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Fuzzy Logic-Driven Assessment Model for Mathematical Proof Grading in Education

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Abstract. Fuzzy logic has become a critical tool in handling uncertainty in decision-making systems, especially those that require subjective human judgment. This paper aims to introduce an advanced fuzzy logic-based grading system for evaluating mathematical proofs with improved consistency, flexibility, and automation of grading processes in mathematical education. This study proposes a refined fuzzy logic system implemented in Python to assess the correctness of students' proofs based on four core questions: assumptions and conclusions, correct definitions, appropriate proof methods, and logical reasoning. The proposed system utilized the fuzzy logic technique of fuzzification, rule-based inference, and defuzzification to evaluate a score between "Correct" and "False" in points from 0 to 10. The system was tested using student data while fuzzy grades were assessed together with the grades assigned by the professor. The results show that the system provides a subtler and more reliable grading procedure, which handles partial correctness better than the traditional binary methods. It addresses the gap in grading the mathematical proof by introducing an adaptive, computationally efficient, and consistent solution.

2020 Mathematics Subject Classifications: 03B52, 03F03, 68T05, 97D20

Key Words and Phrases: Fuzzy logic, Mathematical proof evaluation, Learning and adaptive systems in artificial intelligence, Educational assessment

1. Introduction

Fuzzy logic is a type of many-valued logic in which the truth value of variables can be any real number between 0 and 1. In Boolean logic, the truth value is confined to the integers 0 or 1. The concept of fuzzy logic was first published by mathematician Lotfi Zadeh in 1965 when he developed the fuzzy set theory. Although it derives from infinite-valued logic discovered and studied as early as 1920 by Lukasiewicz and Tarski, fuzzy sets emphasize the way that humans frequently make decisions, making use of non-numerically

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based information. Fuzzy or fuzzy models act as a representation of the lack of clarity and imprecise information and allow for the representation of uncertainty and ambiguity in a mathematical sense. Fuzzy models can identify, interpret, and manipulate uncertain data and information. Fuzzy logic has many applications, such as in control theory and artificial intelligence [1,2].

The data we can glean from various traditional and online educational institutions around the world is growing in terms of data analysis; several initiatives have come to help students, and educators, in virtual-type courses, track and manage academic performance in competency-type learning. The educator assesses student capabilities and keeps track of them [3]. An important foundational component of mathematics is mathematical proof (which is important in substantiating statements and explanations for their correctness). The pivotal element of mathematics is mathematical proof. It is critical to check whether the assertions are valid, and it can also be used to explain why they are correct. However, proofs serve more than just the validation of assertions. Proofs also aid one's understanding of the underlying concepts and develop analytical skills. They are important in linking ideas together; they enable us to communicate and organize mathematical ideas. Proofs also allow us to structure ideas logically, produce mathematical statements, and strengthen reasoning skills and thought processes.

Although traditional methods of instruction and correcting proof privilege specific rules that may not fully accommodate how students think and know, this investigation is aimed at creating a more flexible approach to correcting mathematical proofs while maintaining rigor. In prior research conducted by a pair of researchers, a program was developed to teach university-level students how to write detailed mathematical proof in algebra. The program consisted of three techniques that were directly taught: direct proof, indirect proof, and proof by contradiction. It was also the goal of the program to promote a positive attitude about the role and benefit of mathematical proof in mathematics. The researchers divided the model into four key procedures for writing and teaching mathematical proofs: Identifying Assumptions and Conclusions, Emphasizing the distinction between them and how to derive them from given theorems then, Defining Necessary Terms, Ensuring students understand and can use the definitions required to prove the theorem, after that Selecting the Proof Method: Highlighting the differences between the three proof methods used in the study, when to use each method, and the steps involved in writing proofs for each method and finally, Connecting and Justifying Statements: Using mathematical analysis and reasoning to link proof steps and provide justifications.

The framework highlighted the need to link and explain statements clearly, beginning with the hypothesis, selecting the best proof method, and arranging statements in a logical order. Each statement should connect to the next, creating a clear argument based on basic rules, key theorems, and relevant definitions. The researchers used these four points to evaluate student's proofs and assign grades. The current study uses that framework, using Python to implement a fuzzy system for proof correction by incorporating fuzzy sets and fuzzy inference systems. Specifically, the model is applied to student proofs from three mathematics courses: Introduction to Abstract Algebra (Course 261), Introduction to Foundations of Mathematics (Course 250), and Linear Algebra (Course 111). This research aims to develop a model that can assess students' proofs more accurately and provide meaningful feedback to enhance their learning experience.

Figure 1 below illustrates the proposed fuzzy logic-based framework for evaluating mathematical proofs. The framework outlines the process of decomposing a proof into key components, applying fuzzy logic techniques, and generating a final grade with feedback.

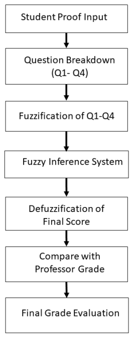

Figure 1: Flowchart of Fuzzy system

The process of making decisions in fuzzy environments involves activities in which the objectives and/or constraints of the process are vaguely defined. A fuzzy decision is conceptualized as the intersection of fuzzy objectives and fuzzy constraints, and a maximizing decision is a fuzzy decision where the membership function of the fuzzy decision has its maximum value. The concepts are investigated under multiple-stage decision processes as well as under deterministic and stochastic assumptions. Fuzzy logic has been applied in many areas, including education, to improve social equity in classification. Unlike traditional binary logic, fuzzy logic allows the degrees of truth value, in this case for a student grade, to range between 0 and 1. This type of logic is ideal for handling the complexities of student evaluations with uncertain reflections on student performance. In Domingo and Martinez [4], the MCQs (multiple choice questionnaires) utilized in the assessments have traditional multiple choice questionnaire scales that typically come with three or four answer options (MCQ-3 or MCQ-4). The use of fuzzy logic enables the professors to grade students for fairness taking into account the initial score of the test by applying the allocation index of difficulty. The allocation index of difficulty provides a more precise method for evaluating whether students are appropriately graded concerning the difficult adjustments of the students that makes distinguishing their performance easier based on their varying levels of difficulty to assess the students as a whole. This allows the professors to adjust their grading process fairly while visually showing each student's true level of difficulty and performance level in their grade assessments. These outcomes further illustrate the variety that exists in the classroom by utilizing linguistic variables and pre-set rules, which makes the grading process and outcomes much more equitable. The application of fuzzy logic keeps the picture of the proportion of degree concerning complexity.

A fuzzy logic framework was proposed to facilitate the evaluation of the exam considering both the difficulty of the questions and the time to answer. The interpretability of fuzzy Logic (FL) allows easier systems design and analysis, making it useful in different fields for rule-based systems [5]. In a study by Vora and Tulshyan [6], fuzzy logic methods were also described as extended from Binary Logic as introduced in the 1960s by Lotfi Zadeh, and it was also highlighted as useful for addressing complex problems with stochastic inputs. It discussed fuzzy logic's main components: fuzzification, fuzzy rules, inference methods, and defuzzification. Since fuzzy logic control (FLC) has low hardware requirements, is robust, and is an efficient way of control, it is beneficial. Tsidylo et al., [7] introduced the developing phase of an intelligent system for evaluating multilevel test tasks and fuzzy logic was applied in MATLAB, which is an important advantage. Their system uses the Sugeno production model, allowing domain experts to interact in careful detail to build practical inference systems at an easy-to-interpret level. Their system has 4 input variables (representing levels of knowledge assimilation) in addition to one initial variable. The system can deal with characterizations of fuzziness such as task difficulty, correctness, given time, and percentage of tasks completed correctly, and is beneficial in providing complete and specific evaluations.

Additionally, several studies look at the use of fuzzy logic in education, more specifically in the use of assessing and evaluating student performance in the classroom. The previous work indicates that a fuzzy logic system has been developed to convert traditional grading into letter grades. With the notion of mathematical proof, one of the evaluation strategies was to discuss the difficulty, importance, and complexity of proof questions based on fuzzy membership functions [8]. In the study by Alajmi and Al-Kandari [9] on Teaching Mathematical Proofs, they outlined a framework to develop a program for teaching mathematical proofs that engaged the students through a discussion and analysis of examples of proofs. When making each proof, the instructors would grade the students by separating the proof-writing steps into identifying the assumptions and conclusions, identifying the definitions correctly, selecting and applying proof methods, logically flowing, and justifying. The framework for writing proofs was shown to have a significant positive impact on university students' proof-writing skills and that it positively impacted students' beliefs about proofs. The students showed better understanding and application of mathematical definitions, showed better logical flow in their proofs, and had more confidence in their ability to write proofs [10]. This study presents an advanced fuzzy logic-based framework for the evaluation of mathematical proof, which is a flexible, consistent, and computationally efficient, alternative to traditional grading by using linguistic variables, fuzzy inference, and real-world grading data [11]. Despite previous systems, there were still difficulties that students' proofs presented in terms of various logical flow and justification. Previous assessment systems also typically oversimplified students' performances to a greenhouse level, not depicting the finer differences necessary to enable meaningful feedback. Specifically, Annabestani et al. (2019) noted that standard four-valued logic methods led to unrealistically similar grades and errors from teachers for more systematic assessments. However, when fuzzy descriptive evaluation systems were introduced, these problems were mitigated, and much more nuanced and fairer assessments were realized that better encapsulated the complexities of learning for students [12]. Similar progress had been achieved within outcome-based education and online time-based engineering courses, where fuzzy logic models of evaluation had provided additional possibilities for evaluation that encompassed multiple performance parameters that included students' possible antecedent misconceptions into consideration which led to more accurate evaluation and better instructional support to students. In summary, these works demonstrate the potential of fuzzy logic systems for accommodating uncertainty and subjectivity in student assessment whereas conventional methods do not [13,14].

Recent trends in fuzzy logic by combining fuzzy logic with Multi-Criteria Decision Making (MCDM) methods provide more evidence that fuzzy logic is suitable for addressing complex, uncertain domains like education. Ejegwa et al. (2024) have already examined the integration of a Fermatean fuzzy distance metric and MCDM to assess A Security Crisis, and the findings from more advanced fuzzy models support the possibilities for the future and suitability of fuzzy models to address uncertainty [15]. Likewise, Kousar, et al. (2024) utilized MCDM for environmental decision-making [16]. Moreover, both Sri, et al. (2024) and Jayakumar, et al. (2024) applied fuzzy MCDM frameworks that dealt with positive/negative attributes and prioritized risk factors respectively [17, 18]. Surya, et al. (2024), on the other hand, proposed new entropy measures for fuzzy hypersoft sets which contributed to multi-attribute decisions. These approaches are very relevant in educational grading, where there are multiple subjective factors with criteria unrelated to the other criteria evaluated or factored for each outcome [19].

In a recent study, fuzzy logic was applied to monitor students' academic achievement in a supervised Outcome-Based Education (OBE) framework. The study comprised a fuzzy inference system incorporating extraneous factors such as direct assessment, indirect assessment, and student stress levels. The fuzzy inference systems drew on both Mamdani and Sugeno methods for approach. The system evaluated student performance in zones (comfort, average, and highly stressed), enabling the evaluation system to provide measures based on stress while also providing targeted intervention where the individual's academic stress is considerably less than normal. A prototype mobile application highlighted the application of a measurement system [20].

Moreover, a recent study introduces a Fuzzy Expert System (FES) to evaluate student academic performance in a supervised laboratory context, including numerous linguistic factors. The system incorporates seven key input variables: stress, motivation, confidence, parental support and availability, self-study hours, punctuality, and peer influence. Utilizing various defuzzification techniques, including centroid, bisector, and mean of maxima methods, the FES provides precise and measurable assessments of student performance. The research concludes that all three defuzzification methods perform equally well, offering educational institutions a robust tool for identifying factors influencing student outcomes and tailoring interventions accordingly[21]. In summary, this study proposes a fuzzy logic-based system developed in Python to approximate grading consistent with professor criteria, aiming to improve the accuracy and meaningfulness of student proof assessments.

This paper is divided into four sections: Section One: Introduction, Section Two: Methodology, Section Three: Results, Section Four: Discussion, Section Five: Conclusion and Section 6 Limitations and Future Directions.

2. Material and Methods

2.1. Data Collection

The data consists of grades from 20 students for four assessment questions (Q1, Q2, Q3, Q4) and the grade assigned by the professor for each student. The scores of the students for every question are in the range 0,0.5,1,1.5,2. The professors' grades, between 0 and 10, provide the basis to judge the performance of the fuzzy logic system. The data has the following structure: Q1, Q2, Q3, Q4: Scores for each student on four assessment questions The individual question scores, which are initially in the range of 0, 0.5, 1, 1.5, and 2, are scaled to a 0 to 10 range to align with the professor's grading scale. Scaling aims to convert the finer granularity of the individual question scores into a more widely used grading scale. The following approach is used to map the raw scores to a 0-10 range: Professor Grade: The final grade assigned by the professor is the fuzzy system's target output.

The fuzzy system is implemented and evaluated using Python in a Jupyter Notebook environment (Hosted on Google Colab). The collected data is stored in a Data Frame, and the fuzzy system is applied to predict the grades based on the inputs, as shown in Figure 2.

2.2. Input and Output Variable

The input variables in the fuzzy logic system are the scores of the four questions (Q1, Q2, Q3, Q4), and the output variable is the final grade, denoted as grade. These variables are represented as fuzzy sets with membership functions. The input fuzzification process classifies the total score q into one of five fuzzy categories: fail, poor, average, good, or excellent. These categories reflect increasing levels of performance. Detailed membership thresholds are provided in Appendix A.

2.3. Fuzzification

Fuzzification is the process of converting crisp inputs (e.g., student scores) into fuzzy values according to the defined fuzzy sets. For each of the question scores (q), the process of fuzzification will associate a membership value to all the five fuzzy categories: fail, poor, average, good, and excellent. This membership value is interpreted as the degree of belongingness of the score to each of the fuzzy sets. Whereas Q1, Q2, Q3, and Q4 represent the individual scores obtained by a student on four assessment questions, each ranging from 0 to 2 (with possible values: 0, 0.5, 1, 1.5, or 2), q denotes the normalized total score calculated by summing the scores of Q1 to Q4 and scaling the result to a 0–10 range The output defuzzification process maps each course grade into five linguistic categories: fail, poor, average, good, or excellent. Each course (Course 1, Course 2, and Course 3) uses customized thresholds to classify student performance. Full rule thresholds are provided in Appendix A.

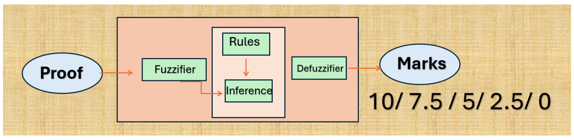

Figure 2: Proposed System

2.4. Defuzzification

The fuzzy output is converted to a crisp value, graded between 0 and 10. In the fuzzy logic system, defuzzification can be performed by the centroid method. The weighted average of the membership functions calculates the centroid of the output fuzzy set.

2.5. Defining the Fuzzy Inference Rules

Fuzzy inference rules identify the output grade after fuzzy inputs. They manifest expert knowledge of how the various student performance combinations should lead to a particular grade. The fuzzy rules assign a fail grade primarily when the first two questions, Q1 and Q2, both have failing evaluations. If Q1 is poor and Q2 fails, combined with a fail in either Q3 or Q4, the final grade is also fail. When Q1 is fail and the other questions are mostly fail or poor, the system consistently assigns a fail grade. Even if Q1 is excellent, the presence of fail grades in Q2, Q3, and Q4 can still result in a fail. In general, if three or more responses are fail, especially involving Q1 or Q2, the overall grade will be fail.

A poor grade is given when Q1 or Q2 shows average or excellent performance, but the other answers fall mostly into fail or poor categories. When both Q1 and Q2 are poor, or when all four answers are poor, the final grade is set to poor. The system also assigns poor grades if Q1 is excellent but combined with poor or fail responses in other questions.

For higher grades, when Q1 and Q2 both have excellent or good evaluations, and the other questions show average to excellent ratings, the overall grade tends to be good or excellent. The fuzzy rules reflect the importance of strong performance in the first two questions and consider the combined quality of all four answers to determine the final grade accurately. These fuzzy rules are detailed in Appendix A.

The performance of the fuzzy logic system is evaluated by comparing the fuzzy grades with the professor's grades. Several metrics are used to assess the system's accuracy:

1. R-squared Value: It calculates the amount of variance in the professor's grades explained by the fuzzy grades. The nearer the R-squared value to 1, the better the model fit.

2. Mean Absolute Error (MAE): MAE represents the average of the absolute differences between the fuzzy grades and the professor's grades. It gives a straightforward measure of the average magnitude of the errors in the predictions, regardless of their direction. Lower MAE values indicate better predictive performance.

3. Root Mean Squared Error (RMSE): RMSE is a commonly used metric that squares the differences before averaging, then takes the square root. It penalizes larger errors more than MAE, making it sensitive to outliers. Lower RMSE values signify more accurate

predictions.

4. Mean Bias Error (MBE): MBE measures the average bias in the predictions, indicating whether the fuzzy system tends to overestimate or underestimate the grades. A value near zero indicates that the system has no consistent bias.

5. The correlation between the fuzzy grades and the professor's grades is also visualized in the correlation heatmap, which gives insight into how well the fuzzy logic system can reproduce the grading patterns.

2.6. Handling Uncertainty

Fuzzy logic inherently accommodates uncertainty, making it ideal for systems where data may not be precise or deterministic. In this system, we use Type-1 fuzzy sets, which allow for the inclusion of vague or incomplete information. The membership functions model this uncertainty, enabling the system to generate reliable outputs even when input data is imprecise.

3. Results

The membership function for input variables of all three courses: Introduction to Abstract Algebra (Course 261), Introduction to Foundations of Mathematics (Course 250), and Linear Algebra (Course111) is consistent to ensure uniformity throughout the evaluation process while the membership function for output variables and rules varies according to the different grading in three courses. The triangular membership function is used to access the input and output variables scores of four questions (Q1, Q2, Q3, Q4). Each question score is categorized into linguistic words; fail, poor, average, good, and excellent as shown in Figure 3.

3.1. Analysis of Course 1

In Figure 4, the triangular membership function of output variables provides a visual representation of how output values are translated into degrees of membership. As shown in Table 1 the students with high scores of 7.5 or above in the fuzzy system perform well. In these cases, the fuzzy grades and professor grades are closely aligned according to the rules if a student gets an excellent score in the first three questions then the final grade would be excellent and ranges above 9 as the students 4, 6, 8, 10, 11, 13, 17, 19 have fuzzy grades excellent shown in Table 1. For the grades that are failing, poor, average, and good the fuzzy system also provides reliable evaluation but in some cases, grades slightly differ from professor grades in student 1 the professor assigned 0 grade because the question 1 score is poor and the rest scores are failing but the fuzzy system provides approximate grade 0.1 also the other students, student 2 the first two question grade is excellent, question 3 grade is average and question 4 grade is failing the professor assigned grade is 3.0 but the fuzzy system is 3.5 both lies in final grade average, these discrepancies are minor and do not impact the overall system evaluation.

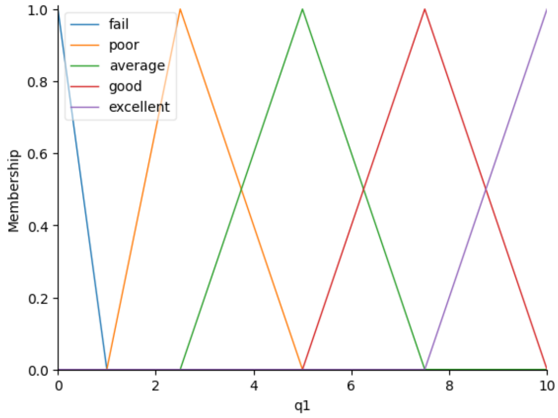

Figure 3: Membership functions for input variables

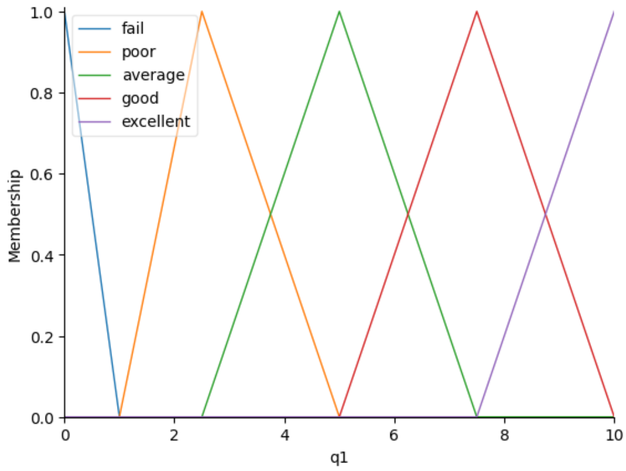

Figure 4: Course 1 Triangular membership function of output variables

To assess the predictive performance of the fuzzy logic system, several evaluation metrics were applied: The R^2 value for Course 1 is 0.94, indicating that 94 percent of the variance in the professor's grades. This signifies excellent model performance and consistent alignment with human grading. The MAE is 0.557 meaning on average fuzzy grades deviate by just over 0.5 from professor grades. This represents a very high level of accuracy in the system's predicted values. The RMSE is 0.801 meaning the degree of error is relatively low and the fuzzy system does not typically make large deviations from the professor's grades. Since RMSE penalizes larger errors more than MAE, the relatively small value confirms model stability and consistency. The MBE is 0.294, suggesting a slight tendency of the fuzzy system to overestimate grades by about 0.29 points on average. However, this bias is small and does not significantly affect the reliability of the evaluation. Additionally, the correlation heat map in Figure 5 confirms that fuzzy grades are highly correlated (0.98) with professor grades, further validating the effectiveness of the fuzzy logic system in mimicking instructor grading behavior.

Table 1: Student Performance: Professor vs. Fuzzy Grading.

Student	Q1	Q2	Q3	Q4	Professor Grade	Fuzzy Grade
1	2.5	0.0	0.0	0.0	0.0	0.1
2	10.0	10.0	5.0	0.0	3.0	3.5
3	0.0	2.5	0.0	0.0	0.0	0.1
4	10.0	10.0	10.0	5.0	7.5	9.5
5	2.5	0.0	0.0	0.0	0.0	0.1
6	10.0	10.0	10.0	5.0	8.0	9.5
7	5.0	5.0	0.0	0.0	1.0	1.0
8	7.5	10.0	10.0	7.5	9.0	9.5
9	2.5	2.5	0.0	0.0	1.0	1.1
10	10.0	10.0	10.0	10.0	10.0	9.5
11	10.0	10.0	10.0	5.0	8.0	9.5
12	10.0	5.0	10.0	5.0	7.0	6.0
13	10.0	10.0	10.0	7.5	9.5	9.5
14	10.0	2.5	5.0	2.5	3.5	3.5
15	7.5	2.5	5.0	5.0	5.5	6.8
16	10.0	5.0	7.5	2.5	5.0	5.1
17	10.0	10.0	10.0	10.0	10.0	9.5
18	10.0	10.0	7.5	5.0	7.0	6.8
19	10.0	10.0	10.0	7.5	9.0	9.5
20	10.0	5.0	5.0	5.0	6.0	5.5

Figure 6 shows a graphical comparison between the fuzzy logic system's predicted grades and the grades assigned to each student by the grading professor for Course 1 (Abstract Algebra). The fuzzy system seems to follow the grades assigned by the professor quite closely. In the high and mid-range performance proportions in the grading system, the fuzzy logic-predicted grades for students 3, 5, 7, 9, 12, and 16 closely resembled those assigned by the professor; therefore, the fuzzy logic grading system is positively related or comparable to the grading system used by the professor. While the differing grades were not the same for all students, the overall grades were very similar. Some students, however, had points that deviated despite grades being limited to less than 2 points (students 2, 10, 11, 12, and 14) while generally remaining in the same overall grade proportion or category (ie: Average, Good, Excellent). Generally, the fuzzy logic grading system follows the same pattern as the grading system used by the professor with rise and fall in the same similar proportions across the student group. This alignment is further supported by the high R-squared value (0.94) and low error metrics (MAE = 0.557, RMSE = 0.801), reinforcing the reliability and validity of the fuzzy logic model as an effective automated grading assistant.

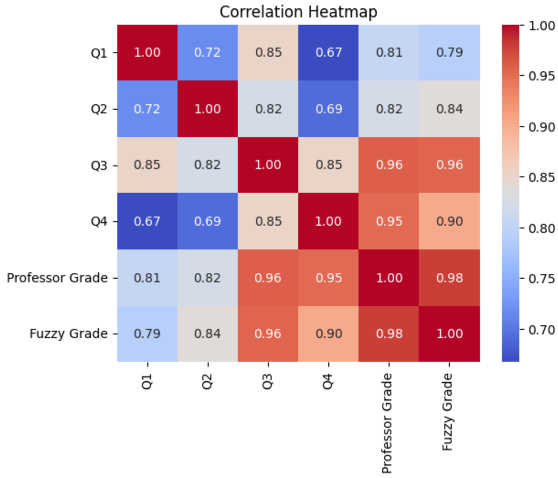

Figure 5: Correlation Heatmap of Professor and Fuzzy system grades

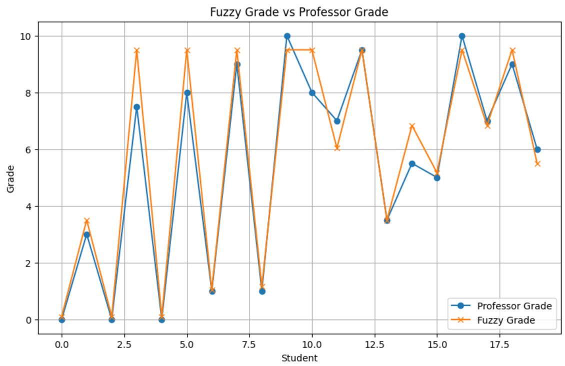

Figure 6: Line Chart of the fuzzy logic system's predicted grades versus the actual grades of Course 1

3.2. Analysis of Course 2

In Table 2, students 2, 6, 13, and 15 perform well as the professor grade is 10 and fuzzy grade is 9.5 both grades lie in the excellent category. Similarly, students 9, and 10 performed well only in Q1 but got average grades in the rest of the questions, the professor's assigned grade was 5 and the fuzzy system grade was the same as the professor's grade. Also, students 4, 5, and 14 got poor grades as their professor grades are 0 and fuzzy grades are slightly higher 0.3 but both lie in the fail category. Students 17 and 20 lie in the good category but their fuzzy grades are slightly higher than the professor's grade. In Figure 7, the triangular membership function of output variables provides a visual representation of how output values are translated into degrees of membership in Course 2

Table 2: Second Set of Student Grades: Professor vs. Fuzzy Grading.

Student	Q1	Q2	Q3	Q4	Professor Grade	Fuzzy Grade
1	5	0.0	0.0	5.0	1.0	0.3
2	10	10.0	10.0	10.0	10.0	9.5
3	10	0.0	10.0	0.0	4.0	4.8
4	5	0.0	0.0	0.0	0.5	0.3
5	0	0.0	0.0	0.0	0.0	0.0
6	10	10.0	10.0	10.0	10.0	9.5
7	10	10.0	5.0	5.0	7.0	6.5
8	10	10.0	5.0	5.0	7.0	6.5
9	10	5.0	5.0	5.0	5.0	5.0
10	10	5.0	5.0	5.0	5.0	5.0
11	10	10.0	5.0	5.0	5.5	6.5
12	10	5.0	0.0	2.5	2.5	3.7
13	10	10.0	10.0	5.0	7.5	8.3
14	0	0.0	0.0	0.0	0.0	0.0
15	10	10.0	10.0	10.0	10.0	9.5
16	5	2.5	2.5	2.5	3.0	2.5
17	10	10.0	10.0	2.5	6.5	8.3
18	0	5.0	5.0	2.5	2.0	2.5
19	10	2.5	2.5	2.5	2.5	2.5
20	10	10.0	10.0	5.0	8.0	8.3

The evaluation metrics for Course 2 ("Introduction to Foundation of Mathematics 250") provide clear support for the fuzzy logic system's accuracy and reliability as compared with the professor's grading. The R-squared value of 0.9519 indicates a very strong correlation between the fuzzy system's grades and those from the professor. The model can account for roughly 95 percent of the variance from the professor's grading. This suggests the fuzzy system is quite robust and consistent in duplicating the grading pattern of the instructor. The Mean Absolute Error (MAE) of 0.5635 and the Root Mean Square Error (RMSE) of 0.7159 are both quite low and indicate that the differences between the two grading methods are small, and in general, would only be by more than one grade point.

The low error values indicate the accuracy of the model and its ability to stay very close to human judgment. The Mean Bias Error (MBE) of 0.1801 indicates a small positive bias in the fuzzy system, suggesting it tends to slightly overestimate student grades compared to the professor's, with the systemic bias being low. To summarize the previous analyses visually, the correlation heat map in Figure 8 presents a correlation coefficient of 0.98 between fuzzy and professor grades, confirming the system's capacity to reconstruct human judgment. This visualization, combined with the strong numerical results and student-level consistency, supports the conclusion that the fuzzy system is a reliable, efficient, and accurate tool for automating grade assessment in Course 2.

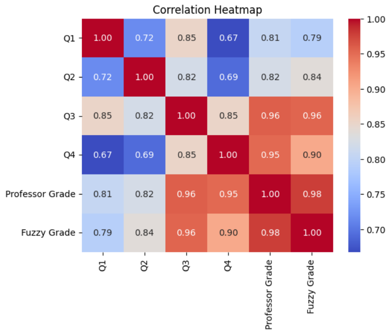

Figure 7: Course 2 Triangular membership function of output variables

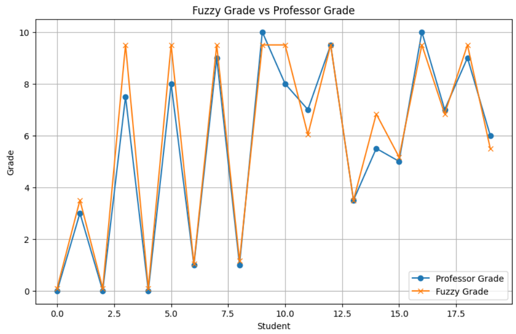

Figure 8: Correlation Heat map of Professor and Fuzzy system grades

In Figure 9, the Fuzzy System Grades and Professor Grades for 20 students in Course 2 Introduction to Foundation of Mathematics 250 are presented. Overall, both lines, representing the fuzzy grades and the professor's grades, follow each other closely for nearly all students. This close visual resemblance reinforces the previously stated high R-squared value (0.9519) and correlation coefficient (0.98) which suggest that the fuzzy system replicates the professors grading with a high degree of accuracy. There are numerous students (students 2, 6, 9, 10, 14, and 15) with grades that are almost identical across the fuzzy and professor grades, demonstrating the fuzzy system's ability to precisely match a human's judgment, particularly when student performance is pronounced and consistent. A few students (3, 11, 12, 13, 17, and 20) had fuzzy grades that were slightly higher than the professor's grade, corresponding to the noted small positive bias (Mean Bias Error = 0.1801). Importantly, these differences do not alter the students' grade categories, indicating the system remains fair and within reasonable bounds.

For students with very low performance, such as students 4, 5, and 14, both the fuzzy system and the professor assigned low grades, with minimal variation between the two. This consistency shows that the fuzzy system is capable of accurately identifying failing performances. Overall, the parallel movement of the two lines throughout the graph confirms that the fuzzy grading system reliably tracks the professor's grading pattern across the full performance spectrum, from failing to excellent students.

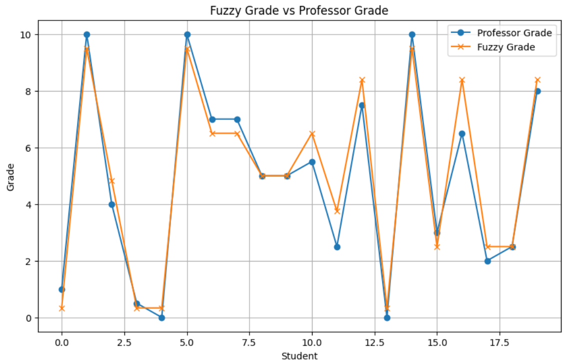

Figure 9: Line Chart of the fuzzy logic system's predicted grades versus the actual grades of Course 2

3.3. Analysis of Course 3

In Table 3, The performance of students 7,11,14, and 20 is excellent as the professor's grade is 10 and Fuzzy grade is 9.5 both grades lie in the excellent category. Students 3,4 and 13 lie in the poor category as their scores for Q1 to Q4 are mostly poor their Fuzzy grades are 1.5 to 2 approximating the professor's grades. Also, students 15 and 18 got average grades as their professor grades are 3 and fuzzy grades are 3.1 both lie in the average category. Students 1, 2, 5, 8, 9, 10, 12, 16, and 16 to 19 lie in the fail category because their scores of Q1 to Q4 are mostly failing their fuzzy grades range from 0 to 0.5 approximating the professor's grades. In Figure 10, the triangular membership function of output variables provides a visual representation of how output values are translated into degrees of membership in Course 3.

Table 3: Third Set of Student Grades: Professor vs. Fuzzy Grading.

Student	Q1	Q2	Q3	Q4	Professor Grade	Fuzzy Grade
1	0.0	0.0	2.5	0.0	0.5	0.5
2	0.0	0.0	0.0	0.0	0.0	0.0
3	5.0	2.5	0.0	0.0	1.5	2.0
4	2.5	2.5	2.5	2.5	2.0	2.0
5	2.5	0.0	0.0	0.0	0.0	0.5
6	10.0	0.0	0.0	0.0	1.0	0.5
7	10.0	10.0	10.0	7.5	9.5	9.5
8	2.5	0.0	2.5	0.0	0.5	0.5
9	2.5	2.5	0.0	0.0	0.5	0.5
10	10.0	0.0	0.0	0.0	1.0	0.5
11	10.0	10.0	10.0	10.0	10.0	9.5
12	5.0	0.0	2.5	0.0	1.0	0.5
13	10.0	0.0	5.0	0.0	1.5	2.0
14	10.0	10.0	10.0	10.0	10.0	9.5
15	10.0	5.0	10.0	0.0	3.0	3.1
16	5.0	0.0	0.0	0.0	1.0	0.5
17	2.5	0.0	0.0	0.0	0.5	0.5
18	10.0	2.5	2.5	2.5	3.0	3.1
19	2.5	2.5	0.0	0.0	0.5	0.5
20	10.0	10.0	10.0	5.0	9.0	9.5

The evaluation metrics for Course 3 (Linear Algebra 111) indicate a highly accurate performance of the fuzzy logic grading system. The R-squared value of 0.9887 demonstrates an exceptionally strong correlation between the professor's grades and the fuzzy system's grades, meaning the system explains nearly 99 percent of the variance in the professor's grading. This is further supported by the Mean Absolute Error (MAE) of 0.2883 and the Root Mean Square Error (RMSE) of 0.3732, both of which indicate that the average deviation between the fuzzy and professor grades is minimal. The Mean Bias Error (MBE) of -0.0117 suggests that the fuzzy system is practically unbiased, slightly underestimating the professor's grades by a negligible margin.

Figure 11 correlation heatmap confirms the strength of this model, showing a nearly perfect correlation coefficient (0.99) between professor and fuzzy grades. Collectively, these results validate the fuzzy system's capability to accurately replicate the human grading process in Course 3, supporting its utility as a reliable automated grading assistant.

Figure 12 presents a visual comparison between the grades assigned by the professor and those estimated by the fuzzy logic system for 20 students in Course 3 (Linear Algebra 111). The fuzzy grades closely track the professor's grades across all students, with both lines generally overlapping or running in parallel. This strong alignment visually supports the high R-squared value (0.9887) reported earlier, indicating a strong correlation between the two grading methods.

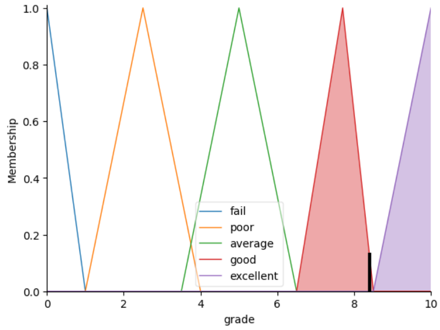

Figure 10: Course 3 Triangular membership function of output variables

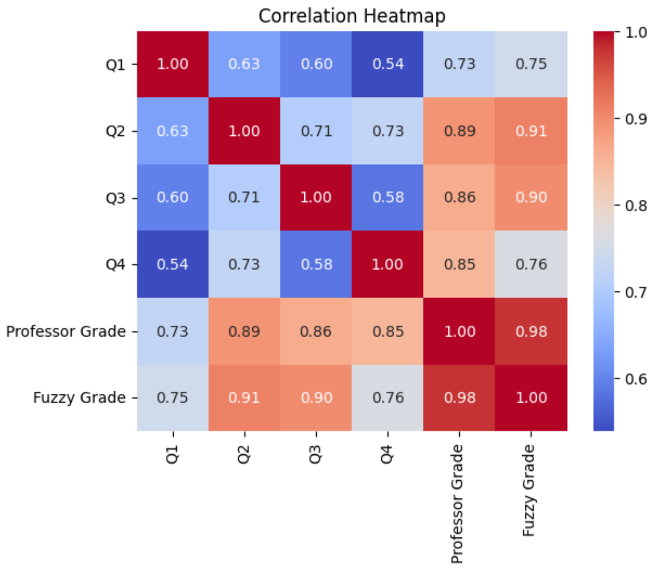

Figure 11: Correlation Heat map of Professor and Fuzzy system grades

For students with high performance—particularly students 6, 10, 11, 14, and 20—the fuzzy system's grades almost perfectly match the professor's evaluations, demonstrating the model's ability to reproduce accurate grading for excellent submissions. Similarly, students who scored poorly (e.g., students 1–5, 8–9, 12, and 16–17) show minimal discrepancies, with both systems consistently assigning low grades near 0 or 0.5. This confirms the system's reliability in assessing both ends of the performance spectrum.

There are minor deviations for students such as 7, 13, 15, and 18, where the fuzzy system slightly overestimates or underestimates the professor's score by approximately 0.5 to 1 point. However, these differences remain within acceptable limits and do not significantly alter the overall grading pattern. The overall behavior of the two lines—rising and falling together across the student indices—further validates that the fuzzy logic model successfully captures the grading trend used by the professor, making it a promising tool for automated and consistent assessment.

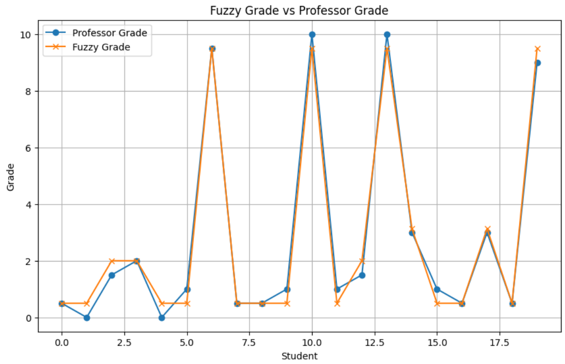

Figure 12: Line Chart of the fuzzy logic system's predicted grades versus the actual grades of Course 3

4. Discussion

The key findings of this study demonstrate that the fuzzy logic-based grade evaluation system aligns closely with professor-assigned grades across all three courses: Introduction to Abstract Algebra (Course 261), Introduction to Foundations of Mathematics (Course 250), and Linear Algebra (Course 111). While the triangular membership functions for output variables varied by course grading criteria, the input membership functions consistently used linguistic categories—fail, poor, average, good, and excellent. The fuzzy system effectively approximates professor grades, with students scoring 7.5 or above performing well and showing close agreement between professor and fuzzy grades across all performance categories. This is supported by high R^2 values ranging from 0.94 to 0.98, strong correlations up to 0.99, and low error metrics (MAE and RMSE), which provide robust statistical validation of the system's accuracy. A fuzzy logic system has been widely used to assess student academic performance that offers a more comprehensive assessment of student performance compared to traditional methods. Mostly, the traditional grading system depends on the crisp number of values or letter grades which may not completely capture the complexities of student's abilities and understandings. Fuzzy logic on the other hand makes it possible to include uncertainties and variations offering a more adaptable and thorough assessment. In a previous study, fuzzy logic and statistical grading were compared. According to the results, fuzzy grading is more flexible than traditional statistical grading and frequently assigns higher or different grades [22].

In a previous study, a fuzzy logic-based grading card was introduced that highlighted the limited use of traditional methods that only rely on crisp numbers, the study demonstrates that the integration of a fuzzy logic system into grading systems can strengthen the evaluation process by considering the complexities of student performance more accurately [23]. Another study highlights the importance of fuzzy logic that evaluates the performance of e-learning students in programming courses. The system uses fuzzy logic to determine the assessment such as attendance in Skype sessions, course marks, and programing lab activities, the results of this system demonstrated that the fuzzy system effectively evaluates student performance and offers a more reliable evaluation than the traditional methods [24]. Additionally, a study demonstrated the advantages of incorporating fuzzy logic in academic settings, particularly its capacity to deal with the inherent ambiguity in performance evaluations. Fuzzy logic provides a more flexible and precise method for assessing student performance compared to traditional grading systems [25].

The fuzzy grading system exhibited distinct advantages over traditional crisp methods through appropriately leveraging the inherent ambiguity in student performance using flexible assessment specific to the assessment criteria of courses. This flexibility is important for the evaluation of student performance in complex academic evaluations since strict thresholds may not accurately reflect a nuanced understanding of course material. Nevertheless, there were some small differences between the fuzzy system and professor grades, which could relate to the subjective nature of the membership functions design or differences in grading from instructor to instructor. Furthermore, the current validation was limited to simple comparisons with instructor grades with no consideration for alternative automated grading methods. Further research needs to compare these findings with wider validation across machine learning methods and larger combinations of distance education datasets, to support greater generalization. Building on these findings, Pathak (2025) utilized a fuzzy expert system for evaluating student overall performance, integrating multiple criteria for assessment, ultimately improving accuracy and objectivity [21]. Similarly, Wardoyo and Yuniarti (2020) were able to adjust fuzzy logic parameters to suit e-learning contexts thus highlighting the adjustable and effective nature of such methods in online education contexts [26].

In the field of online engineering learning, Krouska et al. (2020) are worth mentioning, who adopted fuzzy logic as the basis of an intelligent tutoring system that improved assessments of learners' performance by taking into account task complexity and student working effort, ultimately providing a more accurate performance evaluation by accounted for various contexts. [27] In addition, Amelia et al. (2020) also contributed to the area of education with a meta-analysis of 38 studies. They determined that fuzzy logic can actively emphasize and support the transparency and objectivity of student performance assessments in different learning situations. [28]

The overall similarities of our results compared to prior studies reaffirm its reliability and adaptability in providing performance assessments. However, minor discrepancies between the fuzzy system and professor-assigned grades were noted, potentially due to subjective design choices in membership functions or variability in instructor grading styles. The results also point to potential future studies that could use benchmarks against grading models based on machine learning or utilize a larger dataset with student population characteristics. These types of future studies would increase the extent and accuracy of a computerized grading system. Such efforts could further enhance the generalizability and precision of automated grading systems. While the current study focused on academic evaluation, the underlying fuzzy logic approach is adaptable to various industries where decision-making involves uncertainty, ambiguity, or subjective judgment. For example, fuzzy systems have been successfully applied in healthcare (diagnosis support) [29], finance (credit risk scoring) [30], and human resources (employee performance appraisal) [31]. The ability of fuzzy logic to model imprecise human reasoning makes it a valuable tool beyond education. However, domain-specific customization of membership functions and rules would be necessary for effective implementation in those fields.

5. Limitations and Future Directions

While the fuzzy logic-based grading system has delivered encouraging results, it also opens avenues for further enhancement. Designed specifically for three mathematics courses, the current system demonstrates strong adaptability. Professors can easily modify question weights and fuzzy rules to suit different courses, making the framework highly customizable. However, to expand its applicability beyond mathematics, future developments could focus on building more intuitive interfaces that simplify this customization for a broader range of subjects.

Additionally, because the system reflects the grading patterns of individual instructors, it inherently captures elements of human subjectivity. Rather than being a drawback, this allows the system to mirror real-world academic evaluations. To improve consistency across different users, future work could explore the integration of calibration tools or shared rule repositories that align grading standards across institutions.

The study's use of a relatively small sample size 20 students per course provided a focused setting to demonstrate the system's potential. This foundation now invites larger-scale studies involving more diverse student populations, which would further validate and strengthen the model's reliability.

Finally, while the current system operates based on predefined rules, its performance could be enhanced by incorporating adaptive learning mechanisms. Future iterations may blend fuzzy logic with machine learning, allowing the system to evolve and improve continuously based on feedback and new data. These enhancements would further solidify the system's value as a reliable, scalable, and intelligent grading solution.

6. Conclusion

The fuzzy logic grading system demonstrates high precision and reliability in all three courses, with strong correlations (0.98) and values of R squares ranging from 0.94 to 0.99, confirming its effectiveness in replicating the professor's grading behavior. Low error metrics MAE (as low as 0.2883) and RMSE (as low as 0.3732) indicate minimal deviation between the fuzzy system and instructor grades, supporting the precision of the model in assessing different levels of student performance. Although slight biases were observed (MBE between -0.01 and +0.29), these remain within acceptable limits and do not significantly impact grade categories, maintaining fairness and consistency in evaluation. Visual comparisons and heatmap correlations further validate the fuzzy system's ability to emulate human grading trends, making it a promising tool for scalable, automated assessment in mathematical proof-based courses.

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Code Availability:
The Python implementation of the fuzzy logic system used in this study is available from the corresponding author upon reasonable request.

A. Appendix A

Pseudo code for input variables

IF q is greater or equal to 0 AND q is smaller or equal to 1:
SET q to 'fail'
ELSE IF q is greater than 1 AND q is smaller or equal to 5:
    SET q to 'poor'
ELSE IF q is greater than 5 AND q is smaller or equal to 7.5:
    SET q to 'average'
ELSE IF q is greater than 7.5 AND q is smaller or equal to 10:
    SET q to 'good'
ELSE IF q is greater than 10:
    SET q to 'excellent'
END FOR

Pseudo code for output variables:

For course 1

For each course 1 grade:
If the course grade is greater or equal to 0 AND the course grade is smaller or equal to 0.3:
    SET course grade to 'fail'
ELSE IF course grade is greater than 0.3 AND course grade is smaller or equal to 1.5:
    SET course grade to 'poor'
ELSE IF course grade is greater than 1.5 AND course grade is smaller or equal to 3.5:
    SET course grade to 'average'
ELSE IF course grade is greater than 3.5 AND course grade is smaller or equal to 5:
    SET course grade to 'good'
ELSE IF course grade is greater than 5 AND course grade is smaller or equal to 10:
    SET course grade to 'excellent'
END IF END FOR

For course 2

For each course 2 grade: If the course grade is greater or equal to 0 AND the course grade is smaller or equal to 1:

SET course grade to 'fail'
ELSE IF course grade is greater than 1 AND course grade is smaller or equal to 2.5:
SET course grade to 'poor'
ELSE IF course grade is greater than 2.5 AND course grade is smaller or equal to 4:
SET course grade to 'average'
ELSE IF course grade is greater than 4 AND course grade is smaller or equal to 6.5:
SET course grade to 'good'
ELSE IF course grade is greater than 6.5 AND course grade is smaller or equal to 10:
SET course grade to 'excellent'
END IF

For course 3
For each course 3 grade:
If the course grade is greater or equal to 0 AND the course grade is smaller or equal to 1.5:
SET course grade to 'fail'
ELSE IF course grade is greater than 1.5 AND course grade is smaller or equal to 3:
SET course grade to 'poor'
ELSE IF course grade is greater than 3 AND course grade is smaller or equal to 5.5:
SET course grade to 'average'
ELSE IF course grade is greater than 5.5 AND course grade is smaller or equal to 7.5:
SET course grade to 'good'
ELSE IF course grade is greater than 7.5 AND course grade is smaller or equal to 10:
SET course grade to 'excellent'
END IF
END FOR

Fuzzy rules:
IF the following rules apply
rule1: IF q1['fail'] AND q2['fail'] THEN grade['fail']
rule2: IF q1['poor'] AND q2['fail'] AND (q3['fail'] OR q4['fail']) THEN grade['fail']
rule3: IF q1['excellent'] AND q2['excellent'] AND q3['average'] AND q4['fail'] THEN grade['average']
rule4: IF q1['fail'] AND q2['poor'] AND q3['fail'] AND q4['fail'] THEN grade['fail']
rule5: IF q1['excellent'] AND q2['excellent'] AND q3['excellent'] AND q4['average'] THEN grade['excellent']
rule6: IF q1['average'] AND q2['average'] AND q3['fail'] AND q4['fail'] THEN grade['poor']
rule7: IF q1['good'] AND q2['excellent'] AND q3['excellent'] AND q4['good'] THEN grade['excellent']
rule8: IF (q1['excellent'] OR q2['excellent']) AND (q3['excellent'] OR q4['excellent']) THEN grade['excellent']
rule9: IF q1['excellent'] AND q2['average'] AND q3['excellent'] AND q4['average'] THEN grade['good']

rule10: IF q1['excellent'] AND q2['excellent'] AND q3['excellent'] AND q4['good'] THEN grade['excellent']
rule11: IF q1['excellent'] AND q2['poor'] AND q3['average'] AND q4['poor'] THEN grade['average']
rule12: IF q1['good'] AND q2['poor'] AND q3['average'] AND q4['average'] THEN grade['good']
rule13: IF q1['excellent'] AND q2['excellent'] AND q3['good'] AND q4['poor'] THEN grade['good']
rule14: IF q1['excellent'] AND q2['excellent'] AND q3['excellent'] AND q4['good'] THEN grade['excellent']
rule15: IF q1['excellent'] AND q2['average'] AND q3['average'] AND q4['average'] THEN grade['excellent']
rule16: IF q1['excellent'] AND q2['excellent'] AND q3['excellent'] THEN grade['excellent']
rule17: IF q1['excellent'] AND q2['excellent'] AND q3['excellent'] THEN grade['excellent']
rule18: IF q1['average'] AND q2['average'] AND q3['average'] AND q4['average'] THEN grade['average']
rule19: IF q1['poor'] AND q2['poor'] AND q3['poor'] AND q4['poor'] THEN grade['poor']
rule20: IF q1['excellent'] AND q2['average'] THEN grade['average']
rule21: IF q1['excellent'] AND q2['excellent'] AND q3['good'] AND q4['good'] THEN grade['excellent']
rule22: IF q1['excellent'] AND q2['fail'] AND q3['fail'] AND q4['fail'] THEN grade['fail']
rule23: IF q1['good'] AND q2['good'] AND q3['average'] AND q4['average'] THEN grade['good']
rule24: IF q1['average'] AND q2['average'] THEN grade['fail']
rule25: IF q1['poor'] AND q2['poor'] THEN grade['poor']
rule26: IF q1['fail'] AND q2['fail'] THEN grade['fail']
rule27: IF q1['average'] AND q2['fail'] AND q3['fail'] AND q4['average'] THEN grade['fail']
rule28: IF q1['average'] AND q2['poor'] AND q3['poor'] AND q4['poor'] THEN grade['poor']
rule29: IF q1['excellent'] AND q2['excellent'] AND q3['excellent'] AND q4['poor'] THEN grade['good']
rule30: IF q1['fail'] AND q2['average'] AND q3['average'] AND q4['poor'] THEN grade['poor']
rule31: IF q1['excellent'] AND q2['poor'] AND q3['poor'] AND q4['poor'] THEN grade['poor']
rule32: IF q1['excellent'] AND q2['average'] AND q3['fail'] AND q4['poor'] THEN grade['poor']
rule33: IF (q1['excellent'] OR q2['excellent']) AND (q3['excellent'] OR q4['excellent']) THEN grade['excellent']
rule34: IF q1['excellent'] AND q2['fail'] AND q3['excellent'] AND q4['fail'] THEN grade['poor']
rule35: IF q1['average'] AND q2['fail'] AND q3['fail'] AND q4['fail'] THEN grade['fail']
rule37: IF q1['excellent'] AND q2['excellent'] AND q3['average'] AND q4['average'] THEN grade['average']
rule38: IF q1['excellent'] AND q2['average'] THEN grade['average']
rule39: IF q1['excellent'] AND q2['excellent'] AND q3['excellent'] AND q4['average'] THEN grade['good']
rule40: IF q1['excellent'] AND q2['excellent'] THEN grade['excellent']
rule41: IF q1['excellent'] AND q2['excellent'] AND q3['excellent'] AND q4['average'] THEN grade['excellent']
rule42: IF q1['average'] AND q2['average'] AND q3['average'] AND q4['excellent'] THEN

grade['average']
rule43: IF q1['fail'] AND q2['fail'] THEN grade['fail']
rule44: IF q1['fail'] AND q2['poor'] AND q3['fail'] AND q4['fail'] THEN grade['fail']
rule45: IF q1['fail'] AND q2['fail'] AND q3['fail'] AND q4['fail'] THEN grade['fail']
rule46: IF q1['average'] AND q2['poor'] AND q3['fail'] AND q4['fail'] THEN grade['poor']
rule47: IF q1['poor'] AND q2['poor'] AND q3['poor'] AND q4['poor'] THEN grade['poor']
rule48: IF q1['poor'] AND q2['fail'] AND q3['fail'] AND q4['fail'] THEN grade['fail']
rule49: IF q1['excellent'] AND q2['fail'] AND q3['fail'] AND q4['fail'] THEN grade['fail']
rule50: IF q1['excellent'] AND q2['excellent'] AND q3['excellent'] AND q4['good'] THEN grade['excellent']
rule51: IF q1['poor'] AND q2['fail'] AND q3['poor'] AND q4['fail'] THEN grade['fail']
rule52: IF q1['poor'] AND q2['poor'] AND q3['fail'] AND q4['fail'] THEN grade['fail']
rule53: IF q1['excellent'] AND q2['excellent'] AND q3['excellent'] AND q4['excellent'] THEN grade['excellent']
rule54: IF q1['average'] AND q2['fail'] AND q3['poor'] AND q4['fail'] THEN grade['fail']
rule55: IF q1['excellent'] AND q2['fail'] AND q3['average'] AND q4['fail'] THEN grade['poor']
rule56: IF q1['excellent'] AND q2['average'] AND q3['excellent'] AND q4['fail'] THEN grade['average']
rule57: IF q1['average'] AND q2['fail'] AND q3['fail'] AND q4['fail'] THEN grade['fail']
rule58: IF q1['excellent'] AND q2['poor'] AND q3['poor'] AND q4['poor'] THEN grade['average']
rule59: IF q1['excellent'] AND q2['excellent'] AND q3['excellent'] AND q4['average'] THEN grade['excellent']