EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
2025, Vol. 18, Issue 3, Article Number 6324
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

Separation Axioms in Quadri-Partition Neutrosophic Soft Topological Spaces

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Abstract. This study introduces and explores the concepts of separation axioms within the framework of quadri-partitioned neutrosophic soft topological space (QPNSTS). These spaces are an extension of neutrosophic soft topological structures, designed to handle higher levels of uncertainty through quadri-partitioned neutrosophic soft sets (QPNSs). We presented new definitions and properties of $T_i - spaces (i = 0, 1, 2, 3, 4)$ in this extended context, offering detailed criteria that distinguish these separation conditions. Through a series of illustrative examples, we demonstrate how these conditions behave and interact within the structure of QPNSTS. The relationships among these separation axioms, as well as their connections to other results, are also discussed.

2020 Mathematics Subject Classifications: 54A05
Key Words and Phrases: Neutrosophic soft set, quadri-partitioned neutrosophic soft set, quadri-partitioned neutrosophic soft topological space, quadri-partitioned neutrosophic soft separation axioms.

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DOI: https://doi.org/10.29020/nybg.ejpam.v18i3.6324

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1. Introduction

Fuzzy set theory (FST) is the extension of classical crisp set theory (CST) into a multivariate form. Fuzzy approaches have several advantages over the crisp ones: the main one being that they have more flexible decision frontiers, and thus are characterized by their higher ability to adjust a specific domain of application. The classical CST has sharp frontiers. In classical CST, the numerical value 1 is assigned to the truth value (T) and 0 is assigned to be the false value (F). With the passage of time, the notion of classical CST was modified with the exhibition of fuzzy logic (FL) by Zadeh [1]. In case of FL, there is no limit over T and F as well. It could suppose exactly some fixed values over [0,1]. Pawlak [2] introduced rough set theory (RST). After that Atanassov [3] exhibited the notion of intuitionistic fuzzy set theory (IFST). This theory is actually the rectification of FST. In this theory, the non-membership function was developed to address the short-coming that exists in FST. The author presented all the fundamental operations of this theory and to make it more attractive and suitable, examples were presented. Soft set theory (SST) is supposed to be the starting point of advanced mathematics. This theory is used to reduce the error that exists in the above discussed theories. In this direction, first forwarded mathematician was Molodtsov [4]. Maji et al. [5] studied the theory of soft sets initiated by Molodtsov. The authors defined, some operations, which are equality of two soft sets, subsets and super set of a soft set, complements of a soft set, null soft set, and absolute soft set with examples. Soft binary operations like AND, OR and the operations of union, intersection and De Morgen s laws are defined. Finally, number of results is verified in soft set theory. Yang, deeply studied [5] and pointed out that the assertion $(F, A) \cup \phi = \phi$ needed some sort of attention and finally showed that this assertion is absolutely wrong by posing counterexample [6]. Maji [7] introduced hybrid set which is known as neutrosophic soft set and defined some definitions and operations. Few characteristics of this hybrid are exposed. Deli and Broumi [8] defined relation on neutrosophic soft sets, which show how two neutrosophic soft sets are to be composed. The authors discussed symmetric, transitive and reflexive relations in sense of neutrosophic soft sets. The concept of equivalency, partition, equivalence classes and quotient is installed in terms of neutrosophic soft sets. Irfan et al. pointed out some results in [5] that are not true in general. This issue was addressed by Irfan et al. in [9] by constructing suitable counter example. Shao and Qin [10] defined fuzzy soft lattice and some related properties are derived, which extends the notion of fuzzy lattice to catch the algebraic structures of soft sets. The concept of fuzzy soft ideal over a lattice is presented and the lattice structures of fuzzy soft ideal over a lattice are discussed. Abbas et al. [11] relaxed conditions on parameters which lead them to propose some new concepts that consequently generalize existing comparable notions. The authors introduced the concepts of generalized finite soft equality (gf-soft equality), generalized finite soft union (gf-soft union) and generalized finite soft intersection (gf-soft intersection) of two soft sets. Al-Shami et al. [12] initiated the concept of almost soft compact, approximately soft Lindelof and mildly soft compact spaces. The sufficient conditions for the equivalent among these spaces are addressed. Some results which associate soft hyper connectedness and soft connectedness, respectively with almost soft compactness and mildly soft compactness are verified. Mehmood et al. [13] introduced new operations for union, intersection, and complement using vague soft sets in a novel approach that incorporates both true and false statements. The union is defined as the maximum, and the intersection is defined as the minimum. Based on these operations, vague soft topology is established. Pairwise vague soft open sets and pairwise vague soft closed sets are defined within vague soft bi-topological structures (VSBTS). Additionally, generalized vague soft open sets are introduced in VSBTS in relation to the soft points of the space. Based on these generalized vague soft open sets and separation axioms are defined. Furthermore, these separation axioms are applied to other significant results within VSBTS.

1.1. Literature review

Smarandache [14] exhibited the conception of neutrosophic set (NS). The author added the indeterminacy-membership function. Neutrosophic set is categorized by 3-components that is truth-membership (TM), indeterminacy-membership (IM) and false-membership (FM) functions. Moreover, the neutrosophic set is straight forward generalization of IFST. Ozturk et al. [15] introduced new operations on neutrosophic soft sets and these operations are union, intersection and complements. On the basis of these operations the authors defined neutrosophic soft topology and discussed its basic results. These results were verified through examples. Jaikumar et al. [16] introduced the concept of equitable integrity and strong equitable integrity in single valued neutrosophic graphs. Bera and Mahapatra [17]introduced a strong hybrid structure in form of neutrosophic soft topological space. The notion of interior, closure, neighborhood, boundary and regularity are also introduced in terms of neutrosophic soft set. The concept of base for neutrosophic soft topology and subspace topology on neutrosophic soft set is defined with suitable examples. Moreover, the concept of separation axioms on neutrosophic soft topological space is introduced along with investigation of several characteristics. Jun et al. [18] extended the concept of cubic sets to the neutrosophic sets. The notions of truth-internal (indeterminacy-internal, falsity-internal) neutrosophic cubic sets and truth external (indeterminacy-external, falsity-external) neutrosophic cubic sets are introduced, and related properties are investigated. Al-Omeri and Smarandache [19] linked NCS to topological spaces and exposed a space which is known as neutrosophic topological space. Zhang [20]used bipolar fuzzy set (BFS) in decision making problems. Yaqoob and Ansari [21] introduced the concept of bi-polar $(\lambda, \delta) - fuzification$ of a ternary semi-group and discuss some structural properties of bipolar $(\lambda, \delta) - fuzzy\ ideals$ of a ternary semi-group. Yaqoob et al. [22] applied [9] to many branches of mathematics. The authors initiated a bipolar fuzzy sets in $\Gamma - semi - hyper\ groups$ and some more results. Yaqoob et al. [23] introduced new type of bi-polar fuzzy sets in $\Gamma - semi - hyper\ groups$. Hashim et al. [24]used neutrosophic bipolar fuzzy in many fields of sciences. The authors used this theory in group decision making mode base on hybrid making problems with exact values, interval values and linguistic variables. Eventually, the authors applied these concepts and techniques upon hybrid multi-attributes decision making problem of selecting the best medicine to cure some particular diseases and develop an algorithm

for neutrosophic bi-polar fuzzy hybrid multi-attribute group decision making. Mehmood et al.[25] discussed a new concept of vague soft bi-topological space and its structural behaviors. This approach is based on generalized vague soft open sets, known as vague soft $\beta - open\ sets$. An examples are given to understand the structures. Pair-wise vague soft $\beta - open$ and pair-wise vague soft $\beta - close\ sets$ are also addressed with examples vague soft bi-topological space. Vague soft separation axioms are initiated in (VSBTS) concerning soft point of the space. Other separation axioms are also addressed relative to soft points of the space. Mehmood et al.[13] focused on two crucial techniques that are used for measurements. These technique are introduced in soft sets concerning crisp points of the space. Relevant examples support these strategies. Khattak et al. [26] bounced up the idea of neutrosophic soft b-open sets, neutrosophic soft b-closed sets and their properties. Also the idea of neutrosophic soft b-neighborhood and neutrosophic soft b-separation axioms in neutrosophic soft topological structures are also reflected here.
Gulistan et al. [27] introduced the concept of complex neutrosophic and defined the notion of alpha-cut of complex neutrosophic set. The authors also define the cartesian product of complex neutrosophic subgroups. Furthermore, introduced the concept of image and preimage of complex neutrosophic set and prove some of its properties. Gulistan et al. [28]aimed at discussing a new concept called N-neutrosophic cubic sets aimed at assessing the negative aspect of the internet. The theoretical foundation for soft separation axioms was laid through the study of supra soft sets and their topological properties, including continuity and compactness [29–31]. These early developments provide a springboard for analyzing more complex uncertainty models. Recent advancements involving neutrosophic structures such as (ζ_1, ζ_2)-neutrosophic ideals [32], two-fold fuzzy neutrosophic rings [33], and fuzzy algebras over neutrosophic reals [34] highlight the growing sophistication of algebraic and topological frameworks under multi-valued uncertainty. These insights, when extended to Quadri-Partition Neutrosophic Soft Topological Spaces, enable the formulation and refinement of separation axioms suitable for multi-perspective decision-making and AI-augmented systems [35, 36]. The notion of N-cubic indeterminacy function and N-cubic falsity function has been added with the N-cubic set to obtain a novel concept of N-neutrosophic cubic sets. The novelty of the N-neutrosophic cubic set may be associated with the fact that there is a large range of values to discuss uncertainty and vagueness as compared to the N-cubic set. The basic operations of the N-neutrosophic cubic sets along with some interesting properties are discussed. Analysis has also been focused on examining the negative rating function and aggregating operators of N-neutrosophic cubic sets. Finally, as an application, a numerical example is given to test the applicability of the proposed model.

1.2. Research Gap

Despite the significant contributions of Saeed et al. [37] in advancing the theory of quadri-partitioned neutrosophic soft sets (QPNSS) and quadri-partitioned neutrosophic soft topological spaces (QPNSTS), an important aspect remains underexplored: the investigation of separation axioms within the context of QPNSTS. While the existing work provides a solid foundation for understanding the properties and operations of QPNS spaces, it does not address the explicit development or exploration of separation axioms in this extended framework. Separation axioms are essential in classical topology and its generalizations, as they help define the structure of topological spaces by distinguishing between points and sets. However, the extension of these axioms to QPNSTS, which incorporate higher levels of uncertainty through quadri-partitioned membership attributes, has not been adequately studied or formalized. This gap in the research motivates the present study, which aims to define and explore the various separation axioms within the context of QPNSTS. By investigating these axioms, we seek to enhance the theoretical framework of QPNSS and soft topological spaces, thus extending and enriching the work of Saeed et al. [37] and contributing to the broader understanding of neutrosophic soft topologies.

1.3. Motivation

The following work served as the primary source of motivation for the present study. Saeed et al. [37] introduced an innovative approach to handling indeterminacy by dividing it into two distinct components based on membership: relative truth (RT), which leans toward truth, and relative falsehood (RF), which leans toward falsehood. This refinement enhances the interpretability and accuracy of neutrosophic models by considering both relative truth and relative falsehood, instead of a single indeterminate value. Building on this idea, the authors developed a modified neutrosophic structure known as the quadri-partitioned neutrosophic soft set (QPNSS). This framework incorporates four membership attributes: absolute truth, relative truth, relative falsehood, and absolute falsehood, thereby offering a more comprehensive representation of uncertainty in soft set theory. Several new operations were introduced on QPNSS, including subsets, complement, absolute set, set difference, null set, and logical operations like AND and OR. Furthermore, the concept was extended to define a quadri-partitioned neutrosophic soft topological space (QPNSTS), within which fundamental results were presented. The study also explored key topological concepts such as pre-open (p-open) sets, interior, and closure, as well as advanced properties including QPNS compactness, the reducibility to finite sub-covers, and the behavior of constructs like the intersection of QPNS p-closed sets and QPNS p-compact spaces. This foundational work significantly contributes to the theoretical development of QPNS structures, particularly within the domain of soft topological spaces, and has directly inspired the direction and objectives of the present research.

1.4. Organization of the paper

This work is organized into the following sections. Section 2: This section establishes the core theoretical framework of neutrosophic soft sets (NSS). It rigorously formalizes the definitions of fundamental set-theoretic operations complement, subset, union, intersection, and difference within the context of NSS. Additionally, it delineates special constructs such as the null neutrosophic soft set and the absolute neutrosophic soft set, providing a comprehensive basis for further analytical development. Section 3: In this section, the groundwork is laid for quadri-partitioned neutrosophic soft sets by introducing their core definitions and fundamental properties. It covers essential operations such as union, intersection, complement, subset, and equality. These operations form the basis for our next sections. Section 4: This section introduces and explores separation axioms within the framework of quadri-partitioned neutrosophic soft topological spaces. We define $T_i, (i = 0, 1, 2, 3, 4)$ separation properties for such spaces using quadri-partitioned neutrosophic soft points and open sets. Examples illustrate spaces that satisfy or fail each axiom. Key theorems established relationships between separation axioms and properties like closeness and regularity. Section 5: This section discusses comparative analysis. Section 6: this section introduces the conclusion and future work.

2. Preliminaries

This section establishes the core theoretical framework of neutrosophic soft sets (NSS). It rigorously formalizes the definitions of fundamental set-theoretic operations complement, subset, union, intersection, and difference within the context of NSS. Additionally, it delineates special constructs such as the null neutrosophic soft set and the absolute neutrosophic soft set, providing a comprehensive basis for further analytical development.

Definition 1. *[8] Let $\acute{E}$ be a set of parameters and X be an initial universe set. $P(X)$ represents the collection of all NSSs for x. A set defined by a set-valued function $\tilde{F}$ expressing a mapping $\tilde{F} : \acute{E} \to P(X)$ is then a NSS $(\tilde{F}, \acute{E})$ over X, where $\tilde{F}$ is referred to as the approximate function of the NSS $(\tilde{F}, \acute{E})$. Stated differently, the NSS can be expressed as a collection of ordered pairs:*

$$(\tilde{F}, \acute{E}) = \left\{ \left(\grave{e}, \langle x, T_{\tilde{F}(\grave{e})}(x), I_{\tilde{F}(\grave{e})}(x), F_{\tilde{F}(\grave{e})}(x) \rangle : x \in X \right) : \grave{e} \in \acute{E} \right\}.$$

Here, $T_{\tilde{F}(\grave{e})}(x)$, $I_{\tilde{F}(\grave{e})}(x)$, and $F_{\tilde{F}(\grave{e})}(x)$ are the truth-membership, indeterminacy-membership, and falsity-membership functions of $\tilde{F}(\grave{e})$, respectively, and they all lie within the interval $[0, 1]$. The inequality

$$0 \leq T_{\tilde{F}(\grave{e})}(x) + I_{\tilde{F}(\grave{e})}(x) + F_{\tilde{F}(\grave{e})}(x) \leq 3$$

is evident as the supremum of each T, I, and F is 1 and the infimum is 0. This means that each value is a typical value between 0 and 1.

Definition 2. *[17] Let $(\tilde{F}, \acute{E})$ be a NSS. Then $(\tilde{F}, \acute{E})^c$ is the complement of $(\tilde{F}, \acute{E})$:*

$$(\tilde{F}, \acute{E})^c = \left\{ \left(\grave{e}, \langle x, F_{\tilde{F}(\grave{e})}(x), 1 - I_{\tilde{F}(\grave{e})}(x), T_{\tilde{F}(\grave{e})}(x) \rangle : x \in X \right) : \grave{e} \in \acute{E} \right\}.$$

It is obvious that:

$$\left((\tilde{F}, \acute{E})^c \right)^c = (\tilde{F}, \acute{E}).$$

Definition 3. *[7] Let $(\tilde{F}, \acute{E})$ and $(\tilde{G}, \acute{E})$ be two NSSs. Then $(\tilde{F}, \acute{E})$ is said to be a neutrosophic soft subset of $(\tilde{G}, \acute{E})$ if:*

$$T_{\tilde{F}(\grave{e})}(x) \leq T_{\tilde{G}(\grave{e})}(x), \quad I_{\tilde{F}(\grave{e})}(x) \leq I_{\tilde{G}}(\grave{e})(x), \quad F_{\tilde{F}(\grave{e})}(x) \geq F_{\tilde{G}(\grave{e})}(x), \quad \forall \grave{e} \in \acute{E}, \forall x \in X.$$

It is denoted by:

$$(\tilde{F}, \acute{E}) \subseteq (\tilde{G}, \acute{E}).$$

Definition 4. *[15] Let $(\tilde{F}_1, \acute{E})$ and $(\tilde{F}_2, \acute{E})$ be two NSSs. Then their union is represented by $(\tilde{F}_1, \acute{E}) \Cup (\tilde{F}_2, \acute{E}) = (\tilde{F}_3, \acute{E})$ as:*

$$(\tilde{F}_3, \acute{E}) = \left\{ \left(\grave{e}, \langle x, T_{\tilde{F}_{3(\grave{e})}}(x), I_{\tilde{F}_{3(\grave{e})}}(x), F_{\tilde{F}_{3(\grave{e})}}(x) \rangle : x \in X \right) : \grave{e} \in \acute{E} \right\}.$$

Where:

$$\begin{aligned} T_{\tilde{F}_{3(\grave{e})}}(x) &= \max\{T_{\tilde{F}_{1(\grave{e})}}(x), T_{\tilde{F}_{2(\grave{e})}}(x)\}, \\ I_{\tilde{F}_{3(\grave{e})}}(x) &= \max\{I_{\tilde{F}_{1(\grave{e})}}(x), I_{\tilde{F}_{2(\grave{e})}}(x)\}, \\ F_{\tilde{F}_{3(\grave{e})}}(x) &= \min\{F_{\tilde{F}_{1(\grave{e})}}(x), F_{\tilde{F}_{2(\grave{e})}}(x)\}. \end{aligned}$$

Definition 5. *[15] Let $(\tilde{F}_1, \acute{E})$ and $(\tilde{F}_2, \acute{E})$ be two NSSs. Then their intersection is symbolized by $(\tilde{F}_1, \acute{E}) \Cap (\tilde{F}_2, \acute{E}) = (\tilde{F}_3, \acute{E})$ as:*

$$(\tilde{F}_3, \acute{E}) = \left\{ \left(\grave{e}, \langle x, T_{\tilde{F}_{3(\grave{e})}}(x), I_{\tilde{F}_{3(\grave{e})}}(x), F_{\tilde{F}_{3(\grave{e})}}(x) \rangle : x \in X \right) : \grave{e} \in \acute{E} \right\}.$$

Where:

$$\begin{aligned} T_{\tilde{F}_{3(\grave{e})}}(x) &= \min\{T_{\tilde{F}_{1(\grave{e})}}(x), T_{\tilde{F}_{2(\grave{e})}}(x)\}, \\ I_{\tilde{F}_{3(\grave{e})}}(x) &= \min\{I_{\tilde{F}_{1(\grave{e})}}(x), I_{\tilde{F}_{2(\grave{e})}}(x)\}, \\ F_{\tilde{F}_{3(\grave{e})}}(x) &= \max\{F_{\tilde{F}_{1(\grave{e})}}(x), F_{\tilde{F}_{2(\grave{e})}}(x)\}. \end{aligned}$$

Definition 6. *[15] Let $(\tilde{F}_1, \acute{E})$ and $(\tilde{F}_2, \acute{E})$ be two NSSs. Then the difference operation on them is denoted by $(\tilde{F}_1, \acute{E}) \setminus (\tilde{F}_2, \acute{E}) = (\tilde{F}_3, \acute{E})$ and is defined by:*

$$(\tilde{F}_3, \acute{E}) = \left\{ \left(\grave{e}, \langle x, T_{\tilde{F}_{3(\grave{e})}}(x), I_{\tilde{F}_{3(\grave{e})}}(x), F_{\tilde{F}_{3(\grave{e})}}(x) \rangle : x \in X \right) : \grave{e} \in \acute{E} \right\}.$$

Where:

$$\begin{aligned} T_{\tilde{F}_{3(\grave{e})}}(x) &= \min\{T_{\tilde{F}_{1(\grave{e})}}(x), T_{\tilde{F}_{2(\grave{e})}}(x)\}, \\ I_{\tilde{F}_{3(\grave{e})}}(x) &= \min\{I_{\tilde{F}_{1(\grave{e})}}(x), I_{\tilde{F}_{2(\acute{e})}}(x)\}, \\ F_{\tilde{F}_{3(\grave{e})}}(x) &= \max\{F_{\tilde{F}_{1(\grave{e})}}(x), F_{\tilde{F}_{2(\grave{e})}}(x)\}. \end{aligned}$$

Definition 7. *[15] 1. A NSS $(\tilde{F}, \acute{E})$ is said to be a null NSS if:*

$$T_{\tilde{F}(\grave{e})}(x) = 0, \quad I_{\tilde{F}(\grave{e})}(x) = 0, \quad F_{\tilde{F}(\grave{e})}(x) = 1, \quad \forall \grave{e} \in \acute{E}, \forall x \in X.$$

It is denoted by $0_{(X,\acute{E})}$.

2. A neutrosophic soft set $(\tilde{F}, \acute{E})$ is said to be an absolute NSS if:

$$T_{\tilde{F}(\grave{e})}(x) = 1, \quad I_{\tilde{F}(\grave{e})}(x) = 1, \quad F_{\tilde{F}(\grave{e})}(x) = 0, \quad \forall \grave{e} \in \acute{E}, \forall x \in X.$$

It is symbolized as $1_{(X,\acute{E})}$.

It is obvious that:

$$0_{(X,\acute{E})} = 1_{(X,\acute{E})}, \quad 1_{(X,\acute{E})} = 0_{(X,\acute{E})}.$$

3. Operations on Quadri-Partitioned Neutrosophic Soft Sets

In this section, the groundwork is laid for quadri-partitioned neutrosophic soft sets (QPNSSs) by introducing their core definitions and fundamental properties. It covers essential operations such as union, intersection, complement, subset, and equality. These operations form the basis for our next sections.

Definition 8. *Let $\acute{E}$ be the set of parameters and X be the key set. Let $P(X)$ represent the power set of X. Then, a QPNSS $(\tilde{F}, \acute{E})$ over X is a mapping $\tilde{F} : \acute{E} \to P(X)$, where $\tilde{F}$ is the function of the QPNSS $(\tilde{F}, \acute{E})$. Symbolically,*

$$(\tilde{F}, \acute{E}) = \left\{ \Big(\grave{e}, \langle x, AbT_{\tilde{F}(\grave{e})}(x), ReT_{\tilde{F}(\grave{e})}(x), ReF_{\tilde{F}(\grave{e})}(x), AbF_{\tilde{F}(\grave{e})}(x) \rangle : x \in X \Big) : \grave{e} \in \acute{E} \right\}.$$

Where, $AbT_{\tilde{F}(\grave{e})}(s)$, $ReT_{\tilde{F}(\grave{e})}(s)$, $ReF_{\tilde{F}(\grave{e})}(s)$, and $AbF_{\tilde{F}(\grave{e})}(s)$ belong to the interval $[0, 1]$. Respectively, these functions are called the absolute true-membership, relative true-membership, relative false-membership, and absolute false-membership functions of $\tilde{F}(\grave{e})$. Since the supremum of each function is 1 and the infimum is 0, the inequality

$$0 \leq AbT_{\tilde{F}(e)}(s) + ReT_{\tilde{F}(e)}(s) + ReF_{\tilde{F}(e)}(s) + AbF_{\tilde{F}(e)}(s) \leq 4.$$

is automatically true.

Definition 9. *Let $(\tilde{F}, \acute{E})$ be a QPNSS over the key set X. Then, the complement of $(\tilde{F}, \acute{E})$ is denoted by $(\tilde{F}, \acute{E})^c$ and is defined as follows:*

$$(\tilde{F}, \acute{E})^c = \left\{ \Big(\grave{e}, \langle x, AbF_{\tilde{F}(\grave{e})}(x), ReF_{\tilde{F}(\grave{e})}(x), ReT_{\tilde{F}(\grave{e})}(x), AbT_{\tilde{F}(\grave{e})}(x) \rangle : x \in X \Big) : \grave{e} \in \acute{E} \right\}.$$

So, $\Big((\tilde{F}, \acute{E})^c \Big)^c = (\tilde{F}, \acute{E})$.

Definition 10. *Let $(\tilde{F}, \acute{E})$ and $(\tilde{G}, \acute{E})$ be two QPNSSs over the key set X. Then, $(\tilde{F}, \acute{E}) \subseteq (\tilde{G}, \acute{E})$ if*

$$AbT_{\tilde{F}(\grave{e})}(x) \preceq AbT_{\tilde{G}(\grave{e})}(x),$$

$$\begin{aligned}ReT_{\tilde{F}(\grave{e})}(x) &\preceq ReT_{\tilde{G}(\grave{e})}(x),\\ ReF_{\tilde{F}(\grave{e})}(x) &\succeq ReF_{\tilde{G}(\grave{e})}(x),\\ AbF_{\tilde{F}(\grave{e})}(x) &\succeq AbF_{\tilde{G}(\grave{e})}(x),\end{aligned}$$

for all $\grave{e} \in \acute{E}$ *and* $x \in X$. *If* $(\tilde{F}, \acute{E}) \subseteq (\tilde{G}, \acute{E})$ *and* $(\tilde{F}, \acute{E}) \supseteq (\tilde{G}, \acute{E})$, *then* $(\tilde{F}, \acute{E}) = (\tilde{G}, \acute{E})$.

Definition 11. *Let* $(\tilde{F}, \acute{E})$ *and* $(\tilde{G}, \acute{E})$ *be two QPNSSs over the key set* X *such that* $(\tilde{F}, \acute{E}) \neq (\tilde{G}, \acute{E})$. *Then their union is denoted by* $(\tilde{F}, \acute{E}) \Cup (\tilde{G}, \acute{E}) = (\tilde{H}, \acute{E})$ *and is defined as:*

$$(\tilde{H}, \acute{E}) = \left\{ \left(\grave{e}, \langle x, AbT_{\tilde{H}(\grave{e})}(x), ReT_{\tilde{H}(\grave{e})}(x), ReF_{\tilde{H}(\grave{e})}(x), AbF_{\tilde{H}(\grave{e})}(x)\rangle : x \in X\right) : \grave{e} \in \acute{E} \right\}.$$

where

$$\begin{aligned}AbT_{\tilde{H}(\grave{e})}(x) &= \max\left\{AbT_{\tilde{F}(\grave{e})}(x), AbT_{\tilde{G}(\grave{e})}(x)\right\},\\ ReT_{\tilde{H}(\grave{e})}(x) &= \max\left\{ReT_{\tilde{F}(\grave{e})}(x), ReT_{\tilde{G}(\grave{e})}(x)\right\},\\ ReF_{\tilde{H}(\grave{e})}(x) &= \min\left\{ReF_{\tilde{F}(\grave{e})}(x), ReF_{\tilde{G}(\grave{e})}(x)\right\},\\ AbF_{\tilde{H}(\grave{e})}(x) &= \min\left\{AbF_{\tilde{F}(\grave{e})}(x), AbF_{\tilde{G}(\grave{e})}(x)\right\}.\end{aligned}$$

Definition 12. *Let* $(\tilde{F}, \acute{E})$ *and* $(\tilde{G}, \acute{E})$ *be two QPNSSs over the key set* X *such that* $(\tilde{F}, \acute{E}) \neq (\tilde{G}, \acute{E})$. *Then their intersection is denoted by* $(\tilde{F}, \acute{E}) \Cap (\tilde{G}, \acute{E}) = (\tilde{H}, \acute{E})$ *and is defined as:*

$$(\tilde{H}, \acute{E}) = \left\{ \left(\grave{e}, \langle x, AbT_{\tilde{H}(\grave{e})}(x), ReT_{\tilde{H}(\grave{e})}(x), ReF_{\tilde{H}(\grave{e})}(x), AbF_{\tilde{H}(\grave{e})}(x)\rangle : x \in X\right) : \grave{e} \in \acute{E} \right\}.$$

where

$$\begin{aligned}AbT_{\tilde{H}(\grave{e})}(x) &= \min\left\{AbT_{\tilde{F}(\grave{e})}(x), AbT_{\tilde{G}(\grave{e})}(x)\right\},\\ ReT_{\tilde{H}(\grave{e})}(x) &= \min\left\{ReT_{\tilde{F}(\grave{e})}(x), ReT_{\tilde{G}(\grave{e})}(x)\right\},\\ ReF_{\tilde{H}(\grave{e})}(x) &= \max\left\{ReF_{\tilde{F}(\grave{e})}(x), ReF_{\tilde{G}(\grave{e})}(x)\right\},\\ AbF_{\tilde{H}(\grave{e})}(x) &= \max\left\{AbF_{\tilde{F}(\grave{e})}(x), AbF_{\tilde{G}(\grave{e})}(x)\right\}.\end{aligned}$$

Definition 13. *Let* $(\tilde{F}, \acute{E})$ *and* $(\tilde{G}, \acute{E})$ *be two QPNSSs over the key set* X *such that* $(\tilde{F}, \acute{E}) \neq (\tilde{G}, \acute{E})$. *Then, their difference is denoted by* $(\tilde{H}, \acute{E}) = (\tilde{F}, \acute{E}) \setminus (\tilde{G}, \acute{E})$ *and is defined as:*

$$(\tilde{H}, \acute{E}) = (\tilde{F}, \acute{E}) \Cap (\tilde{G}, \acute{E})^c,$$

such that

$$(\tilde{H}, \acute{E}) = \left\{ \left(\grave{e}, \langle x, AbT_{\tilde{H}(\grave{e})}(x), ReT_{\tilde{H}(\grave{e})}(x), ReF_{\tilde{H}(\grave{e})}(x), AbF_{\tilde{H}(\grave{e})}(x)\rangle\right) : x \in X, \grave{e} \in \acute{E} \right\}.$$

where

$$AbT_{\tilde{H}(\grave{e})}(x) = \min\left\{AbT_{\tilde{F}(\grave{e})}(x), AbT_{\tilde{G}(\grave{e})}(x)\right\},$$
$$ReT_{\tilde{H}(\grave{e})}(x) = \min\left\{ReT_{\tilde{F}(\grave{e})}(x), ReT_{\tilde{G}(\grave{e})}(x)\right\},$$
$$ReF_{\tilde{H}(\grave{e})}(x) = \max\left\{ReF_{\tilde{F}(\grave{e})}(x), ReF_{\tilde{G}(\grave{e})}(x)\right\},$$
$$AbF_{\tilde{H}(\grave{e})}(x) = \max\left\{AbF_{\tilde{F}(\grave{e})}(x), AbF_{\tilde{G}(\grave{e})}(x)\right\}.$$

Definition 14. *Let* $\{(\tilde{F}_i, \acute{E}) : i \in I\}$ *be a family of QPNSSs over the key set* X*. Then,*

$$\bigcup_{i\in I}(\tilde{F}_i, \acute{E}) = \left\{\left(\grave{e}, \langle x, \sup_{i\in I} AbT_{\tilde{F}_{i(\grave{e})}}(x), \sup_{i\in I} ReT_{\tilde{F}_{i(\grave{e})}}(x), \inf_{i\in I} ReF_{\tilde{F}_{i(\grave{e})}}(x), \inf_{i\in I} AbF_{\tilde{F}_{i(\grave{e})}}(x)\rangle\right) : x \in X, \grave{e} \in \acute{E}\right\}.$$

$$\bigcap_{i\in I}(\tilde{F}_i, \acute{E}) = \left\{\left(\grave{e}, \langle x, \inf_{i\in I} AbT_{\tilde{F}_{i(\grave{e})}}(x), \inf_{i\in I} ReT_{\tilde{F}_{i(\grave{e})}}(x), \sup_{i\in I} ReF_{\tilde{F}_{i(\grave{e})}}(x), \sup_{i\in I} AbF_{\tilde{F}_{i(\grave{e})}}(x)\rangle\right) : x \in X, \grave{e} \in \acute{E}\right\}.$$

Definition 15. *Let* $(\tilde{F}, \acute{E})$ *and* $(\tilde{G}, \acute{E})$ *be two QPNSSs over the key set* X*. Then, the "AND" operation on them is denoted by* $(\tilde{F}, \acute{E}) \wedge (\tilde{G}, \acute{E}) = (\tilde{H}, \acute{E} \times \acute{E})$ *and is defined as:*

$$(\tilde{H}, \acute{E} \times \acute{E}) = \Big\{\Big((\grave{e}_1, \grave{e}_2), \langle x, AbT_{\tilde{H}}(\grave{e}_1, \grave{e}_2)(x), ReT_{\tilde{H}}(\grave{e}_1, \grave{e}_2)(x),$$
$$ReF_{\tilde{H}}(\grave{e}_1, \grave{e}_2)(x), AbF_{\tilde{H}}(\grave{e}_1, \grave{e}_2)(x) : x \in X\rangle\Big) : (\grave{e}_1, \grave{e}_2) \in \acute{E} \times \acute{E}\Big\}.$$

where

$$AbT_{\tilde{H}(\grave{e})}(x) = \min\left\{AbT_{\tilde{F}(\grave{e})}(x), AbT_{\tilde{G}(\grave{e})}(x)\right\},$$
$$ReT_{\tilde{H}(\grave{e})}(x) = \min\left\{ReT_{\tilde{F}(\grave{e})}(x), ReT_{\tilde{G}(\grave{e})}(x)\right\},$$
$$ReF_{\tilde{H}(\grave{e})}(x) = \max\left\{ReF_{\tilde{F}(\grave{e})}(x), ReF_{\tilde{G}(\grave{e})}(x)\right\},$$
$$AbF_{\tilde{H}(\grave{e})}(x) = \max\left\{AbF_{\tilde{F}(\grave{e})}(x), AbF_{\tilde{G}(\grave{e})}(x)\right\}.$$

Definition 16. *Let* $(\tilde{F}, \acute{E})$ *and* $(\tilde{G}, \acute{E})$ *be two QPNSSs over the key set* X*. Then, the "OR" operation on them is denoted by* $(\tilde{F}, \acute{E}) \vee (\tilde{G}, \acute{E}) = (\tilde{H}, \acute{E} \times \acute{E})$ *and is defined as:*

$$(\tilde{H}, \acute{E} \times \acute{E}) = \Big\{\Big((\grave{e}_1, \grave{e}_2), \langle x, AbT_{\tilde{H}(\grave{e}_1, \grave{e}_2)}(x), ReT_{\tilde{H}(\grave{e}_1, \grave{e}_2)}(x),$$
$$ReF_{\tilde{H}(\grave{e}_1, \grave{e}_2)}(x), AbF_{\tilde{H}(\grave{e}_1, \grave{e}_2)}(x) : x \in X\rangle\Big) : (\grave{e}_1, \grave{e}_2) \in \acute{E} \times \acute{E}\Big\}.$$

where

$$AbT_{\tilde{H}(\grave{e})}(x) = \max\left\{AbT_{\tilde{F}(\grave{e})}(x), AbT_{\tilde{G}(\grave{e})}(x)\right\},$$
$$ReT_{\tilde{H}(\grave{e})}(x) = \max\left\{ReT_{\tilde{F}(\grave{e})}(x), ReT_{\tilde{G}(\grave{e})}(x)\right\},$$
$$ReF_{\tilde{H}(\grave{e})}(x) = \min\left\{ReF_{\tilde{F}(\grave{e})}(x), ReF_{\tilde{G}(\grave{e})}(x)\right\},$$
$$AbF_{\tilde{H}(\grave{e})}(x) = \min\left\{AbF_{\tilde{F}(\grave{e})}(x), AbF_{\tilde{G}(\grave{e})}(x)\right\}.$$

Definition 17. *QPNSS $(\tilde{F}, \acute{E})$ over the key set X is said to be a null QPNSS if*

$$AbT_{\tilde{F}(\grave{e})}(x) = 0, \quad ReT_{\tilde{F}(\grave{e})}(x) = 0, \quad \forall \grave{e} \in \acute{E}, \forall x \in X,$$

$$ReF_{\tilde{F}(\grave{e})}(x) = 1, \quad AbF_{\tilde{F}(\grave{e})}(x) = 1, \quad \forall \grave{e} \in \acute{E}, \forall x \in X.$$

It is signified as $0_{(X,\acute{E})}$.

Definition 18. *A quadri-partitioned neutrosophic soft set $(\tilde{F}, \acute{E})$ over the key set X is called an absolute QPNSS if*

$$AbT_{\tilde{F}(\grave{e})}(x) = 1, \quad ReT_{\tilde{F}(\grave{e})}(x) = 1, \quad \forall \grave{e} \in \acute{E}, \forall x \in X,$$

$$ReF_{\tilde{F}(\grave{e})}(x) = 0, \quad AbF_{\tilde{F}(\grave{e})}(x) = 0, \quad \forall \grave{e} \in \acute{E}, \forall x \in X.$$

Clearly,

$$0^c_{(X,\acute{E})} = 1_(X, \acute{E}), \quad 1^c_{(X,\acute{E})} = 0_{(X,\acute{E})}.$$

Definition 19. *If the family of all QPNSSs over X is designated as $\mathbb{P}NSS(X)$, then $x^{\grave{e}}_{\langle p_1,p_2,p_3,p_4\rangle}$ is called a QPNS point for every point $x \in X$, $0 \preceq p_1, p_2, p_3, p_4 \preceq 1, \grave{e} \in \acute{E}$, and is given by:*

$$x^{\grave{e}}_{\langle p_1,p_2,p_3,p_4\rangle}(\mathcal{Y}) = \begin{cases} \langle p_1, p_2, p_3, p_4\rangle, & \text{if } \grave{e} = \grave{e}' \text{ and } \mathcal{Y} = x, \\ (0, 0, 0, 1), & \text{if } \grave{e}' \neq \grave{e} \text{ or } \mathcal{Y} \neq x. \end{cases}$$

Definition 20. *Let $(\tilde{F}, \acute{E})$ be a QPNSS over the key set X, $x^{\grave{e}}_{\langle p_1,p_2,p_3,p_4\rangle} \in QPNSS(\tilde{F}, \acute{E})$ if*

$$p_1 \preceq AbT_{\tilde{F}(\grave{e})}(x), \quad p_2 \preceq ReT_{\tilde{F}(\grave{e})}(x), \quad p_3 \succeq ReF_{\tilde{F}(\grave{e})}(x), \quad p_4 \succeq AbF_{\tilde{F}(\grave{e})}(x).$$

Definition 21. *Let the QPNSS $(X, \acute{E})$ be the family of all quadri-partitioned neutrosophic soft sets, and let $\tau \subset QPNSS(X, \acute{E})$. Then, τ is a quadri-partitioned neutrosophic soft topology (QPNST) on $\tilde{X}$ if:*

(i) $0_{(X,\acute{E})}$, $1_{(X,\acute{E})} \in \tau$,

(ii) The union of any number of QPNSSs in τ *belongs to* τ,

(iii) The intersection of a finite number of QPNSSs in τ *belongs to* τ.

Then, $(X, \tau, \acute{E})$ *is said to be a quadri-partitioned neutrosophic soft topological space (QPNSTS) over* X*. Each number of* τ *is said to be a neutrosophic soft open set*

Definition 22. *Let* $(X, \tau, \acute{E})$ *be a QPNSTS over* X *and* $(\tilde{F}, \acute{E})$ *be a QPNSS over* X*. Then* $(\tilde{F}, \acute{E})$ *is said to be a quadri-partitioned neutrosophic soft closed set (QPNSCS) if its complement is a QPNS open set.*

Example 1. *Suppose that* $X = \{x_1, x_2\}$*. Then a quadri-partitioned neutrosophic set*

$$A = \{\langle x_1, 0.1, 0.2, 0.1, 0.5\rangle, \langle x_2, 0.5, 0.2, 0.2, 0.7\rangle\}$$

is the union of quadri-partitioned neutrosophic soft points $x_{1_{(0.1,0.2,0.1,0.5)}}$ *and* $x_{2_{(0.5,0.2,0.2,0.7)}}$. *Now we define the concept of quadri-partitioned neutrosophic soft points for quadri-partitioned neutrosophic soft sets.*

Example 2. *Suppose that the universe set* X *is given by* $X = \{x_1, x_2\}$ *and the set of parameters by* $\acute{E} = \{\grave{e}_1, \grave{e}_2\}$*. Let us consider quadri-partitioned neutrosophic soft set* $(\tilde{F}, \acute{E})$ *over the universe set* X *as follows:*

$$(\tilde{F}, \acute{E}) = \begin{Bmatrix} \grave{e}_1 = (\langle x_1, 0.3, 0.4, 0.3, 0.6\rangle, \langle x_2, 0.4, 0.2, 0.1, 0.8\rangle) \\ \grave{e}_2 = (\langle x_1, 0.4, 0.4, 0.2, 0.8\rangle, \langle x_2, 0.3, 0.4, 0.3, 0.2\rangle) \end{Bmatrix}$$

It is clear that $(\tilde{F}, \acute{E})$ *is the union of its quadri-partitioned neutrosophic soft points*

$$x^{\grave{e}_1}_{1(0.3,0.4,0.3,0.6)},\ x^{\grave{e}_2}_{1(0.4,0.2,0.1,0.8)},\ x^{\grave{e}_1}_{2(0.4,0.4,0.2,0.8)} \text{ and } x^{\grave{e}_2}_{2(0.3,0.4,0.3,0.2)}$$

Here,

$$x^{\grave{e}_1}_{1(0.3,0.4,0.3,0.6)} = \begin{Bmatrix} \grave{e}_1 = (\langle x_1, 0.3, 0.4, 0.3, 0.6\rangle, \langle x_2, 0, 0, 1, 1\rangle) \\ \grave{e}_2 = (\langle x_1, 0, 0, 1, 1\rangle, \langle x_2, 0, 0, 1, 1\rangle) \end{Bmatrix}$$

$$x^{\grave{e}_2}_{1(0.4,0.4,0.2,0.8)} = \begin{Bmatrix} \grave{e}_1 = (\langle x_1, 0, 0, 1, 1\rangle, \langle x_2, 0, 0, 1, 1\rangle) \\ \grave{e}_2 = (\langle x_1, 0.4, 0.4, 0.2, 0.8\rangle, \langle x_2, 0, 0, 1, 1\rangle) \end{Bmatrix}$$

$$x^{\grave{e}_1}_{2(0.4,0.2,0.1,0.8)} = \begin{Bmatrix} \grave{e}_1 = (\langle x_1, 0, 0, 1, 1\rangle, \langle x_2, 0.4, 0.2, 0.1, 0.8\rangle) \\ \grave{e}_2 = (\langle x_1, 0, 0, 1, 1\rangle, \langle x_2, 0, 0, 1, 1\rangle) \end{Bmatrix}$$

$$x^{\grave{e}_2}_{2(0.3,0.4,0.3,0.2)} = \begin{Bmatrix} \grave{e}_1 = (\langle x_1, 0, 0, 1, 1\rangle, \langle x_2, 0, 0, 1, 1\rangle) \\ \grave{e}_2 = (\langle x_1, 0, 0, 1, 1\rangle, \langle x_2, 0.3, 0.4, 0.3, 0.2\rangle) \end{Bmatrix}$$

Definition 23. *Let $(\tilde{F}, \acute{E})$ be a QPNSS over the key set X. We say that $x^{\grave{e}}_{(p_1,p_2,p_3,p_4)} \in (\tilde{F}, \acute{E})$ read as belonging to the quadri-partitioned neutrosophic soft set $(\tilde{F}, \acute{E})$, whenever*

$$p_1 \preceq AbT_{\tilde{F}(\grave{e})}(x), \quad p_2 \preceq ReT_{\tilde{F}(\grave{e})}(x), \quad p_3 \succeq ReF_{\tilde{F}(\grave{e})}(x), \quad p_4 \succeq AbF_{\tilde{F}(\grave{e})}(x).$$

Definition 24. *Let $(X, \tau, \acute{E})$ be a quadri-partitioned neutrosophic soft topological space over X. A quadri-partitioned neutrosophic soft set $(\tilde{F}, \acute{E})$ in $(X, \tau, \acute{E})$ is called QPNS neighbourhood of a QPNS point $x^{\grave{e}}_{(p_1,p_2,p_3,p_4)} \in (\tilde{F}, \acute{E})$, if there exists a QPNS open set $(\tilde{G}, \acute{E})$ such that $x^{\grave{e}}_{(p_1,p_2,p_3,p_4)} \in (\tilde{G}, \acute{E})$.*

Theorem 1. *Let $(X, \tau, \acute{E})$ be a QPNSTS and $(\tilde{F}, \acute{E})$ be a QPNSS over X. Then, $(\tilde{F}, \acute{E})$ is quadri-partitioned neutrosophic soft open set if and only if $(\tilde{F}, \acute{E})$ is a quadri-partitioned neutrosophic soft neighborhood of its quadri-partitioned neutrosophic soft points.*

Proof. Let $(\tilde{F}, \acute{E})$ is a quadri-partitioned neutrosophic soft open set and $x^{\grave{e}}_{(p_1,p_2,p_3,p_4)} \in (\tilde{F}, \acute{E})$. Then $x^{\grave{e}}_{(p_1,p_2,p_3,p_4)} \in (\tilde{F}, \acute{E}) \subset (\tilde{F}, \acute{E})$. Therefore, $(\tilde{F}, \acute{E})$ is a quadri-partitioned neutrosophic soft neighborhood of $x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}$.
Conversely, if $(\tilde{F}, \acute{E})$ is a quadri-partitioned neutrosophic soft neighborhood of its quadri-partitioned neutrosophic soft points. Let $x^{\grave{e}}_{(p_1,p_2,p_3,p_4)} \in (\tilde{F}, \acute{E})$. Since $(\tilde{F}, \acute{E})$ is a quadri-partitioned neutrosophic soft neighborhood of its quadri-partitioned neutrosophic soft point $x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}$, there exist $(\tilde{G}, \acute{E}) \in \tau$ such that $x^{\grave{e}}_{(p_1,p_2,p_3,p_4)} \in (\tilde{G}, \acute{E}) \subset (\tilde{F}, \acute{E})$. Since

$$(\tilde{F}, \acute{E}) = \cup\{x^{\grave{e}}_{(p_1,p_2,p_3,p_4)} : x^{\grave{e}}_{\langle p_1,p_2,p_3,p_4\rangle} \in (\tilde{F}, \acute{E})\}$$

it follows that $(\tilde{F}, \acute{E})$ is a union of quadri-partitioned neutrosophic soft open sets and hence $(\tilde{F}, \acute{E})$ is a quadri-partitioned neutrosophic soft open set. The The neighborhood system of a quadri-partitioned neutrosophic soft point $x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}$, denoted by $U(x^{\grave{e}}_{(p_1,p_2,p_3,p_4),E})$ at $x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}$, is the family of all its neighborhoods in a quadri-partitioned neutrosophic soft topological space.

Theorem 2. *The neighborhood system $U(x^{\grave{e}}_{(p_1,p_2,p_3,p_4),E})$ at $x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}$, in a quadri-partitioned neutrosophic soft topological space $(X, \tau, \acute{E})$ has the following properties:*

1.

$$If\ (\tilde{F}, \acute{E}) \in U(x^{\grave{e}}_{(p_1,p_2,p_3,p_4),E}),\ then\ x^{\grave{e}}_{(p_1,p_2,p_3,p_4)} \in (\tilde{F}, \acute{E});$$

2.

$$If\ (\tilde{F}, \acute{E}) \in U(x^{\grave{e}}_{(p_1,p_2,p_3,p_4),E})\ and\ (\tilde{F}, \acute{E}) \subset (\tilde{H}, \acute{E}), then (\tilde{F}, \acute{E}) \in U(x^{\grave{e}}_{(p_1,p_2,p_3,p_4),E});$$

3.

$$If\ (\tilde{F}, \acute{E}), (\tilde{G}, \acute{E}) \in U(x^{\grave{e}}_{(p_1,p_2,p_3,p_4),E}),\ then\ (\tilde{F}, \acute{E}) \cap (\tilde{G}, \acute{E}) \in U(x^{\grave{e}}_{(p_1,p_2,p_3,p_4),E});$$

4.

$$If\ (\tilde{F}, \acute{E}) \in U(x^{\grave{e}}_{(p_1,p_2,p_3,p_4),E}\ and\ (\tilde{G}, \acute{E}) \in U(x^{\grave{e}}_{(p_1,p_2,p_3,p_4),E}),$$
$$such\ that\ (\tilde{G}, \acute{E}) \in U(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4),E}),\ for\ each\ \in (\tilde{G}, \acute{E}).$$

Proof. Obvious.

Definition 25. *Let $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})$ and $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)})$ be two quadri-partitioned neutrosophic soft points. For the quadri-partitioned neutrosophic soft points $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})$ and $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)})$ over a common universe X, we say that these are distinct quadri-partitioned neutrosophic soft points if*

$$(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \cap (y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)}) = 0_{(X,\acute{E})}.$$

It is clear that $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})$ and $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)})$ are distinct quadri-partitioned neutrosophic soft points if and only if $x \neq y$ or $\grave{e} \neq \grave{e}'$.

4. Quadri-Partitioned Neutrosophic Soft Separation Axioms

This section introduces and explores new separation axioms within the framework of quadri-partitioned neutrosophic soft topological spaces. We define $T_i, (i = 0, 1, 2, 3, 4)$ separation properties for such spaces using quadri-partitioned neutrosophic soft points and open sets. Examples illustrate spaces that satisfy or fail each axiom. Key theorems established relationships between separation axioms and properties like closeness and regularity.

Definition 26. *Let $(X, \tau, \acute{E})$ be a quadri-partitioned neutrosophic soft topological space over X and $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})$ and $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)})$ are distinct quadri-partitioned neutrosophic soft points. If there exist quadri-partitioned neutrosophic soft open sets $(\tilde{F}, \acute{E})$ and $(\tilde{G}, \acute{E})$, such that $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \in (\tilde{F}, \acute{E})$ and*

$$(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \cap (\tilde{G}, \acute{E}) = 0_{(X,\acute{E})}.$$

or $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)}) \in (\tilde{G}, \acute{E})$ and

$$(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)}) \cap (\tilde{F}, \acute{E}) = 0_{(X,\acute{E})}.$$

Then $(X, \tau, \acute{E})$ is called quadri-partitioned neutrosophic soft T_0 space.

Definition 27. *Let $(X, \tau, \acute{E})$ be a quadri-partitioned neutrosophic soft topological space over X and $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})$ and $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)})$ are distinct quadri-partitioned neutrosophic soft points. If there exist quadri-partitioned neutrosophic soft open sets $(\tilde{F}, \acute{E})$ and $(\tilde{G}, \acute{E})$, such that $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \in (\tilde{F}, \acute{E})$ and*

$$(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \cap (\tilde{G}, \acute{E}) = 0_{(X,\acute{E})}.$$

or $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)}) \in (\tilde{G}, \acute{E})$ *and*

$$(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)}) \cap (\tilde{F}, \acute{E}) = 0_{(X,\acute{E})}.$$

Then $(X, \tau, \acute{E})$ *is called quadri-partitioned neutrosophic soft* T_1 *space.*

Definition 28. *Let* $(X, \tau, \acute{E})$ *be a quadri-partitioned neutrosophic soft topological space over* X *and* $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})$ *and* $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)})$ *are distinct quadri-partitioned neutrosophic soft points. If there exist quadri-partitioned neutrosophic soft open sets* $(\tilde{F}, \acute{E})$ *and* $(\tilde{G}, \acute{E})$*, such that* $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \in (\tilde{F}, \acute{E})$, $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)}) \in (\tilde{G}, \acute{E})$ *and*

$$(\tilde{F}, \acute{E}) \cap (\tilde{G}, \acute{E}) = 0_{(X,\acute{E})}.$$

Then $(X, \tau, \acute{E})$ *is called quadri-partitioned neutrosophic soft* T_1 *space.*

Example 3. *Let* $X = \{x_1, x_2\}$ *be a universe set,* $\acute{E} = \{\grave{e}_1, \grave{e}_2\}$ *be a parameters set, and*

$$x^{\grave{e}_1}_{1(0.1,0.3,0.1,0.7)},\ x^{\grave{e}_2}_{1(0.2,0.3,0.2,0.6)},\ x^{\grave{e}_1}_{2(0.3,0.2,0.1,0.5)} \textit{ and } x^{\grave{e}_2}_{2(0.4,0.3,0.1,0.4)}$$

be quadri-partitioned neutrosophic soft points. Then the family

$$\tau = \left\{0_{(X,\acute{E})}, 1_{(X,\acute{E})}, (\tilde{F}_1, \acute{E}), (\tilde{F}_2, \acute{E}), (\tilde{F}_3, \acute{E}), (\tilde{F}_4, \acute{E}), (\tilde{F}_5, \acute{E}), (\tilde{F}_6, \acute{E}), (\tilde{F}_7, \acute{E}), (\tilde{F}_8, \acute{E})\right\},$$

Where

$$(\tilde{F}_1, \acute{E}) = \{x^{\grave{e}_1}_{1(0.1,0.3,0.1,0.7)}\}$$

$$(\tilde{F}_2, \acute{E}) = \{x^{\grave{e}_2}_{1(0.2,0.3,0.2,0.6)}\}$$

$$(\tilde{F}_3, \acute{E}) = \{x^{\grave{e}_1}_{2(0.3,0.2,0.1,0.5)}\}$$

$$(\tilde{F}_4, \acute{E}) = (\tilde{F}_1, \acute{E}) \cup (\tilde{F}_2, \acute{E})$$

$$(\tilde{F}_5, \acute{E}) = (\tilde{F}_1, \acute{E}) \cup (\tilde{F}_3, \acute{E})$$

$$(\tilde{F}_6, \acute{E}) = (\tilde{F}_2, \acute{E}) \cup (\tilde{F}_3, \acute{E})$$

$$(\tilde{F}_7, \acute{E}) = (\tilde{F}_1, \acute{E}) \cup (\tilde{F}_2, \acute{E}) \cup (\tilde{F}_3, \acute{E})$$

$$(\tilde{F}_8, \acute{E}) = (\tilde{F}_1, \acute{E}) \cup (\tilde{F}_2, \acute{E}) \cup (\tilde{F}_3, \acute{E}) \cup (\tilde{F}_4, \acute{E})$$

so

$$(\tilde{F}_8, \acute{E}) = \left\{x^{\grave{e}_1}_{1(0.1,0.3,0.1,0.7)},\ x^{\grave{e}_2}_{1(0.2,0.3,0.2,0.6)},\ x^{\grave{e}_1}_{2(0.3,0.2,0.1,0.5)}, x^{\grave{e}_2}_{2(0.4,0.3,0.1,0.4)}\right\}$$

is a quadri-partitioned neutrosophic soft topology over X. *Hence* $(X, \tau, \acute{E})$ *is a quadri-partitioned neutrosophic soft topological space over* X. *Also* $(X, \tau, \acute{E})$ *is a quadri-partitioned neutrosophic soft* T_0 *space but not a quadri-partitioned neutrosophic soft* T_1 *space because for quadri-partitioned neutrosophic soft points* $x^{\grave{e}_1}_{2(0.1,0.4,0.7)}$ *and* $x^{\grave{e}_2}_{2(0.4,0.4,0.4)}$, $(X, \tau, \acute{E})$ *is not a quadri-partitioned neutrosophic soft* T_1 *space.*

Example 4. *Let* $X = N$ *be a natural number set and* $\acute{E} = \{\grave{e}\}$ *be a parameter set. Here* $n^{\grave{e}}_{(\pounds_n, \int_n, \omega_n, \kappa_n)}$ *are quadri-partitioned neutrosophic soft points. Here we can give* $\{\alpha_n, \beta_n, \gamma_n\}$ *appropriate values and the quadri-partitioned neutrosophic soft points* $n^{\grave{e}}_{(\pounds_n, \int_n, \omega_n, \kappa_n)}$ *and* $m^{\grave{e}}_{(\pounds_m, \int_m, \omega_m, \kappa_m)}$ *are distinct quadri-partitioned neutrosophic soft points if and only if* $m \neq n$. *It is clear that there is one-to-one compatibility between the set of natural numbers and the set of quadri-partitioned neutrosophic soft points* $N^{\grave{e}} = \{n^{\grave{e}}_{(\pounds_n, \int_n, \omega_n, \kappa_n)}\}$.
Then we give co-finite topology on this set. Then quadri-partitioned neutrosophic soft set $(\tilde{F}, \acute{E})$ *is a quadri-partitioned neutrosophic soft open set if and only if the finite quadri-partitioned neutrosophic soft point is discarded from* $N^{\grave{e}}$. *Hence,* $(X, \tau, \acute{E})$ *is a quadri-partitioned neutrosophic soft* $T_1 -$ *space but not a neutrosophic soft* $T_2 -$ *space.*

Example 5. *Let* $X = \{x_1, x_2\}$ *be a universe set,* $\acute{E} = \{\grave{e}_1, \grave{e}_2\}$ *be a parameters set, and*

$$x^{\grave{e}_1}_{1(0.1,0.3,0.1,0.7)},\ x^{\grave{e}_2}_{1(0.2,0.3,0.2,0.6)},\ x^{\grave{e}_1}_{2(0.3,0.2,0.1,0.5)} \text{ and } x^{\grave{e}_2}_{2(0.4,0.3,0.1,0.4)}$$

be quadri-partitioned neutrosophic soft points. Then the family

$$\tau = \begin{Bmatrix} 0_{(X,\acute{E})}, 1_{(X,\acute{E})}, (\tilde{F}_1, \acute{E}), (\tilde{F}_2, \acute{E}), (\tilde{F}_3, \acute{E}), (\tilde{F}_4, \acute{E}), (\tilde{F}_5, \acute{E}), (\tilde{F}_6, \acute{E}), (\tilde{F}_7, \acute{E}), \\ (\tilde{F}_8, \acute{E}), (\tilde{F}_9, \acute{E}), (\tilde{F}_{10}, \acute{E}), (\tilde{F}_{11}, \acute{E}), (\tilde{F}_{12}, \acute{E}), (\tilde{F}_{13}, \acute{E}), (\tilde{F}_{14}, \acute{E}), (\tilde{F}_{15}, \acute{E}) \end{Bmatrix},$$

Where

$$(\tilde{F}_1, \acute{E}) = \{x^{\grave{e}_1}_{1(0.1,0.3,0.1,0.7)}\}$$

$$(\tilde{F}_2, \acute{E}) = \{x^{\grave{e}_2}_{1(0.2,0.3,0.2,0.6)}\}$$

$$(\tilde{F}_3, \acute{E}) = \{x^{\grave{e}_1}_{2(0.3,0.2,0.1,0.5)}\}$$

$$(\tilde{F}_4, \acute{E}) = \{x^{\grave{e}_2}_{2(0.4,0.3,0.1,0.4)}\}$$

$$(\tilde{F}_5, \acute{E}) = (\tilde{F}_1, \acute{E}) \cup (\tilde{F}_2, \acute{E})$$

$$(\tilde{F}_6, \acute{E}) = (\tilde{F}_1, \acute{E}) \cup (\tilde{F}_3, \acute{E})$$

$$(\tilde{F}_7, \acute{E}) = (\tilde{F}_1, \acute{E}) \cup (\tilde{F}_4, \acute{E})$$

$$(\tilde{F}_8, \acute{E}) = (\tilde{F}_2, \acute{E}) \cup (\tilde{F}_3, \acute{E})$$

$$(\tilde{F}_9, \acute{E}) = (\tilde{F}_2, \acute{E}) \cup (\tilde{F}_4, \acute{E})$$

$$(\tilde{F}_{10}, \acute{E}) = (\tilde{F}_3, \acute{E}) \cup (\tilde{F}_4, \acute{E})$$

$$(\tilde{F}_{11}, \acute{E}) = (\tilde{F}_1, \acute{E}) \cup (\tilde{F}_2, \acute{E}) \cup (\tilde{F}_3, \acute{E})$$

$$(\tilde{F}_{12}, \acute{E}) = (\tilde{F}_1, \acute{E}) \cup (\tilde{F}_2, \acute{E}) \cup (\tilde{F}_4, \acute{E})$$

$$(\tilde{F}_{13}, \acute{E}) = (\tilde{F}_2, \acute{E}) \cup (\tilde{F}_3, \acute{E}) \cup (\tilde{F}_4, \acute{E})$$

$$(\tilde{F}_{14}, \acute{E}) = (\tilde{F}_1, \acute{E}) \cup (\tilde{F}_3, \acute{E}) \cup (\tilde{F}_4, \acute{E})$$

so

$$(\tilde{F}_{15}, \acute{E}) = \left\{ x^{\grave{e}_1}_{1(0.1,0.3,0.1,0.7)},\ x^{\grave{e}_2}_{1(0.2,0.3,0.2,0.6)},\ x^{\grave{e}_1}_{2(0.3,0.2,0.1,0.5)}, x^{\grave{e}_2}_{2(0.4,0.3,0.1,0.4)} \right\}$$

is a quadri-partitioned neutrosophic soft topology over X. Hence $(X, \tau, \acute{E})$ is a quadri-partitioned neutrosophic soft topological space over X. Also $(X, \tau, \acute{E})$ is a quadri-partitioned neutrosophic soft T_2 space.

Theorem 3. *Let $(X, \tau, \acute{E})$ is a quadri-partitioned neutrosophic soft topological space over X. Then $(X, \tau, \acute{E})$ is a quadri-partitioned neutrosophic soft T_2 space if and only if each quadri-partitioned neutrosophic soft point is a quadri-partitioned neutrosophic soft closed set.*

Proof. Let $(X, \tau, \acute{E})$ is a quadri-partitioned neutrosophic soft T_1 *space* and $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})$ be an arbitrary quadri-partitioned neutrosophic soft point. We show that $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})^c$ is a quadri-partitioned neutrosophic soft open set. Let $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)}) \in (x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})^c$; then $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})$ *and* $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)})$ are distinct quadri-partitioned neutrosophic soft points. Hence,$x \neq y$ or $\grave{e} \neq \grave{e}'$. Since $(X, \tau, \acute{E})$ is a quadri-partitioned neutrosophic soft T_1 *space* there exist a quadri-partitioned neutrosophic soft open set $(\tilde{G}, \acute{E})$, such that

$$(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)}) \in (\tilde{G}, \acute{E}) \ \textit{and}\ (x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \cap (\tilde{G}, \acute{E}) = 0_{(X,\acute{E})}.$$

Then, since $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \cap (\tilde{G}, \acute{E}) = 0_{(X,\acute{E})}$, we have$(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)}) \in (\tilde{G}, \acute{E}) \subset x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})^c$. This implies that $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})^c$ is a quadri-partitioned neutrosophic soft open set, such that $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})$ is a quadri-partitioned neutrosophic soft closed set. Suppose that each quadri-partitioned neutrosophic soft point $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})$ is a quadri-partitioned neutrosophic soft closed set. Then, $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})^c$ is a quadri-partitioned neutrosophic soft open set. Let $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \cap (y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)}) = 0_{(X,\acute{E})}$. Thus $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)}) \in (x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})$ and

$$(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \cap x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})^c = 0_{(X,\acute{E})}.$$

Therefore, $(X, \tau, \acute{E})$ is a quadri-partitioned neutrosophic soft $T_1 - space$

Theorem 4. *Let $(X, \tau, \acute{E})$ is a quadri-partitioned neutrosophic soft topological space over X. Then $(X, \tau, \acute{E})$ is a quadri-partitioned neutrosophic soft $T_2 - space$ if and only if for distinct quadri-partitioned neutrosophic soft points $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})$ and $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)})$, there exist a quadri-partitioned neutrosophic soft open set $(\tilde{F}, \acute{E})$ containing $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})$ but not $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)})$ such that $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)}) \notin \overline{(\tilde{F}, \acute{E})}$.*

Proof. Let $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})$ and $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)})$ be two quadri-partitioned neutrosophic soft points in quadri- partitioned neutrosophic soft $T_2 - space$, $(X, \tau, \acute{E})$. Then there exist disjoint quadri-partitioned neutrosophic soft open sets $(\tilde{F}, \acute{E})$, $(\tilde{G}, \acute{E})$ such that $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \in (\tilde{F}, \acute{E})$ and $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)}) \in (\tilde{G}, \acute{E})$.
Since

$$(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \cap (y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)}) = 0_{(X,\acute{E})}$$

and

$$(\tilde{F}, \acute{E}) \cap (\tilde{G}, \acute{E}) = 0_{(X,\acute{E})}, \ (y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)}) \notin (\tilde{F}, \acute{E}).$$

It implies that $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)}) \notin \overline{(\tilde{F}, \acute{E})}$. Next suppose that, for distinct quadri-partitioned neutrosophic soft points $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})$ and $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)})$ there exists a quadri-partitioned neutrosophic soft open set $(\tilde{F}, \acute{E})$ containing $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})$ but not $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)})$ such that $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)}) \notin \overline{(\tilde{F}, \acute{E})}$. Then $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)}) \in (\overline{(\tilde{F}, \acute{E})})^c$, such that $(\tilde{F}, \acute{E})$ and $(\overline{(\tilde{F}, \acute{E})})^c$ are disjoint quadri-partitioned neutrosophic soft open sets containing $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})$ and $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)})$ respectively.

Theorem 5. *Let $(X, \tau, \acute{E})$ be a quadri-partitioned neutrosophic soft $T_1 - space$ for every quadri-partitioned neutrosophic soft point $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \in (\tilde{F}, \acute{E}) \in \tau$. If there exist a quadri-partitioned neutrosophic soft open set $(\tilde{F}, \acute{E})$ such that*

$$(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \in (\tilde{G}, \acute{E}) \subset \overline{(\tilde{G}, \acute{E})} \subset (\tilde{F}, \acute{E})$$

Then $(X, \tau, \acute{E})$ be a quadri-partitioned neutrosophic soft $T_1 - space$.

Proof. Suppose

$$(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \cap (y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)}) = 0_{(X,\acute{E})}$$

Since $(X, \tau, \acute{E})$ be a quadri-partitioned neutrosophic soft $T_1 - space$. $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})$ and $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)})$ are quadri-partitioned neutrosophic soft closed sets in τ. Thus

$$(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \in (y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)})^c \in \tau.$$

Then there exist quadri-partitioned neutrosophic soft open set $(\tilde{G}, \acute{E})$ such that

$$(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \in (\tilde{G}, \acute{E}) \subset \overline{(\tilde{G}, \acute{E})} \subset (y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)})^c$$

Hence, we have $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)}) \in (\overline{(\tilde{F}, \acute{E})})^c$, $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \in (\tilde{G}, \acute{E})$ and $(\tilde{G}, \acute{E}) \cap (\overline{(\tilde{F}, \acute{E})})^c$, i.e, $(X, \tau, \acute{E})$ is a quadri-partitioned neutrosophic soft $T_2 - space$

Remark 1. *Let $(X, \tau, \acute{E})$ be a quadri-partitioned neutrosophic soft T_i−space for $i = 1, 2$. For each $x \neq y$, quadri-partitioned neutrosophic points $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})$ and $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)})$ have neighborhood satisfying conditions of $T_i - space$ in quadri-partitioned neutrosophic topological space $(X, \tau_{\grave{e}}, \acute{E})$ for each $\grave{e} in \acute{E}$ because $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})$ and $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)})$ are distinct quadri-partitioned neutrosophic soft points.*

Definition 29. *Let $(X, \tau, \acute{E})$ is a quadri-partitioned neutrosophic soft topological space over X. $(\tilde{F}, \acute{E})$ be a quadri-partitioned neutrosophic soft closed set, and $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \cap (\tilde{F}, \acute{E}) = 0_{(X,\acute{E})}$. If there exist quadri-partitioned neutrosophic soft open sets $(\tilde{G}_1, \acute{E})$ and $(\tilde{G}_2, \acute{E})$ such that $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \cap (\tilde{G})_1, \acute{E})$, $(\tilde{G}_2, \acute{E}) \subset (\tilde{F}, \acute{E})$ and $(\tilde{G}_1, \acute{E}) \cap (\tilde{G}_2, \acute{E}) = 0_{(X,\acute{E})}$, then $(X, \tau, \acute{E})$ is called a quadri-partitioned neutrosophic soft regular and quadri-partitioned neutrosophic soft $T_1 - space$.*

Theorem 6. *Let $(X, \tau, \acute{E})$ be a quadri-partitioned neutrosophic soft topological space over X, $(X, \tau, \acute{E})$ is a quadri-partitioned neutrosophic soft $T_3 - space$ if and only if $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \in (\tilde{F}_1, \acute{E}) \in \tau$, there exist a quadri-partitioned neutrosophic soft open set $(\tilde{G}_1, \acute{E}) \in \tau$ such that*

$$(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \in (\tilde{G}_1, \acute{E}) \subset \overline{(\tilde{G}, \acute{E})} \subset (\tilde{F}, \acute{E})$$

Proof. Let $(X, \tau, \acute{E})$ be a quadri-partitioned neutrosophic soft T_3−*space* and $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \in (\tilde{F}_1, \acute{E}) \in \tau$. Since $(X, \tau, \acute{E})$ is a quadri-partitioned neutrosophic soft $T_3 - space$ for the quadri-partitioned neutrosophic soft points $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})$ and quadri-partitioned neutrosophic soft closed set $(\tilde{F}, \acute{E})^c$, there exist $(\tilde{G}_1, \acute{E})$, $(\tilde{G}_2, \acute{E}) \in \tau$ such that $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \in (\tilde{G}_1, \acute{E})$, $(\tilde{F}, \acute{E})^c \in (\tilde{G}_2, \acute{E})$ and $(\tilde{G}_1, \acute{E}) \cap (\tilde{G}_2, \acute{E}) = 0_{(X,\acute{E})}$. Thus we have

$$(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \in (\tilde{G}_1, \acute{E}) \subset (\tilde{G}, \acute{E})^c \subset (\tilde{F}, \acute{E})$$

Since $(\tilde{G}, \acute{E})^c$ is a quadri-partitioned neutrosophic soft closed set so that $\overline{(\tilde{G}, \acute{E})} \subset (\tilde{G}, \acute{E})^c$.

Conversely let $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \cap (\tilde{H},\acute{E}) = 0_{(X,\acute{E})}$ and $(\tilde{H},\acute{E})$ be a quadri-partitioned neutrosophic soft closed set. Thus, $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \in (\tilde{H},\acute{E})^c$ and from the condition of the theorem, we have

$$(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \in (\tilde{G},\acute{E}) \subset \overline{(\tilde{G},\acute{E})} \subset (\tilde{H},\acute{E})^c.$$

Then, $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \in (\tilde{G},\acute{E})$, $(\tilde{H},\acute{E}) \subset (\overline{(\tilde{G},\acute{E})})^c$ and $(\tilde{G},\acute{E}) \subset (\overline{(\tilde{G},\acute{E})})^c = 0_{(X,\acute{E})}$, i.e, $(X,\tau,\acute{E})$ is a quadri-partitioned neutrosophic soft $T_3 - space$.

Definition 30. *A quadri-partitioned neutrosophic soft topological space $(X,\tau,\acute{E})$ over X is called a d quadri-partitioned neutrosophic soft normal space if for every pair of disjoint quadri-partitioned neutrosophic soft closed sets $(\tilde{F}_1,\acute{E})$, $(\tilde{F}_2,\acute{E})$, there exists disjoint quadri-partitioned neutrosophic soft open sets $(\tilde{G}_1,\acute{E})$, $(\tilde{G}_2,\acute{E})$ such that $(\tilde{G}_1,\acute{E}) \subset (\tilde{G}_2,\acute{E})$ and $(\tilde{F}_2,\acute{E}) \subset (\tilde{G}_2,\acute{E})$. Then, $(X,\tau,\acute{E})$ is said to be quadri-partitioned neutrosophic soft $T_4 - space$ if it is a quadri-partitioned neutrosophic soft normal and quadri-partitioned neutrosophic soft $T_1 - space$.*

Theorem 7. *Let $(X,\tau,\acute{E})$ be a quadri-partitioned neutrosophic soft topological space over X. Then, $(X,\tau,\acute{E})$ is said to be quadri-partitioned neutrosophic soft $T_4 - space$ if and only if, for each quadri-partitioned neutrosophic soft closed set $(\tilde{F},\acute{E})$ and quadri-partitioned neutrosophic soft open set $(\tilde{G},\acute{E})$ with $(\tilde{F},\acute{E}) \subset (\tilde{G},\acute{E})$, there exist a quadri-partitioned neutrosophic soft open set $(\tilde{D},\acute{E})$ such that*

$$(\tilde{F},\acute{E}) \subset (\tilde{D},\acute{E}) \subset \overline{(\tilde{D},\acute{E})} \subset (\tilde{G},\acute{E}).$$

Proof. Let $(X,\tau,\acute{E})$ be a quadri-partitioned neutrosophic soft $T_1 - space$, $(\tilde{F},\acute{E})$ be a quadri-partitioned neutrosophic soft closed set, and $(\tilde{F},\acute{E}) \subset (\tilde{G},\acute{E}) \in \tau$. Then, $(\tilde{G},\acute{E})$ is a quadri-partitioned neutrosophic soft closed set and $(\tilde{F},\acute{E}) \cap (\tilde{G},\acute{E})^c = 0_{(X,\acute{E})}$. Since $(X,\tau,\acute{E})$ is a quadri-partitioned neutrosophic soft $T_4 - space$, there exist quadri-partitioned neutrosophic soft open sets $(\tilde{D}_1,\acute{E})$, $(\tilde{D}_2,\acute{E})$ such that $(\tilde{F},\acute{E}) \subset (\tilde{D}_1,\acute{E})$ and $(\tilde{G},\acute{E})^c \subset (\tilde{D}_2,\acute{E})$ and $(\tilde{D}_1,\acute{E}) \cap (\tilde{D}_2,\acute{E}) = 0_{(X,\acute{E})}$. This implies that

$$(\tilde{F},\acute{E}) \subset (\tilde{D},\acute{E}) \subset (\tilde{D},\acute{E})^c \subset (\tilde{G},\acute{E}).$$

So $(\tilde{D},\acute{E})^c$ is a quadri-partitioned neutrosophic soft closed set and $(\tilde{D}_1,\acute{E}) \subset (\tilde{D}_2,\acute{E})$ is satisfied. Thus

$$(\tilde{F},\acute{E}) \subset (\tilde{D}_1,\acute{E}) \subset (\tilde{D}_1,\acute{E}) \subset (\tilde{G},\acute{E}).$$

is obtained.

Conversely let $(\tilde{F}_1,\acute{E})$, $(\tilde{F}_2,\acute{E})$ be two disjoint quadri-partitioned neutrosophic soft closed sets. then $(\tilde{F}_1,\acute{E}) \subset (\tilde{F}_2,\acute{E})^c$. From the condition of theorem, there exist a quadri-partitioned neutrosophic soft open set $(\tilde{D},\acute{E})$ such that

$$(\tilde{F}_1,\acute{E}) \subset (\tilde{D},\acute{E}) \subset \overline{(\tilde{G},\acute{E})} \subset (\tilde{F},\acute{E})^c.$$

Thus, $(\tilde{D}, \acute{E})$, $(\overline{(\tilde{D}, \acute{E})})^c$ are quadri-partitioned neutrosophic soft open sets and $(\tilde{F}_1, \acute{E}) \subset (\tilde{D}, \acute{E})$, $(\tilde{F}_2, \acute{E}) \subset (\overline{(\tilde{D}, \acute{E})})^c$ and $(\tilde{D}, \acute{E}) \cap (\overline{(\tilde{D}, \acute{E})})^c = 0_{(X,\acute{E})}$ are obtained. Hence, $(X, \tau, \acute{E})$ is a quadri-partitioned neutrosophic soft $T_4 - space$.

Definition 31. *Let $(X, \tau, \acute{E})$ be a quadri-partitioned neutrosophic soft topological space over X and $(\tilde{F}, \acute{E})$ be an arbitrary quadri-partitioned neutrosophic soft set. Then*

$$\tau = \{(\tilde{F}, \acute{E}), (\tilde{H}, \acute{E}) : (\tilde{H}, \acute{E}) \in \tau\}$$

is said to be quadri-partitioned neutrosophic soft topology on $(\tilde{F}, \acute{E})$ and $(X, \tau, \acute{E})$ is called a quadri-partitioned neutrosophic soft topological subspace of $(X, \tau, \acute{E})$.

Theorem 8. *Let $(X, \tau, \acute{E})$ be a quadri-partitioned neutrosophic soft topological space over X. If $(X, \tau, \acute{E})$ is a quadri-partitioned neutrosophic soft $T_i - space$, then the quadri-partitioned neutrosophic soft topological subspace $(X, \tau, \acute{E})$ is a quadri-partitioned neutrosophic soft $T_i - space$ for $i = 0, 1, 2, 3$.*

Proof. Let $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})$, $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)}) \in (X, \tau, \acute{E})$ such that $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)})$, $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)}) = 0_{(X,\acute{E})}$. Thus, there exist quadri-partitioned neutrosophic soft open sets $(\tilde{F}_1, \acute{E})$ *and* $(\tilde{F}_2, \acute{E})$ satisfying the conditions of double-valued neutrosophic $T_i - space$ such that $(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \in (\tilde{F}_1, \acute{E})$ and $(y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)}) \in (\tilde{F}_2, \acute{E})$. Then

$$(x^{\grave{e}}_{(p_1,p_2,p_3,p_4)}) \in (\tilde{F}_1, \acute{E}) \cap (\tilde{F}, \acute{E}) \ and \ (y^{\grave{e}'}_{(p_1,p_2,p_3,p_4)}) \in (\tilde{F}_2, \acute{E}) \cap (\tilde{F}, \acute{E}).$$

Also, the quadri-partitioned neutrosophic soft $T_i - space$ for $i = 0, 1, 2, 3$.

Theorem 9. *Let $(X, \tau, \acute{E})$ be a quadri-partitioned neutrosophic soft topological space over X. If $(X, \tau, \acute{E})$ is a quadri-partitioned neutrosophic soft $T_4 - space$ and $(\tilde{F}, \acute{E})$ is a quadri-partitioned neutrosophic soft closed set in $(X, \tau, \acute{E})$, and then $(X, \tau, \acute{E})$ is a quadri-partitioned neutrosophic soft $T_4 - space$.*

Proof. Let $(X, \tau, \acute{E})$ is a quadri-partitioned neutrosophic soft $T_4 - space$ and $(\tilde{F}, \acute{E})$ is a quadri-partitioned neutrosophic soft closed set in $(X, \tau, \acute{E})$. Let $(\tilde{F}_1, \acute{E})$ *and* $(\tilde{F}_2, \acute{E})$ be two quadri-partitioned neutrosophic soft closed sets in $(X, \tau, \acute{E})$ such that $(\tilde{F}_1, \acute{E}) \cap (\tilde{F}_2, \acute{E}) = 0_{(X,\acute{E})}$. When $(\tilde{F}, \acute{E})$ is a quadri-partitioned neutrosophic soft closed set in $(X, \tau, \acute{E})$, $(\tilde{F}_1, \acute{E})$ *and* $(\tilde{F}_2, \acute{E})$ be two quadri-partitioned neutrosophic soft closed sets in $(X, \tau, \acute{E})$. Since $(X, \tau, \acute{E})$ is a quadri-partitioned neutrosophic soft $T_4 - space$, there exist quadri-partitioned neutrosophic soft open sets $(\tilde{G}_1, \acute{E})$ *and* $(\tilde{G}_2, \acute{E})$ such that $(\tilde{F}_1, \acute{E}) \subset (\tilde{G}_1, \acute{E})$, $(\tilde{F}_2, \acute{E}) \subset (\tilde{G}_2, \acute{E})$ and $(\tilde{G}_1, \acute{E}) \cap (\tilde{G}_2, \acute{E}) = 0_{(X,\acute{E})}$. Then $(\tilde{F}_1, \acute{E}) = (\tilde{G}_1, \acute{E}) \cap (\tilde{F}, \acute{E})$, $(\tilde{F}_2, \acute{E}) = (\tilde{G}_2, \acute{E}) \cap (\tilde{F}, \acute{E})$ and

$$((\tilde{G}_1, \acute{E}) \cap (\tilde{F}, \acute{E})) \cap ((\tilde{G}_2, \acute{E}) \cap (\tilde{F}, \acute{E})) = 0_{(X,\acute{E})}.$$

This implies that $(X, \tau, \acute{E})$ is a quadri-partitioned neutrosophic soft $T_4 - space$.

5. Comparative Analysis

Proposed Work: Focuses on the quadri-partitioned neutrosophic soft topological space (QPNSTS), with an emphasis on separation axioms and their extended behavior in the context of $T_i - spaces$ for $(i = 0, 1, 2, 3, 4)$. This work introduces higher levels of uncertainty through quadri-partitioned neutrosophic soft sets (QPNSs) and aims to explore their theoretical foundations, offering new definitions and detailed criteria.
First Published Work [38]: Deals with the basic theoretical framework of neutrosophic soft sets and introduces the concept of neutrosophic soft points. The focus is more on redefining concepts like neutrosophic soft $T_i spaces$ which are actually separation axioms and exploring their relationships.
Second Published Work [26]: This work introduces the concept of neutrosophic soft b-structures, focusing on the development of b-separation axioms and the theory of neutrosophic soft $b - T_i spaces$. It provides results related to the neutrosophic soft topology and its application in uncertainty modeling.
In conclusion, while the proposed work focuses on extending separation axioms within the QPNSTS framework, both published works concentrate on various aspects of neutrosophic soft sets and their application, with the second published work being the closest in terms of investigating separation axioms but focusing on b-separation axioms.

6. Conclusion and Future Work

In this study, we have introduced and developed the concept of separation axioms within the framework of quadri-partitioned neutrosophic soft topological spaces. By extending classical separation conditions to this more generalized and uncertainty-tolerant environment, we defined and analyzed $T_4 - spaces$ for $(i = 0, 1, 2, 3, 4)$, providing rigorous criteria for each and highlighting their distinctive characteristics. The behavior and interplay of these axioms were examined through carefully constructed examples, reinforcing their theoretical foundations and practical relevance. Furthermore, we investigated key relationships among the proposed axioms and explored their connections to existing results in the broader context of neutrosophic soft topology. These findings not only enrich the theoretical landscape but also offer promising directions for application, particularly in areas requiring nuanced decision-making under uncertainty. Looking ahead, we aim to extend this work to triple-valued neutrosophic soft topological spaces, focusing on more advanced structural properties such as compactness and connectedness. These explorations will be complemented by real-world applications, potentially leveraging advanced machine learning techniques to illustrate the practical utility of the developed theories.

Acknowledgements

(i) We extend our sincere gratitude to Prof. Dr. Arif Mehmood, principal investigator of our collaborative project and leader of a 176-member research team. His exceptional leadership, vision, and unwavering support have been instrumental to our success. His guidance not only shaped our work but also inspired us to maintain professionalism and perseverance throughout this journey. We are truly thankful for his invaluable mentorship.

(ii) The authors extend their appreciation to the Deanship of Scientific Research at Northern Border University, Arar, KSA for funding this research work through the project number "NBU-FFR-2025-2727-09" .

Author Contributions: All authors read and approved the final manuscript.
Funding Statement: The authors received no specific funding for this study.
Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.
Availability of Data and Materials: All the data and materials are provided in the manuscript.

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