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Heptapartitioned Neutrosophic Soft Topological Spaces and Some Graphical Representation of Sternberg's Triangular Theory of Love in terms of Heptapartitioned Neutrosophic Soft Sets with the Applications of Some Advanced Machine Learning Techniques

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Abstract. The most generalized form of the soft neutrosophic set used in this article is the single valued heptapartitioned neutrosophic soft set (HPNSS). A set theoretic research is completed and some operations on them are defined. We went on to do a thorough investigation on HPNSS and discovered a number of odd characteristics. We built heptapartitioned neutrosophic soft topological spaces (HPNSTS). Characterization of triangle theory of love is addressed with the help of HPNSTS. Heat map is developed for young boys and young girls separately. The normalized correlation matrix heat map (NCMHM) for young boys and girls is discussed. The analyzation and visualization of the correlation matrix of a data set containing data for Sternberg's Eight Types of Love for young boys and girls is addressed. K-mean-HPNS-clustering is applied to group the data into K clusters. The elbow approach is used to determine the optimal number of clusters (K) for the K-mean clustering algorithm.

Key Words and Phrases: Neutrosophic soft set, HPNS points, HPNSTS, HPNS p-open set, set, K-Means algorithm, machine learning techniques and Heatmaps.

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1. Introduction

The most significant method for dealing with ambiguity and missing data that arise in many scientific domains is fuzzy set theory (FST) [1]. This theory is only helpful when discussing membership value; it cannot sufficiently address the non-membership value. The main problem with FST was this. This conundrum was expertly resolved by Atanassov [2], who also introduced the concept of intuisionistic fuzzy set theory (IFST). This method captures the value of membership as well as non-membership. The concept of soft set theory (SST) was first proposed by Molodtsov [3] as a novel way to handle nebulous and confusing circumstances. Molodtsov [4] tackled a number of problems, including function smoothness, using soft set theory. Xu et al. [5] initially presented VSST as an expansion of FST. In their discussion of unclear soft set, Huang et al. [6] identified a few inaccurate findings. The authors provided additional new definitions and provided examples to support the wrong finding. The idea of VSTS was first introduced by Chang Wang and Yaya Li [7] under the heading TSVSS. The writers examined the findings in this area and talked about the fundamental ideas of VST. Smarandache [8]extended SS to HSS. The author also presented the hybrids of NHSSs, IFSSs, PHSSs, and crisp FSs. Based on the neutrosophic soft set (NSS), Bera and Mahapatra [9] proposed the idea of a new structure known as NST. This concept opened up a whole new area of mathematics. The writers courteously went over each of the foundational concepts before moving on to the fundamental outcomes and providing pertinent instances for greater comprehension. Ozturk et al. [10] installed extremely operations on NSSs and NSTs based on these unique procedures. Based on the operation outlined in [10], Ozturk et al. [11] disclosed the notions of NS mapping, NS open mapping, and NS homeomorphism. Mehmood et al. [12] created new open sets in NSTS. AL-Nafee [13] not only created a new family of NSSs but also came up with new operations (union and intersection) on neutrosophic sets. Based on the operations described in [14], AL-Nafee et al. [14] constructed NSBTS to finish their previous research. The authors used these fundamental methods to replicate all of the important findings of NSBTS. NSBTS was developed by Dadas and Demiralp [15] using the guidelines provided in [9]. Several aspects of the Menger selection principle were explored by Ljubia et al. [16] in their study of the principle in regard to soft sets. The relationships between the recently suggested soft open coverings were examined by the writers. Using soft s-regular open covers, Al-shami et al. [17] provided a detailed description of near SMSs. Additionally; they demonstrated how they fell into the same class as soft regular spaces that is, soft Menger spaces. Al-shami et al.'s concept of an ISTSs was first presented [18]. Al-shami et al. [17] explored fixed soft points and weak Forms of soft separation axioms. The theory of ISS connected and ISLCS was given by Al-shami et al. [18]. They also discussed the behavior of these spaces as a finite product of soft spaces and under infra soft homeomorphism maps. The authors discussed the essential elements of a soft point and described its main traits. The concepts of open, closed, and homeomorphism mappings in ISTS content were first introduced by Al-shami [18]. Al-shami [18] investigated ISC and ISLSs, demonstrating their main features using a sequence of infra soft closed sets. Examining the "transmission" of these concepts between classical and IST was the author's main focus. The concept of soft bi-operators was first introduced by Baravan et al. [19]. An example from Ali et al. was used to build an effective bipolar soft generalization of the q-rung ortho-pair fuzzy set model [20]. A model known as the FBSESs was introduced by Ali et al. [20]. In order to create a unique discretization of these basic mathematical ideas with an essentially continuous character, John [21] proposed a soft structure over a set. The advantage is that it gave rise to new instruments for applying mathematical analysis technology in practical applications to identify uncertainty or incomplete data. John [21] investigated the qualitative characteristics of specific entities, known as TSs, that remain unchanged under specific types of transformations, known as continuous maps. Additionally, the author discussed a few generalized structures. John [21] has quickly examined a few constructions. Mehmood et al. [12] explored many structures and provided new definitions for NSTSs. Mehmood et al. [22]presented the idea of VSTSs in a novel approach and explored various structures with regard to VS points using VS $\beta - open$ sets.

1.1. Literature review

Mehmood et al. [12, 22] explored various spaces and structures with respect to VS beta open sets and NSTSs. The idea of QPNSS was implemented in [23], and the idea of QPNSTS was presented in [23] based on QPNSS. Both VST and NST are extensions of FST and VST, respectively. The clustering procedure of the triple refined indeterminate NS for personality grouping was examined by Kandasamy and Smarandache [24]. NSS with SPPS was examined by Shil et al. [25]. PPNS and its characteristics are examined by Mallick and Pramanik [26]. Tripathy and Das [27] talked about PPNTS.

These references [28, 29] became the driving force behind the current work. The writers of reference [28] talked about HPNS and its fundamental characteristics. The writers of reference [30] examined the fundamental operators and discussed the idea of HPNSS. This citation [30] inspired us to develop the novel concept of HPNSTS. We completed it with success, introducing the idea of HPNSTS and demonstrating its fundamental theorems and operations. These ideas are supported by the provided examples. The triangular theory of love is installed in [29]. In our work, we are developing along the lines of the recent progress made in the concept of neutrosophic soft topologies; more specifically, we will be using the quadri-partitioned framework proposed in [31] as an initial starting point that will allow us to add heptapartitioned structure to the theory of HPNSTS.

1.2. Motivation

The complexities of human connections have long piqued the interest of both psychology and mathematics. A robust framework that can handle ambiguity and nuance is required to understand emotional dynamics. When it comes to defining the intricate relationships between intimacy, desire, and commitment, three characteristics of love, conventional theories usually fall short. Heptapartitioned neutrosophic soft set theory (HPNSS) provides a versatile mathematical framework to model interactions with varying degrees of falsity, indeterminacy, and truth, making it a promising approach. This approach allows for a more nuanced representation of the emotional states connected to love while also accommodating the ambiguities inherent in human emotions.

Furthermore, the combination of this paradigm with state-of-the-art machine learning techniques can enhance our understanding of relationship dynamics. Through the application of HPNSS and Sternberg's theory to the analysis of emotional interaction data, we are able to identify patterns and insights that may lead to more effective personal development and relationship therapy.

Our ultimate objective is to close the knowledge gap that exists between psychological theory and mathematical modeling in order to promote a greater understanding of love and its complexity. Our goal is to make a positive impact on both the field of mathematics and the practical understanding of human interactions by utilizing innovative methods to illustrate and examine emotional dynamics by using the idea of HPNSS.

1.3. Novelty

The complexities of human interactions have long troubled both psychology and mathematics. It takes a solid framework that can capture fuzziness and nuance to comprehend emotional dynamics. Intimacy, desire, and commitment are three fundamental components of love, and they often have intricate relationships between them that traditional theories are unable to fully capture. Heptapartitioned neutrosophic soft set theory (HPNSS) offers a versatile mathematical framework that may be used to mimic interactions with varying degrees of truth, indeterminacy, and falsehood, making it a viable approach for expressing the subtleties of human emotions.

This method provides for a more detailed explanation of the emotional states associated with love and takes into account the inherent ambiguities in human relationships. Furthermore, integrating HPNSS with cutting-edge machine learning techniques enhances our understanding of link dynamics. We can find trends and insights that could result in better methods for relationship treatment and personal growth by utilizing HPNSS and Sternberg's theory in the study of emotional interaction data. Our ultimate objective is to bridge the knowledge gap between psychological theory and mathematical modeling in order to gain a deeper understanding of the complexities of love.

In section 2, fundamental concepts and conclusions are given. Few operations on HPNSSs are defined in Section 3 along with a summary of their essential properties. Few theorems are also addressed. Examples are given so you can understand the results more easily. In section 4, the PPNSTS concept is illustrated. The terms HPNS semi-open, HPNS pre-open are defined. One of these intriguing HPNS generalized open sets, referred to as the HPNS pre-open set, is selected, and certain fundamentals are then produced based on this description. These consist of the HPNS closer, HPNS exterior, HPNS border, and HPNS interior.

The notion of general distance between two SVHNSs is defined in section 5, along with a discussion of the various distance metrics, including weighted Euclidean and weighted Hamming distances. Furthermore, flowcharts and algorithms are laid out to provide a clear comprehension of the scenario. The heat map for young boys and girls is created individually in section 6. There is discussion of the normalized correlation matrix heat map (NCMHM) for young boys and girls. This paper examines the correlation matrix analysis and visualization of a dataset that includes information for Sternberg's Eight Types of Love for young boys and girls. The 3-D scatter plot, which is a 3D surface plot depicting a correlation matrix, is used to show the correlation coefficients between various forms of love for young boys and girls. In order to find patterns and commonalities across Sternberg's Eight Types of Love, clustering data representing the love types of young boys and girls is examined. The data is repeatedly grouped into K clusters using K mean-SVHNS-clustering. It is examined how to use t-SNE, a dimensionality reduction approach, to visualize and comprehend the clustered data indicating love types among young boys and girls. Additionally, t-SNE visualization of K-Mean is explored. The elbow approach is used to determine the optimal number of clusters (K) for the K-mean clustering algorithm. Section 8 explains the advantages and Section 9 explains the limitations. Finally, Section 10 offers some ideas for further investigation.

2. Preliminaries

This section covers foundational ideas that are required for the upcoming research.

Definition 1. *[32] Let ∂ be a set of parameters and X be an initial universe set. $P(X)$ represents the collection of all NSS for X. A set defined by a set-valued function $\tilde{F}$ expressing a mapping $\tilde{F} : \partial \to P(X)$ is then a NSS $(\tilde{F}, \partial)$ over X, where $\tilde{F}$ is referred to as the approximate function of the NSS $(\tilde{F}, \partial)$. Stated differently, the NSS can be expressed as a collection of ordered pairs;*

$$(\tilde{F}, \partial) = \left\{ \left(e, \langle s, T_{\tilde{F}(e)}(s), I_{\tilde{F}(e)}(s), F_{\tilde{F}(e)}(s) \rangle : s \in X \right) : e \in \partial \right\}.$$

Here, $T_{\tilde{F}(e)}(s)$, $I_{\tilde{F}(e)}(s)$, and $F_{\tilde{F}(e)}(s)$ are the truth-membership, indeterminacy-membership, and falsity-membership functions of $\tilde{F}(e)$, respectively, and they all lie within the interval $[0, 1]$. The inequality

$$0 \leq T_{\tilde{F}(e)}(s) + I_{\tilde{F}(e)}(s) + F_{\tilde{F}(e)}(s) \leq 3$$

is evident as the supremum of each T, I, and F is 1.

Definition 2. *[9] Let $(\tilde{F}, \partial)$ be a NSS . Then $(\tilde{F}, \partial)^c$ is the complement of $(\tilde{F}, \partial)$:*

$$(\tilde{F}, \partial)^c = \left\{ \left(e, \langle s, F_{\tilde{F}(e)}(s), 1 - I_{\tilde{F}(e)}(s), T_{\tilde{F}(e)}(s) \rangle : s \in X \right) : e \in \partial \right\}.$$

Obvious that:

$$\left((\tilde{F},\partial)^c\right)^c = (\tilde{F},\partial).$$

Definition 3. *[33] Let $(\tilde{F},\partial)$ and $(\tilde{G},\partial)$ be two NSSs. Then $(\tilde{F},\partial)$ is said to be a NS subset of $(\tilde{G},\partial)$ if*

$$T_{\tilde{F}(e)}(s) \le T_{\tilde{G}(e)}(s), \quad I_{\tilde{F}(e)}(s) \le I_{\tilde{G}}(e)(s), \quad F_{\tilde{F}(e)}(s) \ge F_{\tilde{G}(e)}(s), \quad \forall e \in \partial, \forall s \in X.$$

It is denoted by:

$$(\tilde{F},\partial) \subseteq (\tilde{G},\partial).$$

Definition 4. *[14] Let $(\tilde{F}_1,\partial)$ and $(\tilde{F}_2,\partial)$ be two NSSs. Then their union is represented by $(\tilde{F}_1,\partial) \cup (\tilde{F}_2,\partial) = (\tilde{F}_3,\partial)$ as:*

$$(\tilde{F}_3,\partial) = \left\{\left(e, \langle s, T_{\tilde{F}_{3(e)}}(s), I_{\tilde{F}_{3(e)}}(s), F_{\tilde{F}_{3(e)}}(s)\rangle : s \in X\right) : e \in \partial\right\}.$$

Where:

$$T_{\tilde{F}_{3(e)}}(s) = \max\{T_{\tilde{F}_{1(e)}}(s), T_{\tilde{F}_{2(e)}}(s)\},$$
$$I_{\tilde{F}_{3(e)}}(s) = \max\{I_{\tilde{F}_{1(e)}}(s), I_{\tilde{F}_{2(e)}}(s)\},$$
$$F_{\tilde{F}_{3(e)}}(s) = \min\{F_{\tilde{F}_{1(e)}}(s), F_{\tilde{F}_{2(e)}}(s)\}.$$

Definition 5. *[10] Let $(\tilde{F}_1,\partial)$ and $(\tilde{F}_2,\partial)$ be two NSSs. Then their intersection is symbolized by $(\tilde{F}_1,\partial) \cap (\tilde{F}_2,\partial) = (\tilde{F}_3,\partial)$ as:*

$$(\tilde{F}_3,\partial) = \left\{\left(e, \langle s, T_{\tilde{F}_{3(e)}}(s), I_{\tilde{F}_{3(e)}}(s), F_{\tilde{F}_{3(e)}}(s)\rangle : s \in X\right) : e \in \partial\right\}.$$

Where:

$$T_{\tilde{F}_{3(e)}}(s) = \min\{T_{\tilde{F}_{1(e)}}(s), T_{\tilde{F}_{2(e)}}(s)\},$$
$$I_{\tilde{F}_{3(e)}}(s) = \min\{I_{\tilde{F}_{1(e)}}(s), I_{\tilde{F}_{2(e)}}(s)\},$$
$$F_{\tilde{F}_{3(e)}}(s) = \max\{F_{\tilde{F}_{1(e)}}(s), F_{\tilde{F}_{2(e)}}(s)\}.$$

Definition 6. *[10] Let $(\tilde{F}_1,\partial)$ and $(\tilde{F}_2,\partial)$ be two NSSs. Then "$(\tilde{F}_1,\partial)$ difference $(\tilde{F}_2,\partial)$" operation on them is denoted by $(\tilde{F}_1,\partial) \setminus (\tilde{F}_2,\partial) = (\tilde{F}_3,\partial)$ and is defined by:*

$$(\tilde{F}_3,e) = \left\{\left(e, \langle s, T_{\tilde{F}_{3(e)}}(s), I_{\tilde{F}_{3(e)}}(s), F_{\tilde{F}_{3(e)}}(s)\rangle : s \in X\right) : e \in \partial\right\}.$$

Where:

$$T_{\tilde{F}_{3(e)}}(s) = \min\{T_{\tilde{F}_{1(e)}}(s), T_{\tilde{F}_{2(e)}}(s)\},$$
$$I_{\tilde{F}_{3(e)}}(s) = \min\{I_{\tilde{F}_{1(e)}}(s), I_{\tilde{F}_{2(e)}}(s)\},$$
$$F_{\tilde{F}_{3(e)}}(s) = \max\{F_{\tilde{F}_{1(e)}}(s), F_{\tilde{F}_{2(e)}}(s)\}.$$

Definition 7. *[10] 1. A NSS $(\tilde{F}_1,\partial)$ is said to be a null NSS if:*

$$T_{\tilde{F}(e)}(s) = 0, \quad I_{\tilde{F}(e)}(s) = 0, \quad F_{\tilde{F}(e)}(s) = 1, \quad \forall e \in \partial, \forall s \in X.$$

It is denoted by $0_{(X,\partial)}$.

2. A NSS $(\tilde{F}_1,\partial)$ is said to be an absolute NSS if:

$$T_{\tilde{F}(e)}(s) = 1, \quad I_{\tilde{F}(e)}(s) = 1, \quad F_{\tilde{F}(e)}(s) = 0, \quad \forall e \in \partial, \forall s \in X.$$

It is symbolized as $1(X,\partial)$.

$$0_{(X,\partial)} = 1_{(X,\partial)}, \quad 1^c_{(X,\partial)} = 0_{(X,\partial)}.$$

is obvious.

3. A New Approach to Operations on Hepta-Partitioned Neutrosophic Soft Sets

Neutrophic set theory (NST), a generality of vague set theory (VST), is regarded as the most appealing theory since it considers the three possible membership values: true, false, and indeterminacy. The principles are all quite obvious, but the third one is particularly fascinating since it addresses uncertainty, which arises in all aspects of daily life. One can make the situation more certain and free of error if the indeterminacy membership is refined. This can be done by splitting the indeterminacy into five pieces that is possible values. These are relative true, relative false, contradiction, unknown (undefined) and ignorance. This section is devoted to the most basic operations of union, intersection, difference, and absolute null, absolute HPNSSs. Theorems and examples are given for better understanding the situation.

Definition 8. *Let ∂ be the set of parameters and π be the key set. Let $P(\pi)$ represent the power set of π. Then, a HPNSS $(\tilde{F}, \partial)$ over π is a mapping $\tilde{F} : \partial \to P(\pi)$, where $\tilde{F}$ is the function of $(\tilde{F}, \partial)$. Symbolically,*

$$(\tilde{F}, \partial) = \left\{ \left(\eta, \langle s, AbT_{\tilde{F}(\eta)}(s), ReT_{\tilde{F}(\eta)}(s), U_{\tilde{F}(\eta)}(s), C_{\tilde{F}(\eta)}(s), G_{\tilde{F}(\eta)}(s), ReF_{\tilde{F}(\eta)}(s), AbF_{\tilde{F}(\eta)}(s) : s \in \pi \rangle \right) : \eta \in \partial \right\}$$

Where, $AbT_{\tilde{F}(\eta)}(s)$, $ReT_{\tilde{F}(\eta)}(s)$,$U_{\tilde{F}(\eta)}(s)$,$C_{\tilde{F}(\eta)}(s)$,$G_{\tilde{F}(\eta)}(s)$, $ReF_{\tilde{F}(\eta)}(s)$, and $AbF_{\tilde{F}(\eta)}(s) \in [0,1]$ respectively, $AbT_{\tilde{F}(\eta)}(s)$, $ReT_{\tilde{F}(\eta)}(s)$,$U_{\tilde{F}(\eta)}(s)$,$C_{\tilde{F}(\eta)}(s)$,$G_{\tilde{F}(\eta)}(s)$, $ReF_{\tilde{F}(\eta)}(s)$,$AbF_{\tilde{F}(\eta)}(s)$ are called the absolute true-membership, relative true-membership,unknown membership, confusion-membership, ignorance-membership, relative false-membership, and absolute false-membership function of $\tilde{F}(\eta)$. Since the supremum of each function is 1 and the infimum is 0 so the inequality;

$$0 \leq AbT_{\tilde{F}(\eta)}(s) + ReT_{\tilde{F}(\eta)}(s) + U_{\tilde{F}(\eta)}(s) + C_{\tilde{F}(\eta)}(s) + G_{\tilde{F}(\eta)}(s) + ReF_{\tilde{F}(\eta)}(s) + AbF_{\tilde{F}(\eta)}(s) \leq 7.$$

is automaticly true.

Definition 9. *Let $(\tilde{F}, \partial)$ be a HPNSS over the key set π. Then, the complement of $(\tilde{F}, \partial)$ is denoted by $(\tilde{F}, \partial)^c$ and is defined as follows:*

$$(\tilde{F}, \partial)^c = \left\{ \left(\begin{matrix} \eta, \langle s, AbF_{\tilde{F}(\eta)}(s), ReF_{\tilde{F}(\eta)}(s), G_{\tilde{F}(\eta)}(s), \\ 1 - C_{\tilde{F}(\eta)}(s), U_{\tilde{F}(\eta)}(s), ReT_{\tilde{F}(\eta)}(s), AbT_{\tilde{F}(\eta)}(s) : s \in \pi \rangle) : \eta \in \partial. \end{matrix} \right. \right\}$$

It follows that $\left((\tilde{F}, \partial)^c \right)^c = (\tilde{F}, \partial)$.

Definition 10. *Let $(\tilde{F}, \partial)$ and $(\tilde{G}, \partial)$ be two HPNSSs over the key set π then, $(\tilde{F}, \partial) \tilde{\subseteq} (\tilde{G}, \partial)$ if*

$$\begin{aligned} AbT_{\tilde{F}(\eta)}(s) &\preceq AbT_{\tilde{G}(\eta)}(s), \\ ReT_{\tilde{F}(\eta)}(s) &\preceq ReT_{\tilde{G}(\eta)}(s), \\ U_{\tilde{F}(\eta)}(s) &\preceq U_{\tilde{G}(\eta)}(s), \\ C_{\tilde{F}(\eta)}(s) &\preceq C_{\tilde{G}(\eta)}(s), \\ G_{\tilde{F}(\eta)}(s) &\preceq G_{\tilde{G}(\eta)}(s), \\ ReF_{\tilde{F}(\eta)}(s) &\succeq ReF_{\tilde{G}(\eta)}(s), \\ AbF_{\tilde{F}(\eta)}(s) &\succeq AbF_{\tilde{G}(\eta)}(s), \end{aligned}$$

$\forall \eta \in \partial, \forall s \in \pi$. *If $(\tilde{F}, \partial) \tilde{\subseteq} (\tilde{G}, \partial)$ and $(\tilde{F}, \partial) \tilde{\supseteq} (\tilde{G}, \partial)$, then $(\tilde{F}, \partial) = (\tilde{G}, \partial)$.*

Definition 11. *Let* $(\tilde{F},\partial)$ *and* $(\tilde{G},\partial)$ *be two HPNSSs over the key set* π *such that* $(\tilde{F},\partial) \neq (\tilde{G},\partial)$*, then their union is denoted by* $(\tilde{F},\partial)\tilde{\Cup}(\tilde{G},\partial) = (\tilde{H},\partial)$ *and is defined as:*

$$(\tilde{H},\partial) = \left\{ \left(\begin{array}{l} \eta, \langle s, AbT_{\tilde{H}(\eta)}(s), ReT_{\tilde{H}(\eta)}(s), U_{\tilde{H}(\eta)}(s), C_{\tilde{H}(\eta)}(s), \\ G_{\tilde{H}(\eta)}(s), ReF_{\tilde{H}(\eta)}(s), AbF_{\tilde{H}(\eta)}(s) : s \in \pi \rangle \end{array} \right) : \eta \in \partial. \right\}$$

where

$$\begin{aligned} AbT_{\tilde{H}(\eta)}(s) &= \max\left\{ AbT_{\tilde{F}(\eta)}(s), AbT_{\tilde{G}(\eta)}(s) \right\}, \\ ReT_{\tilde{H}(\eta)}(s) &= \max\left\{ ReT_{\tilde{F}(\eta)}(s), ReT_{\tilde{G}(\eta)}(s) \right\}, \\ U_{\tilde{H}(\eta)}(s) &= \max\left\{ U_{\tilde{F}(\eta)}(s), U_{\tilde{G}(\eta)}(s) \right\}, \\ C_{\tilde{H}(\eta)}(s) &= \max\left\{ C_{\tilde{F}(\eta)}(s), C_{\tilde{G}(\eta)}(s) \right\}, \\ G_{\tilde{H}(\eta)}(s) &= \min\left\{ G_{\tilde{F}(\eta)}(s), G_{\tilde{G}(\eta)}(s) \right\}, \\ ReF_{\tilde{H}(\eta)}(s) &= \min\left\{ ReF_{\tilde{F}(\eta)}(s), ReF_{\tilde{G}(\eta)}(s) \right\}, \\ AbF_{\tilde{H}(\eta)}(s) &= \min\left\{ AbF_{\tilde{F}(\eta)}(s), AbF_{\tilde{G}(\eta)}(s) \right\}. \end{aligned}$$

Definition 12. *Let* $(\tilde{F},\partial)$ *and* $(\tilde{G},\partial)$ *be two HPNSSs over the key set* π *such that* $(\tilde{F},\partial) \neq (\tilde{G},\partial)$ *then their intersection is denoted by* $(\tilde{F},\partial)\tilde{\Cap}(\tilde{G},\partial) = (\tilde{H},\partial)$ *and is defined as:*

$$(\tilde{H},\partial) = \left\{ \left(\begin{array}{l} \eta, \langle s, AbT_{\tilde{H}(\eta)}(s), ReT_{\tilde{H}(\eta)}(s), U_{\tilde{H}(\eta)}(s), \\ C_{\tilde{H}(\eta)}(s), G_{\tilde{H}(\eta)}(s), ReF_{\tilde{H}(\eta)}(s), AbF_{\tilde{H}(\eta)}(s) : s \in \pi \rangle \end{array} \right) : \eta \in \partial. \right\}$$

where

$$\begin{aligned} AbT_{\tilde{H}(\eta)}(s) &= \min\left\{ AbT_{\tilde{F}(\eta)}(s), AbT_{\tilde{G}(\eta)}(s) \right\}, \\ ReT_{\tilde{H}(\eta)}(s) &= \min\left\{ ReT_{\tilde{F}(\eta)}(s), ReT_{\tilde{G}(\eta)}(s) \right\}, \\ U_{\tilde{H}(\eta)}(s) &= \min\left\{ U_{\tilde{F}(\eta)}(s), U_{\tilde{G}(\eta)}(s) \right\}, \\ C_{\tilde{H}(\eta)}(s) &= \min\left\{ C_{\tilde{F}(\eta)}(s), C_{\tilde{G}(\eta)}(s) \right\}, \\ G_{\tilde{H}(\eta)}(s) &= \max\left\{ G_{\tilde{F}(\eta)}(s), G_{\tilde{G}(\eta)}(s) \right\}, \\ ReF_{\tilde{H}(\eta)}(s) &= \max\left\{ ReF_{\tilde{F}(\eta)}(s), ReF_{\tilde{G}(\eta)}(s) \right\}, \\ AbF_{\tilde{H}(\eta)}(s) &= \max\left\{ AbF_{\tilde{F}(\eta)}(s), AbF_{\tilde{G}(\eta)}(s) \right\}. \end{aligned}$$

Definition 13. *Let* $(\tilde{F},\partial)$ *and* $(\tilde{G},\partial)$ *be two HPNSSs over the key set* π *such that* $(\tilde{F},\partial) \neq (\tilde{G},\partial)$*. Then, their difference is denoted by* $(\tilde{H},\partial) = (\tilde{F},\partial) \setminus (\tilde{G},\partial)$ *and is defined as:*

$$(\tilde{H},\partial) = (\tilde{F},\partial)\tilde{\Cap}(\tilde{G},\partial)^c,$$

$$(\tilde{H},\partial) = \left\{ \left(\begin{array}{l} \eta, \langle s, AbT_{\tilde{H}(\eta)}(s), ReT_{\tilde{H}(\eta)}(s), U_{\tilde{H}(\eta)}(s), \\ C_{\tilde{H}(\eta)}(s), G_{\tilde{H}(\eta)}(s), ReF_{\tilde{H}(\eta)}(s), AbF_{\tilde{H}(\eta)}(s) : s \in \pi \rangle \end{array} \right) : \eta \in \partial. \right\}$$

where

$$AbT_{\tilde{H}(\eta)}(s) = \min\left\{ AbT_{\tilde{F}(\eta)}(s), AbT_{\tilde{G}(\eta)}(s) \right\},$$

$$ReT_{\tilde{H}(\eta)}(s) = \min\left\{ReT_{\tilde{F}(\eta)}(s), ReT_{\tilde{G}(\eta)}(s)\right\},$$
$$U_{\tilde{H}(\eta)}(s) = \min\left\{U_{\tilde{F}(\eta)}(s), U_{\tilde{G}(\eta)}(s)\right\},$$
$$C_{\tilde{H}(\eta)}(s) = \min\left\{C_{\tilde{F}(\eta)}(s), C_{\tilde{G}(\eta)}(s)\right\},$$
$$G_{\tilde{H}(\eta)}(s) = \max\left\{G_{\tilde{F}(\eta)}(s), G_{\tilde{G}(\eta)}(s)\right\},$$
$$ReF_{\tilde{H}(\eta)}(s) = \max\left\{ReF_{\tilde{F}(\eta)}(s), ReF_{\tilde{G}(\eta)}(s)\right\},$$
$$AbF_{\tilde{H}(\eta)}(s) = \max\left\{AbF_{\tilde{F}(\eta)}(s), AbF_{\tilde{G}(\eta)}(s)\right\}.$$

Definition 14. *Let $\{(\tilde{F}_1, \partial) : i \in I\}$ be a family of HPNSSs over the key set π. Then,*

$$\bigcup_{i\in I}(\tilde{F}_1, \partial) = \left\{\left(\begin{matrix}\eta, \langle s, \sup_{i\in I} AbT_{\tilde{F}_{1(\eta)}}(s), \sup_{i\in I} ReT_{\tilde{F}_{1(\eta)}}(s), \sup_{i\in I} U_{\tilde{F}_{1(\eta)}}(s), \sup_{i\in I} C_{\tilde{F}_{1(\eta)}}(s), \\ \inf_{i\in I} G_{\tilde{F}_{1(\eta)}}(s), \inf_{i\in I} ReF_{\tilde{F}_{1(\eta)}}(s), \inf_{i\in I} AbF_{\tilde{F}_{1(\eta)}}(s) : s \in \pi\rangle\right) : \eta \in \partial.\end{matrix}\right.\right\}$$

$$\bigcap_{i\in I}(\tilde{F}_1, \partial) = \left\{\left(\begin{matrix}\eta, \langle s, \inf_{i\in I} AbT_{\tilde{F}_{1(\eta)}}(s), \inf_{i\in I} ReT_{\tilde{F}_{1(\eta)}}(s), \inf_{i\in I} U_{\tilde{F}_{1(\eta)}}(s), \inf_{i\in I} C_{\tilde{F}_{1(\eta)}}(s), \\ \sup_{i\in I} G_{\tilde{F}_{1(\eta)}}(s), \sup_{i\in I} ReF_{\tilde{F}_{1(\eta)}}(s), \sup_{i\in I} AbF_{\tilde{F}_{1(\eta)}}(s) : s \in \pi\rangle\right) : \eta \in \partial.\end{matrix}\right.\right\}$$

Definition 15. *Let $(\tilde{F}, \partial)$ and $(\tilde{G}, \partial)$ be two HPNSSs over the key set π then "AND" operation on them is denoted by $(\tilde{F}, \partial) \wedge (\tilde{G}, \partial) = (\tilde{H}, \partial \times \partial)$ and is defined as:*

$$(\tilde{H}, \partial\times\partial) = \left\{\left(\begin{matrix}(\eta_1, \eta_2), \langle s, AbT_{\tilde{H}}(\eta_1, \eta_2)(s), ReT_{\tilde{H}}(\eta_1, \eta_2)(s), U_{\tilde{H}}(\eta_1, \eta_2)(s) \\ C_{\tilde{H}}(\eta_1, \eta_2)(s), G_{\tilde{H}}(\eta_1, \eta_2)(s), ReF_{\tilde{H}}(\eta_1, \eta_2)(s), AbF_{\tilde{H}}(\eta_1, \eta_2)(s) : s \in \pi\rangle\right) : (\eta_1, \eta_2) \in \partial \times \partial.\end{matrix}\right.\right\}$$

Where,

$$AbT_{\tilde{H}(\eta)}(s) = \min\left\{AbT_{\tilde{F}(\eta)}(s), AbT_{\tilde{G}(\eta)}(s)\right\},$$
$$ReT_{\tilde{H}(\eta)}(s) = \min\left\{ReT_{\tilde{F}(\eta)}(s), ReT_{\tilde{G}(\eta)}(s)\right\},$$
$$U_{\tilde{H}(\eta)}(s) = \min\left\{U_{\tilde{F}(\eta)}(s), U_{\tilde{G}(\eta)}(s)\right\},$$
$$C_{\tilde{H}(\eta)}(s) = \min\left\{C_{\tilde{F}(\eta)}(s), C_{\tilde{G}(\eta)}(s)\right\},$$
$$G_{\tilde{H}(\eta)}(s) = \max\left\{G_{\tilde{F}(\eta)}(s), G_{\tilde{G}(\eta)}(s)\right\},$$
$$ReF_{\tilde{H}(\eta)}(s) = \max\left\{ReF_{\tilde{F}(\eta)}(s), ReF_{\tilde{G}(\eta)}(s)\right\},$$
$$AbF_{\tilde{H}(\eta)}(s) = \max\left\{AbF_{\tilde{F}(\eta)}(s), AbF_{\tilde{G}(\eta)}(s)\right\}.$$

Definition 16. *Let $(\tilde{F}, \partial)$ and $(\tilde{G}, \partial)$ be HPNSSs over the key set π then "OR" operation on them is denoted by $(\tilde{F}, \partial) \vee (\tilde{G}, \partial) = (\tilde{H}, \partial \times \partial)$ and is defined as:*

$$(\tilde{H}, \partial\times\partial) = \left\{\left(\begin{matrix}(\eta_1, \eta_2), \langle s, AbT_{\tilde{H}}(\eta_1, \eta_2)(s), ReT_{\tilde{H}}(\eta_1, \eta_2)(s), U_{\tilde{H}}(\eta_1, \eta_2)(s), \\ C_{\tilde{H}}(\eta_1, \eta_2)(s), G_{\tilde{H}}(\eta_1, \eta_2)(s), ReF_{\tilde{H}}(\eta_1, \eta_2)(s), AbF_{\tilde{H}}(\eta_1, \eta_2)(s) : s \in \pi\rangle\right) : (\eta_1, \eta_2) \in \partial \times \partial.\end{matrix}\right.\right\}$$

Where,

$$AbT_{\tilde{H}(\eta)}(s) = \max\left\{AbT_{\tilde{F}(\eta)}(s), AbT_{\tilde{G}(\eta)}(s)\right\},$$

$$ReT_{\tilde{H}(\eta)}(s) = \max\left\{ReT_{\tilde{F}(\eta)}(s), ReT_{\tilde{G}(\eta)}(s)\right\},$$
$$U_{\tilde{H}(\eta)}(s) = \max\left\{U_{\tilde{F}(\eta)}(s), U_{\tilde{G}(\eta)}(s)\right\},$$
$$C_{\tilde{H}(\eta)}(s) = \max\left\{C_{\tilde{F}(\eta)}(s), C_{\tilde{G}(\eta)}(s)\right\},$$
$$G_{\tilde{H}(\eta)}(s) = \min\left\{G_{\tilde{F}(\eta)}(s), G_{\tilde{G}(\eta)}(s)\right\},$$
$$ReF_{\tilde{H}(\eta)}(s) = \min\left\{ReF_{\tilde{F}(\eta)}(s), ReF_{\tilde{G}(\eta)}(s)\right\},$$
$$AbF_{\tilde{H}(\eta)}(s) = \min\left\{AbF_{\tilde{F}(\eta)}(s), AbF_{\tilde{G}(\eta)}(s)\right\}.$$

Definition 17. *A hepta-partitioned neutrosophic soft set $(\tilde{F}, \partial)$ over the key set π is said to be a null HPNSSs if*

$$AbT_{\tilde{F}(\eta)}(s) = 0, \quad ReT_{\tilde{F}(\eta)}(s) = 0, \quad \forall\eta \in \partial, \forall s \in \pi,$$
$$U_{\tilde{F}(\eta)}(s) = 0, \forall s \in \pi, \quad C_{\tilde{F}(\eta)}(s) = 0, \quad \forall\eta \in \partial, \forall s \in \pi,$$
$$G_{\tilde{F}(\eta)}(s) = 1, \quad \forall\eta \in \partial, \forall s \in \pi,$$
$$ReF_{\tilde{F}(\eta)}(s) = 1, \quad AbF_{\tilde{F}(\eta)}(s) = 1, \quad \forall\eta \in \partial, \forall s \in \pi.$$

It is signified as $0_{(\pi,\partial)}$.

Definition 18. *A hepta-partitioned neutrosophic soft set $(\tilde{F}, \partial)$ over the key set π is called an absolute HPNSSs if*

$$AbT_{\tilde{F}(\eta)}(s) = 1, \quad ReT_{\tilde{F}(\eta)}(s) = 1, \quad \forall\eta \in \partial, \forall s \in \pi,$$
$$U_{\tilde{F}(\eta)}(s) = 1, \quad \forall\eta \in \partial, \forall s \in \pi, \quad C_{\tilde{F}(\eta)}(s) = 1, \quad \forall\eta \in \partial, \forall s \in \pi,$$
$$G_{\tilde{F}(\eta)}(s) = 0, \quad \forall\eta \in \partial, \forall s \in \pi$$
$$ReF_{\tilde{F}(\eta)}(s) = 0, \quad AbF_{\tilde{F}(\eta)}(s) = 0, \quad \forall\eta \in \partial, \forall s \in \pi.$$

Clearly,

$$0^c_{(\pi,\partial)} = 1(\pi, \partial), \quad 1^c_{(\pi,\partial)} = 0_{(\pi,\partial)}.$$

Definition 19. *The family of all HPNSSs over π is designated as $HPNSS(\tilde{\pi})$ then*

$$s^{\eta}_{\langle\Delta_1,\Delta_2,\Delta_3,\Delta_4,\Delta_5,\Delta_6,\Delta_7\rangle}$$

is called a hepta-partitioned neutrosopic soft point, for every point $s \in \tilde{\pi}, 0 \prec \Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6, \Delta_7 \preceq 1, \eta \in \partial$, and is defined as follows:

$$s^{\eta}_{\langle\Delta_1,\Delta_2,\Delta_3,\Delta_4,\Delta_5,\Delta_6,\Delta_7\rangle}\eta^{/(\mathrm{y})} = \begin{cases} \langle\Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6, \Delta_7\rangle, & \text{if } \eta = \eta' \text{ and } \mathrm{y} = s, \\ (0, 0, 0, 0, 0, 0, 1), & \text{if } \eta' \neq \eta \text{ or } \mathrm{y} \neq s. \end{cases}$$

Definition 20. *Let $(\tilde{F}, \partial)$ be a HPNSS over the key set π then*

$$s^{\eta}_{\langle\Delta_1,\Delta_2,\Delta_3,\Delta_4,\Delta_5,\Delta_6,\Delta_7\rangle} \in (\tilde{F}, \partial)$$

if

$$\Delta_1 \preceq AbT_{\tilde{F}(\eta)}(s), \Delta_2 \preceq ReT_{\tilde{F}(\eta)}(s), \Delta_3 \preceq U_{\tilde{F}(\eta)}(s), \Delta_4 \preceq C_{\tilde{F}(\eta)}(s),$$
$$\Delta_5 \succeq G_{\tilde{F}(\eta)}(s), \Delta_6 \succeq ReF_{\tilde{F}(\eta)}(s), \Delta_7 \succeq AbF_{\tilde{F}(\eta)}(s).$$

Theorem 1. *Let $(\tilde{F},\partial)$, $(\tilde{G},\partial)$, and $(\tilde{H},\partial)$ be HPNSSs over the key set X. Then,*

1.
$$(\tilde{F},\partial)\tilde{\Cup}\left[(\tilde{G},\partial)\tilde{\Cup}(\tilde{H},\partial)\right] = \left[(\tilde{F},\partial)\tilde{\Cup}(\tilde{G},\partial)\right]\tilde{\Cup}(\tilde{H},\partial)$$

2.
$$(\tilde{F},\partial)\tilde{\Cap}\left[(\tilde{G},\partial)\tilde{\Cap}(\tilde{H},\partial)\right] = \left[(\tilde{F},\partial)\tilde{\Cap}(\tilde{G},\partial)\right]\tilde{\Cap}(\tilde{H},\partial)$$

3.
$$(\tilde{F},\partial)\tilde{\Cup}\left[(\tilde{G},\partial)\tilde{\Cap}(\tilde{H},\partial)\right] = \left[(\tilde{F},\partial)\tilde{\Cup}(\tilde{G},\partial)\right]\tilde{\Cap}\left[(\tilde{F},\partial)\tilde{\Cup}(\tilde{H},\partial)\right]$$

4.
$$(\tilde{F},\partial)\tilde{\Cap}\left[(\tilde{G},\partial)\tilde{\Cup}(\tilde{H},\partial)\right] = \left[(\tilde{F},\partial)\tilde{\Cap}(\tilde{G},\partial)\right]\tilde{\Cup}\left[(\tilde{F},\partial)\tilde{\Cap}(\tilde{H},\partial)\right]$$

5.
$$(\tilde{F},\partial)\tilde{\Cup}0_{(X,\partial)} = (\tilde{F},\partial)$$

6.
$$(\tilde{F},\partial)\tilde{\Cap}0_{(X,\partial)} = 0_{(X,\partial)}$$

7.
$$(\tilde{F},\partial)\tilde{\Cup}1_{(X,\partial)} = 1_{(X,\partial)}$$

8.
$$(\tilde{F},\partial)\tilde{\Cap}1_{(X,\partial)} = (\tilde{F},\partial)$$

Proof. 1.
$$(\tilde{F},\partial)\tilde{\Cup}\left[(\tilde{G},\partial)\tilde{\Cup}(\tilde{H},\partial)\right] = \left[(\tilde{F},\partial)\tilde{\Cup}(\tilde{G},\partial)\right]\tilde{\Cup}(\tilde{H},\partial)$$

$$y \in (\tilde{F},\partial)\tilde{\Cup}\left[(\tilde{G},\partial)\tilde{\Cup}(\tilde{H},\partial)\right] \text{...........}(i)$$

$$\Rightarrow y \in (\tilde{F},\partial) \text{ or } y \in \left[(\tilde{G},\partial)\tilde{\Cup}(\tilde{H},\partial)\right]$$

$$\Rightarrow y \in (\tilde{F},\partial) \text{ or } y \in \left[\left\{\left(s, \begin{array}{c} \max[\mathrm{AbT}_{\tilde{G}(\eta)}(s), \mathrm{AbT}_{\tilde{H}(\eta)}(s)], \max[\mathrm{ReT}_{\tilde{G}(\eta)}(s), \mathrm{ReT}_{\tilde{H}(\eta)}(s)], \\ \max[\mathrm{U}_{\tilde{G}(\eta)}(s), \mathrm{U}_{\tilde{H}(\eta)}(s)], \max[\mathrm{C}_{\tilde{G}(\eta)}(s), \mathrm{C}_{\tilde{H}(\eta)}(s)], \\ \min[\mathrm{G}_{\tilde{G}(\eta)}(s), \mathrm{G}_{\tilde{H}(\eta)}(s)], \min[\mathrm{ReF}_{\tilde{G}(\eta)}(s), \mathrm{ReF}_{\tilde{H}(\eta)}(s)], \\ \min[\mathrm{AbF}_{\tilde{G}(\eta)}(s), \mathrm{AbF}_{\tilde{H}(\eta)}(s)] \end{array}\right)\right\}\right].$$

$$\Rightarrow y \in \left[\left\{\left(s, \begin{array}{c} \max[\mathrm{AbT}_{\tilde{F}(\eta)}(s), \mathrm{AbT}_{\tilde{G}(\eta)}(s), \mathrm{AbT}_{\tilde{H}(\eta)}(s)], \max[\mathrm{ReT}_{\tilde{F}(\eta)}(s), \mathrm{ReT}_{\tilde{G}(\eta)}(s), \mathrm{ReT}_{\tilde{H}(\eta)}(s)], \\ \max[\mathrm{U}_{\tilde{F}(\eta)}(s), \mathrm{U}_{\tilde{G}(\eta)}(s), \mathrm{U}_{\tilde{H}(\eta)}(s)], \max[\mathrm{C}_{\tilde{F}(\eta)}(s), \mathrm{C}_{\tilde{G}(\eta)}(s), \mathrm{C}_{\tilde{H}(\eta)}(s)], \\ \min[\mathrm{G}_{\tilde{F}(\eta)}(s), \mathrm{G}_{\tilde{G}(\eta)}(s), \mathrm{G}_{\tilde{H}(\eta)}(s)], \min[\mathrm{ReF}_{\tilde{F}(\eta)}(s), \mathrm{ReF}_{\tilde{G}(\eta)}(s), \mathrm{ReF}_{\tilde{H}(\eta)}(s)], \\ \min[\mathrm{AbF}_{\tilde{F}(\eta)}(s), \mathrm{AbF}_{\tilde{G}(\eta)}(s), \mathrm{AbF}_{\tilde{H}(\eta)}(s)] \end{array}\right)\right\}\right].$$

$$\Rightarrow y \in \left[\left\{\left(s, \begin{array}{c} \max[\mathrm{AbT}_{\tilde{F}(\eta)}(s), \mathrm{AbT}_{\tilde{G}(\eta)}(s)], \max[\mathrm{ReT}_{\tilde{F}(\eta)}(s), \mathrm{ReT}_{\tilde{G}(\eta)}(s)], \\ \max[\mathrm{U}_{\tilde{F}(\eta)}(s), \mathrm{U}_{\tilde{G}(\eta)}(s)], \max[\mathrm{C}_{\tilde{F}(\eta)}(s), \mathrm{C}_{\tilde{G}(\eta)}(s)], \\ \min[\mathrm{G}_{\tilde{F}(\eta)}(s), \mathrm{G}_{\tilde{G}(\eta)}(s)], \min[\mathrm{ReF}_{\tilde{F}(\eta)}(s), \mathrm{ReF}_{\tilde{G}(\eta)}(s)], \\ \min[\mathrm{AbF}_{\tilde{F}(\eta)}(s), \mathrm{AbF}_{\tilde{G}(\eta)}(s)] \end{array}\right)\right\}\right] \text{ or } y \in (\tilde{H},\partial)$$

$$\Rightarrow y \in \left[\left\{\left(s, \begin{array}{c} \max[\mathrm{AbT}_{\tilde{F}(\eta)}(s), \mathrm{AbT}_{\tilde{G}(\eta)}(s)], \max[\mathrm{ReT}_{\tilde{F}(\eta)}(s), \mathrm{ReT}_{\tilde{G}(\eta)}(s)], \\ \max[\mathrm{U}_{\tilde{F}(\eta)}(s), \mathrm{U}_{\tilde{G}(\eta)}(s)], \max[\mathrm{C}_{\tilde{F}(\eta)}(s), \mathrm{C}_{\tilde{G}(\eta)}(s)], \\ \min[\mathrm{G}_{\tilde{F}(\eta)}(s), \mathrm{G}_{\tilde{G}(\eta)}(s)], \min[\mathrm{ReF}_{\tilde{F}(\eta)}(s), \mathrm{ReF}_{\tilde{G}(\eta)}(s)], \\ \min[\mathrm{AbF}_{\tilde{F}(\eta)}(s), \mathrm{AbF}_{\tilde{G}(\eta)}(s)] \end{array}\right)\right\}\right] \tilde{\mathbb{U}}(\tilde{H}, \partial)$$

$$y \in \left[(\tilde{F}, \partial)\tilde{\mathbb{U}}(\tilde{G}, \partial)\right] \tilde{\mathbb{U}}(\tilde{H}, \partial)...........(ii)$$

$$\Rightarrow (\tilde{F}, \partial)\tilde{\mathbb{U}} \left[(\tilde{G}, \partial)\tilde{\mathbb{U}}(\tilde{H}, \partial)\right] \subseteq \left[(\tilde{F}, \partial)\tilde{\mathbb{U}}(\tilde{G}, \partial)\right] \tilde{\mathbb{U}}(\tilde{H}, \partial)..............(iii)$$

Similarly,

$$\left[(\tilde{F}, \partial)\tilde{\mathbb{U}}(\tilde{G}, \partial)\right] \tilde{\mathbb{U}}(\tilde{H}, \partial) \subseteq (\tilde{F}, \partial)\tilde{\mathbb{U}} \left[(\tilde{G}, \partial)\tilde{\mathbb{U}}(\tilde{H}, \partial)\right] ....(iv)$$

Hence,

$$(\tilde{F}, \partial)\tilde{\mathbb{U}} \left[(\tilde{G}, \partial)\tilde{\mathbb{U}}(\tilde{H}, \partial)\right] = \left[(\tilde{F}, \partial)\tilde{\mathbb{U}}(\tilde{G}, \partial)\right] \tilde{\mathbb{U}}(\tilde{H}, \partial)$$

In a similar fashion we can prove rest of the results.

Theorem 2. *Let $(\tilde{F}, \partial)$ and $(\tilde{G}, \partial)$ be HPNSSs over the key set π, then 1.*

$$[(\tilde{F}, \partial)\tilde{\mathbb{U}}(\tilde{G}, \partial)]^c = (\tilde{F}, \partial)^c \tilde{\Cap}(\tilde{G}, \partial)^c;$$

2.

$$[(\tilde{F}, \partial)\tilde{\Cap}(\tilde{G}, \partial)]^c = (\tilde{F}, \partial)^c \tilde{\mathbb{U}}(\tilde{G}, \partial)^c.$$

Proof. 1. $\forall \eta \in \partial$, $\forall s \in \pi$,

$$(\tilde{F}, \partial)\tilde{\mathbb{U}}(\tilde{G}, \partial) = \left[\left\{\left(s, \begin{array}{c} \max[\mathrm{AbT}_{\tilde{F}(\eta)}(s), \mathrm{AbT}_{\tilde{G}(\eta)}(s)], \max[\mathrm{ReT}_{\tilde{F}(\eta)}(s), \mathrm{ReT}_{\tilde{G}(\eta)}(s)], \\ \max[\mathrm{U}_{\tilde{F}(\eta)}(s), \mathrm{U}_{\tilde{G}(\eta)}(s)], \max[\mathrm{C}_{\tilde{F}(\eta)}(s), \mathrm{C}_{\tilde{G}(\eta)}(s)], \\ \min[\mathrm{G}_{\tilde{F}(\eta)}(s), \mathrm{G}_{\tilde{G}(\eta)}(s)], \min[\mathrm{ReF}_{\tilde{F}(\eta)}(s), \mathrm{ReF}_{\tilde{G}(\eta)}(s)], \\ \min[\mathrm{AbF}_{\tilde{F}(\eta)}(s), \mathrm{AbF}_{\tilde{G}(\eta)}(s)] \end{array}\right)\right\}\right]$$

$$[(\tilde{F}, \partial)\tilde{\mathbb{U}}(\tilde{G}, \partial)]^c = \left[\left\{\left(s, \begin{array}{c} \min[\mathrm{AbF}_{\tilde{F}(\eta)}(s), \mathrm{AbF}_{\tilde{G}(\eta)}(s)], \min[\mathrm{ReF}_{\tilde{F}(\eta)}(s), \mathrm{ReF}_{\tilde{G}(\eta)}(s)], \\ \min[\mathrm{G}_{\tilde{F}(\eta)}(s), \mathrm{G}_{\tilde{G}(\eta)}(s)], \min[1 - \mathrm{C}_{\tilde{F}(\eta)}(s), 1 - \mathrm{C}_{\tilde{G}(\eta)}(s)], \\ \min[\mathrm{U}_{\tilde{F}(\eta)}(s), \mathrm{U}_{\tilde{G}(\eta)}(s)], \max[\mathrm{ReT}_{\tilde{F}(\eta)}(s), \mathrm{ReT}_{\tilde{G}(\eta)}(s)], \\ \max[\mathrm{AbT}_{\tilde{F}(\eta)}(s), \mathrm{AbT}_{\tilde{G}(\eta)}(s)] \end{array}\right)\right\}\right]$$

Now

$$(\tilde{F}, \partial)^c = \left\{\left(\begin{array}{l} \eta, \langle s, \mathrm{AbF}_{\tilde{F}(\eta)}(s), \mathrm{ReF}_{\tilde{F}(\eta)}(s), \mathrm{G}_{\tilde{F}(\eta)}(s), \\ 1 - \mathrm{C}_{\tilde{F}(\eta)}(s), \mathrm{U}_{\tilde{F}(\eta)}(s), \mathrm{ReT}_{\tilde{F}(\eta)}(s), \mathrm{AbT}_{\tilde{F}(\eta)}(s) \end{array} \right\},$$

$$(\tilde{G}, \partial)^c = \left\{\left(\begin{array}{l} \eta, \langle s, \mathrm{AbF}_{\tilde{G}(\eta)}(s), \mathrm{ReF}_{\tilde{G}(\eta)}(s), \mathrm{G}_{\tilde{G}(\eta)}(s), \\ 1 - \mathrm{C}_{\tilde{G}(\eta)}(s), \mathrm{U}_{\tilde{G}(\eta)}(s), \mathrm{ReT}_{\tilde{G}(\eta)}(s), \mathrm{AbT}_{\tilde{G}(\eta)}(s) \end{array} \right\}.$$

Thus,

$$(\tilde{F}, \partial)^c \tilde{\Cap}(\tilde{G}, \partial)^c = \left[\left\{\left(s, \begin{array}{c} \min[\mathrm{AbF}_{\tilde{F}(\eta)}(s), \mathrm{AbF}_{\tilde{G}(\eta)}(s)], \min[\mathrm{ReF}_{\tilde{F}(\eta)}(s), \mathrm{ReF}_{\tilde{G}(\eta)}(s)], \\ \min[\mathrm{G}_{\tilde{F}(\eta)}(s), \mathrm{G}_{\tilde{G}(\eta)}(s)], \min[1 - \mathrm{C}_{\tilde{F}(\eta)}(s), 1 - \mathrm{C}_{\tilde{G}(\eta)}(s)], \\ \max[\mathrm{U}_{\tilde{F}(\eta)}(s), \mathrm{U}_{\tilde{G}(\eta)}(s)], \max[\mathrm{ReT}_{\tilde{F}(\eta)}(s), \mathrm{ReT}_{\tilde{G}(\eta)}(s)], \\ \max[\mathrm{AbT}_{\tilde{F}(\eta)}(s), \mathrm{AbT}_{\tilde{G}(\eta)}(s)] \end{array}\right)\right\}\right]$$

Therefore,

$$[(\tilde{F},\partial)\tilde{\Cup}(\tilde{G},\partial)]^c = (\tilde{F},\partial)^c\tilde{\Cap}(\tilde{G},\partial)^c$$

2. $\forall\eta \in \partial$, $\forall s \in \pi$,

$$(\tilde{F},\partial)\tilde{\Cap}(\tilde{G},\partial) = \left[\left\{\left(s, \begin{array}{c} \min[\mathrm{AbT}_{\tilde{F}(\eta)}(s), \mathrm{AbT}_{\tilde{G}(\eta)}(s)], \min[\mathrm{ReT}_{\tilde{F}(\eta)}(s), \mathrm{ReT}_{\tilde{G}(\eta)}(s)], \\ \min[\mathrm{U}_{\tilde{F}(\eta)}(s), \mathrm{U}_{\tilde{G}(\eta)}(s)], \min[\mathrm{C}_{\tilde{F}(\eta)}(s), \mathrm{C}_{\tilde{G}(\eta)}(s)], \\ \max[\mathrm{G}_{\tilde{F}(\eta)}(s), \mathrm{G}_{\tilde{G}(\eta)}(s)], \max[\mathrm{ReF}_{\tilde{F}(\eta)}(s), \mathrm{ReF}_{\tilde{G}(\eta)}(s)], \\ \max[\mathrm{AbF}_{\tilde{F}(\eta)}(s), \mathrm{AbF}_{\tilde{G}(\eta)}(s)] \end{array}\right)\right\}\right]$$

$$[(\tilde{F},\partial)\tilde{\Cap}(\tilde{G},\partial)]^c = \left[\left\{\left(s, \begin{array}{c} \max[\mathrm{AbF}_{\tilde{F}(\eta)}(s), \mathrm{AbF}_{\tilde{G}(\eta)}(s)], \max[\mathrm{ReF}_{\tilde{F}(\eta)}(s), \mathrm{ReF}_{\tilde{G}(\eta)}(s)], \\ \max[\mathrm{G}_{\tilde{F}(\eta)}(s), \mathrm{G}_{\tilde{G}(\eta)}(s)], \min[1-\mathrm{C}_{\tilde{F}(\eta)}(s), 1-\mathrm{C}_{\tilde{G}(\eta)}(s)], \\ \min[\mathrm{U}_{\tilde{F}(\eta)}(s), \mathrm{U}_{\tilde{G}(\eta)}(s)], \min[\mathrm{ReT}_{\tilde{F}(\eta)}(s), \mathrm{ReT}_{\tilde{G}(\eta)}(s)], \\ \min[\mathrm{AbT}_{\tilde{F}(\eta)}(s), \mathrm{AbT}_{\tilde{G}(\eta)}(s)] \end{array}\right)\right\}\right]$$

Now

$$(\tilde{F},\partial)^c = \left\{\left(\begin{array}{l} \eta, \langle s, \mathrm{AbF}_{\tilde{F}(\eta)}(s), \mathrm{ReF}_{\tilde{F}(\eta)}(s), \mathrm{G}_{\tilde{F}(\eta)}(s), \\ 1-\mathrm{C}_{\tilde{F}(\eta)}(s), \mathrm{U}_{\tilde{F}(\eta)}(s), \mathrm{ReT}_{\tilde{F}(\eta)}(s), \mathrm{AbT}_{\tilde{F}(\eta)}(s) \end{array}\right\},$$

$$(\tilde{G},\partial)^c = \left\{\left(\begin{array}{l} \eta, \langle s, \mathrm{AbF}_{\tilde{G}(\eta)}(s), \mathrm{ReF}_{\tilde{G}(\eta)}(s), \mathrm{G}_{\tilde{G}(\eta)}(s), \\ 1-\mathrm{C}_{\tilde{G}(\eta)}(s), \mathrm{U}_{\tilde{G}(\eta)}(s), \mathrm{ReT}_{\tilde{G}(\eta)}(s), \mathrm{AbT}_{\tilde{G}(\eta)}(s) \end{array}\right\}.$$

Thus,

$$(\tilde{F},\partial)^c\tilde{\Cup}(\tilde{G},\partial)^c = \left[\left\{\left(s, \begin{array}{c} \max[\mathrm{AbF}_{\tilde{F}(\eta)}(s), \mathrm{AbF}_{\tilde{G}(\eta)}(s)], \max[\mathrm{ReF}_{\tilde{F}(\eta)}(s), \mathrm{ReF}_{\tilde{G}(\eta)}(s)], \\ \max[\mathrm{G}_{\tilde{F}(\eta)}(s), \mathrm{G}_{\tilde{G}(\eta)}(s)], \min[1-\mathrm{C}_{\tilde{F}(\eta)}(s), 1-\mathrm{C}_{\tilde{G}(\eta)}(s)], \\ \min[\mathrm{U}_{\tilde{F}(\eta)}(s), \mathrm{U}_{\tilde{G}(\eta)}(s)], \min[\mathrm{ReT}_{\tilde{F}(\eta)}(s), \mathrm{ReT}_{\tilde{G}(\eta)}(s)], \\ \min[\mathrm{AbT}_{\tilde{F}(\eta)}(s), \mathrm{AbT}_{\tilde{G}(\eta)}(s)] \end{array}\right)\right\}\right]$$

Therefore,

$$[(\tilde{F},\partial)\tilde{\Cap}(\tilde{G},\partial)]^c = (\tilde{F},\partial)^c\tilde{\Cup}(\tilde{G},\partial)^c$$

Theorem 3. *Let $(\tilde{F},\partial)$ and $(\tilde{G},\partial)$ be HPNSSs over the key set π, then 1.*

$$[(\tilde{F},\partial)\tilde{\vee}(\tilde{G},\partial)]^c = (\tilde{F},\partial)^c\tilde{\wedge}(\tilde{G},\partial)^c;$$

2.

$$[(\tilde{F},\partial)\tilde{\wedge}(\tilde{G},\partial)]^c = (\tilde{F},\partial)^c\tilde{\vee}(\tilde{G},\partial)^c.$$

Proof. 1. $\forall(\eta_1,\eta_2) \in \partial\times\partial$, $\forall s \in \pi$,

$$(\tilde{F},\partial)\tilde{\vee}(\tilde{G},\partial) = \left[\left\{\left(s, \begin{array}{c} \max[\mathrm{AbT}_{\tilde{F}(\eta)}(s), \mathrm{AbT}_{\tilde{G}(\eta)}(s)], \max[\mathrm{ReT}_{\tilde{F}(\eta)}(s), \mathrm{ReT}_{\tilde{G}(\eta)}(s)], \\ \max[\mathrm{U}_{\tilde{F}(\eta)}(s), \mathrm{U}_{\tilde{G}(\eta)}(s)], \max[\mathrm{C}_{\tilde{F}(\eta)}(s), \mathrm{C}_{\tilde{G}(\eta)}(s)], \\ \min[\mathrm{G}_{\tilde{F}(\eta)}(s), \mathrm{G}_{\tilde{G}(\eta)}(s)], \min[\mathrm{ReF}_{\tilde{F}(\eta)}(s), \mathrm{ReF}_{\tilde{G}(\eta)}(s)], \\ \min[\mathrm{AbF}_{\tilde{F}(\eta)}(s), \mathrm{AbF}_{\tilde{G}(\eta)}(s)] \end{array}\right)\right\}\right]$$

$$[(\tilde{F},\partial)\tilde{\vee}(\tilde{G},\partial)]^c = \left[\left\{\left(s, \begin{array}{c} \min[\mathrm{AbF}_{\tilde{F}(\eta)}(s), \mathrm{AbF}_{\tilde{G}(\eta)}(s)], \min[\mathrm{ReF}_{\tilde{F}(\eta)}(s), \mathrm{ReF}_{\tilde{G}(\eta)}(s)], \\ \min[\mathrm{G}_{\tilde{F}(\eta)}(s), \mathrm{G}_{\tilde{G}(\eta)}(s)], \max[1-\mathrm{C}_{\tilde{F}(\eta)}(s), 1-\mathrm{C}_{\tilde{G}(\eta)}(s)], \\ \max[\mathrm{U}_{\tilde{F}(\eta)}(s), \mathrm{U}_{\tilde{G}(\eta)}(s)], \max[\mathrm{ReT}_{\tilde{F}(\eta)}(s), \mathrm{ReT}_{\tilde{G}(\eta)}(s)], \\ \max[\mathrm{AbT}_{\tilde{F}(\eta)}(s), \mathrm{AbT}_{\tilde{G}(\eta)}(s)] \end{array}\right)\right\}\right]$$

Now

$$(\tilde{F},\partial)^c = \left\{\left(\begin{array}{l}\eta, \langle s, \mathrm{AbF}_{\tilde{F}(\eta)}(s), \mathrm{ReF}_{\tilde{F}(\eta)}(s), \mathrm{G}_{\tilde{F}(\eta)}(s), \\ 1-\mathrm{C}_{\tilde{F}(\eta)}(s), \mathrm{U}_{\tilde{F}(\eta)}(s), \mathrm{ReT}_{\tilde{F}(\eta)}(s), \mathrm{AbT}_{\tilde{F}(\eta)}(s)\end{array}\right\},$$

$$(\tilde{G},\partial)^c = \left\{\left(\begin{array}{l}\eta, \langle s, \mathrm{AbF}_{\tilde{G}(\eta)}(s), \mathrm{ReF}_{\tilde{G}(\eta)}(s), \mathrm{G}_{\tilde{G}(\eta)}(s), \\ 1-\mathrm{C}_{\tilde{G}(\eta)}(s), \mathrm{U}_{\tilde{G}(\eta)}(s), \mathrm{ReT}_{\tilde{G}(\eta)}(s), \mathrm{AbT}_{\tilde{G}(\eta)}(s)\end{array}\right\}.$$

Thus,

$$(\tilde{F},\partial)^c\tilde{\wedge}(\tilde{G},\partial)^c = \left[\left\{\left(s, \begin{array}{c} \min[\mathrm{AbF}_{\tilde{F}(\eta)}(s), \mathrm{AbF}_{\tilde{G}(\eta)}(s)], \min[\mathrm{ReF}_{\tilde{F}(\eta)}(s), \mathrm{ReF}_{\tilde{G}(\eta)}(s)], \\ \min[\mathrm{G}_{\tilde{F}(\eta)}(s), \mathrm{G}_{\tilde{G}(\eta)}(s)], \max[1-\mathrm{C}_{\tilde{F}(\eta)}(s), 1-\mathrm{C}_{\tilde{G}(\eta)}(s)], \\ \max[\mathrm{U}_{\tilde{F}(\eta)}(s), \mathrm{U}_{\tilde{G}(\eta)}(s)], \max[\mathrm{ReT}_{\tilde{F}(\eta)}(s), \mathrm{ReT}_{\tilde{G}(\eta)}(s)], \\ \max[\mathrm{AbT}_{\tilde{F}(\eta)}(s), \mathrm{AbT}_{\tilde{G}(\eta)}(s)] \end{array}\right)\right\}\right]$$

Therefore,

$$[(\tilde{F},\partial)\tilde{\vee}(\tilde{G},\partial)]^c = (\tilde{F},\partial)^c\tilde{\wedge}(\tilde{G},\partial)^c$$

2. $\forall \eta \in \partial$, $\forall s \in \pi$,

$$(\tilde{F},\partial)\tilde{\wedge}(\tilde{G},\partial) = \left[\left\{\left(s, \begin{array}{c} \min[\mathrm{AbT}_{\tilde{F}(\eta)}(s), \mathrm{AbT}_{\tilde{G}(\eta)}(s)], \min[\mathrm{ReT}_{\tilde{F}(\eta)}(s), \mathrm{ReT}_{\tilde{G}(\eta)}(s)], \\ \min[\mathrm{U}_{\tilde{F}(\eta)}(s), \mathrm{U}_{\tilde{G}(\eta)}(s)], \min[\mathrm{C}_{\tilde{F}(\eta)}(s), \mathrm{C}_{\tilde{G}(\eta)}(s)], \\ \max[\mathrm{G}_{\tilde{F}(\eta)}(s), \mathrm{G}_{\tilde{G}(\eta)}(s)], \max[\mathrm{ReF}_{\tilde{F}(\eta)}(s), \mathrm{ReF}_{\tilde{G}(\eta)}(s)], \\ \max[\mathrm{AbF}_{\tilde{F}(\eta)}(s), \mathrm{AbF}_{\tilde{G}(\eta)}(s)] \end{array}\right)\right\}\right]$$

$$[(\tilde{F},\partial)\tilde{\Cap}(\tilde{G},\partial)]^c = \left[\left\{\left(s, \begin{array}{c} \max[\mathrm{AbF}_{\tilde{F}(\eta)}(s), \mathrm{AbF}_{\tilde{G}(\eta)}(s)], \max[\mathrm{ReF}_{\tilde{F}(\eta)}(s), \mathrm{ReF}_{\tilde{G}(\eta)}(s)], \\ \max[\mathrm{G}_{\tilde{F}(\eta)}(s), \mathrm{G}_{\tilde{G}(\eta)}(s)], \min[1-\mathrm{C}_{\tilde{F}(\eta)}(s), 1-\mathrm{C}_{\tilde{G}(\eta)}(s)], \\ \min[\mathrm{U}_{\tilde{F}(\eta)}(s), \mathrm{U}_{\tilde{G}(\eta)}(s)], \min[\mathrm{ReT}_{\tilde{F}(\eta)}(s), \mathrm{ReT}_{\tilde{G}(\eta)}(s)], \\ \min[\mathrm{AbT}_{\tilde{F}(\eta)}(s), \mathrm{AbT}_{\tilde{G}(\eta)}(s)] \end{array}\right)\right\}\right]$$

Now

$$(\tilde{F},\partial)^c = \left\{\left(\begin{array}{l}\eta, \langle s, \mathrm{AbF}_{\tilde{F}(\eta)}(s), \mathrm{ReF}_{\tilde{F}(\eta)}(s), \mathrm{G}_{\tilde{F}(\eta)}(s), \\ 1-\mathrm{C}_{\tilde{F}(\eta)}(s), \mathrm{U}_{\tilde{F}(\eta)}(s), \mathrm{ReT}_{\tilde{F}(\eta)}(s), \mathrm{AbT}_{\tilde{F}(\eta)}(s)\end{array}\right\},$$

$$(\tilde{G},\partial)^c = \left\{\left(\begin{array}{l}\eta, \langle s, \mathrm{AbF}_{\tilde{G}(\eta)}(s), \mathrm{ReF}_{\tilde{G}(\eta)}(s), \mathrm{G}_{\tilde{G}(\eta)}(s), \\ 1-\mathrm{C}_{\tilde{G}(\eta)}(s), \mathrm{U}_{\tilde{G}(\eta)}(s), \mathrm{ReT}_{\tilde{G}(\eta)}(s), \mathrm{AbT}_{\tilde{G}(\eta)}(s)\end{array}\right\}.$$

Thus,

$$(\tilde{F},\partial)^c\tilde{\vee}(\tilde{G},\partial)^c = \left[\left\{\left(s, \begin{array}{c} \max[\mathrm{AbF}_{\tilde{F}(\eta)}(s), \mathrm{AbF}_{\tilde{G}(\eta)}(s)], \max[\mathrm{ReF}_{\tilde{F}(\eta)}(s), \mathrm{ReF}_{\tilde{G}(\eta)}(s)], \\ \max[\mathrm{G}_{\tilde{F}(\eta)}(s), \mathrm{G}_{\tilde{G}(\eta)}(s)], \min[1-\mathrm{C}_{\tilde{F}(\eta)}(s), 1-\mathrm{C}_{\tilde{G}(\eta)}(s)], \\ \min[\mathrm{U}_{\tilde{F}(\eta)}(s), \mathrm{U}_{\tilde{G}(\eta)}(s)], \min[\mathrm{ReT}_{\tilde{F}(\eta)}(s), \mathrm{ReT}_{\tilde{G}(\eta)}(s)], \\ \min[\mathrm{AbT}_{\tilde{F}(\eta)}(s), \mathrm{AbT}_{\tilde{G}(\eta)}(s)] \end{array}\right)\right\}\right]$$

Therefore,

$$[(\tilde{F},\partial)\tilde{\wedge}(\tilde{G},\partial)]^c = (\tilde{F},\partial)^c\tilde{\vee}(\tilde{G},\partial)^c$$

Example 1. *Let $\pi = \{s_1, s_2, s_3\}$ be the key set and the set of parameters $\partial = \{\eta_1, \eta_2\}$. Let us develop the HPNSSs $(\tilde{F},\partial)$ and $(\tilde{G},\partial)$ over the key set π as follows:*

$$(\tilde{F},\partial) = \begin{bmatrix} \eta_1 = (\langle s_1, \frac{02}{10}, \frac{03}{10}, \frac{03}{10}, \frac{02}{10}, \frac{08}{10}, \frac{07}{10}, \frac{08}{10}\rangle, \\ \langle s_2, \frac{04}{10}, \frac{04}{10}, \frac{04}{10}, \frac{04}{10}, \frac{04}{10}, \frac{06}{10}, \frac{04}{10}\rangle, \\ \langle s_3, \frac{02}{10}, \frac{04}{10}, \frac{04}{10}, \frac{02}{10}, \frac{03}{10} \frac{06}{10}, \frac{03}{10}\rangle), \\ \eta_2 = (\langle s_1, \frac{03}{10}, \frac{02}{10}, \frac{02}{10}, \frac{03}{10}, \frac{06}{10}, \frac{06}{10}, \frac{06}{10}\rangle, \\ \langle s_2, \frac{01}{10}, \frac{05}{10}, \frac{05}{10}, \frac{01}{10} \frac{5}{10}, \frac{06}{10}, \frac{05}{10}\rangle, \\ \langle s_3, \frac{04}{10}, \frac{03}{10}, \frac{03}{10}, \frac{04}{10}, \frac{05}{10} \frac{06}{10}, \frac{05}{10}\rangle) \end{bmatrix}$$

$$(\tilde{G},\partial) = \begin{bmatrix} \eta_1 = (\langle s_1, \frac{04}{10}, \frac{03}{10}, \frac{03}{10}, \frac{04}{10}, \frac{06}{10}, \frac{06}{10}, \frac{06}{10}\rangle, \\ \langle s_2, \frac{04}{10}, \frac{05}{10}, \frac{05}{10}, \frac{04}{10}, \frac{03}{10}, \frac{06}{10}, \frac{03}{10}\rangle, \\ \langle s_3, \frac{03}{10}, \frac{05}{10}, \frac{05}{10}, \frac{03}{10}, \frac{02}{10} \frac{06}{10}, \frac{02}{10}\rangle), \\ \eta_2 = (\langle s_1, \frac{03}{10}, \frac{04}{10}, \frac{04}{10}, \frac{03}{10}, \frac{05}{10} \frac{06}{10}, \frac{05}{10}\rangle, \\ \langle s_2, \frac{02}{10}, \frac{06}{10}, \frac{06}{10}, \frac{02}{10}, \frac{04}{10} \frac{06}{10}, \frac{04}{10}\rangle, \\ \langle s_3, \frac{04}{10}, \frac{06}{10}, \frac{06}{10}, \frac{04}{10}, \frac{03}{10} \frac{06}{10}, \frac{03}{10}\rangle) \end{bmatrix}$$

$$(\tilde{F},\partial)\tilde{\Cup}(\tilde{G},\partial) = \begin{bmatrix} \eta_1 = (\langle s_1, \frac{04}{10}, \frac{03}{10}, \frac{03}{10}, \frac{04}{10}, \frac{06}{10} \frac{06}{10}, \frac{06}{10}\rangle, \\ \langle s_2, \frac{04}{10}, \frac{05}{10}, \frac{05}{10}, \frac{04}{10}, \frac{03}{10}, \frac{06}{10}, \frac{03}{10}\rangle, \\ \langle s_3, \frac{03}{10}, \frac{05}{10}, \frac{05}{10}, \frac{03}{10}, \frac{02}{10} \frac{06}{10}, \frac{02}{10}\rangle), \\ \eta_2 = (\langle s_1, \frac{03}{10}, \frac{04}{10}, \frac{04}{10}, \frac{03}{10}, \frac{05}{10}, \frac{06}{10}, \frac{05}{10}\rangle, \\ \langle s_2, \frac{02}{10}, \frac{06}{10}, \frac{06}{10}, \frac{02}{10}, \frac{04}{10}, \frac{06}{10}, \frac{04}{10}\rangle, \\ \langle s_3, \frac{04}{10}, \frac{06}{10}, \frac{06}{10}, \frac{04}{10}, \frac{03}{10}, \frac{06}{10}, \frac{03}{10}\rangle) \end{bmatrix}$$

$$(\tilde{F},\acute{E})\tilde{\Cap}(\tilde{G},\partial) = \begin{bmatrix} \eta_1 = (\langle s_1, \frac{02}{10}, \frac{03}{10}, \frac{03}{10}, \frac{02}{10}, \frac{08}{10}, \frac{07}{10}, \frac{08}{10}\rangle, \\ \langle s_2, \frac{04}{10}, \frac{04}{10}, \frac{04}{10}, \frac{04}{10}, \frac{04}{10} \frac{06}{10}, \frac{04}{10}\rangle, \\ \langle s_3, \frac{02}{10}, \frac{04}{10}, \frac{04}{10}, \frac{02}{10}, \frac{03}{10} \frac{6}{10}, \frac{3}{10}\rangle), \\ \eta_2 = (\langle s_1, \frac{03}{10}, \frac{02}{10}, \frac{02}{10}, \frac{03}{10}, \frac{06}{10}, \frac{06}{10}, \frac{06}{10}\rangle, \\ \langle s_2, \frac{01}{10}, \frac{05}{10}, \frac{05}{10}, \frac{01}{10}, \frac{05}{10}, \frac{06}{10}, \frac{05}{10}\rangle, \\ \langle s_3, \frac{04}{10}, \frac{03}{10}, \frac{03}{10}, \frac{04}{10}, \frac{05}{10}, \frac{06}{10}, \frac{05}{10}\rangle) \end{bmatrix}$$

$$(\tilde{F},\partial)\tilde{\wedge}(\tilde{G},\partial) = \begin{bmatrix} (\eta_1,\eta_1) = (\langle s_1, \frac{02}{10}, \frac{03}{10}, \frac{03}{10}, \frac{02}{10}, \frac{08}{10}, \frac{07}{10}, \frac{08}{10}\rangle, \\ \langle s_2, \frac{04}{10}, \frac{04}{10}, \frac{04}{10}, \frac{04}{10}, \frac{04}{10}, \frac{06}{10}, \frac{04}{10}\rangle, \\ \langle s_3, \frac{02}{10}, \frac{04}{10}, \frac{04}{10}, \frac{02}{10}, \frac{03}{10}, \frac{06}{10}, \frac{03}{10}\rangle), \\ (\eta_1,\eta_2) = (\langle s_1, \frac{02}{10}, \frac{03}{10}, \frac{03}{10}, \frac{02}{10}, \frac{08}{10}, \frac{07}{10}, \frac{08}{10}\rangle, \\ \langle s_2, \frac{02}{10}, \frac{04}{10}, \frac{04}{10}, \frac{02}{10}, \frac{04}{10}, \frac{06}{10}, \frac{04}{10}\rangle, \\ \langle s_3, \frac{04}{10}, \frac{03}{10}, \frac{03}{10}, \frac{04}{10}, \frac{04}{10}, \frac{06}{10}, \frac{04}{10}\rangle), \\ (\eta_2,\eta_1) = (\langle s_1, \frac{03}{10}, \frac{02}{10}, \frac{02}{10}, \frac{03}{10}, \frac{06}{10}, \frac{07}{10}, \frac{06}{10}\rangle, \\ \langle s_2, \frac{01}{10}, \frac{05}{10}, \frac{05}{10}, \frac{01}{10}, \frac{05}{10}, \frac{06}{10}, \frac{05}{10}\rangle, \\ \langle s_3, \frac{03}{10}, \frac{03}{10}, \frac{03}{10}, \frac{03}{10}, \frac{04}{10}, \frac{06}{10}, \frac{04}{10}\rangle), \\ (\eta_2,\eta_2) = (\langle s_1, \frac{03}{10}, \frac{02}{10}, \frac{02}{10}, \frac{03}{10}, \frac{06}{10}, \frac{06}{10}, \frac{06}{10}\rangle, \\ \langle s_2, \frac{01}{10}, \frac{05}{10}, \frac{05}{10}, \frac{01}{10}, \frac{05}{10}, \frac{06}{10}, \frac{05}{10}\rangle, \\ \langle s_3, \frac{04}{10}, \frac{03}{10}, \frac{03}{10}, \frac{04}{10}, \frac{05}{10}, \frac{06}{10}, \frac{05}{10}\rangle) \end{bmatrix}$$

$$(\tilde{F},\partial)\tilde{\vee}(\tilde{G},\partial) = \begin{bmatrix} (\eta_1,\eta_1) = (\langle s_1, \frac{04}{10}, \frac{03}{10}, \frac{03}{10}, \frac{04}{10}, \frac{06}{10}, \frac{06}{10}, \frac{06}{10}\rangle, \\ \langle s_2, \frac{04}{10}, \frac{05}{10}, \frac{05}{10}, \frac{04}{10}, \frac{03}{10}, \frac{06}{10}, \frac{03}{10}\rangle, \\ \langle s_3, \frac{03}{10}, \frac{05}{10}, \frac{05}{10}, \frac{03}{10}, \frac{02}{10}, \frac{06}{10}, \frac{02}{10}\rangle), \\ (\eta_1,\eta_2) = (\langle s_1, \frac{02}{10}, \frac{03}{10}, \frac{03}{10}, \frac{02}{10}, \frac{08}{10}, \frac{07}{10}, \frac{08}{10}\rangle, \\ \langle s_2, \frac{02}{10}, \frac{04}{10}, \frac{04}{10}, \frac{02}{10}, \frac{04}{10}, \frac{06}{10}, \frac{04}{10}\rangle, \\ \langle s_3, \frac{04}{10}, \frac{03}{10}, \frac{03}{10}, \frac{04}{10}, \frac{04}{10}, \frac{06}{10}, \frac{04}{10}\rangle), \\ (\eta_2,\eta_1) = (\langle s_1, \frac{04}{10}, \frac{3}{10}, \frac{03}{10}, \frac{04}{10}, \frac{06}{10}, \frac{06}{10}, \frac{06}{10}\rangle, \\ \langle s_2, \frac{04}{10}, \frac{05}{10}, \frac{05}{10}, \frac{04}{10}, \frac{03}{10}, \frac{06}{10}, \frac{03}{10}\rangle, \\ \langle s_3, \frac{04}{10}, \frac{05}{10}, \frac{05}{10}, \frac{04}{10}, \frac{02}{10}, \frac{06}{10}, \frac{02}{10}\rangle), \\ (\eta_2,\eta_2) = (\langle s_1, \frac{03}{10}, \frac{04}{10}, \frac{04}{10}, \frac{03}{10}, \frac{05}{10}, \frac{06}{10}, \frac{05}{10}\rangle, \\ \langle s_2, \frac{02}{10}, \frac{06}{10}, \frac{06}{10}, \frac{04}{10}, \frac{04}{10}, \frac{06}{10}, \frac{04}{10}\rangle, \\ \langle s_3, \frac{04}{10}, \frac{06}{10}, \frac{06}{10}, \frac{04}{10}, \frac{03}{10}\frac{06}{10}, \frac{03}{10}\rangle) \end{bmatrix}$$

4. A New Approach to Operations on Heptapartitioned Neutrosophic Soft Topological Space

The notion of HPNSTS is presented in this section. The terms HPNS semi-open, HPNS pre-open, and HPNS $\beta - open$ sets are defined. One of these intriguing HPNS generalized open sets, referred to as the HPNS pre-open set, is selected, and certain fundamentals are then produced based on this description. These consist of the HPNS closer, HPNS exterior, HPNS border, and HPNS interior.

Definition 21. *Let the HPNSS $(\tilde{\pi},\partial)$ be the family of all HPNSSs, and let $\tau \subset HPNSS(\tilde{\pi},\partial)$. Then, τ is a hepta- partitioned neutrosophic soft topology (HPNST) on $\tilde{\pi}$ if:*

(i) $0_{(\langle\pi\rangle,\partial)}$, $1_{(\langle\pi\rangle,\partial)} \in \tau$,

(ii) The union of any number of HPNSSs in τ belongs to τ,

(iii) The intersection of a finite number of HPNSSs in τ belongs to τ.

Then, $(\tilde{\pi},\tau,\partial)$ is said to be a HPNSTS over $\tilde{\pi}$.

Definition 22. *If $(\tilde{\pi},\tau,\partial)$ be a HPNSTS over $\tilde{\pi}$.HPNSS $(\tilde{F},\partial)$ is HPNS neighborhood of HPNS point $s^{\eta}_{\langle\Delta_1,\Delta_2,\Delta_3,\Delta_4,\Delta_5,\Delta_6,\Delta_7\rangle} \in (\tilde{F},\partial)$, if there is a HPNS open set $(\tilde{G},\partial)$ such that $s^{\eta}_{\langle\Delta_1,\Delta_2,\Delta_3,\Delta_4,\Delta_5,\Delta_6,\Delta_7\rangle} \in (\tilde{G},\partial)$.*

Definition 23. *If (π,τ_1,∂) and (π,τ_2,∂) be two HPNSTSs then, $(\pi,\tau_1,\tau_2,\partial)$ is called a HP-NSBTS. If $(\pi,\tau_1,\tau_2,\partial)$ is a HPNSBTS. A HPNS subset $(\tilde{F},\partial)$ is open in $(\pi,\tau_1,\tau_2,\partial)$ if there is a HPNSS open set $(\tilde{G},\partial) \in \tau_1$ and a HPNSS open set $(\tilde{H},\partial) belongsto \tau_2$ such that:*

$$(\tilde{F},\partial) = (\tilde{G},\partial) \Cup (\tilde{H},\partial).$$

Example 2. *Let $\pi = \{s_1,s_2,s_3\}$, $\partial = \{\eta_1,\eta_2\}$ and*

$$\tau_1 = \{0_{(\pi,\partial)}, 1_{(\pi,\partial)}, (\tilde{F},\partial), (\tilde{G},\partial)\},$$
$$\tau_2 = \{0_{(\pi,\partial)}, 1_{(\pi,\partial)}, (\tilde{H},\partial), (\tilde{I},\partial)\}.$$

Where $(\tilde{F},\partial)$, $(\tilde{G},\partial)$, $(\tilde{H},\partial)$, *and* $(\tilde{I},\partial)$ *being HPNSSs are as follows:*

$$(\tilde{F},\partial)=\left[\begin{array}{l}\eta_1=(\langle s_1,\frac{02}{10},\frac{03}{10},\frac{03}{10},\frac{02}{10},\frac{08}{10},\frac{07}{10},\frac{08}{10}\rangle,\\ \quad\langle s_2,\frac{04}{10},\frac{04}{10},\frac{04}{10},\frac{04}{10},\frac{04}{10},\frac{06}{10},\frac{04}{10}\rangle,\\ \quad\langle s_3,\frac{02}{10},\frac{04}{10},\frac{04}{10},\frac{02}{10},\frac{03}{10},\frac{06}{10},\frac{03}{10}\rangle);\\ \eta_2=(\langle s_1,\frac{03}{10},\frac{02}{10},\frac{02}{10},\frac{03}{10},\frac{06}{10},\frac{06}{10},\frac{06}{10}\rangle,\\ \quad\langle s_2,\frac{01}{10},\frac{05}{10},\frac{05}{10},\frac{01}{10},\frac{05}{10},\frac{06}{10},\frac{05}{10}\rangle,\\ \quad\langle s_3,\frac{04}{10},\frac{03}{10},\frac{03}{10},\frac{04}{10},\frac{05}{10},\frac{06}{10},\frac{05}{10}\rangle).\end{array}\right]$$

$$(\tilde{G},\partial)=\left[\begin{array}{l}\eta_1=(\langle s_1,\frac{04}{10},\frac{03}{10},\frac{03}{10},\frac{04}{10},\frac{06}{10},\frac{06}{10},\frac{06}{10}\rangle,\\ \quad\langle s_2,\frac{04}{10},\frac{05}{10},\frac{05}{10},\frac{04}{10},\frac{03}{10},\frac{06}{10},\frac{03}{10}\rangle,\\ \quad\langle s_3,\frac{03}{10},\frac{05}{10},\frac{05}{10},\frac{03}{10},\frac{02}{10},\frac{06}{10},\frac{02}{10}\rangle);\\ \eta_2=(\langle s_1,\frac{03}{10},\frac{04}{10},\frac{04}{10},\frac{03}{10},\frac{05}{10},\frac{06}{10},\frac{05}{10}\rangle,\\ \quad\langle s_2,\frac{02}{10},\frac{06}{10},\frac{06}{10},\frac{02}{10},\frac{04}{10},\frac{06}{10},\frac{04}{10}\rangle,\\ \quad\langle s_3,\frac{04}{10},\frac{06}{10},\frac{06}{10},\frac{04}{10},\frac{03}{10},\frac{06}{10},\frac{03}{10}\rangle).\end{array}\right]$$

$$(\tilde{H},\partial)=\left[\begin{array}{l}\eta_1=(\langle s_1,\frac{06}{10},\frac{06}{10},\frac{06}{10},\frac{06}{10},\frac{02}{10},\frac{06}{10},\frac{02}{10}\rangle,\\ \quad\langle s_2,\frac{06}{10},\frac{06}{10},\frac{06}{10},\frac{06}{10},\frac{02}{10},\frac{06}{10},\frac{02}{10}\rangle,\\ \quad\langle s_3,\frac{04}{10},\frac{06}{10},\frac{06}{10},\frac{04}{10},\frac{01}{10},\frac{06}{10},\frac{01}{10}\rangle),\\ \eta_2=(\langle s_1,\frac{05}{10},\frac{06}{10},\frac{06}{10},\frac{05}{10},\frac{02}{10},\frac{06}{10},\frac{02}{10}\rangle,\\ \quad\langle s_2,\frac{06}{10},\frac{07}{10},\frac{07}{10},\frac{06}{10},\frac{02}{10},\frac{06}{10},\frac{02}{10}\rangle,\\ \quad\langle s_3,\frac{05}{10},\frac{05}{10},\frac{05}{10},\frac{05}{10},\frac{01}{10},\frac{06}{10},\frac{01}{10}\rangle).\end{array}\right]$$

$$(\tilde{I},\partial)=\left[\begin{array}{l}\eta_1=(\langle s_1,\frac{01}{10},\frac{02}{10},\frac{02}{10},\frac{01}{10},\frac{07}{10},\frac{06}{10},\frac{07}{10}\rangle,\\ \quad\langle s_2,\frac{04}{10},\frac{04}{10},\frac{04}{10},\frac{04}{10},\frac{03}{10},\frac{06}{10},\frac{03}{10}\rangle,\\ \quad\langle s_3,\frac{02}{10},\frac{04}{10},\frac{04}{10},\frac{02}{10},\frac{02}{10},\frac{06}{10},\frac{02}{10}\rangle),\\ \eta_2=(\langle s_1,\frac{03}{10},\frac{02}{10},\frac{02}{10},\frac{03}{10},\frac{05}{10},\frac{06}{10},\frac{05}{10}\rangle,\\ \quad\langle s_2,\frac{01}{10},\frac{05}{10},\frac{05}{10},\frac{01}{10},\frac{05}{10},\frac{06}{10},\frac{05}{10}\rangle,\\ \quad\langle s_3,\frac{04}{10},\frac{03}{10},\frac{03}{10},\frac{04}{10},\frac{05}{10},\frac{06}{10},\frac{05}{10}\rangle).\end{array}\right].$$

Theorem 4. *Let* $(\pi,\tau_1,\tau_2,\partial)$ *be a HPNSBTS. Then their intersection that is* $\tau_1 \Cap \tau_2$ *is a HPNSBTS on* π*.*

Proof. The first and third requirements are clear, and we move forward as follows for the second condition.
Let $\{(\tilde{F}_i, \partial); i \in I\} \in \tau_1 \Cap \tau_2$, then $(\tilde{F}_i, \partial) \in \tau_1$ and $(\tilde{F}_i, \partial) \in \tau_2$, as τ_1 and τ_2 are HPNSBTSs on π, then:

$$\Cup_{i\in I}(\tilde{F}_i, \partial) \in \tau_1, \quad \Cup_{i\in I}(\tilde{F}_i, \partial) \in \tau_2.$$

So

$$\Cup_{i\in I}(\tilde{F}_i, \partial) \in \tau_1 \Cap \tau_2.$$

Remark 1. *Let $(\pi, \tau_1, \tau_2, \partial)$ be a HPNSBTS, then their union that ia $\tau_1 \Cup \tau_2$ need not be a HPNSBTS on π.*

Example 3. *Let $\pi = \{s_1, s_2, s_3\}$, $\partial = \{\eta_1, \eta_2\}$,*

$$\tau_1 = \{0_{(\pi,\partial)}, 1_{(\pi,\partial)}, (\tilde{F}, \partial), (\tilde{G}, \partial), (\tilde{H}, \partial)\},$$
$$\tau_2 = \{0_{(\pi,\partial)}, 1_{(\pi,\partial)}, (\tilde{I}, \partial), (\tilde{J}, \partial)\}.$$

Where $(\tilde{F}, \partial)$, $(\tilde{G}, \partial)$, $(\tilde{H}, \partial)$, $(\tilde{I}, \partial)$ and $(\tilde{J}, \partial)$ being HPNS sub set are as follows:

$$(\tilde{F}, \partial) = \begin{bmatrix} \eta_1 = (\langle s_1, \frac{02}{10}, \frac{03}{10}, \frac{03}{10}, \frac{02}{10}, \frac{08}{10}, \frac{08}{10}, \frac{08}{10}\rangle, \\ \langle s_2, \frac{04}{10}, \frac{04}{10}, \frac{04}{10}, \frac{04}{10}, \frac{04}{10}, \frac{04}{10}, \frac{04}{10}\rangle, \\ \langle s_3, \frac{02}{10}, \frac{04}{10}, \frac{04}{10}, \frac{02}{10}, \frac{03}{10}, \frac{03}{10}, \frac{03}{10}\rangle); \\ \eta_2 = (\langle s_1, \frac{03}{10}, \frac{02}{10}, \frac{02}{10}, \frac{03}{10}, \frac{06}{10}, \frac{06}{10}, \frac{06}{10}\rangle, \\ \langle s_2, \frac{01}{10}, \frac{05}{10}, \frac{05}{10}, \frac{01}{10}, \frac{05}{10}, \frac{05}{10}, \frac{05}{10}\rangle, \\ \langle s_3, \frac{04}{10}, \frac{03}{10}, \frac{03}{10}, \frac{04}{10}, \frac{05}{10}, \frac{05}{10}, \frac{05}{10}\rangle). \end{bmatrix}$$

$$(\tilde{G}, \partial) = \begin{bmatrix} \eta_1 = (\langle s_1, \frac{04}{10}, \frac{03}{10}, \frac{03}{10}, \frac{04}{10}, \frac{06}{10}, \frac{06}{10}, \frac{06}{10}\rangle, \\ \langle s_2, \frac{04}{10}, \frac{05}{10}, \frac{05}{10}, \frac{04}{10}, \frac{03}{10}, \frac{03}{10}, \frac{03}{10}\rangle, \\ \langle s_3, \frac{03}{10}, \frac{05}{10}, \frac{05}{10}, \frac{03}{10}, \frac{02}{10}, \frac{02}{10}, \frac{02}{10}\rangle); \\ \eta_2 = (\langle s_1, \frac{03}{10}, \frac{04}{10}, \frac{04}{10}, \frac{03}{10}, \frac{05}{10}, \frac{05}{10}, \frac{05}{10}\rangle, \\ \langle s_2, \frac{02}{10}, \frac{06}{10}, \frac{06}{10}, \frac{02}{10}, \frac{04}{10}, \frac{04}{10}, \frac{04}{10}\rangle, \\ \langle s_3, \frac{04}{10}, \frac{06}{10}, \frac{06}{10}, \frac{04}{10}, \frac{03}{10}, \frac{03}{10}, \frac{03}{10}\rangle). \end{bmatrix}$$

$$(\tilde{H}, \partial) = \begin{bmatrix} \eta_1 = (\langle s_1, \frac{05}{10}, \frac{04}{10}, \frac{04}{10}, \frac{05}{10}, \frac{04}{10}, \frac{04}{10}, \frac{04}{10}\rangle, \\ \langle s_2, \frac{06}{10}, \frac{06}{10}, \frac{06}{10}, \frac{06}{10}, \frac{02}{10}, \frac{02}{10}, \frac{02}{10}\rangle, \\ \langle s_3, \frac{04}{10}, \frac{06}{10}, \frac{06}{10}, \frac{04}{10}, \frac{01}{10}, \frac{01}{10}, \frac{01}{10}\rangle), \\ \eta_2 = (\langle s_1, \frac{04}{10}, \frac{06}{10}, \frac{06}{10}, \frac{04}{10}, \frac{03}{10}, \frac{03}{10}, \frac{03}{10}\rangle, \\ \langle s_2, \frac{03}{10}, \frac{07}{10}, \frac{07}{10}, \frac{03}{10}, \frac{03}{10}, \frac{03}{10}, \frac{03}{10}\rangle, \\ \langle s_3, \frac{05}{10}, \frac{07}{10}, \frac{07}{10}, \frac{05}{10}, \frac{01}{10}, \frac{01}{10}, \frac{01}{10}\rangle). \end{bmatrix}$$

$$(\tilde{I},\partial)=\begin{bmatrix}\eta_1=(\langle s_1,\frac{06}{10},\frac{06}{10},\frac{06}{10},\frac{06}{10},\frac{02}{10},\frac{02}{10},\frac{02}{10}\rangle,\\ \langle s_2,\frac{06}{10},\frac{06}{10},\frac{06}{10},\frac{06}{10},\frac{02}{10},\frac{02}{10},\frac{02}{10}\rangle,\\ \langle s_3,\frac{04}{10},\frac{06}{10},\frac{06}{10},\frac{04}{10},\frac{01}{10},\frac{01}{10},\frac{01}{10}\rangle),\\ \eta_2=(\langle s_1,\frac{05}{10},\frac{06}{10},\frac{06}{10},\frac{05}{10},\frac{02}{10},\frac{02}{10},\frac{02}{10}\rangle,\\ \langle s_2,\frac{06}{10},\frac{07}{10},\frac{07}{10},\frac{06}{10},\frac{02}{10},\frac{02}{10},\frac{02}{10}\rangle,\\ \langle s_3,\frac{05}{10},\frac{05}{10},\frac{05}{10},\frac{05}{10},\frac{01}{10},\frac{01}{10},\frac{01}{10}\rangle).\end{bmatrix}.$$

$$(\tilde{J},\partial)=\begin{bmatrix}\eta_1=(\langle s_1,\frac{01}{10},\frac{02}{10},\frac{02}{10},\frac{01}{10},\frac{07}{10},\frac{07}{10},\frac{07}{10}\rangle,\\ \langle s_2,\frac{04}{10},\frac{04}{10},\frac{04}{10},\frac{04}{10},\frac{03}{10},\frac{03}{10},\frac{03}{10}\rangle,\\ \langle s_3,\frac{02}{10},\frac{04}{10},\frac{04}{10},\frac{02}{10},\frac{02}{10},\frac{02}{10},\frac{02}{10}\rangle),\\ \eta_2=(\langle s_1,\frac{03}{10},\frac{02}{10},\frac{02}{10},\frac{03}{10},\frac{05}{10},\frac{05}{10},\frac{05}{10}\rangle,\\ \langle s_2,\frac{01}{10},\frac{05}{10},\frac{05}{10},\frac{01}{10},\frac{05}{10},\frac{05}{10},\frac{05}{10}\rangle,\\ \langle s_3,\frac{04}{10},\frac{03}{10},\frac{03}{10},\frac{04}{10},\frac{05}{10},\frac{05}{10},\frac{05}{10}\rangle).\end{bmatrix}$$

Here, $\tau_1 \Cup \tau_2 = \{0_{(\tilde{\pi},\partial)}, 1_{(\tilde{\pi},\partial)}, (\tilde{F},\partial), (\tilde{G},\partial), (\tilde{H},\partial), (\tilde{I},\partial), (\tilde{J},\partial)\}$ *is not a HPNSBTS on* $\tilde{\pi}$ *as* $(\tilde{H},\partial) \Cup (\tilde{I},\partial)$ *does not belong to* $\tau_1 \Cup \tau_2$.

Definition 24. *Let* $(\pi,\tau_1,\tau_2,\partial)$ *be a HPNSBTS, then a HPNSS*

$$(\tilde{H},\partial)=\left[\left(\eta,\left\langle s, AbT_{\tilde{H}(\eta)}(s), ReT_{\tilde{H}(\eta)}(s), C_{\tilde{H}(\eta)}(s)ReF_{\tilde{H}(\eta)}(s), AbF_{\tilde{H}(\eta)}(s): s\in\pi\right\rangle\right):\eta\in\partial\right]$$

is a pairwise HPNS open set if there is a HPNS open set $(\tilde{F},\partial)$ *in* τ_1 *and a HPNS open set* $(\tilde{G},\partial)$ *in* τ_2 *such that for all* $s\in\pi$,

$$(\tilde{H},\partial)=(\tilde{F},\partial)\Cup(\tilde{G},\partial)=\left\{\left(\eta,\left\{s,\begin{array}{c}AbT_{\tilde{H}(\eta)}(s)=\max[AbT_{\tilde{F}(\eta)}(s),AbT_{\tilde{G}(\eta)}(s)],\\ ReT_{\tilde{H}(\eta)}(s)=\max[ReT_{\tilde{F}(\eta)}(s),ReT_{\tilde{G}(\eta)}(s)],\\ C_{\tilde{H}(\eta)}(s)=\max[C_{\tilde{F}(\eta)}(s),C_{\tilde{G}(\eta)}(s)],\\ ReF_{\tilde{H}(\eta)}(s)=\min[ReF_{\tilde{F}(\eta)}(s),ReF_{\tilde{G}(\eta)}(s)],\\ AbF_{\tilde{H}(\eta)}(s)=\min[AbF_{\tilde{F}(\eta)}(s),AbF_{\tilde{G}(\eta)}(s)]\end{array}\right\}\right):\eta\in\partial\right\}.$$

This is denoted by HPNSO(π,∂).

Definition 25. *Let* $(\pi,\tau_1,\tau_2,\partial)$ *be a HPNSBTS then a HPNSS.*

$$(\tilde{H},\partial)=\left[\left(\eta,\left\langle\left\langle s,\begin{array}{c}AbT_{\tilde{H}(\eta)}(s), ReT_{\tilde{H}(\eta)}(s), C_{\tilde{H}(\eta)}(s), U_{\tilde{H}(\eta)}(s),\\ G_{\tilde{H}(\eta)}(s), ReF_{\tilde{H}(\eta)}(s), AbF_{\tilde{H}(\eta)}(s): s\in\pi\end{array}\right\rangle\right\rangle\right):\eta\in\partial\right]$$

is called a pairwise HPNSCS if $(\tilde{H},\partial)^c$ *is a pairwise HPNSO.* $(\tilde{H},\partial)$ *is a HPNS closed set if there exists a HPNS closed set* $(\tilde{F},\partial)$ *in* τ_1 *and a HPNS closed set* $(\tilde{G},\partial)$ *in* τ_2 *such that for all*

$s \in \pi$,

$$(\tilde{H}, \partial) = (\tilde{F}, \acute{E}) \Cap (\tilde{G}, \partial) = \left\{ \left(\eta, \left\{ s, \begin{array}{c} AbT_{\tilde{H}(\eta)}(s) = \min[AbT_{\tilde{F}(\eta)}(s), AbT_{\tilde{G}(\eta)}(s)], \\ ReT_{\tilde{H}(\eta)}(s) = \min[ReT_{\tilde{F}(\eta)}(s), ReT_{\tilde{G}(\eta)}(s)], \\ C_{\tilde{H}(\eta)}(s) = \min[C_{\tilde{F}(\eta)}(s), C_{\tilde{G}(\eta)}(s)], \\ U_{\tilde{H}(\eta)}(s) = \min[U_{\tilde{F}(\eta)}(s), U_{\tilde{G}(\eta)}(s)], \\ G_{\tilde{H}(\eta)}(s) = \max[G_{\tilde{F}(\eta)}(s), G_{\tilde{G}(\eta)}(s)], \\ ReF_{\tilde{H}(e)}(s) = \max[ReF_{\tilde{F}(\eta)}(s), ReF_{\tilde{G}(\eta)}(s)], \\ AbF_{\tilde{H}(e)}(s) = \max[AbF_{\tilde{F}(\eta)}(s), AbF_{\tilde{G}(\eta)}(s)] \end{array} \right\} \right) : \eta \in \partial \right\}.$$

This is denoted by HPNSC(π, ∂).

Definition 26. *Let $(\pi, \tau_1, \tau_2, \partial)$ be a HPNSBTS over π, and $(\tilde{\ddot{\Upsilon}}, \partial)$ be a HPNSS then:*

(i) $(\tilde{\ddot{\Upsilon}}, \partial)$ *is HPNS semi-open if* $(\tilde{\ddot{\Upsilon}}, \partial) \subseteq NScl(NSint(\tilde{\ddot{F}}, \partial))$.

(ii) $(\tilde{\ddot{\Upsilon}}, \partial)$ *is HPNS pre-open (p-open) if* $(\tilde{\ddot{\Upsilon}}, \partial) \subseteq NSint(NScl(\tilde{\ddot{\Upsilon}}, \partial))$.

(iii) $(\tilde{\ddot{F}}, \partial)$ *is HPNS $\beta-$open if*

$$(\tilde{\ddot{\Upsilon}}, \partial) \subseteq NScl(NSint(\tilde{\ddot{\Upsilon}}, \partial)) \Cup NSint(NScl(\tilde{\ddot{\Upsilon}}, \partial)),$$

and HPNS $\beta-$close if

$$(\tilde{\ddot{\Upsilon}}, \partial) \supseteq NScl(NSint(\tilde{\ddot{\Upsilon}}, \partial)) \Cap NSint(NScl(\tilde{\ddot{\Upsilon}}, \partial)).$$

Definition 27. *Let $(\pi, \tau_1, \tau_2, \partial)$ be a HPNSBTS over π, and $(\tilde{\ddot{\Upsilon}}, \partial)$ be a HPNS, then interior of $(\tilde{\ddot{\Upsilon}}, \partial)$, denoted by $(\tilde{\ddot{F}}, \partial)^\circ$, is the union of all HPNS p-open sets of $(\tilde{\ddot{F}}, \partial)$. Clearly, $(\tilde{\ddot{F}}, \partial)^\circ$ is the largest HPNS p-open set contained in $(\tilde{\ddot{F}}, \partial)$.*

Definition 28. *Let $(\pi, \tau_1, \tau_2, \partial)$ be a HPNSBTS, and $(\tilde{\ddot{F}}, \partial)$ be a HPNS, the frontier of $(\tilde{\ddot{F}}, \partial)$ denoted by $Fr((\tilde{\ddot{\Upsilon}}, \partial))$, is a HPNS point $s^\eta_{\langle \Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6, \Delta_7 \rangle}$ such that every HPNS p-open set comprising $s^\eta_{\langle \Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6, \Delta_7 \rangle}$ comprises at least one point of $(\tilde{\ddot{\Upsilon}}, \partial)$ and at least one HPNS point of $(\tilde{\ddot{\Upsilon}}, \partial)^c$.*

Definition 29. *If $(\pi, \tau_1, \tau_2, \partial)$ is a HPNSBTS and $(\tilde{\ddot{\Upsilon}}, \partial)$ is a HPNS, then the exterior of $(\tilde{\ddot{\Upsilon}}, \partial)$, denoted by $Ext((\tilde{\ddot{\Upsilon}}, \partial))$, is a HPNS point $s^\eta_{\langle \Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6, \Delta_7 \rangle}$ is called exterior of $(\tilde{\ddot{\Upsilon}}, \partial)$ if $s^\eta_{\langle \Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6, \Delta_7 \rangle}$ is in the interior of $(\tilde{\ddot{\Upsilon}}, \partial)^c$, that is HPNS p-open set $(\tilde{g}, \partial)$ such that*

$$s^\eta_{\langle \Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6, \Delta_7 \rangle} \in (\tilde{g}, \partial) \subseteq (\tilde{\ddot{F}}, \partial)^c.$$

Definition 30. *If $(\tilde{\pi}, \tau_1, \tau_2, \partial)$ and $(\langle \tilde{Y} \rangle, \mathfrak{F}_1, \mathfrak{F}_2, \partial)$ are HPNSBTSs, and $(f, \phi) : (\tilde{\pi}, \tau_1, \tau_2, \partial) \to (\langle \tilde{Y} \rangle, \mathfrak{F}_1, \mathfrak{F}_2, \partial)$ is a HPNS mapping, then (f, ϕ) is said to be a HPNS p-close mapping if the image $(f, \phi)(\tilde{\ddot{F}}, \partial)$ of each HPNS p-closed set $(\tilde{\ddot{F}}, \partial)$ over $\tilde{\pi}$ is a HPNS p-closed set in $\langle \tilde{Y} \rangle$, then (f, ϕ) is said to be a HPNS p-close mapping.*

Theorem 5. *Let $(\pi, \tau_1, \tau_2, \partial)$ be a HPNSBTSs over π and $(\tilde{\ddot{\Upsilon}}, \partial)$ is HPNS subset. $(\tilde{\ddot{\Upsilon}}, \partial)$ is a HPNS p-open set if and only if $(\tilde{\ddot{\Upsilon}}, \partial) = (\tilde{\ddot{\Upsilon}}, \partial)^\circ$.*

Proof. Let $(\tilde{\ddot{\Upsilon}}, \partial)$ be a HPNS p-open set. Then, the biggest HPNS p-open set surrounded by $(\tilde{\ddot{\Upsilon}}, \partial)$ is equal to $(\tilde{\ddot{\Upsilon}}, \partial)$. Hence, $(\tilde{\ddot{\Upsilon}}, \partial) = (\tilde{\ddot{\Upsilon}}, \partial)^\circ$.

Contrariwise, it is recognized that $(\tilde{\ddot{\Upsilon}}, \partial)^\circ$ is a HPNS p-open set, and if $(\tilde{\ddot{\Upsilon}}, \partial) = (\tilde{\ddot{\Upsilon}}, \partial)^\circ$, then $(\tilde{\ddot{\Upsilon}}, \partial)$ is a HPNS p-open set.

Theorem 6. *Let $(\pi, \tau_1, \tau_2, \partial)$ be a HPNSBTS over π, and let $(\tilde{\pounds}, \partial)$ and $(\tilde{\Im}, \partial)$ be HPNS subsets then*

(i) $[(\tilde{\pounds}, \partial)^\circ]^\circ = (\tilde{\pounds}, \partial)^\circ$,

(ii) $(0_{(\langle\tilde{\pi}\rangle,\partial)})^\circ = 0_{(\langle\tilde{\pi}\rangle,\partial)}$ *and* $(1_{(\langle\tilde{\pi}\rangle,\partial)})^\circ = 1_{(\langle\tilde{\pi}\rangle,\partial)}$,

(iii) $(\tilde{\pounds}, \partial) \subseteq (\tilde{\Im}, \partial) \Rightarrow (\tilde{\pounds}, \partial)^\circ \subseteq (\tilde{\Im}, \partial)^\circ$,

(iv) $[(\tilde{\pounds}, \partial) \Cap (\tilde{\Im}, \partial)]^\circ = (\tilde{\pounds}, \partial)^\circ \Cap (\tilde{\Im}, \partial)^\circ$,

(v) $(\tilde{\pounds}, \partial)^\circ \Cup (\tilde{\Im}, \partial)^\circ \subseteq [(\tilde{\pounds}, \partial) \Cup (\tilde{\Im}, \partial)]^\circ$.

Proof.

(i) $(\tilde{\pounds, \partial}^\circ = (\tilde{\Im}, \partial)$ then $(\tilde{\Im}, \partial) \in \tilde{\tau}$ iff $(\tilde{\Im}, \partial) = (\tilde{\pounds, \partial}^\circ$. So $[(\tilde{\pounds}, \partial)^\circ]^\circ = (\tilde{\pounds}, \partial)^\circ$

(ii) Since $0_{(\langle\tilde{\pi}\rangle,\partial)}$ and $1_{(\langle\tilde{\pi}\rangle,\partial)}$ are always HPNS p-open sets, so

$$(0_{(\langle\tilde{\pi}\rangle,\partial)})^\circ = 0_{(\langle\tilde{\pi}\rangle,\partial)}, \quad \text{and} \quad (1_{(\langle\tilde{\pi}\rangle,\partial)})^\circ = 1_{(\langle\tilde{\pi}\rangle,\partial)}.$$

(iii) It is known that $(\tilde{\pounds}, \partial)^\circ \subseteq (\tilde{\pounds}, \partial) \subseteq (\tilde{\Im}, \partial)$ and $(\tilde{\Im}, \partial)^\circ \subseteq (\tilde{\Im}, \partial)$. Since $(\tilde{\Im}, \partial)^\circ$ is the biggest HPNS p-open set enclosed in $(\tilde{\Im}, \partial)$ and so, $(\tilde{\pounds}, \partial)^\circ \subseteq (\tilde{\Im}, \partial)^\circ$.

(iv) Since $(\tilde{\pounds}, \partial) \Cap (\tilde{\Im}, \partial) \subseteq (\tilde{\pounds}, \partial)$ and $(\tilde{\pounds}, \partial) \Cap (\tilde{\Im}, \partial) \subseteq (\tilde{\Im}, \partial)$, then

$$[(\tilde{\pounds}, \partial) \Cap (\tilde{\Im}, \partial)]^\circ \subseteq (\tilde{\pounds}, \partial)^\circ and [(\tilde{\pounds}, \partial) \Cap (\tilde{\Im}, \partial)]^\circ \subseteq (\tilde{\Im}, \partial)^\circ.$$

so,

$$[(\tilde{\pounds}, \partial) \Cap (\tilde{\Im}, \partial)]^\circ \subseteq (\tilde{\pounds}, \partial)^\circ \Cap (\tilde{\Im}, \partial)^\circ.$$

On other way, since $(\tilde{\pounds}, \partial)^\circ \subseteq (\tilde{\pounds}, \partial)$ and $(\tilde{\Im}, \partial)^\circ \subseteq (\tilde{\Im}, \partial)$, then

$$(\tilde{\pounds}, \partial)^\circ \Cap (\tilde{\Im}, \partial)^\circ \subseteq (\tilde{\pounds}, \partial) \Cap (\tilde{\Im}, \partial).$$

also $[(\tilde{\pounds}, \partial) \Cap (\tilde{\Im}, \partial)]^\circ \subseteq (\tilde{\pounds}, \partial) \Cap (\tilde{\Im}, \partial)$ is the biggest HPNS p-open set.
$\Rightarrow (\tilde{\pounds}, \partial)^\circ \Cap (\tilde{\Im}, \partial)^\circ \subseteq [(\tilde{\pounds}, \partial) \Cap (\tilde{\Im}, \partial)]^\circ$
Thus,

$$[(\tilde{\pounds}, \partial) \Cap (\tilde{\Im}, \partial)]^\circ = (\tilde{\pounds}, \partial)^\circ \Cap (\tilde{\Im}, \partial)^\circ.$$

(v) Since $(\tilde{\pounds}, \partial) \subseteq (\tilde{\pounds}, \partial) \Cup (\tilde{\Im}, \partial)$ and $(\tilde{\Im}, \partial) \subseteq (\tilde{\pounds}, \partial) \Cup (\tilde{\Im}, \partial)$, then

$$(\tilde{\pounds}, \partial)^\circ \subseteq [(\tilde{\pounds}, \partial) \Cup (\tilde{\Im}, \partial)]^\circ, \quad \text{and} \quad (\tilde{\Im}, \partial)^\circ \subseteq [(\tilde{\pounds}, \partial) \Cup (\tilde{\Im}, \partial)]^\circ.$$

$\Rightarrow (\tilde{\pounds}, \partial)^\circ \Cup (\tilde{\Im}, \partial)^\circ \subseteq [(\tilde{\pounds}, \partial) \Cup (\tilde{\Im}, \partial)]^\circ$.

Theorem 7. *Let $(\pi, \tau_1, \tau_2, \partial)$ be a HPNSBTS over π, and if $(\tilde{\pounds}, \partial)$ is a HPNS subset, then $(\tilde{\pounds}, \partial)$ is a HPNS p-closer set if and only if $(\tilde{\pounds}, \partial) = \overline{(\tilde{\pounds}, \partial)}$.*

Proof. Let $(\tilde{\pounds},\partial)$ is a HPNS p-closer set, then:

$$(\tilde{\pounds},\partial)^d = (\tilde{\pounds},\partial)$$

this implies

$$(\tilde{\pounds},\partial) \Cup (\tilde{\pounds},\partial)^d \cong (\tilde{\pounds},\partial)$$

$\Rightarrow$

$$\overline{(\tilde{\pounds},\partial)} \cong (\tilde{\pounds},\partial)$$

and conversely let $\overline{(\tilde{\pounds},\partial)} \cong (\tilde{\pounds},\partial)$, this implies that

$$(\tilde{\pounds},\partial) \Cup (\tilde{\pounds},\partial)^d \cong (\tilde{\pounds},\partial)$$

$\Rightarrow$

$$(\tilde{\pounds},\partial)^d = (\tilde{\pounds},\partial)$$

this implies, $(\tilde{\pounds},\partial)$ is a HPNS p-closer set.

Theorem 8. *Let $(\pi,\tau_1,\tau_2,\partial)$ be a HPNSBTS over π, and let $(\tilde{\pounds},\partial)$ and $(\tilde{\Im},\partial)$ be HPNS subsets then*

(i) $\overline{\overline{(\tilde{\pounds},\partial)}} = \overline{(\tilde{\pounds},\partial)}$,

(ii) $\overline{(0_{(\langle\tilde{\pi}\rangle,\partial)})} = 0_{(\tilde{\pi},\partial)}$ *and* $\overline{(1_{(\langle\tilde{\pi}\rangle,\partial)})} = \overline{(1_{(\langle\tilde{\pi}\rangle,\partial)})}$,

(iii) $(\tilde{\pounds},\partial) \subseteq \langle(\tilde{\Im},\partial)\rangle \Rightarrow \overline{(\tilde{\pounds},\partial)} \subseteq \overline{(\tilde{\Im},\partial)}$,

(iv) $\overline{[(\tilde{\pounds},\partial) \Cup (\tilde{\Im},\partial)]} = \overline{(\tilde{\pounds},\partial)} \Cup \overline{(\tilde{\Im},\partial)}$,

(v) $\overline{[(\tilde{\pounds},\partial) \Cap (\tilde{\Im},\partial)]} \subseteq \overline{(\tilde{\pounds},\partial)} \Cap \overline{(\tilde{\Im},\partial)}$.

Proof.

(i) If $\overline{(\tilde{\pounds},\partial)} = (\tilde{\Im},\partial)$ then $(\tilde{\Im},\partial)$ is a HPNS p-closed set.Hence, if $(\tilde{\Im},\partial) = \overline{(\tilde{\Im},\partial)}$. Therefore $\overline{[\overline{(\tilde{\pounds},\partial)}]} = \overline{(\tilde{\pounds},\partial)}$

(ii) Since $0_{(\langle\tilde{\pi}\rangle,\partial)}$ and $1_{(\langle\tilde{\pi}\rangle,\partial)}$ are always HPNS p-closure set so by the above result

$$\overline{(0_{(\langle\tilde{\pi}\rangle,\partial)})} = 0_{(\langle\tilde{\pi}\rangle,\partial)}, \quad \text{and} \quad \overline{(1_{(\langle\tilde{\pi}\rangle,\partial)})} = 1_{(\langle\tilde{\pi}\rangle,\partial)}.$$

(iii) Since $(\tilde{\pounds},\partial) \subseteq \overline{(\tilde{\pounds},\partial)}$ and $(\tilde{\Im},\partial) \subseteq \overline{(\tilde{\Im},\partial)}$, so $(\tilde{\pounds},\partial) \subseteq (\tilde{\Im},\partial) \subseteq \overline{(\tilde{\Im},\partial)}$ .Since $\overline{(\tilde{\Im},\partial)}$ is the smallest HPNS p-closure set covering in $(\tilde{\pounds},\partial)$ then $\overline{(\tilde{\pounds},\partial)} \subseteq \overline{(\tilde{\Im},\partial)}$.

(iv) Since $(\tilde{\pounds},\partial) \subseteq (\tilde{\pounds},\partial) \Cup (\tilde{\Im},\partial)$ and $(\tilde{\Im},\partial) \subseteq 0_{(\langle\tilde{\pi}\rangle,\partial)} \Cup (\tilde{\Im},\partial)$ then

$$\overline{(\tilde{\pounds},\partial)} \subseteq \overline{[(\tilde{\pounds},\partial) \Cup (\tilde{\Im},\partial)]} and \overline{(\tilde{\Im},\partial)} \subseteq \overline{[(\tilde{\pounds},\partial) \Cup (\tilde{\Im},\partial)]}.$$

so,

$$\overline{(\tilde{\pounds},\partial)} \Cup \overline{(\tilde{\Im},\partial)} \subseteq \overline{[(\tilde{\pounds},\partial) \Cup (\tilde{\Im},\partial)]}.$$

Conversely, since $(\tilde{\pounds},\partial) \subseteq \overline{(\tilde{\pounds},\partial)}$ and $(\tilde{\Im},\partial) \subseteq \overline{(\tilde{\Im},\partial)}$, then

$$(\tilde{\pounds},\partial) \Cup (\tilde{\Im},\partial) \subseteq \overline{(\tilde{\pounds},\partial)} \Cup \overline{(\tilde{\Im},\partial)}.$$

Besides, $\overline{[(\tilde{\pounds},\partial) \Cup (\tilde{\Im},\partial)]}$ is the smallest HPNS p-closed set that enclosing $(\tilde{\pounds},\partial) \Cup (\tilde{\Im},\partial)$ therefore, $\overline{[(\tilde{\pounds},\partial) \Cup (\tilde{\Im},\partial)]} \subseteq \overline{(\tilde{\pounds},\partial)} \Cup \overline{(\tilde{\Im},\partial)}$.
Thus, $\overline{[(\tilde{\pounds},\partial) \Cup (\tilde{\Im},\partial)]} = \overline{(\tilde{\pounds},\partial)} \Cup \overline{(\tilde{\Im},\partial)}$.

(v) Since $\langle(0_{(\langle\tilde{\pi}\rangle,\partial)})\rangle \Cap (\tilde{\Im},\partial) \subseteq \overline{(\tilde{\pounds},\partial)} \Cap \overline{(\tilde{\Im},\partial)}$ and $\overline{[(\tilde{\pounds},\partial) \Cap (\tilde{\Im},\partial)]}$ is the smallest HPNS p-closed set that enclosing $(\tilde{\pounds},\partial) \Cap (\tilde{\Im},\partial)$, then $\overline{[(\tilde{\pounds},\partial) \Cap (\tilde{\Im},\partial)]} \subseteq \overline{(\tilde{\pounds},\partial)} \Cap \overline{(\tilde{\Im},\partial)}$.

Theorem 9. *Let $(\pi,\tau_1,\tau_2,\partial)$ be a HPNSBTSs over π, and let $(\tilde{\pounds},\partial)$ be a HPNSS then,*

(i) $[\overline{(\tilde{\pounds},\partial)}]^c = [(\tilde{\pounds},\partial)^c]^\circ$,

(ii) $[(\tilde{\pounds},\partial)^\circ]^c = \overline{[(\tilde{\pounds},\partial)^c]}$.

Proof.

(i)

$$\begin{aligned}\overline{(\tilde{\pounds},\partial)} &= \Cap\{(\tilde{\Im},\partial) \in (\pi,\tau_1,\tau_2,\partial)^c : (\tilde{\Im},\partial) \supseteq (\tilde{\pounds},\partial)\}\\ &\Rightarrow [\overline{(\tilde{\pounds},\partial)}]^c = \left[\Cap\{(\tilde{\Im},\partial) \in (\pi,\tau_1,\tau_2,\partial)^c : (\tilde{\Im},\partial) \supseteq (\tilde{\pounds},\partial)\}\right]^c\\ &= \Cup\{(\tilde{\Im},\partial)^c \in (\pi,\tau_1,\tau_2,\partial) : (\tilde{\Im},\partial)^c \subseteq (\tilde{\pounds},\partial)^c\}\\ &= [(\tilde{\pounds},\partial)^c]^\circ.\end{aligned}$$

(ii)

$$\begin{aligned}(\tilde{\pounds},\partial)^\circ &= \Cup\{(\tilde{\Im},\partial) \in (\pi,\tau_1,\tau_2,\partial) : (\tilde{\Im},\partial) \subseteq (\tilde{\pounds},\partial)\}\\ &\Rightarrow [(\tilde{\pounds},\partial)^\circ]^c = \left[\Cap\{(\tilde{\Im},\partial) \in (pi,\tau_1,\tau_2,\partial) : (\tilde{\Im},\partial) \subseteq (\tilde{\pounds},\partial)\}\right]^c\\ &= \Cap\{(\tilde{\Im},\partial)^c \in (\pi,\tau_1,\tau_2,\partial)^c : (\tilde{\Im},\partial)^c \supseteq (\tilde{\pounds},\partial)^c\}\\ &= \overline{[(\tilde{\pounds},\partial)^c]}.\end{aligned}$$

Theorem 10. *Let $(\pi,\tau_1,\tau_2,\partial)$ be a HPNSBTS over π. If $(\tilde{\pounds},\partial)$ and $(\tilde{\Im},\partial)$ are HPNS subsets, then:*

(i) $\mathrm{Ext}((\tilde{\pounds},\partial) \Cup (\tilde{\Im},\partial)) = \mathrm{Ext}((\tilde{\pounds},\partial)) \Cup \mathrm{Ext}((\tilde{\Im},\partial))$.

(ii) $\mathrm{Ext}((\tilde{\pounds},\partial) \Cap (\tilde{\Im},\partial)) \supseteq \mathrm{Ext}((\tilde{\pounds},\partial)) \Cup \mathrm{Ext}((\tilde{\Im},\partial))$.

(iii) $\mathrm{Fr}((\tilde{\pounds},\partial) \Cup (\tilde{\Im},\partial)) \subseteq \mathrm{Fr}(\tilde{\pounds},\partial) \Cup \mathrm{Fr}(\tilde{\Im},\partial)$.

(iv) $\mathrm{Fr}((\tilde{\pounds},\partial) \Cap (\tilde{\Im},\partial)) \subseteq \mathrm{Fr}(\tilde{\pounds},\partial) \Cup \mathrm{Fr}(\tilde{\Im},\partial)$.

Proof.

(i) Since

$$\begin{aligned}\mathrm{Ext}((\tilde{\pounds},\partial) \Cup (\tilde{\Im},\partial)) &= (((\tilde{\pounds},\partial) \Cup (\tilde{\Im},\partial))^{\mathcal{C}})^\circ\\ &= ((\tilde{\pounds},\partial)^{\mathcal{C}} \Cap (\tilde{\Im},\partial)^{\mathcal{C}})^\circ\\ &= ((\tilde{\pounds},\partial)^{\mathcal{C}})^\circ \Cap ((\tilde{\Im},\partial)^{\mathcal{C}})^\circ\\ &= \mathrm{Ext}((\tilde{\pounds},\partial)) \Cap \mathrm{Ext}((\tilde{\Im},\partial)).\end{aligned}$$

(ii)

$$\begin{aligned}\mathrm{Ext}((\tilde{\pounds},\partial) \Cap (\tilde{\Im},\partial)) &= (((\tilde{\pounds},\partial) \Cap (\tilde{\Im},\partial))^{\mathcal{C}})^\circ\\ &= (((\tilde{\pounds},\partial)^{\mathcal{C}} \Cup (\tilde{\Im},\partial)^{\mathcal{C}}))^\circ\\ &\supseteq ((\tilde{\pounds},\partial)^{\mathcal{C}})^\circ \Cup ((\tilde{\Im},\partial)^{\mathcal{C}})^\circ\\ &= \mathrm{Ext}((\tilde{\pounds},\partial)) \Cup \mathrm{Ext}((\tilde{\Im},\partial)).\end{aligned}$$

(iii)

$$\begin{aligned}
&\mathrm{Fr}((\tilde{\pounds},\partial)\Cup(\tilde{\Im},\partial)) = \overline{(\tilde{\pounds},\partial)\Cup(\tilde{\Im},\partial)} \Cap \overline{((\tilde{\pounds},\partial)\Cup(\tilde{\Im},\partial))^{\mathcal{C}}}\\
&= (\overline{(\tilde{\pounds},\partial)} \Cup \overline{(\tilde{\Im},\partial)}) \Cap \overline{(\tilde{\pounds},\partial)^{\mathcal{C}} \Cap (\tilde{\Im},\partial)^{\mathcal{C}}} \subseteq ((\tilde{\pounds},\partial)) \Cup ((\tilde{\Im},\partial)) \Cap \overline{(\tilde{\pounds},\partial)^{\mathcal{C}}} \Cap \overline{(\tilde{\Im},\partial)^{\mathcal{C}}})\\
&= \{(\overline{((\tilde{\pounds},\partial))}) \Cup ((\tilde{\Im},\partial)) \Cap \overline{(\tilde{\pounds},\partial)^{\mathcal{C}}} \Cap \overline{(\tilde{\Im},\partial)^{\mathcal{C}}}\}\\
&= \{(\overline{(\tilde{\pounds},\partial)} \Cap \overline{(\tilde{\pounds},\partial)^{\mathcal{C}}}) \Cup (\tilde{\Im},\partial) \Cap \overline{(\tilde{\pounds},\partial)^{\mathcal{C}}}\} \Cup \{(\overline{(\tilde{\Im},\partial)^{\mathcal{C}}}) \Cap (\overline{(\tilde{\pounds},\partial)^{\mathcal{C}}} \Cap \overline{(\tilde{\Im},\partial)^{\mathcal{C}}})\}\\
&= \{\mathrm{Fr}((\tilde{\pounds},\partial)) \Cap (\overline{(\tilde{\Im},\partial)^{\mathcal{C}}})\} \Cup \{\mathrm{Fr}((\tilde{\Im},\partial)) \Cap \overline{(\tilde{\pounds},\partial)^{\mathcal{C}}}\}\\
&\subseteq \mathrm{Fr}((\tilde{\pounds},\partial)) \Cup \mathrm{Fr}((\tilde{\Im},\partial)).
\end{aligned}$$

(iv)

$$\begin{aligned}
&\mathrm{Fr}((\tilde{\pounds},\partial)\Cap(\tilde{\Im},\partial)) = \overline{(\tilde{\pounds},\partial)\Cap(\tilde{\Im},\partial)} \Cap \overline{((\tilde{\pounds},\partial)\Cap(\tilde{\Im},\partial))^{\mathcal{C}}}\\
&\subseteq (\overline{(\tilde{\pounds},\partial)} \Cap \overline{(\tilde{\Im},\partial)}) \Cap ((\tilde{\pounds},\partial)^{\mathcal{C}}) \Cup (\tilde{\Im},\partial)^{\mathcal{C}})\\
&= \{(\overline{(\tilde{\pounds},\partial)}) \Cup (\overline{(\tilde{\Im},\partial)}) \Cap \overline{(\tilde{\pounds},\partial)^{\mathcal{C}}}\} \Cup \{(\overline{(\tilde{\pounds},\partial)}) \Cup (\overline{(\tilde{\Im},\partial)}) \Cap \overline{(\tilde{\Im},\partial)^{\mathcal{C}}}\}\\
&= \{\mathrm{Fr}((\tilde{\pounds},\partial)) \Cap (\overline{(\tilde{\Im},\partial)})\} \Cup \{(\overline{(\tilde{\pounds},\partial)}) \Cap \mathrm{Fr}((\tilde{\Im},\partial))\}\\
&\subseteq \mathrm{Fr}((\tilde{\pounds},\partial)) \Cup \mathrm{Fr}((\tilde{\Im},\partial)).
\end{aligned}$$

5. Characterization of Distance Measures and Machine Learning Techniques in Terms of Single Valued Hepta partitioned Neutrosophic Soft Sets.

This section defines the term ”general distance” between two HPNSSs and discusses the various distance measurements, including weighted Euclidean and weighted Hamming distances. Heat map is actually data visualization technique. Heat map is supposed to be very informative structure and as we know that heat maps helps us in understanding the correlation in data (the graphical illustration of data where values are represented by colors). Heat maps are commonly used to visualize data in a two-dimensional space, with colors representing the intensity of the data. In the context of Sternberg's Eight Types of Love, heat maps can be used to display the intensity of each type of love for both young boys and girls. This can help identify patterns and trends in the data. 3D scatter plots are useful for visualizing the distribution of data points in a three-dimensional space. They can be particularly useful when there are a large number of data points, as they can help identify clusters and outliers. In the context of Sternberg's Eight Types of Love, a 3D scatter plot can be used to visualize the correlation matrix, with the z-coordinate representing the correlation values and the x and y coordinates obtained from the mesh grid of the correlation matrix shape. A method for determining the proper value of K (number of clusters) in K-means clustering is the elbow method. The Elbow method is a strategy for determining the appropriate value of K (number of clusters). It creates consistency in the cluster analysis design. Elbow method is used to determine the optimal number of clusters within the data set.

Heat map is developed for young boys and young girls separately. The normalized correlation matrix heat map (NCMHM) for young boys and girls is discussed The analyzation and visualization of the correlation matrix of a data set containing data for Sternberg's Eight Types of Love for young boys and girls is addressed. The 3D surface plot representing a correlation matrix, the 3-D scatter plot is used to visualize the correlation coefficients between different types of love for young boys and girls. The clustering data representing love types of young boys and girls to identify patterns and similarities within Sternberg's 8 Types of Love is studied. K mean-SVHNS- clustering is applied to group the data into K clusters iteratively. Visualizing and interpreting the clustered data representing love types among young boys and girls using t-SNE, a dimensionality reduction technique as well as t-SNE visualization of K-Mean is studied.

Elbow Method is conducted to determine the optimal number of clusters (K) for the K-mean clustering algorithm.

5.1. Characterization of Data Set Depiction

Three elements make up the triangle theory of love: decision/commitment, which is the decision to love someone and the ongoing commitment to sustaining that decision; intimacy, which is the feeling of being bounded, close, and connected in a romantic relationship; and passion, which is the drive toward romance, physical attraction, and sexual consummation. The total strength of these three elements determines how much love one experiences, and the relationship between these strengths determines the type of love one experiences. Interaction between the three parts and the behaviors they induce forms a multitude of unique types of loving experiences. These kinds of love are meant to be the single-valued hepta partitioned neutrosophic set's parameters. To study romantic relationships among young adults, researchers held meetings with individuals from different backgrounds. Participants in the sessions, which took place in community and educational settings, discussed their experiences and feelings for love. Each session was led by a licensed psychologist who created a relaxed atmosphere that encouraged candid conversation. Participants talked about their favorite love affair themes, relationships, and day-to-day experiences while data was being gathered. To foster a laid-back attitude, participants were shown simple gestures, such as sharing presents or messages or expressing their favorites. Next, participants were asked to use their imaginations to mimic romantic phone calls and have simulated talks. A diverse dataset was ensured by conducting ten sessions in both community and school settings. Seven more films of young adults having romantic love encounters were taken from internet platforms in order to increase diversity. The sentiment analysis system for romantic love affairs (RSAS) was built using values assigned to seven membership functions based on the recorded descriptions from these sessions and films.

Definition 31. *If there are two HPNSSs A and B on $X = x_1, x_2, ...x_n$ are symbolized by $A = \{\langle x_i, T_A(x_i), RT_A(x_i), C_A(x_i), U_A(x_i), G_A(x_i), RF_A(x_i), F_A(x_i)\rangle : x_i \in X\}$ and $B = \{\langle x_i, T_B(x_i), RT_B(x_i), C_B(x_i), U_B(x_i), G_B(x_i), RF_B(x_i), F_B(x_i)\rangle : x_i \in X\}$ such that $T_A(x_i)$, $RT_A(x_i), C_A(x_i), U_A(x_i), G_A(x_i), RF_A(x_i), F_A(x_i) \in [0, 1]$ and $T_B(x_i), RT_B(x_i), C_B(x_i), U_B(x_i), G_B(x_i)$, $RF_B(x_i), F_B(x_i) \in [0, 1]$ for every $x_i \in X$. Let $w_i(i = 1, 2, ...n)$ be the weight of elements $x_i(i = 1, 2, ...n)$, $w_i \geq 0(i = 1, 2, ...n)$ and $\sum_{i=1}^{n} w_i = 1$. Then, the generalized HPNSS weighted distance is demarcated as*

$$d_\lambda(A, B) = [\tfrac{1}{7} \sum_{i=1}^{n} w_i\{|T_A(\xi_i) - T_B(\xi_i)|^\lambda + |RTC_A(\xi_i) - RTC_B(\xi_i)|^\lambda + |C_A(\xi_i) - C_B(\xi_i)|^\lambda + |U_A(\xi_i) - U_B(\xi_i)|^\lambda + |G_A(\xi_i) - G_B(\xi_i)|^\lambda + |RF_A(\xi_i) - RF_B(\xi_i)|^\lambda + |F_A(\xi_i) - F_B(\xi_i)|^\lambda\}]^{\frac{1}{\lambda}}$$
with, $\lambda > 0$.

By substituting $\lambda = 1, 2$ in the preceding equation, one can derive the HPNSS weighted Hamming distance and the HPNSS weighted Euclidean distance, respectively. The following is the weighted Hamming distance for single valued heptapartitioned neutrosophic soft sets.

$$d_\lambda(A, B) = [\tfrac{1}{7} \sum_{i=1}^{n} w_i\{|T_A(\xi_i) - T_B(\xi_i)| + |RTC_A(\xi_i) - RTC_B(\xi_i)| + |C_A(\xi_i) - C_B(\xi_i)| + |U_A(\xi_i) - U_B(\xi_i)| + |G_A(\xi_i) - G_B(\xi_i)| + |RF_A(\xi_i) - RF_B(\xi_i)| + |F_A(\xi_i) - F_B(\xi_i)|\}]$$
where, $\lambda = 0$.

Next is heptapartitioned neutrosophic soft set weighted Euclidean distance:

$$d_\lambda(A, B) = [\tfrac{1}{7} \sum_{i=1}^{n} w_i\{|T_A(\xi_i) - T_B(\xi_i)|^2 + |RTC_A(\xi_i) - RTC_B(\xi_i)|^2 + |C_A(\xi_i) - C_B(\xi_i)|^2 + |U_A(\xi_i) - U_B(\xi_i)|^2 + |G_A(\xi_i) - G_B(\xi_i)|^2 + |RF_A(\xi_i) - RF_B(\xi_i)|^2 + |F_A(\xi_i) - F_B(\xi_i)|^2\}]^{\frac{1}{2}}$$
where, $\lambda = 2$.

Table 3 describes the factors used to analyze sentiments of love; the first three are taken from the body of literature, and the expert adds five more characteristics to the original three to better capture the subtleties of young adults' feelings of love.

Table 1: An explanation of Sternberg's love types' parameters

S.No	**Parameter Name**	**Description**
1.	Non Love:	Characterizes a relationship that lacks commitment, passion, and closeness.
2.	Linking(Intimacy only):	Symbolizes intimate connections that are marked by emotional closeness. This case does not have a trivial application. Genuine friendships, in Sternberg's opinion, are defined more by an intimate liking than by extreme enthusiasm or a sustained devotion. Instead, in these kinds of friendships, one feels warm, close, and restricted to the Infatuated Love (Passion only): other. Components: Nonverbal communication, expressive dancing, and shared dreams.
3.	Infatuated Love (Passion only):	Involves strong sexual allure and desire without any long-term or emotional commitment. Enamored love is often referred to as "love at first sight." However, captivated love may stop suddenly if the intimacy and commitment components of love are missing. Components: Playthings utilized, spending time together, and expressing emotions.
4.	Empty Love (Commitment only):	Represents dedication without intimacy or passion, which is common in some arranged marriages. Sometimes, deeper romantic relationships can become empty, with the commitment remaining but the intimacy and passion having faded. Empty love is often the beginning of a union in cultures where planned marriages are the norm. Components: Emotional Tone, Collaborative Creativity, and Feeling Expression.
5.	Romantic Love (Intimacy+ Passion):	Romantic lovers are bound together both physically (passionate arousal) and emotionally (likeness). Components: Equitable positions, a common passion, and emotional gaze.
		Continue to next page

Table 1 – continued from previous page

S.No	Parameter Name	Description
6.	Companionate Love (Intimacy+ Commitment):	It has a strong emotional bond and devotion but not a lot of passion. It is often seen in marriages where there is no longer any desire in the connection, but there is still a deep commitment and loyalty. Companimate love is usually a close personal relationship, without any sexual or physical desire, that you have with someone you live your life with. Components: Associative reasoning, grammatically accurate speech, and related conversions are the components.
7.	Fatuous Love(Passion+ Commitment):	Includes ardent dedication without the intensity of close emotional connection. One example of it would be a frenetic romance and marriage, in which the commitment is primarily motivated by passion without the stabilizing influence of intimacy. Components: sharing of gifts
8.	Consummate Love (Intimacy+ Passion+ Commitment):	Symbolizes the perfect kind of love, which combines the three elements in harmony. It represents the perfect partnership that most people aspire to but probably very few really get to experience, and it is the entire manifestation of love. Sternberg cautions that maintaining a perfect love can be more challenging than finding one. He stresses the importance of putting the components of love into reality. Ideal love might not endure indefinitely. For example, if passion fades over time, it may become companionate love. Components: Sharing gifts

These kinds of love are the limiting circumstances for the triangular theory. Most romantic relationships will fit into one of these categories since the various facets of love are shown continually rather than separately.

6. Characterization of Sternberg's 8- Types of Love in Terms of Example

This part is devoted to an example centered around a 19-year-old pair. The physicalists made the following observations and conducted an interview regarding their romantic relationship while working within the eight decision-making variables. The information is provided in tabular form. The data pertaining to love affairs is reflected in tables 2 and 3, respectively, for young boys and girls.
Below is the produced table 3 for the young boy.

Table 2: Sternberg's Eight Types of Love for Young Boy: An Overview

Love	Abs.T	Re.T	Unknown	Contra	Re.F	Ab.F	Ig
Non Love:	0.4	0.2	0.15	0	0.25	0	0
Linking (Intimacy only):	0.3	0.3	0	0	0.25	0.15	0
Infatuated Love (Passion only):	0	0	0	0	0.25	0.75	0
Empty Love (Commitment only):	0	0.75	0.25	0	0	0	0
Romantic Love (Intimacy + Passion):	0.2	0.2	0.2	0.3	0.1	0	0
Companionate Love (Intimacy + Commitment):	0.15	0	0.3	0	0.25	0	0.3
Fatuous Love (Passion + Commitment):	0.15	0.3	0	0	0.25	0	0.3
Consummate Love (Intimacy + Passion+ Commitment):	0.4	0.2	0.2	0.1	0.1	0	0

Table 3 provides a description of the parameters that were considered.

Table 3: Sternberg's Eight Types of Love for Young Girl

Love	Abs.T	Re.T	Unknown	Contra	Re.F	Ab.F	Ig
Non Love:	0.4	0.2	0.13	0	0.27	0	0
Linking (Intimacy only):	0.31	0.32	0	0	0.27	0.17	0
Infatuated Love (Passion only):	0	0	0	0	0.26	0.70	0
Empty Love (Commitment only):	0	0.70	0.20	0	0	0	0
Romantic Love (Intimacy + Passion):	0.21	0.26	0.29	0.34	0.1	0	0
Companionate Love (Intimacy + Commitment):	0.17	0	0.31	0	0.29	0	0.38
Fatuous Love (Passion + Commitment):	0.18	0.35	0	0	0.20	0	0.36
Consummate Love (Intimacy + Passion+ Commitment):	0.40	0.2	0.27	0.1	0.1	0	0

Table 3 provides a description of the parameters that were considered.

The information presented here is intended to be used in the evaluation of love feelings. Each love feeling is accompanied by a description and a dataset denoted as SVHNS. An overview of Sternberg's triangular theory of love and data integration is provided below.
Table 3 provides fascinating insights into the different types of love that can be discovered by applying Sternberg's theory of love to the available data sets.

Table 4: Example for Heptapartitioned Neutrosophic Soft Set

S.No	**Parameter Name**	**Description**	**SVHNS**
1.	Non Love	Components: Lack of commitment, passion, and closeness	[0.4,0.2,0.15, 0,0.25,0,0]
2.	Liking (Intimacy only):	Components: nopassion, no commitment, intimacy only	[0.3,0.3,0,0,0.25,0.15,0]
3.	Infatuation (Passion only)	Components: intense passion, lack of commitment or intimacy.	[0,0,0,0,0.25,0.75,0]
4.	Empty Love (Commitment only)	Components: no intimacy, no desire, high commitment.	[0,0.75,0.25,0.0,0.0,0.0,0.0]
5.	Romantic Love (Intimacy + Passion)	Components: no commitment, great intimacy and desire.	[0.2,0.20,0.20,0.3,0.1,0,0]

continued on next page

Table4 – continued from previous page

S.No	Parameter Name	Description	SVHNS
6.	Companionate Love (Intimacy + Commitment)	Components: no desire, high levels of intimacy and commitment.	[0.15,0.0,0.3,0,0.25,0,0.3]
7.	Fatuous Love (Passion + Commitment)	Components: No intimacy, high levels of dedication and passion.	[0.15,0.3,0.0,0.0,0.25,0,3]
8.	Consummate Love (Intimacy + Passion + Commitment)	Components: High levels of commitment, passion, and intimacy.	[0.4,0.20,0.20,0.1,0.1,0,0]

Sternberg's triangle theory of love is validated by the various ways in which intimacy, passion, and commitment can be combined, as these forms of love demonstrate. The supplied data sets demonstrate how complex and multifaceted love is, which is consistent with the basic ideas of the theory.

7. Results and Discussions

Many Python libraries were used for data visualization and graphical representation, including pylab, sklearn, matplotlib, pandas, numpy, and seaborn. The previously mentioned methods were visualized using Python programming, and K-means clustering was carried out depending on the elbow curve result. These illustrations have produced logical conclusions and discussed the eight main factors that influence young boys' and girls' romantic relationships. The color scale of a heat map, which makes the connection and associativity features very evident.
Two variables are related when they exhibit a correlation with one another. The association that arises when one attribute increases along with another is known as correlation. By taking into account absolute truth, relative truth, absolute false, relative absolute false, contradiction, unknown (undefined), and ignorance, the technique seeks to understand eight different mindsets. Interesting patterns in love feelings were found in the dataset after applying K-means clustering. Different groups surfaced, each with members who shared a similar inclination toward romance. Deeper understanding of the emotional landscape was made possible by the correlation analysis, which also highlighted the connections between concepts like "Romantic love (intimacy + passion) and Companionate love (intimacy + commitment)." These results highlight the intricacy and connectivity that are intrinsic to love relationships. When used strategically, the Elbow Method identified the best clusters and made a substantial contribution to our comprehension of the heterogeneous nature of romantic relationships. The findings validate the efficacy of sophisticated machine learning methodologies in deciphering the intricacies of human emotions concerning love, particularly within the framework of enchanting romantic encounters. A holistic perspective is offered by the multidimensional analysis, which highlights the requirement of taking a sophisticated approach to comprehending and analyzing the complex emotional states involved in love relationships.

Figure 1 shows a heat map created specifically for young males. To make it simpler to read, the x-axis is turned by 45 degrees and the forms of love are labeled on the ticks. In reality, a heat map is a data visualization method. As can be seen from the graphic, hierarchically clustering heat map of correlation matrix (HCHMCM) allows one to examine correlations within clustered data and also offers insights into the interactions between various components.

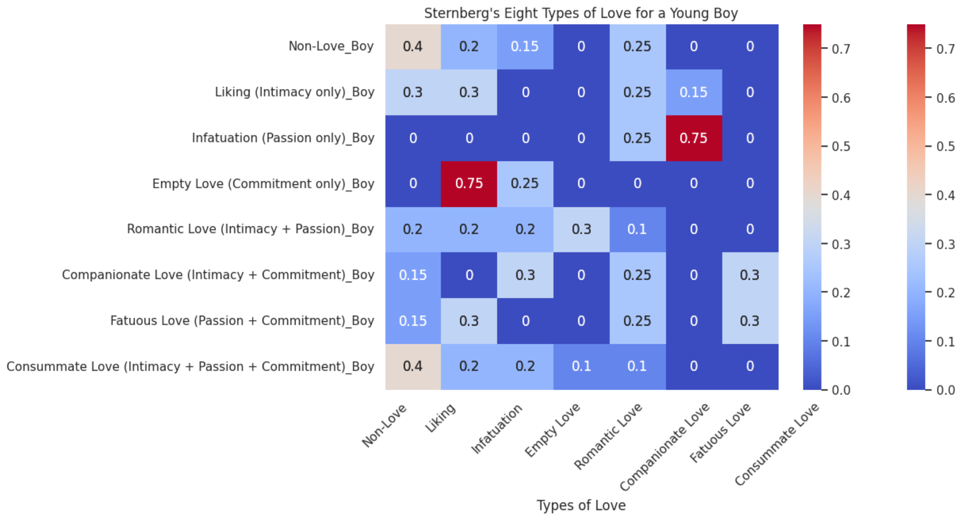

Figure 1: Heat Map for Young Males

In Figure 2, Heat map is developed for young girls. The x-axis ticks are labelled with the types of love, and the x-axis is rotated by 45 degree to make it easier to read.

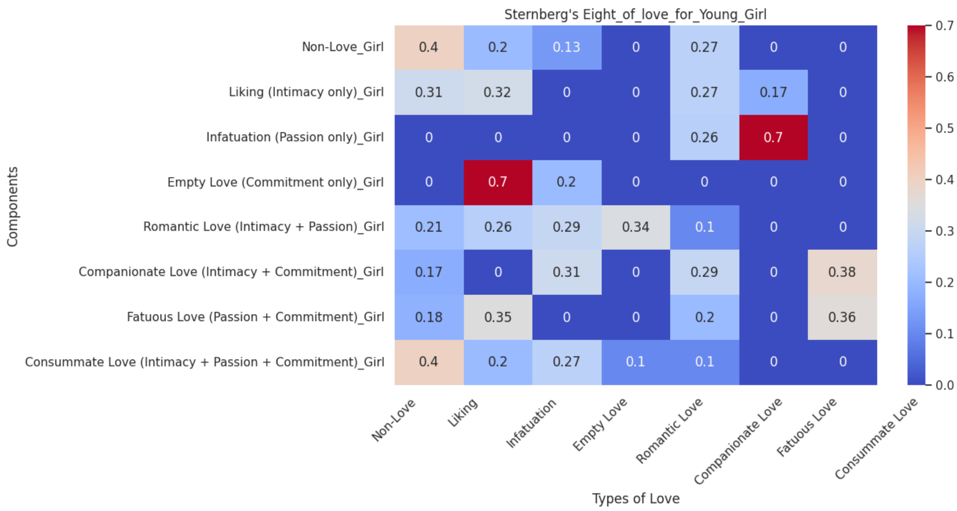

Figure 2: Heat Map for Young Girls

Figure 3 presents a discussion of the normalized correlation matrix heat map (NCMHM) for young boys and girls. This illustrates the symmetry of the data around the off diagonal, which is shown as being dark brown in color. It also shows that the off diagonal is filled with a single element. In reality, a heat map is a data visualization method. As can be seen from the graphic, NCMHM is able to depict correlations within clustered data and also offers insights into the interactions between various components.

The code also calculates the correlation matrix visualizes it using Heat maps. The correlation matrix exhibits the correlation coefficients between each pair of variables, which can guide us to recognize the relationships between the variables. The correlation matrix is then normalized using the Min Max Scalar from the sklearn library to guarantee that the values are ranges between 0 and 1. Moreover we see that light brown shows super strong $+ive$ correlation and which happened to be 0.86 and this is the relative false $MF - RF_{A(x)}$ and the second strong $+ive$ correlation and which happened to be 0.84 and this truth $MF - T_{A(x)}$ etc. The less $+ive$ correlation is 0.2 and this is unknown $MF - U_{A(x)}$ etc. On the diagonal running from bottom right to left, one can observe the correlation between the variables.

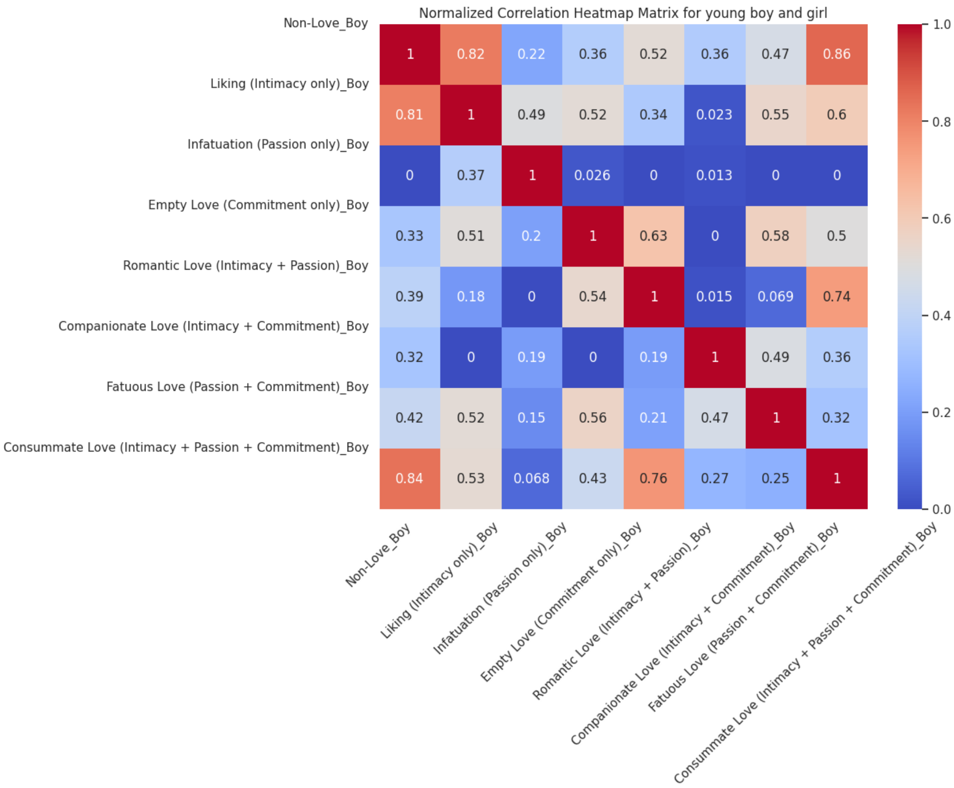

Figure 3: Normalized Correlation Matrix

In Figure 4, the code provided is used to analyze and visualize the correlation matrix of a data set containing data for Sternberg's Eight Types of Love for young boys and girls. The correlation matrix is calculated using the Pearson correlation coefficient method, which measures the linear correlation between two variables. The code provided first imports the necessary libraries, including pandas, sea born, and Min Max Scaler from sklearn. The data for the analysis is defined as two dictionaries, one for boys and one for girls, with each dictionary containing the values for each type of love. Next, the data is combined into a single array and the labels are combined into a separate array. The data is then converted to Data Frames for further processing. The correlation matrix is calculated using the correlation method from the pandas library.

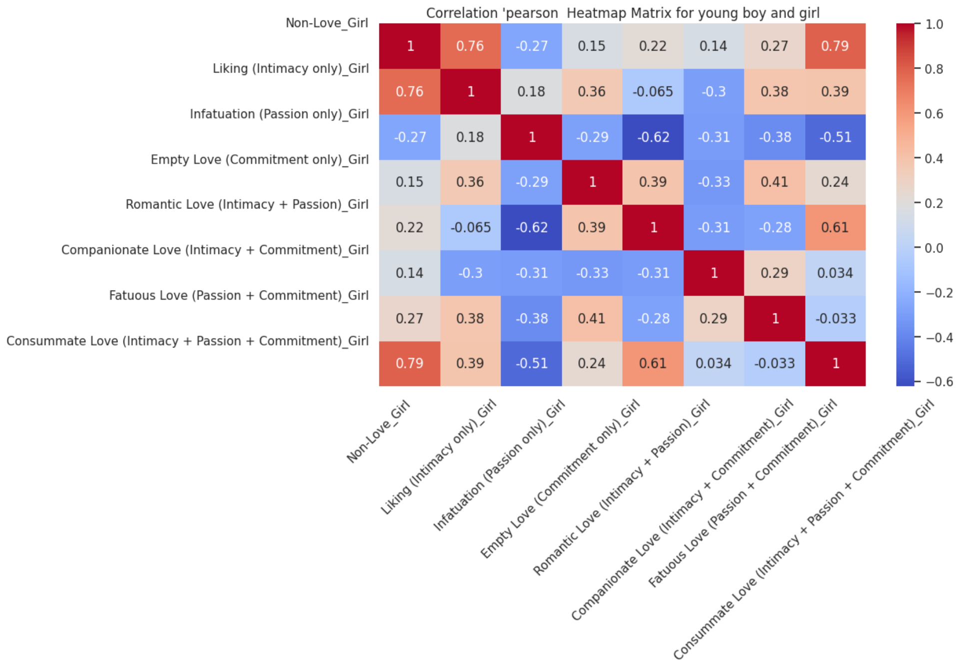

Figure 4: Correlation Pearson Heat Map Matrix

Figure 5, illustrates a 3D surface plot representing a correlation matrix. In this visualization, the height of the surface indicates the correlation coefficient between two variables. The x and y correspond to the variables being analyzed. That is x label correspond to the girls feeling and y label correspond to the boys feeling. The $z - axis$ displays the correlation coefficient values. The plot is created using Matplotlib's plot-surface function from the mpl-toolkits. Mplot3d module. The color scheme chosen for this figure is "coolwarm", where blue represents low correlation and red signifies high correlation. The x and y axis are labeled as X Label and the Y Label respectively, and the figure is titled "3D Surface Plot of Correlation Matrix" This graphical representation aids in identifying the strength of relationship between variables, highlighting both strong and weak correlation within the data set.

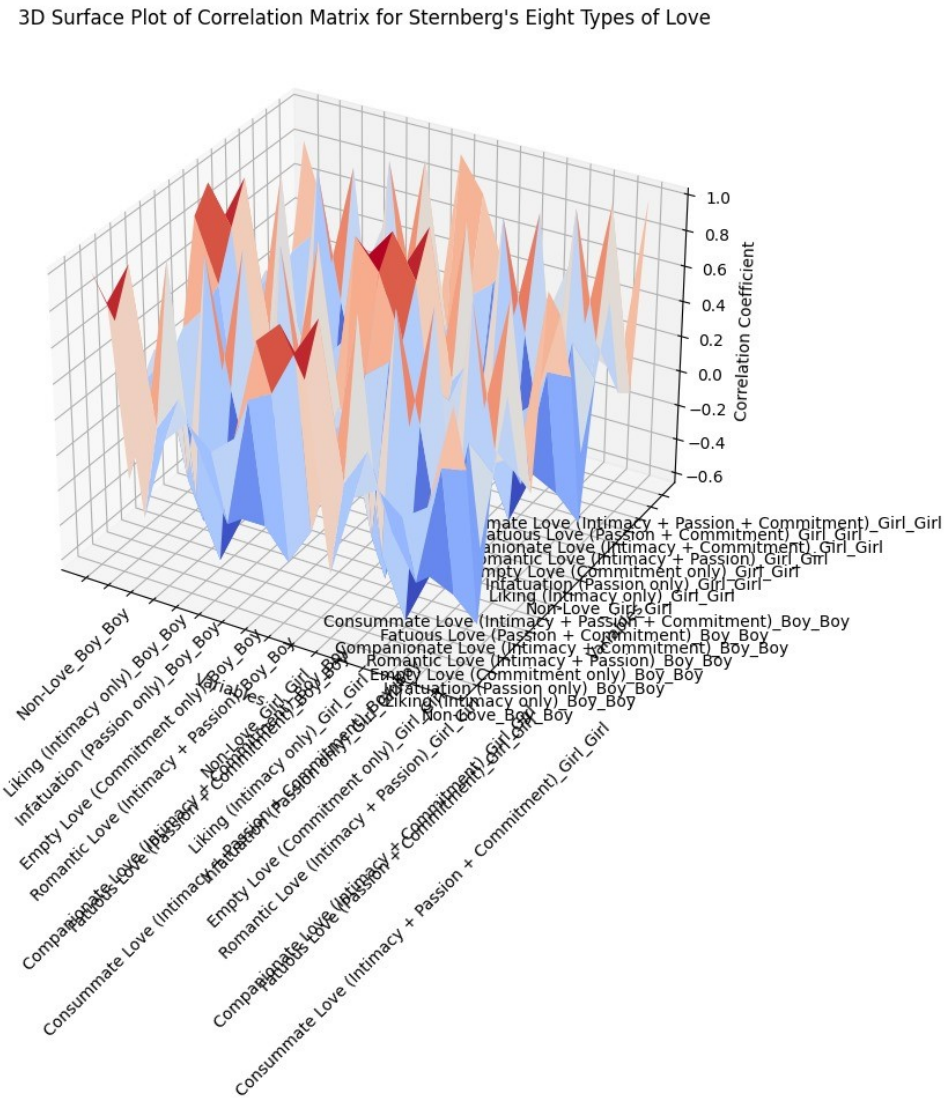

Figure 5: 3D Surface Plot of Correlation Matrix

In Figure 6, the 3-D scatter plot in the code is used to visualize the correlation coefficients between different types of love for young boys and girls. The x, y and z coordinates of the scatter plot correspond to the indices of the correlation, the labels of the of the correlation matrix, and the correlation coefficients, respectively. The color for each point in the scatter plot is determined by the correlation coefficient, with a cool warm color map to represent the values. The color bar on the right side of the plot displays the correlation coefficient values corresponding to each color. This visualization helps to understand the relationships between different types of love and their correlation coefficients in 3D-space.

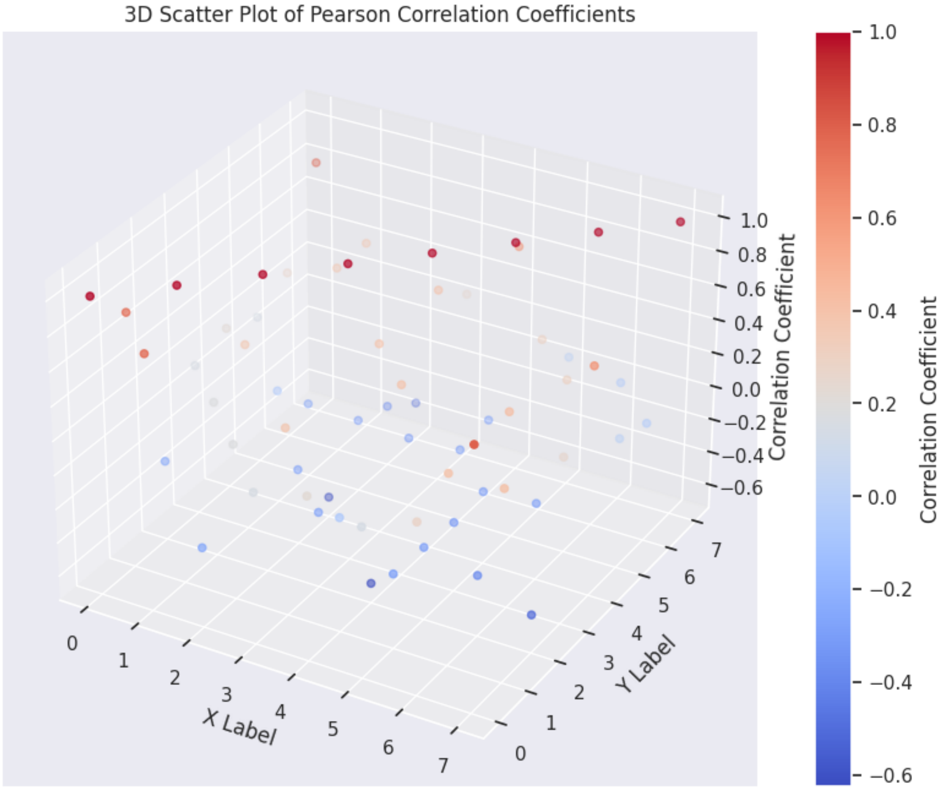

Figure 6: 3D Scatter Plot of Pearson Correlation Coefficients

In Figure 7, the code generates a 3D scatter plot that is useful for visualizing the distribution of different types of love based on their scores in intimacy, passion and commitment. This type of visualization is important for research as it can help researchers identify patterns, trends and relationships between variables. By using 3D scatter plot, researchers can better understand the complex relationships between different types of love and their underlying dimensions. Additionally, the use of color legend can help researchers quickly identify and differentiate between the different types of love, making it easier to interpret the visualization. Overall, the use of 3D scatter plots and color legends can enhance the interpretation and understanding of research findings, making them an important tool for researchers in various fields.

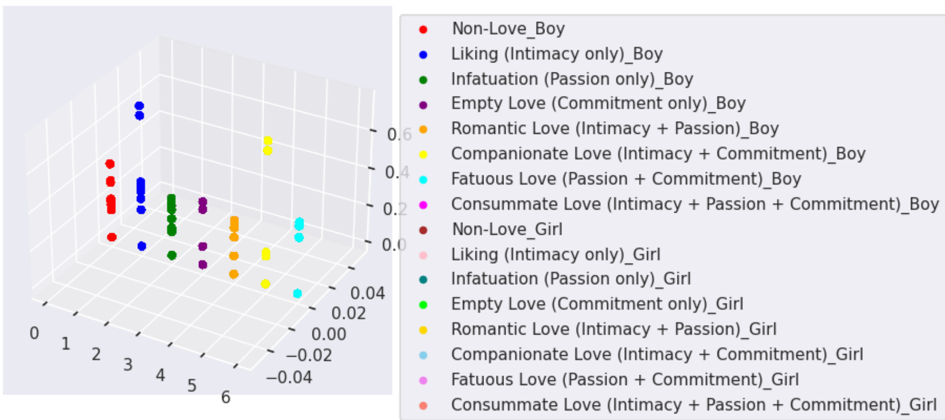

Figure 7: 3D Scatter Plot for Distribution of Love

In Figure 8, the code aims to cluster data representing love types of young boys and girls to identify patterns and similarities within Sternberg's 8 Types of Love. The code aims to cluster

data representing love types of young boys and girls to identify patterns and similarities within Sternberg's 8 Types of Love. K mean-SVHNS- clustering is applied to group the data into K clusters iteratively. The clusters and centroids are normalized using Min Max Scalar for better comparison and analysis. The normalized clusters are visualized in a scatter plot with different colours representing each cluster. Centroids for each cluster are marked with $'x'$ symbols in the plot for easy identification. The method offers a practical and visual way to analyze complex emotional data, potentially contributing to the understanding of love types among young individuals in psychological studies.
K-Mean clustering on feature T for the parameters "Companionate love (intimacy+ commitment)" on the $y-axis$ against "romantic love (intimacy+ commitment)" on the $x-axis$ indicates that there is a higher concentration of points at $x = 0.00$ and $y = 1,0$. The first centroid is c_1 at $x = 0.00$ and $y = 1.0$, and the second centroid is c_2 at $x = 0.08$ and $y = 0.0$. Near $x = 1.0$ and $y = 0.9$, higher point concentrations were also found.

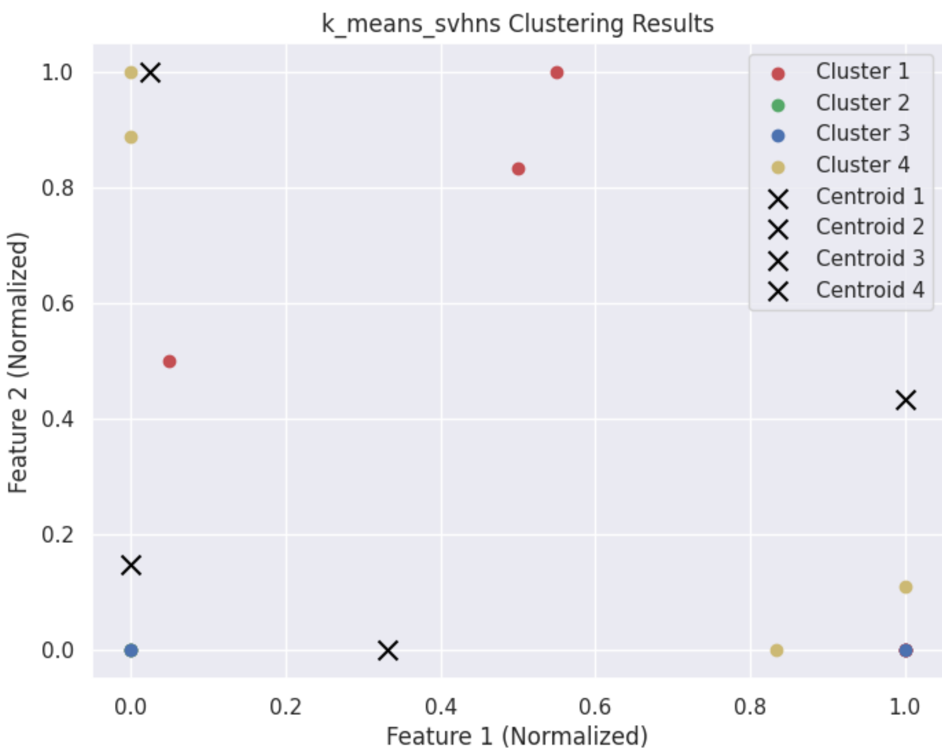

Figure 8: K-Means SVHNS Clustering Results

Figure 9, visualize and interpret the clustered data representing love types among young boys and girls using t-SNE, a dimensionality reduction technique. Initially, data on love types is clustered using the k-mean algorithm to group similar data points for both boys and girls. The clustered data and centroids are normalized for better visualization and comparison in the t-SNE analysis. The t-SNE algorithm is applied to reduce the dimensionality of the data and the clusters in a two-dimensional space. The normalized cluster data is transformed and visualize using t-SNE to represent the clusters in a lower-dimensional space. Each cluster is visually represented by distinct colours in the t-SNE plot. Centroids are marked with x symbols in the t-SNE plot. The t-SNE visualization provides a clearer representation of the clustered data, enabling a better understanding of the relationships and patterns among different types of love among young individuals. The t-SNE visualization enhances the clarity of the clustered data representation, serving as a valuable aid and readers when interpreting the relationships and similarities in Sternberg's 8 Types of Love among young individuals.
Clustering on feature T for the parameters "Companionate love (intimacy+ commitment)" on

the $y-axis$ against "romantic love (intimacy+ commitment)" on the $x-axis$ indicates that a higher concentration of points is located between $x = 0.2$ and $y = 0.38$. The original centroid, c_1 is located at $x = 0.00$ and $y = 0.0$.

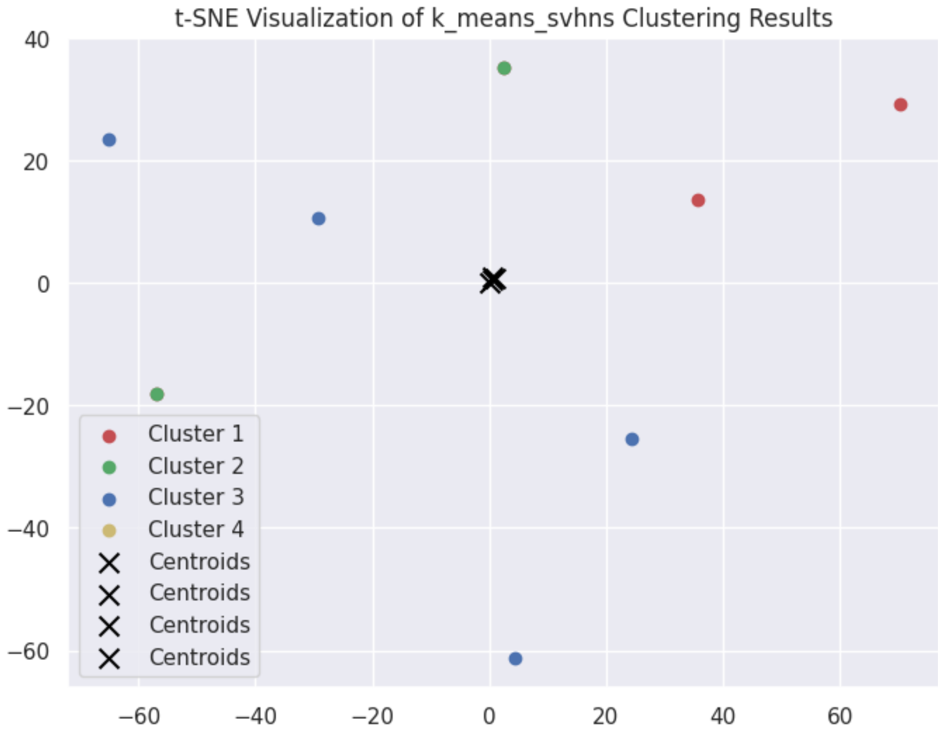

Figure 9: t-SNE Visualization of K-Means SVHNS Clustering Results

Figure 10, in the provided code snippet, an analysis using the Elbow Method is conducted to determine the optimal number of clusters (K) for the K-mean clustering algorithm. The code aims to use the Elbow Method to identify the most suitable number for grouping and analyzing data related to Sternberg's 8 Types of love among young individuals. The Elbow Method is applied to determine the optimal number of clusters by evaluating the inertia (within-cluster sum of squares) for different values of K. An Elbow curve is plotted to demonstrate the relationship between the number of clusters and the inertia values. The K- means algorithm is utilized to cluster the normalized data into varying numbers of groups.

By analyzing the Elbow curve, the point where the inertia value starts decreasing at a slower rate (forming an "elbow") indicates the clusters. The plot showcase the relationship between the number of clusters and inertia, aiding in the selection of an appropriate K value based on the curve's characteristics. The Elbow Technique delivers a systematic tactic to determine the optimal number of clusters for the K-mean algorithm, enhancing the efficiency and effectiveness of clustering analysis on emotional data types among young individuals. The technique offers a structured and data-driven technique to optimize the clustering process, enabling us to make informed decisions regarding the number of clusters for analyzing emotional relationship within Sternberg's 8 Types of Love for young girls. The Elbow curve visualization serves as a valuable tool for us, facilitating a clear understanding of the clustering process and aiding in the selection of an of clusters for subsequent analysis.

Figure 10 reflects Elbow method (this method is used to decide how many clusters it should consider and this method is actually the graph between K and distortion WCSS). Here the K-values are taken along $x-axis$ and the distortion (WCSS) is taken along $y-axis$. We see that there are number of K-values but we encounter the business value of K which is at 4 that is at $k = 4$ the optimal value is 4 because here we see that the drastic change in $y-axis$ value occurred at $k = 4$ the value of $y = 5.4$ as shown in the figure.

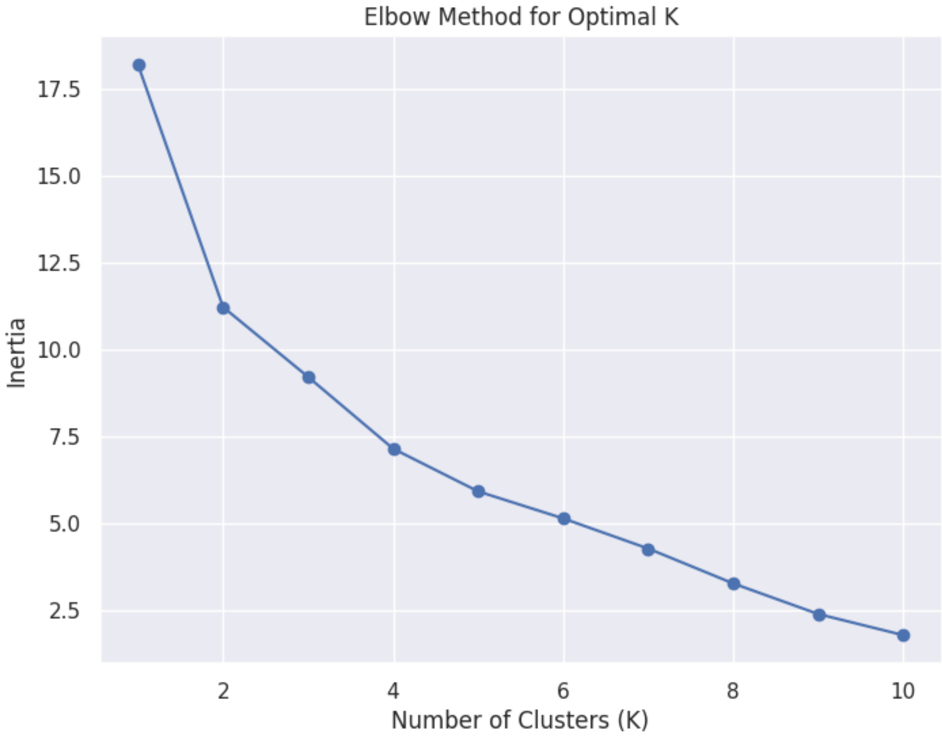

Figure 10: Elbow Method for Optimal K

The comparative analysis is in Figure 11.

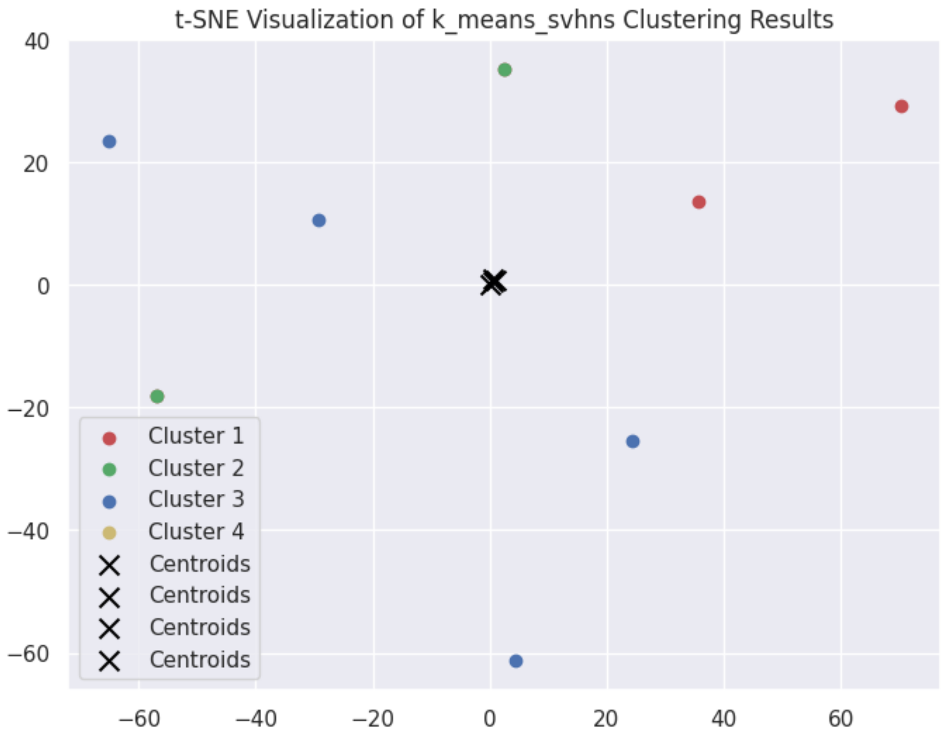

Figure 11: comparative

8. Advantages

Data Visualization: The use of heat maps (Figures 1, 2 and 3) efficiently depicts complex associations, making it easier to discern relationships among different types of love. The readability is improved by the changes, such as turning the x-axis.
Correlation Analysis: Understanding the interactions between various types of love is made easier by the clear picture of relationships provided by the normalized correlation matrix (Figure 3).
3D Visualizations: To make correlations easier to understand in a spatial context, Figures 5, 6 and 7 display correlation coefficients using 3D surface and scatter plots.
Clustering Insights: The K-Mean clustering (Figure 8) offers an organized method for finding patterns in the data, which makes it easier to see how different love kinds are similar.
Dimensionality Reduction: Figure 9's t-SNE display efficiently lowers complexity to make clusters and interactions in lower dimensions more understandable.
Optimal Clustering Determination: The robustness of the clustering study is increased by the Elbow Method (Figure 10), which methodically determines the ideal number of clusters.

9. Limitations

Interpretation Complexity: Heat maps and three-dimensional plots offer detailed data visualizations, but their intricacy could potentially be too much for some readers to handle, necessitating thorough explanation.
Data Dependency: The quality and completeness of the input data have a major impact on how accurately the correlation coefficients and clustering findings are calculated. Conclusions that are misrepresented could be caused by bias or inadequacies in the dataset.
Assumption of Linearity: Assuming linear correlations between variables, the Pearson correlation approach may be unable to capture more intricate interactions.
Subjectivity in Clustering: The number of clusters in a K-Mean cluster must be predetermined, which can be arbitrary and may not accurately represent the actual data structure if proper reasoning isn't provided.
Limited Contextualization: Although the study offers insightful statistical information, additional qualitative research may be necessary to interpret the quantitative results due to the psychological consequences of the findings.
Potential Overfitting: In case the sample size is insufficient, there is a possibility of overfitting to the dataset due to the clustering and dimensionality reduction techniques.
In conclusion, your method of analyzing emotional data connected to love types successfully makes use of contemporary data visualization and clustering approaches. By addressing the noted shortcomings, future studies will be strengthened and a more thorough knowledge of the linkages involved will be provided.

10. Conclusion and Future Work

The reason why ordinary, fuzzy, and ambiguous set theories are considered outdated is that they have numerous flaws when it comes to solving specific situations. The new theory known as the neutrosophic hypothesis greatly enhanced these theories. Fuzzy set theory doesn't know anything about non-membership; it solely deals with membership. Only the vague set theory can be addressed by either membership or non-membership. A vague set theory method is inapplicable not the case of indeterminacy. The sole method available to study this weakness is the neutrosophic set theoretical approach. A new theory called soft set theory was presented in order to lower the error and produce better outcomes. Later, as time went on, soft set theory and fuzzy, hazy, and neutrosophic set theories were combined to create new ideas. As time went on, new structures based on these theories were defined. The structure with more mathematical relevance among these is neutrosophic soft topology. Fuzzy soft topology is an extension of vague soft topology, and both are an extension of neutrosophic soft topology. A branch of Neutrosophy called Neutrosophic Set examines the nature, origin, and scope of neutralities as well as how they interact with other ideational spectra. Recently, NSS a potent general formal framework was been forth. The three membership functions that make up NSS false membership, inderemanancy membership, and genuine membership need to be improved. By dividing the indeterminate concept or value into two groups depending on membership one for indeterminacy leaning towards truth membership and the other for indeterminacy leaning towards false membership we presented a strategy to provide indeterminacy more sensitivity and accuracy. This implies definitely that the indeterminacy situation is divided into two components in order to purify it. A refined neutrosophic set is a modified form of a neutrosophic soft set. This article illustrates the triangle theory of love using the single valued heptapartitioned neutrosophic set (SVHNS), a generalized form of the neutrosophic set. Seven membership functions in this collection are more attuned to issues that come up in day-to-day living. The following are the definitions of membership functions: ignorance, relative false, absolute false, contradiction, absolute true, and relative true. A higher level of accuracy is possible given this level of uncertainty. The refined neutrosophic soft set, an extended version of the Pentapartitioned neutrosophic soft set (PNSS), is defined and demonstrated for the first time in this study. This improved neutrosophic soft set is called "Heptapartitioned neutrosophic soft set (HPNSS)". Anything that can be divided into seven distinct qualities is called "Heptapartitioned." Relative true, contradiction, ignorance, unknown (undefined), and relative false are the five components of indeterminacy, in that order. Being able to more sensitively and accurately depict the hazy, imprecise, inconsistent, and incomplete information that exists in the real world is another advantage of HPNSS, an inclusive example of PPNSS. With this degree of uncertainty, a higher degree of accuracy is feasible. In order to handle indeterminacy more precisely, indeterminacy (I) is divided into five categories based on membership. These two types of indeterminacy one as indeterminacy leaning towards truth membership, which is assumed to be relative true, and the other as indeterminacy leaning towards false membership, which is assumed to be relative false lead only to confusion, indeterminacy that is unknown, and indeterminacy that equally leans towards truth membership and false membership, which is known as ignorance because one can choose the direction.

Relative truth (ReT) refers to an indeterminacy that can be categorized as such when it can be identified as such and has a higher truth value than a false value. Relative falseness (ReF) is the term used to describe indeterminacy that can be determined to be more false than truth, but cannot be categorised as false. Confusion will arise in these situations, as well as in the other two, when the indeterminacy cannot be distinguished as being more true than false or as being more truthful than false. A set theoretic research is completed and some operations on them are defined. We went on to do a thorough investigation on HPNSS and discovered a number of odd characteristics that they caused. Along with a counter example, some equivalent characterizations and their relationships are discussed. We therefore built heptapartitioned neutrosophic soft topological spaces (HPNSTS) towards these newly defined procedures. Basic HPNSTS definitions and theorems were presented.

There are three new definitions provided: HPNS $\beta - open$ sets in HPNSTS, HPNS pre-open (P-open), and heptapartitioned neutrosophic soft (HPNS) semi-open (S-open). Among these, the P-open set, a fascinating HPNS generalized open set, is selected, and several essentials are then produced based on its description. These include the HPNS closer, HPNS exterior, HPNS border, and HPNS interior. A unique heat map is created for young females and boys. There is discussion of the normalized correlation matrix heat map (NCMHM) for young boys and girls. This paper examines the correlation matrix analysis and visualization of a dataset that includes information for Sternberg's Eight Types of Love for young boys and girls. The 3-D scatter plot, which is a 3D surface plot depicting a correlation matrix, is used to show the correlation coefficients between various forms of love for young boys and girls. In order to find patterns and commonalities across Sternberg's Eight Types of Love, clustering data representing the love types of young boys and girls is examined. The data is repeatedly grouped into K clusters using K mean-SVHNS-clustering. It is examined how to use t-SNE, a dimensionality reduction approach, to visualize and comprehend the clustered data indicating love types among young boys and girls. Additionally, t-SNE visualization of K-Mean is explored. The best number of clusters (K) for the K-mean clustering algorithm is found using the elbow method. We will generalize this idea and use the same methods in relation to the spaces' soft points in the future. In their discussion of the Van der Waals equation with beta-time fractional derivatives, Qawaqneh et al. [34] derived dark, bright, and singular soliton solutions by applying the Kudryashov and sine-Gordon expansion methods. Their findings, which are backed by graphical proof, offer fresh perspectives on nonlinear fractional models. Similarly, a fundamental framework for dealing with imprecision and uncertainty in mathematical modeling was presented by Batiha et al. [35]. Judeh and Abu Hammad [36] built on these theoretical advancements by proposing a soft computing-optimized fractional-order PID (FOPID) controller for robotic arm systems. This controller outperformed traditional PID controllers, especially when it came to handling external disturbances and system nonlinearities. Based on these ideas, a more fully developed theory of partitioned neutrosophic soft (HPNS) spaces might in the future incorporate higher-level schemes of partitioning and the extension of classical soft topological properties to suit the processing of more complex decisions and more vague information.

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