EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
2025, Vol. 18, Issue 2, Article Number 6049
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

Petroleum Well Site Selection Using MCRAT Integrated with Rough Set Theory

Shaaban M. Shaaban[1], Anas M. El-Sherif[2], Ahmed A. Hammam[3], Husham M. Attaalfadeel[4,*], Yehya I. Mesalam[5,6]

[1] *Center for Scientific Research and Entrepreneurship, Northern Border University, Arar 73213, Saudi Arabia*
[2] *Engineering College, Northern Border University, Arar 1321, Saudi Arabia*
[3] *General Courses Department, Northern Border University, Arar 1321, Saudi Arabia*
[4] *Department of Mathematics, College of Science, Northern Border University, Arar 91431, Saudi Arabia*
[5] *Industrial Engineering Department, Faculty of Engineering, Zagazig University, Zagazig 44519, Egypt*
[6] *Department of Industrial Engineering, College of Engineering, Northern Border University, Arar 1321, Saudi Arabia*

Abstract. Shale Oil (SO) has emerged as an attractive additional supply of conventional crude oil throughout the world in recent years. Shale oil quality evaluation includes a variety of geochemical parameters. In this research, we propose a novel Integrated shale oil evaluation approach. This method firstly determines the relative weights of parameters using rough set theory. Finally, Multiple Criteria Ranking by Alternative Trace (MCRAT) technique is utilized to calculate the rank of Shale oil wells. Twenty-seven samples of shale oil were collected from various distinct well sites. Twelve different geochemical parameters were examined in the gathered samples in order to determine the grade of the shale oil. The results reveal that shale oil sample 17 geochemical parameters, with shale oil grade I is the best collection parameters concentrations. The suggested approach is compared to five Multi-Criteria Decision Making (MCDM) approaches to demonstrate its effectiveness. The obtained results gained by the MCRAT Integrated with Rough Set Theory (RST) is clear in ideas and easy in computation, which may be effectively applied to address a variety of problems, both similar and different.

2020 Mathematics Subject Classifications: 90B50, 91B06, 62C99, 03E72

Key Words and Phrases: Multi criteria decision making (MCDM), shale oil, multiple criteria ranking by alternative trace (MCRAT), rough set theory (RST)

---

*Corresponding author.
DOI: https://doi.org/10.29020/nybg.ejpam.v18i2.6049

Email addresses: hushamalhassan@nbu.edu.sa (H. M. Attaalfadeel), ymesalam@yahoo.com (Y. I. Mesalam)

1. Introduction

Exploration in the petroleum sector is primarily concerned with locating commercially recoverable reserves. Exploring for petroleum is a complicated and uncertain process that is expensive and complex. An important step in petroleum exploration is the modelling of the petroleum location. The potential modelling is important step to decide where the well will be drilled to avoid the high cost needed in petroleum location because of uncertainty and complexity process. The evaluation of the petroleum location for undiscovered petroleum accumulations involves integrating surface and subsurface datasets, including information on geology, geochemistry, geophysics, petrophysics, and geography.

Numerous data-based approaches have been created and effectively used in mineral potential mapping (MPM) during the last few decades as Decision-tree analysis (DTA) using a geographic information system (GIS) [1], certainty factor C-F model [2], Wildcat modelling [3], Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE II) [4], Elimination and Choice Translation Reality III (ELECTRE III) technique [5], structural surface-restoration and logistic regression (LR) analysis [6], Boost-WofE is new algorithm which using a weighted training sample to implement weights [7], Bayesian Network classifier using Naive Bayes (NB) [8], Random Forests (RF) algorithm [9], index overlay integration method [10], Data Envelopment Analysis (DEA) technique [11], Fuzzy Logic Analysis [12], Analytic Hierarchy Process - Technique for Order Preference by Similarity to Ideal Solution (AHP–TOPSIS) algorithm [13], TOPSIS method [14–16], fuzzy comprehensive assessment model with entropy weights (FCAEW) method [17], extreme learning machine (ELM) [18], maximum entropy (MaxEnt) model [19], weights-of-evidence (WofE) method [20], artificial neural networks (ANN) and random forest(RF) [21], isolation forest model based on data-mining algorithm [22], Fuzzy logic, an outranking technique, and the Artificial Bee Colony (ABC) optimization algorithm are combined to create the optimized fuzzy ELECTRE (OFE) methodology [23], Evaluation based on Distance from Average Solution (EDAS) method [24, 25], Combinative Distance based Assessment (CODAS) method [26, 27], Weighted Aggregated Sum Product Assessment (WASPAS) method [28].

However, a geographical analysis of identified petroleum reservoirs within an area of interest is not required by knowledge-based approaches. Some examples of these methods include Dempster-Shafer belief theory, Boolean logic, outranking techniques, fuzzy logic, and index overlay. Expert knowledge is applied to select the parameters of models [29, 30]. The research [31] provides a knowledge-driven method based on digital maps (GIS) of geology, heat flow, young faults, and young volcanism, to evaluate geothermal systems. Also, [32] provides data-driven methods in order to model petroleum potential in a geographical context [33] provides a hybrid fuzzy weights-of-evidence (WofE) approach that uses information-based fuzzy membership values and data-based conditional probabilities to produce fuzzy predictor patterns. In a geographic information system (GIS) environment [34] integrated primary geological control factors on petroleum occurrence to determine which regions have the most petroleum potential for further investigation using weights of evidence.

Numerous scholars have used a variety of strategies in an effort to evaluate petroleum potential. over the past few years. In [35] Tounsi created an expert system called approximation fuzzy assessment (AFA) based on the strength of fuzzy set and possibility theories for evaluating oil potential in Algeria's Hassi Messaoud areas. The petroleum potential was evaluated in a non-spatial framework, despite the system's ability to manage data with ambiguity. Chen and Osadetz [36] suggested a model-based simulation strategy employing the Fourier transform algorithm to construct a petroleum accumulation map. In [37] the author developed a GIS-based fuzzy multi criteria evaluation model for determining petroleum potential. A fractal model was used by Chen and Osadetz in [38] to replicate how unknown oil and gas deposits would be distributed geographically.

Multi-criteria decision-making (MCDM) techniques have been applied more often in recent years to address a wide range of issues in several scientific domains [39, 40]. One of the scientific domains where the use of MCDM [41, 42] techniques is essential for developing a trustworthy model in the decision-making process is engineering, medical, and business [43, 44]. Accordingly, a vast array of MCDM techniques have been created to address these kinds of challenging issues. As new MCDM techniques advanced quickly, techniques for determining the weights of criteria were also created. This indicates that a considerable portion of the best answer is determined by the weights of the criterion. Criteria weight evaluation is a major issue in MCDM approaches, as it significantly impacts decision-making outcomes. In this paper rough set theory (RST) is used for evaluating the weights of attributes. Professor Pawlak put up a theory of rough sets in 1982 [45], which offers a structured method for handling vague or insufficient data.

Even though previous researchers have conducted a significant amount of research on petroleum well site selection using various MCDM techniques, a straightforward and methodical mathematical approach is still required to help the decision maker choose the right site for a given engineering context.

this study developed an integrated MCRAT Rough Set Theory approach for shale oil quality classification. The integrated model overcomes the limitations of single algorithms and maintains good predictive performance, providing a reliable and practical solution for shale oil classification in engineering context.

The rest of the article is structured as follows: Section 2 explains the process methodology of the proposed approach, while results, validations and discussions are presented in Section 3. Lastly, the final section gives the research results, and the paper concludes with a list of references.

2. Methods

2.1. Preliminaries of rough Set Theory

RST is a mathematical data processing technology presented by Polish academic Pawlak. Incomplete, imprecise, and erroneous data may be processed and mined efficiently using this strategy [45]. RST aids in eliminating redundant features from high-dimensional datasets. RST provides an objective description of uncertain problems without requiring additional information beyond the present dataset. Expert subjectivity and empirical knowledge may be avoided by using a weighting approach based on the knowledge granularity and attribute significance of rough sets. It calculates the weight after analyzing the association between data and indicators objectively. We'll go over the RST related ideas in this part.

2.2. Decision table

In RST, a data set is represented as a table, with each row representing a process, an instance, or a simple object. The observable properties of an item are represented in each of the table's columns [45]. This table is referred to as a decision table (DT). Any 4-tuple $DT = (U, A, V_a, F_a)$ represents a decision table in more formal terms. Where U represents a nonempty finite set of objects, process or cases (data from experiments) called universe, A represents a set of primitive features for objects; $A = C \cup D$, where C represents a condition attribute set (input) and D represents a decision attribute set (output), for each $a \in A$, the set V_a contains all possible values of attribute a, $F_a : U \to V_a$ is called the decision function.

2.3. Indiscernibility relation

Let $DT = (U, A, V_a, F_a)$ be a decision table, $(B \subseteq A)$. The binary relation $IND(B)$ called indiscernibility relation, It gathers indistinguishable objects, defined here as those having the same a (characteristic values) with regard to R. $IND(B)$ is defined by

$$IND(B) = \left\{(x, y) \in U^2 : \forall a \in R, a(x) = a(y)\right\} \tag{1}$$

So, $IND(B)$ is an equivalent relation and $IND(B) = \cap_{a \in B} IND(a)$.

2.4. Criteria importance degree

This section introduces the basic definition of criteria importance [46, 47]. To determine the importance of each criterion, we use the attribute reduction method of RST. By minimizing attributes, we can identify a core attribute, remove unimportant attributes, and establish a relationship between attributes. As an alternative, we determine the attribute contribution degree by measuring the variation size in system structure after reducing one attribute. The criteria weight increases as the variation size increases.

In a DT, we define $Sig_R(c)$ as criteria c important for subset c:

$$Sig_R(c) = 1 - \frac{Card(R \cup \{c\})}{card(R)} \tag{2}$$

Where: $card(R) = card(IND(R))$

In reality, we could run across the following scenario while calculating the contribution degree using formula (2): certain attribute contribution degrees are equal to zero or have values that are inconsistent with the facts. To overcome this issue, we express the enhancement importance relation as:

$$Sig_R(c_i) = \left| Sig_R(c_i) - \frac{1}{n-1} \sum_{j=1(j\neq i)}^{n} \left[\frac{Sig_R(c_{i,}, c_{j,}) - Sig_R(c_i)}{2} \right] \right| \tag{3}$$

2.5. Multiple Criteria Ranking by Alternative Trace (MCRAT) Approach

Katarina et al. (2021), developed multiple criteria ranking by alternative trace (MCRAT) [48]. This approach consists of two stages, first stage involves normalizing, weighting, and determination of a component's "magnitude" after getting an optimal alternative and decomposition of alternatives. The MCRAT techniques steps are defined as follow [44]:

Step 1: define the main parameters and determine the alternatives.

Step 2: Build the decision matrix of X

$$X = [X_{ij}]_{mn} = \begin{bmatrix} & x_{11}x_{12}\dots x_{1n} & \\ & x_{21}x_{22}\dots x_{2n} & \\ \vdots & \vdots \quad\quad \ddots & \vdots \\ & x_{m1}x_{m2}\dots x_{mn} & \end{bmatrix} \tag{4}$$

Where n is the number of parameters and m is the number of selections, and X_{ij} is the outcomes value of i^{th} alternative on j^{th} parameters.

Step 3: Use the formula below to normalize the choice matrix, removing dimensions from various criterion,

$$r_{ij} = \begin{cases} \frac{X_{ij} - minX_{ij}}{\max X_{ij} - minX_{ij}} & if\ j \in B \\ \frac{\max X_{ij} - X_{ij}}{\max X_{ij} - minX_{ij}} & if\ j \in C \end{cases} \tag{5}$$

Where B represents the collection of maximized characteristics, and C represents the subset of minimized characteristics.

Step 4: For each alternative, prepare a weighted normalised matrix U as following:

$$U = [U_{ij}]_{mn} = \begin{bmatrix} & u_{11}u_{12}...u_{1n} & \\ & u_{21}u_{22}...u_{2n} & \\ \vdots & \vdots \quad\quad \ddots & \vdots \\ & u_{m1}u_{m2}...u_{mn} & \end{bmatrix} \tag{6}$$

Step 5: Calculation the optimal alternative

$$q_j = \max\left(u_{ij} \left\lceil 1 \leq j \leq n \right\rceil\right),\ \forall i \in [1,2,3,\dots,m] \tag{7}$$

Then the obtained optimal alternative is calculated using the following formula.

$$Q = \{\ q_1, q_2,\ ..\quad .,\ q_j\ \},\ j\ =\ 1,\ 2,\ ...,\ n \tag{8}$$

Step 6: Decomposition step

This phase entails decomposing the ideal choice into two subgroups or components. The set Q may be represented as the union of the two subsets:

$$Q = Q^{\max} \cup Q^{\min} \tag{9}$$

$$Q \ = \ \{ \ q_1, \ q_2, \ . \ . \ . \ , \ q_k \ \} \cup \ \ \{ \ q_1, \ q_2, \ . \ . \ . \ , \ q_h \ \ \} ; \ k \ + \ h \ = \ j \tag{10}$$

where k indicates the total number of factors to be maximized, and h denotes the total number of parameters to be decreased.

Step 7: Decomposition of the alternative

Similarly, to Step 6 we perform decomposition of each alternative:

$$U = U^{\max} \cup U^{\min}, \ \forall \ i \ \in \ [1, \ 2, \ . \ . \ . \ , \ m] \tag{11}$$

$$Ui \ = \ \{ \ u_{i1}, \ \ u_{i2}, \ . \ . \ . \ , \ \ u_{ik} \ \} \cup \ \{ \ \ u_{i1}, \ \ u_{i2}, \ . \ . \ . \ , \ \ u_{ih} \ \} , \ \forall \ i \ \in \ [1, \ 2, \ . \ . \ . \ , \ m] \tag{12}$$

Step 8: Magnitude of component

Using the following formula to calculate the magnitude

$$Q_k = \sqrt{q_1^2 + q_2^2 + \ldots ..q_k^2 (13)}$$

$$Q_h = \sqrt{q_1^2 + q_2^2 + \ldots ..q_h^2 (14)}$$

Every preference is treated using the same methodology.

$$U_{ik} = \sqrt{u_{i1}^2 + u_{i2}^2 + \ldots ..u_{ik}^2 \ \forall \ i \ \in \ [1, \ 2, \ .., \ m] (15)}$$

$$U_{ih} = \sqrt{u_{i1}^2 + u_{i2}^2 + \ldots ..u_{ih}^2 \ \forall \ i \ \in \ [1, \ 2, \ .., \ m] (16)}$$

Step 9: Ranking Alternative using (MCRAT) approach

Construct the matrix F using the best substitute elements:

$$F = \begin{bmatrix} Q_k & 0 \\ 0 & Q_h \end{bmatrix} \tag{17}$$

Additionally, construct the matrix G_i using various elements:

$$G_i = \begin{bmatrix} U_{ik} & 0 \\ 0 & U_{ih} \end{bmatrix} \ \forall \ i \ \in \ [1, \ 2, \ . \ . \ . \ , \ m] \tag{18}$$

If matrix F and matrix G_i are multiplied to get the matrix T_i:

$$T_i = F \times G_i = \begin{bmatrix} t_{11} & 0 \\ 0 & t_{22} \end{bmatrix} \ \forall \ i \ \in \ [1, \ 2, \ . \ . \ . \ , \ m] \tag{19}$$

The trace of the matrix Ti is obtained by the following formula:

$$tr\,(T_i) = t_{11,i} + t_{22,i}, \ \forall \ i \ \in \ [1, \ 2, \ . \ . \ . \ , \ m] \tag{20}$$

According to the descending order of tr (T_i) the alternatives are ranked

3. Proposed Methodology

The suggested technique involves three fundamental stages:

Stage I. determining criteria to be implemented inside the framework

Stage II. RST weight calculation

Stage III. Using the Multiple Criteria Ranking by Alternative Trace technique, the alternatives are ranked in order of preference.

The general framework of the suggested approach is shown in Fig. 1.

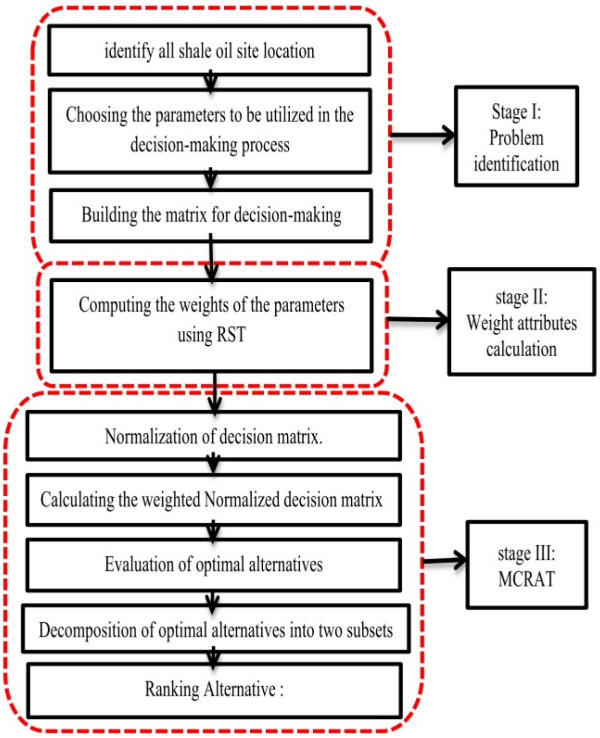

Figure 1: The diagrammatic layout of the Integrated MCRAT Rough Set Theory approach.

4. Shale oil wells Assessment Using the integrated RST and MCRAT Approach

This portion presents a real-world application to verify the effectiveness and performance of the recommended method for evaluating the quality of shale oil wells.

4.1. Information System of Evaluating Shale oil location

Numerous chemical and physical factors are involved in the shale oil site (SOs) evaluation problem. In this paper, twenty-seven shale oil samples were taken from Al-Quseir area of the Red Sea coast of Egypt as shown in table 1 [20]. For each sample, twelve parameter including hydrogen total organic carbon (Cr_1), the amount of thermovaporized-free hydrocarbon (Cr_2), the amount of hydrocarbon compounds originating from kerogen cracking (Cr_3), an indication of residual petroleum potential of the rock (Cr_4), the amount of CO_2 generated through thermal heating (Cr_5), Production Index (Cr_6), Oxygen Index (Cr_7), Hydrogen Index (Cr_8), Rolyzable carbon index (Cr_9), genetic potential (Cr_{10}), vitrinite reflectance (Cr_{11}), and the temperature at which the maximum release of hydrocarbons (Cr_{12}) were investigated.

Table 1: Shale Oil samples information system

Alternative / Criterion	Cr_1 Max	Cr_2 Max	Cr_3 Max	Cr_4 Max	Cr_5 Min	Cr_6 Max	Cr_7 Min	Cr_8 Max	Cr_9 Max	Cr_{10} Max	Cr_{11} Max	Cr_{12} Min
SOs1	19.07	3.04	97.57	3.83	422	553.44	21.35	0.045	97.31	25.48	83.51	0.43
SOs2	22.2	2.99	107.35	5.02	422	521.08	23.83	0.045	107.04	21.38	91.58	0.43
SOs3	23.56	5.19	153.18	6.65	423	695.211	29.65	0.045	155.07	23.03	131.45	0.45
SOs4	23.82	5.38	142.97	5.36	424	642.51	23.62	0.055	145.05	26.67	123.13	0.47
SOs5	20.76	2.86	94.99	4.46	423	494.64	22.72	0.045	94.55	21.30	81.22	0.45
SOs6	19.86	3.27	99.17	4.87	422	539.61	26	0.045	99.14	20.36	85.03	0.43
SOs7	22.69	3.34	104.94	4.88	421	498.58	22.63	0.045	104.98	21.50	89.87	0.41
SOs8	18.97	4.38	101.55	4.6	423	578.72	25.78	0.055	102.63	22.08	87.92	0.45
SOs9	19.22	2.33	60.45	6.17	425	343.84	34.11	0.045	59.48	9.80	52.11	0.49
SOs10	19.03	1.93	59.92	5.62	424	344.29	31.39	0.045	58.55	10.66	51.34	0.47
SOs11	12.38	1.26	22.63	3.2	424	202.67	28.44	0.045	20.59	7.07	19.83	0.47
SOs12	12.09	2.83	68.04	7.1	423	621.75	64.78	0.045	67.57	9.58	58.82	0.45
SOs13	13.48	2.9	67.41	7.74	423	549.27	62.67	0.055	67.01	8.71	58.36	0.45
SOs14	12.14	3.37	68.66	7.24	424	624.66	65.76	0.055	68.73	9.48	59.78	0.47
SOs15	21.73	5.46	139.54	11.27	422	688.7	54.71	0.055	141.7	12.38	120.35	0.43
SOs16	21.23	5.4	140.88	11.7	422	711.93	58.21	0.055	142.98	12.04	121.41	0.43
SOs17	22.15	5.85	180.53	9.43	425	870.4	44.86	0.055	183.08	19.14	154.70	0.49
SOs18	21.21	5.2	170.53	11.35	424	860.27	56.53	0.055	172.43	15.02	145.86	0.47
SOs19	21.11	5.48	139.23	10.9	425	707.8	54.56	0.055	141.41	12.77	120.11	0.49
SOs20	21.72	2.58	91	4.85	421	453.28	23.56	0.045	90.28	18.76	77.67	0.41
SOs21	22.63	3.02	86.68	8.52	421	414.98	39.63	0.045	86.4	10.17	74.45	0.41
SOs22	21.43	3.59	89.86	9.81	423	453.81	48.33	0.055	90.15	9.16	77.56	0.45
SOs23	21.16	3.1	99.47	5.78	423	507.59	28.86	0.045	99.27	17.21	85.13	0.45
SOs24	3.03	1.13	13.05	2.28	420	582.07	120	0.045	10.88	5.72	11.77	0.4
SOs25	2.83	1.05	8.91	0.36	422	404.3	21.17	0.045	6.66	24.75	8.27	0.43
SOs26	3.33	1.05	12.52	0.82	422	481.76	37.27	0.045	10.27	15.27	11.26	0.43
SOs27	2.93	1.29	8.52	0.27	420	361.45	15	0.045	6.51	31.56	8.14	0.4

4.2. Weights calculation of the assessment parameters by RST

The weights of parameters for Shel oil site evaluation are computed using RST. Shale oil information system need to be in discretized form before apply the proposed approach, consequently, equal width technique prior to analysis is used. Where the attribute significant is calculated through calculating the equivalence relation of each parameter using Eq. 2. Finally, using Eq. 3 is applied to calculate the degree of importance of each criterion. Table 2 displays the respective weights of the evaluation criteria.

Table 2: The criteria weight calculation

Cr_1	Cr_2	Cr_3	Cr_4	Cr_5	Cr_6	Cr_7	Cr_8	Cr_9	Cr_{10}	Cr_{11}	Cr_{12}
0.0859	0.0618	0.0928	0.0756	0.0941	0.0943	0.0908	0.0682	0.0929	0.0855	0.0925	0.0655

4.3. Assessment of the available locations of Shale oil wells by MCRAT

Initially, the attributes for shale oil assessment are transformed into dimensionless values using the linear normalization procedure in the MCRAT method. This enables the comparison of all parameters. Table 3 shows shale oil normalized decision matrix. The corresponding weighted normalized matrix is subsequently constructed using Eq. 5, as illustrated in Table 4. After that the optimal alternatives for the given criteria using formula 7 & 8 are obtained. Table 5 shows the Optimal alternative and decomposition of each alternative. Then, the decomposition calculation for each alternative to obtain the optimum solution is executed as shown in table 6. Consequently, the computation of the magnitude for the optimal alternatives and other components using formula section 2.6. is calculated as shown in table 7. Finally, the calculation of the alternative rank using the formula 17-20 is performed. Table 8 shows the alternatives rank.

Table 3: Normalized decision matrix

Alternatives	Cr_1	Cr_2	Cr_3	Cr_4	Cr_5	Cr_6	Cr_7	Cr_8	Cr_9	Cr_{10}	Cr_{11}	Cr_{12}
SOs1	0.8006	0.5197	0.5405	0.3274	0.9953	0.6358	0.7026	0.8182	0.5315	0.8073	0.5398	0.9302
SOs2	0.9320	0.5111	0.5946	0.4291	0.9953	0.5987	0.6295	0.8182	0.5847	0.6777	0.5920	0.9302
SOs3	0.9891	0.8872	0.8485	0.5684	0.9929	0.7987	0.5059	0.8182	0.8470	0.7300	0.8497	0.8889
SOs4	1.0000	0.9197	0.7919	0.4581	0.9906	0.7382	0.6351	1.0000	0.7923	0.8453	0.7960	0.8511
SOs5	0.8715	0.4889	0.5262	0.3812	0.9929	0.5683	0.6602	0.8182	0.5164	0.6749	0.5250	0.8889
SOs6	0.8338	0.5590	0.5493	0.4162	0.9953	0.6200	0.5769	0.8182	0.5415	0.6453	0.5496	0.9302
SOs7	0.9526	0.5709	0.5813	0.4171	0.9976	0.5728	0.6628	0.8182	0.5734	0.6815	0.5810	0.9756
SOs8	0.7964	0.7487	0.5625	0.3932	0.9929	0.6649	0.5818	1.0000	0.5606	0.6996	0.5684	0.8889
SOs9	0.8069	0.3983	0.3348	0.5274	0.9882	0.3950	0.4398	0.8182	0.3249	0.3105	0.3368	0.8163
SOs10	0.7989	0.3299	0.3319	0.4803	0.9906	0.3956	0.4779	0.8182	0.3198	0.3379	0.3318	0.8511
SOs11	0.5197	0.2154	0.1254	0.2735	0.9906	0.2328	0.5274	0.8182	0.1125	0.2241	0.1282	0.8511
SOs12	0.5076	0.4838	0.3769	0.6068	0.9929	0.7143	0.2316	0.8182	0.3691	0.3037	0.3802	0.8889
SOs13	0.5659	0.4957	0.3734	0.6615	0.9929	0.6311	0.2393	1.0000	0.3660	0.2760	0.3772	0.8889
SOs14	0.5097	0.5761	0.3803	0.6188	0.9906	0.7177	0.2281	1.0000	0.3754	0.3005	0.3865	0.8511
SOs15	0.9123	0.9333	0.7729	0.9632	0.9953	0.7912	0.2742	1.0000	0.7740	0.3924	0.7780	0.9302
SOs16	0.8913	0.9231	0.7804	1.0000	0.9953	0.8179	0.2577	1.0000	0.7810	0.3816	0.7848	0.9302
SOs17	0.9299	1.0000	1.0000	0.8060	0.9882	1.0000	0.3344	1.0000	1.0000	0.6067	1.0000	0.8163
SOs18	0.8904	0.8889	0.9446	0.9701	0.9906	0.9884	0.2653	1.0000	0.9418	0.4761	0.9429	0.8511
SOs19	0.8862	0.9368	0.7712	0.9316	0.9882	0.8132	0.2749	1.0000	0.7724	0.4048	0.7764	0.8163
SOs20	0.9118	0.4410	0.5041	0.4145	0.9976	0.5208	0.6367	0.8182	0.4931	0.5946	0.5021	0.9756
SOs21	0.9500	0.5162	0.4801	0.7282	0.9976	0.4768	0.3785	0.8182	0.4719	0.3224	0.4813	0.9756
SOs22	0.8997	0.6137	0.4978	0.8385	0.9929	0.5214	0.3104	1.0000	0.4924	0.2903	0.5014	0.8889
SOs23	0.8883	0.5299	0.5510	0.4940	0.9929	0.5832	0.5198	0.8182	0.5422	0.5454	0.5503	0.8889
SOs24	0.1272	0.1932	0.0723	0.1949	1.0000	0.6687	0.1250	0.8182	0.0594	0.1814	0.0761	1.0000
SOs25	0.1188	0.1795	0.0494	0.0308	0.9953	0.4645	0.7085	0.8182	0.0364	0.7843	0.0534	0.9302
SOs26	0.1398	0.1795	0.0694	0.0701	0.9953	0.5535	0.4025	0.8182	0.0561	0.4839	0.0728	0.9302
SOs27	0.1230	0.2205	0.0472	0.0231	1.0000	0.4153	1.0000	0.8182	0.0356	1.0000	0.0526	1.0000

Table 4: Normalize DM

Alternatives	Cr_1	Cr_2	Cr_3	Cr_4	Cr_5	Cr_6	Cr_7	Cr_8	Cr_9	Cr_{10}	Cr_{11}	Cr_{12}
SOs1	0.0688	0.0321	0.0502	0.0247	0.093654	0.0600	0.0638	0.0558	0.0494	0.0690	0.0499	0.0609
SOs2	0.0801	0.0316	0.0552	0.0324	0.093654	0.0565	0.0572	0.0558	0.0543	0.0579	0.0548	0.0609
SOs3	0.0850	0.0548	0.0787	0.0430	0.093433	0.0753	0.0459	0.0558	0.0787	0.0624	0.0786	0.0582
SOs4	0.0859	0.0568	0.0735	0.0346	0.093212	0.0696	0.0577	0.0682	0.0736	0.0723	0.0736	0.0557
SOs5	0.0749	0.0302	0.0488	0.0288	0.093433	0.0536	0.0599	0.0558	0.0480	0.0577	0.0486	0.0582
SOs6	0.0716	0.0345	0.0510	0.0315	0.093654	0.0585	0.0524	0.0558	0.0503	0.0552	0.0508	0.0609
SOs7	0.0818	0.0353	0.0539	0.0315	0.093876	0.0540	0.0602	0.0558	0.0533	0.0583	0.0537	0.0639
SOs8	0.0684	0.0463	0.0522	0.0297	0.093433	0.0627	0.0528	0.0682	0.0521	0.0598	0.0526	0.0582
SOs9	0.0693	0.0246	0.0311	0.0399	0.092993	0.0373	0.0399	0.0558	0.0302	0.0265	0.0312	0.0535
SOs10	0.0686	0.0204	0.0308	0.0363	0.093212	0.0373	0.0434	0.0558	0.0297	0.0289	0.0307	0.0557
SOs11	0.0446	0.0133	0.0116	0.0207	0.093212	0.0220	0.0479	0.0558	0.0104	0.0192	0.0119	0.0557
SOs12	0.0436	0.0299	0.0350	0.0459	0.093433	0.0674	0.0210	0.0558	0.0343	0.0260	0.0352	0.0582
SOs13	0.0486	0.0306	0.0347	0.0500	0.093433	0.0595	0.0217	0.0682	0.0340	0.0236	0.0349	0.0582
SOs14	0.0438	0.0356	0.0353	0.0468	0.093212	0.0677	0.0207	0.0682	0.0349	0.0257	0.0357	0.0557
SOs15	0.0784	0.0577	0.0717	0.0728	0.093654	0.0746	0.0249	0.0682	0.0719	0.0335	0.0720	0.0609
SOs16	0.0766	0.0570	0.0724	0.0756	0.093654	0.0771	0.0234	0.0682	0.0726	0.0326	0.0726	0.0609
SOs17	0.0799	0.0618	0.0928	0.0609	0.092993	0.0943	0.0304	0.0682	0.0929	0.0519	0.0925	0.0535
SOs18	0.0765	0.0549	0.0877	0.0733	0.093212	0.0932	0.0241	0.0682	0.0875	0.0407	0.0872	0.0557
SOs19	0.0761	0.0579	0.0716	0.0704	0.092993	0.0767	0.0250	0.0682	0.0718	0.0346	0.0718	0.0535
SOs20	0.0783	0.0273	0.0468	0.0313	0.093876	0.0491	0.0578	0.0558	0.0458	0.0508	0.0464	0.0639
SOs21	0.0816	0.0319	0.0446	0.0551	0.093876	0.0450	0.0344	0.0558	0.0438	0.0276	0.0445	0.0639
SOs22	0.0773	0.0379	0.0462	0.0634	0.093433	0.0492	0.0282	0.0682	0.0457	0.0248	0.0464	0.0582
SOs23	0.0763	0.0327	0.0511	0.0373	0.093433	0.0550	0.0472	0.0558	0.0504	0.0466	0.0509	0.0582
SOs24	0.0109	0.0119	0.0067	0.0147	0.094100	0.0631	0.0114	0.0558	0.0055	0.0155	0.0070	0.0655
SOs25	0.0102	0.0111	0.0046	0.0023	0.093654	0.0438	0.0643	0.0558	0.0034	0.0671	0.0049	0.0609
SOs26	0.0120	0.0111	0.0064	0.0053	0.093654	0.0522	0.0365	0.0558	0.0052	0.0414	0.0067	0.0609
SOs27	0.0106	0.0136	0.0044	0.0017	0.094100	0.0392	0.0908	0.0558	0.0033	0.0855	0.0049	0.0655

Table 5: Optimal alternative Decomposition

Alternative	Max				Min	Max	Min	Max				Min
	Cr1	Cr2	Cr3	Cr4	Cr5	Cr6	Cr7	Cr8	Cr9	Cr10	Cr11	Cr12
	q1	q2	q3	q4	q5	q6	q7	q8	q9	q10	q11	q12
Q Max	0.0859	0.0618	0.0928	0.0756		0.0943		0.0682	0.0929	0.0855	0.0925	
Q Min					0.0941		0.0908					0.0655

Table 6: Decomposition of alternatives.

Alternative / Criterion	Max Cr_1	Max Cr_2	Max Cr_3	Max Cr_4	Min Cr_5	Max Cr_6	Min Cr_7	Max Cr_8	Max Cr_9	Max Cr_{10}	Max Cr_{11}	Min Cr_{12}
	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10	U11	U12
SOs1 U^{Max}	0.0688	0.0321	0.0502	0.0247		0.0600		0.0558	0.0494	0.0690	0.0499	
SOs1 U^{Min}					0.093654		0.0638					0.0609
SOs2 U^{Max}	0.0801	0.0316	0.0552	0.0324		0.0565		0.0558	0.0543	0.0579	0.0548	
SOs2 U^{Min}					0.093654		0.0572					0.0609
SOs3 U^{Max}	0.0850	0.0548	0.0787	0.0430		0.0753		0.0558	0.0787	0.0624	0.0786	
SOs3 U^{Min}					0.093433		0.0459					0.0582
SOs4 U^{Max}	0.0859	0.0568	0.0735	0.0346		0.0696		0.0682	0.0736	0.0723	0.0736	
SOs4 U^{Min}					0.093212		0.0577					0.0557
SOs5 U^{Max}	0.0749	0.0302	0.0488	0.0288		0.0536		0.0558	0.0480	0.0577	0.0486	
SOs5 U^{Min}					0.093433		0.0599					0.0582
SOs6 U^{Max}	0.0716	0.0345	0.0510	0.0315		0.0585		0.0558	0.0503	0.0552	0.0508	
SOs6 U^{Min}					0.093654		0.0524					0.0609
SOs7 U^{Max}	0.0818	0.0353	0.0539	0.0315		0.0540		0.0558	0.0533	0.0583	0.0537	
SOs7 U^{Min}					0.093876		0.0602					0.0639
SOs8 U^{Max}	0.0684	0.0463	0.0522	0.0297		0.0627		0.0682	0.0521	0.0598	0.0526	
SOs8 U^{Min}					0.093433		0.0528					0.0582
SOs9 U^{Max}	0.0693	0.0246	0.0311	0.0399		0.0373		0.0558	0.0302	0.0265	0.0312	
SOs9 U^{Min}					0.092993		0.0399					0.0535
SOs10 U^{Max}	0.0686	0.0204	0.0308	0.0363		0.0373		0.0558	0.0297	0.0289	0.0307	
SOs10 U^{Min}					0.093212		0.0434					0.0557
SOs11 U^{Max}	0.0446	0.0133	0.0116	0.0207		0.0220		0.0558	0.0104	0.0192	0.0119	
SOs11 U^{Min}					0.093212		0.0479					0.0557
SOs12 U^{Max}	0.0436	0.0299	0.0350	0.0459		0.0674		0.0558	0.0343	0.0260	0.0352	
SOs12 U^{Min}					0.093433		0.0210					0.0582
SOs13 U^{Max}	0.0486	0.0306	0.0347	0.0500		0.0595		0.0682	0.0340	0.0236	0.0349	
SOs13 U^{Min}					0.093433		0.0217					0.0582
SOs14 U^{Max}	0.0438	0.0356	0.0353	0.0468		0.0677		0.0682	0.0349	0.0257	0.0357	
SOs14 U^{Min}					0.093212		0.0207					0.0557
SOs15 U^{Max}	0.0784	0.0577	0.0717	0.0728		0.0746		0.0682	0.0719	0.0335	0.0720	
SOs16 U^{Min}					0.093654		0.0249					0.0609
SOs16 U^{Max}	0.0766	0.0570	0.0724	0.0756		0.0771		0.0682	0.0726	0.0326	0.0726	
SOs16 U^{Min}					0.093654		0.0234					0.0609
SOs17 U^{Max}	0.0799	0.0618	0.0928	0.0609		0.0943		0.0682	0.0929	0.0519	0.0925	
SOs17 U^{Min}					0.092993		0.0304					0.0535
SOs18 U^{Max}	0.0765	0.0549	0.0877	0.0733		0.0932		0.0682	0.0875	0.0407	0.0872	
SOs18 U^{Min}					0.093212		0.0241					0.0557
SOs19 U^{Max}	0.0761	0.0579	0.0716	0.0704		0.0767		0.0682	0.0718	0.0346	0.0718	
SOs19 U^{Min}					0.092993		0.0250					0.0535
SOs20 U^{Max}	0.0783	0.0273	0.0468	0.0313		0.0491		0.0558	0.0458	0.0508	0.0464	
SOs20 U^{Min}					0.093876		0.0578					0.0639
SOs21 U^{Max}	0.0816	0.0319	0.0446	0.0551		0.0450		0.0558	0.0438	0.0276	0.0445	
SOs21 U^{Min}					0.093876		0.0344					0.0639
SOs22 U^{Max}	0.0773	0.0379	0.0462	0.0634		0.0492		0.0682	0.0457	0.0248	0.0464	
SOs22 U^{Min}					0.093433		0.0282					0.0582
SOs23 U^{Max}	0.0763	0.0327	0.0511	0.0373		0.0550		0.0558	0.0504	0.0466	0.0509	
SOs23 U^{Min}					0.093433		0.0472					0.0582
SOs24 U^{Max}	0.0109	0.0119	0.0067	0.0147		0.0631		0.0558	0.0055	0.0155	0.0070	
SOs24 U^{Min}					0.094100		0.0114					0.0655
SOs25 U^{Max}	0.0102	0.0111	0.0046	0.0023		0.0438		0.0558	0.0034	0.0671	0.0049	
SOs25 U^{Min}					0.093654		0.0643					0.0609
SOs26 U^{Max}	0.0120	0.0111	0.0064	0.0053		0.0522		0.0558	0.0052	0.0414	0.0067	
SOs26 U^{Min}					0.093654		0.0365					0.0609
SOs27 U^{Max}	0.0106	0.0136	0.0044	0.0017		0.0392		0.0558	0.0033	0.0855	0.0049	
SOs27 U^{Min}					0.094100		0.0908					0.0655

Table 7: Magnitude of optimal alternative

Alternatives	Max Qk U_{ik}	Min Qh U_{ik}
Q	0.252122	0.146255
SOs1	0.1590	0.1287
SOs2	0.1647	0.1255
SOs3	0.2081	0.1193
SOs4	0.2068	0.1230
SOs5	0.1540	0.1254
SOs6	0.1569	0.1234
SOs7	0.1643	0.1285
SOs8	0.1675	0.1221
SOs9	0.1227	0.1145
SOs10	0.1207	0.1170
SOs11	0.0833	0.1187
SOs12	0.1298	0.1121
SOs13	0.1346	0.1122
SOs14	0.1379	0.1106
SOs15	0.2040	0.1145
SOs16	0.2056	0.1142
SOs17	0.2365	0.1115
SOs18	0.2284	0.1112
SOs19	0.2031	0.1101
SOs20	0.1497	0.1274
SOs21	0.1500	0.1186
SOs22	0.1597	0.1136
SOs23	0.1560	0.1198
SOs24	0.0891	0.1152
SOs25	0.0991	0.1289
SOs26	0.0892	0.1176
SOs27	0.1110	0.1463

Table 8: Alternative Ranking

Alternatives	Total value	Rank
SOs1	0.0589	11
SOs2	0.0599	10
SOs3	0.0699	4
SOs4	0.0701	3
SOs5	0.0572	13
SOs6	0.0576	12
SOs7	0.0602	8
SOs8	0.0601	9
SOs9	0.0477	22
SOs10	0.0475	23
SOs11	0.0384	27
SOs12	0.0491	21
SOs13	0.0504	19
SOs14	0.0509	18
SOs15	0.0682	6
SOs16	0.0685	5
SOs17	0.0759	1
SOs18	0.0739	2
SOs19	0.0673	7
SOs20	0.0564	16
SOs21	0.0552	17
SOs22	0.0569	14
SOs23	0.0569	15
SOs24	0.0393	26
SOs25	0.0438	24
SOs26	0.0397	25
SOs27	0.0494	20

4.4. Evaluation the suggested approach in comparison to other comprehensive assessment methods

In order to assess the efficacy and reliability of the proposed method, we also compared the shale oil ranking outcomes to those of known optimization techniques like: TOPSIS [14, 39], EDAS [24, 24], Codas [26, 27], and WASPAS [28]. The results of rankings of various approaches are presented in table 9, and Fig.2.

Based on a comparison of ranking results, it appears that ranking outcomes are generally aligned. Based on the slight variations in rankings between established and proposed methods, the proposed method may be a good decision-making tool.

Table 9: Results of shale Oil Ranking

SO. No.	Ranked based on different methods				
	Proposed	CODAS	EDAS	TOPSIS	WASPAS
SOs1	11	12	14	14	14
SOs2	10	8	10	10	10
SOs3	4	3	6	6	6
SOs4	3	7	7	7	7
SOs5	13	16	17	17	15
SOs6	12	14	13	13	13
SOs7	8	9	11	11	11
SOs8	9	10	8	9	8
SOs9	22	22	21	22	21
SOs10	23	23	22	23	22
SOs11	27	27	24	27	24
SOs12	21	20	19	18	20
SOs13	19	19	18	19	19
SOs14	18	18	15	16	17
SOs15	6	5	5	5	5
SOs16	5	4	3	3	4
SOs17	1	1	1	2	1
SOs18	2	2	2	1	2
SOs19	7	6	4	4	3
SOs20	16	17	20	20	18
SOs21	17	15	16	15	16
SOs22	14	11	9	8	9
SOs23	15	13	12	12	12
SOs24	26	21	23	21	23
SOs25	24	25	27	25	26
SOs26	25	26	26	26	25
SOs27	20	24	25	24	27

The Comparison of the rankings shows that the highest-ranked alternatives (SOs17 and SOs18) remain consistent across all MCDM methods, highlighting the reliability of the proposed approach. Moreover, the mid-ranked and lower-ranked alternatives display only minor variations, suggesting that the methodology follows a decision-making pattern similar to traditional MCDM techniques. Although slight discrepancies are observed, particularly in alternatives such as SOs3, SOs4, and SOs22, the overall ranking trend remains largely stable.

Based on the results, it appears that the proposed approach is in line with other established methods. Furthermore, this method effectively handles datasets with multiple input variables, providing an enhanced decision-making process.

5. Conclusions

The evaluation of shale Oil quality is one of the most important aspects in crude oil administration.

This study introduces a new two-stage method to evaluate shale oil quality. It combines Rough Set Theory (RST) with (MCRAT). The approach identifies key geochemical factors and ranks shale oil samples based on quality.

The obtained results are compared with established decision-making methods like CODAS, EDAS, TOPSIS, and WASPAS shows that the proposed method produces similar rankings. The top-ranked samples remain stable across different techniques, confirming the model's reliability. Although some mid- and lower-ranked samples show slight differences, the overall trend aligns with conventional methods.

This approach has several benefits. It reduces computational complexity, handles large datasets efficiently, and is easy to implement. RST helps simplify the evaluation by eliminating unnecessary parameters while preserving accuracy.

Overall, the suggested approach is a practical and reliable tool for assessing shale oil quality. Future studies can apply it to other decision-making problems in the petroleum industry, integrate machine learning techniques, or expand the dataset for better predictions.

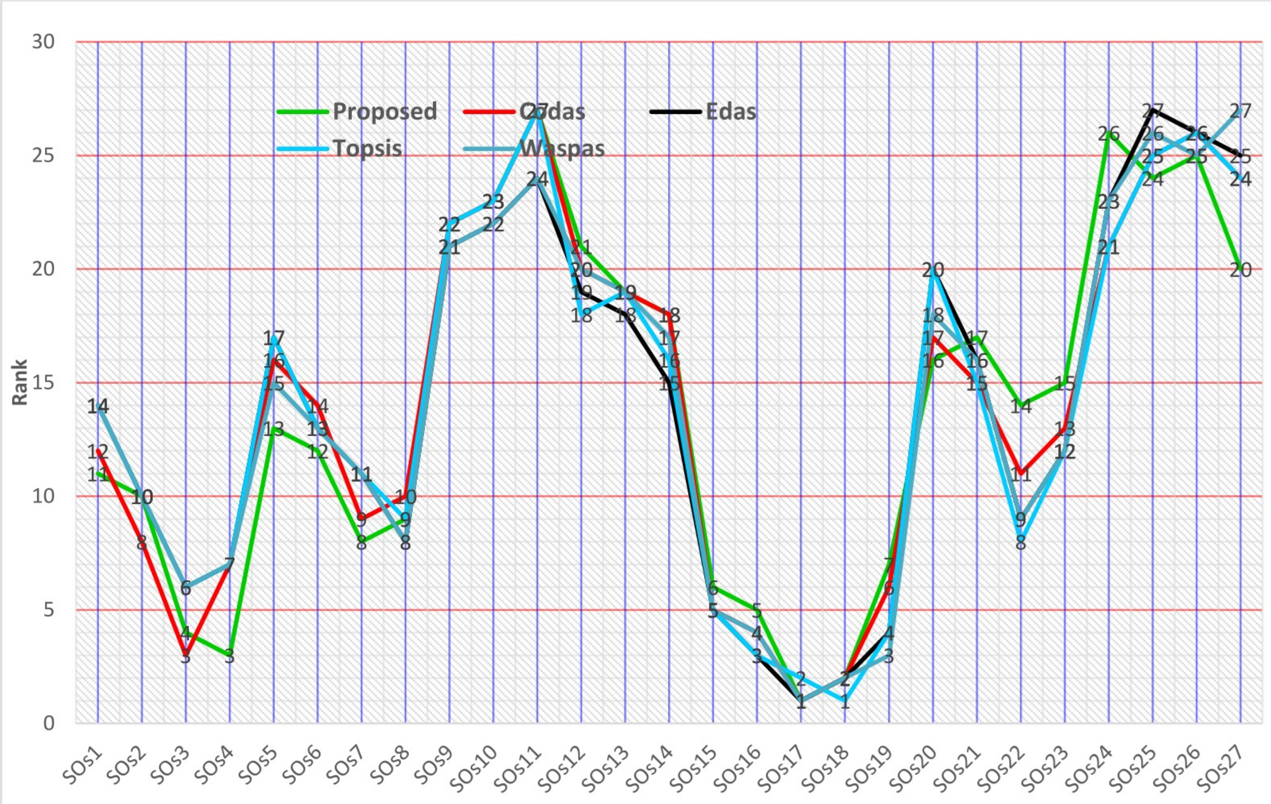

Figure 2: Comparative rankings of suggested method with other MCDM techniques.

Acknowledgements

The authors extend their appreciation to Northern Border University, Saudi Arabia, for supporting this work through project number (NBU-CRP-2025-289).

Conflicts of Interest

The authors declare no conflict of interest.

References

[1] R. K. T. Reddy and G. F. Bonham-Carter. A decision-tree approach to mineral potential mapping in Snow Lake area, Manitoba. *Canadian Journal of Remote Sensing*, 17(2):191–200, 1991.

[2] Y. L. Chen. Indicator pattern combination for mineral resource potential mapping with the general C–F model. *Mathematical Geology*, 35:301–321, 2003.

[3] E. J. M. Carranza. Improved wildcat modelling of mineral prospectivity. *Resource Geology*, 60(2):129–149, 2010.

[4] M. Abedi, S. A. Torabi, G. H. Norouzi, M. Hamzeh, and G. R. Elyasi. Promethee II: a knowledge-driven method for copper exploration. *Computers & Geosciences*, 46:255–263, 2012.

[5] M. Abedi, S. A. Torabi, G. H. Norouzi, and M. Hamzeh. ELECTRE III: a knowledge-driven method for integration of geophysical data with geological and geochemical data in mineral prospectivity mapping. *Journal of Applied Geophysics*, 87:9–18, 2012.

[6] P. Mejía-Herrera, J. J. Royer, G. Caumon, and A. Cheilletz. Curvature attribute from surface-restoration as predictor variable in kupferschiefer copper potentials. *Natural Resources Research*, 24:275–290, 2015.

[7] Q. Cheng. Boost WOFE: a new sequential weights of evidence model reducing the effect of conditional dependency. *Mathematical Geosciences*, 47:591–621, 2015.

[8] A. M. Ibrahim, B. Bennett, and F. Isiaka. The optimisation of Bayesian classifier in predictive spatial modelling for secondary mineral deposits. *Procedia Computer Science*, 61:478–485, 2015.

[9] E. J. M. Carranza and A. G. Laborte. Data-driven predictive mapping of gold prospectivity, Baguio District, Philippines: application of random forests algorithm. *Ore Geology Reviews*, 71:777–787, 2015.

[10] S. Rezaei, M. Lotfi, P. Afzal, M. R. Jafari, M. S. Meigoony, and M. Khalajmasoumi. Investigation of copper and gold prospects using index overlay integration method and multifractal modeling in Saveh 1:100,000 sheet, Central Iran. *Gospodarka Surowcami Mineralnymi*, 31:51–74, 2015.

[11] S. A. Hosseini and M. Abedi. Data envelopment analysis: a knowledge-driven method for mineral prospectivity mapping. *Computers & Geosciences*, 82:111–119, 2015.

[12] B. A. Elliott, R. Verma, and J. R. Kyle. Prospectivity modeling for Cambrian–Ordovician hydraulic fracturing sand resources around the Llano Uplift, Central Texas. *Natural Resources Research*, 25:389–415, 2016.

[13] H. H. Asadi, A. Sansoleimani, M. Fatehi, and E. J. M. Carranza. An AHP–TOPSIS predictive model for district-scale mapping of porphyry Cu–Au potential: a case study from Salafchegan area (Central Iran). *Natural Resources Research*, 25:417–429, 2016.

[14] M. Abedi and G. H. Norouzi. A general framework of TOPSIS method for integration of airborne geophysics, satellite imagery, geochemical and geological data. *International Journal of Applied Earth Observation and Geoinformation*, 46:31–44, 2016.

[15] Z. Zhang, R. Zuo, and Y. Xiong. A comparative study of fuzzy weights of evidence and random forests for mapping mineral prospectivity for skarn-type Fe deposits in the southwestern Fujian metallogenic belt, China. *Science China Earth Sciences*, 59:556–572, 2016.

[16] M. A. El-Baz and Y. I. Mesalam. TOPSIS and multi-choice goal programming approach to supplier selection in supply chain management. *Egyptian Journal of Engineering Sciences and Technology*, 15(2):154–166, 2012.

[17] X. Ding, X. Chong, Z. Bao, Y. Xue, and S. Zhang. Fuzzy comprehensive assessment method based on the entropy weight method and its application in the water environmental safety evaluation of the Heshangshan drinking water source area, Three

Gorges Reservoir Area, China. *Water*, 9:329, 2017.

[18] Y. L. Chen and W. Wu. Mapping mineral prospectivity using an extreme learning machine regression. *Ore Geology Reviews*, 80:200–213, 2017.

[19] Y. Liu, K. Zhou, N. Zhang, and J. Wang. Maximum entropy modeling for orogenic gold prospectivity mapping in the Tangbale-Hatu belt, Western Junggar, China. *Ore Geology Reviews*, 100:133–147, 2018.

[20] Y. Qin and L. Liu. Quantitative 3D association of geological factors and geophysical fields with mineralization and its significance for ore prediction: an example from Anqing Orefield, China. *Minerals*, 8:300, 2018.

[21] T. Sun, F. Chen, L. Zhong, W. Liu, and Y. Wang. GIS-based mineral prospectivity mapping using machine learning methods: a case study from Tongling Ore District, Eastern China. *Ore Geology Reviews*, 109:26–49, 2019.

[22] Y. Chen and W. Wu. Isolation forest as an alternative data-driven mineral prospectivity mapping method with a higher data-processing efficiency. *Natural Resources Research*, 28:31–46, 2019.

[23] M. Hamzeh and F. Karimipour. An ABC-optimized fuzzy ELECTRE approach for assessing petroleum potential at the petroleum system level. *Open Geosciences*, 12:580–597, 2020.

[24] M. K. Ghorabaee, E. K. Zavadskas, M. Amiri, and Z. Turskis. Extended EDAS method for fuzzy multi-criteria decision-making: an application to supplier selection. *International Journal of Computers Communications & Control*, 11:358–371, 2016.

[25] M. K. Ghorabaee, M. Amiri, E. K. Zavadskas, Z. Turskis, and J. Antucheviciene. Stochastic EDAS method for multi-criteria decision-making with normally distributed data. *Journal of Intelligent & Fuzzy Systems*, 33:1627–1638, 2017.

[26] M. K. Ghorabaee, E. K. Zavadskas, Z. Turskis, and J. Antuchevičienė. A new combinative distance-based assessment (CODAS) method for multi-criteria decision-making. *Economic Computation and Economic Cybernetics Studies and Research*, 50:25–44, 2016.

[27] A. K. Mitra. Combinative distance-based assessment (CODAS) approach of multi-criteria decision-making for grading of Tossa jute fibres. *Research Journal of Textile and Apparel*, 2022.

[28] A. Arisantoso, M. H. Somaida, M. Sanwasih, and M. I. Shalahudin. Multi-criteria decision making using the WASPAS method in webcam selection decision support systems. *The IJICS (International Journal of Informatics and Computer Science)*, 7(1):1, 2023.

[29] P. Kraipeerapun, C. C. Fung, and W. Brown. Assessment of uncertainty in mineral prospectivity prediction using interval neutrosophic set. *Proceedings of the International Conference on Computational and Information Sciences*, pages 1074–1079, 2005.

[30] E. J. M. Carranza, H. Wibowo, S. D. Barritt, et al. Spatial data analysis and integration for regional-scale geothermal potential mapping, West Java, Indonesia. *Geothermics*, 37:267–299, 2008.

[31] M. F. Coolbaugh, J. V. Taranik, G. L. Raines, et al. A geothermal GIS for Nevada:

defining regional controls and favorable exploration terrains for extensional geothermal systems. *Geothermal Resources Council Transactions*, 26:485–490, 2002.

[32] E. J. M. Carranza, M. Hale, and C. Faassen. Selection of coherent deposit-type locations and their application in data-driven mineral prospectivity mapping. *Ore Geology Reviews*, 33:536–558, 2008.

[33] A. Porwal, E. J. M. Carranza, and M. Hale. A hybrid fuzzy weights-of-evidence model for mineral potential mapping. *Natural Resources Research*, 15(1):1–14, 2006.

[34] S. S. Zargani, R. A. Vaughan, and A. A. Missallati. Spatial integration of geological datasets for predictive hydrocarbon studies in Murzuq Basin, SW Libya. *Proceedings of the IEEE International Geoscience and Remote Sensing Symposium*, pages 991–993, 2003.

[35] M. Tounsi. An approximate reasoning based technique for oil assessment. *Expert Systems with Applications*, 29:485–491, 2005.

[36] Z. Chen and K. G. Osadetz. Undiscovered petroleum accumulation mapping using model-based stochastic simulation. *Mathematical Geology*, 38:1–16, 2006.

[37] L. Bingham, R. Zurita-Milla, and A. Escalona. Geographic information system-based fuzzy-logic analysis for petroleum exploration with a case study of northern South America. *AAPG Bulletin*, 96:2121–2142, 2012.

[38] Z. Chen and K. Osadetz. Simulating the spatial distribution of undiscovered petroleum accumulations. *Journal of Canadian Petroleum Technology*, 44:24–30, 2005.

[39] P. Wang, Z. Zhu, and Y. Wang. A novel hybrid MCDM model combining the SAW, TOPSIS and GRA methods based on experimental design. *Information Sciences*, 345:27–45, 2016.

[40] A. K. Yadav, K. Singh, N. I. Arshad, M. Ferrara, A. Ahmadian, and Y. I. Mesalam. MADM-based network selection and handover management in heterogeneous network: a comprehensive comparative analysis. *Results in Engineering*, 21, 2024.

[41] J. Gidiagba, L. Tartibu, and M. Okwu. Sustainable supplier selection in the oil and gas industry: an integrated multi-criteria decision making approach. *Procedia Computer Science*, 217:1243–1255, 2023.

[42] M. Pouresmaieli, M. Ataei, A. N. Qarahasanlou, and A. Barabadi. Multi-criteria decision-making methods for sustainable decision-making in the mining industry (a comprehensive study). *Journal of Mining and Environment*, 15(2):683–706, 2024.

[43] N. Kaya. Multi-criteria decision-making methods (MCDM): a bibliometric analysis (1974–2024). *Pressacademia*, 2024.

[44] R. Kumar and D. Pamucar. A comprehensive and systematic review of multi-criteria decision-making (MCDM) methods to solve decision-making problems: two decades from 2004 to 2024. *Spectrum of Decision Making and Applications*, 2(1):178–197, 2025.

[45] Z. Pawlak. Rough sets. *International Journal of Computer & Information Sciences*, 11:341–356, 1982.

[46] L. Dun, H. Pei, and C. Jiang. New methodology of attribute weights acquisition based on rough sets theory. *Systems Engineering and Electronics*, 30(8):1481–1484,

2008.

[47] H. Y. Liang, Y. T. Lee, M. L. You, and K. L. Wen. The weighting analysis of influence factors in clinical skin physiology assessment via rough sets method. *Journal of Bioscience and Biotechnology*, 2(1):39–46, 2010.

[48] K. Urošević, Z. Gligorić, I. Miljanović, Č. Beljić, and M. Gligorić. Novel methods in multiple criteria decision-making process (MCRAT and RAPS)—application in the mining industry. *Mathematics*, 9:1980, 2021.