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A Novel Maclaurin Series Approach to the Sakiadis Flow Problem and Its Fractal Formulation in Fluid Mechanics

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Abstract. This research paper introduces a method utilizing the Maclaurin series to analyze the standard Sakiadis problem. The technique uses the Maclaurin series to describe fluid-mechanics boundary layers, combining the series solution with diagonal Padé approximants for handling the infinity condition. Additionally, the Hausdorff derivative is used to examine the Sakiadis equation's fractal formulation. The current approach is consistent with the previous process and follows the correct balance. This study is an important resource for furthering research in this area and provides insightful information.

2020 Mathematics Subject Classifications: 76D10, 34M10, 76M22

Key Words and Phrases: Maclaurin series method (MSM), Sakiadis equation, Padé approximants, Hausdorff derivative

1. Introduction

Significant engineering applications have investigated laminar, incompressible boundary-layer flows, for example: the smooth removal of plastic sheets, the cooling of an endless metal plate in a cooling tub, and processes in glass and polymer industries. Notably, a fundamental boundary-layer equation arising in fluid mechanics is the Sakiadis equation. In order to solve such fluid-dynamics problems, a wide variety of numerical and theoretical techniques have been used for the Blasius equation. Howarth reported the first numerical results for the Blasius equation using the Runge–Kutta methodology [1]. Chinese professors Liao [2], Yu *et al.* [3], and Ji Huan He [4] employed approximate methods to analyze the Blasius equation. Wazwaz [5, 6] utilized Adomian decomposition and the Variational Iteration Method to tackle the Blasius equation. A connection between the Adomian decomposition method and the homotopy method for treating the Blasius equation was examined in [7]. An approximate analytic solution of the Blasius equation was achieved in

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[8] using Padé approximants. Ganji *et al.* [9] used the homotopy perturbation method to obtain an analytical solution for the Blasius equation. Parand *et al.* [10] tested the sinc–collocation method and Xu *et al.* [11] studied a fixed-point approach to solving the Blasius equation. A two-point block predictor–corrector methodology was developed in [12] for numerically resolving the well-known Blasius and Sakiadis flows. In [13], a generalized version of the Blasius equation was derived by Makhfi and Bebbouchi. Recently, Mutuk [14] employed neural networks in the field of fluid mechanics and treated the Blasius equation. The development and application of the RCW method to the Blasius problem were provided by Rahmanzadeh and Asadi [15].

However, all of the above works [1–15] and other methods highlighted in [16–27] have their own difficulties, shortcomings, and computational challenges. It is therefore essential to introduce a simple and accurate method for solving differential equations in engineering and science, including for non-mathematicians. The Maclaurin series is an elementary strategy that is widely accessible. This paper implements the Maclaurin series strategy for the third-order equation; the clarity of the procedure and the explicit results render the technique appealing for real-world applications. The fundamental purpose of this work is to propose a novel solution of the Sakiadis equation arising in fluid mechanics via the Maclaurin Series Method (MSM). For Newtonian fluids, the Sakiadis equation is essential to the analysis of laminar boundary-layer problems. The basic difficulty presented by the boundary condition at infinity is addressed here. By combining the series obtained using MSM with diagonal Padé approximants, reliable outcomes are obtained [28]. Table 1 illustrates the convergence of MSM. Additionally, this article compares the outcomes with previous solutions [5, 6]. MSM provides a more straightforward and effective route than many decomposition and iterative approaches, which frequently entail considerable complexity. Because it avoids assumptions, discretization, over-linearization, and other constrictive techniques that can substantially alter the original problem, the method is especially well suited to fluid-mechanics problems. With MSM, a high degree of precision is maintained while removing the need for laborious numerical techniques. The results are both promising and accurate, and the suggested method performs effectively.

2. Sakiadis Flow and its solution

We assume a laminar Newtonian fluid flow with $\xi > 0$ throughout the domain. Consider the boundary layer equations in two-dimensional motions [5, 6]:

$$\begin{aligned}
&\frac{\partial G}{\partial x} + \frac{\partial H}{\partial y} = 0,\\
&G\,\frac{\partial G}{\partial x} + H\,\frac{\partial G}{\partial y} = \gamma\,\frac{\partial^2 G}{\partial y^2},\\
&G(x,0) = U_0, \quad H(x,0) = 0, \quad G(x,y) \to 0 \text{ as } y \to \infty.
\end{aligned} \tag{1}$$

Considering the similarity transformation

$$\begin{aligned} &\xi = \sqrt{\tfrac{U_0}{\gamma x}}\, y, \quad G = U_0 g'(\xi), \\ &H = -\tfrac{1}{2}\sqrt{\tfrac{U_0 \gamma}{x}}\left[g(\xi) - \xi g'(\xi)\right], \end{aligned} \tag{2}$$

the Eq. (1) becomes

$$g'''(\xi) \;+\; \frac{1}{2}\, g(\xi)\, g''(\xi) \;=\; 0, \tag{3}$$

$$g(0) = 0, \qquad g'(0) = 1, \qquad g'(\infty) = 0. \tag{4}$$

To illustrate the solution process, we assume that $g''(0) = \beta$ is an unknown constant that needs to be further identified. In Eq. (3), setting $\xi = 0$ yields

$$g'''(0) \;+\; \frac{1}{2}\, g(0)\, g''(0) \;=\; 0. \tag{5}$$

Using Eq. (4) in Eq. (5), we get

$$g'''(0) \;=\; 0. \tag{6}$$

Differentiating Eq. (3) w.r.t ξ, we obtain

$$g^{iv}(\xi) \;+\; \frac{1}{2}\, g'(\xi)\, g''(\xi) \;+\; \frac{1}{2}\, g(\xi)\, g'''(\xi) \;=\; 0. \tag{7}$$

Setting $\xi = 0$ in Eq. (7) results in

$$g^{iv}(0) + \frac{1}{2} g'(0)\, g''(0) + \frac{1}{2} g(0)\, g'''(0) = 0. \tag{8}$$

Using Eq. (4), Eq. (6) into Eq. (8), we get

$$g^{iv}(0) = -\frac{\beta}{2}. \tag{9}$$

Differentiating Eq. (7) w.r.t. ξ, we obtain

$$g^{v}(\xi) + \frac{1}{2} g(\xi)\, g^{iv}(\xi) + g'(\xi)\, g'''(\xi) + \frac{1}{2}\big(g''(\xi)\big)^2 = 0. \tag{10}$$

Setting $\xi = 0$ in Eq. (7) and considering the Eq. (4), Eq. (6) and Eq. (9), we have

$$g^{v}(0) + \frac{1}{2}\, g(0)\, g^{iv}(0) + g'(0)\, g'''(0) + \frac{1}{2}\big(g''(0)\big)^2 = 0, \tag{11}$$

$$g^{v}(0) = -\frac{\beta^2}{2}. \tag{12}$$

By a similar operation via Mathematica, we can obtain

$$g^6(0) = \frac{3\beta}{4},$$
$$g^7(0) = \frac{11\beta^2}{4}, \tag{13}$$
$$g^8(0) = \frac{\beta}{8}\big(-15 + 22\beta^2\big),$$
$$\vdots$$
$$\vdots$$

The Maclaurin series solution can be expressed as

$$\begin{aligned} g(\xi) &= g(0) + g'(0)\frac{\xi}{1!} + g''(0)\frac{\xi^2}{2!} + g'''(0)\frac{\xi^3}{3!} + g^{iv}(0)\frac{\xi^4}{4!} + g^{v}(0)\frac{\xi^5}{5!} + g^{vi}(0)\frac{\xi^6}{6!} \\ &\quad + g^7(0)\frac{\xi^7}{7!} + g^8(0)\frac{\xi^8}{8!} + g^9(0)\frac{\xi^9}{9!} + g^{10}(0)\frac{\xi^{10}}{10!} + \dots, \end{aligned} \tag{14}$$

$$\begin{aligned} g(\xi) \;=\; & \xi \;+\; \beta\,\frac{\xi^2}{2} \;-\; \beta\,\frac{\xi^4}{48} \;-\; \beta^2\,\frac{\xi^5}{240} \;+\; \beta\,\frac{\xi^6}{960} \;+\; 11\beta^2\,\frac{\xi^7}{20160} \\ & +\; \beta\big(-15 + 22\beta^2\big)\,\frac{\xi^8}{322560} \;-\; 129\beta^2\,\frac{\xi^9}{2903040} \;-\; \frac{15}{16}\big(-7\beta + 50\beta^3\big)\,\frac{\xi^{10}}{10!} \\ & -\; \frac{3}{16}\big(-587\beta^2 + 250\beta^4\big)\,\frac{\xi^{11}}{11!} \;+\; \frac{15}{32}\big(-63\beta + 1408\beta^3\big)\,\frac{\xi^{12}}{12!} \\ & +\; \frac{3}{32}\big(-9385\beta^2 + 18598\beta^4\big)\,\frac{\xi^{13}}{13!} \;+\; \frac{3}{64}\big(3465\beta - 199988\beta^3 + 37196\beta^5\big)\,\frac{\xi^{14}}{14!} \\ & -\; \frac{3}{64}\big(-174645\beta^2 + 1017244\beta^4\big)\,\frac{\xi^{15}}{15!} \;-\; \frac{3}{128}\big(45045\beta - 6081294\beta^3 + 5089516\beta^5\big)\,\frac{\xi^{16}}{16!} \\ & -\; \frac{3}{128}\big(3753435\beta^2 - 51909900\beta^4 + 5089516\beta^6\big)\,\frac{\xi^{17}}{17!}\,. \end{aligned} \tag{15}$$

Table 1: MSM $\beta = g''(0)$ solution comparison with ADM, VIM

Padé approximants	**Present method**	**ADM [5]**	**VIM [6]**
[2/2]	0.5773502692	0.5773502692	0.577350693
[3/3]	0.5163977795	0.5163977795	0.5163977793
[4/4]	0.5227030798	0.5227030798	0.5227030796
[5/5]	1.267686100	-	-
[6/6]	0.5217102130	–	0.5217102130
[7/7]	0.5026354150	–	0.5026354150
[8/8]	Complex numbers	–	Complex numbers
[9/9]	Complex numbers	–	Complex numbers
[10/10]	0.4672639966	–	0.4672639966
[11/11]	0.5176098151	–	0.5176098151

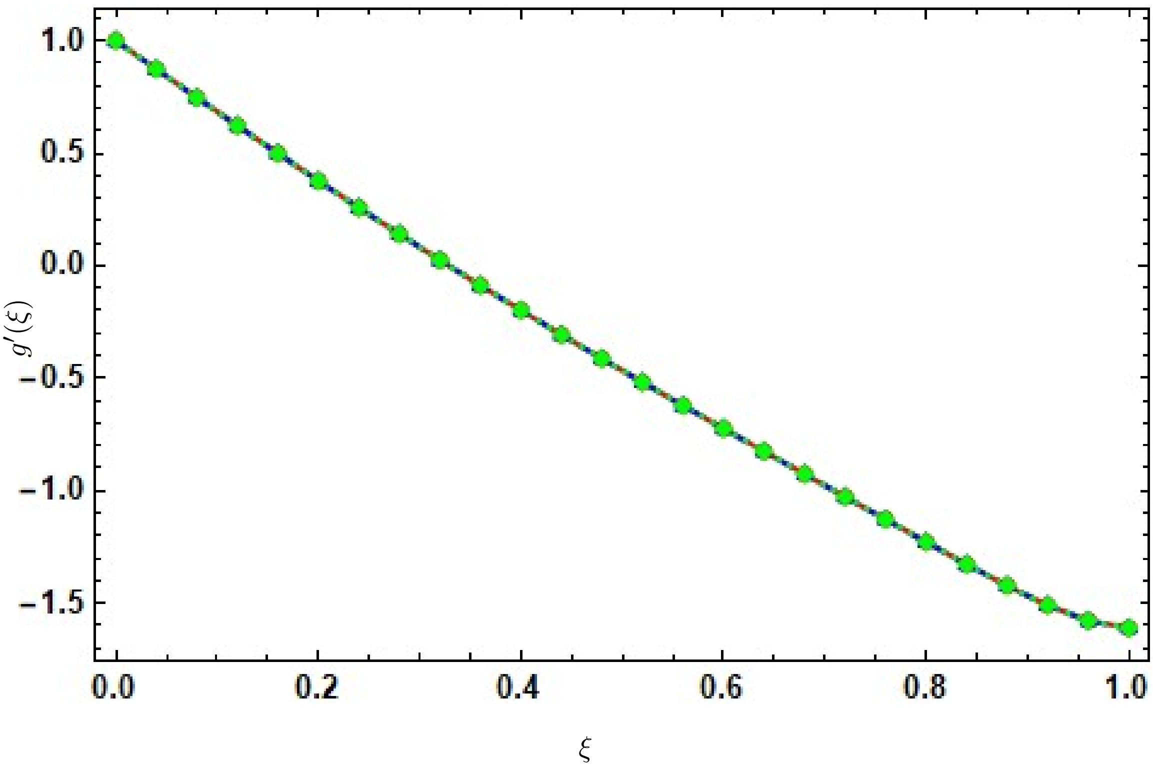

Figure 1: MSM g' VIM g' ADM g'

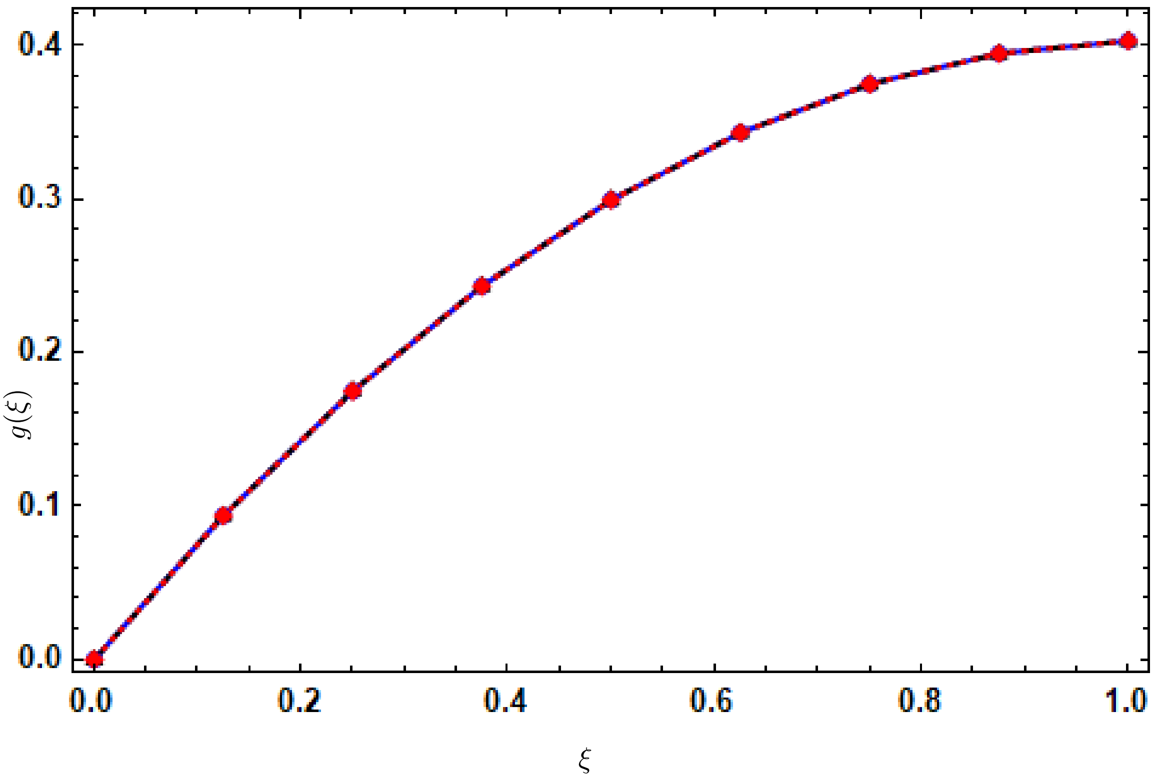

Figure 2: Solution of g via MSM Solution of g via VIM Solution of g via ADM

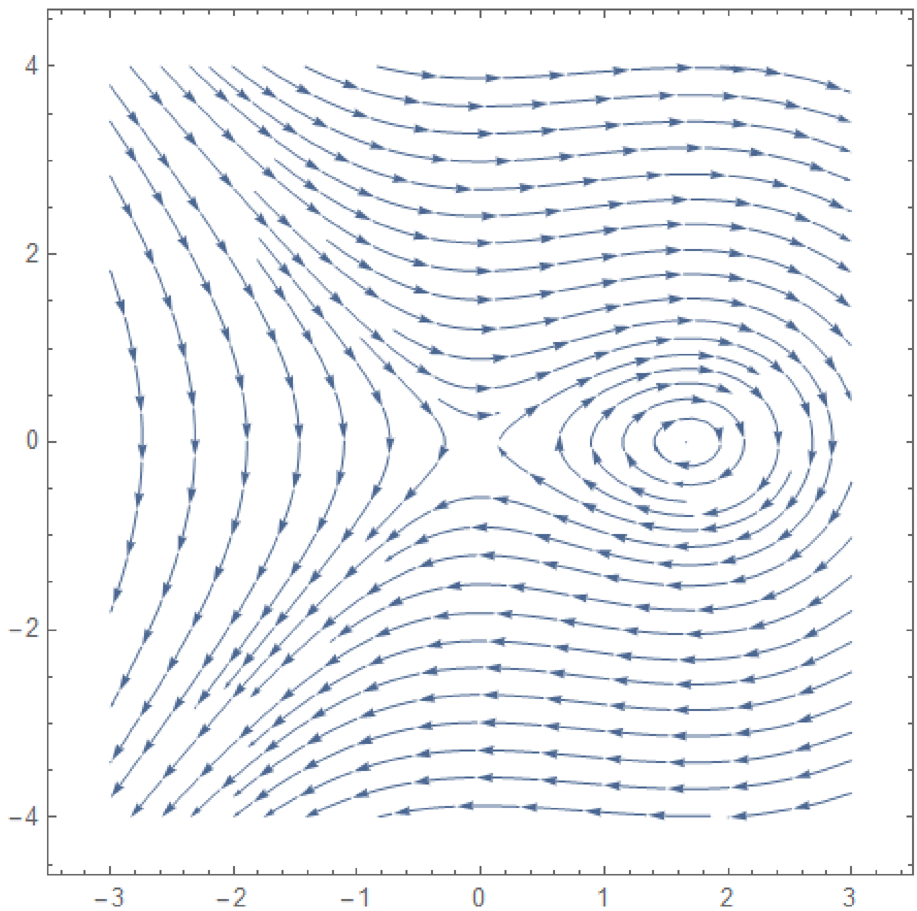

Figure 3: Streamlines of g.

3. Discussion

This paper presents the Sakiadis equation series solution using a novel method from the Maclaurin series (MSM). By adding a similarity variable, Sakiadis developed continuity and momentum equations for a continuous, incompressible laminar flow of fluid. MSM validity is shown in Table 1 and Figs. 1 and 2. Figure 3 reveals streamlines displaying the actual Sakiadis equation phenomena. Streamlines display particle trajectories traveling along the flow paths and provide insight into complex flow characteristics. Figure 3 is a form of communication designed to show the direction in which the fluid particle travels in the flow of fluid at a given level. The following fractal formulation for Eq. (3) may be established in the context of fractal derivative [29, 30]:

$$\frac{d}{d\xi^\alpha}\left(\frac{d}{d\xi^\alpha}\left(\frac{dg}{d\xi^\alpha}\right)\right) + \frac{1}{2}\,g(\xi^\alpha)\,\frac{d}{d\xi^\alpha}\left(\frac{dg}{d\xi^\alpha}\right) = 0, \tag{16}$$

where $\frac{dg}{d\xi^\alpha}$ is the fractal derivative defined as:

$$\frac{dg}{d\xi^\alpha} = \lim_{\xi\to\xi_1}\frac{g(\xi)-g(\xi_1)}{\xi^\alpha-\xi_1^\alpha} \tag{17}$$

Or

$$\frac{dg}{d\xi^\alpha} = \frac{1}{\alpha\,\xi^{\alpha-1}}\frac{dg}{d\xi} \tag{18}$$

With transformation [29, 30]

$$\xi^\alpha = s \tag{19}$$

Using Eq. (19), Eq. (16) analogue to Eq. (3) and one can recover easily the same results obtained in Eq. (15). Provided the traditional calculus, this transformation makes the fractal calculus extremely simple.

4. Conclusion

This article introduces a novel method for calculating the new solution of the Sakiadis equation in the Maclaurin series. Padé approximants handle the infinity condition. This approach is easy to implement, and the outcomes demonstrate that by reducing the size of the calculation, the explanations can convert extra precise. Comparing the current solution to previous solutions [5, 6] shows that there is excellent agreement between the two. Furthermore, the analysis presented here strengthens trust in the MSM's efficiency. Moreover, a fractal model of the Sakiadis equation is introduced. In addition to the demonstrated accuracy and simplicity of the Maclaurin Series Method (MSM), future work may explore its application to more complex fluid models. These include non-Newtonian fluids, fractional derivative formulations, and multi-phase flows. The MSM's adaptability and precision make it a promising tool for solving advanced boundary-layer problems in engineering and applied sciences.

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