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Triangular Intuitionistic Fuzzy Frank Aggregation for Efficient Renewable Energy Project Selection

A. Fahmi[1], A. Hashmi[2], Aziz Khan[3], Aiman Mukheimer[3], Thabet Abdeljawad[3,5,*], Rajermani Thinakaran[4]

[1] *Department of Mathematics and Statistics, Faculty of Sciences, The University of Faisalabad, Pakistan*
[2] *Faculty of Management Sciences, The University of Central Punjab, Lahore City, Pakistan*
[3] *Department of Mathematics and Sciences, Prince Sultan University, P.O. Box 66833, 11586 Riyadh, Saudi Arabia*
[4] *Faculty of Data Science and Information Technology, INTI International University, Negeri Sembilan, Malaysia*
[5] *Department of Mathematics and Applied Mathematics Sefako Makgatho Health Sciences University Garankuwa, Medusa 0204, South Africa*

Abstract. Optimizing renewable energy project selection presents a complex challenge that demands intelligent decision-making under conditions of uncertainty. In multi-criteria decision-making (MCDM), the need to balance numerous conflicting factors makes these techniques invaluable for effective project evaluation and selection. This paper introduces a novel Triangular Intuitionistic Fuzzy Frank (TIFF) framework, which integrates triangular intuitionistic fuzzy averaging and geometric aggregation operators to enhance decision-making in renewable energy project assessment. We develop several new aggregation operators, including: Triangular Intuitionistic Fuzzy Frank Weighted Averaging (TIFFWA), Ordered Weighted Averaging (TIFFOWA), Hybrid Averaging (TIFFHA), Weighted Geometric (TIFFWG), Ordered Weighted Geometric (TIFFOWG), and Hybrid Geometric (TIFFHG). These operators, built upon the Frank t-norm and t-conorm, enable more accurate and adaptive evaluations by effectively managing varying levels of uncertainty. In addition, novel scoring and precision functions are introduced to further refine the decision-making process, yielding more reliable outcomes. A step-by-step methodology is presented for applying the TIFF approach to renewable energy project selection, providing clear guidance for practical implementation. To validate the method, a numerical case study is conducted, demonstrating the superior performance of the TIFF framework compared to existing techniques. The results underscore the method's efficiency, adaptability, and practical value as a robust tool for optimizing renewable energy project decisions under uncertainty.

2020 Mathematics Subject Classifications: 03E72, 90B50, 62C86

Key Words and Phrases: Fuzzy set, Multi-attribute decision making, Triangular Fuzzy Frank Aggregation, energy efficiency

1. Introduction

The drive for sustainable living and growing environmental concerns have increased the need for energy efficiency in buildings, particularly smart houses [1–19]. The development of AI technology has opened up new avenues for energy consumption optimization. However, conventional optimization techniques are severely hampered by the complexity of building systems,

*Corresponding author.
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Email addresses: aliyafahmi@gmail.com (A. Fahmi), arshiahashmee@gmail.com (A. Hashmi), akhan@psu.edu.sa (A. Khan), tabdeljawad@psu.edu.sa (T. Abdeljawad), rajermani.thina@newinti.edu.my (R. Thinakaran)

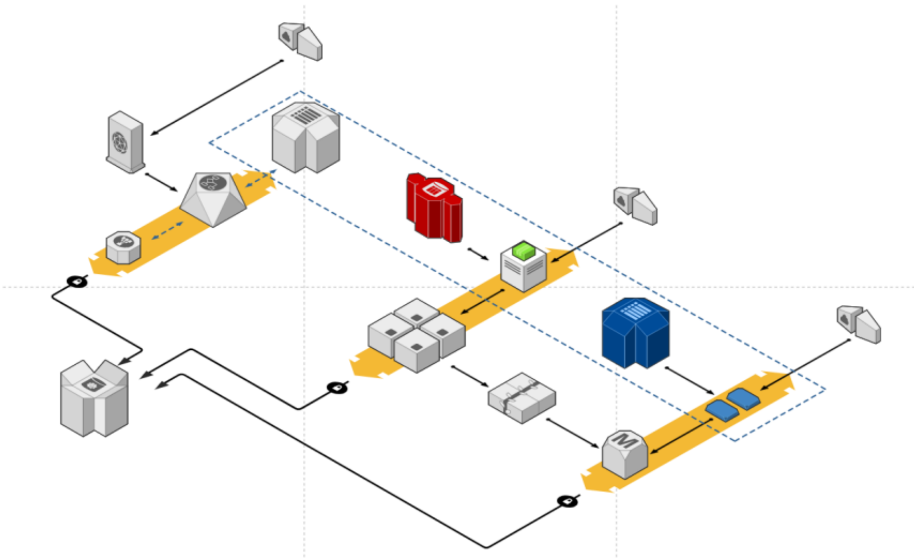

Figure 1, efficient renewable energy project selection.

which include varying occupancy levels and erratic weather. Current methods, such as those based on [20] static models or heuristic algorithms, frequently struggle to adjust to changes in the environment in real time, which restricts their capacity to achieve consistent and ideal energy use [21]. Fuzzy logic was the foundation of many AI-driven techniques for building energy optimization. Although these approaches were successful in controlling uncertainty, they usually depended on oversimplified models that were unable to fully represent the intricacy of smart home environments [22–32]. Systems such as fuzzy-based decision frameworks for assessing HVAC system performance [14] and the adaptive neuro-fuzzy inference system for load forecasting [33] had trouble managing uncertainty in real-time energy management, especially when taking dynamic factors like weather or human behavior into account [33–44].

Figure 1 is efficient renewable energy project selection given as

The concept of a fuzzy set, which can address the issue of uncertainty in various contexts, was developed by Zadeh [16]. Sets with degrees of membership are called FSs, and membership functions with values in the interval [0, 1] are allowed according to FS theory. However, because there could be some reluctant degree, it might not always hold in real life that the degree of nonmembership function is equal to one minus membership function. The intuitionistic FS was initially proposed by Atanassov [45]. Truth and falsity grades are assigned to the constituents of IFSs. The degree of hesitancy about an element's truth and falsity grades within a set must be represented using IFS. IFSs are used in numerous real-world scenarios to solve issues. One example of this is when we toss a coin; there are two possible outcomes, head or tail. Expert opinions were represented using the Basic Uncertain Information (BUI) technique, and in a group decision-making context, these opinions were combined using aggregation operations [29]. The coin will show either way at a time, but not both ways at once. The theory of Pythagorean fuzzy sets to address these types of issues [5]. Because PyFS and IFS have the same structure but different conditions, PyFS is the improved version of IFSs that have overcome their limitations. IFS and PyFS are often closely related. Compared to IFSs, PyFS allows us to measure uncertainty more precisely and adequately. Aggregation operators [28, 40, 43, 46], similarity measures [1, 29, 44, 47], decision-making approaches [12, 48, 49], and various sorts of procedures was a few examples of uses of FS and its extensions. By clustering fuzzy c-numbers, Xu and Li [10] prophesied the reversion of a fuzzy time sequence. This study offers a class of fuzzy clustering procedures specifically tailored for processing fuzzy data [42]. Fuzzy c-number clusterings are novel algorithms designed to handle different kinds of fuzzy data more efficiently, fuzzy number forms such as conventional [50–58], trapezoidal, LR-type, and triangular fuzzy numbers [14]. Akram et al. [59, 60] introduced several PFS-imitable applications. A thorough case study on choosing medical subject experts was used to test an integrated MCDM algorithm that was created using the suggested operators and the combined criterion weight determination model [46, 61–63]. As a result, it can represent the relative relevance of the supplied Pythagorean fuzzy argument and its ordered position. Several interval-valued Furthermore, we provide some interval-valued Pythagorean fuzzy point weighted averaging (IVPF-PWA) operators that can modify the degree of the aggregated arguments with a parameter by combining the interval-valued Pythagorean fuzzy point operators with the IVPFWA operator. In order to address multi-attribute group decision-making under interval-valued Pythagorean fuzzy information, Rahman et al. [4] presented an operator. The value and compatibility of the discussed methodologies and decision support systems were examined [1, 2]. The generalized IVPNFWG operator were presented by Yang et al. [14] to aggregate IVPNF information. Petchimuthu et al. [47], a new concept of Pythagorean neutrosophic normal interval-valued weighted averaging (PNSNIVWA), Pythagorean neutrosophic normal interval-valued weighted geometric (PNSNIVWG), and generalized Pythagorean neutrosophic normal interval-valued weighted averaging (GPNSNIVWA) as well as generalized Pythagorean neutrosophic normal interval-valued weighted geometric (GPNSNIVWG) were discussed by Palanikumar et al. [43] and Peng et al.[44].

Figure 2 gives methods for enhancing as Based on the consistency of the InPLPR, Wang et al. [7] created a decision-making method that includes estimating missing data, enhancing consistency, and evaluating the options. An incomplete probabilistic linguistic term set was presented by Liu et al. [41]. The distinction between absolute and relative knowledge distances in the structural characteristics of hierarchical clustering was examined by Lian et al. [**?**]. Anusha et al. [62] covered the hybridizations in addition to providing an extension of the MSM operators and their requests based on q-rung probabilistic dual hesitant fuzzy sets. Depending on the evaluation values of each choice, it is frequently possible to examine many possibilities to arrive at a comprehensive assessment result, for example by using MADM. A unique approach to selecting robotic systems for homogenous group DM was presented by Bairagi [64]. These operators were utilized to devise a method for handling group decision-making with CPF information, introduced the Artificial intelligence applied in DM to choose a maintenance approach and other research [7–12, 16, 40, 42, 65].

Figure 3 gives the as MCDM method below is

This manuscript is structured as follows: Section 2 introduces the concept of IFSs. Section 3 presents operational rules for TIFSs, including algebraic and Frank operational laws. In Section 4, we propose the TIFFWA, TIFFOWA, TIFFHWA, TIFFWG, TIFFOWG and TIFFHWG operators. Section 5 outlines an optimized MCDM process for the TIF model. Section 6, define the case study and a comparative analysis. Finally, Section 7 concludes the study.

1.1. Contribution of study

The application of triangular intuitionistic Fuzzy Frank Aggregation Operators (TIFFAO) has significantly advanced the efficient selection of renewable energy projects by effectively addressing uncertainty and optimizing project evaluations. The main contributions of this study are as follows:

(a) Operational laws and triangular intuitionistic Fuzzy Numbers (TIFN) are defined to provide a solid mathematical basis for selecting renewable energy projects.

(b) A novel accuracy and scoring function is introduced to improve the precision of decision-making processes in renewable energy project evaluations.

(c) Various TIFN aggregation operators are introduced, such as TIFFWA, TIFFOWA, TIFFHWA, TIFFWG, TIFFOWG and TIFFHWG, tailored for Multi-Criteria Decision-Making (MCDM) in renewable energy project selection. These operators enhance the adaptability and

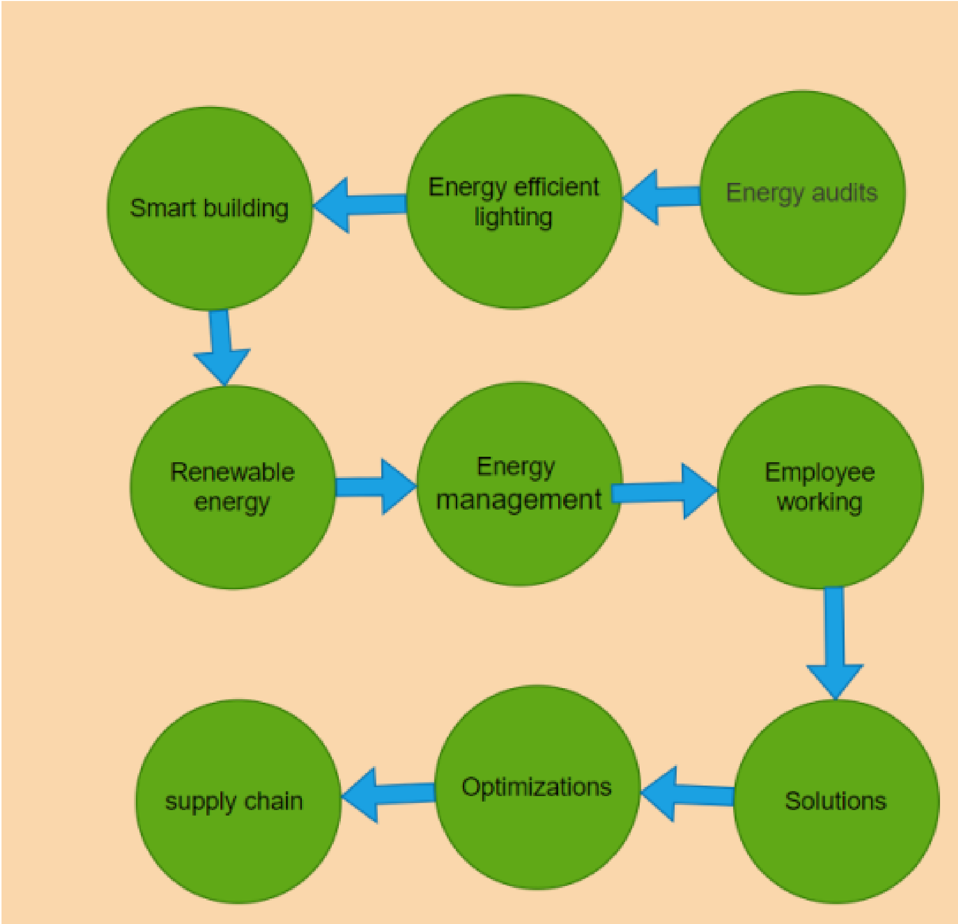

Figure 2, methods for enhancing energy efficiency.

effectiveness of evaluating projects under dynamic conditions.

(d) The TIFFAO framework efficiently processes inconsistent and diverse data, leading to more accurate selection of renewable energy projects, with better forecasting and cost management. This method is particularly useful in evaluating projects where factors like budget, environmental impact, and technological feasibility vary.

(e) The TIFFAO approach excels in managing multi-dimensional evaluations for renewable energy projects, where various factors such as energy efficiency, sustainability, and stakeholder needs must be balanced. By optimizing the selection process, this method minimizes uncertainties and maximizes the long-term benefits of the chosen projects.

This study offers a robust methodology that significantly enhances the selection process for renewable energy projects, providing a more adaptive, efficient, and data-driven approach to making informed decisions.

1.2. Motivation

This study focuses on the challenge of selecting efficient renewable energy projects, providing a foundation for future work in the development of intelligent decision-making systems for energy project evaluation.

The triangular intuitionistic Fuzzy Set (TIFS) framework has been designed to allow energy

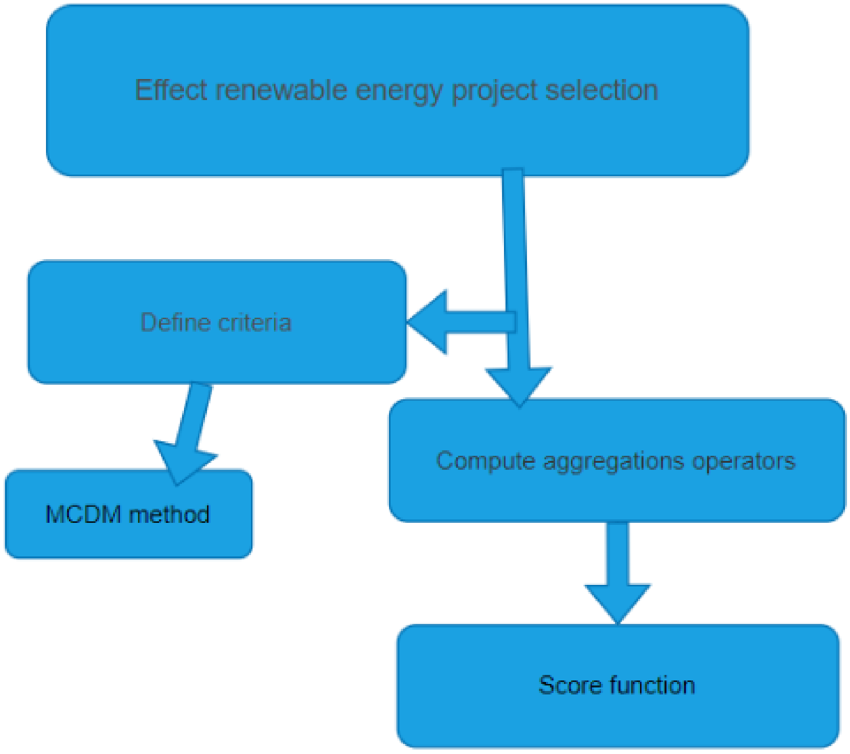

Figure 3, MCDM method.

project evaluators to offer insights more easily. By covering a wider range of information uncertainty, preventing the loss of critical data when converting qualitative project characteristics into quantitative information.

The case study presented can be adapted for use by energy project selection teams, helping them improve decision-making processes through structured evaluation. This method provides a clear path from expert opinions to actionable project selection solutions.

Renewable energy project evaluation data is often complex, uncertain, and incomplete due to varying environmental conditions, technological advancements, and stakeholder considerations. Conventional selection methods struggle with such uncertainty. The triangular intuitionistic Fuzzy Frank Aggregation Operators (TIFFAO) effectively manage and integrate this data, significantly improving the reliability of project selection decisions and enhancing the overall efficiency of renewable energy project evaluation systems.

1.3. Novelty

In this article, we aim to design the following:

i. To define advanced operational laws for triangular intuitionistic Fuzzy Frank statistics, extending traditional operational laws to effectively address the complexity of renewable energy project selection and assess its mathematical properties.

ii. To introduce innovative aggregation operators, such as triangular intuitionistic Fuzzy Frank Aggregation Operators, specifically tailored for optimizing the selection process of renewable energy projects, improving decision-making in dynamic evaluation scenarios.

iii. To propose a Multi-Criteria Decision-Making (MCDM) technique using triangular intuitionistic fuzzy sets (TIF) that helps evaluate various factors influencing renewable energy project selection.

iv. To apply the proposed methodology to solve a real-world problem in renewable energy project selection, demonstrating its practical value and efficiency in real-world decision-making scenarios.

v. To validate the robustness of the proposed approach through a sensitivity analysis, assessing how different influencing factors impact the overall project selection process.

The abbreviation of table 1 is written below.

Abbreviations	Full Name
IFFNs	Intuitionistic fuzzy frank numbers
TIFN	Triangular intuitionistic fuzzy number
TIFFAO	Triangular intuitionistic fuzzy frank aggregation operator
TIFFWA	Triangular intuitionistic fuzzy frank weighted average
TIFFOWA	Triangular intuitionistic fuzzy frank ordered weighted average
TIFFHWA	Triangular intuitionistic fuzzy frank hybrid weighted average
TIFFWG	Triangular intuitionistic fuzzy frank weighted geometric
TIFFOWG	Triangular intuitionistic fuzzy frank ordered weighted geometric
TIFFHWG	Triangular intuitionistic fuzzy frank hybrid weighted geometric

2. Basic ideas

Definition 1. *[16] Considering that $\Phi \neq X$ and let is use a fuzzy set $\gamma = \left\{ \begin{array}{c} \left\langle x, \mu_{\gamma(x)} \right\rangle \\ : x \in X \end{array} \right\}$. An element x in X is represented by the membership function, $\mu_{\gamma(x)}$ is a mapping from X to $[0,1]$.*

Definition 2. *[31] Let $a_1 = [L_1, \kappa_1]$ and $a_2 = [L_2, \kappa_2]$ be the IFSs based on frank operators and $\lambda > 0$, then*

$$a_1 \oplus a_2 = \left[1 - \log_\beta^{\left(1 + \frac{(\beta^{1-L_1}-1)(\beta^{1-L_2}-1)}{\beta-1}\right)}, \log_\beta^{\left(1 + \frac{(\beta^{k_2}-1)(\beta^{\kappa_2}-1)}{\beta-1}\right)} \right];$$

$$a_1 \otimes a_2 = \left[\log_\beta^{\left(1 + \frac{(\beta^{\bar{L_1}}-1)(\beta^{L_2}-1)}{\beta-1}\right)}, 1 - \log_\beta^{\left(1 + \frac{(\beta^{1-K_1}-1)(\beta^{1-\kappa_2}-1)}{\beta-1}\right)} \right];$$

$$a_1^\lambda = \left[\log_\beta^{\left(1 + \frac{(\beta^{L_1}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)}, 1 - \log_\beta^{\left(1 + \frac{(\beta^{1-\kappa_1}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)} \right];$$

$$\lambda a_1 = \left[1 - \log_\beta^{\left(1 + \frac{(\beta^{1-L_1}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)}, \log_\beta^{\left(1 + \frac{(\beta^{1-\kappa_1}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)} \right].$$

Definition 3. *[11] Let $a = [\varsigma, \chi]$ be the IFSs, then the score function is $a = \varsigma_\alpha - \chi_\alpha$.*

Definition 4. *[11] Let $a = [\varsigma, \chi]$ be the IFSs, then the accuracy function is $a = \varsigma_\alpha + \chi_\alpha$.*

3. TIFN and operational laws on Frank

The section address the operational laws of the frank t-norm and t-conorm. The frank operational laws are a collection of axioms that control how operations in TIF logic, such as t-norms and t-conorms.

Definition 5. *Let $a_1 = \left[\begin{array}{c} [C_1, D_1, E_1], \\ [G_1, H_1, L_1] \end{array} \right]$ and $a_2 = \left[\begin{array}{c} [C_2, D_2, E_2], \\ [G_2, H_2, L_2] \end{array} \right]$ be two TIFFNs and $\lambda > 0$, then*

$$a_1 \oplus a_2 = \begin{bmatrix} \begin{bmatrix} 1 - \log_\beta^{\left(1+\frac{(\beta^{1-C_1}-1)(\beta^{1-C_2}-1)}{\beta-1}\right)}, \\ 1 - \log_\beta^{\left(1+\frac{(\beta^{1-D_1}-1)(\beta^{1-D_2}-1)}{\beta-1}\right)}, \\ 1 - \log_\beta^{\left(1+\frac{(\beta^{1-E_1}-1)(\beta^{1-E_2}-1)}{\beta-1}\right)} \end{bmatrix}, \\ \begin{bmatrix} \log_\beta^{\left(1+\frac{(\beta^{G_1}-1)(\beta^{G_2}-1)}{\beta-1}\right)}, \\ \log_\beta^{\left(1+\frac{(\beta^{H_1}-1)(\beta^{H_2}-1)}{\beta-1}\right)}, \\ \log_\beta^{\left(1+\frac{(\beta^{L_1}-1)(\beta^{L_2}-1)}{\beta-1}\right)} \end{bmatrix} \end{bmatrix};$$

$$a_1 \otimes a_2 = \begin{bmatrix} \begin{bmatrix} \log_\beta^{\left(1+\frac{(\beta^{C_1}-1)(\beta^{C_2}-1)}{\beta-1}\right)}, \\ \log_\beta^{\left(1+\frac{(\beta^{D_1}-1)(\beta^{D_2}-1)}{\beta-1}\right)}, \\ \log_\beta^{\left(1+\frac{(\beta^{E_1}-1)(\beta^{E_2}-1)}{\beta-1}\right)} \end{bmatrix}, \\ \begin{bmatrix} 1 - \log_\beta^{\left(1+\frac{(\beta^{1-G_1}-1)(\beta^{1-G_2}-1)}{\beta-1}\right)}, \\ 1 - \log_\beta^{\left(1+\frac{(\beta^{1-H_1}-1)(\beta^{1-H_2}-1)}{\beta-1}\right)}, \\ 1 - \log_\beta^{\left(1+\frac{(\beta^{1-L_1}-1)(\beta^{1-L_2}-1)}{\beta-1}\right)} \end{bmatrix} \end{bmatrix};$$

$$a_1^\lambda = \begin{bmatrix} \begin{bmatrix} \log_\beta^{\left(1+\frac{(\beta^{C_1}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)}, \log_\beta^{\left(1+\frac{(\beta^{D_1}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)}, \\ \log_\beta^{\left(1+\frac{(\beta^{E_1}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)} \end{bmatrix}, \\ \begin{bmatrix} 1 - \log_\beta^{\left(1+\frac{(\beta^{1-G_1}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)}, \\ 1 - \log_\beta^{\left(1+\frac{(\beta^{1-H_1}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)}, \\ 1 - \log_\beta^{\left(1+\frac{(\beta^{1-L_1}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)} \end{bmatrix} \end{bmatrix};$$

$$\lambda a_1 = \begin{bmatrix} \begin{bmatrix} 1 - \log_\beta^{\left(1+\frac{(\beta^{1-C_1}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)}, \\ 1 - \log_\beta^{\left(1+\frac{(\beta^{1-D_1}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)}, \\ 1 - \log_\beta^{\left(1+\frac{(\beta^{1-E_1}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)} \end{bmatrix}, \\ \begin{bmatrix} \log_\beta^{\left(1+\frac{(\beta^{1-G_1}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)}, \\ \log_\beta^{\left(1+\frac{(\beta^{1-H_1}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)}, \\ \log_\beta^{\left(1+\frac{(\beta^{1-L_1}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)} \end{bmatrix} \end{bmatrix}$$

Definition 6. *The TIFNs are* $a = \begin{bmatrix} [C_j, D_j, E_j], \\ [G_j, H_j, L_j] \end{bmatrix}$, *score function* I *is defined as:* $I = \frac{\langle [C_j+D_j+E_j]-[G_j+H_j+L_j] \rangle}{6}$.

Definition 7. *The TIFNs are* $a = \begin{bmatrix} C_j, D_j, E_j, \\ G_j, H_j, L_j \end{bmatrix}$, *accuracy function* M *is defined as:* $M =$

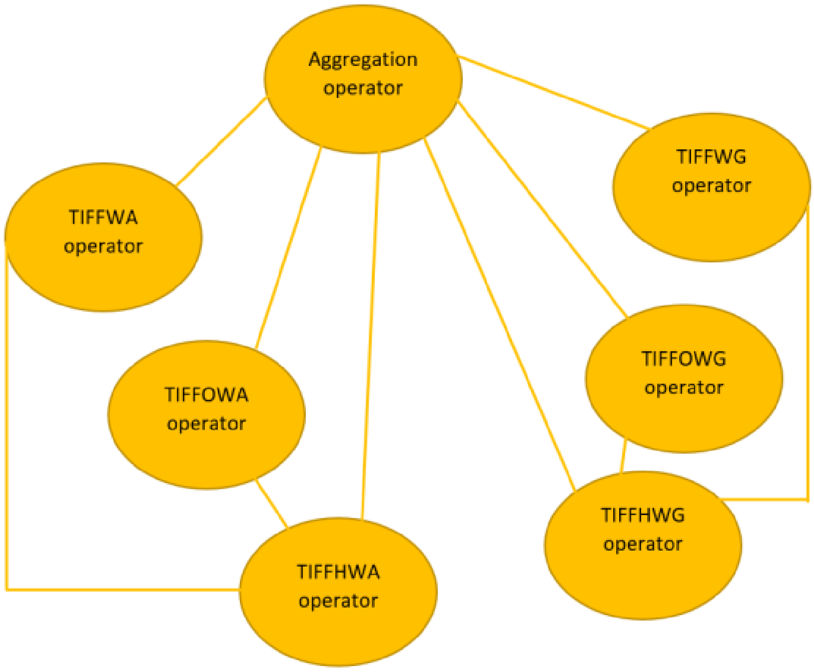

Figure 4 Aggregations operators.

$\frac{\langle [C_j+D_j+E_j]+[G_j+H_j+L_j]\rangle}{6}$.

TIFFWA, TIFFOWA, TIFFHA, TIFFWG, TIFFOWG, and TIFFHG are the aggregation operators shown in Figure 4. The six suggested aggregation operations for processing decision-making data in the form of Triangular Intuitionistic Fuzzy Numbers (TIFNs) within the Frank t-norm framework are depicted in this image. These operators were created especially to combine several criteria or professional judgments while taking into account the degrees of membership, non-membership, and hesitancy that are present in intuitionistic fuzzy settings. Figure 4 is given as below

4. TIFNs aggregation operator based on frank

This section presents the TIFFWA, TIFFOWA, TIFFWHA, TIFFWG, TIFFOWG and TIFFHG operators new methods for TIFNs with several noteworthy characteristics based on frank operators.

4.1. TIFFWA operator

Definition 8. *Let* $k_j = \left[\begin{array}{c} [C_j, D_j, E_j], \\ [G_j, H_j, L_j] \end{array} \right]$ *be the gathering of TIFNs and* $u = (u_1, u_2, ..., u_m)^T$ *is the weight vector with* $u_j \in [0,1]$ *and* $\sum_{j=1}^{m} u_j = 1$. *Then*

$$TIFFWA\,(k_1, k_2, ..., k_m) = \bigoplus_{j=1}^{m} u_j k_j$$

is said TIFFWA operator.

Theorem 1. *The collection of TIFNs are* $a_j = \begin{bmatrix} [C_j, D_j, E_j], \\ [G_j, H_j, L_j] \end{bmatrix}$ *and* $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)^T$ *is the weight vector with* $\lambda_j \in [0,1]$ *and* $\sum_{j=1}^{n} \lambda_j = 1$. *Then it is said TIFFWA operator and*

$TIFFWA(a_1, a_2, ..., a_n) =$

$$\begin{bmatrix} \begin{bmatrix} 1-\log_\beta^{\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-C_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)}, 1-\log_\beta^{\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-D_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)}, \\ 1-\log_\beta^{\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-E_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)} \end{bmatrix}, \\ \begin{bmatrix} \log_\beta^{\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-G_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)}, \\ \log_\beta^{\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-H_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)}, \\ \log_\beta^{\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-L_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)} \end{bmatrix} \end{bmatrix}.$$

Proof. Since $n = 1$,

$$a_1\lambda_1 = \begin{bmatrix} \begin{bmatrix} 1-\log_\beta^{\left(1+\frac{(\beta^{1-C_1}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)}, 1-\log_\beta^{\left(1+\frac{(\beta^{1-D_1}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)}, 1-\log_\beta^{\left(1+\frac{(\beta^{1-E_1}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)} \end{bmatrix}, \\ \begin{bmatrix} \log_\beta^{\left(1+\frac{(\beta^{1-G_1}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)}, \log_\beta^{\left(1+\frac{(\beta^{1-H_1}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)}, \log_\beta^{\left(1+\frac{(\beta^{1-L_1}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)} \end{bmatrix} \end{bmatrix};$$

$$a_2\lambda_2 = \begin{bmatrix} \begin{bmatrix} 1-\log_\beta^{\left(1+\frac{(\beta^{1-C_2}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)}, 1-\log_\beta^{\left(1+\frac{(\beta^{1-D_2}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)}, 1-\log_\beta^{\left(1+\frac{(\beta^{1-E_2}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)} \end{bmatrix}, \\ \begin{bmatrix} \log_\beta^{\left(1+\frac{(\beta^{1-G_2}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)}, \log_\beta^{\left(1+\frac{(\beta^{1-H_2}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)}, \log_\beta^{\left(1+\frac{(\beta^{1-L_2}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)} \end{bmatrix} \end{bmatrix},$$

$n = k$

$$\left[\begin{array}{c}\left[\begin{array}{c}1-\log_{\beta}\left(1+\frac{\prod_{j=1}^{k}\left(\beta^{1-C_{j}}-1\right)^{\lambda_{j}}}{(\beta-1)^{\lambda_{j}-1}}\right),1-\log_{\beta}\left(1+\frac{\prod_{j=1}^{k}\left(\beta^{1-D_{j}}-1\right)^{\lambda_{j}}}{(\beta-1)^{\lambda_{j}-1}}\right),\\ 1-\log_{\beta}\left(1+\frac{\prod_{j=1}^{k}\left(\beta^{1-E_{j}}-1\right)^{\lambda_{j}}}{(\beta-1)^{\lambda_{j}-1}}\right)\end{array}\right],\\ \left[\begin{array}{c}\log_{\beta}\left(1+\frac{\prod_{j=1}^{k}\left(\beta^{1-G_{j}}-1\right)^{\lambda_{j}}}{(\beta-1)^{\lambda_{j}-1}}\right),\log_{\beta}\left(1+\frac{\prod_{j=1}^{k}\left(\beta^{1-H_{j}}-1\right)^{\lambda_{j}}}{(\beta-1)^{\lambda_{j}-1}}\right),\\ \log_{\beta}\left(1+\frac{\prod_{j=1}^{k}\left(\beta^{1-L_{j}}-1\right)^{\lambda_{j}}}{(\beta-1)^{\lambda_{j}-1}}\right)\end{array}\right]\end{array}\right]$$

$n=k+1$

$$\left[\begin{array}{c}\left[\begin{array}{c}1-\log_{\beta}\left(1+\frac{\prod_{j=1}^{k+1}\left(\beta^{1-C_{j}}-1\right)^{\lambda_{j}}}{(\beta-1)^{\lambda_{j}-1}}\right),1-\log_{\beta}\left(1+\frac{\prod_{j=1}^{k+1}\left(\beta^{1-D_{j}}-1\right)^{\lambda_{j}}}{(\beta-1)^{\lambda_{j}-1}}\right),\\ 1-\log_{\beta}\left(1+\frac{\prod_{j=1}^{k+1}\left(\beta^{1-E_{j}}-1\right)^{\lambda_{j}}}{(\beta-1)^{\lambda_{j}-1}}\right)\end{array}\right],\\ \left[\begin{array}{c}\log_{\beta}\left(1+\frac{\prod_{j=1}^{k+1}\left(\beta^{1-G_{j}}-1\right)^{\lambda_{j}}}{(\beta-1)^{\lambda_{j}-1}}\right),\log_{\beta}\left(1+\frac{\prod_{j=1}^{k+1}\left(\beta^{1-H_{j}}-1\right)^{\lambda_{j}}}{(\beta-1)^{\lambda_{j}-1}}\right),\\ \log_{\beta}\left(1+\frac{\prod_{j=1}^{k+1}\left(\beta^{1-L_{j}}-1\right)^{\lambda_{j}}}{(\beta-1)^{\lambda_{j}-1}}\right)\end{array}\right]\end{array}\right]$$

Theorem 2. ***(Idempotency):*** *If* $\widetilde{ZV}=\left[\begin{array}{c}[C_{j},D_{j},E_{j}],\\ [G_{j},H_{j},L_{j}]\end{array}\right]$ *for all* $N=1,2,3,...,m,$ *then*

$$TIFFWA(\widetilde{ZV},\widetilde{ZV},\widetilde{ZV},...,\widetilde{ZV})=\widetilde{ZV}.$$

Proof. *Since* $\widetilde{ZV} = \begin{bmatrix} [C_j, D_j, E_j], \\ [G_j, H_j, L_j] \end{bmatrix}$

$$\left[\begin{array}{c} \left[1-\log_\beta^{\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-C_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)}, 1-\log_\beta^{\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-D_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)}, 1-\log_\beta^{\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-E_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)} \right], \\ \left[\log_\beta^{\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-G_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)}, \log_\beta^{\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-H_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)}, \log_\beta^{\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-L_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)} \right] \end{array}\right]$$

$$= \left[\begin{array}{c} \left[1-\log_\beta^{\left(1+\frac{(\beta^{1-C_j}-1)^{\sum_{j=1}^{n}\lambda_j}}{(\beta-1)^{\sum_{j=1}^{n}\lambda_j-1}}\right)}, 1-\log_\beta^{\left(1+\frac{(\beta^{1-D_j}-1)^{\sum_{j=1}^{n}\lambda_j}}{(\beta-1)^{\sum_{j=1}^{n}\lambda_j-1}}\right)}, 1-\log_\beta^{\left(1+\frac{(\beta^{1-E_j}-1)^{\sum_{j=1}^{n}\lambda_j}}{(\beta-1)^{\sum_{j=1}^{n}\lambda_j-1}}\right)} \right], \\ \left[\log_\beta^{\left(1+\frac{(\beta^{1-G_j}-1)^{\sum_{j=1}^{n}\lambda_j}}{(\beta-1)^{\sum_{j=1}^{n}\lambda_j-1}}\right)}, \log_\beta^{\left(1+\frac{(\beta^{1-H_j}-1)^{\sum_{j=1}^{n}\lambda_j}}{(\beta-1)^{\sum_{j=1}^{n}\lambda_j-1}}\right)}, \log_\beta^{\left(1+\frac{(\beta^{1-L_j}-1)^{\sum_{j=1}^{n}\lambda_j}}{(\beta-1)^{\sum_{j=1}^{n}\lambda_j-1}}\right)} \right] \end{array}\right]$$

$$= \left[\begin{array}{c} \left[1-\log_\beta^{\left(1+\frac{(\beta^{1-C_j}-1)^{1}}{(\beta-1)^{1-1}}\right)}, 1-\log_\beta^{\left(1+\frac{(\beta^{1-D_j}-1)^{1}}{(\beta-1)^{1-1}}\right)}, 1-\log_\beta^{\left(1+\frac{(\beta^{1-E_j}-1)^{1}}{(\beta-1)^{1-1}}\right)} \right], \\ \left[\log_\beta^{\left(1+\frac{(\beta^{1-G_j}-1)^{1}}{(\beta-1)^{1-1}}\right)}, \log_\beta^{\left(1+\frac{(\beta^{1-H_j}-1)^{1}}{(\beta-1)^{1-1}}\right)}, \log_\beta^{\left(1+\frac{(\beta^{1-L_j}-1)^{1}}{(\beta-1)^{1-1}}\right)} \right] \end{array}\right]$$

$$= \left[\begin{array}{c} \left[1-\log_\beta^{\left(1+(\beta^{1-C_j}-1)^{1}\right)}, 1-\log_\beta^{\left(1+(\beta^{1-D_j}-1)^{1}\right)}, 1-\log_\beta^{\left(1+(\beta^{1-E_j}-1)^{1}\right)} \right], \\ \left[\log_\beta^{\left(1+(\beta^{1-G_j}-1)^{1}\right)}, \log_\beta^{\left(1+(\beta^{1-H_j}-1)^{1}\right)}, \log_\beta^{\left(1+(\beta^{1-L_j}-1)^{1}\right)} \right] \end{array}\right]$$

$$\widetilde{ZV} = \begin{bmatrix} [C_j, D_j, E_j], \\ [G_j, H_j, L_j] \end{bmatrix}$$

Theorem 3. ***(Boundedness):*** *If* $Y = \begin{bmatrix} [C_j, D_j, E_j], \\ [G_j, H_j, L_j] \end{bmatrix}$ *be a collection of TIFSs. If* $Y^- = min(Y_1, Y_2, ..., Y_m), Y^+ = \max(Y_1, Y_2, ..., Y_m)$, *then*

$$Y^- \leq TIFFWA(Y_1, Y_2, ..., Y_m) \leq Y^+.$$

Proof. $Y^- = \min(Y_1, Y_2, ..., Y_m) = \begin{bmatrix} [C_j, D_j, E_j], \\ [G_j, H_j, L_j] \end{bmatrix}$ *and*

$Y^+ = \max(Y_1, Y_2, ..., Y_m) = \begin{bmatrix} [C_j, D_j, E_j], \\ [G_j, H_j, L_j] \end{bmatrix}$

Since $min_j(C_j) \leq max_j(C_j); min_j(D_j) \leq max_j(D_j); min_j(E_j) \leq max_j(E_j)$ *and* $min_j(G_j) \leq max_j(G_j); min_j(H_j) \leq max_j(H_j); min_j(L_j) \leq max_j(L_j)$

Which implies that

$$1 - \log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-C_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right) \geq 1 - \log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-(\min C_j)}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right);$$

$$1 - \log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-D_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right) \geq 1 - \log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-(\min D_j)}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right);$$

$$1 - \log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-E_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right) \geq 1 - \log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-(\min E_j)}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right) \text{ and}$$

$$\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-G_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right) \geq \log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-(\max G_j)}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right);$$

$$\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-H_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right) \geq \log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-(\max H_j)}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right);$$

$$\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-L_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right) \geq \log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-(\max L_j)}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)$$

$TIFFWA(Y_1, Y_2, ..., Y_m) = \begin{bmatrix} [C_j, D_j, E_j], \\ [G_j, H_j, L_j] \end{bmatrix}$

$S(Y) = \frac{[[C_j+D_j+E_j]-[G_j+H_j+L_j]]}{6} \leq \frac{[[\max_j C_j+\max_j D_j+\max_j E_j]-[\min_j G_j+\min_j H_j+\min_j L_j]]}{6} = S(Y^-)$

$S(Y) \leq S(Y^-)$ *and*

$S(Y) = \frac{[[C_j+D_j+E_j]-[G_j+H_j+L_j]]}{6} \geq \frac{[[\max_j C_j+\max_j D_j+\max_j E_j]-[\min_j G_j+\min_j H_j+\min_j L_j]]}{6} = S(Y^+)$

$S(Y) \geq S(Y^+)$

which implies that

$TIFFWA(Y_1, Y_2, ..., Y_m) = Y^-$ *and*

$TIFFWA(Y_1, Y_2, ..., Y_m) = Y^+$

4.2. TIFFOWA operator

Definition 9. *Let* $k_j = \begin{bmatrix} [C_j, D_j, E_j], \\ [G_j, H_j, L_j] \end{bmatrix}$ *be the gathering of TIFNs and* $u = (u_1, u_2, ..., u_m)^T$ *is the weight vector with* $u_j \in [0,1]$ *and* $\sum_{j=1}^{m} u_j = 1$*. Then*

$$TIFFOWA\,(k_1, k_2, ..., k_m) = \bigoplus_{j=1}^{m} u_j k_j$$

is said TIFFOWA operator.

Theorem 4. *The collection of TIFNs are* $a_j = \begin{bmatrix} [C_j, D_j, E_j], \\ [G_j, H_j, L_j] \end{bmatrix}$ *and* $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)^T$ *is the weight vector with* $\lambda_j \in [0,1]$ *and* $\sum_{j=1}^{n} \lambda_j = 1$*. Then it is said TIFFOWA operator and* $TIFFOWA(a_1, a_2, ..., a_n) =$

$$\left[\begin{array}{c}\left[\begin{array}{c}1-\log_{\beta}\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-C_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right),\\ 1-\log_{\beta}\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-D_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right),\\ 1-\log_{\beta}\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-E_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)\end{array}\right],\\ \left[\begin{array}{c}\log_{\beta}\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-G_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right),\\ \log_{\beta}\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-H_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right),\\ \log_{\beta}\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-L_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)\end{array}\right]\end{array}\right].$$

Proof:This proof is the same as in Theorem 1

Theorem 5. ***(Idempotency):****If* $\widetilde{ZV} = \left[\begin{array}{c}[C_j, D_j, E_j],\\ [G_j, H_j, L_j]\end{array}\right]$ *for all* $N = 1, 2, 3, ..., m,$ *then* *TIFFOWA*$(\widetilde{ZV}, \widetilde{ZV}, \widetilde{ZV}, ..., \widetilde{ZV}) = \widetilde{ZV}$.

Proof:This proof is the same as in Theorem 2

Theorem 6. ***(Boundedness):****If* $Y^- = min(\widetilde{ZV}_1, \widetilde{ZV}_2, ..., \widetilde{ZV}_m), Y^+ = \max(\widetilde{ZV}_1, \widetilde{ZV}_2, ..., \widetilde{ZV}_m),$ *then*

$$Y^- \leq TIFFOWA(\widetilde{ZV}_1, \widetilde{ZV}_2, ..., \widetilde{ZV}_m) \leq Y^+.$$

Proof:This proof is the same as in Theorem 3.

4.3. TIFFHWA operator

Definition 10. *Let* $k_j = \begin{bmatrix} [C_j, D_j, E_j], \\ [G_j, H_j, L_j] \end{bmatrix}$ *be the gathering of TIFNs and* $u = (u_1, u_2, ..., u_m)^T$ *is the weight vector with* $u_j \in [0,1]$ *and* $\sum_{j=1}^{m} u_j = 1$*. Then*

$$TIFFHWA\,(k_1, k_2, ..., k_m) = \bigoplus_{j=1}^{m} u_j k_j$$

is said TIFFHWA operator.

Theorem 7. *The collection of TIFNs are* $a_j = \begin{bmatrix} [C_j, D_j, E_j], \\ [G_j, H_j, L_j] \end{bmatrix}$ *and* $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)^T$ *is the weight vector with* $\lambda_j \in [0,1]$ *and* $\sum_{j=1}^{n} \lambda_j = 1$*. Then it is said TIFFHWA operator and* $TIFFHWA(a_1, a_2, ..., a_n) =$

$$\left[\begin{array}{c} \left[\begin{array}{c} 1 - \log_\beta\left(1 + \frac{\prod_{j=1}^{n}\left(\beta^{1-C_j} - 1\right)^{\lambda_j}}{(\beta-1)^{\lambda_j - 1}}\right), \\ 1 - \log_\beta\left(1 + \frac{\prod_{j=1}^{n}\left(\beta^{1-D_j} - 1\right)^{\lambda_j}}{(\beta-1)^{\lambda_j - 1}}\right), \\ 1 - \log_\beta\left(1 + \frac{\prod_{j=1}^{n}\left(\beta^{1-E_j} - 1\right)^{\lambda_j}}{(\beta-1)^{\lambda_j - 1}}\right) \end{array}\right], \\ \left[\begin{array}{c} \log_\beta\left(1 + \frac{\prod_{j=1}^{n}\left(\beta^{1-G_j} - 1\right)^{\lambda_j}}{(\beta-1)^{\lambda_j - 1}}\right), \\ \log_\beta\left(1 + \frac{\prod_{j=1}^{n}\left(\beta^{1-H_j} - 1\right)^{\lambda_j}}{(\beta-1)^{\lambda_j - 1}}\right), \\ \log_\beta\left(1 + \frac{\prod_{j=1}^{n}\left(\beta^{1-L_j} - 1\right)^{\lambda_j}}{(\beta-1)^{\lambda_j - 1}}\right) \end{array}\right] \end{array}\right].$$

Proof:This proof is the same as in Theorem 1

Theorem 8. ***(Idempotency):****If* $\widetilde{ZV} = \begin{bmatrix} [C_j, D_j, E_j], \\ [G_j, H_j, L_j] \end{bmatrix}$ *for all* $R = 1, 2, 3, ..., m$, *then* $TIFFHWA(\widetilde{ZV}, \widetilde{ZV}, \widetilde{ZV}, ..., \widetilde{ZV}) = \widetilde{ZV}$.

Proof:This proof is the same as in Theorem 2.

Theorem 9. ***(Boundedness):****If* $Y^{-} = min(\widetilde{ZV}_1, \widetilde{ZV}_2, ..., \widetilde{ZV}_m), Y^{+} = \max(\widetilde{ZV}_1, \widetilde{ZV}_2, ..., \widetilde{ZV}_m)$, *then*

$$Y^{-} \leq TIFFHWA(\widetilde{ZV}_1, \widetilde{ZV}_2, ..., \widetilde{ZV}_m) \leq Y^{+}.$$

Proof:This proof is the same as in Theorem 3.

4.4. TIFFWG operator

Definition 11. *Let* $k_j = \begin{bmatrix} [C_j, D_j, E_j], \\ [G_j, H_j, L_j] \end{bmatrix}$ *be the gathering of TIFNs and* $u = (u_1, u_2, ..., u_m)^T$ *is the weight vector with* $u_j \in [0, 1]$ *and* $\sum_{j=1}^{m} u_j = 1$. *Then*

$$TIFFWG\,(k_1, k_2, ..., k_m) = \bigotimes_{j=1}^{m} k_j^{u_j}$$

is said TIFFWG operator.

Theorem 10. *The collection of TIFNs are* $a_j = \begin{bmatrix} [C_j, D_j, E_j], \\ [G_j, H_j, L_j] \end{bmatrix}$ *and* $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)^T$ *is the weight vector with* $\lambda_j \in [0, 1]$ *and* $\sum_{j=1}^{n} \lambda_j = 1$. *Then it is said TIFFWG operator and* $TIFFWG(a_1, a_2, ..., a_n) =$

$$\left[\begin{array}{c}\left[\begin{array}{c}\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{C_j}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right),\\ \log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{D_j}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right),\\ \log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{E_j}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)\end{array}\right],\\ \left[\begin{array}{c}1-\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-G_j}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right),\\ 1-\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-H_j}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right),\\ 1-\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-L_j}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)\end{array}\right]\end{array}\right].$$

Proof. *Since* $n=1$,

$$a_1\lambda_1=\left[\begin{array}{c}\left[\log_\beta\left(1+\frac{(\beta^{C_1}-1)^{\lambda_1}}{(\beta-1)^{\lambda_1-1}}\right),\log_\beta\left(1+\frac{(\beta^{D_1}-1)^{\lambda_1}}{(\beta-1)^{\lambda_1-1}}\right),\log_\beta\left(1+\frac{(\beta^{E_1}-1)^{\lambda_1}}{(\beta-1)^{\lambda_1-1}}\right)\right],\\ \left[\begin{array}{c}1-\log_\beta\left(1+\frac{(\beta^{1-G_1}-1)^{\lambda}}{(\beta-1)^{\lambda-1}}\right),1-\log_\beta\left(1+\frac{(\beta^{1-H_1}-1)^{\lambda}}{(\beta-1)^{\lambda-1}}\right),\\ 1-\log_\beta\left(1+\frac{(\beta^{1-L_1}-1)^{\lambda}}{(\beta-1)^{\lambda-1}}\right)\end{array}\right]\end{array}\right];$$

$$a_2\lambda_2=\left[\begin{array}{c}\left[\log_\beta\left(1+\frac{(\beta^{C_2}-1)^{\lambda_2}}{(\beta-1)^{\lambda_2-1}}\right),\log_\beta\left(1+\frac{(\beta^{D_2}-1)^{\lambda_2}}{(\beta-1)^{\lambda_2-1}}\right),\log_\beta\left(1+\frac{(\beta^{E_2}-1)^{\lambda_2}}{(\beta-1)^{\lambda_2-1}}\right)\right],\\ \left[\begin{array}{c}1-\log_\beta\left(1+\frac{(\beta^{1-G_2}-1)^{\lambda_2}}{(\beta-1)^{\lambda_2-1}}\right),1-\log_\beta\left(1+\frac{(\beta^{1-H_2}-1)^{\lambda_2}}{(\beta-1)^{\lambda_2-1}}\right),\\ 1-\log_\beta\left(1+\frac{(\beta^{1-L_2}-1)^{\lambda_2}}{(\beta-1)^{\lambda_2-1}}\right)\end{array}\right]\end{array}\right]$$

$n=k$

$$\left[\begin{array}{c} \left[\begin{array}{c} \log_\beta\left(1+\frac{\prod_{j=1}^{k}\left(\beta^{C_j}-1\right)^\lambda}{(\beta-1)^{\lambda-1}}\right), \\ \log_\beta\left(1+\frac{\prod_{j=1}^{k}\left(\beta^{D_j}-1\right)^\lambda}{(\beta-1)^{\lambda-1}}\right), \\ \log_\beta\left(1+\frac{\prod_{j=1}^{k}\left(\beta^{E_j}-1\right)^\lambda}{(\beta-1)^{\lambda-1}}\right) \end{array}\right], \\ \left[\begin{array}{c} 1-\log_\beta\left(1+\frac{\prod_{j=1}^{k}\left(\beta^{1-G_j}-1\right)^\lambda}{(\beta-1)^{\lambda-1}}\right), \\ 1-\log_\beta\left(1+\frac{\prod_{j=1}^{k}\left(\beta^{1-H_j}-1\right)^\lambda}{(\beta-1)^{\lambda-1}}\right), \\ 1-\log_\beta\left(1+\frac{\prod_{j=1}^{k}\left(\beta^{1-L_j}-1\right)^\lambda}{(\beta-1)^{\lambda-1}}\right) \end{array}\right] \end{array}\right]$$

$n = k + 1$

$$\left[\begin{array}{l}\left[\begin{array}{l}\log_\beta\left(1+\frac{\prod_{j=1}^{k+1}(\beta^{C_j}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right),\\ \log_\beta\left(1+\frac{\prod_{j=1}^{k+1}(\beta^{D_j}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right),\\ \log_\beta\left(1+\frac{\prod_{j=1}^{k+1}(\beta^{E_j}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)\end{array}\right],\\ \left[\begin{array}{l}1-\log_\beta\left(1+\frac{\prod_{j=1}^{k+1}(\beta^{1-G_j}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right),\\ 1-\log_\beta\left(1+\frac{\prod_{j=1}^{k+1}(\beta^{1-H_j}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right),\\ 1-\log_\beta\left(1+\frac{\prod_{j=1}^{k+1}(\beta^{1-L_j}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)\end{array}\right]\end{array}\right]$$

Theorem 11. ***(Idempotency):*** *If* $\widetilde{ZV} = \left[\begin{array}{c}[C_j, D_j, E_j],\\ [G_j, H_j, L_j]\end{array}\right]$ *for all* $L = 1, 2, 3, ..., m$, *then* $TIFFWG(\widetilde{ZV}, \widetilde{ZV}, \widetilde{ZV}, ..., \widetilde{ZV}) = \widetilde{ZV}$.

Proof. Since $\widetilde{ZV} = [[C_j, D_j, E_j], [G_j, H_j, L_j]]$

$$\left[\begin{array}{c}\left[\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-C_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right),\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-D_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right),\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-E_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)\right],\\ \left[1-\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-G_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right),1-\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-H_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right),1-\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-L_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)\right]\end{array}\right]$$

$$=\left[\begin{array}{c}\left[\log_\beta\left(1+\frac{(\beta^{1-C_j}-1)^{\sum_{j=1}^{n}\lambda_j}}{(\beta-1)^{\sum_{j=1}^{n}\lambda_j-1}}\right),\log_\beta\left(1+\frac{(\beta^{1-D_j}-1)^{\sum_{j=1}^{n}\lambda_j}}{(\beta-1)^{\sum_{j=1}^{n}\lambda_j-1}}\right),\log_\beta\left(1+\frac{(\beta^{1-E_j}-1)^{\sum_{j=1}^{n}\lambda_j}}{(\beta-1)^{\sum_{j=1}^{n}\lambda_j-1}}\right)\right],\\ \left[1-\log_\beta\left(1+\frac{(\beta^{1-G_j}-1)^{\sum_{j=1}^{n}\lambda_j}}{(\beta-1)^{\sum_{j=1}^{n}\lambda_j-1}}\right),1-\log_\beta\left(1+\frac{(\beta^{1-H_j}-1)^{\sum_{j=1}^{n}\lambda_j}}{(\beta-1)^{\sum_{j=1}^{n}\lambda_j-1}}\right),1-\log_\beta\left(1+\frac{(\beta^{1-L_j}-1)^{\sum_{j=1}^{n}\lambda_j}}{(\beta-1)^{\sum_{j=1}^{n}\lambda_j-1}}\right)\right]\end{array}\right]$$

$$=\left[\begin{array}{c}\left[\log_\beta\left(1+\frac{(\beta^{1-C_j}-1)^1}{(\beta-1)^{1-1}}\right),\log_\beta\left(1+\frac{(\beta^{1-D_j}-1)^1}{(\beta-1)^{1-1}}\right),\log_\beta\left(1+\frac{(\beta^{1-E_j}-1)^1}{(\beta-1)^{1-1}}\right)\right],\\ \left[1-\log_\beta\left(1+\frac{(\beta^{1-G_j}-1)^1}{(\beta-1)^{1-1}}\right),1-\log_\beta\left(1+\frac{(\beta^{1-H_j}-1)^1}{(\beta-1)^{1-1}}\right),1-\log_\beta\left(1+\frac{(\beta^{1-L_j}-1)^1}{(\beta-1)^{1-1}}\right)\right]\end{array}\right]$$

$$=\left[\begin{array}{c}\left[\log_\beta^{\left(1+(\beta^{1-C_j}-1)^1\right)},\log_\beta^{\left(1+(\beta^{1-D_j}-1)^1\right)},\log_\beta^{\left(1+(\beta^{1-E_j}-1)^1\right)}\right],\\ \left[1-\log_\beta^{\left(1+(\beta^{1-G_j}-1)^1\right)},1-\log_\beta^{\left(1+(\beta^{1-H_j}-1)^1\right)},1-\log_\beta^{\left(1+(\beta^{1-L_j}-1)^1\right)}\right]\end{array}\right]$$

$$\widetilde{ZV}=\left[\begin{array}{c}[C_j,D_j,E_j],\\ [G_j,H_j,L_j]\end{array}\right]$$

Theorem 12. ***(Boundedness):*** *If* $Y = \begin{bmatrix} [C_j, D_j, E_j], \\ [G_j, H_j, L_j] \end{bmatrix}$ *be a collection of TIFSs. If* $Y^- = min(Y_1, Y_2, ..., Y_m), Y^+ = \max(Y_1, Y_2, ..., Y_m)$, *then*

$$Y^- \leq TIFFWG(Y_1, Y_2, ..., Y_m) \leq Y^+.$$

Proof. $Y^- = \min(Y_1, Y_2, ..., Y_m) = \begin{bmatrix} [C_j, D_j, E_j], \\ [G_j, H_j, L_j] \end{bmatrix}$ *and*

$Y^+ = \max(Y_1, Y_2, ..., Y_m) = \begin{bmatrix} [C_j, D_j, E_j], \\ [G_j, H_j, L_j] \end{bmatrix}$

Since $min_j(C_j) \leq max_j(C_j); min_j(D_j) \leq max_j(D_j); min_j(E_j) \leq max_j(E_j)$ *and* $min_j(G_j) \leq max_j(G_j); min_j(H_j) \leq max_j(H_j); min_j(L_j) \leq max_j(L_j)$

Which implies that

$$\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-C_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right) \geq \log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-(\min C_j)}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right);$$

$$\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-D_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right) \geq \log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-(\min D_j)}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right);$$

$$\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-E_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right) \geq \log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-(\min E_j)}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right) \text{ and}$$

$$1\text{-}\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-G_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right) \geq 1-\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-(\max G_j)}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right);$$

$$1\text{-}\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-H_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right) \geq 1-\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-(\max H_j)}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right);$$

$$1\text{-}\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-L_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right) \geq 1-\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-(\max L_j)}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)$$

$TIFFWG(Y_1, Y_2, ..., Y_m) = \begin{bmatrix} [C_j, D_j, E_j], \\ [G_j, H_j, L_j] \end{bmatrix}$

$S(Y) = \frac{[[C_j+D_j+E_j]-[G_j+H_j+L_j]]}{6} \leq \frac{[[\max_j C_j+\max_j D_j+\max_j E_j]-[\min_j G_j+\min_j H_j+\min_j L_j]]}{6} = S(Y^-)$

$S(Y) \leq S(Y^-)$ *and*

$S(Y) = \frac{[[C_j+D_j+E_j]-[G_j+H_j+L_j]]}{6} \geq \frac{[[\max_j C_j+\max_j D_j+\max_j E_j]-[\min_j G_j+\min_j H_j+\min_j L_j]]}{6} = S(Y^+)$

$S(Y) \geq S(Y^+)$

which implies that

$TIFFWG(Y_1, Y_2, ..., Y_m) = Y^-$ *and*

$TIFFWG(Y_1, Y_2, ..., Y_m) = Y^+$

4.5. TIFFOWG operator

Definition 12. *Let* $k_j = \begin{bmatrix} [C_j, D_j, E_j], \\ [G_j, H_j, L_j] \end{bmatrix}$ *be the gathering of TIFNs and the weight vector is* $g = (g_1, g_2, ..., g_m)^T$ *with* $g_j \in [0,1]$ *and* $\sum_{j=1}^{m} g_j = 1$. *Then*

$$TIFFOWG\,(k_1, k_2, ..., k_m) = \bigotimes_{j=1}^{m} k_j^{g_j}$$

is said TIFFOWG operator.

Theorem 13. *Let* $a_j = \begin{bmatrix} [C_j, D_j, E_j], \\ [G_j, H_j, L_j] \end{bmatrix}$ *be the collection of TIFNs and the weight vector is* $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)^T$ *with* $\lambda_j \in [0,1]$ *and* $\sum_{j=1}^{n} \lambda_j = 1$. *Then it is said TIFFOWG operator and* $TIFFOWG(a_1, a_2, ..., a_n) =$

$$\left[\begin{array}{c} \left[\begin{array}{c} \log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{C_j}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right), \\ \log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{D_j}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right), \\ \log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{E_j}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right) \end{array}\right], \\ \left[\begin{array}{c} 1-\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-G_j}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right), \\ 1-\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-H_j}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right), \\ 1-\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-L_j}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right) \end{array}\right] \end{array}\right].$$

Proof:This proof is the same as in Theorem 10.

Theorem 14. ***(Idempotency):****If* $\widetilde{BU} = \left[\begin{array}{c}[C_j, D_j, E_j], \\ [G_j, H_j, L_j]\end{array}\right]$ *for all* $r = 1, 2, 3, ..., m,$ *then* $TIFFOWG(BU, BU, BU, ..., BU) = BU.$

This proof is the same as in Theorem 11.

Theorem 15. ***(Boundedness):****If* $Y^- = min(f_1, f_2, ..., f_m)$, $Y^+ = \max(f_1, f_2, ..., f_m)$, *then*

$$Y^- \leq TIFFOWG(f_1, f_2, ..., f_m) \leq Y^+.$$

This proof is the same as in Theorem 12.

4.6. TIFFHWG operator

Definition 13. *The gathering of TIFNs are* $f_j = \left[\begin{array}{c}[C_j, D_j, E_j], \\ [G_j, H_j, L_j]\end{array}\right]$ *and the weight vector is* $u = (u_1, u_2, ..., u_n)^T$ *with* $u_j \in [0, 1]$ *and* $\sum_{j=1}^{n} u_j = 1$, *the associated vector is* $u = (u_1, u_2, ..., u_n)^T$

with $u_j \in [0,1]$ *and* $\sum_{j=1}^{n} u_j = 1$. *Then*

$$TIFFHWG\left(f_1, f_2, ..., f_n\right) = \bigotimes_{j=1}^{m} f_j^{u_j}$$

is said TIFFHWG operator.

Theorem 16. *Let* $a_j = \left[\begin{array}{c}[C_j, D_j, E_j],\\ [G_j, H_j, L_j]\end{array}\right]$ *be the collection of TIFNs and the weight vector is* $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)^T$ *with* $\lambda_j \in [0,1]$ *and* $\sum_{j=1}^{n} \lambda_j = 1$. *Then it is said TIFFHWG operator and* $TIFFHWG(a_1, a_2, ..., a_n) =$

$$\left[\begin{array}{c}\left[\begin{array}{c}\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{C_j}-1)^{\lambda}}{(\beta-1)^{\lambda-1}}\right),\\ \log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{D_j}-1)^{\lambda}}{(\beta-1)^{\lambda-1}}\right),\\ \log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{E_j}-1)^{\lambda}}{(\beta-1)^{\lambda-1}}\right)\end{array}\right],\\ \left[\begin{array}{c}1-\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-G_j}-1)^{\lambda}}{(\beta-1)^{\lambda-1}}\right),\\ 1-\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-H_j}-1)^{\lambda}}{(\beta-1)^{\lambda-1}}\right),\\ 1-\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-L_j}-1)^{\lambda}}{(\beta-1)^{\lambda-1}}\right)\end{array}\right]\end{array}\right].$$

Proof: This proof is the same as in Theorem 10.

Theorem 17. ***(Idempotency):****If* $\widetilde{A} = \left[\begin{array}{c}[C_j, D_j, E_j],\\ [G_j, H_j, L_j]\end{array}\right]$ *for all* $R = 1, 2, 3, ..., m$, *then* $TIFFHWG(\widetilde{A}, \widetilde{A}, \widetilde{A}, ..., \widetilde{A}) = \widetilde{A}$.

Proof:This proof is the same as in Theorem 11.

Theorem 18. ***(Boundedness):****If* $Y^{-} = min(f_1, f_2, ..., f_n)$, $Y^{+} = \max(f_1, f_2, ..., f_n)$, *then*

$$Y^{-} \leq TIFFHWG(f_1, f_2, ..., f_n) \leq Y^{+}.$$

Proof:This proof is the same as in Theorem 12.

5. Proposed technique based on TIFAA operator triangular fuzzy C-mean clustering algorithm

Triangular Fuzzy clustering algorithms are used to group people according to shared characteristics or habits, a process known as user profiling. Users can have their membership degrees a measure of how much they belong to each cluster assigned to them thanks to triangular fuzzy clustering. This strategy is especially helpful in situations where users may simultaneously display traits from several categories. This is a simple overview of how triangular fuzzy clustering for user profiling could be used. Compile pertinent user information that can be utilized for profiling. The demographic data, browsing history, purchasing patterns, and interactions with a website or application are some examples of this data. Choose the characteristics or features that will be utilized to the clustering process. These attributes ought to accurately reflect the traits of the users and have a bearing on the process of profiling. For the given problem, choose a triangular fuzzy clustering algorithm that is suitable triangular Fuzzy C-Means.

Step 1:Describe the TIF decision matrix

Step 2:Describe the TIFFWA operator and $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)$.

TIFFWA$(A_1, A_2, ..., A_n) \ =$

$$\left[\begin{array}{c}\left[\begin{array}{c}1-\log_{\beta}\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-C_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right),\\ 1-\log_{\beta}\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-D_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right),\\ 1-\log_{\beta}\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-E_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)\end{array}\right],\\ \left[\begin{array}{c}\log_{\beta}\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-G_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right),\\ \log_{\beta}\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-H_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right),\\ \log_{\beta}\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-L_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right)\end{array}\right]\end{array}\right]$$

Step 3:The TIFCM algorithms work on TIFFWA operator $U=[U_{ic}]_{i=1\ldots n}^{c=1\ldots c}$

Step 4:Using k-means clusting value is defined as cluster's parameters.

Step 5:Calculate the cluster center to find the centroid

$$M_j=\left[\frac{\sum_{j=1}^{n}(C_j)^{\alpha}\sum_{j=1}^{n}(D_j)^{\alpha}\sum_{j=1}^{n}(E_j)^{\alpha}\xi_j}{\sum_{j=1}^{n}(C_j)^{\alpha}\sum_{j=1}^{n}(D_j)^{\alpha}\sum_{j=1}^{n}(E_j)^{\alpha}},\frac{\sum_{j=1}^{n}(G_j)^{\alpha}\sum_{j=1}^{n}(H_j)^{\alpha}\sum_{j=1}^{n}(L_j)^{\alpha}\upsilon_j}{\sum_{j=1}^{n}(G_j)^{\alpha}\sum_{j=1}^{n}(H_j)^{\alpha}\sum_{j=1}^{n}(L_j)^{\alpha}}\right]$$

n =*Number of alternatives or decision makers;*

α =*Weighting exponent to adjust sensitivity of the aggregation;*

C_j, D_j, E_j ,*Lower, middle, and upper bounds of the membership degree for alternative*

G_j, H_j, L_j ,*Lower, middle, and upper bounds of the non-membership degree for alternative*

ξ_j =*Weight or importance level assigned* $j-th$ *membership value*

υ_j =*Weight or importance level assigned to the* j*-th non-membership value*

Step 6:Find out the distance of each point from the centroid

$$f_j=\langle\|C^{-}-r_j\|+\|D^{-}-r_j\|+\|E^{-}-r_j\|,\|G-r_j\|+\|H-r_j\|+\|L-r_j\|\rangle$$

Step 7:Calculate the score function $\frac{\langle[C_j+D_j+E_j]-[G_j+H_j+L_j]\rangle}{6}$

Step 8:Find the ranking.

6. Case history

Recognize the goal of selecting the most efficient renewable energy projects. Effective project selection requires a well-structured review process, based on data-driven insights, focusing on energy generation potential, sustainability, and cost-effectiveness (not just assumptions or guesswork). This requires proper planning and the implementation of strategies to evaluate and choose optimal projects. Instead of focusing solely on project failures, engage in a constructive conversation about how to improve the selection process. Prioritize development, sustainability, and cost-effectiveness over simply criticizing the system. Share findings and recommendations for enhancing the selection process and suggest next steps for more efficient renewable energy projects. If any of these issues seem familiar, it's because these challenges often arise in renewable energy project selection, leading to higher costs, wasted resources, or missed energy goals. While technology in renewable energy can help optimize energy production, many projects struggle to identify and address inefficiencies in energy generation. These small inefficiencies, if ignored, often escalate into larger, harder-to-manage issues. For example, if energy generation or cost projections deviate unexpectedly, after conducting a thorough analysis, it may be due to inefficient system design, outdated technologies, or inaccurate forecasting models. In your role as a project manager or energy consultant, you might suggest upgrading outdated systems or adjusting project parameters to improve performance. However, do you act on these issues early, or, like many, do you allow minor inefficiencies to escalate into bigger problems that could be harder to address later? Just like medical symptoms serve as early warnings, inefficiencies in renewable energy projects act as warning signs that need immediate attention. If not treated early, these inefficiencies can lead to higher costs, system overload, or even project failure. Addressing issues early enables quick fixes and prevents significant consequences.

PID_1*: Renewable Energy Systems use advanced technologies like AI and machine learning to optimize project performance by analyzing real-time data and predicting future energy production. These systems improve project efficiency by monitoring resource usage, optimizing energy output, and adjusting project settings based on environmental patterns.*

PID_2*: Integration of solar, wind, and energy storage solutions within renewable energy projects introduces complex dynamics. By managing energy from these diverse sources, projects can balance energy production, reduce waste, and enhance sustainability. Efficiently integrating these systems can increase self-sufficiency and minimize reliance on external energy sources.*

PID_3*: Energy Monitoring and Analytics are key to identifying inefficiencies in renewable energy projects. With data-driven insights, project managers can optimize energy generation, pinpoint areas of waste, and make informed decisions about how to maximize energy production. These insights empower project teams to adjust operational strategies and minimize inefficiencies.*

Step 1:Explain the table 2 and 3 TIF decision matrix.

Table 2 of the TIF decision matrix.

	Su	Ra	Ch
PID_1	$\begin{bmatrix} [0.1, 0.2, 0.3], [0.02, 0.04, 0.5] \end{bmatrix}$	$\begin{bmatrix} [0.11, 0.21, 0.33], [0.024, 0.044, 0.11] \end{bmatrix}$	$\begin{bmatrix} [0.21, 0.41, 0.53], [0.14, 0.24, 0.31] \end{bmatrix}$
PID_2	$\begin{bmatrix} [0.21, 0.41, 0.53], [0.14, 0.24, 0.31] \end{bmatrix}$	$\begin{bmatrix} [0.1, 0.2, 0.3], [0.02, 0.04, 0.5] \end{bmatrix}$	$\begin{bmatrix} [0.11, 0.21, 0.33], [0.024, 0.044, 0.11] \end{bmatrix}$
PID_3	$\begin{bmatrix} [0.21, 0.41, 0.53], [0.14, 0.24, 0.31] \end{bmatrix}$	$\begin{bmatrix} [0.11, 0.21, 0.33], [0.024, 0.044, 0.11] \end{bmatrix}$	$\begin{bmatrix} [0.1, 0.2, 0.3], [0.02, 0.04, 0.5] \end{bmatrix}$

TIF decision matrix table 3.

	Su	Ra	Ch
PID_1	$\begin{bmatrix} [0.11, 0.12, 0.13], [0.1, 0.2, 0.12] \end{bmatrix}$	$\begin{bmatrix} [0.1, 0.2, 0.3], [0.03, 0.04, 0.5 \end{bmatrix}$	$\begin{bmatrix} [0.21, 0.22, 0.23], [0.1, 0.3, 0.22] \end{bmatrix}$
PID_2	$\begin{bmatrix} [0.1, 0.2, 0.3], [0.03, 0.04, 0.5 \end{bmatrix}$	$\begin{bmatrix} [0.11, 0.12, 0.13], [0.1, 0.2, 0.12] \end{bmatrix}$	$\begin{bmatrix} [0.21, 0.22, 0.23], [0.1, 0.3, 0.22] \end{bmatrix}$
PID_3	$\begin{bmatrix} [0.21, 0.22, 0.23], [0.1, 0.3, 0.22] \end{bmatrix}$	$\begin{bmatrix} [0.1, 0.2, 0.3], [0.03, 0.04, 0.5 \end{bmatrix}$	$\begin{bmatrix} [0.11, 0.12, 0.13], [0.1, 0.2, 0.12] \end{bmatrix}$

Step 2:Describe the TIFFWA operator and $\xi = (0.26, 0.21, 0.25)$.
TIFFWA operator is in table 4.TIFFWA operator table 4.

	Su	*Ra*	*Ch*
PID_1	[0.1001, 0.3698, 0.1478] [0.1983, 0.1258 0.1092	[0.1452, 0.7541, 0.9451] [0.1675, 0.2587 0.1124	[0.2561, 0.0026, 0.4598 [0.2343, 0.4587, 0.2032]
PID_2	[0.3031, 0.7412, 0.9871] [0.3014, 0.3974, 0.2052]	[0.1698, 0.3012, 0.1231, [0.3453, 0.0198 0.9872	[0.0001, 0.3698, 0.3014 [0.1004, 0.9014, 0.2123
$PIPS_3$	[0.0541, 0.0131, 0.3214], [0.7412, 0.1134, 0.2359]	[0.3211, 0.0123, 0.0212], [0.4569, 0.0556, 0.0411	[0.3698, 0.4569, 0.3051], [0.3124, 0.3084, 0.2082]

Step 3:The TFCM algorithms work on TIFFWA operator $U = [U_{ic}]_{i=1...n}^{c=1...c}$.
TFCM algorithms work on TIFFWA operator in table 5.
TFCM algorithms table 5.

	Su	*Ra*	*Ch*
PID_1	0.4	0.2	0.1
PID_2	0.02	0.12	0.03
PID_3	0.01	0.23	0.01

Step 4:Using k-means clusting value is defined as cluster's parameters in table 6.
k-means clusting table 6

	Su	*Ra*	*Ch*
PID_1	[0.1232, 0.2563, 0.3698], [0.1258, 0.0987, 0.2587]	[0.1232, 0.2563, 0.3698], [0.1258, 0.0987, 0.2587]	[0.1209, 0.1163, 0.9898], [0.8518, 0.0967, 0.1477]
PID_2	[0.1963, 0.2258, 0.3741], [0.1753, 0.3577, 0.9877]	[0.1942, 0.2753, 0.3148], [0.1559, 0.7534, 0.2741]	[0.0002, 0.0143, 0.4568], [0.1008, 0.0904, 0.2014]
PID_3	[0.4432, 0.6963, 0.3448], [0.1148, 0.0753, 0.2369]	[0.1753, 0.2159, 0.3149], [0.1157, 0.0741, 0.2358]	[0.1154, 0.2258, 0.3354], [0.1145, 0.0985, 0.2147]

Step 5:Calculate the cluster center to find the centroid
$C_1 = 0.2345, C_2 = 0.1034, C_3 = 0.3456$.
Step 6:Find out the distance of each point from the centroid in table 7.

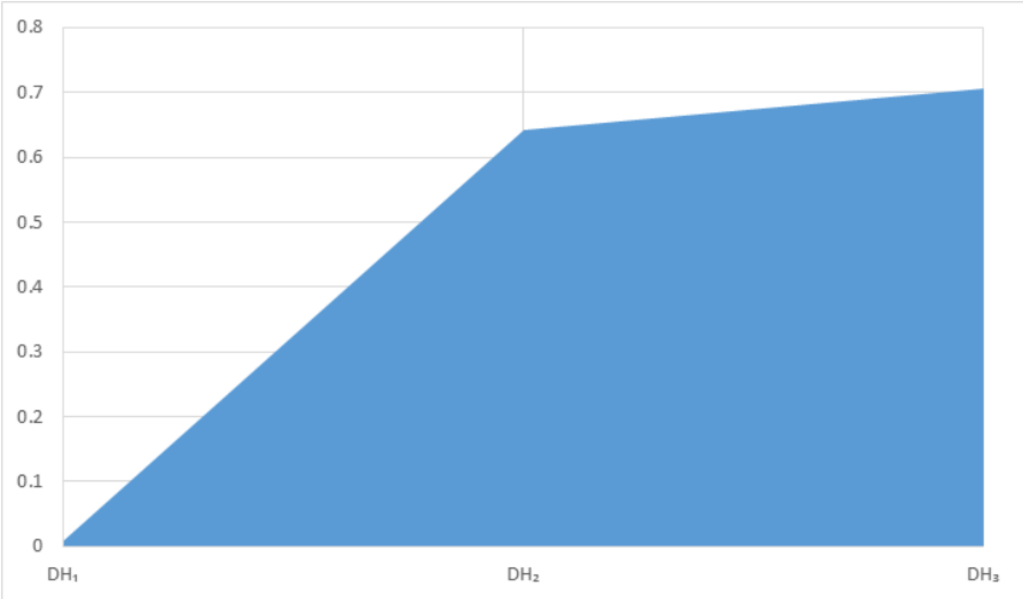

Figure 5 different score functions.

	Centroid table 7
PID_1	$\begin{bmatrix} [0.1256, 0.1016, 0.1852], \\ [0.9874, 0.1984, 0.1397] \end{bmatrix}$
PID_2	$\begin{bmatrix} [0.1036, 0.1298, 0.1987], \\ [0.9012, 0.1387, 0.1989] \end{bmatrix}$
PID_3	$\begin{bmatrix} [0.1003, 0.1009, 0.0002], \\ [0.0694, 0.9874, 0.3219] \end{bmatrix}$

Step 7:Calculate the score function $DH_1 = 0.0091, DH_2 = 0.6412, DH_3 = 0.7056.$

Step 8:Find the ranking $DH_3 > DH_2 > DH_1$ *and* DH_3 *is the best.*

Score function of figure 5 is different ranking. because Figure 5's ranking seems to alter depending on the score function. This disparity results from different evaluation standards being applied: DH_3 *is the first ranking,* DH_2 *is the second ranking and* DH_1 *is the last ranking. Figure 5 is given as below*

6.1. Comparsion technique with existing way

Different existing ways are written below in Table 8. Table 8 Existing techniques.

Methods	*Average operator*	*Geometric operator*
MCDM [46]	$DH_3 > DH_2 > DH_1$	$DH_3 > DH_2 > DH_1$
Similarity measure [20]	$DH_3 > DH_2 > DH_1$	$DH_3 > DH_2 > DH_1$
T-SFS operators [47]	$DH_3 > DH_2 > DH_1$	$DH_3 > DH_2 > DH_1$
Aggregation operators [5]	$DH_3 > DH_2 > DH_1$	$DH_3 > DH_2 > DH_1$
Hamacher operators [46]	$DH_3 > DH_2 > DH_1$	$DH_3 > DH_2 > DH_1$
Particle swarm [50]	$DH_3 > DH_2 > DH_1$	$DH_3 > DH_2 > DH_1$
Intuitionistic operators [11]	$DH_3 > DH_2 > DH_1$	$DH_3 > DH_2 > DH_1$
IFFPA operators [31]	$DH_3 > DH_2 > DH_1$	$DH_3 > DH_2 > DH_1$

6.2. Validity way

In this subsection, we present the proposed method for validating the aggregating operator, as shown in Table 9.

Ways	*Average operator*	*Geometric operator*	*hybrid*
Proposed technique	✓	✓	✓
Muirhead mean-based 2-tuple linguistic [48]	✓	✓	✓
pythagorean fuzzy TOPSIS method [60]	✓	✓	✓
pythagorean Dombi method [59]	✓	✓	✓
Complex Pythagorean fuzzy information [49]	✓	✓	✓
PLTM operators [32]	✓	✓	✓

Table 9 presents a comparative overview of different decision-making techniques based on their use of various aggregation operators—namely, average, geometric, and hybrid operators. The methods listed include both the proposed technique and several existing approaches from the literature.

The "$\surd$" (check mark) symbol in the table indicates that the corresponding method utilizes that specific type of aggregation operator. For example, if a method has a $\surd$ under "Average Operator," it means that the method applies an average-based aggregation approach.

This table is intended to highlight the versatility and comprehensiveness of the proposed technique in comparison to existing methods, demonstrating its capability to support all three types of aggregation operators.

6.3. Results and discussion

There are several reasons why the proposed method is superior to traditional approaches in renewable energy project selection. Conventional models typically rely on fixed, static data and fail to accommodate the dynamic nature of energy markets, fluctuating resource availability, and evolving environmental conditions.

To improve the effectiveness of current energy project evaluation systems, integrating TIFF aggregation operators can process complex, uncertain data such as fluctuating renewable energy production, varying demand, and technology performance metrics.

Provide comprehensive guidelines for energy project evaluators to efficiently use systems based on TIFF aggregation operators. Stress the importance of accurate data interpretation and highlight the advantages of triangular intuitionistic fuzzy methods for optimizing the selection of renewable energy projects.

To validate the reliability and practicality of TIFF aggregation operators in real-world scenarios, conduct pilot evaluations of renewable energy projects across various contexts. Gather critical feedback and data during these evaluations to refine and improve the methodology before large-scale implementation.

Collaborate with regulatory authorities to ensure that the integration of TIFF aggregation operators in renewable energy project selection complies with all necessary standards and regulations.

6.4. Advantages

There are numerous important benefits to using triangular intuitionistic Fuzzy Frank Aggregation Operators in decision-making for selecting renewable energy projects.

- *A clear demonstration of Multi-Criteria Decision-Making (MCDM) evidence in a non-verbal manner using the TIF method. The TIF method has proven to be essential for clarifying uncertain and incomplete project evaluation results.*
- *TIFF aggregation operators successfully integrate imprecise and heterogeneous data, enhancing the accuracy of renewable energy project selection forecasts. This results in more precise project evaluations, particularly in complex scenarios with fluctuating energy generation, variable demand, and diverse environmental conditions.*
- *TIFFAO enables consistent decision-making across various project selection scenarios by combining multiple criteria in a more advanced way. As a result, the reliability of selecting optimal energy projects is increased, and variability in our is minimized.*
- *MCDM becomes more transparent and easier to understand due to TIFFO's structured approach to data aggregation. This promotes more confident project selection decisions and helps stakeholders better comprehend the rationale behind the final selections.*
- *Due to their expertise in renewable energy project evaluation, their ability to make practical judgments, and the effectiveness of the evaluation processes, energy project specialists are often required to guide the optimization of renewable energy systems. Current methods rely heavily on expert judgment to interpret and select viable energy projects.*

6.5. Sensitive study

We define the sensitive study in this subsection, which is represented in table 10 below.

Value	DH_1	DH_2	DH_3
$\Delta = 0, DH_i$	0.0112	0.8967	0.2545
Preference order ranking	3	1	2
$\Delta = 0.1, DH_i$	0.0145	0.6066	0.4014
Preference order ranking	3	1	2
$\Delta = 0.2, DH_i$	0.0123	0.6234	0.4124
Preference order ranking	3	1	2
$\Delta = 0.3, DH_i$	0.0101	0.6765	0.4873
Preference order ranking	3	1	2
$\Delta = 0.4, DH_i$	0.0077	0.8345	0.2098
Preference order ranking	3	1	2
$\Delta = 0.5, DH_i$	0.0145	0.4987	0.2012
Preference order ranking	3	1	2
$\Delta = 0.6, DH_i$	0.1021	0.2345	0.1987
Preference order ranking	3	1	2
$\Delta = 0.7, DH_i$	0.1129	0.3409	0.2983
Preference order ranking	3	1	2
$\Delta = 0.8, DH_i$	0.1983	0.7654	0.5672
Preference order ranking	3	1	2

7. Conclusion

In this section, we introduce triangular intuitionistic fuzzy data based Frank Aggregation Operators applied to the selection of renewable energy projects. The operational laws are defined along with the corresponding score and accuracy functions. This study uses the TIFFWA, TIFFOWA, TIFFHWA, TIFFWG, TIFFOWG and TIFFHWG operators within the Multi-Criteria Decision-Making (MCDM) approach to address challenges in renewable energy project selection. It is demonstrated that these operators enable the MCDM technique to effectively differentiate between various renewable energy alternatives, offering flexibility in making decisions related to project selection and energy efficiency. These operators exhibit essential properties such as commutativity, idempotency, boundedness, associativity, and monotonicity. When combined with clustering techniques like the triangular Fuzzy C-means algorithm, these aggregation operators enhance the accuracy and computational efficiency of the project selection process, making it possible to cluster data effectively for more reliable decision-making.

In the near future, we plan to incorporate artificial intelligence into renewable energy project selection. This includes utilizing neural networks, automation, data analysis, and virtual assistants to enhance decision-making processes related to project assessment. Additionally, we aim to expand the current approach by incorporating Generalized triangular Cubic Fuzzy Frank Aggregation Operators and triangular Cubic Fuzzy Frank Geometric Aggregation Operators, allowing for more advanced, adaptive, and efficient systems for selecting renewable energy projects.

8. Compliance with Ethical Standards

The authors declare that there is no conflict of interests regarding the publication of this paper. Compliance with Ethical Standards: This study is not supported by any source or any organizations.

Ethical approval: This article does not contain any studies with human participants or animals performed by any of the authors.

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