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A New Decision-Making Approach Using Bipolar Interval-Valued Fuzzy Soft Matrices

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Abstract. Based on what the fuzzy set theory has earned a lot of significant interest from researchers and has led to several important extensions. The bipolar fuzzy sets (BFSs) have gained a lot of applications in different fields. The BFS includes extensions such as interval bipolar fuzzy sets, which are BFS but in an interval form. In this article, we introduce a new hypermodel called bipolar interval fuzzy soft sets (BIVFSSs) in matrix form when we extend the bipolar fuzzy soft matrix to effectively and flexibly represent and manipulate uncertain information with increased flexibility. The matrix form gives BIVFSSs more freedom and accuracy in handling data and performing more algebraic operations. Therefore, the present study coined the notion of determinant on a bipolar interval-valued fuzzy soft matrix (BIVFSM) and its properties. Then we presented all the mathematical properties of this form, supported by numerical examples and representations, to explain the mechanism of operation of these tools accurately. Finally, these tools have been applied to solve a multi-attribute decision-making problem by proposing a multi-step algorithm.

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Key Words and Phrases: Fuzzy matrices, interval fuzzy matrices, soft set, decision-making, bipolar fuzzy soft matrices

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1. Introduction

Zadeh in 1965 proposed the theory of fuzzy set (FS) [1] as a mathematical extension of crisp set in order to handle uncertainty in the various real applications. The traditional FS is represented by a well-single functioning membership function (single belonging function) whose starting point is the universal set and whose rest is the closed interval [0,1]. The dynamics of coupling the elements of the comprehensive set with the single membership function has attracted considerable interest from users, especially those who employ these mathematical tools in the decision-making process, i.e., choosing the optimal option from among several presented options. For example, if the comprehensive set contains three cars and one of them is to be chosen, using the single membership function, each of these cars can be assigned a value belonging to the closed interval [0, 1]. Accordingly, the choice will be for the car that obtained a score close to 1. In search of more flexibility, Turksen [2] applied the concept of interval-valued FSs (IV-FSs) when he changed the form of value from single value to interval value, looking for more flexibility to handle uncertainty in the various real applications. The issues that FS dealt with were dealt with by IVFS, who proved to be superior, flexible, and more adaptable in putting data on daily life problems. At present, IV-FSs have attracted wide attention from many researchers, and they have made some achievements. Petry and Yager [3] developed some applications on IV-FS information. Liu et al.[4] developed some common measures on IV-FSs and used these measures in handling DM problems. The soft set (SS) [5] was merged with FS by Cagman [6]. Tripathy et al. [7] founded hybrid models called IV-FSSs when they merged IV-FSs and SSs. By looking to SS structures, it has a clear representation of the elements of the universal set. The SS has many extensions, for example see [8-13]. However, the integration of SSs is not limited to the FSs; rather, it extends to other extensions of the FSs, for example: Abuqamar and Hassan [14,15] defined normal groups on Q-neutrosophic soft sets as a new algebraic approach of FSs . Saeed et al. [16]. modified a new hybrid of the Fermatean neutrosophic soft set. Alkhazaleh [17] introduced the concept of an effective fuzzy soft set, and he showed its operation. In any case, the nature of thinking of the decision maker is characterized by two sides: positive and negative. That is, for every situation, the human mind takes two sides: positive and negative. To illustrate this idea, Zhang [18] suggests the idea of bipolar fuzzy sets (BFS) where every belonging function has two positive and negative sides, called positive belonging function and the negative belonging function, and both of them have a stable location that differs from the other, meaning that the short period that we mentioned previously expands to [-1,1]. Now there are many contributions based on this idea, including bipolar interval -valued fuzzy set (BIVFS) [19] when we change the positive and negative constraints from single to interval form and other works [20 -24]. Al-Sharqi et al [25,26] introduced BNHSS theory applied on DM. On the other hand, the matrix is considered an important algebraic tool for representing data in rows and columns. Matrices have many uses in economics, engineering, medicine, science, and other areas of life. In this work, we aim to benefit from the properties of the matrix by employing it in fuzzy environments. In this work, we aim to present a new fuzzy algebraic concept called bipolar interval-valued fuzzy soft matrices (BIVFSM) when we present the BIVFSS information in matrix form. This structure gives this concept more flexibility and more applicability to other mathematical operations. The research gap between the two approaches, BFSMs and BIVFSSMs, is evident in how they cover data for decision-making problems and which one offers more flexibility than the other. This pushes us in a strong way to handle the shortage that showing in BFSMs. Therefore, the BIVFSSMs use interval values, hence offers a more comprehensive representation of data for decision-making problems Also, an illustration discussed the properties and the determinant of the BIVFSM. In addition, the 3D representations of the BIFM are depicted in this study as well as we will employ these new structures in DM-problems.

Main contributions:

(i) The idea of BIVFSM was defined as an extension of the fuzzy matrix as a combination of BIVFSMs and SSs.

(ii) The basic operations on these structures are explained with some properties and some numerical examples to illustrate the working mechanism.

(iii) The determinant of the BIV-FMs is proposed and shown their efficient by some numerical examples to illustrate the working mechanism.

(iv) A practical application is presented that demonstrates the importance of this concept in solving everyday life problems based on a proposed algorithm.

This article consists of five parts, as follows: Section 2 provides a definition of BIV-FSSs in set form. Section 3 organizes the main definition of BIV-FSMs and their main properties with some illustrative examples. Section 4 shows the definition of the determinant of the BIV-FMs. Section 5. practical application of the decision-making mechanism using the proposed tools.

2. Bipolar Interval-Valued Fuzzy Soft Set (BIV-FSSs)

Here in this section, we provide the official definition of the BIVFSS structure, which combines two prior concepts, namely interval fuzzy set and soft set, under bipolarity properties.

Definition 1. *Assume that X be non – empty universe set and Y be a parametrized set contains all parametrized that represent X element's. Then the constituent $\Gamma_{A_i} = (\Gamma, A_i \subseteq Y)$ is known as an $BIV-FSS$ over non – empty soft universe set (X,Y) where $\Gamma_{A_i} : X \to BIV-FSS(X)$. such that here $BIV-FSS(X)$. denotes all of the family of all $BIV-FSS(X)$. of X and it is given in the following form:*

$$\Gamma_{A_i} = \{(\dot{\tau}, B^{+}(\dot{\tau})(\sigma), B^{-}(\dot{\tau})(\sigma)) \,|\, \forall \dot{\tau} \in X \text{ and } \sigma \in A_i \subseteq Y\}$$
$$= \left\{ \left(\dot{\tau}, \left[B^{+,L}(\dot{\tau})(\sigma), B^{+,U}(\dot{\tau})(\sigma)\right], \left[B^{-,L}(\dot{\tau})(\sigma), B^{-,U}(\dot{\tau})(\sigma)\right]\right) \,\middle|\, \forall \dot{\tau} \in X \right\}$$
$$\textit{where}$$

$$B^{+,L}(\dot{\tau})(\sigma), B^{+,U}(\dot{\tau})(\sigma) : (X, A_i \subseteq Y) \to [0,1],$$
$$B^{-,L}(\dot{\tau})(\sigma), B^{-,U}(\dot{\tau})(\sigma) : (X, A_i \subseteq Y) \to [-1,0].$$

Example 1. *Let* $X = \{\dot{\tau}_1, \dot{\tau}_2, \dot{\tau}_3, \dot{\tau}_4, \dot{\tau}_5\}$ *be a non – empty universe set represent five cars in one car exhibition in KL.* $Y = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$ *be a parametrized set contains all parametrized that represent* X *element's where*
$\sigma_1 = Beautiful$,$\sigma_2 = Costly$,$\sigma_3 = Apply\ Modern\ Technology$ *and* $\sigma_4 = Fuel\ Efficient$ *and* $A_1 \subseteq Y = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$.
Then

$$\Gamma_{A_1} = (\Gamma, A_1) = \left\{ \begin{array}{l} \Gamma(\sigma_1) = \left\{ \begin{array}{l} \dot{\tau}_1, \langle [0.2, 0.5], [-0.6, -0.4] \rangle \\ \dot{\tau}_2, \langle [0.7, 0.9], [-0.5, -0.3] \rangle \\ \dot{\tau}_3, \langle [0.2, 0.8], [-0.8, -0.2] \rangle \\ \dot{\tau}_4, \langle [0.1, 0.4], [-0.1, -0.1] \rangle \\ \dot{\tau}_5, \langle [0.1, 0.4], [-0.4, -0.3] \rangle \end{array} \right\} \\ \Gamma(\sigma_2) = \left\{ \begin{array}{l} \dot{\tau}_1, \langle [0.3, 0.7], [-0.7, -0.1] \rangle \\ \dot{\tau}_2, \langle [0.8, 0.9], [-0.9, -0.9] \rangle \\ \dot{\tau}_3, \langle [0.4, 0.7], [-0.6, -0.5] \rangle \\ \dot{\tau}_4, \langle [0.2, 0.5], [-0.5, -0.3] \rangle \\ \dot{\tau}_5, \langle [0.1, 0.8], [-0.7, -0.7] \rangle \end{array} \right\} \\ \Gamma(\sigma_3) = \left\{ \begin{array}{l} \dot{\tau}_1, \langle [0.3, 0.7], [-0.7, -0.1] \rangle \\ \dot{\tau}_2, \langle [0.8, 0.9], [-0.9, -0.9] \rangle \\ \dot{\tau}_3, \langle [0.4, 0.7], [-0.6, -0.5] \rangle \\ \dot{\tau}_4, \langle [0.2, 0.5], [-0.5, -0.3] \rangle \\ \dot{\tau}_5, \langle [0.1, 0.8], [-0.7, -0.7] \rangle \end{array} \right\} \\ \Gamma(\sigma_4) = \left\{ \begin{array}{l} \dot{\tau}_1, \langle [0.4, 0.5], [-0.3, -0.1] \rangle \\ \dot{\tau}_2, \langle [0.7, 0.9], [-0.5, -0.3] \rangle \\ \dot{\tau}_3, \langle [0.2, 0.7], [-0.5, -0.1] \rangle \\ \dot{\tau}_4, \langle [0.1, 0.5], [-0.5, -0.3] \rangle \\ \dot{\tau}_5, \langle [0.8, 0.8], [-0.7, -0.7] \rangle \end{array} \right\} \end{array} \right\}$$

3. Bipolar Interval-Valued Fuzzy Soft Matrices (BIV-FSMs)

We present the definition of BIVFSS in the form of a matrix in order to generate a new concept called Bipolar interval-valued fuzzy soft matrices (BIV-FSMs) along with numerical examples to show how it works.

Definition 2. *Let* $X = \{\dot{\tau}_1, \dot{\tau}_2, \dot{\tau}_3, \ldots, \dot{\tau}_n\}$ *be a non-empty universe set containing a set of alternatives (rows) and* $Y = \{\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_m\}$ *be a parameterized set containing the attributes (columns) of each component in* X. *Then a BIV-FSM is given by*

$$\Gamma_{A_i} = \{(\dot{\tau}_n, \sigma_m), B^+(\dot{\tau}_n, \sigma_m), B^-(\dot{\tau}_n, \sigma_m)\}$$

where

$$B^{+}(\dot{\tau}_n,\sigma_m)=[B^{+,L}(\dot{\tau}_n,\sigma_m),B^{+,U}(\dot{\tau}_n,\sigma_m)],\quad B^{-}(\dot{\tau}_n,\sigma_m)=[B^{-,L}(\dot{\tau}_n,\sigma_m),B^{-,U}(\dot{\tau}_n,\sigma_m)]$$

and for all $n=1,2,3,\ldots,q$ and $m=1,2,3,\ldots,r$ such that

$$B^{+,L}(\dot{\tau}_n,\sigma_m),B^{+,U}(\dot{\tau}_n,\sigma_m):X\times Y\to[0,1],\quad B^{-,L}(\dot{\tau}_n,\sigma_m),B^{-,U}(\dot{\tau}_n,\sigma_m):X\times Y\to[-1,0].$$

Generally, we indicate the notions of

$$B^{+}(\dot{\tau}_n,\sigma_m)=[B^{+,L}(\dot{\tau}_n,\sigma_m),B^{+,U}(\dot{\tau}_n,\sigma_m)]$$

to $\bar{a}_{(n\times m)}=[\bar{a}^{+,L}_{(n\times m)},\bar{a}^{+,U}_{(n\times m)}]$ and

$$B^{-}(\dot{\tau}_n,\sigma_m)=[B^{-,L}(\dot{\tau}_n,\sigma_m),B^{-,U}(\dot{\tau}_n,\sigma_m)]$$

to $\bar{b}_{(n\times m)}=[\bar{b}^{-,L}_{(n\times m)},\bar{b}^{-,U}_{(n\times m)}]$. Then we have

$$\Gamma_{A_i}=[\bar{a}^{+}_{(n\times m)},\bar{b}^{-}_{(n\times m)}]=$$
$$\begin{bmatrix} \left([\bar{a}^{+,L}_{(1n\times 11)},\bar{a}^{+,U}_{(1n\times 11)}],[\bar{b}^{-,L}_{(1n\times 11)},\bar{b}^{-,U}_{(1n\times 11)}]\right) & \cdots & \left([\bar{a}^{+,L}_{(1n\times m)},\bar{a}^{+,U}_{(1n\times m)}],[\bar{b}^{-,L}_{(1n\times m)},\bar{b}^{-,U}_{(1n\times m)}]\right) \\ \vdots & \ddots & \vdots \\ \left([\bar{a}^{+,L}_{(n1\times 1m)},\bar{a}^{+,U}_{(n1\times 1m)}],[\bar{b}^{-,L}_{(n1\times 1m)},\bar{b}^{-,U}_{(n1\times 1m)}]\right) & \cdots & \left([\bar{a}^{+,L}_{(n\times m)},\bar{a}^{+,U}_{(n\times m)}],[\bar{b}^{-,L}_{(n\times m)},\bar{b}^{-,U}_{(n\times m)}]\right) \end{bmatrix}$$

Definition 3. *Let $\Gamma^1_A=\left\{\left([\bar{a}^{+,L}_{(n\times m)},\bar{a}^{+,U}_{(n\times m)}],[\bar{b}^{-,L}_{(n\times m)},\bar{b}^{-,U}_{(n\times m)}]\right)\right\}$ be BIV-FSMs on $(X\times Y)$. Then Γ^1_A is called a zero matrix if*

$$\Gamma^1_A=\left\{\left([\bar{0}^{+,L}_{(n\times m)},\bar{0}^{+,U}_{(n\times m)}],[\bar{0}^{-,L}_{(n\times m)},\bar{0}^{-,U}_{(n\times m)}]\right)\right\}.$$

Definition 4. *Let $\Gamma^1_A=\left\{\left([\bar{a}^{+,L}_{(n\times m)},\bar{a}^{+,U}_{(n\times m)}],[\bar{b}^{-,L}_{(n\times m)},\bar{b}^{-,U}_{(n\times m)}]\right)\right\}$ be BIV-FSMs on $(X\times Y)$. Then Γ^1_A is called an absolute matrix if*

$$\Gamma^1_A=\left\{\left([\bar{1}^{+,L}_{(n\times m)},\bar{1}^{+,U}_{(n\times m)}],[-1^{-,L}_{(n\times m)},-1^{-,U}_{(n\times m)}]\right)\right\}.$$

Definition 5. *Let $\Gamma^1_A=\left\{\left([\bar{a}^{+,L}_{(n\times m)},\bar{a}^{+,U}_{(n\times m)}],[\bar{b}^{-,L}_{(n\times m)},\bar{b}^{-,U}_{(n\times m)}]\right)\right\}$ be BIV-FSMs on $(X\times Y)$. Then Γ^1_A is called a sequel matrix if the number of rows (n) equals the number of columns (m).*

Definition 6. *Let*

$$\Gamma^1_A=\left\{\left([\bar{a}^{+,L}_{(n\times m)},\bar{a}^{+,U}_{(n\times m)}],[\bar{b}^{-,L}_{(n\times m)},\bar{b}^{-,U}_{(n\times m)}]\right)\right\}$$

and

$$\Gamma^2_A=\left\{\left([\bar{c}^{+,L}_{(n\times m)},\bar{c}^{+,U}_{(n\times m)}],[\bar{d}^{-,L}_{(n\times m)},\bar{d}^{-,U}_{(n\times m)}]\right)\right\}$$

be two BIV-FSMs on $(X \times Y)$*. Then we say that* $\Gamma_A^1 \leq \Gamma_A^2$ *if the values of both matrices are given as follows:*

$$\bar{a}^{+,L}_{(n\times m)} \leq \bar{c}^{+,L}_{(n\times m)}, \quad \bar{a}^{+,U}_{(n\times m)} \leq \bar{c}^{+,U}_{(n\times m)}, \quad \bar{b}^{-,L}_{(n\times m)} \leq \bar{d}^{-,L}_{(n\times m)}, \quad \bar{b}^{-,U}_{(n\times m)} \leq \bar{d}^{-,U}_{(n\times m)}.$$

Definition 7. *Let*

$$\Gamma_A^1 = \left\{ \left([\bar{a}^{+,L}_{(n\times m)}, \bar{a}^{+,U}_{(n\times m)}], [\bar{b}^{-,L}_{(n\times m)}, \bar{b}^{-,U}_{(n\times m)}] \right) \right\}$$

and

$$\Gamma_A^2 = \left\{ \left([\bar{c}^{+,L}_{(n\times m)}, \bar{c}^{+,U}_{(n\times m)}], [\bar{d}^{-,L}_{(n\times m)}, \bar{d}^{-,U}_{(n\times m)}] \right) \right\}$$

be two BIV-FSMs on $(X \times Y)$*. Then we say that* $\Gamma_A^1 = \Gamma_A^2$ *if the values of both matrices are given as follows:*

$$\bar{a}^{+,L}_{(n\times m)} = \bar{c}^{+,L}_{(n\times m)}, \quad \bar{a}^{+,U}_{(n\times m)} = \bar{c}^{+,U}_{(n\times m)}, \quad \bar{b}^{-,L}_{(n\times m)} = \bar{d}^{-,L}_{(n\times m)}, \quad \bar{b}^{-,U}_{(n\times m)} = \bar{d}^{-,U}_{(n\times m)}.$$

Definition 8. *Let*

$$\Gamma_A^1 = \left\{ \left([\bar{a}^{+,L}_{(n\times m)}, \bar{a}^{+,U}_{(n\times m)}], [\bar{b}^{-,L}_{(n\times m)}, \bar{b}^{-,U}_{(n\times m)}] \right) \right\}$$

and

$$\Gamma_A^2 = \left\{ \left([\bar{c}^{+,L}_{(n\times m)}, \bar{c}^{+,U}_{(n\times m)}], [\bar{d}^{-,L}_{(n\times m)}, \bar{d}^{-,U}_{(n\times m)}] \right) \right\}$$

be two BIV-FSMs on $(X \times Y)$*. Then the addition (+) of two BIV-FSMs is denoted as*

$$\Gamma_A^1 + \Gamma_A^2 = \left\{ \left[\frac{\bar{a}^{+,L}_{(n\times m)} + \bar{c}^{+,L}_{(n\times m)}}{2}, \frac{\bar{a}^{+,U}_{(n\times m)} + \bar{c}^{+,U}_{(n\times m)}}{2} \right], \left[\frac{\bar{b}^{-,L}_{(n\times m)} + \bar{d}^{-,L}_{(n\times m)}}{2}, \frac{\bar{b}^{-,U}_{(n\times m)} + \bar{d}^{-,U}_{(n\times m)}}{2} \right] \right\}.$$

Definition 9. *Let*

$$\Gamma_A^1 = \left\{ \left([\bar{a}^{+,L}_{(n\times m)}, \bar{a}^{+,U}_{(n\times m)}], [\bar{b}^{-,L}_{(n\times m)}, \bar{b}^{-,U}_{(n\times m)}] \right) \right\}$$

and

$$\Gamma_A^2 = \left\{ \left([\bar{c}^{+,L}_{(n\times m)}, \bar{c}^{+,U}_{(n\times m)}], [\bar{d}^{-,L}_{(n\times m)}, \bar{d}^{-,U}_{(n\times m)}] \right) \right\}$$

be two BIV-FSMs on $(X \times Y)$*. Then the subtraction (−) of two BIV-FSMs is denoted as*

$$\Gamma_A^1 - \Gamma_A^2 = \left\{ \left[|\bar{a}^{+,L}_{(n\times m)} - \bar{c}^{+,L}_{(n\times m)}|, |\bar{a}^{+,U}_{(n\times m)} - \bar{c}^{+,U}_{(n\times m)}| \right], \left[\frac{\bar{b}^{-,L}_{(n\times m)} + \bar{d}^{-,L}_{(n\times m)}}{2}, \frac{\bar{b}^{-,U}_{(n\times m)} + \bar{d}^{-,U}_{(n\times m)}}{2} \right] \right\}.$$

Definition 10. *Let*

$$\Gamma_A^1 = \left\{ \left([\bar{a}^{+,L}_{(n\times m)}, \bar{a}^{+,U}_{(n\times m)}], [\bar{b}^{-,L}_{(n\times m)}, \bar{b}^{-,U}_{(n\times m)}] \right) \right\}$$

and

$$\Gamma_A^2 = \left\{ \left([\bar{c}^{+,L}_{(n\times m)}, \bar{c}^{+,U}_{(n\times m)}], [\bar{d}^{-,L}_{(n\times m)}, \bar{d}^{-,U}_{(n\times m)}] \right) \right\}$$

be two BIV-FSMs on $(X \times Y)$. Then the multiplication $(\times)$ of two BIV-FSMs is denoted as

$$\Gamma_A^1 \times \Gamma_A^2 = \left\{ \left[|\bar{a}^{+,L}_{(n\times m)} \times \bar{c}^{+,L}_{(n\times m)}|, |\bar{a}^{+,U}_{(n\times m)} \times \bar{c}^{+,U}_{(n\times m)}| \right], \left[-(\bar{b}^{-,L}_{(n\times m)} \times \bar{d}^{-,L}_{(n\times m)}), -(\bar{b}^{-,U}_{(n\times m)} \times \bar{d}^{-,U}_{(n\times m)}) \right] \right\}.$$

Definition 11. *Let*

$$\Gamma_A^1 = \left\{ \left([\bar{a}^{+,L}_{(n\times m)}, \bar{a}^{+,U}_{(n\times m)}], [\bar{b}^{-,L}_{(n\times m)}, \bar{b}^{-,U}_{(n\times m)}] \right) \right\}$$

be a BIV-FSM on $(X \times Y)$. Then the fuzzy scalar multiplication $k \in [0,1]$ of the BIV-FSM is denoted as

$$k\Gamma_A^1 = \left\{ \left[k\bar{a}^{+,L}_{(n\times m)}, k\bar{a}^{+,U}_{(n\times m)} \right], \left[k\bar{b}^{-,L}_{(n\times m)}, k\bar{b}^{-,U}_{(n\times m)} \right] \right\}.$$

Definition 12. *Let*

$$\Gamma_A^1 = \left\{ \left([\bar{a}^{+,L}_{(n\times m)}, \bar{a}^{+,U}_{(n\times m)}], [\bar{b}^{-,L}_{(n\times m)}, \bar{b}^{-,U}_{(n\times m)}] \right) \right\}$$

and

$$\Gamma_A^2 = \left\{ \left([\bar{c}^{+,L}_{(n\times m)}, \bar{c}^{+,U}_{(n\times m)}], [\bar{d}^{-,L}_{(n\times m)}, \bar{d}^{-,U}_{(n\times m)}] \right) \right\}$$

be two BIV-FSMs on $(X \times Y)$. Then the maximum operation between two BIV-FSMs is denoted as $(\Gamma_A^1 \vee \Gamma_A^2)$ and the minimum operation as $(\Gamma_A^1 \wedge \Gamma_A^2)$, respectively, and is given as follows:

$$\Gamma_A^1 \vee \Gamma_A^2 = \left\{ \left(\max[\bar{a}^{+,L}_{(n\times m)}, \bar{c}^{+,L}_{(n\times m)}], \max[\bar{a}^{+,U}_{(n\times m)}, \bar{c}^{+,U}_{(n\times m)}] \right), \left(\min[\bar{b}^{-,L}_{(n\times m)}, \bar{d}^{-,L}_{(n\times m)}], \min[\bar{b}^{-,U}_{(n\times m)}, \bar{d}^{-,U}_{(n\times m)}] \right) \right\}.$$

Example 2. *Take two BIV-FSMs Γ_A^1 and Γ_A^2 given as follows:*

$$\Gamma_A^1 = \begin{bmatrix} ([0.3, 0.4], [-0.2, -0.1]) & ([0.6, 0.8], [-0.6, -0.5]) \\ ([0.1, 0.1], [-0.4, -0.2]) & ([0.7, 0.9], [-0.4, -0.4]) \end{bmatrix}$$

And

$$\Gamma_A^2 = \begin{bmatrix} ([0.5, 0.7], [-0.5, -0.4]) & ([0.3, 0.6], [-0.6, -0.3]) \\ ([0.3, 0.3], [-0.6, -0.6]) & ([0.5, 0.8], [-0.2, -0.0]) \end{bmatrix}$$

Then

$$\Gamma_A^1 \vee \Gamma_A^2 = \begin{bmatrix} ([0.5, 0.7], [-0.5, -0.4]) & ([0.6, 0.8], [-0.6, -0.5]) \\ ([0.3, 0.3], [-0.6, -0.6]) & ([0.7, 0.9], [-0.4, -0.4]) \end{bmatrix}$$

$$\Gamma_A^1 \wedge \Gamma_A^2 = \begin{bmatrix} ([0.3, 0.4], [-0.2, -0.1]) & ([0.3, 0.6], [-0.6, -0.3]) \\ ([0.3, 0.3], [-0.4, -0.2]) & ([0.7, 0.9], [-0.2, -0.0]) \end{bmatrix}$$

Definition 13. *Let*

$$\Gamma_A^1 = \left\{ \left([\bar{a}^{+,L}_{(n\times m)}, \bar{a}^{+,U}_{(n\times m)}], [\bar{b}^{-,L}_{(n\times m)}, \bar{b}^{-,U}_{(n\times m)}] \right) \right\}$$

and

$$\Gamma_A^2 = \left\{ \left([\bar{c}^{+,L}_{(n\times m)}, \bar{c}^{+,U}_{(n\times m)}], [\bar{d}^{-,L}_{(n\times m)}, \bar{d}^{-,U}_{(n\times m)}] \right) \right\}$$

be two BIV-FSMs on $(X \times Y)$. Then the minimum operation between two BIV-FSMs is denoted as $(\Gamma_A^1 \wedge \Gamma_A^2)$ and is given as follows:

$$\Gamma_A^1 \wedge \Gamma_A^2 = \left\{ \left(\min[\bar{a}^{+,L}_{(n\times m)}, \bar{c}^{+,L}_{(n\times m)}], \min[\bar{a}^{+,U}_{(n\times m)}, \bar{c}^{+,U}_{(n\times m)}] \right), \left(\max[\bar{b}^{-,L}_{(n\times m)}, \bar{d}^{-,L}_{(n\times m)}], \max[\bar{b}^{-,U}_{(n\times m)}, \bar{d}^{-,U}_{(n\times m)}] \right) \right\}.$$

Example 3. *Take two BIV-FSMs Γ_A^1 and Γ_A^2 given as follows:*

$$\Gamma_A^1 = \begin{bmatrix} ([0.3, 0.4], [-0.2, -0.1]) & ([0.6, 0.8], [-0.6, -0.5]) \\ ([0.1, 0.1], [-0.4, -0.2]) & ([0.7, 0.9], [-0.4, -0.4]) \end{bmatrix}$$

And

$$\Gamma_A^2 = \begin{bmatrix} ([0.5, 0.7], [-0.5, -0.4]) & ([0.3, 0.6], [-0.6, -0.3]) \\ ([0.3, 0.3], [-0.6, -0.6]) & ([0.5, 0.8], [-0.2, -0.0]) \end{bmatrix}$$

Then

$$\Gamma_A^1 \wedge \Gamma_A^2 = \begin{bmatrix} ([0.3, 0.4], [-0.2, -0.1]) & ([0.3, 0.6], [-0.6, -0.3]) \\ ([0.3, 0.3], [-0.4, -0.2]) & ([0.7, 0.9], [-0.2, -0.0]) \end{bmatrix}$$

Definition 14. *Let Γ_A^1 be a BIV-FSM on the ordered pair $(X \times Y)$. Then $(\Gamma_A^1)^c$ is called the complement of Γ_A^1 and is given as follows:*

$$(\Gamma_A^1)^c = \left[[1 - \bar{a}^{+,U}_{(n\times m)}, (1 - \bar{a}^{+,L}_{(n\times m)})], [-1 - \bar{b}^{-,U}_{(n\times m)}, -1 - \bar{b}^{-,L}_{(n\times m)}] \right]$$

Example 4. *Consider Γ_A^1 that is given in Example 3, then $(\Gamma_A^1)^c$ is given as follows:*

$$(\Gamma_A^1)^c = \begin{bmatrix} ([0.6, 0.7], [-0.9, -0.2]) & ([0.2, 0.4], [-0.5, -0.4]) \\ ([0.9, 0.9], [-0.8, -0.6]) & ([0.1, 0.3], [-0.6, -0.6]) \end{bmatrix}$$

Proposition 1. Let Γ_A^1 be a BIV-FSM on the ordered pair $(X \times Y)$. Then

$$((\Gamma_A^1)^c)^c = \Gamma_A^1.$$

Proof: This proposition is clear based on Definition 14.

Definition 15. *Let Γ_A^1 and Γ_A^2 be two BIV-FSMs on the ordered pair $(X \times Y)$. Then the addition operation between Γ_A^1 and Γ_A^2 is given as follows:*

$$\Gamma_A^1 + \Gamma_A^2 = \left\{ \left(\min[\bar{a}^{+,L}_{(n\times m)}, \bar{c}^{+,L}_{(n\times m)}], \max[\bar{a}^{+,U}_{(n\times m)}, \bar{c}^{+,U}_{(n\times m)}] \right), \left(\min[\bar{b}^{-,L}_{(n\times m)}, \bar{d}^{-,L}_{(n\times m)}], \max[\bar{b}^{-,U}_{(n\times m)}, \bar{d}^{-,U}_{(n\times m)}] \right) \right\}.$$

Example 5. *Take two BIV-FSMs Γ_A^1 and Γ_A^2 given as follows:*

$$\Gamma_A^1 = \begin{bmatrix} ([0.3, 0.4], [-0.2, -0.1]) & ([0.6, 0.8], [-0.6, -0.5]) \\ ([0.1, 0.1], [-0.4, -0.2]) & ([0.7, 0.9], [-0.4, -0.4]) \end{bmatrix}$$

And

$$\Gamma_A^2 = \begin{bmatrix} ([0.5, 0.7], [-0.5, -0.4]) & ([0.3, 0.6], [-0.6, -0.3]) \\ ([0.3, 0.3], [-0.6, -0.6]) & ([0.5, 0.8], [-0.2, -0.0]) \end{bmatrix}$$

Then

$$\Gamma_A^1 + \Gamma_A^2 = \begin{bmatrix} ([0.5, 0.7], [-0.5, -0.4]) & ([0.6, 0.8], [-0.6, -0.5]) \\ ([0.3, 0.3], [-0.6, -0.6]) & ([0.7, 0.9], [-0.4, -0.4]) \end{bmatrix}$$

Proposition 2. Let Γ_A^1, Γ_A^2, and Γ_A^3 be three BIV-FSMs on $(X \times Y)$. Then:

(i) $\Gamma_A^1 + \Gamma_A^1 = \Gamma_A^1$

(ii) $\Gamma_A^1 + \Gamma_A^2 = \Gamma_A^2 + \Gamma_A^1$

(iii) $\Gamma_A^1 + (\Gamma_A^2 + \Gamma_A^3) = (\Gamma_A^1 + \Gamma_A^2) + \Gamma_A^3$

Proof. The proofs of all the above points depend on Definition 16 and the properties mentioned above.

Definition 16. *Let Γ_A^1 and Γ_A^2 given in Definition 3.1 be two BIV-FSMs on $(X \times Y)$. Then the max-min composition between Γ_A^1 and Γ_A^2 is given as follows:*

$$\Gamma_A^1 \odot \Gamma_A^2 = \left\{ \left(\bigvee_{k=1}^{n} \left(mix[\bar{a}^{+,L}_{km}, \bar{c}^{+,L}_{km}] \right), \bigvee_{k=1}^{n} \left(mix[\bar{a}^{+,U}_{km}, \bar{c}^{+,U}_{km}] \right), \bigwedge_{k=1}^{n} \left(\max[\bar{b}^{-,L}_{km}, \bar{d}^{-,L}_{km}] \right), \bigwedge_{k=1}^{n} \left(\max[\bar{b}^{-,U}_{km}, \bar{d}^{-,U}_{km}] \right) \right) \right\}$$

Definition 17. *Let Γ_A^1 and Γ_A^2 given in Definition 3.1 be two BIV-FSMs on $(X \times Y)$. Then the min-mix composition between Γ_A^1 and Γ_A^2 is given as follows:*

$$\Gamma_A^1 \oplus \Gamma_A^2 = \left\{ \left(\bigwedge_{k=1}^{n} \left(\max[\bar{a}^{+,L}_{km}, \bar{c}^{+,L}_{km}] \right), \bigwedge_{k=1}^{n} \left(\max[\bar{a}^{+,U}_{km}, \bar{c}^{+,U}_{km}] \right), \bigvee_{k=1}^{n} \left(\min[\bar{b}^{-,L}_{km}, \bar{d}^{-,L}_{km}] \right), \bigvee_{k=1}^{n} \left(\min[\bar{b}^{-,U}_{km}, \bar{d}^{-,U}_{km}] \right) \right) \right\}$$

4. Determinant of the Bipolar Interval-Valued Fuzzy Soft Matrices

In this section of this article, we discuss the definition of determinants of BIV-FSM, where any ordinary matrix has a determinant whose output values are within the closed interval [1,-1].

Definition 18. *Let Γ_A^1 given in Definition 2 be a BIV-FSM on $(X \times Y)$. Then the bipolar interval-valued fuzzy soft determinant of Γ_A^1 is given as follows:*

$$\left|\Gamma_A^1\right| = \begin{vmatrix} \left(\left[\ddot{a}_{1\times 1}^{+,L}, \ddot{a}_{1\times 1}^{+,U}\right], \left[\ddot{b}_{1\times 1}^{-,L}, \ddot{b}_{1\times 1}^{-,U}\right]\right) & \cdots & \left(\left[\ddot{a}_{1\times m}^{+,L}, \ddot{a}_{1\times m}^{+,U}\right], \left[\ddot{b}_{1\times m}^{-,L}, \ddot{b}_{1\times m}^{-,U}\right]\right) \\ \left(\left[\ddot{a}_{2\times 1}^{+,L}, \ddot{a}_{2\times 1}^{+,U}\right], \left[\ddot{b}_{2\times 1}^{-,L}, \ddot{b}_{2\times 1}^{-,U}\right]\right) & \cdots & \left(\left[\ddot{a}_{2\times m}^{+,L}, \ddot{a}_{2\times m}^{+,U}\right], \left[\ddot{b}_{2\times m}^{-,L}, \ddot{b}_{2\times m}^{-,U}\right]\right) \\ & \vdots \; \vdots \; \vdots & \\ \left(\left[\ddot{a}_{n\times 1}^{+,L}, \ddot{a}_{n\times 1}^{+,U}\right], \left[\ddot{b}_{n\times 1}^{-,L}, \ddot{b}_{n\times 1}^{-,U}\right]\right) & \cdots & \left(\left[\ddot{a}_{n\times m}^{+,L}, \ddot{a}_{n\times m}^{+,U}\right], \left[\ddot{b}_{n\times m}^{-,L}, \ddot{b}_{n\times m}^{-,U}\right]\right) \end{vmatrix}$$

$$= \left(\left[\bigwedge_{k=1}^{n} \bar{a}_{(k,k)}^{+,L}, \bigvee_{k=1}^{n} \bar{a}_{(k,k)}^{+,U}\right], \left[\bigwedge_{k=1}^{n} \bar{b}_{(k,k)}^{-,L}, \bigvee_{k=1}^{n} \bar{b}_{(k,k)}^{-,U}\right]\right)$$
$$= \left([\bar{a}^{+,L}, \bar{a}^{+,U}], [\bar{b}^{-,L}, \bar{b}^{-,U}]\right)$$

Example 6. *Take the BIV-FSM Γ_A^1 given as follows:*

$$\Gamma_A^1 = \begin{bmatrix} ([0.3, 0.4], [-0.2, -0.1]) & ([0.6, 0.8], [-0.6, -0.5]) \\ ([0.1, 0.1], [-0.4, -0.2]) & ([0.7, 0.9], [-0.4, -0.4]) \end{bmatrix}$$

Then the determinant is calculated as follows:

$$\left|\Gamma_A^1\right| = ([\wedge (\wedge (0.3, 0.1), \wedge (0.6, 0.7)), \vee (\vee (0.4, 0.1), \vee (0.8, 0.9))],$$
$$[\wedge (\wedge (-0.2, -0.4), \wedge (-0.6, -0.4)), \vee (\vee (-0.1, -0.2), \vee (-0.5, -0.4))])$$

$$= ([0.1, 0.9], [-0.6, -0.1])$$

Proposition 3. Let Γ_A^1 and Γ_A^2 given in Definition 2 be two BIV-FSMs on $(X \times Y)$, and let k be a real number. Then the following properties hold:

(i) $k \cdot |\Gamma_A^1| = |k \cdot \Gamma_A^1|$

(ii) $|\Gamma_A^1 \cdot \Gamma_A^2| = |\Gamma_A^1| \cdot |\Gamma_A^2|$

Proof. The proof of these points is directly based on the definition of the determinant of the BIV-FSM.

5. Decision-making approach using Bipolar Interval-Valued Fuzzy Soft Matrices

In this section, we will present a scenario for a decision-making problem and apply the tools proposed in this work.

Example 8. To assist Mr. Xu in choosing a suitable colored car from among a number of available cars, we assume the following:

Let $X = \{\tau_1, \tau_2, \tau_3, \tau_4\}$ represent four cars and $Y = \{\sigma_1, \sigma_2, \sigma_3\}$ where σ_1 = red car, σ_2 = white car, and σ_3 = black car.

To help Mr. Xu, we will organize the following algorithm:

Algorithm

Step 1. Building BIV-FSM matrices Γ_A^1, Γ_A^2, and Γ_A^3 of all available cars based on the opinions of experts who evaluated each of these cars as follows:

$$\Gamma_A^1 = \begin{bmatrix} ([0.5,0.7],[-0.5,-0.4]) & ([0.2,0.4],[-0.6,-0.2]) & ([0.2,0.2],[-0.7,-0.1]) \\ ([0.5,0.5],[-0.8,-0.7]) & ([0.2,0.5],[-0.8,-0.6]) & ([0.3,0.9],[-0.5,-0.4]) \\ ([0.5,0.7],[-0.5,-0.4]) & ([0.4,0.6],[-0.9,-0.4]) & ([0.1,0.9],[-0.5,-0.4]) \\ ([0.2,0.6],[-0.4,-0.1]) & ([0.4,0.8],[-0.3,-0.2]) & ([0.1,0.5],[-0.6,-0.2]) \end{bmatrix}_{(4\times 3)}$$

For car σ_1.

$$\Gamma_A^2 = \begin{bmatrix} ([0.1,0.4],[-0.6,-0.2]) & ([0.3,0.4],[-0.6,-0.2]) & ([0.2,0.2],[-0.7,-0.1]) \\ ([0.6,0.9],[-0.8,-0.7]) & ([0.2,0.5],[-0.6,-0.6]) & ([0.2,0.6],[-0.4,-0.4]) \\ ([0.5,0.7],[-0.5,-0.4]) & ([0.4,0.6],[-0.9,-0.4]) & ([0.9,0.9],[-0.5,-0.7]) \\ ([0.1,0.8],[-0.4,-0.1]) & ([0.4,0.8],[-0.3,-0.2]) & ([0.1,0.5],[-0.6,-0.2]) \end{bmatrix}_{(4\times 3)}$$

For car σ_2.

$$\Gamma_A^3 = \begin{bmatrix} ([0.4,0.8],[-0.3,-0.2]) & ([0.1,0.5],[-0.6,-0.2]) & ([0.2,0.2],[-0.7,-0.1]) \\ ([0.5,0.7],[-0.5,-0.4]) & ([0.2,0.5],[-0.8,-0.6]) & ([0.2,0.6],[-0.4,-0.4]) \\ ([0.5,0.7],[-0.5,-0.4]) & ([0.4,0.6],[-0.9,-0.4]) & ([0.9,0.9],[-0.5,-0.7]) \\ ([0.1,0.8],[-0.4,-0.1]) & ([0.4,0.8],[-0.3,-0.2]) & ([0.1,0.5],[-0.6,-0.2]) \end{bmatrix}_{(4\times 3)}$$

For car σ_3.

Step 2. Calculate the determinant of all three BIV-FSMs based on the above definition as follows:

$$|\Gamma_A^1| = ([0.2,0.9],[-0.5,-0.1]),$$
$$|\Gamma_A^2| = ([0.1,0.8],[-0.6,-0.4]),$$
$$|\Gamma_A^3| = ([0.4,0.9],[-0.5,-0.4])$$

Step 3. Find the score value of all $|\Gamma_A^1|, |\Gamma_A^2|, |\Gamma_A^3|$ from the following formula:

$$Q_i = (a^{+,L} + a^{+,U}) \cdot (-(b^{-,L} \times b^{-,U}))$$

Then we get:

$$Q_1 = (0.2 + 0.9)(-(-0.5 \times -0.1)) = 0.055$$

$$Q_2 = (0.1 + 0.8)(-(-0.6 \times -0.4)) = 0.216$$

$$Q_3 = (0.4 + 0.9)(-(-0.5 \times -0.4)) = 0.260$$

Step 4. Rank the values of Q_i and choose the maximum value, which is Q_3, meaning Mr. Xu will choose the third car.

6. Comparison Discussion

Human thinking is characterized by both positive and negative aspects of every situation it encounters in daily life. To address this, the concept of bipolarity emerged as a mathematical concept that translates this thinking into the language of positive and negative numbers, with positive numbers representing the positive nature of such thinking, while negative numbers represent the negative nature of such thinking. In this work, we presented the basic definition of the concept of BIVFSSMs based on the matrix structure in order to benefit from the mathematical effectiveness of the matrix system. This concept is characterized by its importance and flexibility in covering data related to the decision-making problem, which is characterized by uncertainty, unlike the previous concepts explained in the previous studies section. For example, our proposed concept is characterized by a mathematical structure, namely the interval, which works to provide sufficient flexibility in visualising the data, which is missing in all the previous concepts discussed in Section 2. In addition, the advantage of the existence of the soft set that characterizes our proposed concept works to provide a clearer perception of the criteria on which the data is built. Therefore, we can say that we have overcome the existing research gap in BFSSMs and introduced a new, more effective concept in dealing with data uncertainty.

7. Conclusions

The extensions of FSs, such as IVFS and BFS, have various applications in numerous real-life situations. These concepts have the mathematical potential to be further developed by combining them with other mathematical tools. Therefore, to exploit this feature and to overcome the research gap between the two approaches, BFSMs and BIVFSSMs, is evident in how they cover data for decision-making problems and which one offers more flexibility than the other. This pushes us in a strong way to handle the shortage that is showing in BFSMs. Therefore, the BIVFSSMs use interval values, hence offering a more comprehensive representation of data for decision-making problems. In this work, we presented a hybrid concept called BIVFSS under the matrix system effect. This algebraic concept works to show the positive and negative sides of decision-making data in a matrix. Accordingly, based on this concept, the definitions and basic operations associated with it were presented, supported by illustrative examples. We proposed the definition of determining BIVFSSMs and show how it works along with some related applications. Finally, a multi-step algorithm was presented to help in optimal selection, as this algorithm works on defining the determinants of BIV-FSMs. Also, as future studies, these tools can be applied with [25-29] a number of current research works such as [30-33].

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