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Characterizations of Contra-(τ_1, τ_2)-continuous Functions

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Abstract. This paper introduces a new class of functions between bitopological spaces, namely contra-(τ_1, τ_2)-continuous functions. Furthermore, several characterizations and some properties concerning contra-(τ_1, τ_2)-continuous functions are investigated.

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1. Introduction

The field of the mathematical science which goes under the name of topology is concerned with all questions directly or indirectly related to continuity. Viriyapong and Boonpok [1] investigated some characterizations of (Λ, sp)-continuous functions by utilizing the notions of (Λ, sp)-open sets and (Λ, sp)-closed sets due to Boonpok and Khampakdee [2]. Dungthaisong et al. [3] introduced and studied the concept of $g_{(m,n)}$-continuous functions. Duangphui et al. [4] introduced and investigated the notion of $(\mu, \mu')^{(m,n)}$-continuous functions. Moreover, several characterizations of almost (Λ, p)-continuous functions, strongly $\theta(\Lambda, p)$-continuous functions, almost strongly $\theta(\Lambda, p)$-continuous functions, $\theta(\Lambda, p)$-continuous functions, weakly (Λ, b)-continuous functions, $\theta(\star)$-precontinuous functions, $(\Lambda, p(\star))$-continuous functions, $\star$-continuous functions, θ-$\mathscr{I}$-continuous functions, almost (g, m)-continuous functions, pairwise almost M-continuous functions, faintly (τ_1, τ_2)-continuous functions, $\delta(\tau_1, \tau_2)$-continuous functions and almost nearly (τ_1, τ_2)-continuous functions were presented in [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17] and [18], respectively. The notions of contra-continuity and strong S-closedness in topological spaces were introduced by Dontchev [19]. Dontchev [19] obtained very interesting and important results concerning contra-continuity, compactness, S-closedness and strong S-closedness. Dontchev and Noiri [20] introduced and studied the concept of RC-continuity

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between topological spaces which is weaker than contra-continuity. Jafari and Noiri [21] introduced and investigated a new class of functions called contra-super-continuous functions which lies between classes of RC-continuous functions and contra-continuous functions. Jafari and Noiri [22] introduced a new class of function called contra-precontinuous functions which is weaker than contra-continuous functions and studied several basic properties of contra-precontinuous functions. Furthermore, the present authors [22] defined contra-preclosed graphs and investigated relations between contra-precontinuity and contra-preclosed graphs. Ekici [23] introduced and studied a new class of functions called almost contra-precontinuous functions which generalize classes of regular set-connected functions [24], contra-precontinuous functions [22], contra-continuous functions [19], almost s-continuous functions [25] and perfectly continuous functions [26]. Al-Omari and Noorani [27] introduced the concept of almost contra ω-continuous functions via the notion of ω-open sets and investigated several characterizations of contra ω-continuous functions and almost contra ω-continuous functions. Noiri and Popa [28] introduced the of contra m-continuous functions as functions from a set satisfying some minimal conditions into a topological space and investigated some characterizations and the relationships between contra m-continuity and other related generalized forms of continuity. It turns out that the contra m-continuity is a unified form of several modifications of weak contra-continuity due to Baker [29]. On the other hand, the present authors introduced and studied the notions of (τ_1, τ_2)-continuous functions [30], almost (τ_1, τ_2)-continuous functions [31], weakly (τ_1, τ_2)-continuous functions [32], quasi $\theta(\tau_1, \tau_2)$-continuous functions [33], almost quasi (τ_1, τ_2)-continuous functions [34], weakly quasi (τ_1, τ_2)-continuous functions [35], almost weakly (τ_1, τ_2)-continuous functions [36] and almost contra-(Λ, sp)-continuous functions [37]. In this paper, we introduce the concept of contra-(τ_1, τ_2)-continuous functions. We also investigate some characterizations of contra-(τ_1, τ_2)-continuous functions.

2. Preliminaries

Throughout the present paper, spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or simply X and Y) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a bitopological space (X, τ_1, τ_2). The closure of A and the interior of A with respect to τ_i are denoted by τ_i-Cl(A) and τ_i-Int(A), respectively, for $i = 1, 2$. A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-*closed* [38] if $A = \tau_1\text{-Cl}(\tau_2\text{-Cl}(A))$. The complement of a $\tau_1\tau_2$-closed set is called $\tau_1\tau_2$-*open*. The intersection of all $\tau_1\tau_2$-closed sets of X containing A is called the $\tau_1\tau_2$-*closure* [38] of A and is denoted by $\tau_1\tau_2$-Cl(A). The union of all $\tau_1\tau_2$-open sets of X contained in A is called the $\tau_1\tau_2$-*interior* [38] of A and is denoted by $\tau_1\tau_2$-Int(A).

Lemma 1. [38] *Let A and B be subsets of a bitopological space (X, τ_1, τ_2). For the $\tau_1\tau_2$-closure, the following properties hold:*

(1) $A \subseteq \tau_1\tau_2\text{-}Cl(A)$ and $\tau_1\tau_2\text{-}Cl(\tau_1\tau_2\text{-}Cl(A)) = \tau_1\tau_2\text{-}Cl(A)$.

(2) If $A \subseteq B$, then $\tau_1\tau_2\text{-}Cl(A) \subseteq \tau_1\tau_2\text{-}Cl(B)$.

(3) $\tau_1\tau_2$-*Cl*(A) *is* $\tau_1\tau_2$-*closed.*

(4) A *is* $\tau_1\tau_2$-*closed if and only if* $A = \tau_1\tau_2$-*Cl*(A).

(5) $\tau_1\tau_2$-*Cl*$(X - A) = X - \tau_1\tau_2$-*Int*$(A)$.

A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)r$-*open* [39] (resp. $(\tau_1, \tau_2)s$-*open* [40], $(\tau_1, \tau_2)p$-*open* [40], $(\tau_1, \tau_2)\beta$-*open* [40]) if $A = \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A))$ (resp. $A \subseteq \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A))$, $A \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A))$, $A \subseteq \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A)))$). The complement of a $(\tau_1, \tau_2)r$-open (resp. $(\tau_1, \tau_2)s$-open, $(\tau_1, \tau_2)p$-open, $(\tau_1, \tau_2)\beta$-open) set is said to be $(\tau_1, \tau_2)r$-*closed* (resp. $(\tau_1, \tau_2)s$-*closed*, $(\tau_1, \tau_2)p$-*closed*, $(\tau_1, \tau_2)\beta$-*closed*). A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\alpha(\tau_1, \tau_2)$-*open* [41] if $A \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A)))$. The complement of an $\alpha(\tau_1, \tau_2)$-open set is said to be $\alpha(\tau_1, \tau_2)$-*closed*. Let A be a subset of a bitopological space (X, τ_1, τ_2). The set

$$\cap\{G \mid A \subseteq G \text{ and } G \text{ is } \tau_1\tau_2\text{-open}\}$$

is called the $\tau_1\tau_2$-*kernel* [38] of A and is denoted by $\tau_1\tau_2$-*ker*(A).

Lemma 2. [38] *For subsets A, B of a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) $A \subseteq \tau_1\tau_2$-*ker*(A).

(2) If $A \subseteq B$, *then* $\tau_1\tau_2$-*ker*$(A) \subseteq \tau_1\tau_2$-*ker*$(B)$.

(3) If A *is* $\tau_1\tau_2$-*open, then* $\tau_1\tau_2$-*ker*$(A) = A$.

(4) $x \in \tau_1\tau_2$-*ker*(A) *if and only if* $A \cap H \neq \emptyset$ *for every* $\tau_1\tau_2$-*closed set* H *containing* x.

Let A be a subset of a bitopological space (X, τ_1, τ_2). The intersection of all $(\tau_1, \tau_2)p$-closed sets of X containing A is called the $(\tau_1, \tau_2)p$-*closure* [42] of A and is denoted by (τ_1, τ_2)-pCl(A). The union of all $(\tau_1, \tau_2)p$-open sets of X contained in A is called the $(\tau_1, \tau_2)p$-*interior* [42] of A and is denoted by (τ_1, τ_2)-pInt(A).

Lemma 3. *For a subset A of a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) (τ_1, τ_2)-*pCl*$(A) = \tau_1\tau_2$-*Cl*$(\tau_1\tau_2$-*Int*$(A)) \cup A$ [42];

(2) (τ_1, τ_2)-*pInt*$(A) = \tau_1\tau_2$-*Int*$(\tau_1\tau_2$-*Cl*$(A)) \cap A$ [36].

3. Characterizations of contra-(τ_1, τ_2)-continuous functions

In this section, we introduce the concept of contra-(τ_1, τ_2)-continuous functions. Furthermore, some characterizations of contra-(τ_1, τ_2)-continuous functions are discussed.

Definition 1. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be contra-(τ_1, τ_2)-continuous if $f^{-1}(V)$ is $\tau_1\tau_2$-closed in X for every $\sigma_1\sigma_2$-open set V of Y.*

Theorem 1. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) f is contra-(τ_1, τ_2)-continuous;

(2) $f^{-1}(K)$ is $\tau_1\tau_2$-open in X for every $\sigma_1\sigma_2$-closed set K of Y;

(3) for each $x \in X$ and each $\sigma_1\sigma_2$-closed set K of Y containing $f(x)$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $f(U) \subseteq K$;

(4) $f(\tau_1\tau_2\text{-}Cl(A)) \subseteq \sigma_1\sigma_2\text{-}ker(f(A))$ for every subset A of X;

(5) $\tau_1\tau_2\text{-}Cl(f^{-1}(B)) \subseteq f^{-1}(\sigma_1\sigma_2\text{-}ker(B))$ for every subset B of Y.

Proof. (1) $\Rightarrow$ (2): The proof is obvious.

(2) $\Rightarrow$ (3): Let $x \in X$ and K be any $\sigma_1\sigma_2$-closed set of Y containing $f(x)$. By (2), $f^{-1}(K)$ is $\tau_1\tau_2$-open in X. Then, we have $x \in \tau_1\tau_2\text{-Int}(f^{-1}(K))$ and therefore there exists a $\tau_1\tau_2$-open set U of X containing x such that $U \subseteq f^{-1}(K)$. Thus, $f(U) \subseteq K$.

(3) $\Rightarrow$ (4): Let A be any subset of X. Let $x \in \tau_1\tau_2\text{-Cl}(A)$ and K be any $\sigma_1\sigma_2$-closed set of Y containing $f(x)$. Then by (3), there exists a $\tau_1\tau_2$-open set U of X containing x such that $f(U) \subseteq K$; hence $U \subseteq f^{-1}(K)$. Since $x \in \tau_1\tau_2\text{-Cl}(A)$, $U \cap A \neq \emptyset$ and so $\emptyset \neq f(U \cap A) \subseteq f(U) \cap f(A) \subseteq K \cap f(A)$. By Lemma 2, we have $f(x) \in \sigma_1\sigma_2\text{-}ker(f(A))$ and hence $f(\tau_1\tau_2\text{-Cl}(A)) \subseteq \sigma_1\sigma_2\text{-}ker(f(A))$.

(4) $\Rightarrow$ (5): Let B be any subset of Y. By (4) and Lemma 2, we have

$$f(\tau_1\tau_2\text{-Cl}(f^{-1}(B))) \subseteq \sigma_1\sigma_2\text{-}ker(f(f^{-1}(B))) \subseteq \sigma_1\sigma_2\text{-}ker(B)$$

and hence $\tau_1\tau_2\text{-Cl}(f^{-1}(B)) \subseteq f^{-1}(\sigma_1\sigma_2\text{-}ker(B))$.

(5) $\Rightarrow$ (1): Let V be any $\sigma_1\sigma_2$-open set of Y. Then by (5) and Lemma 2, we have $\tau_1\tau_2\text{-Cl}(f^{-1}(V)) \subseteq f^{-1}(\sigma_1\sigma_2\text{-}ker(V)) = f^{-1}(V)$ and so $f^{-1}(V)$ is $\tau_1\tau_2$-closed in X. This shows that f is contra-(τ_1, τ_2)-continuous.

Definition 2. [32] *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be weakly (τ_1, τ_2)-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open set V of Y containing $f(x)$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $f(U) \subseteq \sigma_1\sigma_2\text{-}Cl(V)$. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be weakly (τ_1, τ_2)-continuous if f has this property at each point of X.*

Theorem 2. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is contra-(τ_1, τ_2)-continuous, then f is weakly (τ_1, τ_2)-continuous.*

Proof. Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$. Then, $\sigma_1\sigma_2\text{-Cl}(V)$ is a $\sigma_1\sigma_2$-closed set of Y containing $f(x)$. Since f is contra-(τ_1, τ_2)-continuous, by Theorem 1 there exists a $\tau_1\tau_2$-open set U of X containing x such that $f(U) \subseteq \sigma_1\sigma_2\text{-Cl}(V)$. This shows that f is weakly (τ_1, τ_2)-continuous.

Definition 3. [31] *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be almost (τ_1, τ_2)-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open set V of Y containing $f(x)$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $f(U) \subseteq \sigma_1\sigma_2$-Int($\sigma_1\sigma_2$-Cl($V$)). A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be almost (τ_1, τ_2)-continuous if f has this property at each point of X.*

Definition 4. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called almost (τ_1, τ_2)-open if $f(U) \subseteq \sigma_1\sigma_2$-Int($\sigma_1\sigma_2$-Cl($f(U)$)) for every $\tau_1\tau_2$-open set U of X.*

Theorem 3. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is contra-(τ_1, τ_2)-continuous and almost (τ_1, τ_2)-open, then f is almost (τ_1, τ_2)-continuous.*

Proof. Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$. Then, $\sigma_1\sigma_2$-Cl(V) is a $\sigma_1\sigma_2$-closed set of Y containing $f(x)$. Since f is contra-(τ_1, τ_2)-continuous, by Theorem 1 there exists a $\tau_1\tau_2$-open set U of X containing x such that $f(U) \subseteq \sigma_1\sigma_2$-Cl($V$). Since f is almost (τ_1, τ_2)-open, we have $f(U) \subseteq \sigma_1\sigma_2$-Int($\sigma_1\sigma_2$-Cl($f(U)$)) $\subseteq \sigma_1\sigma_2$-Int($\sigma_1\sigma_2$-Cl($V$)) and hence f is almost (τ_1, τ_2)-continuous.

Recall that a bitopological space (X, τ_1, τ_2) is said to be *almost (τ_1, τ_2)-regular* [43] if for each $(\tau_1, \tau_2)r$-closed set F and each $x \notin F$, there exist disjoint $\tau_1\tau_2$-open sets U and V such that $x \in U$ and $F \subseteq V$.

Lemma 4. [43] *A bitopological space (X, τ_1, τ_2) is almost (τ_1, τ_2)-regular if and only if for each $x \in X$ and each $(\tau_1, \tau_2)r$-open set U with $x \in U$, there exists a $\tau_1\tau_2$-open set V such that $x \in V \subseteq \tau_1\tau_2$-Cl($V$) $\subseteq U$.*

Theorem 4. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is contra-(τ_1, τ_2)-continuous and (Y, σ_1, σ_2) is almost (σ_1, σ_2)-regular, then f is almost (τ_1, τ_2)-continuous.*

Proof. Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$. Then, we have $\sigma_1\sigma_2$-Int($\sigma_1\sigma_2$-Cl(V)) is $(\sigma_1, \sigma_2)r$-open in Y. Since (Y, σ_1, σ_2) is almost (σ_1, σ_2)-regular, by Lemma 4 there exists a $\sigma_1\sigma_2$-open set W of Y such that

$$f(x) \in W \subseteq \sigma_1\sigma_2\text{-Cl}(W) \subseteq \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)).$$

Since f is contra-(τ_1, τ_2)-continuous and $\sigma_1\sigma_2$-Cl(W) is $\sigma_1\sigma_2$-closed in Y, by Theorem 1 there exists a $\tau_1\tau_2$-open set U of X containing x such that

$$f(U) \subseteq \sigma_1\sigma_2\text{-Cl}(W) \subseteq \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)).$$

Thus, f is almost (τ_1, τ_2)-continuous.

Lemma 5. [31] *For a function $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) f is almost (τ_1, τ_2)-continuous at $x \in X$;

(2) $x \in \tau_1\tau_2\text{-}Int(f^{-1}(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V))))$ *for every* $\sigma_1\sigma_2$*-open set* V *of* Y *containing* $f(x)$*;*

(3) $x \in \tau_1\tau_2\text{-}Int(f^{-1}(V))$ *for every* $(\sigma_1, \sigma_2)r$*-open set* V *of* Y *containing* $f(x)$*;*

(4) for each $(\sigma_1, \sigma_2)r$*-open set* V *of* Y *containing* $f(x)$*, there exists a* $\tau_1\tau_2$*-open set* U *of* X *containing* x *such that* $f(U) \subseteq V$*.*

Recall that a bitopological space (X, τ_1, τ_2) is said to be (τ_1, τ_2)-*extremally disconnected* [41] if the $\tau_1\tau_2$-closure of every $\tau_1\tau_2$-open set U of X is $\tau_1\tau_2$-open.

Lemma 6. [41] *For a bitopological space* (X, τ_1, τ_2)*, the following properties are equivalent:*

(1) (X, τ_1, τ_2) *is* (τ_1, τ_2)*-extremally disconnected;*

(2) every $(\tau_1, \tau_2)r$*-open set of* X *is* $\tau_1\tau_2$*-closed;*

(3) every $(\tau_1, \tau_2)r$*-closed set of* X *is* $\tau_1\tau_2$*-open.*

Theorem 5. *If* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is contra-*(τ_1, τ_2)*-continuous and* (Y, σ_1, σ_2) *is* (σ_1, σ_2)*-extremally disconnected, then* f *is almost* (τ_1, τ_2)*-continuous.*

Proof. Let $x \in X$ and V be any $(\sigma_1, \sigma_2)r$-open set of Y containing $f(x)$. Since (Y, σ_1, σ_2) is (σ_1, σ_2)-extremally disconnected, by Lemma 6 we have V is $\sigma_1\sigma_2$-clopen. Since f is contra-(τ_1, τ_2)-continuous, by Theorem 1 there exists a $\tau_1\tau_2$-open set U of X containing x such that $f(U) \subseteq V$. Thus by Lemma 5, f is almost (τ_1, τ_2)-continuous.

Definition 5. [30] *A function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is called* (τ_1, τ_2)*-continuous at a point* $x \in X$ *if for each* $\sigma_1\sigma_2$*-open set* V *of* Y *containing* $f(x)$*, there exists a* $\tau_1\tau_2$*-open set* U *of* X *containing* x *such that* $f(U) \subseteq V$*. A function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is called* (τ_1, τ_2)*-continuous if* f *has this property at each point of* X*.*

Lemma 7. [30] *For a function* $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$*, the following properties are equivalent:*

(1) f *is* (τ_1, τ_2)*-continuous;*

(2) $f^{-1}(V)$ *is* $\tau_1\tau_2$*-open in* X *for every* $\sigma_1\sigma_2$*-open set* V *of* Y*;*

(3) $f(\tau_1\tau_2\text{-}Cl(A)) \subseteq \sigma_1\sigma_2\text{-}Cl(f(A))$ *for every subset* A *of* X*;*

(4) $\tau_1\tau_2\text{-}Cl(f^{-1}(B)) \subseteq f^{-1}(\sigma_1\sigma_2\text{-}Cl(B))$ *for every subset* B *of* Y*;*

(5) $f^{-1}(\sigma_1\sigma_2\text{-}Int(B)) \subseteq \tau_1\tau_2\text{-}Int(f^{-1}(B))$ *for every subset* B *of* Y*;*

(6) $f^{-1}(K)$ *is* $\tau_1\tau_2$*-closed in* X *for every* $\sigma_1\sigma_2$*-closed set* K *of* Y*.*

Definition 6. *A function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is said to satisfy the* (τ_1, τ_2)*-interiority condition if* $\tau_1\tau_2\text{-}Int(f^{-1}(\sigma_1\sigma_2\text{-}Cl(V))) \subseteq f^{-1}(V)$ *for each* $\sigma_1\sigma_2$*-open set* V *of* Y*.*

Theorem 6. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is contra-(τ_1, τ_2)-continuous and satisfy the (τ_1, τ_2)-interiority condition, then f is (τ_1, τ_2)-continuous.*

Proof. Let V be any $\sigma_1\sigma_2$-open set of Y. Since f is contra-(τ_1, τ_2)-continuous, by Theorem 1 we have

$$\begin{aligned} f^{-1}(V) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(V)) &= \tau_1\tau_2\text{-Int}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(V))) \\ &= \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Int}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(V)))) \\ &\subseteq \tau_1\tau_2\text{-Int}(f^{-1}(V)) \subseteq f^{-1}(V) \end{aligned}$$

and hence $f^{-1}(V)$ is $\tau_1\tau_2$-open in X. By Lemma 7, f is (τ_1, τ_2)-continuous.

Definition 7. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to have a contra (τ_1, τ_2)-closed graph if for each $(x, y) \in (X \times Y) - G(f)$, there exist a $\tau_1\tau_2$-open set U of X containing x and a $\sigma_1\sigma_2$-closed set F of Y containing y such that $(U \times F) \cap G(f) = \emptyset$.*

Lemma 8. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ has a contra (τ_1, τ_2)-closed graph if and only if for each $(x, y) \in (X \times Y) - G(f)$, there exist a $\tau_1\tau_2$-open set U of X containing x and a $\sigma_1\sigma_2$-closed set F of Y containing y such that $f(U) \cap F = \emptyset$.*

Definition 8. [44] *A bitopological space (X, τ_1, τ_2) is said to be $\tau_1\tau_2$-Urysohn if for each pair of distinct points x and y in X, there exist $\tau_1\tau_2$-open sets U and V such that $x \in U$, $y \in V$ and $\tau_1\tau_2$-Cl$(U) \cap \tau_1\tau_2$-Cl$(V) = \emptyset$.*

Theorem 7. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is contra-(τ_1, τ_2)-continuous and (Y, σ_1, σ_2) is $\sigma_1\sigma_2$-Urysohn, then $G(f)$ is contra (τ_1, τ_2)-closed.*

Proof. Let $(x, y) \in (X \times Y) - G(f)$. Then, $y \neq f(x)$. Since (Y, σ_1, σ_2) is $\sigma_1\sigma_2$-Urysohn, there exist $\sigma_1\sigma_2$-open sets V and W of Y containing y and $f(x)$, respectively, such that $\sigma_1\sigma_2$-Cl$(V) \cap \sigma_1\sigma_2$-Cl$(W) = \emptyset$. Since f is contra-(τ_1, τ_2)-continuous, by Theorem 1 there exists a $\tau_1\tau_2$-open set U of X containing x such that $f(U) \subseteq \sigma_1\sigma_2$-Cl$(W)$. This implies that $f(U) \cap \sigma_1\sigma_2$-Cl$(V) = \emptyset$ and by Lemma 8, $G(f)$ is contra (τ_1, τ_2)-closed.

Recall that a bitopological space (X, τ_1, τ_2) is said to be (τ_1, τ_2)-T_1 [45] if for any pair of distinct points x, y in X, there exist $\tau_1\tau_2$-open sets U and V such that $x \in U$, $y \notin U$ and $y \in V$, $x \notin V$.

Theorem 8. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is contra-(τ_1, τ_2)-continuous and (Y, σ_1, σ_2) is (σ_1, σ_2)-T_1, then $G(f)$ is contra (τ_1, τ_2)-closed.*

Proof. Let $(x, y) \in (X \times Y) - G(f)$. Then, $y \neq f(x)$. Since (Y, σ_1, σ_2) is (σ_1, σ_2)-T_1, there exists a $\sigma_1\sigma_2$-open set V of Y such that $f(x) \in V$ and $y \notin V$. Since f is (τ_1, τ_2)-continuous, there exists a $\tau_1\tau_2$-open set U of X containing x such that $f(U) \subseteq V$. Thus, $f(U) \cap (Y - V) = \emptyset$ and $Y - V$ is a $\sigma_1\sigma_2$-closed set of Y containing y. This shows that $G(f)$ is contra (τ_1, τ_2)-closed.

Definition 9. [46] *A bitopological space (X, τ_1, τ_2) is said to be (τ_1, τ_2)-T_2 if for any pair of distinct points x, y in X, there exist disjoint $\tau_1\tau_2$-open sets U and V of X containing x and y, respectively.*

Theorem 9. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a contra-(τ_1, τ_2)-continuous injection with a contra (τ_1, τ_2)-closed graph, then (X, τ_1, τ_2) is (τ_1, τ_2)-T_2.*

Proof. Let x and y be any distinct points of X. Since f is injective, $f(x) \neq f(y)$. Then, we have $(x, f(y)) \in (X \times Y) - G(f)$. Since $G(f)$ is contra (τ_1, τ_2)-closed, by Lemma 8 there exist a $\tau_1\tau_2$-open set U of X containing x and a $\sigma_1\sigma_2$-closed set K of Y containing $f(y)$ such that $f(U) \cap K = \emptyset$. Since f is contra-(τ_1, τ_2)-continuous, by Theorem 1 there exists a $\tau_1\tau_2$-open set U_0 of X containing y such that $f(U_0) \subseteq K$. Thus, $f(U) \cap f(U_0) = \emptyset$ and hence $U \cap U_0 = \emptyset$. This shows that (X, τ_1, τ_2) is (τ_1, τ_2)-T_2.

Theorem 10. *Let (X, τ_1, τ_2) be a bitopological space. If for each pair of distinct points x and x' in X, there exists a function f of (X, τ_1, τ_2) into a $\sigma_1\sigma_2$-Urysohn space (Y, σ_1, σ_2) such that $f(x) \neq f(x')$ and f is contra-(τ_1, τ_2)-continuous at x and x', then (X, τ_1, τ_2) is (τ_1, τ_2)-T_2.*

Proof. Let x and x' be any distinct points of X. Then by the hypothesis, there exists a $\sigma_1\sigma_2$-Urysohn space (Y, σ_1, σ_2) and a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ which satisfies the conditions of this theorem. Let $y = f(x)$ and $y' = f(x')$. Then, $y \neq y'$. Since (Y, σ_1, σ_2) is $\sigma_1\sigma_2$-Urysohn, there exist $\sigma_1\sigma_2$-open sets V and W of Y containing y and y', respectively, such that $\sigma_1\sigma_2\text{-Cl}(V) \cap \sigma_1\sigma_2\text{-Cl}(W) = \emptyset$. Since f is contra-(τ_1, τ_2)-continuous at x and x', by Theorem 1 there exist $\tau_1\tau_2$-open sets U and U' of X containing x and x', respectively, such that $f(U) \subseteq \sigma_1\sigma_2\text{-Cl}(V)$ and $f(U') \subseteq \sigma_1\sigma_2\text{-Cl}(W)$. This implies that $U \cap U' = \emptyset$. Thus, (X, τ_1, τ_2) is (τ_1, τ_2)-T_2.

Corollary 1. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a contra-(τ_1, τ_2)-continuous injection and (Y, σ_1, σ_2) is $\sigma_1\sigma_2$-Urysohn, then (X, τ_1, τ_2) is (τ_1, τ_2)-T_2.*

Proof. For each pair of distinct points x and x' in X, f is a contra-(τ_1, τ_2)-continuous function of (X, τ_1, τ_2) into a $\sigma_1\sigma_2$-Urysohn space (Y, σ_1, σ_2) such that $f(x) \neq f(x')$ because f is injective. Thus by Theorem 10, (X, τ_1, τ_2) is (τ_1, τ_2)-T_2.

Definition 10. *A bitopological space (X, τ_1, τ_2) is said to be ultra-$\tau_1\tau_2$-Huasdorff if for each pair of distinct points x and y in X, there exist $\tau_1\tau_2$-clopen sets U and V of X containing x and y, respectively, such that $U \cap V = \emptyset$.*

Theorem 11. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a contra-(τ_1, τ_2)-continuous injection and (Y, σ_1, σ_2) is ultra-$\sigma_1\sigma_2$-Hausdorff, then (X, τ_1, τ_2) is (τ_1, τ_2)-T_2.*

Proof. Let x and y be any distinct points in X. Then, since f is injective, $f(x) \neq f(y)$. Moreover, since (Y, σ_1, σ_2) is ultra-$\sigma_1\sigma_2$-Hausdorff, there exist $\sigma_1\sigma_2$-clopen sets V and W of Y containing x and y, respectively, such that $V \cap W = \emptyset$. Since f is contra-(τ_1, τ_2)-continuous, by Theorem 1 there exist $\tau_1\tau_2$-open sets U and G of X containing x and y, respectively, such that $f(U) \subseteq V$ and $f(G) \subseteq W$. Thus, $U \cap G = \emptyset$ and hence (X, τ_1, τ_2) is (τ_1, τ_2)-T_2.

Recall that a bitopological space (X, τ_1, τ_2) is said to be $\tau_1\tau_2$-*compact* [38] if every cover of X by $\tau_1\tau_2$-open sets of X has a finite subcover. A bitopological space (X, τ_1, τ_2) is said to be *quasi* (τ_1, τ_2)-$\mathscr{H}$-*closed* [47] if every $\tau_1\tau_2$-open cover $\{U_\gamma \mid \gamma \in \nabla\}$, there exists a finite subset ∇_0 of ∇ such that $X = \cup\{\tau_1\tau_2\text{-Cl}(U_\gamma) \mid \gamma \in \nabla_0\}$.

Definition 11. *A bitopological space (X, τ_1, τ_2) is said to be strongly S-$\tau_1\tau_2$-closed if every cover of X by $\tau_1\tau_2$-closed sets of X has a finite subcover.*

Definition 12. *A bitopological space (X, τ_1, τ_2) is said to be S-$\tau_1\tau_2$-closed if every $(\tau_1, \tau_2)s$-open cover $\{U_\gamma \mid \gamma \in \nabla\}$, there exists a finite subset ∇_0 of ∇ such that*

$$X = \cup\{\tau_1\tau_2\text{-}Cl(U_\gamma) \mid \gamma \in \nabla_0\}.$$

Theorem 12. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a contra-(τ_1, τ_2)-continuous surjection and (X, τ_1, τ_2) is $\tau_1\tau_2$-compact, then (Y, σ_1, σ_2) is strongly S-$\sigma_1\sigma_2$-closed.*

Proof. Suppose that (X, τ_1, τ_2) is $\tau_1\tau_2$-compact. Let $\{V_\gamma \mid \gamma \in \nabla\}$ be any cover of Y by $\sigma_1\sigma_2$-closed sets of Y. For each $x \in X$, there exists $\gamma(x) \in \nabla$ such that $f(x) \in V_{\gamma(x)}$. Since f is contra-(τ_1, τ_2)-continuous, by Theorem 1 there exists a $\tau_1\tau_2$-open set $U(x)$ containing x such that $f(U(x)) \subseteq V_{\gamma(x)}$. The family $\{U(x) \mid x \in X\}$ is a cover of X by $\tau_1\tau_2$-open sets. Since (X, τ_1, τ_2) is $\tau_1\tau_2$-compact, there exists a finite number of pints, say, $x_1, x_2, x_3, ..., x_n$ in X such that $X = \cup\{U(x_k) \mid x_k \in X; 1 \leq k \leq n\}$. Thus, $Y = f(X) = \cup\{f(U(x_k)) \mid x_k \in X; 1 \leq k \leq n\} \subseteq \cup\{V_{\gamma(x_k)} \mid x_k \in X; 1 \leq k \leq n\}$. This shows that (Y, σ_1, σ_2) is strongly S-$\sigma_1\sigma_2$-closed.

Corollary 2. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a contra-(τ_1, τ_2)-continuous surjection and (X, τ_1, τ_2) is $\tau_1\tau_2$-compact, then (Y, σ_1, σ_2) is S-$\sigma_1\sigma_2$-closed and hence quasi (σ_1, σ_2)-$\mathscr{H}$-closed.*

Recall that a bitopological space (X, τ_1, τ_2) is said to be $\tau_1\tau_2$-*connected* [38] if X cannot be written as the union of two nonempty disjoint $\tau_1\tau_2$-open sets.

Theorem 13. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a contra-(τ_1, τ_2)-continuous surjection and (X, τ_1, τ_2) is $\tau_1\tau_2$-connected, then (Y, σ_1, σ_2) is $\sigma_1\sigma_2$-connected.*

Proof. Assume that (Y, σ_1, σ_2) is not $\sigma_1\sigma_2$-connected. Then, there exist $\sigma_1\sigma_2$-open sets V and W of Y such that $V \cap W = \emptyset$ and $V \cup W = Y$. Thus, we have $f^{-1}(V) \cap f^{-1}(W) = \emptyset$ and $f^{-1}(V) \cup f^{-1}(W) = X$. Since f is surjective, $f^{-1}(V) \neq \emptyset$ and $f^{-1}(W) \neq \emptyset$. Since f is contra-(τ_1, τ_2)-continuous and V, W are $\sigma_1\sigma_2$-clopen sets, by Theorem 1 we have $f^{-1}(V)$ and $f^{-1}(W)$ are $\tau_1\tau_2$-open in X. Therefore, (X, τ_1, τ_2) is not $\tau_1\tau_2$-connected.

The $\tau_1\tau_2$-*frontier* [31] of a subset A of a bitopological space (X, τ_1, τ_2), denoted by $\tau_1\tau_2$-fr(A), is defined by

$$\tau_1\tau_2\text{-fr}(A) = \tau_1\tau_2\text{-Cl}(A) \cap \tau_1\tau_2\text{-Cl}(X - A) = \tau_1\tau_2\text{-Cl}(A) - \tau_1\tau_2\text{-Int}(A).$$

Theorem 14. *The set of all points $x \in X$ at which a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is not contra-(τ_1, τ_2)-continuous is identical with the union of the $\tau_1\tau_2$-frontier of the inverse images of $\sigma_1\sigma_2$-closed sets of Y containing $f(x)$.*

Proof. Suppose that f is not contra-(τ_1, τ_2)-continuous at $x \in X$. Then, there exists a $\sigma_1\sigma_2$-closed set K of Y containing $f(x)$ such that $f(U) \cap (Y - K) \neq \emptyset$ for every $\tau_1\tau_2$-open set U of X containing x. Thus, $x \in \tau_1\tau_2\text{-Cl}(f^{-1}(Y - K)) = \tau_1\tau_2\text{-Cl}(X - f^{-1}(K))$. On the other hand, we have $x \in f^{-1}(K) \subseteq \tau_1\tau_2\text{-Cl}(f^{-1}(K))$ and hence $x \in \tau_1\tau_2\text{-fr}(f^{-1}(K))$.

Conversely, suppose that f is contra-(τ_1, τ_2)-continuous at $x \in X$. Let K be any $\sigma_1\sigma_2$-closed set of Y containing $f(x)$. By Theorem 1, $x \in f^{-1}(K) = \tau_1\tau_2\text{-Int}(f^{-1}(K))$. Thus, $x \notin \tau_1\tau_2\text{-fr}(f^{-1}(K))$ for every $\sigma_1\sigma_2$-closed set K of Y containing $f(x)$. This completes the proof.

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