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Heptapartitioned Neutrosophic Soft Topologies and Machine Learning Techniques for Exploring Romantic Feelings

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Abstract. In this paper, we introduced the concept of the heptapartitioned neutrosophic soft set (HP-NSS), a novel extension and generalization of the neutrosophic soft set theory. To improve accuracy, the indeterminacy is divided into five additional possibilities, which are as follows: absolute true, relative true, contradiction, unknown, and ignorance, relative false and absolute false. We extend the concept of neutrosophic soft topological spaces by introducing heptapartitioned neutrosophic soft topological spaces (HPNSTS), a novel generalization that incorporates seven distinct partitions to model uncertainty, vagueness, and indeterminacy in topological structures. Three new definitions are introduced and these definitions are p-open, pre-open and semi-open sets. Special attention is focused on p-open sets, and a number of results related to p-open sets are addressed. Machine learning and graphical algorithms, such as K-Means clustering, Heat maps, Elbow method, Feature correlation, 2D-normalized t-SNE, and parallel coordinates of 3D T-SNE, were used and visualized for a real-world application involving the romantic feelings of young boys and girls across various dimensions.

2020 Mathematics Subject Classifications: 03E72, 54A40, 62H30
Key Words and Phrases: NSS, Heptapartitioned neutrosophic SS, Distance Measures, K-Means algorithm, Machine Learning Techniques, SVHNS Applications

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1. Introduction

Fuzzy set theory, introduced by Zadeh [1], was a revolutionary approach to handling uncertainty and imprecision in systems and models where traditional (classical) set theory was not adequate. The theory emerged as a response to the limitations of classical set theory, which assumes a binary membership: an element either belongs to a set or does not. Zadeh's fuzzy set theory, on the other hand, allows for partial membership where an element can belong to a set to some degree between 0 and 1. The concept of interval-valued fuzzy sets (IVFS) was proposed by Turksen [2] to address the representation of combined concepts where linguistic connectives (such as conjunction, disjunction, and implication) and variables are assumed to be fuzzy. In this framework, instead of assigning a single degree of membership to an element, each element in the set is represented by an interval of membership values, which reflects the inherent vagueness and uncertainty in the interpretation of these concepts. Atanassov [3] proposed the intuitionistic fuzzy set (IFS), which extends the classical fuzzy set theory by introducing an additional component. Unlike traditional fuzzy sets, which only use a membership degree to represent the degree of truth, intuitionistic fuzzy sets also incorporate a degree of falsity (non-membership). The contrast intensification operator for intuitionistic fuzzy sets of root type, used to enhance the clarity and quality of gray and color images using Java, was studied in [4]. It compares various types of fuzzy sets, including classical fuzzy sets, intuitionistic fuzzy sets, second-type intuitionistic fuzzy sets, and root-type intuitionistic fuzzy sets, showing how the root-type version improves image enhancement. We propose a new solution for a multi-criteria decision-making problem using similarity measures for intuitionistic fuzzy sets was proposed in [5]. Our approach is shown to be more effective than the method in [6], which fails in certain situations. The neutrosophic set (NS), introduced by Smarandache [7], is an advanced concept designed to model situations involving uncertainty, imprecision, and indeterminacy. It is a more generalized form of fuzzy sets (FS) and intuitionistic fuzzy sets (IFS), and it includes three key components to describe the membership of elements. In the context of Neutrosophic Sets (NS), for applications and practical use cases, readers can refer to studies and research found in references [8-10], which explore how NS can be effectively applied in various fields. Complex fuzzy sets and complex intuitionistic fuzzy sets face challenges in handling imprecise, indeterminate, inconsistent, and incomplete information, especially in periodic setups. To address this, the complex neutrosophic set (CNS)) is introduced. The CNS is an extension of the neutrosophic set, where the membership functions (truth, indeterminacy, and falsity) are complex-valued. Each function combines a real-valued amplitude term (truth, indeterminacy, falsity) with a corresponding phase term, allowing for more flexible representation of uncertainty. The set-theoretic operations on CNS are introduced in [11] including complement, union, intersection, complex neutrosophic product, Cartesian product, distance measures, and d-equalities. It also presents a potential application for the CNS, highlighting the shortcomings of existing methods and demonstrating the superior performance of CNS through graphical comparisons. The CNS is shown to offer a more healthy solution for modeling complex, uncertain, and periodic data than traditional fuzzy and intuitionistic fuzzy approaches. On the other hand, Molodtsov introduced the concept of soft sets [12], which is a new mathematical framework designed to handle uncertainty and imprecision. Soft sets provide a more flexible approach to uncertainty by associating each element of the universe with a set of parameters rather than a single membership value, as seen in classical fuzzy or intuitionistic fuzzy sets. This allows soft sets to model uncertain information more effectively and is particularly useful in scenarios where traditional set theories struggle. Maji et al.[13] expanded on Molodtsov's work by applying soft sets to various fields, including decision-making, image processing, and optimization. They also introduced the concept of fuzzy soft sets [14], which combine the flexibility of soft sets with the concept of fuzziness, allowing for the modeling of not only uncertainty but also partial membership in the set. This extension enabled a richer representation of real-world problems involving both vagueness and uncertainty. The development of hybrids of soft sets [15-19] further expanded

the theory, combining soft sets with other types of fuzzy sets, intuitionistic fuzzy sets, and even neutrosophic sets to tackle increasingly complex problems. These hybrid models offer more robust ways of handling different degrees and types of uncertainty, making soft set theory highly versatile and applicable across a wide range of disciplines.
In 2011, Alkhazaleh and Salleh defined the concept of a soft expert set, in which the opinions of all experts could be represented in a single model, and they provided an application of this concept in decision-making problems [20]. Arokia et al. [21] studied fuzzy parameterizations for decision-making in risk management systems using the soft expert set. Arokia and Arockiarani [22] provided a fusion of the soft expert set and matrix models. Alkhazaleh and Salleh [23] extended the concept of the soft expert set in terms of fuzzy sets and demonstrated its application. Bashir and Salleh [24] introduced the concept of a fuzzy parameterized soft expert set. Bashir et al. [25] discussed the possibility of fuzzy soft expert sets. Alhazaymeh et al. [26] explored the application of generalized vague soft expert sets in decision-making. Broumi and Smarandache [27] extended the soft expert set concept in terms of intuitionistic fuzzy sets. Abu Qamar and Hassan [28,29] presented the idea of Q-neutrosophic soft relations and introduced entropy measures of distance and similarity. Sahin et al. [30] proposed the idea of neutrosophic soft expert sets, while Ulucay et al. [31] introduced the concept of the generalized neutrosophic soft expert set for multiple-criteria decision-making. Al-Quran and Hassan[32] introduced the neutrosophic vague soft expert set theory and further developed it into the complex neutrosophic soft expert set [33,34]. Qayyum et al. [35] proposed the concept of cubic soft expert sets for a more general approach. Finally, Ziemba and Becker [36] presented an analysis of the digital divide using fuzzy forecasting, offering a new approach in decision-making.

1.1. Literature review

The concept of neutrosophic sets, particularly their extensions and applications, has gained significant attention in recent years due to their ability to model uncertainty, vagueness, and indeterminacy in various real-world problems. Several key advancements in this area are discussed in the following literature. Chatterjee et al. [37] introduced the notion of quadripartitioned single valued neutrosophic sets (QSVNS) and explored various set-theoretic operations associated with them. They proposed definitions for distance measures, similarity measures, and entropy, which were subsequently applied to a pattern recognition problem, demonstrating the utility of QSVNS in real-world applications. Mallick and Pramanik [38] further extended the concept by introducing the pentapartitioned neutrosophic set, a new variant that refines the basic structure of neutrosophic sets. In their study, they discussed several fundamental properties of the pentapartitioned neutrosophic set, laying the groundwork for subsequent research in this direction. In a similar vein, a heptapartitioned neutrosophic set was introduced by the authors in [39], providing an even more refined approach to dealing with indeterminate information. The study explored various properties of this set and proposed theoretical advancements that extended the versatility of neutrosophic logic. An important extension of neutrosophic sets was the quadripartitioned single-valued bipolar neutrosophic set (QSVBNS), introduced in [40]. This concept is a generalization that combines both quadripartitioned neutrosophic sets and bipolar neutrosophic sets, with inherent symmetry in its structure. Several operations and set-theoretic studies were conducted on QSVBNS, and new similarity and distance measures were defined. Furthermore, the paper demonstrated the practical application of QSVBNS in a multi-criteria decision-making problems, showing that QSVBNS outperforms fuzzy sets and bipolar fuzzy sets in terms of flexibility and effectiveness. The single-valued refined neutrosophic set (SVRNS), as introduced by [41], represents an important extension of the traditional neutrosophic set, aimed at enhancing its ability to model more complex, real-world uncertainty. SVRNS incorporates six membership functions, extending the classical neutrosophic set by considering not only truth, indeterminacy, and falsity but also imaginary aspects. These functions include complex values, truth tending towards complex and falsity tending towards complex. This expanded framework allows SVRNS to handle cases where conventional models fail to adequately represent ambiguity or imagination. In particular, SVRNS was applied to the study of imaginative pretend play in children aged 1 to 10 years, a domain that involves highly imaginative and often intangible concepts. The study demonstrated that SVRNS outperforms other neutrosophic sets in modeling this type of data, making it a more effective tool for child psychology studies. The application of machine learning algorithms, such as K-mean clustering and parallaxes coordinate analysis, in conjunction with SVRNS further facilitated the exploration of children's mental abilities and their imaginative play. These algorithms helped establish correlations between various determinants of imaginative play and cognitive development. The integration of SVRNS with machine learning tools allowed the researchers to make more precise and logical conclusions about the psychological aspects of play. Additionally, the study provided a comparison of different algorithms, illustrating the relative effectiveness of the methods used. Moving beyond SVRNS, [42] introduced the double-valued neutrosophic set (DVNS) as a new variant designed to improve the sensitivity and precision of handling indeterminacy. Unlike SVRNS, which uses a broader spectrum of membership functions, DVNS specifically classifies indeterminacy into two types: one leaning towards truth and the other towards falsity. This nuanced distinction makes DVNS a powerful tool for dealing with inconsistent or conflicting information. The paper detailed the properties and axioms of DVNS, establishing its superiority over traditional single-valued neutrosophic sets, fuzzy sets, and intuitionistic fuzzy sets in dealing with uncertain information. A significant contribution was the generalized distance measure developed for DVNS, which allows for more accurate clustering of data represented by double-valued neutrosophic information. The double-valued neutrosophic minimum spanning tree (DVS-MST) clustering algorithm was proposed to handle data in which indeterminate information is represented. The effectiveness of this algorithm was demonstrated through various illustrative examples and was compared with other clustering methods such as single-valued neutrosophic minimum spanning tree,intuitionistic fuzzy minimum spanning tree, and fuzzy minimum spanning tree. The results showed that DVN-MST offers better performance in clustering tasks, particularly when dealing with uncertain or incomplete data. In the context of multi-criteria decision-making (MCDM), the cross-entropy measure based on indeterminacy was applied to a double-rated indeterminacy set framework [43]. This approach allows for the evaluation and ranking of alternatives in decision-making scenarios, even when the criteria values are uncertain or imprecise. The proposed method was illustrated through practical examples, demonstrating its ability to handle uncertainty more effectively than traditional MCDM methods.

Further developments in the field have focused on the TRINS (a subset of refined neutrosophic sets) [44]. TRINS provides a more robust framework for communicating ambiguous, inconsistent, and fragmented information in various real-world applications. The ability of TRINS to process uncertain and incomplete data accurately is particularly valuable in complex decision-making contexts. Several studies [45-47] have extended this work, providing new examples and applications in fields such as data analysis and cognitive science. Notably, neutrosophic cognitive mappings (NCMs), based on neutrosophic logic, have been used to model the cognitive processes involved in children's Imaginative play, illustrating the utility of neutrosophic theory in understanding cognitive behavior [48, 49]. Beyond psychological applications, the principles of fuzzy logic and neutrosophic theory have been applied to real-world systems, such as nonlinear supply chains. A new fuzzy robust control strategy was proposed in [50] to handle the complexities of nonlinear supply chain systems, particularly in the presence of lead times. The strategy uses Takagi-Sugeno fuxxy control models and ensures the stability of the system despite the fluctuating demands and supply disruptions. This fuzzy control approach is more effective than traditional methods, especially when dealing with uncertainties in customer demand and supply chain dynamics. The proposed strategy is compared with standard robust H8 control strategies, and simulations demonstrate its superior performance in ensuring system stability. Further work by Zhang et al. [51] extended these fuzzy control strategies to more dynamic environments, developing a fuzzy switched strategy for supply chain networks with uncertain customer demand and production lead times. This approach, incorporating inhibition rates (?), helps mitigate the impact of uncertainty on the system's performance. The fuzzy robust strategy ensures the stable operation of the supply chain network at a low cost, outperforming traditional strategies in simulations.

In addition, [52] explored a fuzzy emergency model for supply chain systems facing disruptions due to emergency incidents. The model uses Takagi-Sugeno fuzzy systems to manage the switching between different emergency strategies, ensuring that the supply chain returns to normal operation with minimal costs. The proposed fuzzy robust emergency strategy ensures that the system remains stable and operational during supply disruptions, further demonstrating the utility of fuzzy logic in complex real- world systems. In the field of control theory, Sarwar and Li [53] examined second-order nonlinear boundary value problems and the existence of common solutions through fuzzy mappings, offering valuable insights into nonlinear dynamics. Similarly, Xia et al. [54] addressed the stabilization of chaotic systems using sampled-data controllers and Takkagi-Sugeno fuzzy models , showing that fuzzy control strategies can effectively manage nonlinear systems subject to delays and uncertainties. Finally, Gao et al. [55] developed a sliding mode control (SMC) strategy to stabilize semi-Markov jump fuzzy systems, demonstrating the robustness and flexibility of fuzzy controllers in real-time applications. In conclusion, the advances in neutrosophic sets and fuzzy control strategies outlined in the literature provide powerful tools for addressing uncertainty and indeterminacy across various domains. From psychological studies involving imaginative play in children to nonlinear supply chain management and fuzzy control systems, these approaches offer valuable insights and practical solutions to complex, real- world problems characterized by vagueness, inconsistency, and uncertainty. As these methods continue to evolve, they will likely find even broader applications in diverse fields such as artificial intelligence, decision-making, and control systems. Hatamleh et al. [64] studied different weighted operators such as generalized averaging and generalized geometric based on trigonometric $\wp$-rung interval-valued approach and in addition to this some examples were given for clear understanding. Shihadeh et al. [65] discussed algebraic structures towards different (α, β) intuitionistic fuzzy ideals and it characterization of an ordered ternary semigroups. Hatamleh et al. [64, 67] studied operators via weighted averaging and geometric approach using trigonometric neutrosophic interval valued set and its extension and characterization of interaction aggregating operators setting interval valued Pythagorean neutrosophic set. Hatamleh et al. [63, 68] discussed applications of complex interval valued picture fuzzy soft relations. . El-Sheikh and Abd El-latif [69] discussed decompositions of some types of supra soft sets and soft continuity and cited some excellent examples for clear understating the concept. Abd El-latif and Hosny, [70] discussed the eye catching concept of soft separation axioms and give examples. Abd El-latif and Hosny discussed some more structures in [69, 71].

The Challenge in Earlier Studies

Earlier studies in the field of uncertainty and indeterminacy often faced limitations in capturing the complexity and multi-dimensionality of psychological data. Traditional models, such as neutrosophic sets, provide a framework for handling uncertainty but typically fall short in addressing the full spectrum of indeterminate states, especially in real-world scenarios. The challenge is further amplified when dealing with complex psychological phenomena, such as romantic feelings, which involve a range of emotional states that cannot be easily captured by standard methods. In this paper, we address these challenges by introducing the concept of the heptapartitioned neutrosophic soft set (HPNSS), a novel extension of neutrosophic soft set theory. This approach improves accuracy by dividing indeterminacy into seven distinct categories: absolute true, relative true, contradiction, unknown, ignorance, relative false, and absolute false. This expansion allows for a more nuanced representation of uncertainty and indeterminacy, particularly in complex psychological studies. Furthermore, we extend neutrosophic soft topological spaces into heptapartitioned neutrosophic soft topological spaces (HPNSTS), which incorporates seven partitions to model uncertainty, vagueness, and indeterminacy in topological structures. These new partitions enable a more accurate representation of psychological data that reflects the diverse and multi-dimensional nature of human emotions.

While earlier research focused on isolated factors, our approach incorporates a holistic view by leveraging machine learning algorithms such as K-means clustering, heat maps, and t-SNE to analyze romantic feelings across various emotional dimensions. This methodology provides a deeper insight into the complex relationships between psychological variables, opening new avenues for understanding emotional dynamics in university students. The integration of these advanced techniques allows for a more comprehensive analysis of romantic feelings, particularly in the context of emotional disorders or relationship break-ups.

This paper is structured into nine sections. Section 1 provides an introduction to the study, literature review, and organized into six parts: Research gap, motivation, research questions, research objectives, significance of the study, and scope of the study. Each part addresses a specific aspect of the research, setting the foundation for the study's focus and approach. Section 2 revisits the fundamental concepts relevant to the study. Section 3 explores operations on hepta-partitioned neutrosophic soft sets and their applications in hepta-partitioned neutrosophic soft topological spaces, covering set operators and discussing their properties. Section 4 presents machine learning techniques in the context of single-valued hepta-partitioned neutrosophic soft sets and data set representation. Section 5 focuses on techniques for assessing romantic feelings, divided into two subsections: 5.1, which presents expert views on evaluation parameters for romantic feelings, and 5.2, which characterizes 16 key feelings through real-life examples. Section 6 presents results from data analyzed using Python libraries, with K-means clustering and the elbow method identifying key patterns in romantic feelings. Heat maps and t-SNE visualizations reveal correlations and complex relationships among 16 emotional traits, highlighting the effectiveness of advanced machine learning techniques in understanding romantic emotions. Section 7 discusses the advantages, while Section 8 addresses the limitations. Finally, Section 9 concludes the paper and outlines potential directions for future research.

Research Gap

In psychological research, particularly in understanding the emotional and mental states of young individuals, uncertainty and indeterminacy are inherent. Traditional methods often struggle to model and represent the imprecise, vague, and contradictory nature of such data. This limitation is especially evident in studies of complex emotional phenomena such as romantic relationships, where psychological variables can be highly subjective and context-dependent. Existing models, including neutrosophic soft sets and their extensions, fail to effectively address these complexities, resulting in inadequate representation of emotional states, relationships, and personal experiences. To address this gap, this paper introduces the heptapartitioned neutrosophic soft sets (HPNSS), a new extension of neutrosophic soft set theory. HPNSS incorporates seven distinct partitions to improve accuracy in modeling uncertainty, vagueness, and indeterminacy, thus offering a more robust framework for analyzing psychological data. The study also extends the concept of neutrosophic soft topological spaces (HPNSTS) to capture the nuanced relationships between psychological variables, particularly in the context of romantic feelings among young adults.

Motivation

The motivation for this study stems from the need to model the complex, uncertain, and often contradictory nature of psychological and emotional data. Traditional methods of data analysis in psychology, including fuzzy sets and single-valued neutrosophic sets, are often insufficient when dealing with real-world problems that involve uncertainty and imprecision, such as emotional disorders, relationships, and mental health. The Heptapartitioned Neutrosophic Soft Set (HPNSS) provides an advanced methodology to handle indeterminacy and uncertainty by categorizing it into seven distinct possibilities: absolute true, relative true, contradiction, unknown, ignorance, relative false, and absolute false. This offers a more nuanced approach to understanding psychological phenomena, particularly in the study of young adults' emotional states, such as romantic feelings. Additionally, employing machine learning techniques like K-means clustering and t-SNE visualizations enhances the capacity to identify patterns and relationships between multiple psychological variables. The introduction of HPNSS and its application to real-world psychological data serves as a crucial step toward more accurate and meaningful analyses in the field of psychology.

Research Questions

1. How does the introduction of seven distinct partitions for indeterminacy in the HPNSS improve the modeling of uncertainty, vagueness, and indeterminacy in comparison to existing neutrosophic soft sets?
2. What are the implications of using HPNSS in representing topological structures, and how does it compare with traditional soft set theory or neutrosophic soft set theory in terms of accuracy and flexibility?
3.What are the mathematical properties of p-open sets in the context of HPNSTS, and how do they influence the topological analysis of uncertain and imprecise data?
4. What are the advantages of using the Heptapartitioned Neutrosophic Soft Set (HPNSS) over traditional neutrosophic sets in modeling indeterminacy in psychological data?
5. How can the HPNSS framework be applied to analyze the romantic feelings of young adults in a university setting?
6.What relationships exist between various psychological variables (such as emotional state, relationship dynamics, and romantic feelings) among university students aged 18-25 years?
7. To what extent do machine learning algorithms like K-means clustering and t-SNE contribute to the understanding of complex emotional states and their relationships in psychological studies?

Research Objectives

1.To introduce the concept of Heptapartitioned Neutrosophic Soft Set (HPNSS) as a novel extension of neutrosophic soft set theory, designed to improve the modeling of uncertainty, vagueness, and indeterminacy.
2.To extend the concept of neutrosophic soft topological spaces (HPNSTS) and introduce new definitions such as *b open, pre-open, and semi-open sets, with a focus on p-open sets.
3.To apply the HPNSS framework in a real-world study focusing on university students aged 18 to 25 years, exploring their romantic feelings and emotional states.
4.To analyze psychological variables using advanced machine learning techniques such as K-means clustering, Heat maps, and t-SNE, with the aim of uncovering patterns and relationships among the data.
5.To evaluate the effectiveness of the HPNSS model and machine learning algorithms in providing new insights into emotional disorders and relationship dynamics among young adults.

Significance of Study

This study is significant because it addresses a gap in current psychological research methodologies by offering a more robust and nuanced approach to modeling emotional states and relationships. By introducing the Heptapartitioned Neutrosophic Soft Set (HPNSS), the study enhances the ability to capture and analyze indeterminate, vague, and contradictory data in psychological research. This approach allows for more accurate representation of complex emotional phenomena, such as romantic feelings and emotional disorders, which are often difficult to quantify. Moreover, the integration of advanced machine learning techniques provides new ways of identifying and visualizing relationships between multiple psychological variables. The findings of this research have practical implications for understanding emotional and psychological dynamics in young adults, particularly in the context of romantic relationships, breakups, and emotional disorders.

Scope of Study

This study focuses on the application of the Heptapartitioned Neutrosophic Soft Set (HPNSS)

and related topological concepts to psychological data collected from university students in Pakistan, specifically individuals aged 18 to 25 years. The scope includes:
1.Theoretical development of the HPNSS and HPNSTS frameworks, including the introduction of new definitions and results related to p-open sets.
2. Empirical analysis of the emotional states and romantic feelings of young adults, incorporating data from the Psychology Department at Peshawar University.
3. Machine learning applications, including K-means clustering, Heat maps, Elbow method, Feature correlation, and t-SNE visualizations, to analyze and visualize the relationships between psychological variables.
4. Exploration of psychological phenomena, with particular attention to emotional disorders, romantic relationships, and emotional states, within a university context.
This study does not extend beyond the university demographic and is limited to the analysis of psychological data in the context of romantic relationships, emotional states, and related variables. Further research could expand the scope to include other demographics or more diverse emotional conditions.

2. Preliminaries

This section covers the basic ideas that are required for the upcoming research.

Definition 1. *[57] Let E be a set of parameters and X is initial universe set. $P(X)$ represents the collection of all NSs for X. A set defined by a set valued function $\tilde{F}$ expressing a mapping $\tilde{F}: E \to P(X)$ is then a neutrosophic soft set $(\tilde{F}, E)$ over X, where $\tilde{F}$ is referred to as the approximate function of the neutrosophic soft set $(\tilde{F}, E)$. Stated differently, the neutrosophic soft set can be expressed as a collection of ordered pairs, $(\tilde{F}, E) = \left\{ \left(e, \langle x, T_{\tilde{F}(e)}(x), I_{\tilde{F}(e)}(x), F_{\tilde{F}(e)}(x) \rangle \right) : x \in X, e \in E \right\}$. $T_{\tilde{F}(e)}(x), I_{\tilde{F}(e)}(x), F_{\tilde{F}(e)}(x) \in [0,1]$ are the truth-membership, indeterminacy-membership, and falsity-membership functions of $\tilde{F}(e)$, respectively, and they all lie within the interval $[0,1]$. The inequality $0 \leq T_{\tilde{F}(e)}(x) + I_{\tilde{F}(e)}(x) + F_{\tilde{F}(e)}(x) \leq 3$ is evident as the supremum of each T, I, and F is 1.*

Definition 2. *[58] Let $(\tilde{F}, E)$ be a neutrosophic soft set. Then $(\tilde{F}, E)^c$ is the complement of $(\tilde{F}, E)$:*

$$(\tilde{F}, E)^c = \left\{ \left(e, \langle x, F_{\tilde{F}(e)}(x), 1 - I_{\tilde{F}(e)}(x), T_{\tilde{F}(e)}(x) \rangle \right) : x \in X, e \in E \right\}.$$

Obvious that:

$$\left((\tilde{F}, E)^c \right)^c = (\tilde{F}, E).$$

Definition 3. *[59] Let $(\tilde{F}, E)$ and $(\tilde{G}, E)$ be two neutrosophic soft sets. Then $(\tilde{F}, E)$ is said to be a neutrosophic soft subset of $(\tilde{G}, E)$ if:*

$$T_{\tilde{F}(e)}(x) \leq T_{\tilde{G}(e)}(x), \quad I_{\tilde{F}(e)}(x) \leq I_{\tilde{G}}(e)(x), \quad F_{\tilde{F}(e)}(x) \geq F_{\tilde{G}(e)}(x), \quad \forall e \in E, \forall x \in X.$$

It is denoted by:

$$(\tilde{F}, E) \subseteq (\tilde{G}, E).$$

Definition 4. *[60] Let $(\tilde{F}_1, E)$ and $(\tilde{F}_2, E)$ be two neutrosophic soft sets. Then their union is represented by $(\tilde{F}_1, E) \cup (\tilde{F}_2, E) = (\tilde{F}_3, E)$ as:*

$$(\tilde{F}_3, E) = \left\{ \left(e, \langle x, T_{\tilde{F}_{3(e)}}(x), I_{\tilde{F}_{3(e)}}(x), F_{\tilde{F}_{3(e)}}(x) \rangle \right) : x \in X, e \in E \right\}.$$

Where:

$$T_{\tilde{F}_{3(e)}}(x) = \max\{T_{\tilde{F}_{1(e)}}(x), T_{\tilde{F}_{2(e)}}(x)\},$$

$$I_{\tilde{F}_{3(e)}}(x) = \max\{I_{\tilde{F}_{1(e)}}(x), I_{\tilde{F}_{2(e)}}(x)\},$$
$$F_{\tilde{F}_{3(e)}}(x) = \min\{F_{\tilde{F}_{1(e)}}(x), F_{\tilde{F}_{2(e)}}(x)\}.$$

Definition 5. *[61] Let $(\tilde{F}_1, E)$ and $(\tilde{F}_2, E)$ be two neutrosophic soft sets. Then their intersection is symbolized by $(\tilde{F}_1, E) \cap (\tilde{F}_2, E) = (\tilde{F}_3, E)$ as:*

$$(\tilde{F}_3, E) = \left\{ \left(e, \langle x, T_{\tilde{F}_{3(e)}}(x), I_{\tilde{F}_{3(e)}}(x), F_{\tilde{F}_{3(e)}}(x) \rangle \right) : x \in X, e \in E \right\}.$$

Where:

$$T_{\tilde{F}_{3(e)}}(x) = \min\{T_{\tilde{F}_{1(e)}}(x), T_{\tilde{F}_{2(e)}}(x)\},$$
$$I_{\tilde{F}_{3(e)}}(x) = \min\{I_{\tilde{F}_{1(e)}}(x), I_{\tilde{F}_{2(e)}}(x)\},$$
$$F_{\tilde{F}_{3(e)}}(x) = \max\{F_{\tilde{F}_{1(e)}}(x), F_{\tilde{F}_{2(e)}}(x)\}.$$

Definition 6. *[61] Let $(\tilde{F}_1, E)$ and $(\tilde{F}_2, E)$ be two neutrosophic soft sets. Then the difference operation on them is denoted by $(\tilde{F}_1, E) \setminus (\tilde{F}_2, E) = (\tilde{F}_3, E)$ and is defined by:*

$$(\tilde{F}_3, e) = \left\{ \left(e, \langle x, T_{\tilde{F}_{3(e)}}(x), I_{\tilde{F}_{3(e)}}(x), F_{\tilde{F}_{3(e)}}(x) \rangle \right) : x \in X, e \in E \right\}.$$

Where:

$$T_{\tilde{F}_{3(e)}}(x) = \min\{T_{\tilde{F}_{1(e)}}(x), T_{\tilde{F}_{2(e)}}(x)\},$$
$$I_{\tilde{F}_{3(e)}}(x) = \min\{I_{\tilde{F}_{1(e)}}(x), I_{\tilde{F}_{2(e)}}(x)\},$$
$$F_{\tilde{F}_{3(e)}}(x) = \max\{F_{\tilde{F}_{1(e)}}(x), F_{\tilde{F}_{2(e)}}(x)\}.$$

Definition 7. *[61] 1. A neutrosophic soft set $(\tilde{F}_1, E)$ is said to be a null neutrosophic soft set if:*

$$T_{\tilde{F}(e)}(x) = 0, \quad I_{\tilde{F}(e)}(x) = 0, \quad F_{\tilde{F}(e)}(x) = 1, \quad \forall e \in E, \forall x \in X.$$

It is denoted by $0_{(X,E)}$.

2. A neutrosophic soft set $(\tilde{F}_1, E)$ is said to be an absolute neutrosophic soft set if:

$$T_{\tilde{F}(e)}(x) = 1, \quad I_{\tilde{F}(e)}(x) = 1, \quad F_{\tilde{F}(e)}(x) = 0, \quad \forall e \in E, \forall x \in X.$$

It is symbolized as $1(X, E)$.

Obvious that:

$$0_{(X,E)} = 1_{(X,E)}, \quad 1^c_{(X,E)} = 0_{(X,E)}.$$

3. Operations on Hepta-Partitioned Neutrosophic Soft Sets and Their Application in Heptapartitioned Neutrosophic Soft Topological Spaces

Neutrosophic set theory (NST), a generality of vague set theory (VST), is regarded as the most appealing theory since it considers the three possible membership values: true, false, and indeterminacy. The principles are all quite obvious, but the third one is particularly fascinating since it addresses uncertainty, which arises in all aspects of daily life. One can make the situation more certain and free of error if the indeterminacy membership is refined. This can be done by splitting the indeterminacy into five pieces that is possible values. These are relative true, relative false, contradiction, unknown (undefined) and ignorance. This section is devoted to the most basic operations of union, intersection, difference, and absolute null, absolute HPNNNs. Theorems and examples are given for better understanding the situation.

The notion of HPNSTS is presented in this section. The terms HPNS semi-open, HPNS pre-open and HPNS $*_b$open sets are defined. One of these intriguing HPNS generalized open sets, referred to as the HPNS pre-open set, is selected, and certain fundamentals are then produced based on this description. These consist of the HPNS closure, HPNS exterior, HPNS boundary, and HPNS interior.

Definition 8. *Let E be the set of parameters and X be the key set. Let $P(X)$ represent the power set of X. Then, a hepta-partitioned neutrosophic soft set $(\tilde{F}, E)$ over X is a mapping $\tilde{F} : E \to P(X)$, where $\tilde{F}$ is the function of $(\tilde{F}, E)$. Symbolically,*

$$(\tilde{F}, E) = \left\{ \left(e, \langle x, AbT_{\tilde{F}(e)}(x), ReT_{\tilde{F}(e)}(x), U_{\tilde{F}(e)}(x), C_{\tilde{F}(e)}(x), G_{\tilde{F}(e)}(x), ReF_{\tilde{F}(e)}(x), AbF_{\tilde{F}(e)}(x) : x \in X \rangle \right) : e \in E \right\}$$

Where, $AbT_{\tilde{F}(e)}(x)$, $ReT_{\tilde{F}(e)}(x)$, $U_{\tilde{F}(e)}(x)$, $C_{\tilde{F}(e)}(x)$, $G_{\tilde{F}(e)}(x)$, $ReF_{\tilde{F}(e)}(x)$, and $AbF_{\tilde{F}(e)}(x) \in [0, 1]$ respectively, $AbT_{\tilde{F}(e)}(x)$, $ReT_{\tilde{F}(e)}(x)$, $U_{\tilde{F}(e)}(x)$, $C_{\tilde{F}(e)}(x)$, $G_{\tilde{F}(e)}(x)$, $ReF_{\tilde{F}(e)}(x)$, $AbF_{\tilde{F}(e)}(x)$ are called the absolute true-membership, relative true-membership,unknown membership, confusion-membership, ignorance-membership, relative false-membership, and absolute false-membership function of $\tilde{F}(e)$. Since the supremum of each function is 1 and the infimum is 0 so the inequality;

$$0 \leq AbT_{\tilde{F}(e)}(x) + ReT_{\tilde{F}(e)}(x) + U_{\tilde{F}(e)}(x) + C_{\tilde{F}(e)}(x) + G_{\tilde{F}(e)}(x) + ReF_{\tilde{F}(e)}(x) + AbF_{\tilde{F}(e)}(x) \leq 7.$$

Definition 9. *Let $(\tilde{F}, E)$ be a hepta-partitioned neutrosophic soft set over the key set X. Then, the complement of $(\tilde{F}, E)$ is denoted by $(\tilde{F}, E)^c$ and is defined as follows:*

$$(\tilde{F}, E)^c = \left\{ \left(\begin{matrix} e, \langle x, AbF_{\tilde{F}(e)}(x), ReF_{\tilde{F}(e)}(x), G_{\tilde{F}(e)}(x), \\ 1 - C_{\tilde{F}(e)}(x), U_{\tilde{F}(e)}(x), ReT_{\tilde{F}(e)}(x), AbT_{\tilde{F}(e)}(x) : x \in X \rangle \right) : e \in E. \end{matrix} \right\}$$

It follows that $\left((\tilde{F}, E)^c \right)^c = (\tilde{F}, E)$.

Definition 10. *Let $(\tilde{F}, E)$ and $(\tilde{G}, E)$ be two hepta-partitioned neutrosophic soft sets over the key set X then, $(\tilde{F}, E) \subseteq (\tilde{G}, E)$ if*

$$\begin{aligned} AbT_{\tilde{F}(e)}(x) &\preceq AbT_{\tilde{G}(e)}(x), \\ ReT_{\tilde{F}(e)}(x) &\preceq ReT_{\tilde{G}(e)}(x), \\ U_{\tilde{F}(e)}(x) &\preceq U_{\tilde{G}(e)}(x), \\ C_{\tilde{F}(e)}(x) &\preceq C_{\tilde{G}(e)}(x), \\ G_{\tilde{F}(e)}(x) &\preceq G_{\tilde{G}(e)}(x), \\ ReF_{\tilde{F}(e)}(x) &\succeq ReF_{\tilde{G}(e)}(x), \\ AbF_{\tilde{F}(e)}(x) &\succeq AbF_{\tilde{G}(e)}(x), \end{aligned}$$

for all $e \in E$ and $x \in X$. If $(\tilde{F}, E) \subseteq (\tilde{G}, E)$ and $(\tilde{F}, E) \supseteq (\tilde{G}, E)$, then $(\tilde{F}, E) = (\tilde{G}, E)$.

Definition 11. *Let $(\tilde{F}, E)$ and $(\tilde{G}, E)$ be two hepta-partitioned neutrosophic soft sets over the key set X such that $(\tilde{F}, E) \neq (\tilde{G}, E)$, then their union is denoted by $(\tilde{F}, E) \tilde{\Cup} (\tilde{G}, E) = (\tilde{H}, E)$ and is defined as:*

$$(\tilde{H}, E) = \left\{ \left(\begin{matrix} e, \langle x, AbT_{\tilde{H}(e)}(x), ReT_{\tilde{H}(e)}(x), U_{\tilde{H}(e)}(x), C_{\tilde{H}(e)}(x), \\ G_{\tilde{H}(e)}(x), ReF_{\tilde{H}(e)}(x), AbF_{\tilde{H}(e)}(x) : x \in X \rangle \right) : e \in E. \end{matrix} \right\}$$

where

$$AbT_{\tilde{H}(e)}(x) = \max \left\{ AbT_{\tilde{F}(e)}(x), AbT_{\tilde{G}(e)}(x) \right\},$$

$$ReT_{\tilde{H}(e)}(x) = \max\left\{ReT_{\tilde{F}(e)}(x), ReT_{\tilde{G}(e)}(x)\right\},$$
$$U_{\tilde{H}(e)}(x) = \max\left\{U_{\tilde{F}(e)}(x), U_{\tilde{G}(e)}(x)\right\},$$
$$C_{\tilde{H}(e)}(x) = \max\left\{C_{\tilde{F}(e)}(x), C_{\tilde{G}(e)}(x)\right\},$$
$$G_{\tilde{H}(e)}(x) = \min\left\{G_{\tilde{F}(e)}(x), G_{\tilde{G}(e)}(x)\right\},$$
$$ReF_{\tilde{H}(e)}(x) = \min\left\{ReF_{\tilde{F}(e)}(x), ReF_{\tilde{G}(e)}(x)\right\},$$
$$AbF_{\tilde{H}(e)}(x) = \min\left\{AbF_{\tilde{F}(e)}(x), AbF_{\tilde{G}(e)}(x)\right\}.$$

Definition 12. *Let $(\tilde{F}, E)$ and $(\tilde{G}, E)$ be two hepta-partitioned neutrosophic soft sets over the key set X such that $(\tilde{F}, E) \neq (\tilde{G}, E)$ then their intersection is denoted by $(\tilde{F}, E)\tilde{\Cap}(\tilde{G}, E) = (\tilde{H}, E)$ and is defined as:*

$$(\tilde{H}, E) = \left\{ \left(\begin{array}{l} e, \langle x, AbT_{\tilde{H}(e)}(x), ReT_{\tilde{H}(e)}(x), U_{\tilde{H}(e)}(x), \\ C_{\tilde{H}(e)}(x), G_{\tilde{H}(e)}(x), ReF_{\tilde{H}(e)}(x), AbF_{\tilde{H}(e)}(x) : x \in X \rangle \right) : e \in E. \end{array} \right\}$$

where

$$AbT_{\tilde{H}(e)}(x) = \min\left\{AbT_{\tilde{F}(e)}(x), AbT_{\tilde{G}(e)}(x)\right\},$$
$$ReT_{\tilde{H}(e)}(x) = \min\left\{ReT_{\tilde{F}(e)}(x), ReT_{\tilde{G}(e)}(x)\right\},$$
$$U_{\tilde{H}(e)}(x) = \min\left\{U_{\tilde{F}(e)}(x), U_{\tilde{G}(e)}(x)\right\},$$
$$C_{\tilde{H}(e)}(x) = \min\left\{C_{\tilde{F}(e)}(x), C_{\tilde{G}(e)}(x)\right\},$$
$$G_{\tilde{H}(e)}(x) = \max\left\{G_{\tilde{F}(e)}(x), G_{\tilde{G}(e)}(x)\right\},$$
$$ReF_{\tilde{H}(e)}(x) = \max\left\{ReF_{\tilde{F}(e)}(x), ReF_{\tilde{G}(e)}(x)\right\},$$
$$AbF_{\tilde{H}(e)}(x) = \max\left\{AbF_{\tilde{F}(e)}(x), AbF_{\tilde{G}(e)}(x)\right\}.$$

Definition 13. *Let $(\tilde{F}, E)$ and $(\tilde{G}, E)$ be two hepta-partitioned neutrosophic soft sets over the key set X such that $(\tilde{F}, E) \neq (\tilde{G}, E)$. Then, their difference is denoted by $(\tilde{H}, E) = (\tilde{F}, E) \setminus (\tilde{G}, E)$ and is defined as:*

$$(\tilde{H}, E) = (\tilde{F}, E)\tilde{\Cap}(\tilde{G}, E)^c,$$

$$(\tilde{H}, E) = \left\{ \left(\begin{array}{l} e, \langle x, AbT_{\tilde{H}(e)}(x), ReT_{\tilde{H}(e)}(x), U_{\tilde{H}(e)}(x), \\ C_{\tilde{H}(e)}(x), G_{\tilde{H}(e)}(x), ReF_{\tilde{H}(e)}(x), AbF_{\tilde{H}(e)}(x) : x \in X \rangle \right) : e \in E. \end{array} \right\}$$

where

$$AbT_{\tilde{H}(e)}(x) = \min\left\{AbT_{\tilde{F}(e)}(x), AbT_{\tilde{G}(e)}(x)\right\},$$
$$ReT_{\tilde{H}(e)}(x) = \min\left\{ReT_{\tilde{F}(e)}(x), ReT_{\tilde{G}(e)}(x)\right\},$$
$$U_{\tilde{H}(e)}(x) = \min\left\{U_{\tilde{F}(e)}(x), U_{\tilde{G}(e)}(x)\right\},$$
$$C_{\tilde{H}(e)}(x) = \min\left\{C_{\tilde{F}(e)}(x), C_{\tilde{G}(e)}(x)\right\},$$
$$G_{\tilde{H}(e)}(x) = \max\left\{G_{\tilde{F}(e)}(x), G_{\tilde{G}(e)}(x)\right\},$$
$$ReF_{\tilde{H}(e)}(x) = \max\left\{ReF_{\tilde{F}(e)}(x), ReF_{\tilde{G}(e)}(x)\right\},$$
$$AbF_{\tilde{H}(e)}(x) = \max\left\{AbF_{\tilde{F}(e)}(x), AbF_{\tilde{G}(e)}(x)\right\}.$$

Definition 14. *Let $\{(\tilde{F}_1, E) : i \in I\}$ be a family of hepta-partitioned neutrosophic soft sets over the key set X. Then,*

$$\bigcup_{i\in I}(\tilde{F}_1, E) = \left\{ \left(\begin{array}{l} e, \langle x, \sup_{i\in I} AbT_{\tilde{F}_{1(e)}}(x), \sup_{i\in I} ReT_{\tilde{F}_{1(e)}}(x), \sup_{i\in I} U_{\tilde{F}_{1(e)}}(x), \sup_{i\in I} C_{\tilde{F}_{1(e)}}(x), \\ \inf_{i\in I} G_{\tilde{F}_{1(e)}}(x), \inf_{i\in I} ReF_{\tilde{F}_{1(e)}}(x), \inf_{i\in I} AbF_{\tilde{F}_{1(e)}}(x) : x \in X\rangle) : e \in E. \end{array} \right. \right\}$$

$$\bigcap_{i\in I}(\tilde{F}_1, E) = \left\{ \left(\begin{array}{l} e, \langle x, \inf_{i\in I} AbT_{\tilde{F}_{1(e)}}(x), \inf_{i\in I} ReT_{\tilde{F}_{1(e)}}(x), \inf_{i\in I} U_{\tilde{F}_{1(e)}}(x), \inf_{i\in I} C_{\tilde{F}_{1(e)}}(x), \\ \sup_{i\in I} G_{\tilde{F}_{1(e)}}(x), \sup_{i\in I} ReF_{\tilde{F}_{1(e)}}(x), \sup_{i\in I} AbF_{\tilde{F}_{1(e)}}(x) : x \in X\rangle) : e \in E. \end{array} \right. \right\}$$

Definition 15. *Let $(\tilde{F}, E)$ and $(\tilde{G}, E)$ be two hepta-partitioned neutrosophic soft sets over the key set X then "AND" operation on them is denoted by $(\tilde{F}, E) \wedge (\tilde{G}, E) = (\tilde{H}, E \times E)$ and is defined as:*

$$(\tilde{H}, E\times E) = \left\{ \left(\begin{array}{l} (e_1, e_2), \langle x, AbT_{\tilde{H}}(e_1, e_2)(x), ReT_{\tilde{H}}(e_1, e_2)(x), U_{\tilde{H}}(e_1, e_2)(x) \\ C_{\tilde{H}}(e_1, e_2)(x), G_{\tilde{H}}(e_1, e_2)(x), ReF_{\tilde{H}}(e_1, e_2)(x), AbF_{\tilde{H}}(e_1, e_2)(x) : x \in X\rangle) : (e_1, e_2) \in E \times E. \end{array} \right. \right\}$$

Where,

$$\begin{aligned} AbT_{\tilde{H}(e)}(x) &= \min\left\{ AbT_{\tilde{F}(e)}(x), AbT_{\tilde{G}(e)}(x) \right\}, \\ ReT_{\tilde{H}(e)}(x) &= \min\left\{ ReT_{\tilde{F}(e)}(x), ReT_{\tilde{G}(e)}(x) \right\}, \\ U_{\tilde{H}(e)}(x) &= \min\left\{ U_{\tilde{F}(e)}(x), U_{\tilde{G}(e)}(x) \right\}, \\ C_{\tilde{H}(e)}(x) &= \min\left\{ C_{\tilde{F}(e)}(x), C_{\tilde{G}(e)}(x) \right\}, \\ G_{\tilde{H}(e)}(x) &= \max\left\{ G_{\tilde{F}(e)}(x), G_{\tilde{G}(e)}(x) \right\}, \\ ReF_{\tilde{H}(e)}(x) &= \max\left\{ ReF_{\tilde{F}(e)}(x), ReF_{\tilde{G}(e)}(x) \right\}, \\ AbF_{\tilde{H}(e)}(x) &= \max\left\{ AbF_{\tilde{F}(e)}(x), AbF_{\tilde{G}(e)}(x) \right\}. \end{aligned}$$

Definition 16. *Let $(\tilde{F}, E)$ and $(\tilde{G}, E)$ be two hepta-partitioned neutrosophic soft sets over the key set X then "OR" operation on them is denoted by $(\tilde{F}, E) \vee (\tilde{G}, E) = (\tilde{H}, E \times E)$ and is defined as:*

$$(\tilde{H}, E\times E) = \left\{ \left(\begin{array}{l} (e_1, e_2), \langle x, AbT_{\tilde{H}}(e_1, e_2)(x), ReT_{\tilde{H}}(e_1, e_2)(x), U_{\tilde{H}}(e_1, e_2)(x), \\ C_{\tilde{H}}(e_1, e_2)(x), G_{\tilde{H}}(e_1, e_2)(x), ReF_{\tilde{H}}(e_1, e_2)(x), AbF_{\tilde{H}}(e_1, e_2)(x) : x \in X\rangle) : (e_1, e_2) \in E \times E. \end{array} \right. \right\}$$

Where,

$$\begin{aligned} AbT_{\tilde{H}(e)}(x) &= \max\left\{ AbT_{\tilde{F}(e)}(x), AbT_{\tilde{G}(e)}(x) \right\}, \\ ReT_{\tilde{H}(e)}(x) &= \max\left\{ ReT_{\tilde{F}(e)}(x), ReT_{\tilde{G}(e)}(x) \right\}, \\ U_{\tilde{H}(e)}(x) &= \max\left\{ U_{\tilde{F}(e)}(x), U_{\tilde{G}(e)}(x) \right\}, \\ C_{\tilde{H}(e)}(x) &= \max\left\{ C_{\tilde{F}(e)}(x), C_{\tilde{G}(e)}(x) \right\}, \\ G_{\tilde{H}(e)}(x) &= \min\left\{ G_{\tilde{F}(e)}(x), G_{\tilde{G}(e)}(x) \right\}, \end{aligned}$$

$$ReF_{\tilde{H}(e)}(x) = \min\left\{ReF_{\tilde{F}(e)}(x), ReF_{\tilde{G}(e)}(x)\right\},$$
$$AbF_{\tilde{H}(e)}(x) = \min\left\{AbF_{\tilde{F}(e)}(x), AbF_{\tilde{G}(e)}(x)\right\}.$$

Definition 17. *A hepta-partitioned neutrosophic soft set $(\tilde{F}, E)$ over the key set X is said to be a null hepta-partitioned neutrosophic soft set if*

$$AbT_{\tilde{F}(e)}(x) = 0, \quad ReT_{\tilde{F}(e)}(x) = 0, \quad \forall e \in E, \forall x \in X,$$
$$U_{\tilde{F}(e)}(x) = 0, \forall x \in X, \quad C_{\tilde{F}(e)}(x) = 0, \quad \forall e \in E, \forall x \in X,$$
$$G_{\tilde{F}(e)}(x) = 1, \quad \forall e \in E, \forall x \in X,$$
$$ReF_{\tilde{F}(e)}(x) = 1, \quad AbF_{\tilde{F}(e)}(x) = 1, \quad \forall e \in E, \forall x \in X.$$

It is signified as $0_{(X,E)}$.

Definition 18. *A hepta-partitioned neutrosophic soft set $(\tilde{F}, E)$ over the key set X is called an absolute hepta-partitioned neutrosophic soft set if*

$$AbT_{\tilde{F}(e)}(x) = 1, \quad ReT_{\tilde{F}(e)}(x) = 1, \quad \forall e \in E, \forall x \in X,$$
$$U_{\tilde{F}(e)}(x) = 1, \quad \forall e \in E, \forall x \in X, \quad C_{\tilde{F}(e)}(x) = 1, \quad \forall e \in E, \forall x \in X,$$
$$G_{\tilde{F}(e)}(x) = 0, \quad \forall e \in E, \forall x \in X$$
$$ReF_{\tilde{F}(e)}(x) = 0, \quad AbF_{\tilde{F}(e)}(x) = 0, \quad \forall e \in E, \forall x \in X.$$

Clearly,

$$0^c_{(X,E)} = 1(X, E), \quad 1^c_{(X,E)} = 0_{(X,E)}.$$

Definition 19. *The family of all hepta-partitioned neutrosophic soft sets over X is designated as $HPNSS(\widetilde{X})$ then*

$$x^e_{\langle\Delta_1,\Delta_2,\Delta_3,\Delta_4,\Delta_5,\Delta_6,\Delta_7\rangle}$$

is called a hepta-partitioned neutrosopic soft point, for every point $x \in \widetilde{X}, 0 \prec \Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6, \Delta_7 \preceq 1, e \in E$, and is defined as follows:

$$x^e_{\langle\Delta_1,\Delta_2,\Delta_3,\Delta_4,\Delta_5,\Delta_6,\Delta_7\rangle} e^{/(\mathrm{y})} = \begin{cases} \langle\Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6, \Delta_7\rangle, & \text{if } e = e' \text{ and } \mathrm{y} = x, \\ (0, 0, 0, 0, 0, 0, 1), & \text{if } e' \neq e \text{ or } \mathrm{y} \neq x. \end{cases}$$

Definition 20. *Let $(\tilde{F}, E)$ be a hepta-partitioned neutrosophic soft set over the key set X then*

$$x^e_{\langle\Delta_1,\Delta_2,\Delta_3,\Delta_4,\Delta_5,\Delta_6,\Delta_7\rangle} \in (\tilde{F}, E)$$

if

$$\Delta_1 \preceq AbT_{\tilde{F}(e)}(x), \Delta_2 \preceq ReT_{\tilde{F}(e)}(x), \Delta_3 \preceq U_{\tilde{F}(e)}(x), \Delta_4 \preceq C_{\tilde{F}(e)}(x),$$
$$\Delta_5 \succeq G_{\tilde{F}(e)}(x), \Delta_6 \succeq ReF_{\tilde{F}(e)}(x), \Delta_7 \succeq AbF_{\tilde{F}(e)}(x).$$

Example 1. *Let $X = x_1, x_2, x_3$ be the key set and the set of parameters $E = \{e_1, e_2\}$. Let us develop the hepta-partitioned neutrosophic soft sets $(\tilde{F}, E)$ and $(\tilde{G}, E)$ over the key set X as follows:*

$$(\tilde{F},E)=\begin{bmatrix} e_1=(\langle x_1,\frac{02}{10},\frac{03}{10},\frac{03}{10},\frac{02}{10},\frac{08}{10},\frac{07}{10},\frac{08}{10}\rangle, \\ \langle x_2,\frac{04}{10},\frac{04}{10},\frac{04}{10},\frac{04}{10},\frac{04}{10},\frac{06}{10},\frac{04}{10}\rangle, \\ \langle x_3,\frac{02}{10},\frac{04}{10},\frac{04}{10},\frac{02}{10},\frac{03}{10}\frac{06}{10},\frac{03}{10}\rangle), \\ e_2=(\langle x_1,\frac{03}{10},\frac{02}{10},\frac{02}{10},\frac{03}{10},\frac{06}{10},\frac{06}{10},\frac{06}{10}\rangle, \\ \langle x_2,\frac{01}{10},\frac{05}{10},\frac{05}{10},\frac{01}{10}\frac{5}{10},\frac{06}{10},\frac{05}{10}\rangle, \\ \langle x_3,\frac{04}{10},\frac{03}{10},\frac{03}{10},\frac{04}{10},\frac{05}{10}\frac{06}{10},\frac{05}{10}\rangle) \end{bmatrix}$$

$$(\tilde{G},E)=\begin{bmatrix} e_1=(\langle x_1,\frac{04}{10},\frac{03}{10},\frac{03}{10},\frac{04}{10},\frac{06}{10},\frac{06}{10},\frac{06}{10}\rangle, \\ \langle x_2,\frac{04}{10},\frac{05}{10},\frac{05}{10},\frac{04}{10},\frac{03}{10},\frac{06}{10},\frac{03}{10}\rangle, \\ \langle x_3,\frac{03}{10},\frac{05}{10},\frac{05}{10},\frac{03}{10},\frac{02}{10}\frac{06}{10},\frac{02}{10}\rangle), \\ e_2=(\langle x_1,\frac{03}{10},\frac{04}{10},\frac{04}{10},\frac{03}{10},\frac{05}{10}\frac{06}{10},\frac{05}{10}\rangle, \\ \langle x_2,\frac{02}{10},\frac{06}{10},\frac{06}{10},\frac{02}{10},\frac{04}{10}\frac{06}{10},\frac{04}{10}\rangle, \\ \langle x_3,\frac{04}{10},\frac{06}{10},\frac{06}{10},\frac{04}{10},\frac{03}{10}\frac{06}{10},\frac{03}{10}\rangle) \end{bmatrix}$$

$$(\tilde{F},E)\tilde{\Cup}(\tilde{G},E)=\begin{bmatrix} e_1=(\langle x_1,\frac{04}{10},\frac{03}{10},\frac{03}{10},\frac{04}{10},\frac{06}{10}\frac{06}{10},\frac{06}{10}\rangle, \\ \langle x_2,\frac{04}{10},\frac{05}{10},\frac{05}{10},\frac{04}{10},\frac{03}{10},\frac{06}{10},\frac{03}{10}\rangle, \\ \langle x_3,\frac{03}{10},\frac{05}{10},\frac{05}{10},\frac{03}{10},\frac{02}{10}\frac{06}{10},\frac{02}{10}\rangle), \\ e_2=(\langle x_1,\frac{03}{10},\frac{04}{10},\frac{04}{10},\frac{03}{10},\frac{05}{10},\frac{06}{10},\frac{05}{10}\rangle, \\ \langle x_2,\frac{02}{10},\frac{06}{10},\frac{06}{10},\frac{02}{10},\frac{04}{10},\frac{06}{10},\frac{04}{10}\rangle, \\ \langle x_3,\frac{04}{10},\frac{06}{10},\frac{06}{10},\frac{04}{10},\frac{03}{10},\frac{06}{10},\frac{03}{10}\rangle) \end{bmatrix}$$

$$(\tilde{F},\acute{E})\tilde{\Cap}(\tilde{G},E)=\begin{bmatrix} e_1=(\langle x_1,\frac{02}{10},\frac{03}{10},\frac{03}{10},\frac{02}{10},\frac{08}{10},\frac{07}{10},\frac{08}{10}\rangle, \\ \langle x_2,\frac{04}{10},\frac{04}{10},\frac{04}{10},\frac{04}{10},\frac{04}{10}\frac{06}{10},\frac{04}{10}\rangle, \\ \langle x_3,\frac{02}{10},\frac{04}{10},\frac{04}{10},\frac{02}{10},\frac{03}{10}\frac{6}{10},\frac{3}{10}\rangle), \\ e_2=(\langle x_1,\frac{03}{10},\frac{02}{10},\frac{02}{10},\frac{03}{10},\frac{06}{10},\frac{06}{10},\frac{06}{10}\rangle, \\ \langle x_2,\frac{01}{10},\frac{05}{10},\frac{05}{10},\frac{01}{10},\frac{05}{10},\frac{06}{10},\frac{05}{10}\rangle, \\ \langle x_3,\frac{04}{10},\frac{03}{10},\frac{03}{10},\frac{04}{10},\frac{05}{10},\frac{06}{10},\frac{05}{10}\rangle) \end{bmatrix}$$

$$(\tilde{F},E)\tilde{\wedge}(\tilde{G},E)=\begin{bmatrix} (e_1,e_1)=(\langle x_1,\frac{02}{10},\frac{03}{10},\frac{03}{10},\frac{02}{10},\frac{08}{10},\frac{07}{10},\frac{08}{10}\rangle, \\ \langle x_2,\frac{04}{10},\frac{04}{10},\frac{04}{10},\frac{04}{10},\frac{04}{10},\frac{06}{10},\frac{04}{10}\rangle, \\ \langle x_3,\frac{02}{10},\frac{04}{10},\frac{04}{10},\frac{02}{10},\frac{03}{10},\frac{06}{10},\frac{03}{10}\rangle), \\ (e_1,e_2)=(\langle x_1,\frac{02}{10},\frac{03}{10},\frac{03}{10},\frac{02}{10},\frac{08}{10},\frac{07}{10},\frac{08}{10}\rangle, \\ \langle x_2,\frac{02}{10},\frac{04}{10},\frac{04}{10},\frac{02}{10},\frac{04}{10},\frac{06}{10},\frac{04}{10}\rangle, \\ \langle x_3,\frac{04}{10},\frac{03}{10},\frac{03}{10},\frac{04}{10},\frac{04}{10},\frac{06}{10},\frac{04}{10}\rangle), \\ (e_2,e_1)=(\langle x_1,\frac{03}{10},\frac{02}{10},\frac{02}{10},\frac{03}{10},\frac{06}{10},\frac{07}{10},\frac{06}{10}\rangle, \\ \langle x_2,\frac{01}{10},\frac{05}{10},\frac{05}{10},\frac{01}{10},\frac{05}{10},\frac{06}{10},\frac{05}{10}\rangle, \\ \langle x_3,\frac{03}{10},\frac{03}{10},\frac{03}{10},\frac{03}{10},\frac{04}{10},\frac{06}{10},\frac{04}{10}\rangle), \\ (e_2,e_2)=(\langle x_1,\frac{03}{10},\frac{02}{10},\frac{02}{10},\frac{03}{10},\frac{06}{10},\frac{06}{10},\frac{06}{10}\rangle, \\ \langle x_2,\frac{01}{10},\frac{05}{10},\frac{05}{10},\frac{01}{10},\frac{05}{10},\frac{06}{10},\frac{05}{10}\rangle, \\ \langle x_3,\frac{04}{10},\frac{03}{10},\frac{03}{10},\frac{04}{10},\frac{05}{10},\frac{06}{10},\frac{05}{10}\rangle) \end{bmatrix}$$

$$(\tilde{F},E)\tilde{\vee}(\tilde{G},E)=\begin{bmatrix} (e_1,e_1)=(\langle x_1,\frac{04}{10},\frac{03}{10},\frac{03}{10},\frac{04}{10},\frac{06}{10},\frac{06}{10},\frac{06}{10}\rangle, \\ \langle x_2,\frac{04}{10},\frac{05}{10},\frac{05}{10},\frac{04}{10},\frac{03}{10},\frac{06}{10},\frac{03}{10}\rangle, \\ \langle x_3,\frac{03}{10},\frac{05}{10},\frac{05}{10},\frac{03}{10},\frac{02}{10},\frac{06}{10},\frac{02}{10}\rangle), \\ (e_1,e_2)=(\langle x_1,\frac{02}{10},\frac{03}{10},\frac{03}{10},\frac{02}{10},\frac{08}{10},\frac{07}{10},\frac{08}{10}\rangle, \\ \langle x_2,\frac{02}{10},\frac{04}{10},\frac{04}{10},\frac{02}{10},\frac{04}{10},\frac{06}{10},\frac{04}{10}\rangle, \\ \langle x_3,\frac{04}{10},\frac{03}{10},\frac{03}{10},\frac{04}{10},\frac{04}{10},\frac{06}{10},\frac{04}{10}\rangle), \\ (e_2,e_1)=(\langle x_1,\frac{04}{10},\frac{3}{10},\frac{03}{10},\frac{04}{10},\frac{06}{10},\frac{06}{10},\frac{06}{10}\rangle, \\ \langle x_2,\frac{04}{10},\frac{05}{10},\frac{05}{10},\frac{04}{10},\frac{03}{10},\frac{06}{10},\frac{03}{10}\rangle, \\ \langle x_3,\frac{04}{10},\frac{05}{10},\frac{05}{10},\frac{04}{10},\frac{02}{10},\frac{06}{10},\frac{02}{10}\rangle), \\ (e_2,e_2)=(\langle x_1,\frac{03}{10},\frac{04}{10},\frac{04}{10},\frac{03}{10},\frac{05}{10},\frac{06}{10},\frac{05}{10}\rangle, \\ \langle x_2,\frac{02}{10},\frac{06}{10},\frac{06}{10},\frac{04}{10},\frac{04}{10},\frac{06}{10},\frac{04}{10}\rangle, \\ \langle x_3,\frac{04}{10},\frac{06}{10},\frac{06}{10},\frac{04}{10},\frac{03}{10}\frac{06}{10},\frac{03}{10}\rangle) \end{bmatrix}$$

Definition 21. *Let the hepta-partitioned neutrosophic soft set* $(\tilde{X}, E)$ *be the family of all hepta-partitioned neutrosophic soft sets, and let* $\sigma \subset HPNSS(\tilde{X}, E)$. *Then,* σ *is a hepta-partitioned neutrosophic soft topology (HPNST) on* $\tilde{X}$ *if:*

(i) $0_{(\langle X\rangle,E)}$, $1_{(\langle X\rangle,E)} \in \sigma$,

(ii) The union of any number of hepta-partitioned neutrosophic soft sets in σ *belongs to* σ,

(iii) The intersection of a finite number of hepta-partitioned neutrosophic soft sets in σ *belongs to* σ.

Then, $(\tilde{X}, \sigma, E)$ *is said to be a hepta-partitioned neutrosophic soft topological space (HPNSTS) over* $\tilde{X}$.

Definition 22. *If* $(\tilde{X}, \sigma, E)$ *be a hepta-partitioned neutrosophic soft sets over* $\tilde{X}$.*HPNSS* $(\tilde{F}, E)$ *is hepta-partitioned neutrosophic soft neighborhood of HPNS point* $x^e_{\langle \Delta_1,\Delta_2,\Delta_3,\Delta_4,\Delta_5,\Delta_6,\Delta-7\rangle} \in (\tilde{F}, E)$, *if there is a HPNS open set* $(\tilde{G}, E)$ *such that* $x^e_{\langle \Delta_1,\Delta_2,\Delta_3,\Delta_4,\Delta_5,\Delta_6,\Delta-7\rangle} \in (\tilde{G}, E)$.

Definition 23. *If* (X, σ_1, E) *and* (X, σ_2, E) *be two hepta-partitioned neutrosophic soft topological spaces (HPNSTSs) then,* $(X, \sigma_1, \sigma_2, E)$ *is called a hepta-partitioned neutrosophic soft bitopological space (HPNSBTS). If* $(X, \sigma_1, \sigma_2, E)$ *is a HPNSBTS. A HPNS subset* $(\tilde{F}, E)$ *is open in* $(X, \sigma_1, \sigma_2, E)$ *if there is a HPNSS open set* $(\tilde{G}, E) \in \sigma_1$ *and a HPNSS open set* $(\tilde{H}, E) belongs to \sigma_2$ *such that:*

$$(\tilde{F}, E) = (\tilde{G}, E) \Cup (\tilde{H}, E).$$

Example 2. *Let* $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ *and*

$$\sigma_1 = \{0(X, E), 1(X, E), (\tilde{F}, E), (\tilde{G}, E)\},$$
$$\sigma_2 = \{0(X, E), 1(X, E), (\tilde{H}, E), (\tilde{I}, E)\}.$$

Where $(\tilde{F}, E)$, $(\tilde{G}, E)$, $(\tilde{H}, E)$, *and* $(\tilde{I}, E)$ *being HPNSSs are as follows:*

$$(\tilde{F}, E) = \begin{bmatrix} e_1 = (\langle x_1, \frac{02}{10}, \frac{03}{10}, \frac{03}{10}, \frac{02}{10}, \frac{08}{10}, \frac{07}{10}, \frac{08}{10}\rangle, \\ \langle x_2, \frac{04}{10}, \frac{04}{10}, \frac{04}{10}, \frac{04}{10}, \frac{04}{10}, \frac{06}{10}, \frac{04}{10}\rangle, \\ \langle x_3, \frac{02}{10}, \frac{04}{10}, \frac{04}{10}, \frac{02}{10}, \frac{03}{10}, \frac{06}{10}, \frac{03}{10}\rangle); \\ e_2 = (\langle x_1, \frac{03}{10}, \frac{02}{10}, \frac{02}{10}, \frac{03}{10}, \frac{06}{10}, \frac{06}{10}, \frac{06}{10}\rangle, \\ \langle x_2, \frac{01}{10}, \frac{05}{10}, \frac{05}{10}, \frac{01}{10}, \frac{05}{10}, \frac{06}{10}, \frac{05}{10}\rangle, \\ \langle x_3, \frac{04}{10}, \frac{03}{10}, \frac{03}{10}, \frac{04}{10}, \frac{05}{10}, \frac{06}{10}, \frac{05}{10}\rangle). \end{bmatrix}$$

$$(\tilde{G}, E) = \begin{bmatrix} e_1 = (\langle x_1, \frac{04}{10}, \frac{03}{10}, \frac{03}{10}, \frac{04}{10}, \frac{06}{10}, \frac{06}{10}, \frac{06}{10}\rangle, \\ \langle x_2, \frac{04}{10}, \frac{05}{10}, \frac{05}{10}, \frac{04}{10}, \frac{03}{10}, \frac{06}{10}, \frac{03}{10}\rangle, \\ \langle x_3, \frac{03}{10}, \frac{05}{10}, \frac{05}{10}, \frac{03}{10}, \frac{02}{10}, \frac{06}{10}, \frac{02}{10}\rangle); \\ e_2 = (\langle x_1, \frac{03}{10}, \frac{04}{10}, \frac{04}{10}, \frac{03}{10}, \frac{05}{10}, \frac{06}{10}, \frac{05}{10}\rangle, \\ \langle x_2, \frac{02}{10}, \frac{06}{10}, \frac{06}{10}, \frac{02}{10}, \frac{04}{10}, \frac{06}{10}, \frac{04}{10}\rangle, \\ \langle x_3, \frac{04}{10}, \frac{06}{10}, \frac{06}{10}, \frac{04}{10}, \frac{03}{10}, \frac{06}{10}, \frac{03}{10}\rangle). \end{bmatrix}$$

$$(\tilde{H},E)=\begin{bmatrix} e_1=(\langle x_1,\frac{06}{10},\frac{06}{10},\frac{06}{10},\frac{06}{10},\frac{02}{10},\frac{06}{10},\frac{02}{10}\rangle, \\ \langle x_2,\frac{06}{10},\frac{06}{10},\frac{06}{10},\frac{06}{10},\frac{02}{10},\frac{06}{10},\frac{02}{10}\rangle, \\ \langle x_3,\frac{04}{10},\frac{06}{10},\frac{06}{10},\frac{04}{10},\frac{01}{10},\frac{06}{10},\frac{01}{10}\rangle), \\ e_2=(\langle x_1,\frac{05}{10},\frac{06}{10},\frac{06}{10},\frac{05}{10},\frac{02}{10},\frac{06}{10},\frac{02}{10}\rangle, \\ \langle x_2,\frac{06}{10},\frac{07}{10},\frac{07}{10},\frac{06}{10},\frac{02}{10},\frac{06}{10},\frac{02}{10}\rangle, \\ \langle x_3,\frac{05}{10},\frac{05}{10},\frac{05}{10},\frac{05}{10},\frac{01}{10},\frac{06}{10},\frac{01}{10}\rangle). \end{bmatrix}$$

$$(\tilde{I},E)=\begin{bmatrix} e_1=(\langle x_1,\frac{01}{10},\frac{02}{10},\frac{02}{10},\frac{01}{10},\frac{07}{10},\frac{06}{10},\frac{07}{10}\rangle, \\ \langle x_2,\frac{04}{10},\frac{04}{10},\frac{04}{10},\frac{04}{10},\frac{03}{10},\frac{06}{10},\frac{03}{10}\rangle, \\ \langle x_3,\frac{02}{10},\frac{04}{10},\frac{04}{10},\frac{02}{10},\frac{02}{10},\frac{06}{10},\frac{02}{10}\rangle), \\ e_2=(\langle x_1,\frac{03}{10},\frac{02}{10},\frac{02}{10},\frac{03}{10},\frac{05}{10},\frac{06}{10},\frac{05}{10}\rangle, \\ \langle x_2,\frac{01}{10},\frac{05}{10},\frac{05}{10},\frac{01}{10},\frac{05}{10},\frac{06}{10},\frac{05}{10}\rangle, \\ \langle x_3,\frac{04}{10},\frac{03}{10},\frac{03}{10},\frac{04}{10},\frac{05}{10},\frac{06}{10},\frac{05}{10}\rangle). \end{bmatrix}.$$

$$(\tilde{J},E)=\begin{bmatrix} e_1=(\langle x_1,\frac{01}{10},\frac{02}{10},\frac{02}{10},\frac{01}{10},\frac{07}{10},\frac{07}{10},\frac{07}{10}\rangle, \\ \langle x_2,\frac{04}{10},\frac{04}{10},\frac{04}{10},\frac{04}{10},\frac{03}{10},\frac{03}{10},\frac{03}{10}\rangle, \\ \langle x_3,\frac{02}{10},\frac{04}{10},\frac{04}{10},\frac{02}{10},\frac{02}{10},\frac{02}{10},\frac{02}{10}\rangle), \\ e_2=(\langle x_1,\frac{03}{10},\frac{02}{10},\frac{02}{10},\frac{03}{10},\frac{05}{10},\frac{05}{10},\frac{05}{10}\rangle, \\ \langle x_2,\frac{01}{10},\frac{05}{10},\frac{05}{10},\frac{01}{10},\frac{05}{10},\frac{05}{10},\frac{05}{10}\rangle, \\ \langle x_3,\frac{04}{10},\frac{03}{10},\frac{03}{10},\frac{04}{10},\frac{05}{10},\frac{05}{10},\frac{05}{10}\rangle). \end{bmatrix}$$

Theorem 1. *Let (X,σ_1,σ_2,E) be a HPNSBTS. Then their intersection that is $\sigma_1 \Cap \sigma_2$ is a HPNSBTS on X.*

Proof. The first and third requirements are clear, and we move forward as follows for the second condition.
Let $\{(\tilde{F}_i,E); i \in I\} \in \sigma_1 \Cap \sigma_2$. Then $(\tilde{F}_i,E) \in \sigma_1$ and $(\tilde{F}_i,E) \in \sigma_2$. Since σ_1 and σ_2 are HPNSBTSs on X, it follows that:

$$\Cup_{i\in I}(\tilde{F}_i,E) \in \sigma_1, \quad \Cup_{i\in I}(\tilde{F}_i,E) \in \sigma_2.$$

So

$$\Cup_{i\in I}(\tilde{F}_i,E) \in \sigma_1 \Cap \sigma_2.$$

Remark 1. *Let (X,σ_1,σ_2,E) be a HPNSBTS, then their union that ia $\sigma_1 \Cup \sigma_2$ need not be a HPNSBTS on X.*

Example 3. *Let* $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$,

$$\sigma_1 = \{0(X,E), 1(X,E), (\tilde{F},E), (\tilde{G},E), (\tilde{H},E)\},$$
$$\sigma_2 = \{0(X,E), 1(X,E), (\tilde{I},E), (\tilde{J},E)\}.$$

Where $(\tilde{F},E)$, $(\tilde{G},E)$, $(\tilde{H},E)$, $(\tilde{I},E)$ *and* $(\tilde{J},E)$ *being HPNS sub set are as follows:*

$$(\tilde{F},E) = \begin{bmatrix} e_1 = (\langle x_1, \frac{02}{10}, \frac{03}{10}, \frac{03}{10}, \frac{02}{10}, \frac{08}{10}, \frac{08}{10}, \frac{08}{10}\rangle, \\ \langle x_2, \frac{04}{10}, \frac{04}{10}, \frac{04}{10}, \frac{04}{10}, \frac{04}{10}, \frac{04}{10}, \frac{04}{10}\rangle, \\ \langle x_3, \frac{02}{10}, \frac{04}{10}, \frac{04}{10}, \frac{02}{10}, \frac{03}{10}, \frac{03}{10}, \frac{03}{10}\rangle); \\ e_2 = (\langle x_1, \frac{03}{10}, \frac{02}{10}, \frac{02}{10}, \frac{03}{10}, \frac{06}{10}, \frac{06}{10}, \frac{06}{10}\rangle, \\ \langle x_2, \frac{01}{10}, \frac{05}{10}, \frac{05}{10}, \frac{01}{10}, \frac{05}{10}, \frac{05}{10}, \frac{05}{10}\rangle, \\ \langle x_3, \frac{04}{10}, \frac{03}{10}, \frac{03}{10}, \frac{04}{10}, \frac{05}{10}, \frac{05}{10}, \frac{05}{10}\rangle). \end{bmatrix}$$

$$(\tilde{G},E) = \begin{bmatrix} e_1 = (\langle x_1, \frac{04}{10}, \frac{03}{10}, \frac{03}{10}, \frac{04}{10}, \frac{06}{10}, \frac{06}{10}, \frac{06}{10}\rangle, \\ \langle x_2, \frac{04}{10}, \frac{05}{10}, \frac{05}{10}, \frac{04}{10}, \frac{03}{10}, \frac{03}{10}, \frac{03}{10}\rangle, \\ \langle x_3, \frac{03}{10}, \frac{05}{10}, \frac{05}{10}, \frac{03}{10}, \frac{02}{10}, \frac{02}{10}, \frac{02}{10}\rangle); \\ e_2 = (\langle x_1, \frac{03}{10}, \frac{04}{10}, \frac{04}{10}, \frac{03}{10}, \frac{05}{10}, \frac{05}{10}, \frac{05}{10}\rangle, \\ \langle x_2, \frac{02}{10}, \frac{06}{10}, \frac{06}{10}, \frac{02}{10}, \frac{04}{10}, \frac{04}{10}, \frac{04}{10}\rangle, \\ \langle x_3, \frac{04}{10}, \frac{06}{10}, \frac{06}{10}, \frac{04}{10}, \frac{03}{10}, \frac{03}{10}, \frac{03}{10}\rangle). \end{bmatrix}$$

$$(\tilde{H},E) = \begin{bmatrix} e_1 = (\langle x_1, \frac{05}{10}, \frac{04}{10}, \frac{04}{10}, \frac{05}{10}, \frac{04}{10}, \frac{04}{10}, \frac{04}{10}\rangle, \\ \langle x_2, \frac{06}{10}, \frac{06}{10}, \frac{06}{10}, \frac{06}{10}, \frac{02}{10}, \frac{02}{10}, \frac{02}{10}\rangle, \\ \langle x_3, \frac{04}{10}, \frac{06}{10}, \frac{06}{10}, \frac{04}{10}, \frac{01}{10}, \frac{01}{10}, \frac{01}{10}\rangle), \\ e_2 = (\langle x_1, \frac{04}{10}, \frac{06}{10}, \frac{06}{10}, \frac{04}{10}, \frac{03}{10}, \frac{03}{10}, \frac{03}{10}\rangle, \\ \langle x_2, \frac{03}{10}, \frac{07}{10}, \frac{07}{10}, \frac{03}{10}, \frac{03}{10}, \frac{03}{10}, \frac{03}{10}\rangle, \\ \langle x_3, \frac{05}{10}, \frac{07}{10}, \frac{07}{10}, \frac{05}{10}, \frac{01}{10}, \frac{01}{10}, \frac{01}{10}\rangle). \end{bmatrix}$$

$$(\tilde{I},E) = \begin{bmatrix} e_1 = (\langle x_1, \frac{06}{10}, \frac{06}{10}, \frac{06}{10}, \frac{06}{10}, \frac{02}{10}, \frac{02}{10}, \frac{02}{10}\rangle, \\ \langle x_2, \frac{06}{10}, \frac{06}{10}, \frac{06}{10}, \frac{06}{10}, \frac{02}{10}, \frac{02}{10}, \frac{02}{10}\rangle, \\ \langle x_3, \frac{04}{10}, \frac{06}{10}, \frac{06}{10}, \frac{04}{10}, \frac{01}{10}, \frac{01}{10}, \frac{01}{10}\rangle), \\ e_2 = (\langle x_1, \frac{05}{10}, \frac{06}{10}, \frac{06}{10}, \frac{05}{10}, \frac{02}{10}, \frac{02}{10}, \frac{02}{10}\rangle, \\ \langle x_2, \frac{06}{10}, \frac{07}{10}, \frac{07}{10}, \frac{06}{10}, \frac{02}{10}, \frac{02}{10}, \frac{02}{10}\rangle, \\ \langle x_3, \frac{05}{10}, \frac{05}{10}, \frac{05}{10}, \frac{05}{10}, \frac{01}{10}, \frac{01}{10}, \frac{01}{10}\rangle). \end{bmatrix}.$$

$$(\tilde{J},E)=\begin{bmatrix} e_1=(\langle x_1,\frac{01}{10},\frac{02}{10},\frac{02}{10},\frac{01}{10},\frac{07}{10},\frac{07}{10},\frac{07}{10}\rangle,\\ \langle x_2,\frac{04}{10},\frac{04}{10},\frac{04}{10},\frac{04}{10},\frac{03}{10},\frac{03}{10},\frac{03}{10}\rangle,\\ \langle x_3,\frac{02}{10},\frac{04}{10},\frac{04}{10},\frac{02}{10},\frac{02}{10},\frac{02}{10},\frac{02}{10}\rangle),\\ e_2=(\langle x_1,\frac{03}{10},\frac{02}{10},\frac{02}{10},\frac{03}{10},\frac{05}{10},\frac{05}{10},\frac{05}{10}\rangle,\\ \langle x_2,\frac{01}{10},\frac{05}{10},\frac{05}{10},\frac{01}{10},\frac{05}{10},\frac{05}{10},\frac{05}{10}\rangle,\\ \langle x_3,\frac{04}{10},\frac{03}{10},\frac{03}{10},\frac{04}{10},\frac{05}{10},\frac{05}{10},\frac{05}{10}\rangle). \end{bmatrix}$$

Here, $\sigma_1 \Cup \sigma_2 = \{0_{(\tilde{X},E)}, 1_{(\tilde{X},E)}, (\tilde{F},E), (\tilde{G},E), (\tilde{H},E), (\tilde{I},E), (\tilde{J},E)\}$ is not a HPNSBTSs on $\tilde{X}$ as $(\tilde{H},E) \Cup (\tilde{I},E)$ does not belong to $\sigma_1 \Cup \sigma_2$.

Definition 24. *Let (X,σ_1,σ_2,E) be a HPNSBTS, then a HPNSS*

$$(\tilde{H},E)=\left[\left(e,\left\langle x, AbT_{\tilde{H}(e)}(x), ReT_{\tilde{H}(e)}(x), C_{\tilde{H}(e)}(x) ReF_{\tilde{H}(e)}(x), AbF_{\tilde{H}(e)}(x) : x\in X\right\rangle\right) : e\in E\right]$$

is a pairwise PPNS open set if there is a PPNS open set $(\tilde{F},E)$ in σ_1 and a PPNS open set $(\tilde{G},E)$ in σ_2 such that for all $x\in X$,

$$(\tilde{H},E)=(\tilde{F},E)\Cup(\tilde{G},E)=\left\{\left(e,\left\{x,\begin{array}{c} AbT_{\tilde{H}(e)}(x)=\max[AbT_{\tilde{F}(e)}(x), AbT_{\tilde{G}(e)}(x)],\\ ReT_{\tilde{H}(e)}(x)=\max[ReT_{\tilde{F}(e)}(x), ReT_{\tilde{G}(e)}(x)],\\ C_{\tilde{H}(e)}(x)=\max[C_{\tilde{F}(e)}(x), C_{\tilde{G}(e)}(x)],\\ ReF_{\tilde{H}(e)}(x)=\min[ReF_{\tilde{F}(e)}(x), ReF_{\tilde{G}(e)}(x)],\\ AbF_{\tilde{H}(e)}(x)=\min[AbF_{\tilde{F}(e)}(x), AbF_{\tilde{G}(e)}(x)] \end{array}\right\}\right) : e\in E\right\}.$$

This is denoted by HPNSO(X,E).

Definition 25. *Let (X,σ_1,σ_2,E) be a HPNSBTS then a HPNSS.*

$$(\tilde{H},E)=\left[\left(e,\left\langle\left\langle x,\begin{array}{c} AbT_{\tilde{H}(e)}(x), ReT_{\tilde{H}(e)}(x), C_{\tilde{H}(e)}(x), U_{\tilde{H}(e)}(x),\\ G_{\tilde{H}(e)}(x), ReF_{\tilde{H}(e)}(x), AbF_{\tilde{H}(e)}(x) : x\in X \end{array}\right\rangle\right\rangle\right) : e\in E\right]$$

is called a pairwise HPNSCS if $(\tilde{H},E)^c$ is a pairwise HPNSO. $(\tilde{H},E)$ is a HPNS closed set if there exists a HPNS closed set $(\tilde{F},E)$ in σ_1 and a HPNS closed set $(\tilde{G},E)$ in σ_2 such that for all $x\in X$,

$$(\tilde{H},E)=(\tilde{F},\acute{E})\Cap(\tilde{G},E)=\left\{\left(e,\left\{x,\begin{array}{c} AbT_{\tilde{H}(e)}(x)=\min[AbT_{\tilde{F}(e)}(x), AbT_{\tilde{G}(e)}(x)],\\ ReT_{\tilde{H}(e)}(x)=\min[ReT_{\tilde{F}(e)}(x), ReT_{\tilde{G}(e)}(x)],\\ C_{\tilde{H}(e)}(x)=\min[C_{\tilde{F}(e)}(x), C_{\tilde{G}(e)}(x)],\\ U_{\tilde{H}(e)}(x)=\min[U_{\tilde{F}(e)}(x), U_{\tilde{G}(e)}(x)],\\ G_{\tilde{H}(e)}(x)=\min[G_{\tilde{F}(e)}(x), G_{\tilde{G}(e)}(x)],\\ ReF_{\tilde{H}(e)}(x)=\max[ReF_{\tilde{F}(e)}(x), ReF_{\tilde{G}(e)}(x)],\\ AbF_{\tilde{H}(e)}(x)=\max[AbF_{\tilde{F}(e)}(x), AbF_{\tilde{G}(e)}(x)] \end{array}\right\}\right) : e\in E\right\}.$$

This is denoted by HPNSC(X,E).

Definition 26. *Let $(X, \sigma_1, \sigma_2, E)$ be a HPNSBTS over X, and $(\tilde{\ddot{F}}, E)$ be a HPNSS then:*

(i) $(\tilde{\ddot{F}}, E)$ *is HPNS semi-open if* $(\tilde{\ddot{F}}, E) \subseteq NScl(NSint(\tilde{\ddot{F}}, E))$.

(ii) $(\tilde{\ddot{F}}, E)$ *is HPNS pre-open (p-open) if* $(\tilde{\ddot{F}}, E) \subseteq NSint(NScl(\tilde{\ddot{F}}, E))$.

(iii) $(\tilde{\ddot{F}}, E)$ *is HPNS $*b$ open if*

$$(\tilde{\ddot{F}}, E) \subseteq NScl(NSint(\tilde{\ddot{F}}, E)) \Cup NSint(NScl(\tilde{\ddot{F}}, E)),$$

and HPNS $*_b$ close if

$$(\tilde{\ddot{F}}, E) \supseteq NScl(NSint(\tilde{\ddot{F}}, E)) \Cap NSint(NScl(\tilde{\ddot{F}}, E)).$$

Definition 27. *Let $(X, \sigma_1, \sigma_2, E)$ be a HPNSBTS over X, and $(\tilde{\ddot{F}}, E)$ be a HPNS, then interior of $(\tilde{\ddot{F}}, E)$, denoted by $(\tilde{\ddot{F}}, E)^\circ$, is the union of all HPNS p-open sets of $(\tilde{\ddot{F}}, E)$. Clearly, $(\tilde{\ddot{F}}, E)^\circ$ is the largest HPNS p-open set contained in $(\tilde{\ddot{F}}, E)$.*

Definition 28. *Let $(X, \sigma_1, \sigma_2, E)$ be a HPNSBTS, and $(\tilde{\ddot{F}}, E)$ be a HPNS, the frontier of $(\tilde{\ddot{F}}, E)$ denoted by $Fr((\tilde{\ddot{F}}, E))$, is a HPNS point $x^e_{\langle \Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6, \Delta_7 \rangle}$ such that every HPNS p-open set comprising $x^e_{\langle \Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6, \Delta_7 \rangle}$ comprises at least one point of $(\tilde{\ddot{F}}, E)$ and at least one HPNS point of $(\tilde{\ddot{F}}, E)^c$.*

Definition 29. *If $(X, \sigma_1, \sigma_2, E)$ is a HPNSBTS and $(\tilde{\ddot{F}}, E)$ is a HPNS, then the exterior of $(\tilde{\ddot{F}}, E)$, denoted by $Ext((\tilde{\ddot{F}}, E))$, is a HPNS point $x^e_{\langle \Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6, \Delta_7 \rangle}$ is called exterior of $(\tilde{\ddot{F}}, E)$ if $x^e_{\langle \Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6, \Delta_7 \rangle}$ is in the interior of $(\tilde{\ddot{F}}, E)^c$, that is HPNS p-open set $(\tilde{g}, E)$ such that*

$$x^e_{\langle \Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6, \Delta_7 \rangle} \in (\tilde{g}, E) \subseteq (\tilde{\ddot{F}}, E)^c.$$

Definition 30. *If $(\tilde{X}, \sigma_1, \sigma_2, E)$ and $(\langle \tilde{Y} \rangle, \mathfrak{F}_1, \mathfrak{F}_2, E)$ are HPNSBTSs, and $(f, \phi) : (\tilde{X}, \sigma_1, \sigma_2, E) \to (\langle \tilde{Y} \rangle, \mathfrak{F}_1, \mathfrak{F}_2, E)$ is a HPNS mapping, then (f, ϕ) is said to be a HPNS p-close mapping if the image $(f, \phi)(\tilde{\ddot{F}}, E)$ of each HPNS p-closed set $(\tilde{\ddot{F}}, E)$ over $\tilde{X}$ is a HPNS p-closed set in $\langle \tilde{Y} \rangle$.*

Theorem 2. *Let $(X, \sigma_1, \sigma_2, E)$ be a HPNSBTSs over X and $(\tilde{\ddot{F}}, E)$ is PPNS subset. Then, $(\tilde{\ddot{F}}, E)$ is a PPNS p-open set if and only if $(\tilde{\ddot{F}}, E) = (\tilde{\ddot{F}}, E)^\circ$.*

Proof. Let $(\tilde{\ddot{F}}, E)$ be a HPNS p-open set. Then, the largest HPNS p-open set surrounded by $(\tilde{\ddot{F}}, E)$ is equal to $(\tilde{\ddot{F}}, E)$. Hence, $(\tilde{\ddot{F}}, E) = (\tilde{\ddot{F}}, E)^\circ$.

Contrariwise, it is known that $(\tilde{\ddot{F}}, E)^\circ$ is a HPNS p-open set, and if $(\tilde{\ddot{F}}, E) = (\tilde{\ddot{F}}, E)^\circ$, then $(\tilde{\ddot{F}}, E)$ is a HPNS p-open set.

Theorem 3. *Let $(X, \sigma_1, \sigma_2, E)$ be a HPNSBTS over X, and let $(\tilde{\pounds}, E)$ and $(\tilde{\Im}, E)$ be HPNS subsets then*

(i) $[(\tilde{\pounds}, E)^\circ]^\circ = (\tilde{\pounds}, E)^\circ$,

(ii) $(0_{(\langle \tilde{X} \rangle, E)})^\circ = 0_{(\langle \tilde{X} \rangle, E)}$ *and* $(1_{(\langle \tilde{X} \rangle, E)})^\circ = 1_{(\langle \tilde{X} \rangle, E)}$,

(iii) $(\tilde{\pounds}, E) \subseteq (\tilde{\Im}, E) \Rightarrow (\tilde{\pounds}, E)^\circ \subseteq (\tilde{\Im}, E)^\circ$,

(iv) $[(\tilde{\pounds}, E) \Cap (\tilde{\Im}, E)]^\circ = (\tilde{\pounds}, E)^\circ \Cap (\tilde{\Im}, E)^\circ$,

(v) $(\tilde{\pounds}, E)^{\circ} \Cup (\tilde{\Im}, E)^{\circ} \subseteq [(\tilde{\pounds}, E) \Cup (\tilde{\Im}, E)]^{\circ}$.

Proof.

(i) $(\pounds,\tilde{} E^{\circ} = (\tilde{\Im}, E)$ then $(\tilde{\Im}, E) \in \tilde{\sigma}$ iff $(\tilde{\Im}, E) = (\pounds,\tilde{} E^{\circ}$. So $[(\tilde{\pounds}, E)^{\circ}]^{\circ} = (\tilde{\pounds}, E)^{\circ}$

(ii) Since $0_{(\langle\tilde{X}\rangle,E)}$ and $1_{(\langle\tilde{X}\rangle,E)}$ are always HPNS p-open sets, so

$$(0_{(\langle\tilde{X}\rangle,E)})^{\circ} = 0_{(\langle\tilde{X}\rangle,E)}, \quad \text{and} \quad (1_{(\langle\tilde{X}\rangle,E)})^{\circ} = 1_{(\langle\tilde{X}\rangle,E)}.$$

(iii) It is known that $(\tilde{\pounds}, E)^{\circ} \subseteq (\tilde{\pounds}, E) \subseteq (\tilde{\Im}, E)$ and $(\tilde{\Im}, E)^{\circ} \subseteq (\tilde{\Im}, E)$. Since $(\tilde{\Im}, E)^{\circ}$ is the biggest HPNS p-open set enclosed in $(\tilde{\Im}, E)$ and so, $(\tilde{\pounds}, E)^{\circ} \subseteq (\tilde{\Im}, E)^{\circ}$.

(iv) Since $(\tilde{\pounds}, E) \Cap (\tilde{\Im}, E) \subseteq (\tilde{\pounds}, E)$ and $(\tilde{\pounds}, E) \Cap (\tilde{\Im}, E) \subseteq (\tilde{\Im}, E)$, then

$$[(\tilde{\pounds}, E) \Cap (\tilde{\Im}, E)]^{\circ} \subseteq (\tilde{\pounds}, E)^{\circ} and [(\tilde{\pounds}, E) \Cap (\tilde{\Im}, E)]^{\circ} \subseteq (\tilde{\Im}, E)^{\circ}.$$

so,

$$[(\tilde{\pounds}, E) \Cap (\tilde{\Im}, E)]^{\circ} \subseteq (\tilde{\pounds}, E)^{\circ} \Cap (\tilde{\Im}, E)^{\circ}.$$

On other way, since $(\tilde{\pounds}, E)^{\circ} \subseteq (\tilde{\pounds}, E)$ and $(\tilde{\Im}, E)^{\circ} \subseteq (\tilde{\Im}, E)$, then

$$(\tilde{\pounds}, E)^{\circ} \Cap (\tilde{\Im}, E)^{\circ} \subseteq (\tilde{\pounds}, E) \Cap (\tilde{\Im}, E).$$

also $[(\tilde{\pounds}, E) \Cap (\tilde{\Im}, E)]^{\circ} \subseteq (\tilde{\pounds}, E) \Cap (\tilde{\Im}, E)$ is the biggest PPNS p-open set.
$\Rightarrow (\tilde{\pounds}, E)^{\circ} \Cap (\tilde{\Im}, E)^{\circ} \subseteq [(\tilde{\pounds}, E) \Cap (\tilde{\Im}, E)]^{\circ}$
Thus,

$$[(\tilde{\pounds}, E) \Cap (\tilde{\Im}, E)]^{\circ} = (\tilde{\pounds}, E)^{\circ} \Cap (\tilde{\Im}, E)^{\circ}.$$

(v) Since $(\tilde{\pounds}, E) \subseteq (\tilde{\pounds}, E) \Cup (\tilde{\Im}, E)$ and $(\tilde{\Im}, E) \subseteq (\tilde{\pounds}, E) \Cup (\tilde{\Im}, E)$, then

$$(\tilde{\pounds}, E)^{\circ} \subseteq [(\tilde{\pounds}, E) \Cup (\tilde{\Im}, E)]^{\circ}, \quad \text{and} \quad (\tilde{\Im}, E)^{\circ} \subseteq [(\tilde{\pounds}, E) \Cup (\tilde{\Im}, E)]^{\circ}.$$

$\Rightarrow (\tilde{\pounds}, E)^{\circ} \Cup (\tilde{\Im}, E)^{\circ} \subseteq [(\tilde{\pounds}, E) \Cup (\tilde{\Im}, E)]^{\circ}$.

Theorem 4. *Let $(X, \sigma_1, \sigma_2, E)$ be a HPNSBTS over X, and if $(\tilde{\pounds}, E)$ is a HPNS subset, then $(\tilde{\pounds}, E)$ is a HPNS p-closer set if and only if $(\tilde{\pounds}, E) = \overline{(\tilde{\pounds}, E)}$.*

Proof. Let $(\tilde{\pounds}, E)$ is a HPNS p-closer set, then:

$$(\tilde{\pounds}, E)^{d} = (\tilde{\pounds}, E)$$

this implies that

$$(\tilde{\pounds}, E) \Cup (\tilde{\pounds}, E)^{d} \cong (\tilde{\pounds}, E)$$

$\Rightarrow$

$$\overline{(\tilde{\pounds}, E)} \cong (\tilde{\pounds}, E)$$

and conversely let $\overline{(\tilde{\pounds}, E)} \cong (\tilde{\pounds}, E)$, this implies that

$$(\tilde{\pounds}, E) \Cup (\tilde{\pounds}, E)^{d} \cong (\tilde{\pounds}, E)$$

$\Rightarrow$

$$(\tilde{\pounds}, E)^{d} = (\tilde{\pounds}, E)$$

this implies, $(\tilde{\pounds}, E)$ is a HPNS p-closer set.

Theorem 5. *Let $(X, \sigma_1, \sigma_2, E)$ be a HPNSBTS over X, and let $(\tilde{\mathcal{L}}, E)$ and $(\tilde{\Im}, E)$ be HPNS subsets then*

(i) $\overline{\overline{(\tilde{\mathcal{L}}, E)}} = \overline{(\tilde{\mathcal{L}}, E)}$,

(ii) $\overline{(0_{(\langle\tilde{X}\rangle,E)})} = 0_{(\tilde{X},E)}$ *and* $\overline{(1_{(\langle\tilde{X}\rangle,E)})} = \overline{(1_{(\langle\tilde{X}\rangle,E)})}$,

(iii) $(\tilde{\mathcal{L}}, E) \subseteq \langle(\tilde{\Im}, E)\rangle \Rightarrow \overline{(\tilde{\mathcal{L}}, E)} \subseteq \overline{(\tilde{\Im}, E)}$,

(iv) $\overline{[(\tilde{\mathcal{L}}, E) \Cup (\tilde{\Im}, E)]} = \overline{(\tilde{\mathcal{L}}, E)} \Cup \overline{(\tilde{\Im}, E)}$,

(v) $\overline{[(\tilde{\mathcal{L}}, E) \Cap (\tilde{\Im}, E)]} \subseteq \overline{(\tilde{\mathcal{L}}, E)} \Cap \overline{(\tilde{\Im}, E)}$.

Proof.

(i) If $\overline{(\tilde{\mathcal{L}}, E)} = (\tilde{\Im}, E)$ then $(\tilde{\Im}, E)$ is a HPNS p-closer set.Hence, if $(\tilde{\Im}, E) = \overline{(\tilde{\Im}, E)}$. Therefore $\overline{[\overline{(\tilde{\mathcal{L}}, E)}]} = \overline{(\tilde{\mathcal{L}}, E)}$

(ii) Since $0_{(\langle\tilde{X}\rangle,E)}$ and $1_{(\langle\tilde{X}\rangle,E)}$ are always HPNS p-closure set so by the above result

$$\overline{(0_{(\langle\tilde{X}\rangle,E)})} = 0_{(\langle\tilde{X}\rangle,E)}, \quad \text{and} \quad \overline{(1_{(\langle\tilde{X}\rangle,E)})} = 1_{(\langle\tilde{X}\rangle,E)}.$$

(iii) Since $(\tilde{\mathcal{L}}, E) \subseteq \overline{(\tilde{\mathcal{L}}, E)}$ and $(\tilde{\Im}, E) \subseteq \overline{(\tilde{\Im}, E)}$, so $(\tilde{\mathcal{L}}, E) \subseteq \underline{(\tilde{\Im}, E)} \subseteq \overline{\underline{(\tilde{\Im}, E)}}$ .Since $\overline{(\tilde{\Im}, E)}$ is the smallest HPNS p-closure set covering in $(\tilde{\mathcal{L}}, E)$ then $\overline{(\tilde{\mathcal{L}}, E)} \subseteq \overline{(\tilde{\Im}, E)}$.

(iv) Since $(\tilde{\mathcal{L}}, E) \subseteq (\tilde{\mathcal{L}}, E) \Cup (\tilde{\Im}, E)$ and $(\tilde{\Im}, E) \subseteq 0_{(\langle\tilde{X}\rangle,E)} \Cup (\tilde{\Im}, E)$ then

$$\overline{(\tilde{\mathcal{L}}, E)} \subseteq \overline{[(\tilde{\mathcal{L}}, E) \Cup (\tilde{\Im}, E)]} and \overline{(\tilde{\Im}, E)} \subseteq \overline{[(\tilde{\mathcal{L}}, E) \Cup (\tilde{\Im}, E)]}.$$

so,

$$\overline{(\tilde{\mathcal{L}}, E)} \Cup \overline{(\tilde{\Im}, E)} \subseteq \overline{[(\tilde{\mathcal{L}}, E) \Cup (\tilde{\Im}, E)]}.$$

Conversely, since $(\tilde{\mathcal{L}}, E) \subseteq \overline{(\tilde{\mathcal{L}}, E)}$ and $(\tilde{\Im}, E) \subseteq \overline{(\tilde{\Im}, E)}$, then

$$(\tilde{\mathcal{L}}, E) \Cup (\tilde{\Im}, E) \subseteq \overline{(\tilde{\mathcal{L}}, E)} \Cup \overline{(\tilde{\Im}, E)}.$$

Besides, $\overline{[(\tilde{\mathcal{L}}, E) \Cup (\tilde{\Im}, E)]}$ is the smallest HPNS p-closed set that enclosing $(\tilde{\mathcal{L}}, E) \Cup (\tilde{\Im}, E)$ therefore, $\overline{[(\tilde{\mathcal{L}}, E) \Cup (\tilde{\Im}, E)]} \subseteq \overline{(\tilde{\mathcal{L}}, E)} \Cup \overline{(\tilde{\Im}, E)}$.
Thus, $\overline{[(\tilde{\mathcal{L}}, E) \Cup (\tilde{\Im}, E)]} = \overline{(\tilde{\mathcal{L}}, E)} \Cup \overline{(\tilde{\Im}, E)}$.

(v) Since $\langle(0_{(\langle\tilde{X}\rangle,E)})\rangle \Cap (\tilde{\Im}, E) \subseteq \overline{(\tilde{\mathcal{L}}, E)} \Cap \overline{(\tilde{\Im}, E)}$ and $\overline{[(\tilde{\mathcal{L}}, E) \Cap (\tilde{\Im}, E)]}$ is the smallest HPNS p-closed set that enclosing $(\tilde{\mathcal{L}}, E) \Cap (\tilde{\Im}, E)$, then $\overline{[(\tilde{\mathcal{L}}, E) \Cap (\tilde{\Im}, E)]} \subseteq \overline{(\tilde{\mathcal{L}}, E)} \Cap \overline{(\tilde{\Im}, E)}$.

Theorem 6. *Let $(X, \sigma_1, \sigma_2, E)$ be a HPNSBTSs over X, and let $(\tilde{\mathcal{L}}, E)$ be a HPNSS then,*

(i) $[(\tilde{\mathcal{L}}, E)]^c = [(\tilde{\mathcal{L}}, E)^c]^\circ$,

(ii) $[(\tilde{\mathcal{L}}, E)^\circ]^c = \overline{[(\tilde{\mathcal{L}}, E)^c]}$.

Proof.

(i)

$$\begin{aligned}\overline{(\tilde{\mathcal{L}},E)} &= \Cap\{(\tilde{\Im},E)\in(X,\sigma_1,\sigma_2,E)^c : (\tilde{\Im},E)\supseteq(\tilde{\mathcal{L}},E)\}\\ &\Rightarrow [\overline{(\tilde{\mathcal{L}},E)}]^c = \Big[\Cap\{(\tilde{\Im},E)\in(X,\sigma_1,\sigma_2,E)^c : (\tilde{\Im},E)\supseteq(\tilde{\mathcal{L}},E)\}\Big]^c\\ &= \Cup\{(\tilde{\Im},E)^c\in(X,\sigma_1,\sigma_2,E) : (\tilde{\Im},E)^c\subseteq(\tilde{\mathcal{L}},E)^c\}\\ &= [(\tilde{\mathcal{L}},E)^c]^\circ.\end{aligned}$$

(ii)

$$\begin{aligned}(\tilde{\mathcal{L}},E)^\circ &= \Cup\{(\tilde{\Im},E)\in(X,\sigma_1,\sigma_2,E) : (\tilde{\Im},E)\subseteq(\tilde{\mathcal{L}},E)\}\\ &\Rightarrow [(\tilde{\mathcal{L}},E)^\circ]^c = \Big[\Cap\{(\tilde{\Im},E)\in(pi,\sigma_1,\sigma_2,E) : (\tilde{\Im},E)\subseteq(\tilde{\mathcal{L}},E)\}\Big]^c\\ &= \Cap\{(\tilde{\Im},E)^c\in(X,\sigma_1,\sigma_2,E)^c : (\tilde{\Im},E)^c\supseteq(\tilde{\mathcal{L}},E)^c\}\\ &= \overline{[(\tilde{\mathcal{L}},E)^c]}.\end{aligned}$$

Theorem 7. *Let (X,σ_1,σ_2,E) be a HPNSBTS over X. If $(\tilde{\mathcal{L}},E)$ and $(\tilde{\Im},E)$ are HPNS subsets, then:*

(i) $\mathrm{Ext}((\tilde{\mathcal{L}},E)\Cup(\tilde{\Im},E)) = \mathrm{Ext}((\tilde{\mathcal{L}},E))\Cup\mathrm{Ext}((\tilde{\Im},E))$.

(ii) $\mathrm{Ext}((\tilde{\mathcal{L}},E)\Cap(\tilde{\Im},E)) \supseteq \mathrm{Ext}((\tilde{\mathcal{L}},E))\Cup\mathrm{Ext}((\tilde{\Im},E))$.

(iii) $\mathrm{Fr}((\tilde{\mathcal{L}},E)\Cup(\tilde{\Im},E)) \subseteq \mathrm{Fr}(\tilde{\mathcal{L}},E)\Cup\mathrm{Fr}(\tilde{\Im},E)$.

(iv) $\mathrm{Fr}((\tilde{\mathcal{L}},E)\Cap(\tilde{\Im},E)) \subseteq \mathrm{Fr}(\tilde{\mathcal{L}},E)\Cup\mathrm{Fr}(\tilde{\Im},E)$.

Proof.

(i) Since

$$\begin{aligned}\mathrm{Ext}((\tilde{\mathcal{L}},E)\Cup(\tilde{\Im},E)) &= (((\tilde{\mathcal{L}},E)\Cup(\tilde{\Im},E))^{\mathcal{C}})^\circ\\ &= ((\tilde{\mathcal{L}},E)^{\mathcal{C}}\Cap(\tilde{\Im},E)^{\mathcal{C}})^\circ\\ &= ((\tilde{\mathcal{L}},E)^{\mathcal{C}})^\circ\Cap((\tilde{\Im},E)^{\mathcal{C}})^\circ\\ &= \mathrm{Ext}((\tilde{\mathcal{L}},E))\Cap\mathrm{Ext}((\tilde{\Im},E)).\end{aligned}$$

(ii)

$$\begin{aligned}\mathrm{Ext}((\tilde{\mathcal{L}},E)\Cap(\tilde{\Im},E)) &= (((\tilde{\mathcal{L}},E)\Cap(\tilde{\Im},E))^{\mathcal{C}})^\circ\\ &= (((\tilde{\mathcal{L}},E)^{\mathcal{C}}\Cup(\tilde{\Im},E)^{\mathcal{C}}))^\circ\\ &\supseteq ((\tilde{\mathcal{L}},E)^{\mathcal{C}})^\circ\Cup((\tilde{\Im},E)^{\mathcal{C}})^\circ\\ &= \mathrm{Ext}((\tilde{\mathcal{L}},E))\Cup\mathrm{Ext}((\tilde{\Im},E)).\end{aligned}$$

(iii)

$$\mathrm{Fr}((\tilde{\mathcal{L}},E)\Cup(\tilde{\Im},E)) = \overline{(\tilde{\mathcal{L}},E)\Cup(\tilde{\Im},E)}\Cap\overline{((\tilde{\mathcal{L}},E)\Cup(\tilde{\Im},E))^{\mathcal{C}}}$$

$$\begin{aligned}&= (\overline{(\tilde{\mathcal{L}},E)}\Cup\overline{(\tilde{\Im},E)})\Cap\overline{(\tilde{\mathcal{L}},E)^{\mathcal{C}}\Cap(\tilde{\Im},E)^{\mathcal{C}}} \subseteq (\overline{(\tilde{\mathcal{L}},E)})\Cup((\tilde{\Im},E))\Cap\overline{(\tilde{\mathcal{L}},E)^{\mathcal{C}}}\Cap\overline{(\tilde{\Im},E)^{\mathcal{C}}})\\ &= \{(\overline{(\tilde{\mathcal{L}},E)})\Cup((\tilde{\Im},E))\Cap\overline{(\tilde{\mathcal{L}},E)^{\mathcal{C}}}\Cap\overline{(\tilde{\Im},E)^{\mathcal{C}}}\}\\ &= \{(\overline{(\tilde{\mathcal{L}},E)}\Cap\overline{(\tilde{\mathcal{L}},E)^{\mathcal{C}}})\Cup(\tilde{\Im},E)\Cap\overline{(\tilde{\mathcal{L}},E)^{\mathcal{C}}}\}\Cup\{(\overline{(\tilde{\Im},E)^{\mathcal{C}}})\Cap(\overline{(\tilde{\mathcal{L}},E)^{\mathcal{C}}}\Cap\overline{(\tilde{\Im},E)^{\mathcal{C}}})\}\\ &= \{\mathrm{Fr}((\tilde{\mathcal{L}},E))\Cap(\overline{(\tilde{\Im},E)^{\mathcal{C}}})\}\Cup\{\mathrm{Fr}((\tilde{\Im},E))\Cap\overline{(\tilde{\mathcal{L}},E)^{\mathcal{C}}}\}\\ &\subseteq \mathrm{Fr}((\tilde{\mathcal{L}},E))\Cup\mathrm{Fr}((\tilde{\Im},E)).\end{aligned}$$

(iv)

$$\mathrm{Fr}((\tilde{\mathcal{L}},E)\Cap(\tilde{\Im},E))=\overline{(\tilde{\mathcal{L}},E)\Cap(\tilde{\Im},E)}\Cap\overline{((\tilde{\mathcal{L}},E)\Cap(\tilde{\Im},E))^{\mathcal{C}}}$$
$$\subseteq(\overline{(\tilde{\mathcal{L}},E)}\Cap\overline{(\tilde{\Im},E)})\Cap((\tilde{\mathcal{L}},E)^{\mathcal{C}})\Cup(\tilde{\Im},E)^{\mathcal{C}})$$
$$=\{(\overline{(\tilde{\mathcal{L}},E)})\Cup(\overline{(\tilde{\Im},E)})\Cap\overline{(\tilde{\mathcal{L}},E)^{\mathcal{C}}}\}\Cup\{(\overline{(\tilde{\mathcal{L}},E)})\Cup(\overline{(\tilde{\Im},E)})\Cap\overline{(\tilde{\Im},E)^{\mathcal{C}}}\}$$
$$=\{\mathrm{Fr}((\tilde{\mathcal{L}},E))\Cap(\overline{(\tilde{\Im},E)})\}\Cup\{(\overline{(\tilde{\mathcal{L}},E)})\Cap\mathrm{Fr}((\tilde{\Im},E))\}$$
$$\subseteq\mathrm{Fr}((\tilde{\mathcal{L}},E))\Cup\mathrm{Fr}((\tilde{\Im},E)).$$

4. Machine Learning Techniques in Terms of Single Valued Heptapartitioned Neutrosophic Soft Sets and Data-Set Representation

The elbow technique is one way to get the suitable value of K (number of clusters) in K-means clustering. The Elbow method is a way for calculating the appropriate K (number of clusters). It promotes consistency in the cluster analysis design. The elbow approach is used to calculate the appropriate number of clusters within a data set. This could indicate natural groupings of people with comparable data sets, allowing for a greater understanding of the variety of data experiences. A heat map is a graphical representation of a data set. The t-SNE technique is cast-off to visualize data between two or three components in 3D space. Parallel coordinates are used to visually represent relationships between many dimensions after the data has been standardized to enable fair comparisons. A matrix displays the relationship between two characteristics, with each value represented by a color. Investigate how individuals with high ratings in one dimension tend to score in others, providing insights into the relationship between several dating dimensions. Researchers conducted sessions with young adults from varied backgrounds to study love feelings in young adults. Participants in the sessions, which took place in educational institutions and community settings, exchanged romantic ideas and stories. Each session was led by a competent psychologist who established an environment conducive to open dialogue. During data collection, participants deliberated their favorite love themes, relationships, and daily activities. To foster a relaxed mood, attendees were urged to bring their favorite sharing-gift delicacies or chocolates. Participants were subsequently encouraged to engage in simulated romantic interactions, imagining phone calls. Ten sessions were held at educational institutions, and two in community settings, yielding a diverse sample. To enhance diversity, seven additional films were obtained from web sources, depicting young adults in similar love circumstances. The recorded descriptions and videos of these conversations were critical in assigning values to seven membership functions, which served as the foundation for developing the romantic sentiment analysis system (RSAS).

Definition 31. *If there are two heptapartitioned neutrosophic soft sets A and B on $X = \xi_1, \xi_2, ...\xi_n$ are symbolized by*
$A=\{\langle\xi_i,T_A(\xi_i),RT_A(\xi_i),C_A(\xi_i),U_A(\xi_i),G_A(\xi_i),RF_A(\xi_i),F_A(\xi_i)\rangle:\xi_i\in X\}$ *and*
$B=\{\langle\xi_i,T_B(\xi_i),RT_B(\xi_i),C_B(\xi_i),U_B(\xi_i),G_B(\xi_i),RF_B(\xi_i),F_B(\xi_i)\rangle:\xi_i\in X\}$ *such that* $T_A(\xi_i)$, $RT_A(\xi_i),C_A(\xi_i),U_A(\xi_i),G_A(\xi_i),RF_A(\xi_i),F_A(\xi_i)\in[0,1]$ *and* $T_B(\xi_i),RT_B(\xi_i),C_B(\xi_i),U_B(\xi_i),G_B(\xi_i)$, $RF_B(\xi_i),F_B(\xi_i)\in[0,1]$ *for every* $\xi_i\in X$. *Let* $w_i(i=1,2,....n)$ *is weight of elements* $\xi_i(i=1,2,....n)$, $w_i\geq 0(i=1,2,....n)$ *and* $\sum_{i=1}^{n}w_i=1$. *Then, the heptapartitioned neutrosophic soft set weighted distance is demarcated as*

$d_\lambda(A,B)=[\frac{1}{7}\sum_{i=1}^{n}w_i\{|T_A(\xi_i)-T_B(\xi_i)|^\lambda+|RTC_A(\xi_i)-RTC_B(\xi_i)|^\lambda+|C_A(\xi_i)-C_B(\xi_i)|^\lambda+|U_A(\xi_i)-U_B(\xi_i)|^\lambda+|G_A(\xi_i)-G_B(\xi_i)|^\lambda+|RF_A(\xi_i)-RF_B(\xi_i)|^\lambda+|F_A(\xi_i)-F_B(\xi_i)|^\lambda\}]^{\frac{1}{\lambda}}$
with, $\lambda>0$.

By substituting $\lambda = 1, 2$ in the preceding equation, one can derive the heptapartitioned neutrosophic soft set weighted Hamming distance and the heptapartitioned neutrosophic soft set weighted Euclidean distance, respectively. The following is the weighted Hamming distance for single valued heptapartitioned neutrosophic soft sets.

$d_\lambda(A,B) = [\frac{1}{7}\sum_{i=1}^{n} w_i\{|T_A(\xi_i) - T_B(\xi_i)| + |RTC_A(\xi_i) - RTC_B(\xi_i)| + |C_A(\xi_i) - C_B(\xi_i)| + |U_A(\xi_i) - U_B(\xi_i)| + |G_A(\xi_i) - G_B(\xi_i)| + |RF_A(\xi_i) - RF_B(\xi_i)| + |F_A(\xi_i) - F_B(\xi_i)|\}]$
where, $\lambda = 0$.
Next is heptapartitioned neutrosophic soft set weighted Euclidean distance:

$d_\lambda(A,B) = [\frac{1}{7}\sum_{i=1}^{n} w_i\{|T_A(\xi_i) - T_B(\xi_i)|^2 + |RTC_A(\xi_i) - RTC_B(\xi_i)|^2 + |C_A(\xi_i) - C_B(\xi_i)|^2 + |U_A(\xi_i) - U_B(\xi_i)|^2 + |G_A(\xi_i) - G_B(\xi_i)|^2 + |RF_A(\xi_i) - RF_B(\xi_i)|^2 + |F_A(\xi_i) - F_B(\xi_i)|^2\}]^{\frac{1}{2}}$
where, $\lambda = 2$.

Table 1: Explanation of Romantic Feelings in Terms of Parameters

S.No	**Parameter Name**	**Description**
1.	Shared Dreams (SD)	The shared dreams and aspirations within the context of romantic feelings, whether based on real or imaginative situations or settings.
2.	Expressive Dance (ED)	Movements expressing romantic feelings, indicating cognitive patterns and emotional involvement.
3.	Non-verbal Communication (NVC)	Non-verbal expressions using body parts (hands, head) to convey romantic ideas or emotions.
4.	Emotional Expression (EE)	Movement of facial muscles for non-verbal communication, reflecting romantic emotions.
5.	Quality Time Together (QTT)	Duration and nature of social interactions during romantic moments, influence the depth of emotional connection.
6.	Play Materials Used(PMU)	Objects or symbols representing romantic elements in the context of romantic feelings.
7.	Collaborative Creativity (CC)	Partner's approach to using provided elements, offering insights into shared imaginative experiences in romantic feelings.
8.	Expressing Feelings (EF)	Vocal or non-vocal expression of romantic feelings and emotions by both partners.
9.	Emotional Tone (ET)	Tone reflecting the mood and state of mind of both partners during romantic interactions.
10.	Mutual Roles (MR)	The roles both partners assume and assign to each other within the context of romantic feelings.
11.	Shared Enthusiasm (SE)	Extent of the partners' involvement and shared excitement during romantic activities.
12.	Emotional Gaze (EG)	Movement of the eyes expressing emotions and connection during romantic moments.
13.	Associative Thinking (AT)	Mental process by which both partners form associations and create shared romantic experiences.

Continued from previous page

Table 1 – continued from previous page

S.No	Parameter Name	Description
14.	Grammaticality Correct Expressions (GCE)	Ability to construct grammatically correct sentences expressing romantic feelings with proper structure and syntax.
15.	Connected Conversations (CC)	Whether sentences formed during romantic interactions are related to each other, enhances the depth of connection.
16.	Gift Sharing (GS)	The act of sharing symbolic or imaginary gifts within the context of romantic feelings.

5. Techniques for Assessing Romantic Feelings: Characterizing 16 Key Feelings through Real-Life Examples

5.1. The Expert Views on the Evaluation Parameters for Romantic Feelings

The article uses a truth membership function with a range of [0,1] to describe several aspects that affect how romantic encounters are evaluated. In terms of complexity and interpretability, these aspects include things like creative topics, expressive dance, nonverbal communication, emotional expression, quality time, and others that help to increase truth membership. While certain elements, like play materials and gift-giving, lack professional opinions, others, like emotional tone or associative thinking, include different levels of complexity or indeterminacy that make them difficult to understand. When assessing love experiences, the overall method incorporates truth, indeterminacy, and complexity. These are listed below:

1. Shared Dreams (SD): An imaginative theme based on real situations increases the truth membership function. Degrees of complexity and indeterminacy are considered within the [0,1] range.
2. Expressive Dance (ED): Truth membership increases with physical movements expressing romantic feelings. Complex and indeterminate values from [0,1] if movements are challenging to interpret.
3. Non-verbal Communication (NVC): Truth membership increases with non-verbal expressions. Body parts conveying romantic ideas contribute. Complex and indeterminate values are assigned within [0,1].
4. Emotional Expression (EE): Facial movements reflecting romantic emotions increase truth membership. Complex and indeterminacy values are assigned within [0,1] for difficult interpretations.
5. Quality Time Together (QTT): Truth membership increases with social interactions during romantic moments. Indeterminate and complex values from [0,1] for challenging interpretations.
6. Play Materials Used (PMU): No specific expert views provided. The data set represents objects or symbols representing romantic elements.
7. Collaborative Creativity (CC): Truth membership increases with a partner's imaginative use of elements. Indeterminate and complex values from [0,1] for difficult interpretations.
8. Expressing Feelings (EF): Truth membership increases with vocal/non-vocal expressions. Indeterminate and complex values from [0,1] for difficult interpretations.
9. Emotional Tone (ET): Truth membership increases with the tone reflecting the mood. Indeterminate and complex values from [0,1] for challenging interpretations.
10. Mutual Roles (MR): Truth membership increases with realistic role identification. Indeterminate and complex values from [0,1] for difficult interpretations.
11. Shared Enthusiasm (SE): Truth membership increases with partners' involvement and ex-

citement. No specific views on indeterminacy or complexity.
12. Emotional Gaze (EG): Truth membership increases with eye movements expressing emotions. Complex and indeterminacy values are assigned within [0,1] for difficult interpretations.
13. Associative Thinking (AT): Truth membership increases with the mental process of forming associations. Indeterminate and complex values from [0,1] for challenging interpretations.
14. Grammatically Correct Expressions (GCE): Truth membership increases with grammatically correct expressions. Indeterminate and complex values from [0,1] for challenging linguistics.
15. Connected Conversations (CC): Truth membership increases with related sentences during romantic interactions. Indeterminate and complex values from [0,1] for challenging coherence.
16. Gift sharing (GS): No specific expert views provided. The data set represents the act of sharing symbolic or imaginary gifts within romantic feelings.
These expert views guide the evaluation process, incorporating truth, indeterminacy, and complexity considerations for each romantic parameter.

5.2. Characterizing 16 Key Feelings through Real-Life Examples

This section focuses on an example with a young couple of 19 years old, with physicalists conducting observations and interviews about their love relationship as they progressed through the domain of 16 choice factors. The developed Table 2 is shown below.

Table 2: Explanation for 16 Feelings

Feeling	**Ab.T**	**Re.T**	**Ab.F**	**Re.F**	**Contra**	**Unknown**	**Ig**
Shared Dreams	0.4	0.2	0.15	0	0.25	0	0
Expressive Dance	0.3	0.3	0	0	0.25	0.15	0
Non-verbal Communication	0	0	0	0	0.25	0.75	0
Emotional Expression	0	0.75	0.25	0	0	0	0
Quality Time Together	0.2	0.2	0.2	0.3	0.1	0	0
Play Materials Used	0.15	0	0.3	0	0.25	0	0.3
Collaborative Creativity	0.15	0.3	0	0	0.25	0	0.3
Expressing Feeling	0.4	0.2	0.2	0.1	0.1	0	0
Emotional Tone	0.2	0.25	0.25	0	0.2	0	0.1
Mutual Roles	0.5	0	0.25	0	0.25	0	0
Shared Enthusiasm	0.5	0.25	0.25	0	0	0	0
Emotional Gaze	0	0	0.5	0	0.5	0	0
Associative Thinking	0.75	0	0	0	0.25	0	0
Grammatically Correct Expressions	0.75	0	0.25	0	0	0	0
Connected Conversations	0.15	0	0.3	0.25	0	0	0.3
Gift sharing	0.15	0	0.3	0.25	0	0.2	0.1

Table 2 explains the discussed parameters. The provided material provides a set of parameters for analyzing love sensations, each with a description and a corresponding dataset represented as SVHNS. Table 3 provides a concise summary.

Table 3: Example for Heptapartitioned Neutrosophic Soft Set

S.No	**Parameter Name**	**Description**	**SVHNS**
1.	Shared Dreams (SD)	Dreams and aspirations shared within romantic feelings, real or imaginative	[0.4,0.2,0.15, 0,0.25,0,0]
2.	Expressive Dance (ED)	Movements expressing romantic feeling, indicating cognitive patterns and emotional involvement	[0.3,0.3,0, 0,0.25,0.15,0]
3.	Non-verbal Communication (NVC)	Non-verbal expressions using body parts to convey romantic ideas or emotions	[0,0,0, 0,0.25,0.75,0]
4.	Emotional Expression (EE)	Movements of facial muscles for non- verbal communication, reflecting romantic emotions.	[0,0.75, 0.25,0,0,0,0]
5.	Quality Time Together (QTT)	Duration and nature of social interactions during romantic moments	[0.2,0.20,0.20,0.3,0.1,0,0]
6.	Play Materials Used (PMU)	Objects or symbols representing romantic elements	[0.15,0,0.3, 0,0.25,0 .0.3]
7.	Collaborative Creativity (CC)	Partner's approach to using provided elements, offering insights into shared imaginative experiences	[0.15,0.3, 0,0,0.25,0 .0.3]
8.	Expressing Feeling (EF)	Vocal or non-vocal expression of romantic feelings and emotions	[0.4,0.20, 0.20,0.1,0.1,0 ,0]
9.	Emotional Tone (ET)	Tone reflects the mood and state of mind during romantic interactions	[0.2,0.25, 0.25,0,0.2,0 .0.1]
10.	Mutual Roles (MR)	Roles both partners assume within romantic feelings	[0.5,0, 0.25, 0,0.25,0,0]
11.	Shared Enthusiasm (SE)	Extent of partners involvement and shared excitement during romantic activities	[0.5,0.25, 0.25,0,0 ,0,0]
12.	Emotional Gaze (EG)	Movement of the eyes expressing emotions and connection during	romantic moments [0,0.0.5, 0,0,0.5 ,0,0]
13.	Associative Thinking (AT)	Mental process by which both partners from associations and create shared romantic experiences	[0.75,0, 0,0 ,0.25,0,0]
14.	Grammatically Correct Expressions(GCE)	Ability to construct grammatically correct sentences expressing romantic feelings	[0.75,0, 0.25,0,0 ,0,0]
15.	Connected Conversations (CC)	Whether sentences formed during romantic interactions are related	[0.15,0, 0.3,0.25,0 ,0,0.3]
16.	Gift Sharing	The data set represents the act of sharing symbolic or imaginary gifts within romantic feelings.	[0.15, 0, 0.3, 0.25, 0, 0.2, 0.1]

Similarly, the expert assisted in the creation of SVHNS tuples for the remaining data sets. The SVHNS sets are then analyzed with machine learning methods.

6. Results and Discussions

Data was represented and visualized via a variety of Python modules, including pandas, numpy, matplotlib, sklearn, seaborn, and pylab. The aforementioned methodologies were demonstrated using Python programming, and the elbow curve data was clustered using K-means algorithms. These images inspired logical implications, and sixteen distinct traits were discovered as influencing young boys' and girls' love impulses. A heat map's color scheme clearly shows correlation and associativity, with blue correlation for darker shades and reddish correlation for lighter shades. A correlation between two variables shows that they are related. The word "reddish correlation" refers to the relationship between an increase in one attribute and another. A blue link arises when one quality improves while another deteriorates. The heat map clearly depicts the correlation and associativity elements using a color scale, with darker shades indicating a blue correlation and lighter shades indicating a red link. The Elbow method is strategically used to identify ideal clusters, considerably adding to a more comprehensive knowledge of the range of romantic feelings. This advanced methodology seeks to comprehend 16 distinct attitudes by taking into consideration absolute truth, relative truth, absolute false, relative false, contradiction, unknown (undefined), and ignorance. K- means clustering of the data set revealed significant patterns in love sensations. Distinct groups evolved, each defined by people who shared similar love interests. The correlation study added to our understanding of the emotional landscape by showing the relationships between elements like "quality time together" and "expressing feelings." These results highlight the complexities and interdependence of romantic relationships. Visualization methods such as t-SNE and 3D visualization allow us to discover specific patterns in the data. Parallel coordinates allowed for a visual portrayal of the complicated relationships between many emotional traits. When utilized strategically, the Elbow method discovered ideal clusters, greatly increasing our understanding of the variety of romantic feelings. The findings lend support to modern machine-learning methodologies for comprehending the complexities of human emotions, particularly in love relationships. The multidimensional analysis provides a holistic perspective, emphasizing the significance of using a sophisticated approach to investigate and comprehend complicated emotional states. The multidimensional analysis presents a comprehensive view, highlighting the importance of employing sophisticated methods to examine and comprehend complex emotional states.
Figure 1 depicts the Encrypted K-Mean clustering algorithm used to feature T for the parameters' shared dreams' on the $y-axis$ and 'expressive dancing' on the $x-axis$, with the highest concentration of points finding near $x = 0.15$ and $y = 00.38$. The first centroid, c_1 is located at $x = 00.00$ and $y = 00.00$, whereas the second centroid, c_2 ,is at $x = 00.00$ and $y = 00.8$. After using K-mean clustering, we got close to c_3, as indicated in the image.

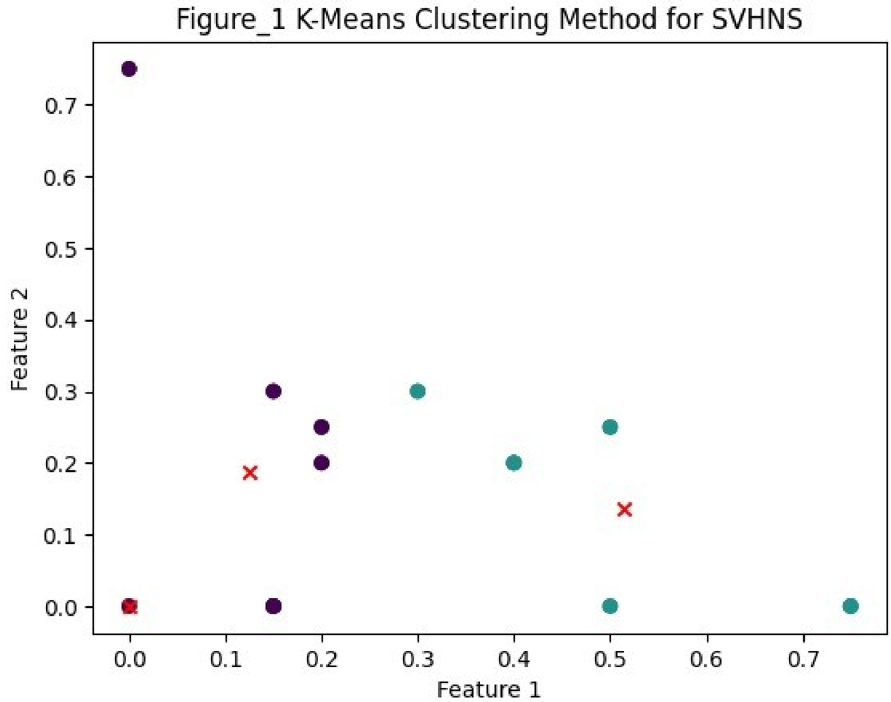

Figure 1: K-Mean Clustering Method

The Figure 2 has been devoted to heat map and as we know that heat maps helps us in understanding the correlation in data (the graphical illustration of data where values are represented by colors) and in particular here correlation heat map has been reflected for the given data. The picture represents thirty two combined feeling of male and female. Here we see that the dominant colour is dark red and also at second number the influential colour is sky colour.

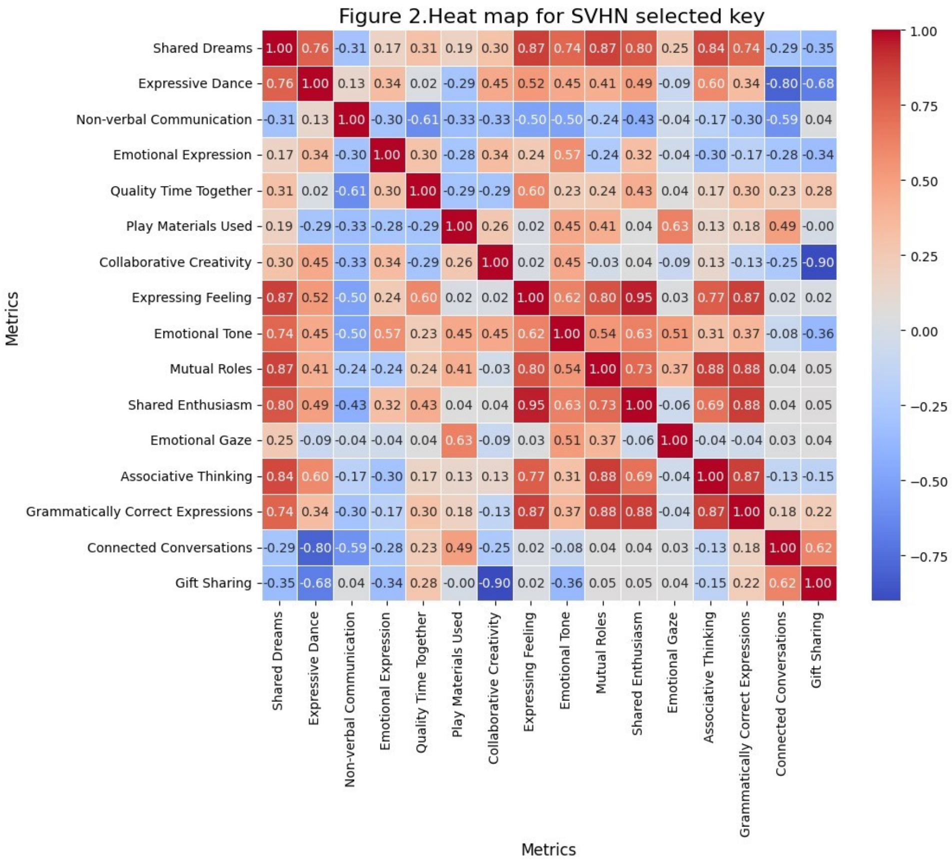

Figure 2: Heat Map for SVHN Selected Key

Figure 3 reflects encrypted Elbow method (this method is used to decide how many clusters it should consider and this method is actually the graph between K and distortion WCSS). Here the K-values are taken along x-axis and the distortion (WCSS) is taken along y-axis. We see that there are number of K- values but we encounter the business value of K which is at 4 that is at $k = 6$ the optimal value is 4 because here we see that the drastic change in y-axis value occurred at k=6 the value of $y = 10.3$ as shown in the figure.

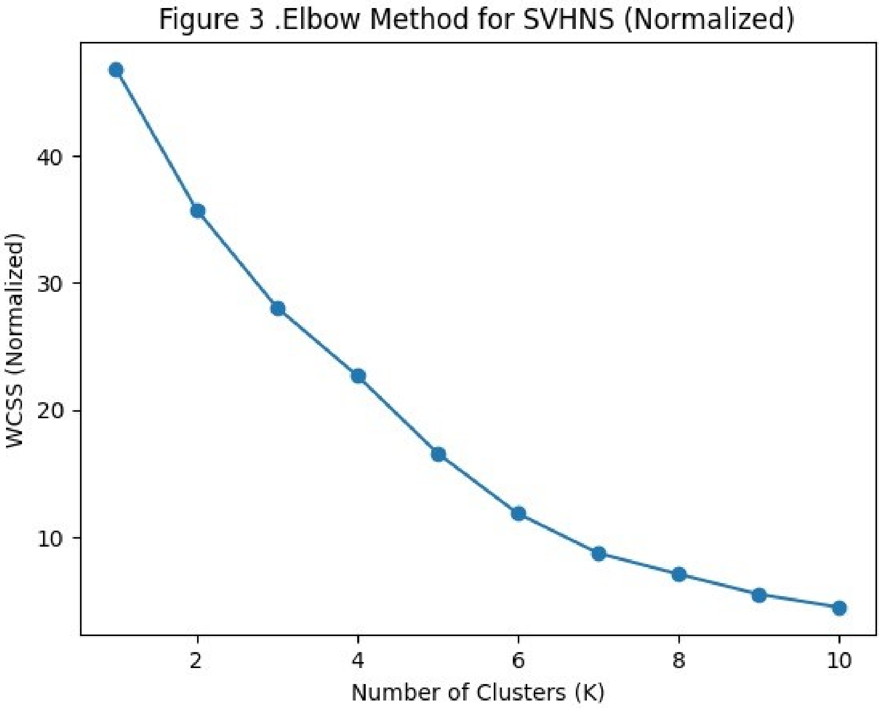

Figure 3: Elbow Normalized Method for SVHN Data Set

Figure 4depicts the correlation matrix for feature T, revealing that the off diagonal is filled with a single element one and that dark green, like the sky, has a strong negative correlation. The positive and negative correlations are symmetric about the off diagonal, as shown by the dark red color. The highest positive score is 00.22, indicating a strong link between the connected items and the others. We can see that there are also negative values, with the highest being −00.047, indicating a strong negative association between the connected items and the rest. The correlation of variables with themselves can be seen diagonally, from bottom right to left.

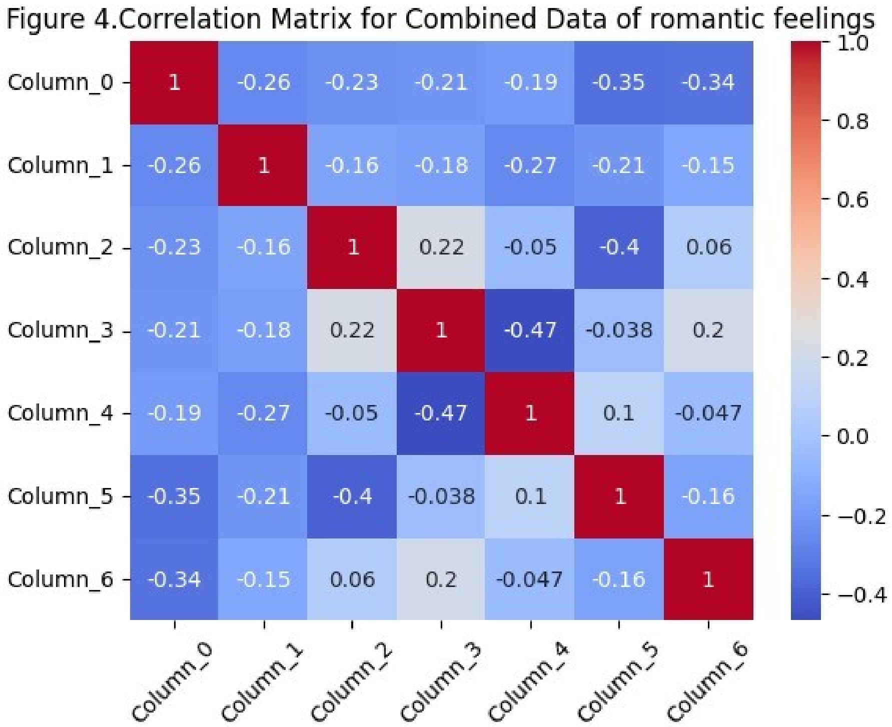

Figure 4: Feature Correlation Analysis for SVHN Data Set

The concentration of points at $(x = -0.2, y = 0.08)$ in Figure 5, representing a t-SNE visualization, suggests that this region of the plot contains data points that share significant similarity in terms of the features being analyzed (e.g., "shared dreams" and "expressive dancing"). This clustering reveals patterns in the data that could represent distinct subgroups of individuals with specific emotional or cognitive characteristics, such as those related to romantic feeling or emotional health in young adults. The concentration of points in this region also emphasizes the ability of t-SNE to uncover hidden relationships in complex, high-dimensional data and offers valuable insights for further psychological analysis and research.

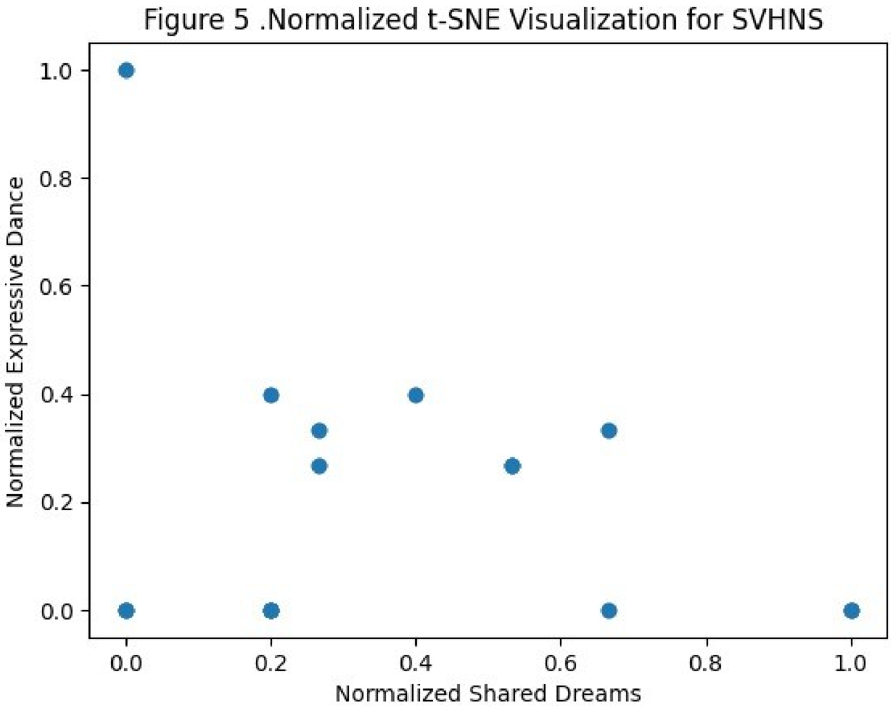

Figure 5: 2D-Normalized t-SNE SVHN Data Set

Figure 6 reflects the resulting 3D scatter plot, which shows how the 16 parameters (representing various romantic behaviors and expressions) are grouped into 3 clusters after applying K-Means clustering and PCA. This helps identify patterns and relationships between the parameters based on their feature values. The different clusters are shown based on the given data in the figure.

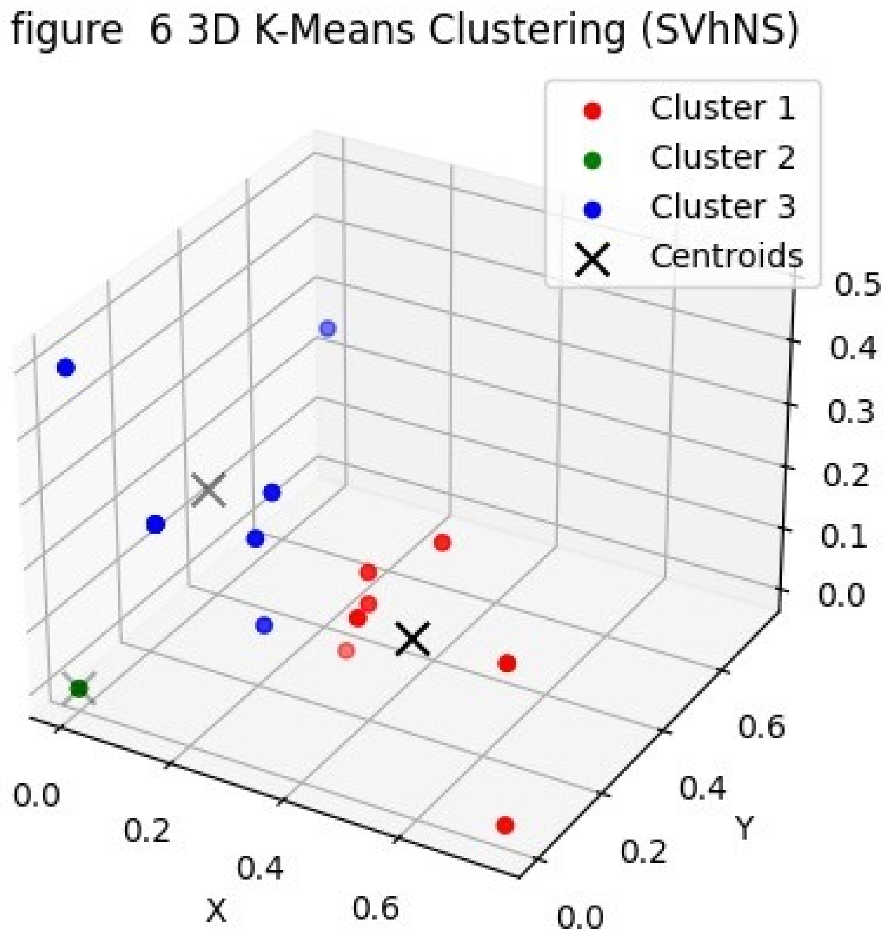

Figure 6: Reflects 3D K-Means Clustering SVHNS

Figure 7 reflects the 3D PCA of K-Means Clustering. The SVHNS data set, which comprises seven- dimensional feature vectors representing amorous behaviors, was subjected to 3D PCA and K-Means Clustering. After data standardization, K-Means clustering grouped related parameters into three clusters, while PCA reduced the dimensions to three for easier presentation.The results, displayed in a three- dimensional scatter plot, revealed connections and patterns among diverse amorous emotions, offering insights into the relationships between different behaviors. The clustering results are shown in the given figure.

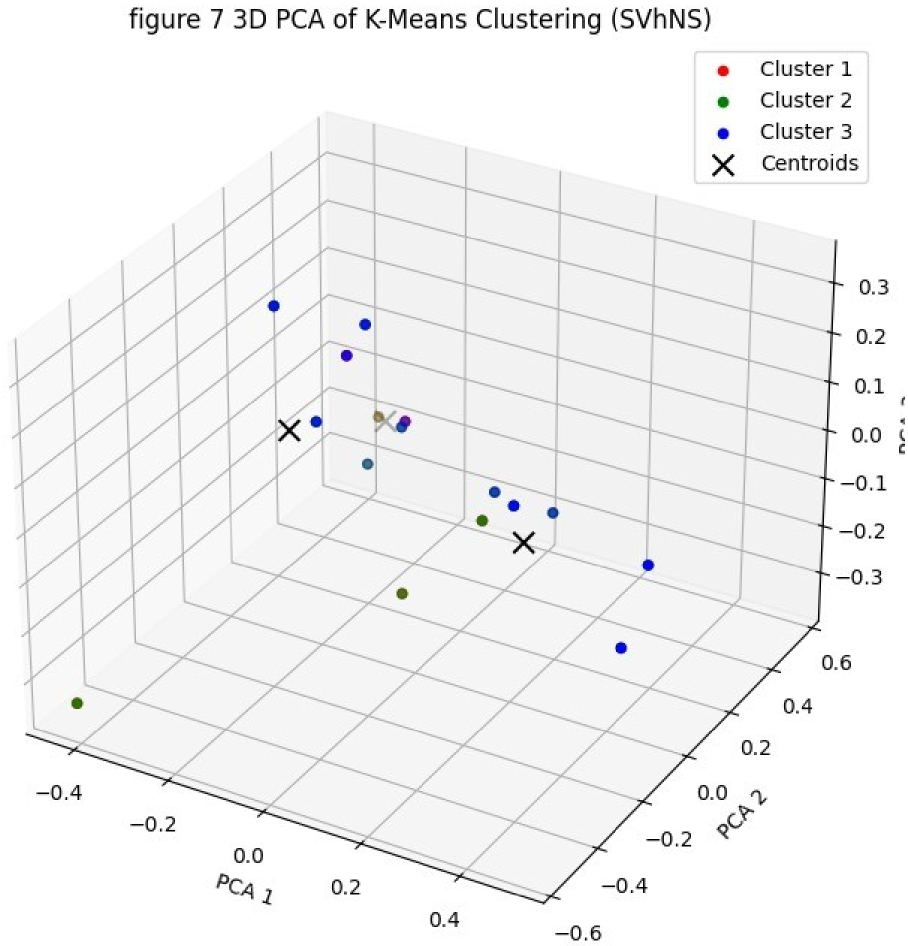

Figure 7: Reflects 3D-PCA of K-Mean Clustering

Figure 8 reflects parallel coordinates of 3D t- SNE of Scaled SVHN data set. A seven-dimensional vector representing the feature values (SVHNS) is used to store the input data as a dictionary, with each row denoting a parameter (such as ”Shared Dreams” or ”Expressive Dance”). We use a standard scaler to scale the feature vectors to have zero mean and unit variance because t-SNE is sensitive to the feature scale. forecasts. The seven-dimensional data is reduced to three dimensions for simpler presentation using the TSNE method from sklearn.manifold. The pandas plotting $parallel_coordinatesfunction$ is used to plot the 3D features that are produced after t-SNE is applied. One of the parameters is represented by each line in the parallel coordinates plot, and one of the t-SNE dimensions (Dimension 1, Dimension 2, Dimension 3) is represented by each vertical line. Following t-SNE transformation, the plot that results will display the relationships between the dataset’s parameters (rows) in three dimensions. This makes it easier to see any groups or trends among the parameters based on their t-SNE projections.

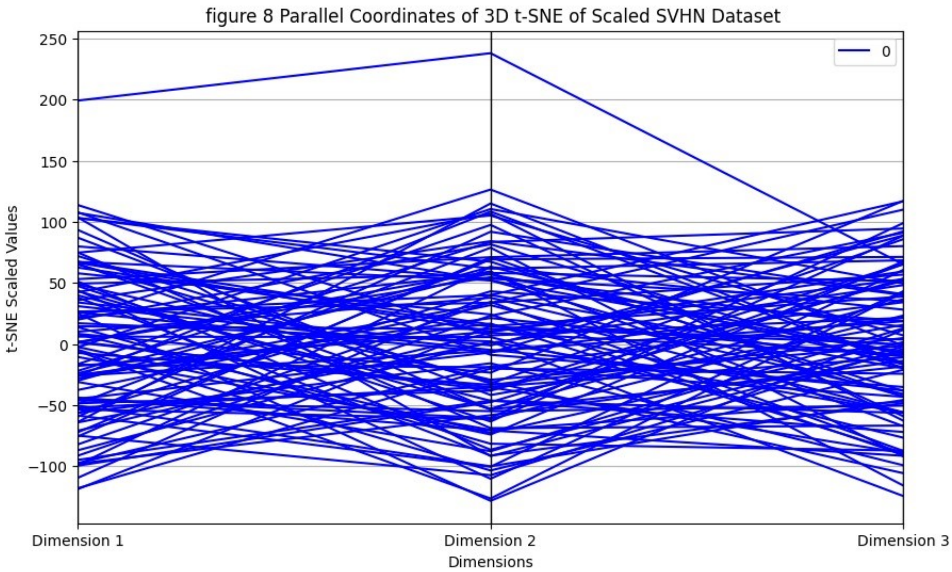

Figure 8: reflects Parallel coordinates of 3D t-SNE of Scaled SVHN Data Set

7. Comparative Analysis

The comparative analysis presented in Tables 4, 5, and 6 offers a detailed evaluation of the proposed methods in relation to the established techniques discussed in [41]. It examines key performance factors such as correlation, visibility, associativity, dynamicity, and scalability. The Heat Map excels at visualizing strong correlations but lacks in visibility and scalability. K-Means provides strong associativity and scalability but offers weaker correlation analysis. t-SNE shows medium visibility and strong dynamicity, especially in 2D/3D projections, though it struggles with scalability. The PAC Graph stands out for its strong visibility and dynamicity, making it ideal for complex data relationships, but it has weaker correlation analysis and scalability compared to K-Means and t-SNE. The choice of method depends on the specific analytical requirements.
The following Table 4 provides a detailed comparative analysis of the proposed methods, contrasting them with the established techniques discussed in [41]. This comparison highlights the strengths and weaknesses of each approach, offering insights into how the proposed methods perform relative to the established techniques across various key factors such as efficiency, accuracy, and adaptability.

Factor	**SVRNS (Single Valued Refined Neutrosophic Set)**	**HPNSS (Heptapartitioned Single Valued Neutrosophic Set)**
1. Membership Function Complexity	Introduces six membership functions: Truth, Indeterminacy, Falsity, plus complex-valued memberships. Emphasis on imaginary and indeterminate aspects, making it sensitive to real-world issues like Children's imagination.	Uses seven partitions for indeterminacy, including absolute true, relative true, contradiction, unknown, ignorance, relative false and absolute false. More granular representation of uncertainty and vagueness.

2. Domain of Application	Primarily applied to childrens cognitive development and imaginary play. Focus on ages 1-10 years, where imagination and creativity are significant factors.	Applied young adults (18-25 years), particularly focusing on romantic feelings and emotional relationships. Targets psychological studies of emotional distress, relationship dynamics and mental health.
3. Data Modeling and Uncertainty Representation	Focus on complex, imaginary, and indeterminate aspects of children's mental states. Uses complex-valued memberships to model uncertainty in imaginative play.	Models uncertainty with seven distinct categories of indeterminacy. Provides a fine-tuned and multifaceted approach to represent uncertainty and vagueness, particularly in romantic relationships and emotional states.
4. Computational Algorithms	Uses machine learning algorithms like K-means clustering along with parallel coordinates for visualization, Focus on analyzing relationships between imaginative play and cognitive abilities of children.	Implements a broader set of algorithms including K-means clustering. Heatmaps Elbow method. Feature correlation, 2D-normalized t-SNE and Parallel Coordinates of 3D t-SNE. Aims to analyze the multidimensional relationships in romantic emotions and emotional distress.
5. Visualization and Data Representation	Parallel coordinates and K-means clustering provide insights into the cognitive development and imaginative play of children. Visualization methods are tailored to capturing the complex and uncertain nature of children's psychological states.	Uses advanced visualization techniques like t-SNE, Heatmaps and parallel coordinates to analyze and represent and emotional states, romantic relationships and mental health of young adults. Provides multidimensional views of psychological data emphasizing the relationships between romantic feelings and emotional distress.

This table provides a clear, side-by-side comparison of the two models based on different factors, highlighting their unique focus and applications.

The following Table 5 provides a detailed comparative analysis of the proposed methods in relation to the established techniques discussed in [41]. This comparison focuses on key performance factors, including correlation, visibility, associativity, dynamicity, and scalability, to highlight the strengths and limitations of each approach across these dimensions.

Aspect	**SVRNS (Single Valued Refined Neutrosophic Set)**	**HPNSS (Heptapartitioned Single Valued Neutrosophic Set)**

1. Correlation	**Strengths:** Linear Correlations: Effective for capturing simple, linear relationships in straightforward data sets. Simplicity: Uses six membership functions, making it easy to calculate and understand basic correlations. **Weaknesses:** Limited by Simplicity: May miss complex correlations involving multiple factors. Lack of Nuance: Struggles to capture subtleties in multidimensional emotional data.	**Strengths:** Multidimensional Correlations: Excels in analyzing complex, non-linear relationships. Advanced Algorithms: Methods like t- SNE, Heatmaps, and Feature Correlation uncover hidden relationships. **Weaknesses:** Complexity: May result in overfitting and challenges in interpretation, especially for smaller data sets.
2. Visibility	**Strengths:** Clear Visualizations: Intuitive, easy-to-understand visualizations for non-experts. Simple Data Interpretation: Uses basic clustering techniques like K-means for clear trend visualization. **Weaknesses:** Less Detailed: May fail to illustrate intricate emotional data or complex psychological states.	**Strengths:** Richer Visualizations: Detailed 2D/3D t-SNE and parallel coordinates capture multidimensional data. Heatmaps & Clustering: Provide insights into how multiple psychological factors interact. **Weaknesses:** Interpretability Issues: Complex visualizations can overwhelm users, especially non-experts.
3. Associativity	**Strengths:** Simple Associations: Good for basic relationships between truth, indeterminacy, and falsity. Easy to Interpret: Associations are simple and straightforward to understand. **Weaknesses:** Limited to Simple Data: Struggles with multidimensional associations.	**Strengths:** Complex Associativity: Can model complex relationships between multiple uncertain and emotional variables. Multiple Uncertainty Types: Captures complex patterns of emotional states, mental health, etc. **Weaknesses:** Complexity in Interpretation: Detailed associations can be difficult to interpret, especially with noise.
4. Dynamicity	**Strengths:** Captures Simple Dynamics: Suitable for modeling basic dynamic changes in cognitive growth or play. **Weaknesses:** Limited for Complex Dynamic Data: Struggles with rapidly changing or multi-factorial data.	**Strengths:** Handles Complex Dynamic Data: Excellent for modeling complex emotional evolution or psychological. **Weaknesses:** Potentially Overcomplicated: May not be ideal for real-time dynamic modeling or simpler systems.

5. Scalability	**Strengths:** Easily Scalable for Simple Data: Works well for small, straightforward datasets. Low Computational Cost: Requires less computational power, ideal for smaller studies. **Weaknesses:** Limited Scalability for Large Datasets: Struggles with large, complex datasets.	**Strengths:** Highly Scalable for Complex Data: Handles large, high-dimensional datasets with many variables. Advanced Computational Methods: Uses algorithms like K-means and t-SNE, making it scalable to large data. **Weaknesses:** High Computational Requirements: Complexity of algorithms requires significant computational resources.

This table summarizes the key points in the areas of Correlation, Visibility, Associativity, Dynamicity, and Scalability for both SVRNS and HPNSS models.

The comparative analysis in Table 7 highlights the strengths and limitations of each method across key factors. The Heat Map excels in visualizing strong correlations but falls short in terms of visibility and scalability. K-Means offers strong associativity and scalability but provides weaker correlation analysis. t- SNE shows medium visibility and strong dynamicity, especially in 2D/3D projections, but struggles with scalability. The PAC Graph stands out for its strong visibility and dynamicity, making it ideal for representing complex relationships and capturing changes in data, though it offers weaker correlation analysis and scalability compared to K-Means and t-SNE. Each method serves a different purpose, and the choice depends on the specific needs of the analysis.

Factors	**Heat Map**	**K-Means**	**t-SNE (2D/3D)**	**K-PAC Graph**
Correlation	Strong correlation visualization.	Weak correlation analysis.	Weak correlation analysis using t-SNE.	Weak correlation analysis using PAC Graph.
Visibility	Visibility in terms of the factor is weak	Medium visibility of the factor	Medium visibility using t-SNE	Strong visibility using PAC Graph.
Associativity	Strong associativity representation.	Strong associativity analysis.	Strong associativity analysis using t-SNE	Medium associativity analysis using PAC Graph.
Dynamacity	Medium dynamicity.	Strong dynamicity analysis.	Strong dynamicity analysis using T-SNE.	Very Strong dynamicity analysis using PAC Graph.
Scalability	Medium scalability.	Strong scalability analysis.	Strong scalability analysis using t-SNE	Medium scalability analysis using PAC Graph.

Graphical Representation of Table 7, in terms of Factors and Methods are given in Figure 9

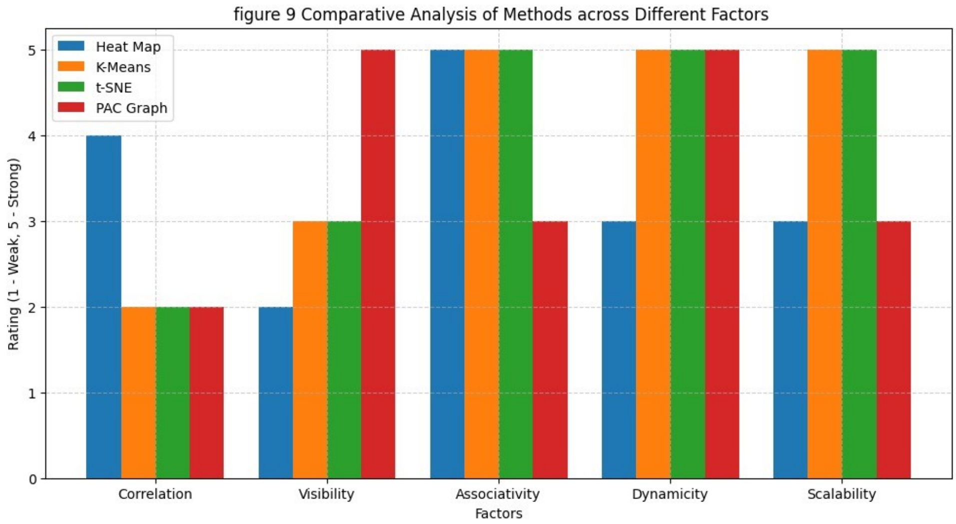

Figure 9: Analysis Level

8. Advantages

1. Comprehensive Analysis: The use of multiple Python libraries (pandas, numpy, sklearn, seaborn) enables a thorough and multidimensional analysis of the data, providing insights into complex emotional traits and behaviors.
2. Clustering and Pattern Detection: K-means clustering, along with the Elbow method, effectively identifies meaningful groups within the data, revealing distinct patterns in romantic feelings among young boys and girls. This clustering helps in understanding emotional similarities and differences.
3. Correlation Insights: The heat map and correlation matrix offer a clear visualization of relationships between emotional traits, helping to identify which factors are positively or negatively associated (e.g., "quality time together" and "expressing feelings").
4.Visualization Techniques: Advanced visualization methods like t-SNE, 3D plots, and parallel coordinates allow for better understanding of high-dimensional data. These tools help uncover hidden patterns and interactions between emotional traits that might not be easily detected otherwise.
5. Dynamicity and Scalability: The analysis indicates that the dataset is highly dynamic and scalable, making it adaptable to changes in size or complexity. The dataset can handle increased volume or complexity without a significant decrease in performance.
6. Effective Use of Machine Learning: The use of machine learning techniques like K-means clustering, t-SNE, and PAC Graph provides deeper insights into the dynamics of human emotions, particularly in romantic relationships, which can lead to more informed conclusions and predictions.

9. Limitations

1. Interpretation Complexity: While the clustering and correlation techniques provide valuable insights, the interpretation of complex emotional data is still challenging. The relationships between emotional traits are not always straightforward, and the nuances of human emotions may be oversimplified.
2. Data Quality and Bias: The dataset may contain biases or inaccuracies that could affect the clustering results. For instance, if the data is self-reported or incomplete, the findings may not accurately reflect the true emotional landscape.

3. Limited Scope: The analysis focuses primarily on emotional traits related to romantic feelings among young boys and girls. It may not generalize well to other age groups or cultural contexts, limiting its broader applicability.
4. Overfitting Risk in Clustering: K-means clustering, while useful, may be prone to overfitting if not carefully tuned. The chosen number of clusters (K) can greatly influence the results, and improper selection of K could lead to misleading conclusions.
5. Ambiguity in Correlations: The correlation analysis provides a broad view of relationships between variables, but it does not capture causality or deeper behavioral insights. Correlation does not imply causation, and some relationships might be coincidental or influenced by hidden factors.
6.Technical Limitations: The analysis relies on advanced techniques like t-SNE and PAC Graph, which can be computationally intensive. As the dataset grows, these methods may require more computational power and time, limiting their scalability in large datasets.
7. Lack of Expert Domain Input: While the data is processed through machine learning methods, the absence of expert domain knowledge (e.g., psychologists or relationship experts) could result in missing or misinterpreted emotional nuances that machines cannot fully understand.
In summary, while the study offers valuable insights and advantages in uncovering complex emotional patterns in romantic relationships, it faces limitations related to data interpretation, scope, and technical challenges, which need to be addressed for more robust and generalizable conclusions.

10. Conclusion and Future Work

In this paper, we introduced the concept of the HPNSS, an innovative extension and generalization of neutrosophic soft set theory. By expanding the indeterminacy into seven distinct categories as absolute true, relative true, contradiction, unknown, ignorance, relative false, and absolute false. we improved the accuracy in modeling uncertainty, vagueness, and indeterminacy in complex systems. We further extended the concept of neutrosophic soft topological spaces, presenting the novel HPNSTS to represent uncertainty in topological structures. This extension led to the introduction of new set definitions, including p open, pre-open, and semi-open sets, with a particular focus on p-open sets, and several related results. The data set used for this study was collected from the Psychology Department at Peshawar University, focusing on individuals aged 18 to 25. The analysis aimed to uncover various psychological patterns and variables in this demographic, providing a deeper understanding of the emotional and romantic dynamics within university life. This data-driven approach, enhanced by the incorporation of HPNSS, enabled a more refined analysis of the inherent uncertainty and imprecision in psychological research. For the practical application of these methods, machine learning and graphical techniques, such as K- Means clustering, Heat maps, the Elbow method, Feature correlation, 2D-normalized t-SNE, and parallel coordinates of 3D T-SNE, were employed to explore the romantic feelings of young boys and girls. These algorithms helped to identify and visualize key patterns and relationships between various factors influencing romantic emotions. The proposed methodologies offer valuable new dimensions for psychological investigations, particularly in understanding emotional disorders or break-ups in romantic relationships among university students. The integration of these advanced techniques provides a comprehensive framework for examining complex emotional states, thus contributing to a more nuanced and holistic understanding of romantic feelings and relationships.

Future Work

1. Given the promising results of applying HPNSS in understanding uncertainty and indeterminacy in emotional and psychological data, further research is recommended to explore its broader applicability across different domains. Future studies could investigate how

HPNSS can be extended to other areas of psychology, social sciences, and decision-making models where uncertainty plays a crucial role.

2. To enhance the theoretical foundation, it would be beneficial to integrate HPNSS with other psychological models, such as cognitive-behavioral theories or emotional intelligence frameworks. This could help create a more robust, interdisciplinary approach to understanding complex emotional dynamics, especially in young adults and university students.

3. The current study was limited to a dataset from the Psychology Department at Peshawar University, focusing on individuals aged 18 to 25. Future research should expand the dataset to include a more diverse demographic, including individuals from different age groups, cultural backgrounds, and educational contexts. This would help generalize the findings and enhance the robustness of the analysis.

4. To further enrich the analysis, it is recommended to incorporate multi-modal data sources such as surveys, interviews, physiological data (e.g., heart rate, skin conductance), and social media activity. Combining these data types could provide a more holistic view of romantic emotions and psychological states.

5. While K-means clustering provided valuable insights into the patterns of romantic feelings, further refinement of clustering techniques (e.g., hierarchical clustering or DBSCAN) could improve the granularity of the analysis. This could help uncover more subtle or complex emotional groupings that K-means may not fully capture.

6. Although t-SNE and 3D visualizations were useful for exploring patterns in the data, employing additional advanced visualization techniques, such as interactive visualizations or graph-based models, could allow for deeper exploration and better communication of complex emotional dynamics in real-time settings.

7. To gain a better understanding of how romantic feelings evolve over time, conducting longitudinal studies that track emotional changes in the same individuals over an extended period would be valuable. This approach could provide insights into how romantic emotions and psychological factors develop and change, particularly in response to major life events like breakups or relationship transitions.

8. Given the cultural differences that can influence romantic feelings and psychological states, it would be useful to conduct cross-cultural studies comparing how young people from different cultural backgrounds experience and express romantic emotions. This could reveal universal patterns as well as cultural-specific dynamics that could further inform emotional well-being interventions.

In conclusion, these recommendations aim to enhance the depth and applicability of the current research, encouraging the continued development of advanced methodologies for understanding emotional dynamics and improving psychological well-being. By expanding the scope and refining the techniques used, future studies can provide even more comprehensive insights into the complexities of romantic feelings and human emotions.

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References

[1] Zadeh, L.A. *Fuzzy sets.* Inf. Control **1965**, *8*, 338–353.
[2] Turksen, I.B. *Interval valued fuzzy sets based on normal forms.* Fuzzy Sets Syst. **1986**, *20*, 191–210.
[3] Atanassov, K.T. *Intuitionistic fuzzy sets.* Fuzzy Sets Syst. **1986**, *20*, 87–96.
[4] Palaniappan, N.; Srinivasan, R. *Applications of intuitionistic fuzzy sets of root type in image processing.* In *Proceedings of the Annual Meeting of the North American Fuzzy Information Processing Society*, Cincinnati, OH, USA, 14–17 June **2009**; pp. 1–5.
[5] Szmidt, E.; Kacprzyk, J. *An application of intuitionistic fuzzy set similarity measures to a multi-criteria decision making problem.* In *Proceedings of the Artificial Intelligence and Soft Computing–ICAISC*, Zakopane, Poland, 25–29 June **2006**; pp. 314–323.
[6] Vlachos, I.K.; Sergiadis, G.D. *Intuitionistic fuzzy information: Applications to pattern recognition.* Pattern Recognit. Lett. **2007**, *28*, 197–206.
[7] Smarandache, F. *A unifying field in logics, neutrosophic logic.* Multiple Valued Logic **2002**, *8*, 385–438.
[8] Karaaslan, F. *Multi-criteria decision-making method based on similarity measures under single-valued neutrosophic refined and interval neutrosophic refined environments.* Int. J. Intell. Syst. **2018**, *33*, 928–952.
[9] Karaaslan, F.; Davvaz, B. *Properties of single-valued neutrosophic graphs.* J. Intell. Fuzzy Syst. **2018**, *34*, 57–79.
[10] Karaaslan, F. *Correlation coefficients of single-valued neutrosophic refined soft sets and their applications in clustering analysis.* Neural Comput. Appl. **2017**, *28*, 2781–2793.
[11] Ali, M.; Smarandache, F. *Complex neutrosophic set.* Neural Comput. Appl. **2017**, *28*, 1817–1834.
[12] Molodtsov, D.A. *Soft set theory–first results.* Comput. Math. Appl. **1999**, *37*, 19–31.
[13] Maji, P.K.; Roy, A.R. *Soft set theory.* Comput. Math. Appl. **2003**, *45*, 555–562.
[14] Maji, P.K.; Biswas, R.; Roy, A.R. *Fuzzy soft sets.* J. Fuzzy Math. **2001**, *9*, 589–602.
[15] Cagman, N.; Enginoglu, S.; Citak, F. *Fuzzy soft set theory and its application.* Iran. J. Fuzzy Syst. **2011**, *8*, 137–147.
[16] Feng, F.; Jun, Y.B.; Liu, X.; Li, L. *An adjustable approach to fuzzy soft set based decision making.* J. Comput. Appl. Math. **2010**, *234*, 10–20.
[17] Gorzalczany, M.B. *A method of inference in approximate reasoning based on interval valued fuzzy sets.* Fuzzy Sets Syst. **1987**, *21*, 1–17.
[18] Kong, Z.; Gao, L.; Wang, L. *Comment on A fuzzy soft set theoretic approach to decision making problems.* J. Comput. Appl. Math. **2009**, *223*, 540–542.
[19] Roy, A.R.; Maji, P.K. *A fuzzy soft set theoretic approach to decision making problems.* J. Comput. Appl. Math. **2007**, *203*, 412–418.
[20] Alkhazaleh, S.; Salleh, A.R. *Soft expert sets.* Adv. Decis. Sci. **2011**, *2011*, 757868.
[21] Arokia Lancy, A.; Tamilarasi, C.; Arockiarani, I. *Fuzzy parameterization for decision making in risk management system via soft expert set.* Int. J. Innov. Res. Stud. **2013**, *2*, 339–344.
[22] Arokia Lancy, A.; Arockiarani, I. *A fusion of soft expert set and matrix models.* Int. J. Res. Eng. Technol. **2013**, *2*, 531–535.
[23] Alkhazaleh, S.; Salleh, A.R. *Fuzzy soft expert set and its application.* Appl. Math. **2014**, *5*, 1349–1368.
[24] Bashir, M.; Salleh, A.R. *Fuzzy parameterized soft expert set.* Abstr. Appl. Anal. **2012**,

2012, 258361.

[25] Bashir, M.; Salleh, A.R. *Possibility fuzzy soft expert set.* Open J. Appl. Sci. **2012**, *12*, 208–211.

[26] Alhazaymeh, K.; Hassan, N. *Application of generalized vague soft expert set in decision making.* Int. J. Pure Appl. Math. **2014**, *93*, 361–367.

[27] Broumi, S.; Smarandache, F. *Intuitionistic fuzzy soft expert sets and its application in decision making.* J. New Theory **2015**, *1*, 89–105.

[28] Abu Qamar, M.; Hassan, N. *Q-neutrosophic soft relation and its application in decision making.* Entropy **2018**, *20*, 172.

[29] Abu Qamar, M.; Hassan, N. *Entropy measures of distance and similarity of Q-neutrosophic soft sets and some applications.* Entropy **2018**, *20*, 672.

[30] Sahin, M.; Alkhazaleh, S.; Ulucay, V. *Neutrosophic soft expert sets.* Appl. Math. **2015**, *6*, 116–127.

[31] Ulucay, V.; Sahin, M.; Hassan, N. *Generalized neutrosophic soft expert set for multiple-criteria decision-making.* Symmetry **2018**, *10*, 437.

[32] Al-Quran, A.; Hassan, N. *Neutrosophic vague soft expert set theory.* J. Intell. Fuzzy Syst. **2016**, *30*, 3691–3702.

[33] Al-Quran, A.; Hassan, N. *The complex neutrosophic soft expert set and its application in decision making.* J. Intell. Fuzzy Syst. **2018**, *34*, 569–582.

[34] Al-Quran, A.; Hassan, N. *The complex neutrosophic soft expert relation and its multiple attribute decision-making method.* Entropy **2018**, *20*, 101.

[35] Qayyum, A.; Abdullah, S.; Aslam, M. *Cubic soft expert sets and their application in decision making.* J. Intell. Fuzzy Syst. **2016**, *31*, 1585–1596.

[36] Ziemba, P.; Becker, J. *Analysis of the digital divide using fuzzy forecasting.* Symmetry **2019**, *11*, 166.

[37] Chatterjee, R.; Majumdar, P.; Samanta, S.K. *On some similarity measures and entropy on quadripartitioned single valued neutrosophic sets.* J. Intell. Fuzzy Syst. **2016**, *30*, 2475–2485.

[38] Mallick, R.; Pramanik, S. *Pentapartitioned neutrosophic set and its properties.* Neutrosophic Sets Syst. **2020**, *36*, 184–192.

[39] Radha, R.; Stanis Arul Mary, A. *Heptapartitioned neutrosophic set.* Int. J. Creat. Res. Thoughts **2021**, *9*, 222–230.

[40] Subhadip, R.; Jeong-Gon, L.; Anita, P.; Syamal, K.S. *Similarity measures of quadripartitioned single valued bipolar neutrosophic sets and its application in multi-criteria decision making problems.* Symmetry **2020**, *12*, 1012.

[41] Vasantha, W.B.; Ilanthenral, K.; Smarandache, F.; Vinayak, D.; Shivam, G. *Study of imaginative play in children using single-valued refined neutrosophic sets.* Symmetry **2020**, *12*, 402. 10.3390/sym12030402

[42] Kandasamy, I. *Double-valued neutrosophic sets, their minimum spanning trees, and clustering algorithm.* J. Intell. Syst. **2018**, *27*, 163–182.

[43] Kandasamy, I.; Smarandache, F. *Multicriteria decision making using double refined indeterminacy neutrosophic cross entropy and indeterminacy based cross entropy.* Appl. Mech. Mater. **2016**, *859*, 129–143.

[44] Kandasamy, I.; Smarandache, F. *Triple refined indeterminate neutrosophic sets for personality classification.* In *Proceedings of the 2016 IEEE Symposium Series on Computational Intelligence (SSCI)*, Athens, Greece, 6–9 December **2016**; IEEE: Piscataway, NJ, USA, **2016**; pp. 1–8.

[45] Kandasamy, I.; Vasantha, W.B.; Obbineni, J.; Smarandache, F. *Indeterminate Likert scaling.* Soft Comput. **2019**, 1–10.

[46] Kandasamy, I.; Vasantha, W.; Mathur, N.; Bisht, M.; Smarandache, F. *Chapter 6 Sentiment analysis of the Me Too movement using neutrosophy: Application of single-valued*

neutrosophic sets. In *Optimization Theory Based on Neutrosophic and Plithogenic Sets*; Elsevier: Amsterdam, The Netherlands, **2020**.

[47] Kandasamy, I.; Vasantha, W.; Obbineni, J.M.; Smarandache, F. *Sentiment analysis of tweets using refined neutrosophic sets*. Comput. Ind. **2020**, *115*, 103180.

[48] Kandasamy, W.B.; Kandasamy, I.; Smarandache, F.; Devvrat, V.; Ghildiyal, S. *Study of imaginative play in children using neutrosophic cognitive maps model*. Neutrosophic Sets Syst. **2019**, *30*, 241–252.

[49] Smarandache, F. *A Unifying Field in Logics: Neutrosophic Logic. Neutrosophic Set, Probability, and Statistics*; American Research Press: Rehoboth, NM, USA, **2000**.

[50] Zhang, S.; Hou, Y.; Zhang, S.; Zhang, M. *Fuzzy control model and simulation for nonlinear supply chain system with lead times*. Complexity **2017**, *2017*, 2017634.

[51] Zhang, S.; Zhang, C.; Zhang, S.; Zhang, M. *Discrete switched model and fuzzy robust control of dynamic supply chain network*. Complexity **2018**, *2018*, 3495096.

[52] Zhang, S.; Zhang, P.; Zhang, M. *Fuzzy emergency model and robust emergency strategy of supply chain system under random supply disruptions*. Complexity **2019**, *2019*, 3092514.

[53] Sarwar, M.; Li, T. *Fuzzy fixed point results and applications to ordinary fuzzy differential equations in complex valued metric spaces*. Hacettepe J. Math. Stat. **2019**, *48*, 1712–1728.

[54] Xia, Y.; Wang, J.; Meng, B.; Chen, X. *Further results on fuzzy sampled-data stabilization of chaotic nonlinear systems*. Appl. Math. Comput. **2020**, *379*, 125225.

[55] Gao, M.; Zhang, L.; Qi, W.; Cao, J.; Cheng, J.; Kao, Y.; Yan, X. *SMC for semi-Markov jump TS fuzzy systems with time delay*. Appl. Math. Comput. **2020**, *374*, 125001.

[56] Zhang, S.; Zhang, M. *Mitigation of Bullwhip Effect in Closed-Loop Supply Chain Based on Fuzzy Robust Control Approach*. Complexity **2020**, *2020*, 1085870.

[57] Deli, I.; Broumi, S. *Neutrosophic soft relations and some properties*. Ann. Fuzzy Math. Inform. **2015**, *9*, 169–182.

[58] Bera, T.; Mahapatra, N.K. *Introduction to neutrosophic soft topological space*. Opsearch **2017**, *54*, 841–867.

[59] Maji, P.K. *Neutrosophic soft set*. Ann. Fuzzy Math. Inform. **2013**, *5*, 157–168.

[60] Al-Nafee, A.B.; Broumi, S.; Smarandache, F. *Neutrosophic soft bitopological spaces*. Int. J. Neutrosophic Sci. **2021**, *14*, 47–56.

[61] Ozturk, T.Y.; Aras, C.G.; Bayramov, S. *A new approach to operations on neutrosophic soft sets and to neutrosophic soft topological spaces*. Commun. Math. Appl. **2019**, *10*, 481–493.

[62] Ahmad, A.; Hatamleh, R.; Matarneh, K.; Al-Husban, A. *On the irreversible k-threshold conversion number for some graph products and neutrosophic graphs*. Int. J. Neutrosophic Sci. **2025**, *25*, 183–196.

[63] Hatamleh, R.; Al-Husban, A.; Sundareswari, K.; Balaj, G.; Palanikumar, M. *Complex tangent trigonometric approach applied to* (γ, σ)*-rung fuzzy set using weighted averaging, geometric operators and its extension*. Commun. Appl. Nonlinear Anal. **2025**, *32*, 133–144.

[64] Hatamleh, R.; Al-Husban, A.; Palanikumar, M.; Sundareswari, K. *Different weighted operators such as generalized averaging and generalized geometric based on trigonometric* $\wp$*-rung interval-valued approach*. Commun. Appl. Nonlinear Anal. **2025**, *32*, 91–101.

[65] Shihadeh, A.; Hatamleh, R.; Palanikumar, M.; Al-Husban, A. *New algebraic structures towards different* (α, β) *intuitionistic fuzzy ideals and its characterization of an ordered ternary semigroup*. Commun. Appl. Nonlinear Anal. **2025**, *32*, 568–578.

[66] Hatamleh, R.; Heilat, A.S.; Palanikumar, M.; Al-Husban, A. *Different operators via weighted averaging and geometric approach using trigonometric neutrosophic interval-valued set and its extension*. Neutrosophic Sets Syst. **2025**, *80*, 194–213.

[67] Hatamleh, R.; Heilat, A.S.; Palanikumar, M.; Al-Husban, A. *Characterization of interaction aggregating operators setting interval-valued Pythagorean neutrosophic set*. Neutrosophic Sets Syst. **2025**, *81*, 285–305.

[68] Hatamleh, R.; Al-Husban, A.; Zubair, S.A.M.; Elamin, M.; Saeed, M.M.; Abdolmaleki, E.;

et al. *AI-assisted wearable devices for promoting human health and strength using complex interval-valued picture fuzzy soft relations*. Eur. J. Pure Appl. Math. **2025**, *18*, 5523–5523.

[69] El-Sheikh, S.A.; Abd El-Latif, A.M. *Decompositions of some types of supra soft sets and soft continuity*. Int. J. Math. Trends Technol. **2014**, *9*, 37–56.

[70] Abd El-Latif, A.M. *On soft supra compactness in supra soft topological spaces*. Tbilisi Math. J. **2018**, *11*, 169–178.

[71] Abd El-Latif, A.M.; Hosny, R.A. *Supra open soft sets and associated soft separation axioms*. Int. J. Adv. Math. **2017**, *6*, 68–81.