DAM) and Real-Time Market (RTM) across both training and testing datasets are presented.

Electricity Market	Data Set	Year	Month		HMPOM	TFRHMP GAPSOM	HMPOM	TFRHMP GAPSOM	HMPOM	TFRHMP GAPSOM
			From	To	MSE		RMSE		MAPE	
Day Ahead Market	Training Set	2021	Jan	June	4.74E+05	2.20E+05	688.226	2.20E+05	17.7031	10.5891
			July	Dec	1.82E+06	1.43E+06	1.35E+03	1.20E+03	22.3853	18.2118
Real Time Market	Training Set	2021	Jan	June	6.47E+05	3.47E+05	804.4875	588.7862	18.6139	13.8118
			July	Dec	5.48E+06	3.98E+06	2.34E+03	1.99E+03	50.0792	26.5601
	Testing Set	2022	Jan	June	5.67E+06	3.16E+06	2.38E+03	1.78E+03	34.1303	23.244

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