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An Approach to Closure and Kernel Defined Through N-Points

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Abstract. In this paper, we introduce novel definitions of closure and kernel of a neutrosophic set (abbreviated as N-set) based on the newly formulated concept of a N-point. Building upon these foundational ideas, we develop and analyze the key properties of these new notions, which naturally lead to the construction of two previously unstudied N-topologies. These contributions offer fresh insights into the topological behavior of neutrosophic structures. We believe that the proposed framework not only deepens the theoretical understanding of N-sets but also opens new avenues for their application in various fields involving uncertainty, imprecision, and indeterminacy.

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1. Introduction

Neutrosophic sets (briefly N-sets) were introduced by F. Smarandache in 2010 [1] as a generalization of classical and intuitionistic fuzzy sets. Unlike their predecessors, N-sets are characterized by the presence of three independent membership functions: truth (T), indeterminacy (I), and falsity (F), each of which varies independently within the unit interval $[0, 1]$. This allows N-sets to effectively model uncertainty, inconsistency, and incompleteness in a way that classical and intuitionistic frameworks cannot, making them particularly suitable for complex real-world applications in decision-making, artificial intelligence, and information systems. Today, researchers have contributed significantly to neutrosophic theory, driving the development of new concepts such as neutrosophic

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convex structures [2] and Pythagorean neutrosophic closure [3], among others. For the above, neutrosophic set theory represents a field of research in constant and rapid growth.

Following the introduction of N-sets, researchers began exploring their topological aspects. In particular, Salama and Alblowi [4] introduced the concept of N-topological spaces, laying the groundwork for a new branch of neutrosophic topology. This development has stimulated a growing body of research aiming to extend classical topological concepts such as open and closed sets, continuity, and convergence into the neutrosophic setting. Subsequent contributions by Ray [5], Subasree and Basari [6, 7], among others, have enriched this field by proposing new forms of neutrosophic continuity, compactness, separation axioms, and bases for neutrosophic topologies (briefly N-topologies). Recently, in 2024, Açikgöz and Esenbel [8] introduced the concept of neutrosophic preclosure and neutrosophic strong semiclosure to study new classes of open sets and explore the properties of modifications of key topological notions such as connectivity and continuity in neutrosophic topological spaces (briefly N-topological spaces). In this year, Tyagi and Kumar Gupta [9] studied the neutrosophic λ-closed sets, generalizing closed and pre-closed neutrosophic sets within N-topological spaces, along with relevant concepts and its properties. Despite this progress, certain fundamental constructs in topology, such as the *closure* and *kernel* operators, have not been thoroughly studied in the context of N-topological spaces. Closure and kernel play pivotal roles in classical and fuzzy topology by characterizing the boundaries and interior structures of sets, as well as influencing the properties of continuity, connectedness, and compactness. Their proper generalization to N-sets is therefore essential for a deeper understanding of neutrosophic topological behavior.

In this paper, we introduce novel definitions of closure and kernel for N-sets, employing the concept of a N-point as the central building block. We then systematically explore the fundamental properties of these new notions and demonstrate how they lead to the construction of two previously unexamined N-topologies. In comparison with most of the studies carried out in N-topological spaces, where variations of the closure and kernel operators have been defined by means of global tools, our study incorporates the novelty of defining these operators by means of a local tool such as the notion of N-point. With this, we intend to fill the gap in the existing literature and bring new perspectives to the theory of N-topological spaces. Our results not only enrich the mathematical framework of N-sets, but also offer potential for future applications in areas where uncertainty and indeterminacy play a central role.

2. Neutrosophic sets

Throughout this paper, let X be a nonempty set, called *universe of discourse.*

Definition 1. [1] A *neutrosophic set* (briefly N-set) N on X is an object of the form

$$N = \{\langle x, \mu_N(x), \sigma_N(x), \gamma_N(x)\rangle : x \in X\},$$

where $\mu_N, \sigma_N, \gamma_N$ are functions from X to $[0, 1]$.

We denote by $\mathcal{N}(X)$ the collection of all N-sets over X.

Definition 2. [10] For $N, M \in \mathcal{N}(X)$ we define the following:

(1) (Inclusion) N is called a *neutrosophic subset* of M, denoted by $N \sqsubseteq M$, if $\mu_N(x) \leq \mu_M(x)$, $\sigma_N(x) \geq \sigma_M(x)$ and $\gamma_N(x) \geq \gamma_M(x)$ for all $x \in X$. Also, we can say that M is a neutrosophic super set of N.

(2) (Equality) N is called *neutrosophic equal* to M, denoted by $N = M$, if $N \sqsubseteq M$ and $M \sqsubseteq N$.

(3) (Universal set) N is called the *neutrosophic universal set*, denoted by $\widetilde{X}$, if $\mu_N(x) = 1, \sigma_N(x) = 0$ and $\gamma_N(x) = 0$ for all $x \in X$.

(4) (Empty set) N is called *neutrosophic empty set*, denoted by $\widetilde{\emptyset}$, if $\mu_N(x) = 0, \sigma_N(x) = 1$ and $\gamma_N(x) = 1$ for all $x \in X$.

(5) (Intersection) The *neutrosophic intersection* of N and M, denoted by $N \sqcap M$, is defined as

$$N \sqcap M = \{(x, \mu_N(x) \wedge \mu_M(x), \sigma_N(x) \vee \sigma_M(x), \gamma_N(x) \vee \gamma_M(x)\rangle : x \in X\}.$$

(6) (Union) The *neutrosophic union* of N and M, denoted by $N \sqcup M$, is defined as

$$N \sqcup M = \{\langle x, \mu_N(x) \vee \mu_M(x), \sigma_N(x) \wedge \sigma_M(x), \gamma_N(x) \wedge \gamma_M(x)\rangle : x \in X\}.$$

(7) (Complement) The *neutrosophic complement* of N, denoted by N^c, is defined as

$$N^c = \{\langle x, \gamma_N(x), 1 - \sigma_N(x), \mu_N(x)\rangle : x \in X\}.$$

Proposition 1. [10] If $N, M \in \mathcal{N}(X)$, then we have the following properties:

(1) $N \sqcap N = N$ and $N \sqcup N = N$.

(2) $N \sqcap M = M \sqcap N$ and $N \sqcup M = M \sqcup N$.

(3) $N \sqcap \tilde{\emptyset} = \tilde{\emptyset}$ and $N \sqcap \tilde{X} = N$.

(4) $N \sqcup \tilde{\emptyset} = N$ and $N \sqcup \tilde{X} = \tilde{X}$.

(5) $N \sqcap (M \sqcap O) = (N \sqcap M) \sqcap O$ and $N \sqcup (M \sqcup O) = (N \sqcup M) \sqcup O$.

(6) $(N^c)^c = N$.

The union and intersection operations given in Definition 2 can be extended as follows.

Definition 3. [4] For $\{N_j : j \in J\} \subseteq \mathcal{N}(X)$ we define the following operations:

(1) (Arbitrary intersection) The *arbitrary neutrosophic intersection* of the collection $\{N_j : j \in J\}$, denoted by $\bigsqcap_{j\in J} N_j$, is defined as

$$\bigsqcap_{j\in J} N_j = \left\{ \left\langle x, \inf_{j\in J} \mu_{N_j}(x), \sup_{j\in J} \sigma_{N_j}(x), \sup_{j\in J} \gamma_{N_j}(x) \right\rangle : x \in X \right\}.$$

(2) (Arbitrary union) The *arbitrary neutrosophic union* of the collection $\{N_j : j \in J\}$, denoted by $\bigsqcup_{j\in J} N_j$, is defined as

$$\bigsqcup_{j\in J} N_j = \left\{ \left\langle x, \sup_{j\in J} \mu_{N_j}(x), \inf_{j\in J} \sigma_{N_j}(x), \inf_{j\in J} \gamma_{N_j}(x) \right\rangle : x \in X \right\}.$$

Proposition 2. [10] If $\{N_j : j \in J\} \subseteq \mathcal{N}(X)$ and $M \in \mathcal{N}(X)$, then we have the following properties:

(1) $M \sqcap \left(\bigsqcup_{j\in J} N_j \right) = \bigsqcup_{j\in J} (M \sqcap N_j)$.

(2) $M \sqcup \left(\bigsqcap_{j\in J} N_j \right) = \bigsqcap_{j\in J} (M \sqcup N_j)$.

(3) $\left(\bigsqcap_{j\in J} N_j \right)^c = \bigsqcup_{j\in J} N_j^c$.

(4) $\left(\bigsqcup_{j\in J} N_j \right)^c = \bigsqcap_{j\in J} N_j^c$.

Definition 4. [10] A *neutrosophic topology* (briefly N-topology) on a set X is a collection $\tau \subseteq \mathcal{N}(X)$ which satisfies the following conditions:

(1) $\widetilde{\emptyset}$ and $\widetilde{X}$ are in τ.

(2) The intersection of two N-sets belonging to τ is in τ.

(3) The union of any collection of N-sets belonging to τ is in τ.

A set X for which a N-topology τ has been defined is called a *N-topological space* and is denoted as a pair (X, τ). If $N \in \tau$, then N is called a *N-open set* and if $N^c \in \tau$, then N is called a *N-closed set*. We denote by τ^c the collection of all N-closed sets in the N-topological space (X, τ).

Proposition 3. [10] Let (X, τ) be a N-topological space. Then, the following conditions hold:

(1) $\widetilde{\emptyset}$ and $\widetilde{X}$ are in τ^c.

(2) The union of two N-sets belonging to τ^c is in τ^c.

(3) The intersection of any collection of N-sets belonging to τ^c is in τ^c.

Definition 5. [10] Let (X, τ) be a N-topological space and $N \in \mathcal{N}(X)$. The N-*closure* of N, denoted by $Cl(N)$, is defined as

$$Cl(N) = \bigsqcap \{F \in \mathcal{N}(X) : N \sqsubseteq F \text{ and } F \in \tau^c\}.$$

Proposition 4. [10] Let (X, τ) be a N-topological space and $N, M \in \mathcal{N}(X)$. Then, the following conditions hold:

(1) $N \sqsubseteq Cl(N)$.

(2) $Cl(Cl(N)) = Cl(N)$.

(3) $Cl(N \sqcup M) = Cl(N) \sqcup Cl(M)$.

(4) $Cl(\widetilde{\emptyset}) = \widetilde{\emptyset}$.

(5) $Cl(\widetilde{X}) = \widetilde{X}$.

(6) If $N \sqsubseteq M$, then $Cl(N) \sqsubseteq Cl(M)$.

(7) $Cl(N \sqcap M) \sqsubseteq Cl(N) \sqcap Cl(M)$.

(8) $N \in \tau^c$ if and only if $N = Cl(N)$.

Definition 6. [5] A N-set $M = \{\langle x, \mu_M(x), \sigma_M(x), \gamma_M(x)\rangle : x \in X\}$ is called a N-*point* if for any element $y \in X$, $\mu_M(y) = a$, $\sigma_M(y) = b$, $\gamma_M(y) = c$ for $y = x$ and $\mu_M(y) = 0$, $\sigma_M(y) = 1$, $\gamma_M(y) = 1$ for $y \neq x$, where $a \in (0, 1]$ and $b, c \in [0, 1)$. In this case, the N-point M is denoted by $M^x_{a,b,c}$ or simply by $x_{a,b,c}$. Also, x is called the *support* of the N-point $x_{a,b,c}$. The N-point $x_{1,0,0}$ is called a N-*crisp point.*

Definition 7. [5] Let $N \in \mathcal{N}(X)$. A N-point $x_{a,b,c}$ is said to belong to N, denoted by $x_{a,b,c} \in N$, if $\mu_N(x) \geq a$, $\sigma_N(x) \leq b$ and $\gamma_N(x) \leq c$.

Remark 1. *It is important to note that* $\widetilde{\emptyset}$ *is not the only* N*-set that does not have points belonging to it. For example, if* $X = \{x, y\}$*, then* $N = \{\langle x, 0, 0.5, 1\rangle, \langle y, 0, 0.4, 1\rangle\}$ *is a* N*-set over* X *for which there are not* N*-points belonging to it.*

Lemma 1. [5] Let $N, M \in \mathcal{N}(X)$. Then, we have:

(1) $N = \bigsqcup\{x_{a,b,c} : x_{a,b,c} \in N\}$.

(2) If $x_{a,b,c} \in N$ and $N \sqsubseteq M$, then $x_{a,b,c} \in M$.

Definition 8. [11] A N-*ideal* on a set X is a nonempty collection $\mathcal{L} \subseteq \mathcal{N}(X)$, which satisfies the following conditions:

(1) $N \in \mathcal{L}$ and $M \sqsubseteq N$ imply that $M \in \mathcal{L}$. (Hereditary property)

(2) $N, M \in \mathcal{L}$ imply that $N \sqcup M \in \mathcal{L}$. (Finite additivity property)

Given a N-topological space (X, τ), a N-ideal $\mathcal{L}$ on X and $N \in \mathcal{N}(X)$, the N-*local function* [12] of N, denoted by $N^\star(\mathcal{L}, \tau)$, is defined as

$$N^\star(\mathcal{L}, \tau) = \bigsqcup\{x_{a,b,c} \in \mathcal{N}(X) : U \sqcap N \notin \mathcal{L} \text{ for every } U \in \tau(x_{a,b,c})\},$$

where $\tau(x_{a,b,c}) = \{U \in \tau : x_{a,b,c} \in U\}$. We will denote $N^\star(\mathcal{L}, \tau)$ by $N^\star$ or $N^\star(\mathcal{L})$.

Theorem 1. [12] Let (X, τ) be a N-topological space with two N-ideals $\mathcal{L}$, $\mathcal{L}'$ on X. If $N, M \in \mathcal{N}(X)$, then the following properties hold:

(1) If $N \sqsubseteq M$, then $N^\star \sqsubseteq M^\star$.

(2) If $\mathcal{L} \subseteq \mathcal{L}'$, then $N^\star(\mathcal{L}') \sqsubseteq N^\star(\mathcal{L})$.

(3) $N^\star = Cl(N^\star) \sqsubseteq Cl(N)$ ($N^\star$ is a N-closed set).

(4) $(N^\star)^\star \sqsubseteq N^\star$.

(5) $(N \sqcup M)^\star = N^\star \sqcup M^\star$.

(6) $(N \sqcap M)^\star \sqsubseteq N^\star \sqcap M^\star$.

(7) If $M \in \mathcal{L}$, then $(N \sqcup M)^\star = N^\star$.

3. N-point-closure

The concept of N-closure introduced by Karatas and Kuru [10] has inspired recent research in the neutrosophic environment, which has extended the related theory of neutrosophic topological spaces to other contexts, some of which can be found in references [8] and [3]. Motivated by these recent advances, in this section, we introduce and investigate the concept of N-point-closure using N-points, which is independent of the concept of N-closure, as we show in two examples below. First, we establish new results related to the concepts of N-points and N-closure.

Proposition 5. Let $N, M \in \mathcal{N}(X)$. If $N \sqcap M = \widetilde{\emptyset}$, then $M \sqsubseteq N^c$ and $N \sqsubseteq M^c$.

Proof. We will only show the neutrosophic inclusion $M \sqsubseteq N^c$, because the other neutrosophic inclusion is shown in the same way. Assume that $N \sqcap M = \widetilde{\emptyset}$. Then,

$$\{\langle x, \mu_N(x) \wedge \mu_M(x), \sigma_N(x) \vee \sigma_M(x), \gamma_N(x) \vee \gamma_M(x)\rangle : x \in X\} = \widetilde{\emptyset},$$

so the following equalities hold:

(1) $\mu_N(x) \wedge \mu_M(x) = 0$, $\forall x \in X$.

(2) $\sigma_N(x) \vee \sigma_M(x) = 1$, $\forall x \in X$.

(3) $\gamma_N(x) \vee \gamma_M(x) = 1$, $\forall x \in X$.

From equality (1), we have $\gamma_N(x) = 1 - \mu_N(x) \geq \mu_M(x)$, $\forall x \in X$. From equality (2), let us observe that one of the quantities $\sigma_N(x)$ or $\sigma_M(x)$ is equal to 1 for an arbitrary $x \in X$. Since $0 \leq \sigma_N(x) \leq 1$ and $0 \leq \sigma_M(x) \leq 1$, $\forall x \in X$, it follows that $\sigma_N(x) + \sigma_M(x) \geq 1$, $\forall x \in X$. Thus, $\sigma_M(x) \geq 1 - \sigma_N(x)$, $\forall x \in X$. Using a similar reasoning from equality (3), we show that $\gamma_M(x) \geq 1 - \gamma_N(x) = \mu_N(x)$, $\forall x \in X$. Therefore,

$$M = \{\langle x, \mu_M(x), \sigma_M(x), \gamma_M(x)\rangle : x \in X\} \sqsubseteq \{\langle x, \gamma_N(x), 1 - \sigma_N(x), \mu_N(x)\rangle : x \in X\} = N^c.$$

In the following example, we show that the converse of Proposition 5, in general, is not true.

Example 1. Let $X = \{x, y\}$ and $O, U \in \mathcal{N}(X)$ such that

$$N = \{\langle x, 0.4, 0.8, 0.6\rangle, \langle y, 0.6, 0.7, 0.4\rangle\},$$
$$M = \{\langle x, 0.3, 0.3, 0.7\rangle, \langle y, 0.4, 0.3, 0.6\rangle\}.$$

Then,

$$M \sqsubseteq N^c = \{\langle x, 0.6, 0.2, 0.4\rangle, \langle y, 0.4, 0.3, 0.6\rangle\},$$

but

$$N \sqcap M = \{\langle x, 0.3, 0.8, 0.7\rangle, \langle y, 0.4, 0.7, 0.6\rangle\} \neq \widetilde{\emptyset}.$$

Remark 2. In Example 1, we have $N \sqcup N^c = \{\langle x, 0.6, 0.2, 0.4\rangle, \langle y, 0.6, 0.3, 0.4\rangle\} \neq \widetilde{X}$ and $N \sqcap N^c = \{\langle x, 0.4, 0.8, 0.6\rangle, \langle y, 0.4, 0.7, 0.6\rangle\} \neq \widetilde{\emptyset}$. Thus, we deduce that the equalities $N \sqcup N^c = \widetilde{X}$ and $N \sqcap N^c = \widetilde{\emptyset}$ are not satisfied, in general.

Proposition 6. Let $N, M \in \mathcal{N}(X)$. Then, the following properties are equivalent:

(1) $N \sqsubseteq M$.

(2) $x_{a,b,c} \in N$ implies that $x_{a,b,c} \in M$.

Proof. The proof follows directly from Lemma 1.

Definition 9. An application $\Upsilon : \mathcal{N}(X) \to \mathcal{N}(X)$ is called a *N-closure operator* if it satisfies the following conditions:

(1) $N \sqsubseteq \Upsilon(N)$, (expansivity)

(2) $\Upsilon(\Upsilon(N)) = \Upsilon(N)$, (idempotency)

(3) $\Upsilon(N \sqcup M) = \Upsilon(N) \sqcup \Upsilon(M)$, (additivity)

(4) $\Upsilon(\widetilde{\emptyset}) = \widetilde{\emptyset}$, (non-spontaneous creation)

whenever $M, N \in \mathcal{N}(X)$.

Proposition 7. If $\Upsilon : \mathcal{N}(X) \to \mathcal{N}(X)$ is a N-closure operator, then the collection $\tau(\Upsilon) = \{N \in \mathcal{N}(X) : \Upsilon(N^c) = N^c\}$ is a N-topology on X and Υ is the N-closure in the N-topological space $(X, \tau(\Upsilon))$.

Proof. We verify that $\tau(\Upsilon)$ satisfies the conditions of a N-topology on X. Indeed:
(1) $\widetilde{X} \in \tau(\Upsilon)$ because by the non-spontaneous creation of Υ, we have $\Upsilon(\widetilde{X}^c) = \Upsilon(\widetilde{\emptyset}) = \widetilde{\emptyset} = \widetilde{X}^c$. Also, $\widetilde{\emptyset} \in \tau(\Upsilon)$ because $\widetilde{X} \sqsubseteq \Upsilon(\widetilde{X}) \sqsubseteq \widetilde{X}$ by expansivity of Υ and the fact that $\Upsilon(\widetilde{X}) \in \mathcal{N}(X)$.
(2) Let $\{M_j : j \in J\} \subseteq \mathcal{N}(X)$. Then, $\Upsilon(M_j^c) = M_j^c$ for each $j \in J$. By expansivity Υ, we have $\left(\bigsqcup_{j\in J} M_j\right)^c \sqsubseteq \Upsilon\left(\left(\bigsqcup_{j\in J} M_j\right)^c\right)$. For other neutrosophic inclusion, let us note that $\left(\bigsqcup_{j\in J} M_j\right)^c \sqsubseteq M_j^c$, for each $j \in J$. By idempotency of Υ, it follows that

$$\Upsilon\left(\left(\bigsqcup_{j\in J} M_i\right)^c\right) \sqsubseteq \Upsilon(M_j^c) = M_j^c \text{ for each } j \in J.$$

Thus, $\Upsilon\left(\left(\bigsqcup_{j\in J} M_j\right)^c\right) \sqsubseteq \bigsqcap_{j\in J} M_j^c = \left(\bigsqcup_{j\in J} M_j\right)^c$ and hence, $\Upsilon\left(\left(\bigsqcup_{j\in J} M_j\right)^c\right) = \left(\bigsqcup_{j\in J} M_j\right)^c$. This shows that $\bigsqcup_{j\in J} M_j$ belongs to $\tau(\Upsilon)$.
(3) Suppose that $N, M \in \mathcal{N}(X)$. Then, $\Upsilon(N^c) = N^c$ and $\Upsilon(M^c) = M^c$ and so, by additivity of Υ, we get that $\Upsilon((N \sqcap M)^c) = \Upsilon(N^c \sqcup M^c) = \Upsilon(N^c) \sqcup \Upsilon(M^c) = N^c \sqcup M^c = (N \sqcap M)^c$, which proves that $N \sqcap M$ belongs to $\tau(\Upsilon)$.

From (1)-(3), we conclude that $\tau(\Upsilon)$ is a N-topology on X. Now, we will prove the remainder of the statement.

First, $\Upsilon(N) \in \tau^c(\Upsilon)$, because $\Upsilon(N) = \Upsilon(\Upsilon(N))$ by (2). Since $N \sqsubseteq \Upsilon(N)$ and $Cl(N)$ is the smallest N-$\tau(\Upsilon)$-closed set containing N, it follows that $Cl(N) \sqsubseteq \Upsilon(N)$. Secondly, $N \sqsubseteq Cl(N)$ and $Cl(N) \sqsubseteq \Upsilon(N)$ imply that $Cl(N) \sqsubseteq \Upsilon(Cl(N)) = Cl(N)$. Therefore, $\Upsilon(N) = Cl(N)$ whatever is $N \in \mathcal{N}(X)$.

Let $\mathcal{N}_p(X) = \{N \in \mathcal{N}(X) : \text{there exists a } N\text{-point } x_{a,b,c} \in N\}$ and let $\mathcal{N}'(X) = \{\widetilde{\emptyset}\} \cup \mathcal{N}_p(X)$. In the remainder of this paper, we will use the definitions and results described in the previous section, restricted to the collection $\mathcal{N}'(X)$.

Definition 10. Let (X,τ) be a N-topological space and $N \in \mathcal{N}'(X)$. The N*-point-closure* of N, denoted by $Cl_p(N)$, is defined as

$$Cl_p(N) = \bigsqcup\{x_{a,b,c} \in \mathcal{N}'(X) : U \sqcap N \neq \widetilde{\emptyset} \text{ for every } U \in \tau(x_{a,b,c})\}.$$

Remark 3. In general, it is not true that $Cl(N) = Cl_p(N)$ for each $N \in \mathcal{N}'(X)$. Moreover, none of the neutrosophic inclusions $Cl(N) \sqsubseteq Cl_p(N)$ and $Cl_p(N) \sqsubseteq Cl(N)$ is true in general, as we can see in the following two examples.

Example 2. Let $X = \{x,y\}$ and $O,U \in \mathcal{N}'(X)$ such that

$$O = \{\langle x, 0.5, 0.5, 0.5\rangle, \langle y, 0.4, 0.4, 0.6\rangle\},$$
$$U = \{\langle x, 0.6, 0.6, 0.4\rangle, \langle y, 0.3, 0.3, 0.7\rangle\}.$$

Observe that

$$\tau = \{\tilde{\emptyset}, \tilde{X}, O, U, O \sqcap U, O \sqcup U\}$$

is a N-topology on X. Then, the collection of all N-closed sets on X is

$$\tau^c = \left\{\tilde{X}, \tilde{\emptyset}, O^c, U^c, (O \sqcap U)^c, (O \sqcup U)^c\right\},$$

where

$$O^c = \{\langle x, 0.5, 0.5, 0.5\rangle, \langle y, 0.6, 0.6, 0.4\rangle\},$$
$$U^c = \{\langle x, 0.4, 0.4, 0.6\rangle, \langle y, 0.7, 0.7, 0.3\rangle\},$$
$$(O \sqcap U)^c = \{\langle x, 0.5, 0.4, 0.5\rangle, \langle y, 0.7, 0.6, 0.3\rangle\},$$
$$(O \sqcup U)^c = \{\langle x, 0.4, 0.5, 0.6\rangle, \langle y, 0.6, 0.7, 0.4\rangle\}.$$

Consider the N-set $N = \{\langle x, 0.1, 0.8, 0.9\rangle, \langle y, 0.4, 0.9, 0.6\rangle\}$ and the N-point $x_{0.4,0.3,0.6}$. Then, $Cl(N) = (O \sqcup U)^c$ and $\widetilde{X}$ is the only N-open set to which $x_{0.4,0.3,0.6}$ belongs. Since $\widetilde{X} \sqcap N \neq \widetilde{\emptyset}$, it follows that $x_{0.4,0.3,0.6}$ belongs to $Cl_p(N)$, but $x_{0.4,0.3,0.6}$ does not belong to $Cl(N) = \{\langle x, 0.4, 0.5, 0.6\rangle, \langle y, 0.6, 0.7, 0.4\rangle\}$.

Example 3. Let $X = \{x,y\}$ and $O,U \in \mathcal{N}'(X)$ such that

$$O = \{\langle x, 0.2, 0.4, 0.8\rangle, \langle y, 0.4, 0.6, 0.6\rangle\},$$
$$U = \{\langle x, 0, 0.3, 1\rangle, \langle y, 0.1, 1, 0.9\rangle\}.$$

Consider the N-topology on X given by

$$\tau = \{\tilde{\emptyset}, \tilde{X}, O, U, O \sqcap U, O \sqcup U\}.$$

The collection of all N-closed sets on X is

$$\tau^c = \left\{\tilde{X}, \tilde{\emptyset}, O^c, U^c, (O \sqcap U)^c, (O \sqcup U)^c\right\},$$

where

$$\begin{aligned} O^c &= \{\langle x, 0.8, 0.6, 0.2\rangle, \langle y, 0.6, 0.4, 0.4\rangle\},\\ U^c &= \{\langle x, 1, 0.7, 0\rangle, \langle y, 0.9, 0, 0.1\rangle\},\\ (O \sqcap U)^c &= \{\langle x, 1, 0.6, 0\rangle, \langle y, 0.9, 0, 0.1\rangle\},\\ (O \sqcup U)^c &= \{\langle x, 0.8, 0.7, 0.2\rangle, \langle y, 0.6, 0.4, 0.4\rangle\}. \end{aligned}$$

Consider the N-set $N = \{\langle x, 0.1, 1, 0.9\rangle, \langle y, 0, 0.3, 1\rangle\}$ and the N-point $y_{0.1,1,0.9}$. Then, $y_{0.1,1,0.9} \in Cl(N) = U^c$, but $U \in \tau(y_{0.1,1,0.9})$ and $U \sqcap N = \widetilde{\emptyset}$, which implies that $y_{0.1,1,0.9}$ does not belong to $Cl_p(N)$.

Proposition 8. Let (X, τ) be a N-topological space and $N, M \in \mathcal{N}'(X)$. Then, the following conditions hold:

(1) $N \sqsubseteq Cl_p(N)$.

(2) $Cl_p(Cl_p(N)) = Cl_p(N)$.

(3) If $N \sqsubseteq M$, then $Cl_p(N) \sqsubseteq Cl_p(M)$.

(4) $Cl_p(N \sqcap M) \sqsubseteq Cl_p(N) \sqcap Cl_p(M)$.

(5) $Cl_p(N \sqcup M) = Cl_p(N) \sqcup Cl_p(M)$.

(6) $Cl_p(\widetilde{\emptyset}) = \widetilde{\emptyset}$.

(7) $Cl_p(\widetilde{X}) = \widetilde{X}$.

Proof. (1) It is obvious from Definition 10.

(2) The inclusion $Cl_p(N) \sqsubseteq Cl_p(Cl_p(N))$ is an immediate consequence of part (1). For the other inclusion, suppose that $x_{a,b,c} \in Cl_p(Cl_p(N))$ and let $O \in \tau(x_{a,b,c})$. Then, $O \sqcap Cl_p(N) \neq \widetilde{\emptyset}$, which implies that there exists a N-point $y_{u,v,w} \in Cl_p(N)$ and $O \in \tau(y_{u,v,w})$. Thus, $O \sqcap N \neq \widetilde{\emptyset}$ and hence, $x_{a,b,c} \in Cl_p(N)$.

(3) Suppose that $N, M \in \mathcal{N}'(X)$ are such that $N \sqsubseteq M$. If $x_{a,b,c} \notin Cl_p(M)$, then there exists $U \in \tau(x_{a,b,c})$ such that $U \sqcap M = \widetilde{\emptyset}$, which implies that $U \sqcap N \sqsubseteq U \sqcap M = \widetilde{\emptyset}$ and so, $U \sqcap N = \widetilde{\emptyset}$, which proves that $x_{a,b,c} \notin Cl_p(N)$. Therefore, $Cl_p(N) \sqsubseteq Cl_p(M)$ whenever $N \sqsubseteq M$.

(4) The proof follows from part (3).

(5) The inclusion $Cl_p(N) \sqcup Cl_p(M) \sqsubseteq Cl_p(N \sqcup M)$ is an immediate consequence of part (1). Suppose that $x_{a,b,c} \notin Cl_p(N) \sqcup Cl_p(M)$. Then, there exist $O, U \in \tau(x_{a,b,c})$ such that $O \sqcap N = \widetilde{\emptyset}$ and $U \sqcap M = \widetilde{\emptyset}$. Putting $V = O \sqcap U$, we have $V \in \tau(x_{a,b,c})$ and

$$\begin{aligned} V \sqcap (N \sqcup M) &= (V \sqcap N) \sqcup (V \sqcap M) = (O \sqcap U \sqcap N) \sqcup (O \sqcap U \sqcap M)\\ &\sqsubseteq (O \sqcap N) \sqcup (U \sqcap M) = \widetilde{\emptyset} \sqcup \widetilde{\emptyset} = \widetilde{\emptyset}, \end{aligned}$$

which implies that $V \sqcap (N \sqcup M) = \widetilde{\emptyset}$ and hence, $x_{a,b,c} \notin Cl_p(N \sqcup M)$.

(6) and (7) are deduced from the Definition 10.

In the following example, we show that the converse of the part (4) of Proposition 8 is not true in general.

Example 4. Let (X, τ) be the N-topological space given in Example 3. Consider the N-sets $N = \{\langle x, 0.1, 1, 0.9\rangle, \langle y, 0, 0.3, 1\rangle\}$ and $M = \{\langle x, 0, 0.3, 1\rangle, \langle y, 0.1, 1, 0.9\rangle\}$. Then, $M \sqcap N = \widetilde{\emptyset}$ and so (by part (6) of Proposition 8) $Cl_p(M \sqcap N) = \widetilde{\emptyset}$. On the other hand, as $\widetilde{X}$ is the only N-open set to which the N-point $x_{0.3,1,0.7}$ belongs and $\widetilde{X} \sqcap N \neq \widetilde{\emptyset}$, $\widetilde{X} \sqcap M \neq \widetilde{\emptyset}$, we obtain that $x_{0.3,1,0.7} \in Cl_p(M) \sqcap Cl_p(N)$, which implies that $Cl_p(M) \sqcap Cl_p(N) \neq \widetilde{\emptyset}$. Therefore, the inclusion $Cl_p(M) \sqcap Cl_p(N) \sqsubseteq Cl_p(M \sqcap N)$ is not satisfied.

Remark 4. From Proposition 8 we infer that Cl_p satisfies the conditions of Definition 9. Thus, by Proposition 7, we get that the collection $\tau_p = \{N \in \mathcal{N}'(X) : Cl_p(N^c) = N^c\}$ is a N-topology on X and Cl_p is the N-closure in the N-topological space (X, τ_p). We say that a N-set N is N-τ_p-open if $N \in \tau_p$. The complement of a N-τ_p-open set, we will call it a N-τ_p-closed set. Observe that a N-set M is τ_p-closed if and only if $Cl_p(M) = M$.

Remark 5. If we restrict the definition of N-ideal to the collection $\mathcal{N}'(X)$, then we can correct some results given in [12], as we show in the following:

(1) If (X, τ) is a N-topological space and $\mathcal{L}$ is a N-ideal on X, then $\widetilde{\emptyset}^{\star} = \widetilde{\emptyset}$, because for every N-point $x_{a,b,c} \in \mathcal{N}'(X)$ and every $U \in \tau(x_{a,b,c})$, $\widetilde{\emptyset} \sqcap U = \widetilde{\emptyset} \in \mathcal{L}$.

(2) For each $N \in \mathcal{N}'(X)$, $Cl^{\star}(N)$ is defined as the neutrosophic union of N with $N^{\star}$; that is, $Cl^{\star}(N) = N \sqcup N^{\star}$ (see [12]). Observe that, by Proposition 7, we have $Cl^{\star}$ is a N-closure operator. Using this fact, we denote by $\tau^{\star}$ (or $\tau^{\star}(\mathcal{L})$) to the N-topology generated by $Cl^{\star}$, that is, $\tau^{\star} = \{N \in \mathcal{N}'(X) : Cl^{\star}(N^c) = N^c\}$.

(3) If $\mathcal{L} = \{\widetilde{\emptyset}\}$, then for each $N \in \mathcal{N}'(X)$, $N^{\star} = Cl_p(N)$ and hence, $Cl^{\star}(N) = N \sqcup N^{\star} = N \sqcup Cl_p(N) = Cl_p(N)$, i.e. $Cl^{\star}(N) = Cl_p(N)$. Therefore, $\tau^{\star}(\{\widetilde{\emptyset}\}) = \tau_p$.

(4) By using the result of the previous part, we correct part (3) of Theorem 1 as follows:

$$N^{\star} = Cl_p(N^{\star}) \sqsubseteq Cl_p(N) \quad (N^{\star} \text{ is a } N\text{-}\tau_p\text{-closed set}).$$

The following is the proof of the above statement. Since $\{\widetilde{\emptyset}\} \subseteq \mathcal{L}$ for each N-ideal $\mathcal{L}$ on X, we have $N^{\star}(\mathcal{L}) \subseteq N^{\star}(\{\widetilde{\emptyset}\}) = Cl_p(N)$ for each $N \in \mathcal{N}'(X)$. Suppose that $x_{a,b,c} \in Cl_p(N^{\star})$ and let $U \in \tau(x_{a,b,c})$ arbitrary. Then, $U \sqcap N^{\star} \neq \widetilde{\emptyset}$ and so, there exists a N-point $y_{u,v,w} \in U \sqcap N^{\star}$, which implies that $y_{u,v,w} \in U$ and $y_{u,v,w} \in N^{\star}$. Since $U \in \tau(y_{u,v,w})$, it follows that $U \sqcap N \notin \mathcal{L}$ and so $x_{a,b,c} \in N^{\star}$. On the other hand, since $N^{\star} \sqsubseteq Cl_p(N^{\star})$, we conclude that $N^{\star} = Cl_p(N^{\star})$.

(5) If $\mathcal{L} = \mathcal{N}'(X)$, then for any $N \in \mathcal{N}'(X)$, $N^{\star} = \widetilde{\emptyset}$ and so, $Cl^{\star}(N) = N \sqcup \widetilde{\emptyset} = N$. Therefore, $\tau^{\star}(\mathcal{N}'(X)) = \mathcal{N}'(X)$.

(6) Since $N^\star = Cl_p(N^\star) \sqsubseteq Cl_p(N)$, we have $Cl^\star(N) \sqsubseteq Cl_p(N)$ for each $N \in \mathcal{N}'(X)$. Hence, if N is a N-τ_p-closed set, then $Cl^\star(N) \sqsubseteq Cl_p(N) = N$, which implies that $Cl^\star(N) = N$ and so, N is N-$\tau^\star$-closed. Thus, every N-τ_p-open set is N-$\tau^\star$-open; i.e. $\tau_p \subseteq \tau^\star$.

4. N-point-kernel

In 2017, S. Jafari and N. Rajesh [13] introduced the notion of N-kernel in a N-topological space (X, τ) as follows: the N-*kernel* of $N \in \mathcal{N}(X)$, denoted by $Ker(N)$, is defined as $Ker(N) = \bigsqcap\{U \in \mathcal{N}(X) : N \sqsubseteq U \text{ and } U \in \tau\}$. In this section, we introduce and study the concept of N-point-kernel of $N \in \mathcal{N}'(X)$ by using N-points, which is distinct from the notion given in [13], as we can see in Examples 5 and 6 below.

Definition 11. Let (X, τ) be a N-topological space and $N \in \mathcal{N}'(X)$. The N*-point-kernel* of N, denoted by $Ker_p(N)$, is defined as

$$Ker_p(N) = \bigsqcup\{x_{a,b,c} \in \mathcal{N}'(X) : F \sqcap N \neq \widetilde{\emptyset} \text{ for every } F \in \tau^c(x_{a,b,c})\},$$

where $\tau^c(x_{a,b,c}) = \{F \in \tau^c : x_{a,b,c} \in F\}$.

Remark 6. Given $N \in \mathcal{N}'(X)$ we can see that, in general, is not true $Ker(N) \neq Ker_p(N)$.

Example 5. Let (X, τ) be the N-topological space given in Example 2. Consider the N-set $N = \{\langle x, 0.1, 0.8, 0.9\rangle, \langle y, 0.4, 0.9, 0.6\rangle\}$ and the N-point $x_{0.4,0.3,0.6}$. Then, $Ker(N) = O$ and $\widetilde{X}$ is the only N-closed set to which $x_{0.4,0.3,0.6}$ belongs. Since $\widetilde{X} \sqcap N \neq \widetilde{\emptyset}$, it follows that $x_{0.4,0.3,0.6}$ belongs to $Ker_p(N)$, but $x_{0.4,0.3,0.6}$ does not belong to $Ker(N) = \{\langle x, 0.5, 0.5, 0.5\rangle, \langle y, 0.4, 0.4, 0.6\rangle\}$.

Example 6. Let $X = \{x, y\}$ and $O, U \in \mathcal{N}'(X)$ such that

$$O = \{\langle x, 0.8, 0.6, 0.2\rangle, \langle y, 0.6, 0.4, 0.4\rangle\},$$
$$U = \{\langle x, 1, 0.7, 0\rangle, \langle y, 0.9, 0, 0.1\rangle\}.$$

Consider the N-topology on X given by

$$\tau = \{\tilde{\emptyset}, \tilde{X}, O, U, O \sqcap U, O \sqcup U\}.$$

The collection of all N-closed sets on X is

$$\tau^c = \left\{\tilde{X}, \tilde{\emptyset}, O^c, U^c, (O \sqcap U)^c, (O \sqcup U)^c\right\},$$

where

$$O^c = \{\langle x, 0.2, 0.4, 0.8\rangle, \langle y, 0.4, 0.6, 0.6\rangle\},$$
$$U^c = \{\langle x, 0, 0.3, 1\rangle, \langle y, 0.1, 1, 0.9\rangle\},$$
$$(O \sqcap U)^c = \{\langle x, 0.2, 0.3, 0.8\rangle, \langle y, 0.4, 0.6, 0.6\rangle\},$$
$$(O \sqcup U)^c = \{\langle x, 0, 0.4, 1\rangle, \langle y, 0.1, 1, 0.9\rangle\}.$$

Consider the N-set $N = \{\langle x, 0.1, 1, 0.9\rangle, \langle y, 0, 0.3, 1\rangle\}$ and the N-point $y_{0.1,1,0.9}$. Then, $y_{0.1,1,0.9} \in Ker(N) = U$, but $U^c \in \tau^c(y_{0.1,1,0.9})$ and $U^c \sqcap N = \widetilde{\emptyset}$, which implies that $y_{0.1,1,0.9}$ does not belong to $Ker_p(N)$.

Proposition 9. Let $N, M \in \mathcal{N}'(X)$ and $\{N_\alpha : \alpha \in \Delta\} \subseteq \mathcal{N}'(X)$. Then, the following properties hold:

(1) $N \sqsubseteq Ker_p(N)$.

(2) Si $N \sqsubseteq M$, entonces $Ker_p(N) \sqsubseteq Ker_p(M)$.

(3) $Ker_p(Ker_p(N)) = Ker_p(N)$.

(4) $Ker_p\left(\bigsqcup_{\alpha\in\Delta} N_\alpha\right) = \bigsqcup_{\alpha\in\Delta} Ker_p\left(N_\alpha\right)$.

(5) $Ker_p\left(\bigsqcap_{\alpha\in\Delta} N_\alpha\right) \sqsubseteq \bigsqcap_{\alpha\in\Delta} Ker_p\left(N_\alpha\right)$.

(6) $Ker_p(\widetilde{\emptyset}) = \widetilde{\emptyset}$.

(7) $Ker_p(\widetilde{X}) = \widetilde{X}$.

Proof. (1) It is clear from Definition 11.
(2) Suppose that $x_{a,b,c} \notin Ker_p(M)$. Then, there exists $F \in \tau^c(x_{a,b,c})$ such that $F \sqcap M = \widetilde{\emptyset}$. Since $N \sqsubseteq M$, we have $F \sqcap N \sqsubseteq F \sqcap M = \widetilde{\emptyset}$ and so, $F \sqcap N = \widetilde{\emptyset}$. Therefore, $x_{a,b,c} \notin Ker_p(N)$.
(3) By part (1), we have $Ker_p(N) \sqsubseteq Ker_p(Ker_p(N))$. To demonstrate the opposite inclusion, suppose that $x_{a,b,c} \in Ker_p(Ker_p(N))$ and let $F \in \tau^c(x_{a,b,c})$. Then, $F \sqcap Ker_p(N) \neq \widetilde{\emptyset}$, which implies that there exists a N-point $y_{u,v,w} \in Ker_p(N)$ and $F \in \tau^c(y_{u,v,w})$. Thus, $F \sqcap N \neq \widetilde{\emptyset}$ and hence, $x_{a,b,c} \in Ker_p(N)$.
(4) Since $N_\alpha \sqsubseteq \bigsqcup_{\alpha\in\Delta} N_\alpha$ for each $\alpha \in \triangle$, by part (2) it follows that $Ker_p(N_\alpha) \sqsubseteq Ker_p\left(\bigsqcup_{\alpha\in\Delta} N_\alpha\right)$ for each $\alpha \in \triangle$. Therefore, $\bigsqcup_{\alpha\in\Delta} Ker_p(N_\alpha) \sqsubseteq Ker_p\left(\bigsqcup_{\alpha\in\Delta} N_\alpha\right)$. For the other inclusion, suppose that $x_{a,b,c} \notin \bigsqcup_{\alpha\in\Delta} Ker_p(N_\alpha)$. Then, $x \notin Ker_p(N_\alpha)$ for every $\alpha \in \Delta$ and so, there exists $F_\alpha \in \tau^c(x_{a,b,c})$ such that $F_\alpha \sqcap N_\alpha = \widetilde{\emptyset}$ for each $\alpha \in \Delta$. Putting $F = \bigsqcap_{\alpha\in\Delta} F_\alpha$, we have $F \in \tau^c(x_{a,b,c})$ and $F \sqcap \left(\bigsqcup_{\alpha\in\Delta} N_\alpha\right) = \bigsqcup_{\alpha\in\Delta} (F \sqcap N_\alpha) \sqsubseteq \bigsqcup_{\alpha\in\Delta} (F_\alpha \sqcap N_\alpha) = \widetilde{\emptyset}$. Thus, $F \sqcap \left(\bigsqcup_{\alpha\in\Delta} N_\alpha\right) = \widetilde{\emptyset}$ and hence, $x_{a,b,c} \notin Ker_p\left(\bigsqcup_{\alpha\in\Delta} N_\alpha\right)$, which shows that $Ker_p\left(\bigsqcup_{\alpha\in\Delta} N_\alpha\right) \sqsubseteq \bigsqcup_{\alpha\in\Delta} Ker_p(N_\alpha)$.

(5) Since $\bigsqcap_{\alpha\in\Delta} N_\alpha \sqsubseteq N_\alpha$ for each $\alpha \in \Delta$, by using part (2), we have $Ker_p\left(\bigsqcap_{\alpha\in\Delta} N_\alpha\right) \sqsubseteq Ker_p(N_\alpha)$ for each $\alpha \in \Delta$, which implies that $Ker_p\left(\bigsqcap_{\alpha\in\Delta} N_\alpha\right) \sqsubseteq \bigsqcap_{\alpha\in\Delta} Ker_p(N_\alpha)$.
(6) and (7) are immediate consequences of Definition 11.

In the following example, we show that the converse of the part (4) of Proposition 9 is not true in general.

Example 7. Let (X, τ) be the N-topological space given in Example 5. Consider the N-sets $N = \{\langle x, 0.1, 1, 0.9\rangle, \langle y, 0, 0.3, 1\rangle\}$ and $M = \{\langle x, 0, 0.3, 1\rangle, \langle y, 0.1, 1, 0.9\rangle\}$. Then, $M \sqcap N = \widetilde{\emptyset}$ and so (by part (6) of Proposition 9) $Ker_p(M \sqcap N) = \widetilde{\emptyset}$. On the other hand, as $\widetilde{X}$ is the only N-closed set to which the N-point $x_{0.3,1,0.7}$ belongs and $\widetilde{X} \sqcap N \neq \widetilde{\emptyset}$, $\widetilde{X} \sqcap M \neq \widetilde{\emptyset}$, we obtain that $x_{0.3,1,0.7} \in Ker_p(M) \sqcap Ker_p(N)$, which implies that $Ker_p(M) \sqcap Ker_p(N) \neq \widetilde{\emptyset}$. Therefore, the inclusion $Ker_p(M) \sqcap Ker_p(N) \sqsubseteq Ker_p(M \sqcap N)$ is not satisfied.

Remark 7. By Proposition 9, we have Ker_p satisfies the conditions of Definition 9 and by Proposition 7, we conclude that $\tau_k = \{N \in \mathcal{N}'(X) : Ker_p(N^c) = N^c\}$ is a N-topology on X and Ker_p is the N-closure in the N-topological space (X, τ_k). The elements of τ_k are called N-τ_k-open sets and their complements are said to be N-τ_k-closed sets. It is clear that M is N-τ_k-closed if and only if $Ker_p(M) = M$.

5. Conclusion

The concept of N-set has been the cornerstone of neutrosophic science, which has found its place in contemporary research, since this science means development and applications of neutrosophic logic, set, measure, integral, probability, etc., and their applications in any field of knowledge. In this research it was possible to verify that, in some cases, neutrosophic set theory does not behave like classical set theory; for example, the union of a N-set with its neutrosophic complement is not equal to the neutrosophic universe and the neutrosophic empty set is not the only N-set that does not contain N-points. Also, the notions of closure and kernel of an N-set were introduced by means of the concept of N-point, the main properties of these notions were discussed and two new N-topologies related to the introduced notions were generated. The results presented here constitute a contribution to the theory of N-topological spaces and may be useful to extend this area of knowledge by developing new investigations involving N-functions as has been done in the works of S. Das and B.C. Tripathy [14], S. F. Matar and A.A. Hijab [15], P. Basker and B. Said [16].

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