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Multifunctions Between Temporal Picture Fuzzy Ideal Structures

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Abstract. This paper joins the notion of multifunctions to the notion of a temporal picture fuzzy modal topological structures (TPFMTS) using ideals. In this paper, we introduce the notion of temporal picture fuzzy local function and TPF-ideal topological spaces. Also, we introduce the concepts of TPF^u or TPF^l $\mathfrak{L}^P$-continuous, almost $\mathfrak{L}^P$-continuous, weakly $\mathfrak{L}^P$-continuous and almost weakly $\mathfrak{L}^P$-continuous multifunctions. Several properties and characterizations of the presented multifunctions and their types of continuity are established. Some examples are given to explain the correct implications between these notions.

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1. Introduction

Fuzzification is a crucial tool for addressing humanistic systems in our real-life problems. The first paper on fuzzy set theory was authored by Zadeh in 1965 ([1]). This approach of fuzzy sets FS has been widely applied by many scholars. Fuzzy sets described the positivism of an element ϱ of a universal set $\aleph$ to a subset $\mathbb{G} \subseteq \aleph$ by the membership value $\omega_{\mathbb{G}}(\varrho)$, and posited that the negativism of that element $\varrho \in \aleph$ to the set $\mathbb{G}$ is $1 - \omega_{\mathbb{G}}(\varrho)$. Atanassov in [2] based his theory of intuitionistic fuzzy sets IFS on the notion of the negativism $\varpi_{\mathbb{G}}(\varrho)$ of an element $\varrho \in \aleph$ to a subset $\mathbb{G} \subseteq \aleph$ that may range from $[0, 1]$ and need not be the complement of the positivism of that element $\varrho \in \aleph$ to $\mathbb{G}$. The values $\omega_{\mathbb{G}}(\varrho)$ and $\varpi_{\mathbb{G}}(\varrho)$ represent the positivism and negativism of each $\varrho \in \aleph$ to $\mathbb{G}$, respectively, with the condition that $0 \leq \omega_{\mathbb{G}}(\varrho) + \varpi_{\mathbb{G}}(\varrho) \leq 1$. In this way, Atanassov encompassed all the fuzzy sets FS as a special case of his theory whenever $\omega_{\mathbb{G}}(\varrho) + \varpi_{\mathbb{G}}(\varrho) = 1$. Intuitionistic fuzzy sets IFS

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are more meaningful and applicable to our real-life cases. Cuong in [3] initiated the theory of picture fuzzy sets PFS by adding the neutralism of an element $\varrho \in \aleph$ to the subset $\mathbb{G}$, represented by $\sigma_{\mathbb{G}}(\varrho)$. This definition is conditioned with $0 \leq \omega_{\mathbb{G}}(\varrho) + \varpi_{\mathbb{G}}(\varrho) + \sigma_{\mathbb{G}}(\varrho) \leq 1$. In case where $\sigma_{\mathbb{G}}(\varrho) = 0$ for all $\varrho \in \aleph$, then we revert to intuitionistic sets $\mathbb{G}$ in IFS. Moreover, if $\varpi_{\mathbb{G}}(\varrho) = 1 - \omega_{\mathbb{G}}(\varrho)$ for all $\varrho \in \aleph$, then we revert to a fuzzy set $\mathbb{G}$ in FS. There are several simple modifications for the intuitionistic fuzzy sets [2], which we shall not discuss here. These modifications include Pythagorean fuzzy sets [4, 5], Fermatean fuzzy sets [6], Spherical fuzzy sets [7], q-rung orthopair fuzzy sets [8–10] and q-rung orthopair picture fuzzy sets [11]. Moreover, $(\varsigma, \varkappa)$-fuzzy local function, continuous multifunctions and DF-ideal topological space are found in [12–14]. All these definitions, starting from fuzzy sets, have applications in image processing, decision theory, uncertainty modeling, and beyond, as in [5–7, 15–17].

In this paper, we compile the definitions and notions from regions of general topology, of standard modal logic [16, 18–21] and of picture fuzziness, and represent the concept of a temporal picture fuzzy modal topological structure, briefly TPFMTS. Continuous functions between picture fuzzy topological spaces were discussed in [22]. Some types of continuity of multifunctions were studied in [23, 24]. Also, we presented the general form of temporal picture fuzzy continuous multifunctions. In this paper, we merge the classical definitions of multifunctions in general topology and the standard modal logic [16, 19–21, 25] with the notion of picture fuzzy sets, further expanding into the realm of TPFMTS.

The motivations of this paper are as follow. Section 1 is an introduction. Section 2 is given for the basics. Section 3 presents the main definition of temporal picture fuzzy local functions that are joined to a temporal picture fuzzy ideal. Section 4 investigates the notions of temporal picture upper and temporal picture lower almost -$\mathfrak{L}^P$-continuity and introduces many characteristic properties of these defined multifunctions. Section 5 investigates the notions of temporal picture upper and temporal picture lower weak -$\mathfrak{L}^P$-continuity and discusses its properties, as well as investigates the implications associated with the previous definitions of temporal picture upper and temporal picture lower almost-$\mathfrak{L}^P$-continuity. Section 6 investigates the notions of temporal picture upper and temporal picture lower almost weak -$\mathfrak{L}^P$-continuity, and discusses its properties, as well as investigates the implications associated with the previous definitions. Section 7 presents the conclusion.

The research on TPFMTS has several important applications in various domains: Decision Making, Pattern Recognition, Artificial Intelligence, Information Retrieval and Data Mining. TPFMTS address critical gaps in handling uncertainty, imprecision, and neutrality, which are inherent in real-life problems across diverse domains. To bridge these gaps, Picture Fuzzy Sets PFS were introduced, adding a neutrality component to the membership and non-membership values, thereby enabling a more nuanced representation of uncertainty. TPFMTS expands upon these concepts by the integration in modal logic and general topology using the picture fuzzy sets. This integration introduces global operators, such as closure and interior, which modify classical topological and modal relationships. These global operators facilitate a robust analysis of fuzzy sets under modal and topological constraints, providing a suitable tools for theoretical exploration and practical application.

The study of TPFMTS not only extends the theory of fuzzy sets but also establishes a wide platform for addressing modern computational challenges. Its ability to integrate neutrality, positivity, and negativity within a unified framework lays the foundation for further exploration and application of TPFMTS in dynamic systems, hybrid models, and emerging technologies, positioning it as a cornerstone of modern mathematical and computational innovation.

2. Preliminaries

Through the paper, denote $I = [0, 1], I_0 = (0, 1]$ and $I_1 = [0, 1)$. Let a universe set $\aleph$, be fixed. An PFS $\mathbb{G}$ in $\aleph$ is an object of the following form: $\mathbb{G} = \{\langle \varrho, \omega_{\mathbb{G}}(\varrho), \varpi_{\mathbb{G}}(\varrho), \sigma_{\mathbb{G}}(\varrho)\rangle \,|\varrho \in \aleph\}$, where $\omega_{\mathbb{G}}(\varrho) \in I$ is called the degree of positive membership of ϱ in $\mathbb{G}$, $\varpi_{\mathbb{G}}(\varrho) \in I$ is called the degree of negative membership of ϱ in $\mathbb{G}$, $\sigma_{\mathbb{G}}(\varrho) \in I$ is called the degree of neutral membership of ϱ in $\mathbb{G}$, and where $\omega_{\mathbb{G}}(\varrho)$, $\varpi_{\mathbb{G}}(\varrho)$ and $\sigma_{\mathbb{G}}(\varrho)$ satisfy the following condition: $0 \leq \omega_{\mathbb{G}}(\varrho) + \varpi_{\mathbb{G}}(\varrho) + \sigma_{\mathbb{G}}(\varrho) \leq 1$ for all $\varrho \in \aleph$. Let $\mathfrak{G}$ be a non-empty set (finite or infinite), called the temporal scale with upper and lower boundaries. The elements of $\mathfrak{G}$ are called "time-moments". we define the temporal PFS (TPFS) as follows:

$\mathbb{G}(\mathfrak{G}) = \{\langle\langle \varrho, \mathfrak{g}\rangle, \omega_{\mathbb{G}}(\langle \varrho, \mathfrak{g}\rangle), \varpi_{\mathbb{G}}(\langle \varrho, \mathfrak{g}\rangle), \sigma_{\mathbb{G}}(\langle \varrho, \mathfrak{g}\rangle)\rangle \,|\, \langle \varrho, \mathfrak{g}\rangle \in \aleph \times \mathfrak{G}\}$, where

(1) $\mathbb{G} \subseteq \aleph$ is a fixed set,

(2) $\omega_{\mathbb{G}}(\langle \varrho, \mathfrak{g}\rangle) + \varpi_{\mathbb{G}}(\langle \varrho, \mathfrak{g}\rangle) + \sigma_{\mathbb{G}}(\langle \varrho, \mathfrak{g}\rangle) \leq 1$ for every $\langle \varrho, \mathfrak{g}\rangle \in \aleph \times \mathfrak{G}$,

(3) $\omega_{\mathbb{G}}(\langle \varrho, \mathfrak{g}\rangle), \varpi_{\mathbb{G}}(\langle \varrho, \mathfrak{g}\rangle)$ and $\sigma_{\mathbb{G}}(\langle \varrho, \mathfrak{g}\rangle)$ are the degree of positive membership, negative membership and neutral membership, respectively, of the element $\varrho \in \aleph$ at the time-moment $\mathfrak{g} \in \mathfrak{G}$.

NOTE: each ordinary PFS can be regarded as a TPFS for which $\mathfrak{G}$ is a singleton set.

Additionally, we mentioned that all operations and operators on the PFSs can be defined for the TPFSs. However, the opposite is also true: each TPFS $\mathbb{G}(\mathfrak{G})$is a standard PFS, but over universe $\aleph \times \mathfrak{G}$. For this reason, we can re-define all operations, relations, and operators defined over standard PFSs, now over TPFSs.

Definition 2.1. [22, 26] Let $\aleph$ be a nonempty set, $\mathfrak{G}$ time-scale, $\mathbb{G}(\mathfrak{G}) = \{\langle\langle \varrho, \mathfrak{g}\rangle, \omega_{\mathbb{G}}(\langle \varrho, \mathfrak{g}\rangle), \varpi_{\mathbb{G}}(\langle \varrho, \mathfrak{g}\rangle), \sigma_{\mathbb{G}}(\langle \varrho, \mathfrak{g}\rangle)\rangle$ $|\, \langle \varrho, \mathfrak{g}\rangle \in \aleph \times \mathfrak{G}\}$ and $\mathbb{H}(\mathfrak{G}) = \{\langle\langle \varrho, \mathfrak{g}\rangle, \omega_{\mathbb{H}}(\langle \varrho, \mathfrak{g}\rangle), \varpi_{\mathbb{H}}(\langle \varrho, \mathfrak{g}\rangle), \sigma_{\mathbb{H}}(\langle \varrho, \mathfrak{g}\rangle)\rangle \,|\, \langle \varrho, \mathfrak{g}\rangle$ $\in \aleph \times \mathfrak{G}\}$. Then,

(1) $\mathbb{G}(\mathfrak{G}) \subseteq \mathbb{H}(\mathfrak{G})$iff $\forall \langle \varrho, \mathfrak{g}\rangle \in \aleph \times \mathfrak{G}$, $\omega_{\mathbb{G}}(\langle \varrho, \mathfrak{g}\rangle) \leq \omega_{\mathbb{H}}(\langle \varrho, \mathfrak{g}\rangle)$, $\varpi_{\mathbb{G}}(\langle \varrho, \mathfrak{g}\rangle) \geq \varpi_{\mathbb{H}}(\langle \varrho, \mathfrak{g}\rangle)$ and $\sigma_{\mathbb{G}}(\langle \varrho, \mathfrak{g}\rangle) \leq \sigma_{\mathbb{H}}(\langle \varrho, \mathfrak{g}\rangle)$ or $\sigma_{\mathbb{G}}(\langle \varrho, \mathfrak{g}\rangle) \geq \sigma_{\mathbb{H}}(\langle \varrho, \mathfrak{g}\rangle)$.

(2) $\mathbb{G}(\mathfrak{G}) \cup \mathbb{H}(\mathfrak{G}) = \{\langle\langle \varrho, \mathfrak{g}\rangle, \omega_{\mathbb{G}}(\langle \varrho, \mathfrak{g}\rangle) \vee \omega_{\mathbb{H}}(\langle \varrho, \mathfrak{g}\rangle), \varpi_{\mathbb{G}}(\langle \varrho, \mathfrak{g}\rangle) \wedge \varpi_{\mathbb{H}}(\langle \varrho, \mathfrak{g}\rangle), \sigma_{\mathbb{G}}(\langle \varrho, \mathfrak{g}\rangle) \wedge \sigma_{\mathbb{H}}(\langle \varrho, \mathfrak{g}\rangle)\rangle \,|$ $\langle \varrho, \mathfrak{g}\rangle \in \aleph \times \mathfrak{G}\}\}$.

(3) $\mathbb{G}(\mathfrak{G}) \cap \mathbb{H}(\mathfrak{G}) = \{\langle\langle \varrho, \mathfrak{g}\rangle, \omega_{\mathbb{G}}(\langle \varrho, \mathfrak{g}\rangle) \wedge \omega_{\mathbb{H}}(\langle \varrho, \mathfrak{g}\rangle), \varpi_{\mathbb{G}}(\langle \varrho, \mathfrak{g}\rangle) \vee \varpi_{\mathbb{H}}(\langle \varrho, \mathfrak{g}\rangle), \sigma_{\mathbb{G}}(\langle \varrho, \mathfrak{g}\rangle) \wedge \sigma_{\mathbb{H}}(\langle \varrho, \mathfrak{g}\rangle)\rangle \,|$ $\langle \varrho, \mathfrak{g}\rangle \in \aleph \times \mathfrak{G}\}\}$.

(4) $\dashv \mathbb{G}(\mathfrak{G}) = \{\langle\langle \varrho, \mathfrak{g}\rangle, \varpi_{\mathbb{G}}(\langle \varrho, \mathfrak{g}\rangle), \omega_{\mathbb{G}}(\langle \varrho \mathfrak{g}\rangle), \sigma_{\mathbb{G}}(\langle \varrho, \mathfrak{g}\rangle)\rangle \,|\, \langle \varrho, \mathfrak{g}\rangle \in \aleph \times \mathfrak{G}\}$.

(5) $\mathbb{G}(\mathfrak{G}) \barwedge \mathbb{H}(\mathfrak{G}) = \{\langle\langle \varrho, \mathfrak{g}\rangle, 0, 1, 0\rangle \,|\, \langle \varrho, \mathfrak{g}\rangle \in \aleph \times \mathfrak{G}\}$ if $\mathbb{G}(\mathfrak{G}) \subseteq \mathbb{H}(\mathfrak{G})$, and $\mathbb{G}(\mathfrak{G}) \barwedge \mathbb{H}(\mathfrak{G}) = \mathbb{G}(\mathfrak{G}) \cap (\dashv \mathbb{H}(\mathfrak{G}))$ otherwise.

(6) $\sharp(\mathfrak{G}) = \{\langle\langle \varrho, \mathfrak{g}\rangle, 1, 0, 0\rangle \,|\, \langle \varrho, \mathfrak{g}\rangle \in \aleph \times \mathfrak{G}\}, \flat(\mathfrak{G}) = \{\langle\langle \varrho, \mathfrak{g}\rangle, 0, 1, 0\rangle \,|\, \langle \varrho, \mathfrak{g}\rangle \in \aleph \times \mathfrak{G}\}$.

(7) $\natural(\mathfrak{G}) = \{\langle\langle \varrho, \mathfrak{g}\rangle, 0, 0, 1\rangle \,|\, \langle \varrho, \mathfrak{g}\rangle \in \aleph \times \mathfrak{G}\}, \mho(\mathfrak{G}) = \{\langle\langle \varrho, \mathfrak{g}\rangle, 0, 0, 0\rangle \,|\, \langle \varrho, \mathfrak{g}\rangle \in \aleph \times \mathfrak{G}\}$.

The map $\mathfrak{F}:\aleph\times\mathfrak{G}\nrightarrow\Upsilon\times\mathfrak{G}$ is called a temporal picture fuzzy multifunction ($TPFM$, for short) for any $\langle\varrho,\mathfrak{g}\rangle\in\aleph\times\mathfrak{G}$, $\mathfrak{F}(\langle\varrho,\mathfrak{g}\rangle)\in\left(I^3\right)^{\Upsilon\times\mathfrak{G}}$. The degree of membership of $\langle\zeta,\mathfrak{g}\rangle\in\Upsilon\times\mathfrak{G}$ at the time-moment $\mathfrak{g}\in\mathfrak{G}$ is denoted by: $\mathfrak{F}(\langle\varrho,\mathfrak{g}\rangle)(\langle\zeta,\mathfrak{g}\rangle)=\Psi_{\mathfrak{F}}(\langle\varrho,\mathfrak{g}\rangle,\langle\zeta,\mathfrak{g}\rangle)$. The domain of $\mathfrak{F}$, denoted by $D(\mathfrak{F})$ and the range of $\mathfrak{F}$, denoted by $R(\mathfrak{F})$, are defined by: for any $\langle\varrho,\mathfrak{g}\rangle\in\aleph\times\mathfrak{G}$ and $\langle\zeta,\mathfrak{g}\rangle\in(\Upsilon\times\mathfrak{G})$, $D(\mathfrak{F})(\langle\varrho,\mathfrak{g}\rangle)=\bigcup_{\langle\zeta,\mathfrak{g}\rangle\in(\Upsilon\times\mathfrak{G})}\Psi_{\mathfrak{F}}(\langle\varrho,\mathfrak{g}\rangle,\langle\zeta,\mathfrak{g}\rangle)$ and $R(\mathfrak{F})(\langle\zeta,\mathfrak{g}\rangle)=\bigcup_{\langle\varrho,\mathfrak{g}\rangle\in\aleph\times\mathfrak{G}}\Psi_{\mathfrak{F}}(\langle\varrho,\mathfrak{g}\rangle,\langle\zeta,\mathfrak{g}\rangle)$. $\mathfrak{F}$ is called crisp iff $\Psi_{\mathfrak{F}}(\langle\varrho,\mathfrak{g}\rangle,\langle\zeta,\mathfrak{g}\rangle)=\sharp(\mathfrak{G})$ $\forall$ $\langle\varrho,\mathfrak{g}\rangle\in\aleph\times\mathfrak{G}$ and $\langle\zeta,\mathfrak{g}\rangle\in(\Upsilon\times\mathfrak{G})$. $\mathfrak{F}$ is called Normalized ($NTPFM$, for short) iff $\forall$ $\langle\varrho,\mathfrak{g}\rangle\in\aleph\times\mathfrak{G}$, there exists $\langle\zeta_0,\mathfrak{g}\rangle\in(\Upsilon\times\mathfrak{G})$ such that $\Psi_{\mathfrak{F}}(\langle\varrho,\mathfrak{g}\rangle,\langle\zeta_0,\mathfrak{g}\rangle)=\sharp(\mathfrak{G})$. $\mathfrak{F}$ is called surjective iff $R(\mathfrak{F})\langle\zeta,\mathfrak{g}\rangle=\sharp(\mathfrak{G})$ $\forall$ $\langle\zeta,\mathfrak{g}\rangle\in(\Upsilon\times\mathfrak{G})$. The inverse of $\mathfrak{F}$ denoted by $\mathfrak{F}^-:\Upsilon\nrightarrow\aleph$ is a $TPFM$ defined by: $\mathfrak{F}^-(\langle\zeta,\mathfrak{g}\rangle)(\langle\varrho,\mathfrak{g}\rangle)=\mathfrak{F}(\langle\varrho,\mathfrak{g}\rangle)((\langle\zeta,\mathfrak{g}\rangle))=\Psi_{\mathfrak{F}}(\langle\varrho,\mathfrak{g}\rangle,\langle\zeta,\mathfrak{g}\rangle)$. One easily verifies that $D(\mathfrak{F}^-)=R(\mathfrak{F})$ and $D(\mathfrak{F})=R(\mathfrak{F}^-)$. The image $\mathfrak{F}\mathbb{G}(\mathfrak{G}))$ of $\mathbb{G}(\mathfrak{G})\in\left(I^3\right)^{\aleph\times\mathfrak{G}}$, the lower inverse $\mathfrak{F}^l(\mathbb{U}(\mathfrak{G}))$ and the upper inverse $\mathfrak{F}^u(\mathbb{U}(\mathfrak{G}))$ of $\mathbb{U}(\mathfrak{G})\in\left(I^3\right)^{\Upsilon\times\mathfrak{G}}$ are defined respectively (see [8, 27]) as follows:

$$\mathfrak{F}(\mathbb{G}(\mathfrak{G}))\langle\zeta,\mathfrak{g}\rangle=\bigcup_{\langle\varrho,\mathfrak{g}\rangle\in\aleph\times\mathfrak{G}}\left[\Psi_{\mathfrak{F}}(\langle\varrho,\mathfrak{g}\rangle,\langle\zeta,\mathfrak{g}\rangle)\cap\mathbb{G}(\mathfrak{G})(\langle\varrho,\mathfrak{g}\rangle)\right],$$

$$\mathfrak{F}^l(\mathbb{U}(\mathfrak{G}))\langle\varrho,\mathfrak{g}\rangle=\bigcup_{\langle\zeta,\mathfrak{g}\rangle\in(\Upsilon\times\mathfrak{G})}\left[\Psi_{\mathfrak{F}}(\langle\varrho,\mathfrak{g}\rangle,\langle\zeta,\mathfrak{g}\rangle)\cap\mathbb{U}(\mathfrak{G})\langle\zeta,\mathfrak{g}\rangle\right],$$

$$\mathfrak{F}^u(\mathbb{U}(\mathfrak{G}))\langle\varrho,\mathfrak{g}\rangle=\bigcap_{\langle\zeta,\mathfrak{g}\rangle\in(\Upsilon\times\mathfrak{G})}\left[\dashv\Psi_{\mathfrak{F}}(\langle\varrho,\mathfrak{g}\rangle,\langle\zeta,\mathfrak{g}\rangle)\cup\mathbb{U}(\mathfrak{G})\langle\zeta,\mathfrak{g}\rangle\right].$$

Definition 2.2. [12] A temporal picture fuzzy topology on $\aleph$ is a map τ: $\left(I^3\right)^{\aleph\times\mathfrak{G}}\to I^3$ defined by
$\tau(\mathbb{G}(\mathfrak{G}))=\langle\omega_\tau(\mathbb{G}(\mathfrak{G})),\varpi_\tau(\mathbb{G}(\mathfrak{G})),\sigma_\tau(\mathbb{G}(\mathfrak{G}))\rangle$ which satisfies the following properties:
(1) $\tau(\flat(\mathfrak{G}))=\tau(\sharp(\mathfrak{G}))=\langle 1,0,0\rangle$.
(2) $\tau(\mathbb{G}\cap\mathbb{H})\geq\tau(\mathbb{G})\wedge\tau(\mathbb{H})$, for each $\mathbb{G},\mathbb{H}\in\left(I^3\right)^{\aleph\times\mathfrak{G}}$.
(3) $\tau(\bigcup_{i\in\Gamma}\mathbb{G}_i)\geq\bigwedge_{i\in\Gamma}\tau(\mathbb{G}_i)$, for each $\mathbb{G}_i\in\left(I^3\right)^{\aleph\times\mathfrak{G}}$, $i\in\Gamma$.

The pair $(\aleph,\tau)$ is called a temporal picture fuzzy topological space in Šostak's sense. For any $\mathbb{G}(\mathfrak{G})\in\left(I^3\right)^{\aleph\times\mathfrak{G}}$ the number $\omega_\tau(\mathbb{G}(\mathfrak{G}))$ is called the openness degree at a certain times , $\varpi_\tau(\mathbb{G}(\mathfrak{G}))$ is called the non openness degree at a certain times, while $\sigma_\tau(\mathbb{G}(\mathfrak{G}))$ is called the neutral degree at a certain times. For $\mathbb{G}(\mathfrak{G})\in\left(I^3\right)^{\aleph\times\mathfrak{G}}$

$$\begin{aligned} cl_\tau(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle) &= \bigcap\{\mathbb{H}(\mathfrak{G})\in\left(I^3\right)^{\aleph\times\mathfrak{G}}:\mathbb{G}(\mathfrak{G})\subseteq\mathbb{H}(\mathfrak{G}),\tau(\dashv\mathbb{H}(\mathfrak{G}))\geq\langle\varsigma,\varkappa,\vartheta\rangle\},\\ int_\tau(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle) &= \bigcup\{\mathbb{H}(\mathfrak{G})\in\left(I^3\right)^{\aleph\times\mathfrak{G}}:\mathbb{G}(\mathfrak{G})\supseteq\mathbb{H}(\mathfrak{G}),\tau(\mathbb{H}(\mathfrak{G}))\geq\langle\varsigma,\varkappa,\vartheta\rangle\}.\end{aligned}$$

Definition 2.3. [12] Let $\mathfrak{F}:(\aleph,\tau)\nrightarrow(\Upsilon,\sigma)$ be a $TPFM$, $\varsigma\in I_0,\varkappa\in I_1$ and $\vartheta\in I_1$. Then, $\mathfrak{F}$ is called:

(1) TPF^uS-continuous at a fuzzy point $\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$ iff $\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in \mathfrak{F}^u(\mathbb{U}(\mathfrak{G}))$ for each $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$, $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle \varsigma,\varkappa,\vartheta\rangle$ there exists $\mathbb{G}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$, $\tau(\mathbb{G}(\mathfrak{G})) \geq \langle \varsigma,\varkappa,\vartheta\rangle$ and $\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in \mathbb{G}(\mathfrak{G})$, such that $\mathbb{G}(\mathfrak{G}) \cap D(\mathfrak{F}) \subseteq \mathfrak{F}^u(\mathbb{U}(\mathfrak{G}))$.

(2) TPF^lS-continuous at a fuzzy point $\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$ iff $\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in \mathfrak{F}^l(\mathbb{U}(\mathfrak{G}))$ for each $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$, $\sigma\mathbb{U}(\mathfrak{G})) \geq \langle \varsigma,\varkappa,\vartheta\rangle$, there exists $\mathbb{G}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$, $\tau(\mathbb{G}(\mathfrak{G})) \geq \langle \varsigma,\varkappa,\vartheta\rangle$, and $\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in \mathbb{G}(\mathfrak{G})$, such that $\mathbb{G}(\mathfrak{G}) \subseteq \mathfrak{F}^l(\mathbb{U}(\mathfrak{G}))$.

Remark 2.1. (1) If $\mathfrak{F}$ is $NTPFM$, then $\mathfrak{F}$ is TPF^uS -continuous at a fuzzy point $\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$ iff
$\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in \mathfrak{F}^u(\mathbb{U}(\mathfrak{G}))$ for each $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$, $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle \varsigma,\varkappa,\vartheta\rangle$ there exists
$\mathbb{G}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$, $\tau(\mathbb{G}(\mathfrak{G})) \geq \langle \varsigma,\varkappa,\vartheta\rangle$, and $\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in \mathbb{G}(\mathfrak{G})$, such that $\mathbb{G}(\mathfrak{G}) \subseteq \mathfrak{F}^u(\mathbb{U}(\mathfrak{G}))$.

Definition 2.4. [13] The map $\mathfrak{L}^P$: $(I^3)^{\aleph\times\mathfrak{G}} \to I^3$ is called temporal picture fuzzy ideal on $\aleph$ if it satisfies the following conditions for $\mathbb{G}(\mathfrak{G}), \mathbb{H}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$:

(1) $\mathfrak{L}^P(\flat(\mathfrak{G})) = \langle 1,0,0\rangle, \mathfrak{L}^P(\sharp(\mathfrak{G})) = \langle 0,1,0\rangle$.
(2) $\mathbb{G}(\mathfrak{G}) \subseteq \mathbb{H}(\mathfrak{G}) \Rightarrow \mathfrak{L}^P(\mathbb{G}(\mathfrak{G})) \geq \mathfrak{L}^P(\mathbb{H}(\mathfrak{G}))$.
(3) $\mathfrak{L}^P(\mathbb{G}(\mathfrak{G}) \cup \mathbb{H}(\mathfrak{G})) \geq \mathfrak{L}^P(\mathbb{G}(\mathfrak{G})) \wedge \mathfrak{L}^P(\mathbb{H}(\mathfrak{G}))$.

Also, $\mathfrak{L}^P$ is called proper if $\mathfrak{L}^P(\sharp(\mathfrak{G})) = \langle 0,1,0\rangle$ and there exists $\mathbb{G}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$ such that
$\mathfrak{L}^P(\mathbb{G}(\mathfrak{G})) > \langle 0,1,0\rangle$.

If $\mathfrak{L}_1^P$ and $\mathfrak{L}_2^P$ are temporal picture fuzzy ideals on $\aleph \times \mathfrak{G}$, we say that $\mathfrak{L}_1^P$ is finer than $\mathfrak{L}_2^P$ ($\mathfrak{L}_2^P$ is coarser than $\mathfrak{L}_1^P$), denoted by $\mathfrak{L}_2^P \subseteq \mathfrak{L}_1^P$, iff $\mathfrak{L}_2^P(\mathbb{G}(\mathfrak{G})) \leq \mathfrak{L}_1^P(\mathbb{G}(\mathfrak{G}))$ $\forall \mathbb{G}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$. Let us define the special picture fuzzy ideals $\mathfrak{L}^{P0}$, $\mathfrak{L}^{P1}$ by

$$\mathfrak{L}^{P0}(\mathbb{G}(\mathfrak{G})) = \begin{cases} \langle 1,0,0\rangle & \text{if } \mathbb{G}(\mathfrak{G}) = \flat(\mathfrak{G}), \\ \langle 0,1,0\rangle & \text{otherwise,} \end{cases} \quad \text{and} \quad \mathfrak{L}^{P1}(\mathbb{G}(\mathfrak{G})) = \begin{cases} \langle 0,1,0\rangle & \text{if } \mathbb{G}(\mathfrak{G}) = \sharp(\mathfrak{G}), \\ \langle 1,0,0\rangle & \text{otherwise.} \end{cases}$$

3. Temporal picture fuzzy local functions

Definition 3.1. Let $(\aleph, \tau, \mathfrak{L}^P)$ be a temporal picture fuzzy ideal topological space, $\mathbb{G}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$, $\varsigma \in I_0$, $\varkappa \in I_1$ and $\vartheta \in I_1$. Then the $\langle \varsigma,\varkappa,\vartheta\rangle$-temporal fuzzy local function $\Phi(\mathbb{G}(\mathfrak{G}), \langle \varsigma,\varkappa,\vartheta\rangle)$ of $\mathbb{G}(\mathfrak{G})$ defined as follows: $\Phi(\mathbb{G}(\mathfrak{G}), \langle \varsigma,\varkappa,\vartheta\rangle) = \bigcap\{\mathbb{H}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}: \mathfrak{L}^P(\mathbb{G}(\mathfrak{G}) \barwedge \mathbb{H}(\mathfrak{G})) \geq \langle \varsigma,\varkappa,\vartheta\rangle, \tau(\dashv \mathbb{H}(\mathfrak{G})) \geq \langle \varsigma,\varkappa,\vartheta\rangle\}$.

Remark 3.1. (1) If we take $\mathfrak{L}^P = \mathfrak{L}^{P0}$ for each $\mathbb{G}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$ we have
$\Phi(\mathbb{G}(\mathfrak{G}), \langle \varsigma,\varkappa,\vartheta\rangle) = \bigcap\{\mathbb{H}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}} : \mathbb{G}(\mathfrak{G}) \subseteq \mathbb{H}(\mathfrak{G}), \tau(\dashv \mathbb{H}(\mathfrak{G})) \geq \langle \varsigma,\varkappa,\vartheta\rangle\} = cl_\tau(\mathbb{G}(\mathfrak{G}), \langle \varsigma,\varkappa,\vartheta\rangle)$.

(2) If we take $\mathfrak{L}^P = \mathfrak{L}^{P1}$ (resp. $\mathfrak{L}^P(\mathbb{G}(\mathfrak{G})) \geq \langle \varsigma,\varkappa,\vartheta\rangle$) for each $\mathbb{G}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$ we have
$\Phi(\mathbb{G}(\mathfrak{G}), \langle \varsigma,\varkappa,\vartheta\rangle) = \flat(\mathfrak{G})$.

We will occasionally write $\Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)$ or $\Phi(\mathbb{G}(\mathfrak{G}),\mathfrak{L}^P,\langle\varsigma,\varkappa,\vartheta\rangle)$ for $\Phi(\mathbb{G}(\mathfrak{G}),\mathfrak{L}^P,\tau,\langle\varsigma,\varkappa,\vartheta\rangle)$.

Theorem 3.1. *Let $(\aleph,\tau,\mathfrak{L}^P)$ be a temporal picture fuzzy ideal topological space and $\mathfrak{L}_1^P$,$\mathfrak{L}_2^P$ be two temporal picture fuzzy ideals on $\aleph$. Then for any set $\mathbb{G}(\mathfrak{G}),\mathbb{H}(\mathfrak{G})\in(I^3)^{\aleph\times\mathfrak{G}}$, $\varsigma\in I_0,\varkappa\in I_1$ and $\vartheta\in I_1$.*

(1) $\Phi(\flat(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)=\flat(\mathfrak{G})$.

(2) *If* $\mathbb{G}(\mathfrak{G})\subseteq\mathbb{H}(\mathfrak{G})$*then* $\Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)\subseteq\Phi(\mathbb{H}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)$.

(3) *If* $\mathfrak{L}_2^P\subseteq\mathfrak{L}_1^P$ *then* $\Phi(\mathbb{G}(\mathfrak{G}),\mathfrak{L}_1^P,\langle\varsigma,\varkappa,\vartheta\rangle)\subseteq\Phi(\mathbb{G}(\mathfrak{G}),\mathfrak{L}_2^P,\langle\varsigma,\varkappa,\vartheta\rangle)$.

(4) $\Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)=cl(\Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle),\langle\varsigma,\varkappa,\vartheta\rangle)\subseteq cl(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)$.

(5) $\Phi(\Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle),\langle\varsigma,\varkappa,\vartheta\rangle)\subseteq\Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)$ *and* $\dashv(\Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle))\neq\Phi(\dashv\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)$.

(6) $\Phi(\mathbb{G}(\mathfrak{G})\cup\mathbb{H}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)\supseteq\Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)\cup\Phi(\mathbb{H}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)$ *and* $\Phi(\mathbb{G}(\mathfrak{G})\cap\mathbb{H}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)$
$\subseteq\Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)\cap\Phi(\mathbb{H}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)$.

(7) *If* $\mathfrak{L}^P(\mathbb{H}(\mathfrak{G}))\geq\langle\varsigma,\varkappa,\vartheta\rangle$ *then* $\Phi(\mathbb{G}(\mathfrak{G})\cup\mathbb{H}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)\supseteq\Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)$.

Proof. (1) From Definition 3.1, we have $\Phi(\flat(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)=\flat(\mathfrak{G})$.

(2) Suppose that $\Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)\nsubseteq\Phi(\mathbb{H}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)$ If $\mathbb{G}(\mathfrak{G})\subseteq\mathbb{H}(\mathfrak{G})$. By the definition of
$\Phi(\mathbb{H}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)$,there exists $\mathbb{K}(\mathfrak{G})\in(I^3)^{\aleph\times\mathfrak{G}}$ with $\Phi(\mathbb{H}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)\subseteq\mathbb{K}(\mathfrak{G})$, $\mathfrak{L}^P(\mathbb{H}(\mathfrak{G})\barwedge\mathbb{K}(\mathfrak{G}))\geq\langle\varsigma,\varkappa,\vartheta\rangle$, $\tau(\dashv\mathbb{K}(\mathfrak{G}))\geq\langle\varsigma,\varkappa,\vartheta\rangle$. Such that $\Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)\nsubseteq\mathbb{K}(\mathfrak{G})$. Since $\mathbb{G}(\mathfrak{G})\subseteq\mathbb{H}(\mathfrak{G})$implise $\mathbb{G}(\mathfrak{G})\barwedge\mathbb{K}(\mathfrak{G})\subseteq\mathbb{H}(\mathfrak{G})\barwedge\mathbb{K}(\mathfrak{G})$, $\mathfrak{L}^P(\mathbb{G}(\mathfrak{G})\barwedge\mathbb{K}(\mathfrak{G}))\geq\mathfrak{L}^P(\mathbb{H}(\mathfrak{G})\barwedge\mathbb{K}(\mathfrak{G}))\geq\langle\varsigma,\varkappa,\vartheta\rangle$. Hence $\Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)\subseteq\mathbb{K}(\mathfrak{G})$, it is a contradiction. Then, $\Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)\subseteq\Phi(\mathbb{H}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)$.

(3) Suppose that $\Phi(\mathbb{G}(\mathfrak{G}),\mathfrak{L}_1^P,\langle\varsigma,\varkappa,\vartheta\rangle)\nsubseteq\Phi(\mathbb{G}(\mathfrak{G}),\mathfrak{L}_2^P,\langle\varsigma,\varkappa,\vartheta\rangle)$ if $\mathfrak{L}_2^P\subseteq\mathfrak{L}_1^P$. By the definition of
$\Phi(\mathbb{G}(\mathfrak{G}),\mathfrak{L}_2^P,\langle\varsigma,\varkappa,\vartheta\rangle)$ there exists $\mathbb{K}(\mathfrak{G})\in(I^3)^{\aleph\times\mathfrak{G}}$ with $\Phi(\mathbb{G}(\mathfrak{G}),\mathfrak{L}_2^P,\langle\varsigma,\varkappa,\vartheta\rangle)\subseteq\mathbb{K}(\mathfrak{G})$, $\mathfrak{L}_2^P(\mathbb{G}(\mathfrak{G})\barwedge\mathbb{K}(\mathfrak{G}))\geq\langle\varsigma,\varkappa,\vartheta\rangle$, $\tau(\dashv\mathbb{K}(\mathfrak{G}))\geq\langle\varsigma,\varkappa,\vartheta\rangle$ such that $\Phi(\mathbb{G}(\mathfrak{G}),\mathfrak{L}_1^P,\langle\varsigma,\varkappa,\vartheta\rangle)\nsubseteq\mathbb{K}(\mathfrak{G})$. Since $\mathfrak{L}_2^P\subseteq\mathfrak{L}_1^P$ implies $\mathfrak{L}_1^P(\mathbb{G}(\mathfrak{G})\barwedge\mathbb{K}(\mathfrak{G}))\geq\mathfrak{L}_2^P(\mathbb{G}(\mathfrak{G})\barwedge\mathbb{K}(\mathfrak{G}))\geq\langle\varsigma,\varkappa,\vartheta\rangle$. Hence, $\Phi(\mathbb{G}(\mathfrak{G}),\mathfrak{L}_1^P,\langle\varsigma,\varkappa,\vartheta\rangle)\subseteq\mathbb{K}(\mathfrak{G})$, it is a contradiction. Then, $\Phi(\mathbb{G}(\mathfrak{G}),\mathfrak{L}_1^P,\langle\varsigma,\varkappa,\vartheta\rangle)\subseteq\Phi(\mathbb{G}(\mathfrak{G}),\mathfrak{L}_2^P,\langle\varsigma,\varkappa,\vartheta\rangle)$.

(4) From Definition 3.1, we have $\Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)=cl(\Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle),\langle\varsigma,\varkappa,\vartheta\rangle)$.

Since $\mathfrak{L}^{P0}\subseteq\mathfrak{L}^P$ for any picture fuzzy ideal $\mathfrak{L}^P$, $\Phi(\mathbb{G}(\mathfrak{G}),\mathfrak{L}^P,\langle\varsigma,\varkappa,\vartheta\rangle)\subseteq\Phi(\mathbb{G}(\mathfrak{G}),\mathfrak{L}^{P0},\langle\varsigma,\varkappa,\vartheta\rangle)=cl(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)$. Thus,
$\Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)=cl(\Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle),\langle\varsigma,\varkappa,\vartheta\rangle)\subseteq cl(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)$.

(5) By (4), we have $\Phi(\Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle),\langle\varsigma,\varkappa,\vartheta\rangle)=cl(\Phi(\Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle),\langle\varsigma,\varkappa,\vartheta\rangle),\langle\varsigma,\varkappa,\vartheta\rangle)$ $\subseteq cl(\Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle),\langle\varsigma,\varkappa,\vartheta\rangle)=\Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)$. In general the converse is not true as shown in next Example 3.1.

(6) Since $\mathbb{G}(\mathfrak{G})\subseteq\mathbb{G}(\mathfrak{G})\cup\mathbb{H}(\mathfrak{G})$ and $\mathbb{H}(\mathfrak{G})\subseteq\mathbb{G}(\mathfrak{G})\cup\mathbb{H}(\mathfrak{G})$implies $\Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)\subseteq\Phi(\mathbb{G}(\mathfrak{G})\cup\mathbb{H}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)$ and $\Phi(\mathbb{H}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)\subseteq\Phi(\mathbb{G}(\mathfrak{G})\cup\mathbb{H}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)$. Thus, $\Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)\cup\Phi(\mathbb{H}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)\subseteq\Phi(\mathbb{G}(\mathfrak{G})\cup\mathbb{H}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)$. Also, since $\mathbb{G}(\mathfrak{G})\cap\mathbb{H}(\mathfrak{G})\subseteq\mathbb{G}(\mathfrak{G})$and $\mathbb{G}(\mathfrak{G})\cap\mathbb{H}(\mathfrak{G})\subseteq\mathbb{H}(\mathfrak{G})$ implies $\Phi(\mathbb{G}(\mathfrak{G})\cap\mathbb{H}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)\subseteq\Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)$ and $\Phi(\mathbb{G}(\mathfrak{G})\cap\mathbb{H}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)\subseteq\Phi(\mathbb{H}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)$. Thus, $\Phi(\mathbb{G}(\mathfrak{G})\cap\mathbb{H}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)\subseteq\Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)\cap\Phi(\mathbb{H}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)$.

(7) Since $\mathfrak{L}^P(\mathbb{H}(\mathfrak{G})) \supseteq \langle\varsigma,\varkappa,\vartheta\rangle$ implies $\Phi(\mathbb{H}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle) = \langle 0,1,0\rangle$.
Thus,$\Phi(\mathbb{G}(\mathfrak{G}) \cup \mathbb{H}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle) \supseteq \Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle) \cup \Phi(\mathbb{H}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle) \supseteq \Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)$.

The following example shows that generally $\Phi(\Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle),\langle\varsigma,\varkappa,\vartheta\rangle) \neq \Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)$ and
$\daleth(\Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)) \neq \Phi(\daleth\,\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)$ for any $\mathbb{G}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$, $\varsigma \in I_0, \varkappa \in I_1$ and $\vartheta \in I_1$.

Example 3.1. *Let* $\aleph = \{\varrho_1,\varrho_2\}$, $\mathfrak{G} = \{\mathfrak{g}_1,\mathfrak{g}_2\}$, *Define* τ, $\mathfrak{L}^P$: $(I^3)^{\aleph\times\mathfrak{G}} \to I^3$ *as follows:*

$$\tau(\mathbb{G}(\mathfrak{G})) = \begin{cases} \langle 1,0,0\rangle & \text{if } \mathbb{G}(\mathfrak{G}) \in \{\flat(\mathfrak{G}),\sharp(\mathfrak{G})\}, \\ \langle 0.4,0.3,0.2\rangle & \text{if } \mathbb{G}(\mathfrak{G}) = \mathbb{G}_1(\mathfrak{G}), \\ \langle 0.5,0.2,0.3\rangle & \text{if } \mathbb{G}(\mathfrak{G}) = \mathbb{G}_2(\mathfrak{G}), \\ \langle 0,1,0\rangle & o.w, \end{cases}$$

$$\mathfrak{L}^P(\mathbb{G}(\mathfrak{G})) = \begin{cases} \langle 1,0,0\rangle & \text{if } \mathbb{G}(\mathfrak{G}) = \flat(\mathfrak{G}), \\ \langle 0.7,0.2,0.1\rangle & \text{if } \flat(\mathfrak{G}) \subset \mathbb{G}(\mathfrak{G}) \subseteq \langle\langle\varrho,\mathfrak{g}\rangle,0.4,0.3,0.3\rangle,\langle\varrho,\mathfrak{g}\rangle \in \aleph\times\mathfrak{G}\}, \\ \langle 0,1,0\rangle & o.w. \end{cases}$$

where
$\mathbb{G}_1(\mathfrak{G}) = \left\{\begin{array}{c} \langle\langle\varrho,\mathfrak{g}_1\rangle,0.3,0.3,0.2\rangle,\varrho\in\aleph \\ \langle\langle\varrho,\mathfrak{g}_2\rangle,0.35,0.4,0.25\rangle,\varrho\in\aleph \end{array}\right\}$, $\mathbb{G}_2(\mathfrak{G}) = \left\{\begin{array}{c} \langle\langle\varrho,\mathfrak{g}_1\rangle,0.55,0.25,0.2\rangle,\varrho\in\aleph \\ \langle\langle\varrho,\mathfrak{g}_2\rangle,0.4,0.35,0.25\rangle,\varrho\in\aleph \end{array}\right\}$,
$\mathbb{G}_3(\mathfrak{G}) = \left\{\begin{array}{c} \langle\langle\varrho,\mathfrak{g}_1\rangle,0.3,0.3,0\rangle,\varrho\in\aleph \\ \langle\langle\varrho,\mathfrak{g}_2\rangle,0.4,0.35,0\rangle,\varrho\in\aleph \end{array}\right\}$ *and* $\mathbb{H}(\mathfrak{G}) = \left\{\begin{array}{c} \langle\langle\varrho,\mathfrak{g}_1\rangle,0.45,0.35,0.2\rangle,\varrho\in\aleph \\ \langle\langle\varrho,\mathfrak{g}_2\rangle,0.44,0.25,0.2\rangle,\varrho\in\aleph \end{array}\right\}$.
Then, $\flat(\mathfrak{G}) = \Phi(\Phi(\mathbb{H}(\mathfrak{G}),\langle 0.4,0.3,0.2\rangle),\langle 0.4,0.3,0.2\rangle) \neq \Phi(\mathbb{H}(\mathfrak{G}),\langle 0.4,0.3,0.2\rangle)$
$= \left\{\begin{array}{c} \langle\langle\varrho,\mathfrak{g}_1\rangle,0.3,0.3,0\rangle,\varrho\in\aleph \\ \langle\langle\varrho,\mathfrak{g}_2\rangle,0.4,0.35,0\rangle,\varrho\in\aleph \end{array}\right\}$,
$\sharp(\mathfrak{G}) = \daleth(\Phi(\mathbb{G}_3(\mathfrak{G}),\langle 0.4,0.3,0.2\rangle)) \neq \Phi(\daleth\,\mathbb{G}_3(\mathfrak{G}),\langle 0.4,0.3,0.2\rangle) = \flat(\mathfrak{G})$.

Definition 3.2. Let $(\aleph,\tau,\mathfrak{L}^P)$ be a temporal picture fuzzy ideal topological space. Then, for each $\mathbb{G}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$, $\varsigma \in I_0, \varkappa \in I_1$ and $\vartheta \in I_1$, we define an operator $cl^*: (I^3)^{\aleph\times\mathfrak{G}} \times I^3 \to (I^3)^{\aleph\times\mathfrak{G}}$ as follows:

$$cl^*_\tau(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle) = \mathbb{G}(\mathfrak{G}) \cup \Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle).$$

Now, if $\mathfrak{L}^P = \mathfrak{L}^{P0}$ then
$cl^*_\tau(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle) = \mathbb{G}(\mathfrak{G}) \cup \Phi(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle) = \mathbb{G}(\mathfrak{G}) \cup cl_\tau(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle) = cl_\tau(\mathbb{G}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)$.

Theorem 3.2. *Let* $(\aleph,\tau,\mathfrak{L}^P)$ *be a temporal picture fuzzy ideal topological space. Then for any fuzzy set* $\mathbb{G}(\mathfrak{G}),\mathbb{H}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$, $\varsigma \in I_0, \varkappa \in I_1$ *and* $\vartheta \in I_1$, *the operator* $cl^*_\tau: (I^3)^{\aleph\times\mathfrak{G}} \times I^3 \to (I^3)^{\aleph\times\mathfrak{G}}$ *satisfies the following properties:*

(1) $cl^*_\tau(\flat(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) = \flat(\mathfrak{G})$.
(2) $\mathbb{G}(\mathfrak{G}) \subseteq cl^*_\tau(\mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) \subseteq cl(\mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)$.
(3) If $\mathbb{G}(\mathfrak{G}) \subseteq \mathbb{H}(\mathfrak{G})$, then $cl^*_\tau(\mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) \subseteq cl^*_\tau(\mathbb{H}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)$.
(4) $cl^*_\tau(\mathbb{G}(\mathfrak{G}) \cup \mathbb{H}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) \supseteq cl^*_\tau(\mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) \cup cl^*_\tau(\mathbb{H}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)$.
(5) $cl^*_\tau(\mathbb{G}(\mathfrak{G}) \cap \mathbb{H}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) \subseteq cl^*_\tau(\mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) \cap cl^*_\tau(\mathbb{H}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)$.

Proof. (1) Since $cl^*_\tau(\flat(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) = \flat \cup \Phi(\flat(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)$ and $\Phi(\flat(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) = \flat(\mathfrak{G})$ implies
$cl^*_\tau(\flat(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) = \flat(\mathfrak{G})$.

(2) $cl^*_\tau(\mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) = \mathbb{G}(\mathfrak{G}) \cup \Phi(\mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)$ implies $\mathbb{G}(\mathfrak{G}) \subseteq cl^*_\tau(\mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)$, Since $\mathbb{G}(\mathfrak{G}) \subseteq cl(\mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)$ and from Theorem 3.1(4), we have $\Phi(\mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) \subseteq cl(\mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)$ implies
$cl^*_\tau(\mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) \subseteq cl(\mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)$. Thus, $\mathbb{G}(\mathfrak{G}) \subseteq cl^*_\tau(\mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) \subseteq cl(\mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)$.

(3) From $\mathbb{G}(\mathfrak{G}) \subseteq \mathbb{H}(\mathfrak{G})$ and Theorem 3.1(2), we have $\mathbb{G}(\mathfrak{G}) \cup \Phi(\mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) \subseteq \mathbb{H}(\mathfrak{G}) \cup \Phi(\mathbb{H}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)$, i.e., $cl^*_\tau(\mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) \subseteq cl^*_\tau(\mathbb{H}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)$.

(4) Since $\mathbb{G}(\mathfrak{G}) \subseteq \mathbb{G}(\mathfrak{G}) \cup \mathbb{H}(\mathfrak{G})$ and $\mathbb{H}(\mathfrak{G}) \subseteq \mathbb{G}(\mathfrak{G}) \cup \mathbb{H}(\mathfrak{G})$ implies $cl^*_\tau(\mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) \subseteq cl^*_\tau(\mathbb{G}(\mathfrak{G}) \cup \mathbb{H}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)$ and $cl^*_\tau(\mathbb{H}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) \subseteq cl^*_\tau(\mathbb{G}(\mathfrak{G}) \cup \mathbb{H}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)$. Thus, $cl^*_\tau(\mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) \cup cl^*_\tau(\mathbb{H}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) \subseteq cl^*_\tau(\mathbb{G}(\mathfrak{G}) \cup \mathbb{H}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)$.

(5) $\mathbb{G}(\mathfrak{G}) \cap \mathbb{H}(\mathfrak{G}) \subseteq \mathbb{G}(\mathfrak{G})$ and $\mathbb{G}(\mathfrak{G}) \cap \mathbb{H}(\mathfrak{G}) \subseteq \mathbb{H}(\mathfrak{G})$ implies
$cl^*_\tau(\mathbb{G}(\mathfrak{G}) \cap \mathbb{H}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) \subseteq cl^*_\tau(\mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)$ and $cl^*_\tau(\mathbb{G}(\mathfrak{G}) \cap \mathbb{H}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) \subseteq cl^*_\tau(\mathbb{H}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)$. Thus, $cl^*_\tau(\mathbb{G}(\mathfrak{G}) \cap \mathbb{H}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) \subseteq cl^*_\tau(\mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) \cap cl^*_\tau(\mathbb{H}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)$.

Theorem 3.3. *Let $(\aleph, \tau, \mathfrak{L}^P)$ be a picture fuzzy ideal topological space. Then, for each $\mathbb{G}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$,
$\varsigma \in I_0, \varkappa \in I_1$ and $\vartheta \in I_1$, we define an operator $int^*_\tau : (I^3)^{\aleph\times\mathfrak{G}} \times I^3 \to (I^3)^{\aleph\times\mathfrak{G}}$ as follows:*

$$int^*_\tau(\mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) = \mathbb{G}(\mathfrak{G}) \cap \dashv (\Phi(\dashv \mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)).$$

For $\mathbb{G}(\mathfrak{G})$, $\mathbb{H}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$, the operator int^* satisfies the following properties:
(1) $int^*_\tau(\sharp(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) = \sharp(\mathfrak{G})$.
(2) $int(\mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) \subseteq int^*_\tau(\mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) \subseteq \mathbb{G}(\mathfrak{G})$.
(3) *If* $\mathbb{G}(\mathfrak{G}) \subseteq \mathbb{H}(\mathfrak{G})$, *then* $int^*_\tau(\mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) \subseteq int^*_\tau(\mathbb{H}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)$.
(4) $int^*_\tau(\mathbb{G}(\mathfrak{G}) \cap \mathbb{H}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) \subseteq int^*_\tau(\mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) \cap int^*_\tau(\mathbb{H}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)$.
(5) $int^*_\tau(\sharp, \langle\varsigma,\varkappa,\vartheta\rangle) = int(\mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)$ *if* $\mathfrak{L}^P = \mathfrak{L}^{P0}$.
(6) $int^*_\tau(\dashv \mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle) = \dashv (cl^*_\tau(\mathbb{G}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle))$.

Proof. It is similarly proved as the proof of Theorem 3.2.

Definition 3.3. Let $\mathfrak{F} : (\aleph, \tau, \mathfrak{L}^P) \nleftrightarrow (\Upsilon, \sigma)$ be a $TPFM$, $\varsigma \in I_0, \varkappa \in I_1$ and $\vartheta \in I_1$. Then, $\mathfrak{F}$ is called:

(1) TPF^u $\mathfrak{L}^P$-continuous at a fuzzy point $\langle\varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$ iff $\langle\varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathfrak{F}^u(\mathbb{U}(\mathfrak{G}))$ for each $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$, $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$ there exists $\mathbb{G}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$, $\tau(\mathbb{G}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$ and $\langle\varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathbb{G}(\mathfrak{G})$ such that $\mathbb{G}(\mathfrak{G}) \cap D(\mathfrak{F}) \subseteq \Phi(\mathfrak{F}^u(\mathbb{U}(\mathfrak{G})), \langle\varsigma,\varkappa,\vartheta\rangle)$.

(2) TPF^l $\mathfrak{L}^P$-continuous at a fuzzy point $\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$ iff $\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in \mathfrak{F}^l(\mathbb{U}(\mathfrak{G}))$ for each $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$, $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle \varsigma,\varkappa,\vartheta\rangle$ there exists $\mathbb{G}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$, $\tau(\mathbb{G}(\mathfrak{G})) \geq \langle \varsigma,\varkappa,\vartheta\rangle$ and $\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in \mathbb{G}(\mathfrak{G})$ such that $\mathbb{G}(\mathfrak{G}) \subseteq \Phi(\mathfrak{F}^l(\mathbb{U}(\mathfrak{G})), \langle \varsigma,\varkappa,\vartheta\rangle)$.

(3) TPF^u $\mathfrak{L}^P$-continuous (resp. TPF^l $\mathfrak{L}^P$-continuous) iff it is TPF^u $\mathfrak{L}^P$-continuous (resp. TPF^l $\mathfrak{L}^P$-continuous) at every fuzzy point $\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$.

Remark 3.2. (1) If $\mathfrak{F}$ is $NTPFM$, then $\mathfrak{F}$ isTPF^u $\mathfrak{L}^P$-continuous at a fuzzy point $\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$ iff
$\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in \mathfrak{F}^u(\mathbb{U}(\mathfrak{G}))$ for each $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$, $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle \varsigma,\varkappa,\vartheta\rangle$ there exists
$\mathbb{G}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$, $\tau(\mathbb{G}(\mathfrak{G})) \geq \langle \varsigma,\varkappa,\vartheta\rangle$ and $\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in \mathbb{G}(\mathfrak{G})$ such that $\mathbb{G}(\mathfrak{G}) \subseteq \Phi(\mathfrak{F}^u(\mathbb{U}(\mathfrak{G})), \langle \varsigma,\varkappa,\vartheta\rangle)$.

(2) TPF^u (resp. TPF^l) $\mathfrak{L}^P$-continuity and TPF^u (resp. TPF^lS)-continuity are independent notions as shown by Example 3.2.

Theorem 3.4. *Let* $\mathfrak{F} : (\aleph, \tau, \mathfrak{L}^P) \multimap (\Upsilon, \sigma)$ *be a* $TPFM$ *(resp.* $NTPFM$*), Then* $\mathfrak{F}$ *is* TPF^l *(resp.* TPF^u*)* $\mathfrak{L}^P$*-continuous iff* $\mathfrak{F}^l(\mathbb{U}(\mathfrak{G})) \subseteq int_\tau\left(\Phi(\mathfrak{F}^l(\mathbb{U}(\mathfrak{G})), \langle \varsigma,\varkappa,\vartheta\rangle), \langle \varsigma,\varkappa,\vartheta\rangle\right)$ *(resp.* $\mathfrak{F}^u(\mathbb{U}(\mathfrak{G}) \subseteq int_\tau(\Phi(\mathfrak{F}^u(\mathbb{U}(\mathfrak{G})), \langle \varsigma,\varkappa,\vartheta\rangle), \langle \varsigma,\varkappa,\vartheta\rangle))$ *for each* $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$, $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle \varsigma,\varkappa,\vartheta\rangle$, $\varsigma \in I_0, \varkappa \in I_1$ *and* $\vartheta \in I_1$.

Proof. ($\Rightarrow$) Let $\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$, $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$, $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle \varsigma,\varkappa,\vartheta\rangle$ and $\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in \mathfrak{F}^l(\mathbb{U}(\mathfrak{G}))$. Then, there exists $\mathbb{G}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$, $\tau(\mathbb{G}(\mathfrak{G})) \geq \langle \varsigma,\varkappa,\vartheta\rangle$ and $\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in \mathbb{G}(\mathfrak{G})$ such that
$\mathbb{G}(\mathfrak{G}) \subseteq \Phi(\mathfrak{F}^l(\mathbb{U}(\mathfrak{G})), \langle \varsigma,\varkappa,\vartheta\rangle)$. Thus, $\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in \mathbb{G}(\mathfrak{G}) \subseteq int_\tau(\Phi(\mathfrak{F}^l(\mathbb{U}(\mathfrak{G})), \langle \varsigma,\varkappa,\vartheta\rangle), \langle \varsigma,\varkappa,\vartheta\rangle)$, and hence

$$\mathfrak{F}^l(\mathbb{U}(\mathfrak{G})) \subseteq int_\tau(\Phi(\mathfrak{F}^l(\mathbb{U}(\mathfrak{G})), \langle \varsigma,\varkappa,\vartheta\rangle), \langle \varsigma,\varkappa,\vartheta\rangle).$$

($\Leftarrow$) Let $\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$, $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$, $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle \varsigma,\varkappa,\vartheta\rangle$ and $\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in \mathfrak{F}^l(\mathbb{U}(\mathfrak{G}))$. Then, $F^l(\mathbb{U}(\mathfrak{G})) \subseteq int_\tau(\Phi(\mathfrak{F}^l(\mathbb{U}(\mathfrak{G})), \langle \varsigma,\varkappa,\vartheta\rangle), \langle \varsigma,\varkappa,\vartheta\rangle)$ and hence,

$$\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in int_\tau(\Phi(\mathfrak{F}^l(\mathbb{U}(\mathfrak{G})), \langle \varsigma,\varkappa,\vartheta\rangle), \langle \varsigma,\varkappa,\vartheta\rangle) \subseteq \Phi(\mathfrak{F}^l(\mathbb{U}(\mathfrak{G})), \langle \varsigma,\varkappa,\vartheta\rangle).$$

Thus, $\mathfrak{F}$ is TPF^l $\mathfrak{L}^P$-continuous. Other case is similarly proved.

The following examples show that there is no relation between TPF^u (TPF^l) S-continuous and TPF^u (TPF^l) $\mathfrak{L}^P$-continuous multifunctions.

Example 3.2. *Let* $\aleph = \{\varrho_1, \varrho_2\}$, $\Upsilon = \{\zeta_1, \zeta_2, \}$, $\mathfrak{G} = \{\mathfrak{g}_1, \mathfrak{g}_2\}$ *and* $\mathfrak{F} : \aleph \multimap \Upsilon$ *be a* $TPFM$ *defined by*
$\Psi_{\mathfrak{F}}(\langle \varrho, \mathfrak{g}\rangle, \langle \zeta, \mathfrak{g}\rangle)$ *as:*

$\Psi_{\mathfrak{F}}(\langle \varrho, \mathfrak{g}\rangle, \langle \zeta, \mathfrak{g}\rangle)$	$\langle \zeta_1, \mathfrak{g}_1\rangle$	$\langle \zeta_1, \mathfrak{g}_2\rangle$	$\langle \zeta_2, \mathfrak{g}_1\rangle$	$\langle \zeta_2, \mathfrak{g}_2\rangle$
$\langle \varrho_1, \mathfrak{g}_1\rangle$	$\langle 0.2, 0.15, 0.3\rangle$	$\langle 0.15, 0.3, 0.5\rangle$	$\langle 1, 0, 0\rangle$	$\langle 0, 1, 0\rangle$
$\langle \varrho_1, \mathfrak{g}_2\rangle$	$\langle 1, 0, 0\rangle$	$\langle 0.1, 0.15, 0.4\rangle$	$\langle 0.3, 0.25, 0.1\rangle$	$\langle 0.3, 0.2, 0.3\rangle$
$\langle \varrho_2, \mathfrak{g}_1\rangle$	$\langle 1, 0, 0\rangle$	$\langle 0.3, 0.3, 0.4\rangle$	$\langle 0.2, 0.2, 0.2\rangle$	$\langle 0.3, 0.25, 0.2\rangle$
$\langle \varrho_2, \mathfrak{g}_2\rangle$	$\langle 0.5, 0.15, 0.15\rangle$	$\langle 0.25, 0.2, 0.1\rangle$	$\langle 1, 0, 0\rangle$	$\langle 0.2, 0.1, 0.2\rangle$.

Define temporal picture fuzzy topologies τ_1, τ_2*:* $\left(I^3\right)^{\aleph\times\mathfrak{G}} \to I^3$, σ_1, σ_2*:* $\left(I^3\right)^{\Upsilon\times\mathfrak{G}} \to I^3$, *and temporal picture fuzzy ideals* $\mathfrak{L}_1^P$, $\mathfrak{L}_2^P$: $\left(I^3\right)^{\Upsilon\times\mathfrak{G}} \to I^3$ *as:*

$$\tau_1(\mathbb{G}(\mathfrak{G})) = \begin{cases} \langle 1,0,0\rangle, & \mathbb{G}(\mathfrak{G}) \in \{\flat(\mathfrak{G}), \sharp(\mathfrak{G})\} \\ \langle 0.6,0.3,0.1\rangle, & \mathbb{G}(\mathfrak{G}) = \mathbb{G}_1(\mathfrak{G}) \\ \langle 0,1,0\rangle, & o.w, \end{cases}$$

$$\tau_2(\mathbb{G}(\mathfrak{G})) = \begin{cases} \langle 1,0,0\rangle, & \mathbb{G}(\mathfrak{G}) \in \{\flat(\mathfrak{G}), \sharp(\mathfrak{G})\} \\ \langle 0.4,0.15,0.35\rangle, & \mathbb{G}(\mathfrak{G}) = \mathbb{G}_2(\mathfrak{G}) \\ \langle 0,1,0\rangle, & o.w, \end{cases}$$

$$\mathfrak{L}_1^P(\mathbb{G}(\mathfrak{G})) = \begin{cases} \langle 1,0,0\rangle, & \mathbb{G}(\mathfrak{G}) = \flat(\mathfrak{G}) \\ \langle 0.4,0.2,0.3\rangle, & \flat(\mathfrak{G}) \subset \mathbb{G}(\mathfrak{G}) \subseteq \langle\langle\varrho,\mathfrak{g}\rangle, 0.4, 0.1, 0.4\rangle, \\ & \langle\varrho,\mathfrak{g}\rangle \in \aleph\times\mathfrak{G} \\ \langle 0,1,0\rangle, & o.w, \end{cases}$$

$$\mathfrak{L}_2^P(\mathbb{G}(\mathfrak{G})) = \begin{cases} \langle 1,0,0\rangle, & \mathbb{G}(\mathfrak{G}) = \flat(\mathfrak{G}) \\ \langle 0.5,0.15,0.3\rangle, & \flat(\mathfrak{G}) \subset \mathbb{G}(\mathfrak{G}) \subseteq \langle\langle\varrho,\mathfrak{g}\rangle, 0.25, 0.25, 0.5\rangle, \\ & \langle\varrho,\mathfrak{g}\rangle \in \aleph\times\mathfrak{G} \\ \langle 0,1,0\rangle, & o.w, \end{cases}$$

$$\sigma(\mathbb{U}(\mathfrak{G})) = \begin{cases} \langle 1,0,0\rangle, & \mathbb{U}(\mathfrak{G}) \in \{\flat(\mathfrak{G}), \sharp(\mathfrak{G})\} \\ \langle 0.3,0.3,0.4\rangle, & \mathbb{U}(\mathfrak{G}) = \mathbb{U}_1(\mathfrak{G}) \\ \langle 0,1,0\rangle, & o.w. \end{cases}$$

where

$\mathbb{G}_1(\mathfrak{G}) = \left\{ \begin{array}{l} \langle\langle\varrho,\mathfrak{g}_1\rangle, 0.3, 0.33, 0.2\rangle, \\ \langle\langle\varrho,\mathfrak{g}_2\rangle, 0.3, 0.33, 0.3\rangle, \end{array} , \varrho\in\aleph \right\}$, $\mathbb{G}_2(\mathfrak{G}) = \left\{ \begin{array}{l} \langle\langle\varrho,\mathfrak{g}_1\rangle, 0.5, 0.4, 0.1\rangle, \\ \langle\langle\varrho,\mathfrak{g}_2\rangle, 0.6, 0.2, 0.1\rangle, \end{array} , \varrho\in\aleph \right\}$
and $\mathbb{U}_1(\mathfrak{G}) = \left\{ \begin{array}{l} \langle\langle\zeta,\mathfrak{g}_1\rangle, 0.3, 0.33, 0.35\rangle, \\ \langle\langle\zeta,\mathfrak{g}_2\rangle, 0.33, 0.33, 0.33\rangle, \end{array} , \zeta\in\Upsilon \right\}$.

Then, (1) $\mathfrak{F} : (\aleph, \tau_1, \ell_1^P) \nrightarrow (\Upsilon, \sigma)$ is TPF^uS (resp. TPF^lS)-continuous but it is not TPF^u (resp. TPF^l) ℓ^P-continuous because

$\mathfrak{F}^u(\mathbb{U}_1(\mathfrak{G})) = \left\{ \begin{array}{l} \langle\langle\varrho,\mathfrak{g}_1\rangle, 0.3, 0.33, 0\rangle, \\ \langle\langle\varrho,\mathfrak{g}_2\rangle, 0.3, 0.33, 0\rangle, \end{array} , \varrho\in\aleph \right\} \subseteq int_\tau(\mathfrak{F}^u(\mathbb{U}_1(\mathfrak{G}), \langle 0.3, 0.3, 0.4\rangle)$
$= \left\{ \begin{array}{l} \langle\langle\varrho,\mathfrak{g}_1\rangle, 0.3, 0.33, 0\rangle, \\ \langle\langle\varrho,\mathfrak{g}_2\rangle, 0.3, 0.33, 0\rangle, \end{array} , \varrho\in\aleph \right\}$,
$\mathfrak{F}^l(\mathbb{U}_1(\mathfrak{G})) = \left\{ \begin{array}{l} \langle\langle\varrho,\mathfrak{g}_1\rangle, 0.3, 0.33, 0\rangle, \\ \langle\langle\varrho,\mathfrak{g}_2\rangle, 0.3, 0.33, 0\rangle, \end{array} , \varrho\in\aleph \right\} \subseteq int_\tau(\mathfrak{F}^l(\mathbb{U}_1(\mathfrak{G}), \langle 0.3, 0.3, 0.4\rangle)$

$= \left\{ \begin{array}{l} \langle\langle\varrho, \mathfrak{g}_1\rangle, 0.3, 0.33, 0\rangle, \\ \langle\langle\varrho, \mathfrak{g}_2\rangle, 0.3, 0.33, 0\rangle, \end{array}, \varrho \in \aleph \right\}$,

but

$\mathfrak{F}^u(\mathbb{U}_1(\mathfrak{G})) = \left\{ \begin{array}{l} \langle\langle\varrho, \mathfrak{g}_1\rangle, 0.3, 0.33, 0\rangle, \\ \langle\langle\varrho, \mathfrak{g}_2\rangle, 0.3, 0.33, 0\rangle, \end{array}, \varrho \in \aleph \right\} \not\subseteq int_\tau(\Phi(\mathfrak{F}^u(\mathbb{U}_1(\mathfrak{G})), \langle 0.3, 0.3, 0.4\rangle), \langle 0.3, 0.3, 0.4\rangle)$
$= \flat(\mathfrak{G})$,

$\mathfrak{F}^l(\mathbb{U}_1(\mathfrak{G})) = \left\{ \begin{array}{l} \langle\langle\varrho, \mathfrak{g}_1\rangle, 0.3, 0.33, 0\rangle, \\ \langle\langle\varrho, \mathfrak{g}_2\rangle, 0.3, 0.33, 0\rangle, \end{array}, \varrho \in \aleph \right\} \not\subseteq int_\tau(\Phi(\mathfrak{F}^l(\mathbb{U}_1(\mathfrak{G})), \langle 0.3, 0.3, 0.4\rangle), \langle 0.3, 0.3, 0.4\rangle)$
$= \flat(\mathfrak{G})$.

(2) $\mathfrak{F} : (\aleph, \tau_2, \ell_2^P) \looparrowright (\Upsilon, \sigma)$ is TPF^u (resp. TPF^l) ℓ^P-continuous but it is not TPF^uS (resp.TPF^lS)-continuous, because,

$\mathfrak{F}^u(\mathbb{U}_1(\mathfrak{G})) = \left\{ \begin{array}{l} \langle\langle\varrho, \mathfrak{g}_1\rangle, 0.3, 0.33, 0\rangle, \\ \langle\langle\varrho, \mathfrak{g}_2\rangle, 0.3, 0.33, 0\rangle, \end{array}, \varrho \in \aleph \right\} \subseteq int_\tau(\Phi(\mathfrak{F}^u(\mathbb{U}_1(\mathfrak{G})), \langle 0.3, 0.3, 0.4\rangle), \langle 0.3, 0.3, 0.4\rangle)$
$= \sharp(\mathfrak{G})$,

$\mathfrak{F}^l(\mathbb{U}_1(\mathfrak{G})) = \left\{ \begin{array}{l} \langle\langle\varrho, \mathfrak{g}_1\rangle, 0.3, 0.33, 0\rangle, \\ \langle\langle\varrho, \mathfrak{g}_2\rangle, 0.3, 0.33, 0\rangle, \end{array}, \varrho \in \aleph \right\} \subseteq int_\tau(\Phi(\mathfrak{F}^l(\mathbb{U}_1(\mathfrak{G})), \langle 0.3, 0.3, 0.4\rangle), \langle 0.3, 0.3, 0.4\rangle)$
$= \sharp(\mathfrak{G})$,

but

$\mathfrak{F}^u(\mathbb{U}_1(\mathfrak{G})) = \left\{ \begin{array}{l} \langle\langle\varrho, \mathfrak{g}_1\rangle, 0.3, 0.33, 0\rangle, \\ \langle\langle\varrho, \mathfrak{g}_2\rangle, 0.3, 0.33, 0\rangle, \end{array}, \varrho \in \aleph \right\} \not\subseteq int_\tau(\mathfrak{F}^u(\mathbb{U}_1(\mathfrak{G}), \langle 0.3, 0.3, 0.4\rangle) = \flat(\mathfrak{G})$,

$\mathfrak{F}^l(\mathbb{U}_1(\mathfrak{G})) = \left\{ \begin{array}{l} \langle\langle\varrho, \mathfrak{g}_1\rangle, 0.3, 0.33, 0\rangle, \\ \langle\langle\varrho, \mathfrak{g}_2\rangle, 0.3, 0.33, 0\rangle, \end{array}, \varrho \in \aleph \right\} \not\subseteq int_\tau(\mathfrak{F}^l(\mathbb{U}_1(\mathfrak{G}), \langle 0.3, 0.3, 0.4\rangle) = \flat(\mathfrak{G})$

.

4. Temporal picture fuzzy almost continuous multifunctions

Definition 4.1. Let $\mathfrak{F} : (\aleph, \tau, \mathfrak{L}^P) \looparrowright (\Upsilon, \sigma)$ be a $TPFM$, $\varsigma \in I_0, \varkappa \in I_1$ and $\vartheta \in I_1$. Then, $\mathfrak{F}$ is called:

(1) TPF^uA $\mathfrak{L}^P$-continuous at a fuzzy point $\langle\varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$ iff $\langle\varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathfrak{F}^u(\mathbb{U}(\mathfrak{G}))$ for each $\mathbb{U}(\mathfrak{G}) \in \left(I^3\right)^{\Upsilon\times\mathfrak{G}}$, $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma, \varkappa, \vartheta\rangle$ there exists $\mathbb{G}(\mathfrak{G}) \in \left(I^3\right)^{\aleph\times\mathfrak{G}}$, $\tau(\mathbb{G}(\mathfrak{G})) \geq \langle\varsigma, \varkappa, \vartheta\rangle$ and $\langle\varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathbb{G}(\mathfrak{G})$
such that $\mathbb{G}(\mathfrak{G}) \cap D(\mathfrak{F}) \subseteq \mathfrak{F}^u(int_\sigma(cl_\sigma^*(\mathbb{U}(\mathfrak{G}), \langle\varsigma, \varkappa, \vartheta\rangle, \langle\varsigma, \varkappa, \vartheta\rangle)))$.

(2) TPF^lA $\mathfrak{L}^P$-continuous at a fuzzy point $\langle\varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$ iff $\langle\varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathfrak{F}^l(\mathbb{U}(\mathfrak{G}))$ for each $\mathbb{U}(\mathfrak{G}) \in \left(I^3\right)^{\Upsilon\times\mathfrak{G}}$, $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma, \varkappa, \vartheta\rangle$ there exists $\mathbb{G}(\mathfrak{G}) \in \left(I^3\right)^{\aleph\times\mathfrak{G}}$, $\tau(\mathbb{G}(\mathfrak{G})) \geq \langle\varsigma, \varkappa, \vartheta\rangle$ and $\langle\varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathbb{G}$ such that $\mathbb{G}(\mathfrak{G}) \subseteq \mathfrak{F}^l(int_\sigma(cl_\sigma^*(\mathbb{U}(\mathfrak{G}), \langle\varsigma, \varkappa, \vartheta\rangle), \langle\varsigma, \varkappa, \vartheta\rangle)))$.

(3) TPF^uA $\mathfrak{L}^P$-continuous (resp. TPF^lA $\mathfrak{L}^P$-continuous) iff it is TPF^uA $\mathfrak{L}^P$-continuous (resp. TPF^lA $\mathfrak{L}^P$-continuous) at every fuzzy point $\langle\varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$.

Remark 4.1. (1) If $\mathfrak{F}$ is $NTPFM$, then $\mathfrak{F}$ is TPF^uA $\mathfrak{L}^P$-continuous at a fuzzy point $\langle\varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$ iff $\langle\varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathfrak{F}^u(\mathbb{U}(\mathfrak{G}))$ for each $\mathbb{U}(\mathfrak{G}) \in \left(I^3\right)^{\Upsilon\times\mathfrak{G}}$, $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma, \varkappa, \vartheta\rangle$ there exists
$\mathbb{G}(\mathfrak{G}) \in \left(I^3\right)^{\aleph\times\mathfrak{G}}$, $\tau(\mathbb{G}(\mathfrak{G})) \geq \langle\varsigma, \varkappa, \vartheta\rangle$ and $\langle\varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathbb{G}(\mathfrak{G})$ such that
$\mathbb{G}(\mathfrak{G}) \subseteq \mathfrak{F}^u(int_\sigma(cl_\sigma^*(\mathbb{U}(\mathfrak{G}), \langle\varsigma, \varkappa, \vartheta\rangle), \langle\varsigma, \varkappa, \vartheta\rangle)))$.

(2) TPF^uS (resp. TPF^lS)-continuity $\Rightarrow$ TPF^uA (resp. TPF^lA) $\mathfrak{L}^P$-continuity $\Rightarrow$ TPF^uA (resp. TPF^lA) -continuity.

(3) TPF^uA (resp. TPF^lA) $\mathfrak{L}^{P0}$-continuity $\Leftrightarrow$ TPF^uA (resp. TPF^lA) -continuity.

Theorem 4.1. *For a $TPFM$ $\mathfrak{F} : (\aleph, \tau) \nrightarrow (\Upsilon, \sigma, \mathfrak{L}^P)$, $\mathbb{U}(\mathfrak{G}) \in \left(I^3\right)^{\Upsilon\times\mathfrak{G}}$, $\varsigma \in I_0, \varkappa \in I_1$ and $\vartheta \in I_1$, the following statements are equivalent:*

(1) *$\mathfrak{F}$ is TPF^lA $\mathfrak{L}^P$-continuous.*

(2) $\mathfrak{F}^l(\mathbb{U}(\mathfrak{G})) \subseteq int_\tau\left(\mathfrak{F}^l(int_\sigma(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle)), \langle\varsigma,\varkappa,\vartheta\rangle\right)$,*if* $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$.

(3) $cl_\tau\left(\mathfrak{F}^u(cl_\sigma(int^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle)), \langle\varsigma,\varkappa,\vartheta\rangle\right) \subseteq \mathfrak{F}^u(\mathbb{U}(\mathfrak{G}))$, *if* $\sigma(\dashv\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$.

Proof. (1) $\Longrightarrow$ (2) Let $\langle\varrho,\mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$, $\mathbb{U}(\mathfrak{G}) \in \left(I^3\right)^{\Upsilon\times\mathfrak{G}}$, $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$ and $\langle\varrho,\mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathfrak{F}^l(\mathbb{U}(\mathfrak{G}))$. Then, there exists $\mathbb{G}(\mathfrak{G}) \in \left(I^3\right)^{\aleph\times\mathfrak{G}}$, $\tau(\mathbb{G}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$ and $\langle\varrho,\mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathbb{G}(\mathfrak{G})$ such that
$\mathbb{G}(\mathfrak{G}) \subseteq \mathfrak{F}^l(int_\sigma(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle))$.

Thus, $\langle\varrho,\mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathbb{G}(\mathfrak{G}) \subseteq int_\tau\mathfrak{F}^l(int_\sigma(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle)), \langle\varsigma,\varkappa,\vartheta\rangle)$, and hence

$$\mathfrak{F}^l(\mathbb{U}(\mathfrak{G})) \subseteq int_\tau\left(\mathfrak{F}^l(int_\sigma(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle)), \langle\varsigma,\varkappa,\vartheta\rangle\right).$$

(2) $\Longrightarrow$ (3) Let $\mathbb{U}(\mathfrak{G}) \in \left(I^3\right)^{\Upsilon\times\mathfrak{G}}$ with $\sigma(\dashv\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$. Then by (2),

$$\begin{aligned}\dashv\mathfrak{F}^u(\mathbb{U}(\mathfrak{G})) &= \mathfrak{F}^l(\dashv\mathbb{U}(\mathfrak{G})) \subseteq int_\tau\left(\mathfrak{F}^l(int_\sigma(cl^*_\sigma(\dashv\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle)), \langle\varsigma,\varkappa,\vartheta\rangle\right)\\ &= \dashv cl_\tau\left(\mathfrak{F}^u(cl_\sigma(int^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle)), \langle\varsigma,\varkappa,\vartheta\rangle\right).\end{aligned}$$

Thus $cl_\tau\left(\mathfrak{F}^u(cl_\sigma(int^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle)), \langle\varsigma,\varkappa,\vartheta\rangle\right) \subseteq \mathfrak{F}^u(\mathbb{U}(\mathfrak{G}))$.

(3) $\Longrightarrow$ (1) Let $\langle\varrho,\mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$, $\mathbb{U}(\mathfrak{G}) \in \left(I^3\right)^{\Upsilon\times\mathfrak{G}}$, $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$ and $\langle\varrho,\mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathfrak{F}^l(\mathbb{U}(\mathfrak{G}))$. Then by (3), we have

$$\begin{aligned}&\dashv\left[int_\tau\left(\mathfrak{F}^l(int_\sigma(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle))\right), \langle\varsigma,\varkappa,\vartheta\rangle)\right]\\ =\ & cl_\tau\left(\mathfrak{F}^u(cl_\sigma(int^*_\sigma(\dashv\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle)), \langle\varsigma,\varkappa,\vartheta\rangle\right) \subseteq \mathfrak{F}^u(\dashv\mathbb{U}(\mathfrak{G})) = \dashv\mathfrak{F}^l(\mathbb{U}(\mathfrak{G})),\end{aligned}$$

and $\mathfrak{F}^l(\mathbb{U}(\mathfrak{G})) \subseteq int_\tau\left(\mathfrak{F}^l(int_\sigma(cl^*_\sigma(\mathbb{K}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle))\right), \langle\varsigma,\varkappa,\vartheta\rangle)$. Therefore,
$\langle\varrho,\mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in int_\tau\left(\mathfrak{F}^l(int_\sigma(cl^*_\sigma(\mathbb{K}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle))\right), \langle\varsigma,\varkappa,\vartheta\rangle)$
$\subseteq \mathfrak{F}^l(int_\sigma(cl^*_\sigma(\mathbb{K}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle))$. Thus, $\mathfrak{F}$ is TPF^lA $\mathfrak{L}^P$-continuous.

The following theorem is similarly proved as the proof of Theorem 4.1.

Theorem 4.2. *For a $NTPFM$ $\mathfrak{F} : (\aleph, \tau) \nrightarrow (\Upsilon, \sigma, \mathfrak{L}^P)$, $\mathbb{U}(\mathfrak{G}) \in \left(I^3\right)^{\Upsilon\times\mathfrak{G}}$, $\varsigma \in I_0, \varkappa \in I_1$ and $\vartheta \in I_1$, the following statements are equivalent:*

(1) *$\mathfrak{F}$ is TPF^uA $\mathfrak{L}^P$-continuous.*

(2) $\mathfrak{F}^u(\mathbb{U}(\mathfrak{G})) \subseteq int_\tau\left(\mathfrak{F}^u(int_\sigma(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle)), \langle\varsigma,\varkappa,\vartheta\rangle\right)$, *if* $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$.

(3) $cl_\tau\left(\mathfrak{F}^l(cl_\sigma(int^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle)), \langle\varsigma,\varkappa,\vartheta\rangle\right) \subseteq \mathfrak{F}^l(\mathbb{U}(\mathfrak{G}))$, *if* $\sigma(\dashv\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$.

The following examples show that the inverse implications in Remark 4.1(2) are not satisfied.

Example 4.1. *Let* $\aleph=\{\varrho_1,\varrho_2\}$, $\Upsilon=\{\zeta_1,\zeta_2,\}$, $\mathfrak{G}=\{\mathfrak{g}_1,\mathfrak{g}_2\}$ *and* $\mathfrak{F}:\aleph\nrightarrow\Upsilon$ *be a TPFM defined by*
$\Psi_{\mathfrak{F}}(\langle\varrho,\mathfrak{g}\rangle,\langle\zeta,\mathfrak{g}\rangle)$ *as* :

$\Psi_{\mathfrak{F}}(\langle\varrho,\mathfrak{g}\rangle,\langle\zeta,\mathfrak{g}\rangle)$	$\langle\zeta_1,\mathfrak{g}_1\rangle$	$\langle\zeta_1,\mathfrak{g}_2\rangle$	$\langle\zeta_2,\mathfrak{g}_1\rangle$	$\langle\zeta_2,\mathfrak{g}_2\rangle$
$\langle\varrho_1,\mathfrak{g}_1\rangle$	$\langle 0,1,0\rangle$	$\langle 0.2,0.3,0.5\rangle$	$\langle 1,0,0\rangle$	$\langle 0.25,0.25,0.5\rangle$
$\langle\varrho_1,\mathfrak{g}_2\rangle$	$\langle 0.3,0.15,0.55\rangle$	$\langle 0.25,0.3,0.4\rangle$	$\langle 1,0,0\rangle$	$\langle 0.3,0.5,0.2\rangle$
$\langle\varrho_2,\mathfrak{g}_1\rangle$	$\langle 1,0,0\rangle$	$\langle 0.3,0.3,0.4\rangle$	$\langle 0.25,0.5,0.25\rangle$	$\langle 0.15,0.15,0.15\rangle$
$\langle\varrho_2,\mathfrak{g}_2\rangle$	$\langle 1,0,0\rangle$	$\langle 0.4,0.1,0.3\rangle$	$\langle 0.2,0.3,0.1\rangle$	$\langle 0.4,0.2,0.4\rangle$.

Define temporal picture fuzzy topologies τ: $\left(I^3\right)^{\aleph\times\mathfrak{G}}\to I^3$, σ: $\left(I^3\right)^{\Upsilon\times\mathfrak{G}}\to I^3$, *and temporal picture fuzzy ideal* $\mathfrak{L}^P$: $\left(I^3\right)^{\Upsilon\times\mathfrak{G}}\to I^3$ *as:*

$$\tau(\mathbb{G}(\mathfrak{G})) = \begin{cases} \langle 1,0,0\rangle, & \mathbb{G}(\mathfrak{G})\in\{\flat(\mathfrak{G}),\sharp(\mathfrak{G})\} \\ \langle 0.6,0.1,0.3\rangle, & \mathbb{G}(\mathfrak{G})=\mathbb{G}_1(\mathfrak{G}) \\ \langle 0,1,0\rangle, & o.w, \end{cases}$$

$$\sigma(\mathbb{U}(\mathfrak{G})) = \begin{cases} \langle 1,0,0\rangle, & \mathbb{U}(\mathfrak{G})\in\{\flat(\mathfrak{G}),\sharp(\mathfrak{G})\} \\ \langle 0.31,0.31,0.38\rangle, & \mathbb{U}(\mathfrak{G})=\mathbb{U}_1(\mathfrak{G}) \\ \langle 0,1,0\rangle, & o.w. \end{cases}$$

$$\mathfrak{L}^P(\mathbb{U}(\mathfrak{G})) = \begin{cases} \langle 1,0,0\rangle, & \mathbb{U}(\mathfrak{G})=\flat(\mathfrak{G}) \\ \langle 0.4,0.25,0.35\rangle, & \flat(\mathfrak{G})\subset\mathbb{U}(\mathfrak{G})\subseteq\langle\langle\zeta,\mathfrak{g}\rangle,0.3,0.2,0.1\rangle, \\ & \langle\zeta,\mathfrak{g}\rangle\in\Upsilon\times\mathfrak{G} \\ \langle 0,1,0\rangle, & o.w, \end{cases}$$

where

$$\mathbb{G}_1(\mathfrak{G})=\left\{\begin{array}{l}\langle\langle\varrho,\mathfrak{g}_1\rangle,0.6,0.2,0.1\rangle, \\ \langle\langle\varrho,\mathfrak{g}_2\rangle,0.5,0.3,0.2\rangle,\end{array},\varrho\in\aleph\right\},\ \mathbb{G}_2(\mathfrak{G})=\left\{\begin{array}{l}\langle\langle\varrho,\mathfrak{g}_1\rangle,0.4,0.4,0\rangle, \\ \langle\langle\varrho,\mathfrak{g}_2\rangle,0.4,0.4,0\rangle,\end{array},\varrho\in\aleph\right\}$$

$$\text{and}\ \ \mathbb{U}_1(\mathfrak{G})=\left\{\begin{array}{l}\langle\langle\zeta,\mathfrak{g}_1\rangle,0.4,0.4,0.2\rangle, \\ \langle\langle\zeta,\mathfrak{g}_2\rangle,0.44,0.41,0.15\rangle,\end{array},\zeta\in\Upsilon\right\}.$$

Then, $\mathfrak{F}:(\aleph,\tau)\nrightarrow(\Upsilon,\sigma,\mathfrak{L}^P)$ is TPF^uA (resp. TPF^lA) $\mathfrak{L}^P$-continuous but is not TPF^uS
(resp. TPF^lS)-continuous, because $\mathfrak{F}^u(\mathbb{U}_1(\mathfrak{G}))=\mathbb{G}_2(\mathfrak{G})$
$\subseteq int_\tau\left(\mathfrak{F}^u(int_\sigma(cl^*_\sigma(\mathbb{U}_1(\mathfrak{G}),\langle 0.31,0.31,0.38\rangle),\langle 0.31,0.31,0.38\rangle)),\langle 0.31,0.31,0.38\rangle\right)=\sharp(\mathfrak{G})$,
$\mathfrak{F}^l(\mathbb{U}_1(\mathfrak{G}))=\mathbb{G}_2(\mathfrak{G})$
$\subseteq int_\tau\left(\mathfrak{F}^l(int_\sigma(cl^*_\sigma(\mathbb{U}_1(\mathfrak{G}),\langle 0.31,0.31,0.38\rangle),\langle 0.31,0.31,0.38\rangle))\right),\langle 0.31,0.31,0.38\rangle)=\sharp(\mathfrak{G})$,

but
$\mathfrak{F}^u(\mathbb{U}_1(\mathfrak{G})) = \mathbb{G}_2(\mathfrak{G}) \nsubseteq int_\tau(\mathfrak{F}^u(\mathbb{U}_1(\mathfrak{G}))), \langle 0.31, 0.31, 0.38\rangle) = \flat(\mathfrak{G})$,
$\mathfrak{F}^l(\mathbb{U}_1(\mathfrak{G})) = \mathbb{G}_2(\mathfrak{G}) \nsubseteq int_\tau\left(\mathfrak{F}^l(\mathbb{U}_1(\mathfrak{G}))), \langle 0.31, 0.31, 0.38\rangle\right) = \flat(\mathfrak{G})$.

Example 4.2. *From the Example 4.1, define temporal picture fuzzy ideal $\mathfrak{L}^P\colon \left(I^3\right)^{\Upsilon\times\mathfrak{G}} \to I^3$ as follows:*

$$\mathfrak{L}^P(\mathbb{U}(\mathfrak{G})) = \begin{cases} \langle 1,0,0\rangle, & \mathbb{U}(\mathfrak{G}) = \flat(\mathfrak{G}) \\ \langle 0.55, 0.25, 0.2\rangle, & \flat(\mathfrak{G}) \subset \mathbb{U}(\mathfrak{G}) \subseteq \left\{ \begin{array}{l} \langle\langle\zeta,\mathfrak{g}_1\rangle, 0.4, 0.4, 0.1\rangle, \\ \langle\langle\zeta,\mathfrak{g}_2\rangle, 0.44, 0.41, 0.1\rangle, \end{array} ,\zeta\in\Upsilon \right\} \\ \langle 0,1,0\rangle, & o.w, \end{cases}$$

Then, $\mathfrak{F} : (\aleph,\tau) \nleftrightarrow (\Upsilon,\sigma,\mathfrak{L}^P)$ is TPF^uA (resp. TPF^lA)-continuous but is not TPF^uA (resp. TPF^lA) $\mathfrak{L}^P$-continuous because

$\mathfrak{F}^u(\mathbb{U}_1(\mathfrak{G})) = \mathbb{G}_2(\mathfrak{G})$
$\subseteq int_\tau(\mathfrak{F}^u(int_\sigma(cl_\sigma(\mathbb{U}_1(\mathfrak{G}), \langle 0.31, 0.31, 0.38\rangle), \langle 0.31, 0.31, 0.38\rangle))), \langle 0.31, 0.31, 0.38\rangle) = \sharp(\mathfrak{G})$,
$\mathfrak{F}^l(\mathbb{U}_1(\mathfrak{G})) = \mathbb{G}_2(\mathfrak{G})$
$\subseteq int_\tau\left(\mathfrak{F}^l(int_\sigma(cl_\sigma(\mathbb{U}_1(\mathfrak{G}), \langle 0.31, 0.31, 0.38\rangle), \langle 0.31, 0.31, 0.38\rangle))\right), \langle 0.31, 0.31, 0.38\rangle) = \sharp(\mathfrak{G})$,
but
$\mathfrak{F}^u(\mathbb{U}_1(\mathfrak{G})) = \mathbb{G}_2(\mathfrak{G})$
$\nsubseteq int_\tau(\mathfrak{F}^u(int_\sigma(cl^*_\sigma(\mathbb{U}_1(\mathfrak{G}), \langle 0.31, 0.31, 0.38\rangle), \langle 0.31, 0.31, 0.38\rangle))), \langle 0.31, 0.31, 0.38\rangle) = \flat(\mathfrak{G})$,
$\mathfrak{F}^l(\mathbb{U}_1(\mathfrak{G})) = \mathbb{G}_2(\mathfrak{G})$
$\nsubseteq int_\tau\left(\mathfrak{F}^l(int_\sigma(cl^*_\sigma(\mathbb{U}_1(\mathfrak{G}), \langle 0.31, 0.31, 0.38\rangle), \langle 0.31, 0.31, 0.38\rangle))\right), \langle 0.31, 0.31, 0.38\rangle) = \flat(\mathfrak{G})$.

Theorem 4.3. *For a $TPFM$ $\mathfrak{F} : (\aleph,\tau) \nleftrightarrow (\Upsilon,\sigma,\mathfrak{L}^P)$, $\mathbb{U}(\mathfrak{G}) \in \left(I^3\right)^{\Upsilon\times\mathfrak{G}}$, $\varsigma\in I_0, \varkappa\in I_1$ and $\vartheta\in I_1$, the following statements are equivalent:*
(1) *$\mathfrak{F}$ is TPF^lA $\mathfrak{L}^P$-continuous.*
(2) *$\tau\left(\mathfrak{F}^l(\mathbb{U}(\mathfrak{G}))\right) \geq \langle\varsigma,\varkappa,\vartheta\rangle$ if $\mathbb{U}(\mathfrak{G}) = int_\sigma(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle)$.*
(3) *$\tau\left(\mathfrak{F}^l(int_\sigma(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle))\right) \geq \langle\varsigma,\varkappa,\vartheta\rangle$ if $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$.*

Proof. (1) $\Longrightarrow$ (2) If $\mathbb{U}(\mathfrak{G}) = int_\sigma(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle)$, then $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$. By Theorem 4.1(2), $\mathfrak{F}^l(\mathbb{U}(\mathfrak{G})) \subseteq int_\tau\left(\mathfrak{F}^l(int_\sigma(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle))\right), \langle\varsigma,\varkappa,\vartheta\rangle) = int_\tau\left(\mathfrak{F}^l(\mathbb{U}(\mathfrak{G})), \langle\varsigma,\varkappa,\vartheta\rangle\right)$. Thus, $\tau\left(\mathfrak{F}^l(\mathbb{U}(\mathfrak{G}))\right) \geq \langle\varsigma,\varkappa,\vartheta\rangle$.

(2) $\Leftrightarrow$ (3) Obvious.

(3) $\Longrightarrow$ (1) Let $\langle\varrho,\mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$, $\mathbb{U}(\mathfrak{G}) \in \left(I^3\right)^{\Upsilon\times\mathfrak{G}}$, $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$ and $\langle\varrho,\mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathfrak{F}^l(\mathbb{U}(\mathfrak{G}))$. Then by (3) and $\mathbb{U}(\mathfrak{G}) \subseteq int_\sigma(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle)$, $\tau\left(\mathfrak{F}^l(int_\sigma(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle))\right) \geq \langle\varsigma,\varkappa,\vartheta\rangle$ and
$\langle\varrho,\mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathfrak{F}^l(\mathbb{U}(\mathfrak{G})) \subseteq \mathfrak{F}^l(int_\sigma(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle))$. Thus, $\mathfrak{F}$ is TPF^lA $\mathfrak{L}^P$-continuous.

The following theorems are similarly proved as the proof of Theorem 4.3.

Theorem 4.4. *For a TPFM* $\mathfrak{F}:(\aleph,\tau)\multimap(\Upsilon,\sigma,\mathfrak{L}^P)$, $\mathbb{U}(\mathfrak{G})\in\left(^{3}\right)^{\Upsilon\times\mathfrak{G}}$, $\varsigma\in I_0,\varkappa\in I_1$ *and* $\vartheta\in I_1$, *the following statements are equivalent:*

(1) $\mathfrak{F}$ *is* $TPF^lA\ \mathfrak{L}^P$*-continuous.*

(2) $\tau\left(\dashv\mathfrak{F}^u\left(\mathbb{U}(\mathfrak{G})\right)\right)\geq\langle\varsigma,\varkappa,\vartheta\rangle$, *if* $\mathbb{U}(\mathfrak{G})=cl_\sigma(int^*_\sigma(\mathbb{U}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle),\langle\varsigma,\varkappa,\vartheta\rangle)$.

(3) $\tau\left(\dashv\mathfrak{F}^u(cl_\sigma(int^*_\sigma(\mathbb{U}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle),\langle\varsigma,\varkappa,\vartheta\rangle))\right)\geq\langle\varsigma,\varkappa,\vartheta\rangle$ *if* $\sigma(\dashv\mathbb{U}(\mathfrak{G}))\geq\langle\varsigma,\varkappa,\vartheta\rangle$.

Theorem 4.5. *For a NTPFM* $\mathfrak{F}:(\aleph,\tau)\multimap(\Upsilon,\sigma,\mathfrak{L}^P)$, $\mathbb{U}(\mathfrak{G})\in\left(I^3\right)^{\Upsilon\times\mathfrak{G}}$, $\varsigma\in I_0,\varkappa\in I_1$ *and* $\vartheta\in I_1$, *the following statements are equivalent:*

(1) $\mathfrak{F}$ *is* $TPF^uA\ \mathfrak{L}^P$*-continuous.*

(2) $\tau\left(\mathfrak{F}^u\left(\mathbb{U}(\mathfrak{G})\right)\right)\geq\langle\varsigma,\varkappa,\vartheta\rangle$, *if* $\mathbb{U}(\mathfrak{G})=int_\sigma(cl^*_\sigma(\mathbb{U}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle),\langle\varsigma,\varkappa,\vartheta\rangle)$.

(3) $\tau\left(\mathfrak{F}^u(int_\sigma(cl^*_\sigma(\mathbb{U}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle),\langle\varsigma,\varkappa,\vartheta\rangle))\right)\geq\langle\varsigma,\varkappa,\vartheta\rangle$ *if* $\sigma(\mathbb{U}(\mathfrak{G}))\geq\langle\varsigma,\varkappa,\vartheta\rangle$.

Theorem 4.6. *For a NTPFM* $\mathfrak{F}:(\aleph,\tau)\multimap(\Upsilon,\sigma,\mathfrak{L}^P)$, $\mathbb{U}(\mathfrak{G})\in\left(I^3\right)^{\Upsilon\times\mathfrak{G}}$, $\varsigma\in I_0,\varkappa\in I_1$ *and* $\vartheta\in I_1$, *the following statements are equivalent:*

(1) $\mathfrak{F}$ *is* $TPF^uA\ \mathfrak{L}^P$*-continuous.*

(2) $\tau\left(\dashv\mathfrak{F}^l\left(\mathbb{U}(\mathfrak{G})\right)\right)\geq\langle\varsigma,\varkappa,\vartheta\rangle$, *if* $\mathbb{U}(\mathfrak{G})=cl_\sigma(int^*_\sigma(\mathbb{U}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle),\langle\varsigma,\varkappa,\vartheta\rangle)$.

(3) $\tau\left(\dashv\mathfrak{F}^l(cl_\sigma(int^*_\sigma(\mathbb{U}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle),\langle\varsigma,\varkappa,\vartheta\rangle))\right)\geq\langle\varsigma,\varkappa,\vartheta\rangle$ *if* $\sigma(\dashv\mathbb{U}(\mathfrak{G}))\geq\langle\varsigma,\varkappa,\vartheta\rangle$.

Theorem 4.7. *Let* $\mathfrak{F}:(\aleph,\tau)\multimap(\Upsilon,\sigma,\mathfrak{L}^P)$ *be a TPFM. Then,* $\mathfrak{F}$ *is* $TPF^lA\ \mathfrak{L}^P$*-continuous iff*
$cl_\tau\left(\mathfrak{F}^u\left(\mathbb{U}(\mathfrak{G})\right),\langle\varsigma,\varkappa,\vartheta\rangle\right)\subseteq\mathfrak{F}^u(cl_\sigma(\mathbb{U}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle))$ *for any* $\mathbb{U}(\mathfrak{G})\in\left(I^3\right)^{\Upsilon\times\mathfrak{G}}$ *with* $\mathbb{U}(\mathfrak{G})\subseteq cl_\sigma(int^*_\sigma(\mathbb{U}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle),\langle\varsigma,\varkappa,\vartheta\rangle),\varsigma\in I_0,\varkappa\in I_1$ *and* $\vartheta\in I_1$.

Proof. ($\Rightarrow$) Let $\mathfrak{F}$ be a $TPF^lA\ \mathfrak{L}^P$-continuous. Then for any $\mathbb{U}(\mathfrak{G})\in\left(I^3\right)^{\Upsilon\times\mathfrak{G}}$ with $\mathbb{U}(\mathfrak{G})\subseteq cl_\sigma(int^*_\sigma(\mathbb{U}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle),\langle\varsigma,\varkappa,\vartheta\rangle)=\mathbb{K}(\mathfrak{G})$ (say) where $\mathbb{K}(\mathfrak{G})=cl_\sigma(int^*_\sigma(\mathbb{K}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle),\langle\varsigma,\varkappa,\vartheta\rangle)$. By Theorem 3.6, $\tau\left(\dashv\mathfrak{F}^u\left(\mathbb{K}(\mathfrak{G})\right)\right)\geq\langle\varsigma,\varkappa,\vartheta\rangle$, and thus

$cl_\tau\left(\mathfrak{F}^u\left(\mathbb{U}(\mathfrak{G})\right),\langle\varsigma,\varkappa,\vartheta\rangle\right)\subseteq cl_\tau\left(\mathfrak{F}^u\left(\mathbb{K}(\mathfrak{G})\right),\langle\varsigma,\varkappa,\vartheta\rangle\right)=\mathfrak{F}^u(cl_\sigma(int^*_\sigma(\mathbb{K}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle),\langle\varsigma,\varkappa,\vartheta\rangle))$
$\subseteq\mathfrak{F}^u(cl_\sigma(\mathbb{U}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle))$.

($\Leftarrow$) Let $\mathbb{U}(\mathfrak{G})\in\left(I^3\right)^{\Upsilon\times\mathfrak{G}}$ with $\mathbb{U}(\mathfrak{G})=cl_\sigma(int^*_\sigma(\mathbb{U}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle),\langle\varsigma,\varkappa,\vartheta\rangle)$. Then, $\mathbb{U}(\mathfrak{G})\subseteq cl_\sigma(int^*_\sigma(\mathbb{U}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle),\langle\varsigma,\varkappa,\vartheta\rangle)$ and $cl_\tau\left(\mathfrak{F}^u\left(\mathbb{U}(\mathfrak{G})\right),\langle\varsigma,\varkappa,\vartheta\rangle\right)\subseteq\mathfrak{F}^u(cl_\sigma(\mathbb{U}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle))=\mathfrak{F}^u\left(\mathbb{U}(\mathfrak{G})\right)$. Therefore, we obtain $\tau\left(\dashv\mathfrak{F}^u\left(\mathbb{U}(\mathfrak{G})\right)\right)\geq\langle\varsigma,\varkappa,\vartheta\rangle$. Thus by Theorem 4.1, $\mathfrak{F}$ is $TPF^lA\ \mathfrak{L}^P$-continuous.

The following theorem is similarly proved as the proof of Theorem 4.7.

Theorem 4.8. *Let* $\mathfrak{F}:(\aleph,\tau)\multimap(\Upsilon,\sigma,\mathfrak{L}^P)$ *be a NTPFM. Then* $\mathfrak{F}$ *is* $TPF^uA\ \mathfrak{L}^P$*-continuous iff*
$cl_\tau\left(\mathfrak{F}^l\left(\mathbb{U}(\mathfrak{G})\right),\langle\varsigma,\varkappa,\vartheta\rangle\right)\subseteq\mathfrak{F}^l(cl_\sigma(\mathbb{U}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle))$ *for any* $\mathbb{U}(\mathfrak{G})\in\left(I^3\right)^{\Upsilon\times\mathfrak{G}}$ *with* $\mathbb{U}(\mathfrak{G})\subseteq cl_\sigma(int^*_\sigma(\mathbb{U}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle),\langle\varsigma,\varkappa,\vartheta\rangle)$, $\varsigma\in I_0,\varkappa\in I_1$ *and* $\vartheta\in I_1$.

5. Temporal picture fuzzy weakly continuous multifunctions

Definition 5.1. Let $\mathfrak{F}:(\aleph,\tau)\multimap(\Upsilon,\sigma,\mathfrak{L}^P)$ be a $TPFM$, $\varsigma\in I_0,\varkappa\in I_1$ and $\vartheta\in I_1$. Then, $\mathfrak{F}$ is called:

(1) TPF^uW $\mathfrak{L}^P$-continuous at a fuzzy point $\langle \varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$ iff $\langle \varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathfrak{F}^u(\mathbb{U}(\mathfrak{G}))$ for each $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$, $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$ there exists $\mathbb{G}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$, $\tau(\mathbb{G}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$ and $\langle \varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathbb{G}(\mathfrak{G})$ such that $\mathbb{G}(\mathfrak{G}) \cap D(\mathfrak{F}) \subseteq \mathfrak{F}^u(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)))$.

(2) TPF^lW $\mathfrak{L}^P$-continuous at a fuzzy point $\langle \varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$ iff $\langle \varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathfrak{F}^l(\mathbb{U}(\mathfrak{G}))$ for each $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$, $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$ there exists $\mathbb{G}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$, $\tau(\mathbb{G}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$ and $\langle \varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathbb{G}(\mathfrak{G})$ such that $\mathbb{G}(\mathfrak{G}) \subseteq \mathfrak{F}^l(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)))$.

(3) TPF^uW $\mathfrak{L}^P$-continuous(resp. TPF^lW $\mathfrak{L}^P$-continuous) iff it is TPF^uW $\mathfrak{L}^P$-continuous(resp. TPF^lW $\mathfrak{L}^P$-continuous) at every fuzzy point $\langle \varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$.

Remark 5.1. (1) If $\mathfrak{F}$ is $NTPFM$, then $\mathfrak{F}$ is TPF^uW $\mathfrak{L}^P$-continuous at a fuzzy point $\langle \varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$ iff
$\langle \varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathfrak{F}^u(\mathbb{U}(\mathfrak{G}))$ for each $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$, $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$ there exists
$\mathbb{G}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$, $\tau(\mathbb{G}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$ and $\langle \varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathbb{G}$ such that $\mathbb{G}(\mathfrak{G}) \subseteq \mathfrak{F}^u(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)))$.

(2) TPF^uA (resp. TPF^lA) $\mathfrak{L}^P$-continuity $\Rightarrow TPF^uW$ (resp. TPF^lW) $\mathfrak{L}^P$-continuity $\Rightarrow TPF^uW$ (resp. TPF^lW)-continuity.

(3) TPF^uW (resp. TPF^lW) $\mathfrak{L}^{P0}$-continuity $\Leftrightarrow TPF^uW$ (resp. TPF^lW)-continuity.

Theorem 5.1. *A $TPFM$ $\mathfrak{F} : (\aleph, \tau) \nrightarrow (\Upsilon, \sigma, \mathfrak{L}^P)$ is TPF^lW $\mathfrak{L}^P$-continuous iff $\mathfrak{F}^l(\mathbb{U}(\mathfrak{G})) \subseteq int_\tau(\mathfrak{F}^l(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle))), \langle\varsigma,\varkappa,\vartheta\rangle)$ for each $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$ with $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$, $\varsigma \in I_0, \varkappa \in I_1$ and $\vartheta \in I_1$.*

Proof. ($\Rightarrow$) Let $\langle \varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$, $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$with $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$ and $\langle \varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathfrak{F}^l(\mathbb{U}(\mathfrak{G}))$. Then, there exists $\mathbb{G}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$, $\tau(\mathbb{G}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$ and $\langle \varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathbb{G}(\mathfrak{G})$ such that
$\mathbb{G}(\mathfrak{G}) \subseteq \mathfrak{F}^l(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)))$. Thus, $\langle \varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathbb{G}(\mathfrak{G}) \subseteq int_\tau(\mathfrak{F}^l(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle))), \langle\varsigma,\varkappa,\vartheta\rangle)$ and hence $\mathfrak{F}^l(\mathbb{U}(\mathfrak{G})) \subseteq int_\tau(\mathfrak{F}^l(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle))), \langle\varsigma,\varkappa,\vartheta\rangle)$.

($\Leftarrow$) Let $\langle \varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$,$\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$with $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$ and $\langle \varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathfrak{F}^l(\mathbb{U}(\mathfrak{G}))$. Then, $\langle \varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathfrak{F}^l(\mathbb{U}(\mathfrak{G})) \subseteq int_\tau(\mathfrak{F}^l(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle))), \langle\varsigma,\varkappa,\vartheta\rangle)$. Thus, $\langle \varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathbb{G}(\mathfrak{G}) \subseteq int_\tau(\mathfrak{F}^l(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle))), \langle\varsigma,\varkappa,\vartheta\rangle) \subseteq \mathfrak{F}^l(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)))$. Hence, $\mathfrak{F}$ is TPF^uW $\mathfrak{L}^P$-continuous.

The following theorem is similarly proved as the proof of Theorem 5.1.

Theorem 5.2. *A $NTPFM$ $\mathfrak{F} : (\aleph, \tau) \nrightarrow (\Upsilon, \sigma, \mathfrak{L}^P)$ is TPF^uW $\mathfrak{L}^P$-continuous iff $\mathfrak{F}^u((\mathbb{U}(\mathfrak{G})) \subseteq int_\tau(\mathfrak{F}^u(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle))), \langle\varsigma,\varkappa,\vartheta\rangle)$ for each $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$ with $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$, $\varsigma \in I_0, \varkappa \in I_1$ and $\vartheta \in I_1$.*

The following examples shows that generally a TPF^uW $\mathfrak{L}^P$-continuous and TPF^lW $\mathfrak{L}^P$-continuous (resp. a TPF^uW continuous and TPF^lW continuous) multifunction need not be either a TPF^uA $\mathfrak{L}^P$-continuous (resp. TPF^uW $\mathfrak{L}^P$-continuous) multifunction or TPF^lA $\mathfrak{L}^P$-continuous (resp. TPF^lW $\mathfrak{L}^P$-continuous) multifunction.

Example 5.1. *From the Example* 3.2,

$\mathfrak{F}:(\aleph,\tau)\nrightarrow(\Upsilon,\sigma,\mathfrak{L}^P)$ *is* TPF^uW *(resp.* TPF^lW*)-continuous but is not* TPF^uW *(resp.* TPF^lW*)* $\mathfrak{L}^P$*-continuous because*

$\mathfrak{F}^u(\mathbb{U}_1(\mathfrak{G})) = \mathbb{G}_2(\mathfrak{G}) \subseteq int_\tau(\mathfrak{F}^u(cl_\sigma(\mathbb{U}_1(\mathfrak{G}),\langle 0.31,0.31,0.38\rangle)),\langle 0.31,0.31,0.38\rangle) = \sharp(\mathfrak{G})$,

$\mathfrak{F}^l(\mathbb{U}_1(\mathfrak{G})) = \mathbb{G}_2(\mathfrak{G}) \subseteq int_\tau\left(\mathfrak{F}^l(cl_\sigma(\mathbb{U}_1(\mathfrak{G}),\langle 0.31,0.31,0.38\rangle))\right),\langle 0.31,0.31,0.38\rangle) = \sharp(\mathfrak{G})$,

but

$\mathfrak{F}^u(\mathbb{U}_1(\mathfrak{G})) = \mathbb{G}_2(\mathfrak{G}) \nsubseteq int_\tau(\mathfrak{F}^u(cl^*_\sigma(\mathbb{U}_1(\mathfrak{G}),\langle 0.31,0.31,0.38\rangle)),\langle 0.31,0.31,0.38\rangle) = \flat(\mathfrak{G})$,

$\mathfrak{F}^l(\mathbb{U}_1(\mathfrak{G})) = \mathbb{G}_2(\mathfrak{G}) \nsubseteq int_\tau\left(\mathfrak{F}^l(cl^*_\sigma(\mathbb{U}_1(\mathfrak{G}),\langle 0.31,0.31,0.38\rangle))\right),\langle 0.31,0.31,0.38\rangle) = \flat(\mathfrak{G})$,

Example 5.2. *Let* $\aleph=\{\varrho_1,\varrho_2\}$, $\Upsilon=\{\zeta_1,\zeta_2,\}$, $\mathfrak{G}=\{\mathfrak{g}_1,\mathfrak{g}_2\}$ *and* $\mathfrak{F}:\aleph\nrightarrow\Upsilon$ *be a* $TPFM$ *defined by*
$\Psi_{\mathfrak{F}}(\langle\varrho,\mathfrak{g}\rangle,\langle\zeta,\mathfrak{g}\rangle)$ *as:*

$\Psi_{\mathfrak{F}}(\langle\varrho,\mathfrak{g}\rangle,\langle\zeta,\mathfrak{g}\rangle)$	$\langle\zeta_1,\mathfrak{g}_1\rangle$	$\langle\zeta_1,\mathfrak{g}_2\rangle$	$\langle\zeta_2,\mathfrak{g}_1\rangle$	$\langle\zeta_2,\mathfrak{g}_2\rangle$
$\langle\varrho_1,\mathfrak{g}_1\rangle$	$\langle 0.2,0.3,0.4\rangle$	$\langle 1,0,0\rangle$	$\langle 0.2,0.5,0.1\rangle$	$\langle 0.6,0.2,0.1\rangle$
$\langle\varrho_1,\mathfrak{g}_2\rangle$	$\langle 0.4,0.2,0.4\rangle$	$\langle 1,0,0\rangle$	$\langle 0.3,0.6,0.1\rangle$	$\langle 0.3,0.3,0.3\rangle$
$\langle\varrho_2,\mathfrak{g}_1\rangle$	$\langle 0.2,0.2,0.2\rangle$	$\langle 0.1,0.1,0.1\rangle$	$\langle 1,0,0\rangle$	$\langle 0.44,0.2,0.1\rangle$
$\langle\varrho_2,\mathfrak{g}_2\rangle$	$\langle 0.5,0.1,0.2\rangle$	$\langle 0.1,0.1,0.1\rangle$	$\langle 0.2,0.3,0.5\rangle$	$\langle 1,0,0\rangle$.

Define temporal picture fuzzy topologies τ: $\left(I^3\right)^{\aleph\times\mathfrak{G}}\to I^3$, σ: $\left(I^3\right)^{\Upsilon\times\mathfrak{G}}\to I^3$, *and temporal picture fuzzy ideal* $\mathfrak{L}^P$: $\left(I^3\right)^{\Upsilon\times\mathfrak{G}}\to I^3$ *as:*

$$\tau(\mathbb{G}(\mathfrak{G})) = \begin{cases} \langle 1,0,0\rangle, & \mathbb{G}(\mathfrak{G})\in\{\flat(\mathfrak{G}),\sharp(\mathfrak{G})\} \\ \langle 0.6,0.2,0.2\rangle, & \mathbb{G}(\mathfrak{G})=\mathbb{G}_1(\mathfrak{G}) \\ \langle 0,1,0\rangle, & o.w, \end{cases}$$

$$\sigma(\mathbb{U}(\mathfrak{G})) = \begin{cases} \langle 1,0,0\rangle, & \mathbb{U}(\mathfrak{G})\in\{\flat(\mathfrak{G}),\sharp(\mathfrak{G})\} \\ \langle 0.4,0.45,0.15\rangle, & \mathbb{U}(\mathfrak{G})=\mathbb{U}_1(\mathfrak{G}) \\ \langle 0,1,0\rangle, & o.w. \end{cases}$$

$$\mathfrak{L}^P(\mathbb{U}(\mathfrak{G})) = \begin{cases} \langle 1,0,0\rangle, & \mathbb{U}(\mathfrak{G})=\flat(\mathfrak{G}) \\ \langle 0.3,0.1,0.6\rangle, & \left\{\begin{array}{l}\langle\langle\zeta,\mathfrak{g}_1\rangle,0.44,0.4,0.16\rangle, \\ \langle\langle\zeta,\mathfrak{g}_2\rangle,0.44,0.41,0.15\rangle,\end{array},\zeta\in\Upsilon\right\}\subseteq\mathbb{U}(\mathfrak{G})\subset\sharp(\mathfrak{G}) \\ \langle 0,1,0\rangle, & o.w, \end{cases}$$

where $\mathbb{G}_1(\mathfrak{G}) = \left\{\begin{array}{l}\langle\langle\varrho,\mathfrak{g}_1\rangle,0.44,0.41,0\rangle, \\ \langle\langle\varrho,\mathfrak{g}_2\rangle,0.44,0.41,0\rangle,\end{array},\varrho\in\aleph\right\}$ *and* $\mathbb{U}_1(\mathfrak{G}) = \left\{\begin{array}{l}\langle\langle\zeta,\mathfrak{g}_1\rangle,0.4,0.44,0.2\rangle, \\ \langle\langle\zeta,\mathfrak{g}_2\rangle,0.41,0.44,0.15\rangle,\end{array},\zeta\in\Upsilon\right\}$.

Then, $\mathfrak{F}:(\aleph,\tau)\nrightarrow(\Upsilon,\sigma,\mathfrak{L}^P)$ is TPF^uW (resp. TPF^lW) $\mathfrak{L}^P$-continuous but is not TPF^uA (resp. TPF^lA)-continuous because

$\mathfrak{F}^u(\mathbb{U}_1(\mathfrak{G})) = \left\{ \begin{array}{l} \langle\langle\varrho, \mathfrak{g}_1\rangle, 0.4, 0.44, 0\rangle, \\ \langle\langle\varrho, \mathfrak{g}_2\rangle, 0.4, 0.44, 0\rangle, \end{array}, \varrho \in \aleph \right\}$
$\subseteq int_\tau\left(\mathfrak{F}^u(cl^*_\sigma(\mathbb{U}_1(\mathfrak{G}), \langle 0.4, 0.45, 0.15\rangle)), \langle 0.4, 0.45, 0.15\rangle\right) = \mathbb{G}_1(\mathfrak{G})$,
$\mathfrak{F}^l(\mathbb{U}_1(\mathfrak{G})) = \left\{ \begin{array}{l} \langle\langle\varrho, \mathfrak{g}_1\rangle, 0.41, 0.44, 0\rangle, \\ \langle\langle\varrho, \mathfrak{g}_2\rangle, 0.41, 0.44, 0\rangle, \end{array}, \varrho \in \aleph \right\}$
$\subseteq int_\tau\left(\mathfrak{F}^l(cl^*_\sigma(\mathbb{U}_1(\mathfrak{G}), \langle 0.4, 0.45, 0.15\rangle)), \langle 0.4, 0.45, 0.15\rangle\right) = \mathbb{G}_1(\mathfrak{G})$,
but
$\mathfrak{F}^u(\mathbb{U}_1(\mathfrak{G})) = \left\{ \begin{array}{l} \langle\langle\varrho, \mathfrak{g}_1\rangle, 0.4, 0.44, 0\rangle, \\ \langle\langle\varrho, \mathfrak{g}_2\rangle, 0.4, 0.44, 0\rangle, \end{array}, \varrho \in \aleph \right\}$
$\nsubseteq int_\tau\left(\mathfrak{F}^u(int_\sigma(cl^*_\sigma(\mathbb{U}_1(\mathfrak{G}), \langle 0.4, 0.45, 0.15\rangle), \langle 0.4, 0.45, 0.15\rangle))\right), \langle 0.4, 0.45, 0.15\rangle) = \flat(\mathfrak{G})$,
$\mathfrak{F}^l(\mathbb{U}_1(\mathfrak{G})) = \left\{ \begin{array}{l} \langle\langle\varrho, \mathfrak{g}_1\rangle, 0.41, 0.44, 0\rangle, \\ \langle\langle\varrho, \mathfrak{g}_2\rangle, 0.41, 0.44, 0\rangle, \end{array}, \varrho \in \aleph \right\}$
$\nsubseteq int_\tau\left(\mathfrak{F}^l(int_\sigma(cl^*_\sigma(\mathbb{U}_1(\mathfrak{G}), \langle 0.4, 0.45, 0.15\rangle), \langle 0.4, 0.45, 0.15\rangle))\right), \langle 0.4, 0.45, 0.15\rangle) = \flat(\mathfrak{G})$.

Theorem 5.3. *A TPFM* $\mathfrak{F} : (\aleph, \tau) \nrightarrow (\Upsilon, \sigma, \mathfrak{L}^P)$ *is* TPF^lW $\mathfrak{L}^P$*-continuous iff* $cl_\tau(\mathfrak{F}^u(int^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma, \varkappa, \vartheta\rangle)), \langle\varsigma, \varkappa, \vartheta\rangle) \subseteq \mathfrak{F}^u(\mathbb{U}(\mathfrak{G}))$ *for each* $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$ *with* $\sigma(\urcorner\,\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma, \varkappa, \vartheta\rangle$, $\varsigma \in I_0, \varkappa \in I_1$ *and* $\vartheta \in I_1$.

Proof. ($\Rightarrow$) Let $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$ with $\sigma(\urcorner\,\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma, \varkappa, \vartheta\rangle$. Then by Theorem 4.7,

$$\begin{aligned} \urcorner\,\mathfrak{F}^u(\mathbb{U}(\mathfrak{G})) &= \mathfrak{F}^l(\urcorner\,\mathbb{U}(\mathfrak{G})) \subseteq int_\tau(\mathfrak{F}^l(cl^*_\sigma(\urcorner\,\mathbb{U}(\mathfrak{G}), \langle\varsigma, \varkappa, \vartheta\rangle)), \langle\varsigma, \varkappa, \vartheta\rangle) \\ &= \urcorner\,cl_\tau(\mathfrak{F}^u(int^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma, \varkappa, \vartheta\rangle)), \langle\varsigma, \varkappa, \vartheta\rangle). \end{aligned}$$

Thus, $cl_\tau(\mathfrak{F}^u(int^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma, \varkappa, \vartheta\rangle)), \langle\varsigma, \varkappa, \vartheta\rangle) \subseteq \mathfrak{F}^u(\mathbb{U}(\mathfrak{G}))$.

($\Leftarrow$) Let $\langle\varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$, $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$ with $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma, \varkappa, \vartheta\rangle$ and $\xi_t \in \mathfrak{F}^l(\mathbb{U}(\mathfrak{G}))$. Then,

$$\begin{aligned} \urcorner int_\tau(\mathfrak{F}^l(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma, \varkappa, \vartheta\rangle)), \langle\varsigma, \varkappa, \vartheta\rangle) &= cl_\tau(\mathfrak{F}^u(int^*_\sigma(\urcorner\,\mathbb{U}(\mathfrak{G}), \langle\varsigma, \varkappa, \vartheta\rangle)), \langle\varsigma, \varkappa, \vartheta\rangle) \\ &\subseteq \mathfrak{F}^u(\urcorner\,\mathbb{U}(\mathfrak{G})) = \urcorner\,\mathfrak{F}^l(\mathbb{U}(\mathfrak{G})), \end{aligned}$$

and hence $\mathfrak{F}^l(\mathbb{U}(\mathfrak{G})) \subseteq int_\tau(\mathfrak{F}^l(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma, \varkappa, \vartheta\rangle)), \langle\varsigma, \varkappa, \vartheta\rangle)$. Thus, it is TPF^lW $\mathfrak{L}^P$-continuous.

The following theorem is similarly proved as the proof of Theorem 5.3.

Theorem 5.4. *A NTPFM* $\mathfrak{F} : (\aleph, \tau) \nrightarrow (\Upsilon, \sigma, \mathfrak{L}^P)$ *is* TPF^uW $\mathfrak{L}^P$*-continuous iff* $cl_\tau(\mathfrak{F}^l(int^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma, \varkappa, \vartheta\rangle)), \langle\varsigma, \varkappa, \vartheta\rangle) \subseteq \mathfrak{F}^l(\mathbb{U}(\mathfrak{G}))$ *for each* $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$ *with* $\sigma(\urcorner\,\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma, \varkappa, \vartheta\rangle$, $\varsigma \in I_0, \varkappa \in I_1$ *and* $\vartheta \in I_1$.

Theorem 5.5. *If* $\mathfrak{F} : (\aleph, \tau) \nrightarrow (\Upsilon, \sigma, \mathfrak{L}^P)$ *is* $NTPF^uW$ $\mathfrak{L}^P$*-continuous and* $\mathfrak{F}(\mathbb{G}(\mathfrak{G})) \subseteq int_\sigma(cl^*_\sigma(\mathfrak{F}(\mathbb{G}(\mathfrak{G})), \langle\varsigma, \varkappa, \vartheta\rangle), \langle\varsigma, \varkappa, \vartheta\rangle)$ *for each* $\mathbb{G}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$ *then* $\mathfrak{F}$ *is* TPF^uA $\mathfrak{L}^P$*-continuous.*

Proof. Let $\langle\varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$, $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$, $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma, \varkappa, \vartheta\rangle$ and $\langle\varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathfrak{F}^u(\mathbb{U}(\mathfrak{G}))$. Then, there exists $\mathbb{G}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$ with $\tau(\mathbb{G}(\mathfrak{G})) \geq \langle\varsigma, \varkappa, \vartheta\rangle$ and $\langle\varrho, \mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in$

$\mathbb{G}(\mathfrak{G})$ such that
$\mathbb{G}(\mathfrak{G}) \subseteq \mathfrak{F}^u(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle))$, then $\mathfrak{F}(\mathbb{G}(\mathfrak{G})) \subseteq \mathfrak{F}(\mathfrak{F}^u(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle))) \subseteq cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)$.
Since $\mathfrak{F}(\mathbb{G}(\mathfrak{G})) \subseteq int_\sigma(cl^*_\sigma(\mathfrak{F}(\mathbb{G}(\mathfrak{G})), \langle\varsigma,\varkappa,\vartheta\rangle),$
$\langle\varsigma,\varkappa,\vartheta\rangle) \subseteq int_\sigma(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle)$, hence
$\mathbb{G}(\mathfrak{G}) \subseteq \mathfrak{F}^u(\mathfrak{F}(\mathbb{G}(\mathfrak{G}))) \subseteq \mathfrak{F}^u(int_\sigma(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle))$. Then, $\mathfrak{F}$ is TPF^uA $\mathfrak{L}^P$-continuous.

Theorem 5.6. *Let* $\mathfrak{F} : (\aleph,\tau) \nrightarrow (\Upsilon,\sigma,\mathfrak{L}^P)$ *be a* TPF^lW $\mathfrak{L}^P$*-continuous. Then,*
$\mathfrak{F}^l(\mathbb{U}(\mathfrak{G})) \subseteq int_\tau(\mathfrak{F}^l(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)), \langle\varsigma,\varkappa,\vartheta\rangle)$ *for any* $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$ *with*
$\mathbb{U}(\mathfrak{G}) \subseteq int_\sigma(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle)$*,* $\varsigma \in I_0, \varkappa \in I_1$ *and* $\vartheta \in I_1$.

Proof. Let $\mathfrak{F}$ be a TPF^lW $\mathfrak{L}^P$-continuous and $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$ with
$\mathbb{U}(\mathfrak{G}) \subseteq int_\sigma(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle)$.
Then, if $\langle\varrho,\mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathfrak{F}^l(\mathbb{U}(\mathfrak{G})) \subseteq \mathfrak{F}^l(int_\sigma(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle))$, there exists
$\mathbb{G}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$,
$\tau(\mathbb{G}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$ and $\langle\varrho,\mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathbb{G}(\mathfrak{G})$ such that

$$\mathbb{G}(\mathfrak{G}) \subseteq (\mathfrak{F}^l(cl^*_\sigma(int_\sigma(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle))) \subseteq \mathfrak{F}^l(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)).$$

Thus, $\mathbb{G}(\mathfrak{G}) \subseteq int_\tau(\mathfrak{F}^l(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)), \langle\varsigma,\varkappa,\vartheta\rangle)$ and
$\mathfrak{F}^l(\mathbb{U}(\mathfrak{G})) \subseteq int_\tau(\mathfrak{F}^l(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)), \langle\varsigma,\varkappa,\vartheta\rangle)$.

The following theorem is similarly proved as the proof of Theorem 5.6.

Theorem 5.7. *Let* $\mathfrak{F} : (\aleph,\tau) \nrightarrow (\Upsilon,\sigma,\mathfrak{L}^P)$ *be a* $NTPF^uW$ $\mathfrak{L}^P$*-continuous . Then,*
$\mathfrak{F}^u((\mathbb{U}(\mathfrak{G})) \subseteq int_\tau(\mathfrak{F}^u(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)), \langle\varsigma,\varkappa,\vartheta\rangle)$ *for any* $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$ *with*
$\mathbb{U}(\mathfrak{G}) \subseteq int_\sigma(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle), \langle\varsigma,\varkappa,\vartheta\rangle)$*,* $\varsigma \in I_0, \varkappa \in I_1$ *and* $\vartheta \in I_1$.

6. Temporal picture fuzzy almost weakly continuous multifunctions

Definition 6.1. Let $\mathfrak{F} : (\aleph,\tau) \nrightarrow (\Upsilon,\sigma,\mathfrak{L}^P)$ be a $TPFM$, $\varsigma \in I_0, \varkappa \in I_1$ and $\vartheta \in I_1$.
Then, $\mathfrak{F}$ is called:

(1) TPF^uAW $\mathfrak{L}^P$-continuous at a fuzzy point $\langle\varrho,\mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$ iff $\langle\varrho,\mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathfrak{F}^u(\mathbb{U}(\mathfrak{G}))$ for each $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$, $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$ there exists $\mathbb{G}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$, $\tau(\mathbb{G}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$ and $\langle\varrho,\mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathbb{G}(\mathfrak{G})$ such that $\mathbb{G}(\mathfrak{G}) \cap D(\mathfrak{F}) \subseteq cl_\tau(\mathfrak{F}^u(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)), \langle\varsigma,\varkappa,\vartheta\rangle)$.

(2) TPF^lAW $\mathfrak{L}^P$-continuous at a fuzzy point $\langle\varrho,\mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$ iff $\langle\varrho,\mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathfrak{F}^l(\mathbb{U}(\mathfrak{G}))$ for each $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$, $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$ there exists $\mathbb{G}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$, $\tau(\mathbb{G}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$ and $\langle\varrho,\mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in \mathbb{G}(\mathfrak{G})$ such that $\mathbb{G}(\mathfrak{G}) \subseteq cl_\tau(\mathfrak{F}^l(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma,\varkappa,\vartheta\rangle)), \langle\varsigma,\varkappa,\vartheta\rangle)$.

(3) TPF^uAW $\mathfrak{L}^P$-continuous(resp. TPF^lAW $\mathfrak{L}^P$-continuous) iff it is TPF^uAW $\mathfrak{L}^P$-continuous
(resp. TPF^lAW $\mathfrak{L}^P$-continuous) at every fuzzy point $\langle\varrho,\mathfrak{g}\rangle_{\langle\varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$.

Remark 6.1. (1) If $\mathfrak{F}$ is $NTPFM$, then $\mathfrak{F}$ is TPF^uAW $\mathfrak{L}^P$-continuous at a fuzzy point $\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$ iff $\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in \mathfrak{F}^u(\mathbb{U}(\mathfrak{G}))$ for each $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$, $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle \varsigma,\varkappa,\vartheta\rangle$ there exists $\mathbb{G}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$,
$\tau(\mathbb{G}(\mathfrak{G})) \geq \langle \varsigma,\varkappa,\vartheta\rangle$ and $\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in \mathbb{G}(\mathfrak{G})$ such that $\mathbb{G}(\mathfrak{G}) \subseteq cl_\tau(\mathfrak{F}^u(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle \varsigma,\varkappa,\vartheta\rangle)), \langle \varsigma,\varkappa,\vartheta\rangle)$.

(2) TPF^uW (resp. TPF^lW) $\mathfrak{L}^P$-continuity $\Rightarrow$ TPF^uAW (resp. TPF^lAW) $\mathfrak{L}^P$-continuity $\Rightarrow$ TPF^uAW (resp. TPF^lAW)-continuity.

(3) TPF^uAW (resp. TPF^lAW) $\mathfrak{L}^{P0}$-continuity $\Leftrightarrow$ TPF^uAW (resp. TPF^lAW)-continuity.

Theorem 6.1. *For a $TPFM$ $\mathfrak{F} : (\aleph, \tau) \looparrowright (\Upsilon, \sigma, \mathfrak{L}^P)$, $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$, $\varsigma \in I_0, \varkappa \in I_1$ and $\vartheta \in I_1$,the following statements are equivalent:*

(1) *$\mathfrak{F}$ is TPF^lAW $\mathfrak{L}^P$-continuous.*

(2) $\mathfrak{F}^l((\mathbb{U}(\mathfrak{G})) \subseteq int_\tau(cl_\tau(\mathfrak{F}^l(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle \varsigma,\varkappa,\vartheta\rangle)), \langle \varsigma,\varkappa,\vartheta\rangle), \langle \varsigma,\varkappa,\vartheta\rangle)$,*if* $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle \varsigma,\varkappa,\vartheta\rangle$.

(3) $cl_\tau(int_\tau(\mathfrak{F}^u(int^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle \varsigma,\varkappa,\vartheta\rangle)), \langle \varsigma,\varkappa,\vartheta\rangle), \langle \varsigma,\varkappa,\vartheta\rangle) \subseteq \mathfrak{F}^u(\mathbb{U}(\mathfrak{G}))$, *if* $\sigma(\dashv \mathbb{U}(\mathfrak{G})) \geq \langle \varsigma,\varkappa,\vartheta\rangle$.

Proof. (1) $\Longrightarrow$ (2) Let $\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$, $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$,$\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle \varsigma,\varkappa,\vartheta\rangle$ and $\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in \mathfrak{F}^l(\mathbb{U}(\mathfrak{G}))$. Then, there exists $\mathbb{G}(\mathfrak{G}) \in (I^3)^{\aleph\times\mathfrak{G}}$, $\tau(\mathbb{G}(\mathfrak{G})) \geq \langle \varsigma,\varkappa,\vartheta\rangle$ and $\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in \mathbb{G}(\mathfrak{G})$ such that
$\mathbb{G}(\mathfrak{G}) \subseteq cl_\tau(\mathfrak{F}^l(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle \varsigma,\varkappa,\vartheta\rangle)), \langle \varsigma,\varkappa,\vartheta\rangle)$. Thus,
$\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in \mathbb{G}(\mathfrak{G}) \subseteq int_\tau(cl_\tau(\mathfrak{F}^l(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle \varsigma,\varkappa,\vartheta\rangle)), \langle \varsigma,\varkappa,\vartheta\rangle), \langle \varsigma,\varkappa,\vartheta\rangle)$, and hence $\mathfrak{F}^l(\mathbb{U}(\mathfrak{G})) \subseteq int_\tau(cl_\tau(\mathfrak{F}^l(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle \varsigma,\varkappa,\vartheta\rangle)), \langle \varsigma,\varkappa,\vartheta\rangle), \langle \varsigma,\varkappa,\vartheta\rangle)$.

(2) $\Longrightarrow$ (3) Let $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$ with $\sigma(\dashv \mathbb{U}(\mathfrak{G})) \geq \langle \varsigma,\varkappa,\vartheta\rangle$. Then by (2),

$$\begin{aligned} \dashv \mathfrak{F}^u(\mathbb{U}(\mathfrak{G})) &= \mathfrak{F}^l(\dashv \mathbb{U}(\mathfrak{G})) \subseteq int_\tau(cl_\tau(\mathfrak{F}^l(cl^*_\sigma(\dashv \mathbb{U}(\mathfrak{G}), \langle \varsigma,\varkappa,\vartheta\rangle)), \langle \varsigma,\varkappa,\vartheta\rangle), \langle \varsigma,\varkappa,\vartheta\rangle) \\ &= \dashv cl_\tau(int_\tau(\mathfrak{F}^u(int^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle \varsigma,\varkappa,\vartheta\rangle)), \langle \varsigma,\varkappa,\vartheta\rangle), \langle \varsigma,\varkappa,\vartheta\rangle, \end{aligned}$$

thus $cl_\tau(int_\tau(\mathfrak{F}^u(int^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle \varsigma,\varkappa,\vartheta\rangle)), \langle \varsigma,\varkappa,\vartheta\rangle), \langle \varsigma,\varkappa,\vartheta\rangle \subseteq \mathfrak{F}^u(\mathbb{U}(\mathfrak{G}))$.

(3) $\Longrightarrow$ (1) Let $\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in D(\mathfrak{F})$, $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$, $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle \varsigma,\varkappa,\vartheta\rangle$ and $\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in \mathfrak{F}^l(\mathbb{U}(\mathfrak{G}))$. Then by (3), we have

$$\begin{aligned} &\dashv int_\tau(cl_\tau(\mathfrak{F}^l(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle \varsigma,\varkappa,\vartheta\rangle)), \langle \varsigma,\varkappa,\vartheta\rangle), \langle \varsigma,\varkappa,\vartheta\rangle) \\ = \;& cl_\tau(int_\tau(\mathfrak{F}^u(int^*_\sigma(\dashv \mathbb{U}(\mathfrak{G}), \langle \varsigma,\varkappa,\vartheta\rangle)), \langle \varsigma,\varkappa,\vartheta\rangle), \langle \varsigma,\varkappa,\vartheta\rangle \subseteq \mathfrak{F}^u(\dashv \mathbb{U}(\mathfrak{G})) = \dashv \mathfrak{F}^l(\mathbb{U}(\mathfrak{G})), \end{aligned}$$

and hence $\mathfrak{F}^l((\mathbb{U}(\mathfrak{G})) \subseteq int_\tau(cl_\tau(\mathfrak{F}^l(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle \varsigma,\varkappa,\vartheta\rangle)), \langle \varsigma,\varkappa,\vartheta\rangle), \langle \varsigma,\varkappa,\vartheta\rangle)$. Therefore,
$\langle \varrho, \mathfrak{g}\rangle_{\langle \varsigma,\varkappa,\vartheta\rangle} \in int_\tau(cl_\tau(\mathfrak{F}^l(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle \varsigma,\varkappa,\vartheta\rangle)), \langle \varsigma,\varkappa,\vartheta\rangle), \langle \varsigma,\varkappa,\vartheta\rangle)$
$\subseteq cl_\tau(\mathfrak{F}^l(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle \varsigma,\varkappa,\vartheta\rangle)), \langle \varsigma,\varkappa,\vartheta\rangle)$. Thus, $\mathfrak{F}$ is $TPF^lAW\mathfrak{L}^P$-continuous.

The following theorem is similar to Theorem 4.5.

Theorem 6.2. *For a $NTPFM$ $\mathfrak{F} : (\aleph, \tau) \looparrowright (\Upsilon, \sigma, \mathfrak{L}^P)$, $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$, $\varsigma \in I_0$, $\varkappa \in I_1$ and $\vartheta \in I_1$, the following statements are equivalent:*

(1) $\mathfrak{F}$ *is* TPF^uAW $\mathfrak{L}^P$*-continuous.*

(2) $\mathfrak{F}^u((\mathbb{U}(\mathfrak{G})) \subseteq int_\tau(cl_\tau(\mathfrak{F}^u(cl^*_\sigma(\mathbb{U}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)),\langle\varsigma,\varkappa,\vartheta\rangle),\langle\varsigma,\varkappa,\vartheta\rangle)$*,if* $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$.

(3) $cl_\tau\left(int_\tau(\mathfrak{F}^l(int^*_\sigma(\mathbb{U}(\mathfrak{G}),\langle\varsigma,\varkappa,\vartheta\rangle)),\langle\varsigma,\varkappa,\vartheta\rangle),\langle\varsigma,\varkappa,\vartheta\rangle\right) \subseteq \mathfrak{F}^l(\mathbb{U}(\mathfrak{G}))$*, if* $\sigma(\daleth\,\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma,\varkappa,\vartheta\rangle$.

The following example shows that generally a TPF^uAW continuous and TPF^lAW continuous (resp. TPF^uAW $\mathfrak{L}^P$-continuous and TPF^lAW $\mathfrak{L}^P$-continuous) need not be either a TPF^uAW $\mathfrak{L}^P$-continuous (resp. TPF^uW $\mathfrak{L}^P$-continuous) or TPF^lAW $\mathfrak{L}^P$-continuous (resp. TPF^lW $\mathfrak{L}^P$-continuous).

Example 6.1. *From the Example* 4.1*, define temporal picture fuzzy topologies* $\tau:\left(I^3\right)^{\aleph\times\mathfrak{G}} \to I^3$ *and temporal picture fuzzy ideal* $\mathfrak{L}^P$*:* $\left(I^3\right)^{\Upsilon\times\mathfrak{G}} \to I^3$ *as follows:*

$$\tau(\mathbb{G}(\mathfrak{G})) = \begin{cases} \langle 1,0,0\rangle, & \mathbb{G}(\mathfrak{G}) \in \{\flat(\mathfrak{G}),\sharp(\mathfrak{G})\} \\ \langle 0.6,0.2,0.2\rangle, & \mathbb{G}(\mathfrak{G}) = \mathbb{G}_1(\mathfrak{G}) \\ \langle 0,1,0\rangle, & o.w, \end{cases}$$

$$\mathfrak{L}^P(\mathbb{U}(\mathfrak{G})) = \begin{cases} \langle 1,0,0\rangle, & \mathbb{U}(\mathfrak{G}) = \flat(\mathfrak{G}) \\ \langle 0.55,0.25,0.2\rangle, & \flat(\mathfrak{G}) \subset \mathbb{U}(\mathfrak{G}) \subseteq \left\{ \begin{array}{l} \langle\langle\zeta,\mathfrak{g}_1\rangle,0.4,0.4,0.1\rangle, \\ \langle\langle\zeta,\mathfrak{g}_2\rangle,0.44,0.41,0.1\rangle, \end{array} ,\zeta\in\Upsilon \right\} \\ \langle 0,1,0\rangle, & o.w, \end{cases}$$

where $\mathbb{G}_1(\mathfrak{G}) = \left\{ \begin{array}{l} \langle\langle\varrho,\mathfrak{g}_1\rangle,0.3,0.5,0.2\rangle, \\ \langle\langle\varrho,\mathfrak{g}_2\rangle,0.2,0.6,0.1\rangle, \end{array} ,\varrho\in\aleph \right\}$

Then, (1) $\mathfrak{F}:(\aleph,\tau) \nrightarrow (\Upsilon,\sigma,\mathfrak{L}^P)$ is TPF^uAW (resp. TPF^lAW)-continuous but is not TPF^uAW (resp. TPF^lAW) $\mathfrak{L}^P$-continuous because

$\mathfrak{F}^u(\mathbb{U}_1(\mathfrak{G})) = \mathbb{G}_2(\mathfrak{G})$

$\subseteq int_\tau(cl_\tau(\mathfrak{F}^u(cl_\sigma(\mathbb{U}_1(\mathfrak{G}),\langle 0.31,0.31,0.38\rangle)),\langle 0.31,0.31,0.38\rangle),\langle 0.31,0.31,0.38\rangle) = \sharp(\mathfrak{G})$,

$\mathfrak{F}^l(\mathbb{U}_1(\mathfrak{G})) = \mathbb{G}_2(\mathfrak{G})$

$\subseteq int_\tau(cl_\tau\left(\mathfrak{F}^l(cl_\sigma(\mathbb{U}_1(\mathfrak{G}),\langle 0.31,0.31,0.38\rangle))\right),\langle 0.31,0.31,0.38\rangle),\langle 0.31,0.31,0.38\rangle) = \sharp(\mathfrak{G})$,

but

$\mathfrak{F}^u(\mathbb{U}_1(\mathfrak{G})) = \mathbb{G}_2(\mathfrak{G})$

$\nsubseteq int_\tau(cl_\tau(\mathfrak{F}^u(cl^*_\sigma(\mathbb{U}_1(\mathfrak{G}),\langle 0.31,0.31,0.38\rangle)),\langle 0.31,0.31,0.38\rangle),\langle 0.31,0.31,0.38\rangle)$

$= \left\{ \begin{array}{l} \langle\langle\varrho,\mathfrak{g}_1\rangle,0.3,0.5,0\rangle, \\ \langle\langle\varrho,\mathfrak{g}_2\rangle,0.2,0.6,0\rangle, \end{array} ,\varrho\in\aleph \right\}$,

$\mathfrak{F}^l(\mathbb{U}_1(\mathfrak{G})) = \mathbb{G}_2(\mathfrak{G})$

$\nsubseteq int_\tau(cl_\tau\left(\mathfrak{F}^l(cl^*_\sigma(\mathbb{U}_1(\mathfrak{G}),\langle 0.31,0.31,0.38\rangle))\right),\langle 0.31,0.31,0.38\rangle),\langle 0.31,0.31,0.38\rangle)$

$= \left\{ \begin{array}{l} \langle\langle\varrho,\mathfrak{g}_1\rangle,0.3,0.5,0\rangle, \\ \langle\langle\varrho,\mathfrak{g}_2\rangle,0.2,0.6,0\rangle, \end{array} ,\varrho\in\aleph \right\}$,

(2) For $\mathbb{G}_1(\mathfrak{G}) = \left\{ \begin{array}{l} \langle\langle\varrho, \mathfrak{g}_1\rangle, 0.5, 0.4, 0\rangle, \\ \langle\langle\varrho, \mathfrak{g}_2\rangle, 0.5, 0.4, 0\rangle, \end{array}, \varrho \in \aleph \right\}$ then $\mathfrak{F} : (\aleph, \tau) \nleftrightarrow (\Upsilon, \sigma, \mathfrak{L}^P)$ is TPF^uAW (resp.
TPF^lAW) $\mathfrak{L}^P$-continuous but is not TPF^uW (resp. TPF^lW) $\mathfrak{L}^P$-continuous because
$\mathfrak{F}^u(\mathbb{U}_1(\mathfrak{G})) = \mathbb{G}_2(\mathfrak{G}) \subseteq int_\tau(cl_\tau(\mathfrak{F}^u(cl^*_\sigma(\mathbb{U}_1(\mathfrak{G}), \langle 0.31, 0.31, 0.38\rangle)), \langle 0.31, 0.31, 0.38\rangle), \langle 0.31, 0.31, 0.38\rangle) = \sharp(\mathfrak{G})$,
$\mathfrak{F}^l(\mathbb{U}_1(\mathfrak{G})) = \mathbb{G}_2(\mathfrak{G}) \subseteq int_\tau(cl_\tau(\mathfrak{F}^l(cl^*_\sigma(\mathbb{U}_1(\mathfrak{G}), \langle 0.31, 0.31, 0.38\rangle)), \langle 0.31, 0.31, 0.38\rangle), \langle 0.31, 0.31, 0.38\rangle) = \sharp(\mathfrak{G})$, but
$\mathfrak{F}^u(\mathbb{U}_1(\mathfrak{G})) = \mathbb{G}_2(\mathfrak{G}) \nsubseteq int_\tau(\mathfrak{F}^u(cl^*_\sigma(\mathbb{U}_1(\mathfrak{G}), \langle 0.31, 0.31, 0.38\rangle)), \langle 0.31, 0.31, 0.38\rangle) = \flat(\mathfrak{G})$,
$\mathfrak{F}^l(\mathbb{U}_1(\mathfrak{G})) = \mathbb{G}_2(\mathfrak{G}) \nsubseteq int_\tau(\mathfrak{F}^l(cl^*_\sigma(\mathbb{U}_1(\mathfrak{G}), \langle 0.31, 0.31, 0.38\rangle)), \langle 0.31, 0.31, 0.38\rangle) = \flat(\mathfrak{G})$.

Theorem 6.3. *Let $\mathfrak{F} : (\aleph, \tau) \nleftrightarrow (\Upsilon, \sigma, \mathfrak{L}^P)$ be a $NTPFM$, $\mathfrak{F}$ be TPF^uAW $\mathfrak{L}^P$-continuous and TPF^lA $\mathfrak{L}^P$-continuous. Then, $\mathfrak{F}$ is TPF^uW $\mathfrak{L}^P$-continuous.*

Proof. Let $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$ with $\sigma(\mathbb{U}(\mathfrak{G})) \geq \langle\varsigma, \varkappa, \vartheta\rangle$ and $\mathfrak{F}$ be TPF^uAW $\mathfrak{L}^P$-continuous. Then by Theorem 6.1(1),

$$\mathfrak{F}^u(\mathbb{U}(\mathfrak{G})) \subseteq int_\tau(cl_\tau(\mathfrak{F}^u(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma, \varkappa, \vartheta\rangle)), \langle\varsigma, \varkappa, \vartheta\rangle), \langle\varsigma, \varkappa, \vartheta\rangle).$$

Since $cl_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma, \varkappa, \vartheta\rangle) = cl_\sigma(int^*_\sigma(cl_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma, \varkappa, \vartheta\rangle), \langle\varsigma, \varkappa, \vartheta\rangle), \langle\varsigma, \varkappa, \vartheta\rangle)$, it follows from Theorem 4.3(2) that $\tau(\dashv \mathfrak{F}^u(cl_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma, \varkappa, \vartheta\rangle))) \geq \langle\varsigma, \varkappa, \vartheta\rangle$, then $\tau(\dashv \mathfrak{F}^u(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma, \varkappa, \vartheta\rangle))) \geq \langle\varsigma, \varkappa, \vartheta\rangle$, and $\mathfrak{F}^u(\mathbb{U}(\mathfrak{G})) \subseteq int_\tau(\mathfrak{F}^u(cl^*_\sigma(\mathbb{U}(\mathfrak{G}), \langle\varsigma, \varkappa, \vartheta\rangle)), \langle\varsigma, \varkappa, \vartheta\rangle)$. Thus by Theorem 5.4, $\mathfrak{F}$ is TPF^uW $\mathfrak{L}^P$-continuous.

The following theorem is similarly proved as the proof of Theorem 6.3.

Theorem 6.4. *Let $\mathfrak{F} : (\aleph, \tau) \nleftrightarrow (\Upsilon, \sigma, \mathfrak{L}^P)$ be a $NTPFM$, $\mathfrak{F}$ be TPF^lAW $\mathfrak{L}^P$-continuous and TPF^uA $\mathfrak{L}^P$-continuous. Then $\mathfrak{F}$ is TPF^lW $\mathfrak{L}^P$-continuous.*

An applications $\mathfrak{M}$, $\mathfrak{N}$, $id_{\aleph\times\mathfrak{G}} : (I^3)^{\aleph\times\mathfrak{G}} \times I^3 \to (I^3)^{\aleph\times\mathfrak{G}}$ are operators on $\aleph$ and $\mathfrak{W}$, $\mathfrak{V}$, $id_{\Upsilon\times\mathfrak{G}} : (I^3)^{\Upsilon\times\mathfrak{G}} \times I^3 \to (I^3)^{\Upsilon\times\mathfrak{G}}$ are operators on Υ.

Definition 6.2. (1) Let $\mathfrak{F} : (\aleph, \tau, \mathfrak{L}^P) \nleftrightarrow (\Upsilon, \sigma)$ be a $TPFM$. Then, $\mathfrak{F}$ is $TPF^l(\mathfrak{M}, \mathfrak{N}, \mathfrak{W}, \mathfrak{V}, \mathfrak{L}^P)$-continuous iff
for every $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$, $\varsigma \in I_0, \varkappa \in I_1$ and $\vartheta \in I_1$,

$$\mathfrak{L}^P[\mathfrak{M}(\mathfrak{F}^l(\mathfrak{V}(\mathbb{U}(\mathfrak{G}), \langle\varsigma, \varkappa, \vartheta\rangle)), \langle\varsigma, \varkappa, \vartheta\rangle) \bar{\wedge} \mathfrak{N}(\mathfrak{F}^l(\mathfrak{W}(\mathbb{U}(\mathfrak{G}), \langle\varsigma, \varkappa, \vartheta\rangle)), \langle\varsigma, \varkappa, \vartheta\rangle)] \geq \sigma(\mathbb{U}(\mathfrak{G})).$$

(2) Let $\mathfrak{F} : (\Xi, \tau, \mathfrak{L}^P_1) \nleftrightarrow (\Upsilon, \sigma, \mathfrak{L}^P_2)$ be a $NTPFM$. Then $\mathfrak{F}$ is $TPF^u(\mathfrak{M}, \mathfrak{N}, \mathfrak{W}, \mathfrak{V}, \mathfrak{L}^P)$-continuous iff
for every $\mathbb{U}(\mathfrak{G}) \in (I^3)^{\Upsilon\times\mathfrak{G}}$, $\varsigma \in I_0, \varkappa \in I_1$ and $\vartheta \in I_1$,

$$\mathfrak{L}^P[\mathfrak{M}(\mathfrak{F}^u(\mathfrak{V}(\mathbb{U}(\mathfrak{G}), \langle\varsigma, \varkappa, \vartheta\rangle)), \langle\varsigma, \varkappa, \vartheta\rangle) \bar{\wedge} \mathfrak{N}(\mathfrak{F}^u(\mathfrak{W}(\mathbb{U}(\mathfrak{G}), \langle\varsigma, \varkappa, \vartheta\rangle)), \langle\varsigma, \varkappa, \vartheta\rangle)] \geq \sigma(\mathbb{U}(\mathfrak{G})).$$

Remark 6.2. 1. TPF^l (resp. $NTPF^u$) $\mathfrak{L}^P$-continuous multifunction $\Leftrightarrow$ TPF^l (resp. $NTPF^u$) $(id_{\aleph\times\mathfrak{G}}, int_\tau(\Phi_\tau), int_\sigma, id_{\Upsilon\times\mathfrak{G}}, \mathfrak{L}^{P0})$-continuous multifunction.

2. TPF^lA (resp. $NTPF^uA$) $\mathfrak{L}^P$-continuous multifunction$\Leftrightarrow$ TPF^l (resp. $NTPF^u$) $(id_{\aleph\times\mathfrak{G}}, int_\tau, int_\sigma(cl^*_\sigma), id_{\Upsilon\times\mathfrak{G}}, \mathfrak{L}^{P0})$-continuous multifunction.

3. TPF^lW (resp. $NTPF^uW$) $\mathfrak{L}^P$-continuous multifunction$\Leftrightarrow$ TPF^l (resp. $NTPF^u$) $(id_{\aleph\times\mathfrak{G}}, int_\tau, cl^*_\sigma, id_{\Upsilon\times\mathfrak{G}}, \mathfrak{L}^{P0})$-continuous multifunction.

4. TPF^lAW (resp. $NTPF^uAW$) $\mathfrak{L}^P$-continuous multifunction$\Leftrightarrow$ TPF^l (resp. $NTPF^u$) $(id_{\aleph\times\mathfrak{G}}, int_\tau(cl_\tau), cl^*_\sigma, id_{\Upsilon\times\mathfrak{G}}, \mathfrak{L}^{P0})$-continuous multifunction.

7. Conclusion

This paper submitted the notions of TPF^u or TPF^l-continuous, TPF^u or TPF^l almost continuous, TPF^u or TPF^l weakly continuous and TPF^u or TPF^l almost weakly continuous multifunctions depending on a TPF-ideal. Some characterizations of these types of TPF-continuous multifunctions are proved, and many examples are submitted to explain the allowed implications between these types of TPFcontinuity. That is, the variety of continuity of TPF-multifunctions based on TPF-ideals and the implications in between are meaningful and have been discussed in detail. In future work, we will generalize these notions to wider forms of TPF-semi continuity. Also, we will try to study the variety of TPF-continuity in the fuzzy soft set theory using special operators.

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References

[1] Lotfi Asker Zadeh. Fuzzy sets. *Information and control*, 8(3):338–353, 1965.

[2] K. Atanassov. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20(1):87–96, 1986.

[3] B.C. Cuong. Picture fuzzy sets. *Journal of Computer Science and Cybernetics*, 30(4):409–409, 2014.

[4] Olgun Murat, Ünver Mehmet, and Şeyhmus Yardımcı. Pythagorean fuzzy points and applications in pattern recognition and pythagorean fuzzy topologies. *Soft computing*, 25(7):5225–5232, 2021.

[5] R.R. Yager. Pythagorean fuzzy subsets. *Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS)*, pages 57–61, 2013.

[6] T. Senapati and R.R. Yager. Fermatean fuzzy weighted averaging/geometric operators and its application in multi-criteria decision making methods. *Eng. Appl. Artif. Intell.*, 85:112–121, 2019.

[7] Ashraf Shahzaib, Abdullah Saleem, Mahmood Tahir, Ghani Fazal, and Mahmood Tariq. Spherical fuzzy sets and their applications in multi-attribute decision making problems. *Journal of Intelligent & Fuzzy Systems*, 36(3):2829–2844, 2019.

[8] T. Al-shami and A. Mhemdi. Generalized frame for orthopair fuzzy sets: (m, n)-fuzzy sets and their applications to multi-criteria decision-making methods. *Information*, 14(1):56, 2023.

[9] Garg Harish and Atef Mohammed. Cq-ROFRS: covering q-rung orthopair fuzzy rough sets and its application to multi-attribute decision-making process. *Complex & Intelligent Systems*, 8(3):2349–2370, 2022.

[10] R.R. Yager. Generalized orthopair fuzzy sets. *IEEE Transactions on Fuzzy Systems*, 25(5):1222–1230, 2016.

[11] L. Li, R. Zhang, J. Wang, X. Shang, and K. Bai. A novel approach to multi-attribute group decision-making with q-rung picture linguistic information. *Symmetry*, 10(5):172, 2018.

[12] M.N. Abu_Shugair, A.A. Abdallah, M. Alzoubi, S.E. Abbas, and Ismail Ibedou. Picture fuzzy multifunctions and modal topological structures. *AIMS Mathematics*, 10(3):7430–7448, 2025.

[13] D.L. Shi, M.N. Abu_Shugair, S.E. Abbas, and Ismail Ibedou. Picture fuzzy modal ideal multifunctions. *European Journal of Pure and Applied Mathematics*, 18(2):5956, 2025.

[14] M.N. Abu_Shugair, A.A. Abdallah, S.E. Abbas, E. El-Sanowsy, and I. Ibedou. Double fuzzy ideal multifunctions. *Mathematics*, 12(8):1128, 2024.

[15] K. Atanassov and R. Tsvetkov. New intuitionistic fuzzy operations, operators and topological structures. *Iranian Journal of Fuzzy Systems*, 20(7):37–53, 2023.

[16] Blackburn Patrick, van Benthem, Johan Fak, and Wolter Frank. *Handbook of modal logic*, volume 3. Elsevier, 2006.

[17] I. Silambarasan. Some algebraic properties of picture fuzzy sets. *Bull. Int. Math. Virtual Inst.*, 11(3):429–442, 2021.

[18] K. Atanassov. Intuitionistic fuzzy modal topological structure. *Mathematics*, 10(18):3313, 2022.

[19] R. Feys. *Modal Logics*. Gauthier, Paris, France, 1965.

[20] M. Fitting and R. Mendelsohn. *First-order modal logic*. Springer, 1998.

[21] Mints Grigori. A short introduction to modal logic. *University of Chicago Press, USA*, 1992.

[22] Abdul Razaq, Ibtisam Masmali, Harish Garg, and Umer Shuaib. Picture fuzzy topological spaces and associated continuous functions. *AIMS Mathematics*, 7(8):14840–14861, 2022.

[23] Chawalit Boonpok Monchaya Chiangpradit, Areeyuth Sama-Ae. Quasi s-(τ_1, τ_2)-continuity for multifunctions. *European J. of Pure and Applied Mathematics*, 18(1):5634, 2025.

[24] Prapart Pue-on, Areeyuth Sama-Ae, and Chawalit Boonpok. Quasi $\theta(\tau_1, \tau_2)$-continuity for multifunctions. *European J. of Pure and Applied Mathematics*, 18(1):5717, 2025.

[25] K. Atanassov, N. Angelova, and T. Pencheva. On two intuitionistic fuzzy modal topological structures. *Axioms*, 12(5):408, 2023.

[26] K. Atanassov. On intuitionistic fuzzy temporal topological structures. *Axioms*, 12:182, 2023.

[27] I. Alshammari, P. Mani, C. Ozel, and H Garg. Multiple attribute decision making algorithm via picture fuzzy nano topological spaces. *Symmetry*, 13:69, 2021.