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Spherical Picture Fuzzy Sets with Application to Multicriteria Decision-Making

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Abstract. *Since the introduction of fuzzy sets by Zadeh in 1965 [1], a lot of new theories regarding imprecision and uncertainty have been introduced. Some of these theories are extensions of fuzzy set theory, other try to handle imprecision and uncertainty in different way. The extensions of ordinary fuzzy sets are classified into two broad categories: 1. Intuitionistic fuzzy sets [2] and their versions, 2. Neutrosophic sets [3] and their versions. The first group extensions can be defined by a membership degree and a non-membership degree, whereas the second class of extensions can be defined by a membership degree (truthiness), a non-membership degree (falsity), and a hesitancy degree (indeterminacy). Spherical and picture fuzzy sets fall into the same group because of the definition of membership functions. The squared sum of membership, nonmembership, and hesitancy degrees is equal to or less than 1.0 in spherical fuzzy sets whereas it is valid for the first degree sum in picture fuzzy sets. In this paper, we unify the concepts of picture fuzzy set and spherical fuzzy set into a broad class and name it as spherical picture fuzzy set ($SPFS$). In $SPFSs$, every element of the universe is represented by a sphere. This unique geometrical representation is more adaptable and adequate for handling ambiguity in multi criteria decision-making. A new distance measure of spherical picture fuzzy sets is illustrated, and it is shown that it satisfies conditions of the distance measure. Besides investigating the structural properties of $SPFS$, set-theoretical operations along with some basic algebraic operations and aggregation operators are discussed. One of the most popular multi-criteria decision-making techniques, $TOPSIS$, is expanded to its $SPFS$ form. To demonstrate the effectiveness and feasibility of the proposed $SPFS$-$TOPSIS$ methodology for managing inherent vagueness in the given data, a numerical case study is analyzed wherein the methodology is applied to the pandemic hospital site selection problem.*

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Key Words and Phrases: Spherical picture fuzzy set, Hesitancy degree, Multi-criteria decision making

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1. Introduction

The notion of fuzzy sets or fuzzy logic was first presented by Zadeh [1] in 1965. In that work Zadeh was implicitly advancing the thesis that one of the reasons humans are better at control than currently existing machines is that they are able to make effective decisions on the basis of imprecise linguistic information. Hence it should be possible to improve the performance of electromechanical controllers by modeling the way in which humans reason with this type of information. Usual fuzzy sets are described by membership degree ($\mathfrak{MND}$) and non-membership degree ($\mathfrak{NMND}$) from [0,1], which respectively show how strongly or weakly an element of universe of discourse is associated with the set. Flexibility in the boundary of fuzzy set is useful in tackling vagueness and uncertainty. Many researchers introduced several new extensions of ordinary fuzzy sets by describing membership functions [1],[4],[2],[5],[6],[3],[7],[8],[9],[10],[11],[12],[13]. Atanassov [2] presented the notion of intuitionistic fuzzy sets ($IFSs$) which consist of $\mathfrak{MD}$ and $\mathfrak{NMD}$ whose sum cannot exceed 1. In $IFSs$ hesitancy of experts is taken into account. Torra [7] introduced hesitant fuzzy sets ($HFSs$) to work with a set of potential membership values of an element in a fuzzy set. F.E Boran et al. [14] introduced Pythagorean fuzzy sets ($PyFSs$) by adding a rather large ground of $\mathfrak{MND}\mathfrak{s}$ and $\mathfrak{NMND}\mathfrak{s}$.

The concept of neutrosophic sets was introduced by Smarandache [3]. In these sets, degree of truthfulness, falsity, and indeterminacy are linked with every element of the universe of discourse such that sum of these degrees cannot exceed 3. Coung [8] introduced picture fuzzy sets ($PFSs$) extending $IFSs$. $PFSs$ were further extended by Gündoğdu and Kahraman [10] by initiating the notion of spherical fuzzy sets ($SFSs$) where each element is associated $\mathfrak{MD}$ and $\mathfrak{NMD}$ and $\mathfrak{HD}$. Circular Intuitionistic fuzzy sets (C-$IFSs$) was developed by Atanassov [15]. In Fig 1, the new extensions of usual fuzzy sets are displayed historically.

In this article, we present the idea of spherical picture fuzzy set ($SPFS$), which extends and unifies the notions of C-$IFSs$ and PFS. In $SPFSs$, every element of the universe is surrounded by a sphere. This unique geometric form is more adaptable and adequate to handle fuzzy information consisting of hesitance or ignorance. $SPFSs$ can reduce data loss and more accurately capture the inherent ambiguity of objective problems in mathematical terms. Furthermore, by incorporating a radius parameter, spherical picture fuzzy sets can effectively integrate multiple vague data elements into a single $SPFS$. This approach can help in streamlining complex multi-attribute group decision-making processes. Combining affiliation, neutral attitude, non-affiliation, and radius into a spherical picture fuzzy set helps represent information more fully and makes it easier to handle vague or uncertain situations. For further details about multi-attribute group decision-making [16] and [17] should be consulted.

Ranking of alternatives is one of the main steps in multi-attribute group decision-making processes. Score functions and distance evaluations are valuable tools for ranking. Chen [18] and other scholars initially proposed the score function within the framework of $IFSs$. Çakir [19] extended score function to C-$IFSs$. As fuzzy multi-attribute decision-making has evolved, an increasing number of score functions have been presented and used in

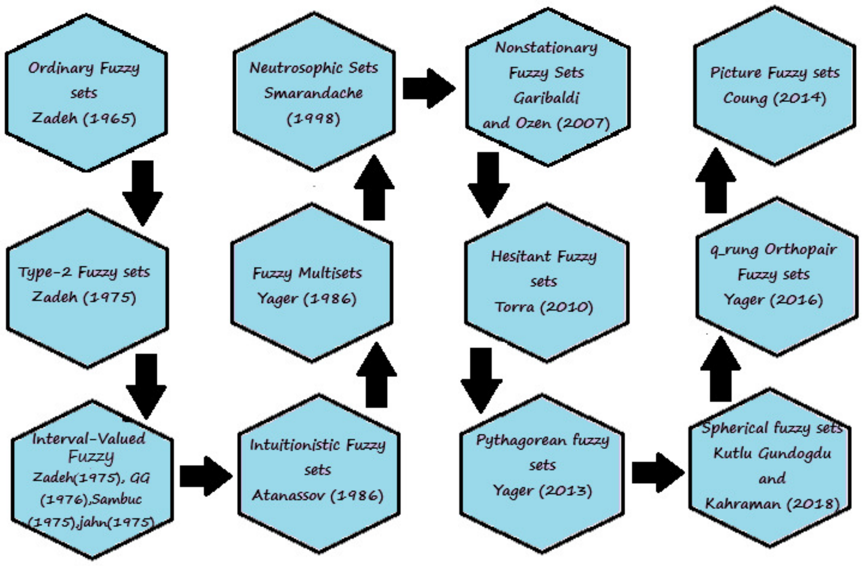

Figure 1: Evolution of fuzzy set

decision-making processes. On the other hand, distance measures play an important role in $IFSs$. One of the main objectives of this research is to define and explore distance measures within spherical picture fuzzy sets. Recently some researchers have worked on fuzzy analysis [20].

2. Preliminaries

In this section some terms and definitions are provided which will be used in the main work of this manuscript.

Definition 1. *[1] Fuzzy set A in a universe of discourse Y is set of order pairs of the form*

$$A = \{(y, \xi_A(y)), y \in Y\}.$$

Value of the membership function $\xi_A(y)$ at $y \in Y$ represents grade of membership of y in A.

Definition 2. *[2] An IFS A on a universal set Y is an object of the form*

$$A = \{(y, \xi_A(y), \rho_A(y)|y \in Y)\},$$

where $\xi_A(y), \rho_A(y) \in [0,1]$ are called the $\mathfrak{MD}$ and $\mathfrak{NMD}$ of y in A. ξ_A and ρ_A satisfy the following condition:

$$\xi_A(y) + \rho_A(y) \leq 1, \forall y \in Y.$$

Definition 3. *[8] A $PFSs$ A on a universe Y is an object in the form of*

$$A = \{(y, \xi_A(y), \psi_A(y), \rho_A(y))|y \in Y\},$$

where $\xi_A(y), \psi_A(y), \rho_A(y) \in [0,1]$ are called positive membership degree ($\mathfrak{MD}$), neutral membership degree ($\mathfrak{N_tMD}$) and negative membership degree ($\mathfrak{ND}$) of y in A respectively, and

$$\xi_A(y) + \psi_A(y) + \rho_A(y) \leq 1, \forall y \in Y.$$

Definition 4. *[8] Let $A = \{(y, \xi_A(y), \psi_A(y), \rho_A(y))|y \in Y\}$, be a $PFSs$ then score function is defined as $\xi(y) + \psi(y) - \rho(y)$.*

Definition 5. *[8] Distances between two $PFSs$ A and B, in $Y = \{y_1, y_2, ..., y_n\}$ are:*

(i) The normalized Harming distance
$H(A,B) = \frac{1}{n}\sum_{i=1}^{n}(|\xi_A(y_i) - \xi_B(y_i)| + |\psi_A(y_i) - \psi_B(y_i)| + |\rho_A(y_i) - \rho_B(y_i)|)$

(ii) The normalized Euclidean distance
$E(A,B) = (\frac{1}{n}\sum_{i=1}^{n}((\xi_A(y_i) - \xi_B(y_i))^2 + (\psi_A(y_i) - \psi_B(y_i))^2 + (\rho_A(y_i) - \rho_B(y_i))^2))^{\frac{1}{2}}$.

3. Definition and Properties of Spherical Picture Fuzzy Sets

In this section some basic concepts and concerning spherical picture fuzzy set are presented for the development of main contents.

Definition 6. *Let Y be universe of discourse. Spherical picture fuzzy set ($SPFS$) is an object of the form*

$$S = \{\langle y, \xi(y), \psi(y), \rho(y); \gamma\rangle | y \in Y\},$$

with

$$0 \leq \xi(y) + \psi(y) + \rho(y) \leq 1,$$

where $\xi : Y \to [0,1], \psi : Y \to [0,1]$ and $\rho : Y \to [0,1]$ represent the membership, neutral, and non-membership functions of S and $\gamma \in [0,1]$ is radius of the sphere surrounding $y \in Y$,

Each element y is $SPFS$ is represented by a sphere with center $(\xi(y), \psi(y), \rho(y))$ and radius γ, whereas in PFS it is represented just by a point. PFS is a special type of $SPFS$ with $\gamma = 0$. On the other hand a spherical picture fuzzy set with $\gamma > 0$ cannot be expressed as ordinary picture fuzzy set. Thus spherical picture fuzzy set is proper extension of PFS.

$\phi(y) = 1 - \xi(y) - \psi(y) - \rho(y)$ is known as degree of hesitancy of $y \in Y$ with respect to S.

Geometrical representation of $SPFS$ is given in Fig 2.

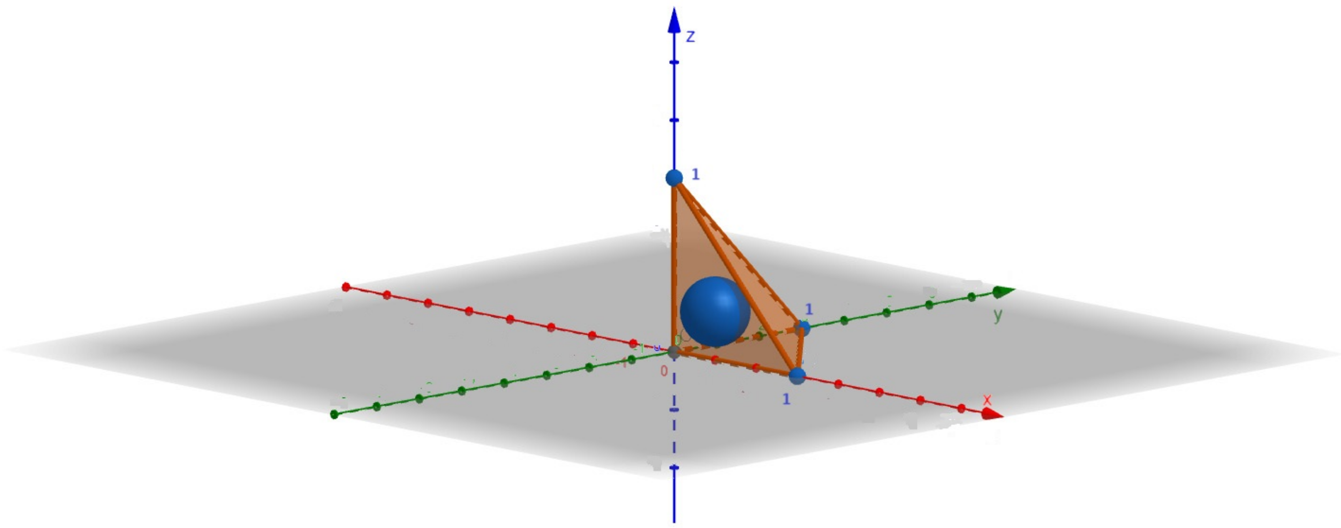

Figure 2: SPFS geometrical representation

Definition 7. *Let* $P^* = \{\langle l, m, n\rangle | l, m, n \in [0,1]$ *and* $l + m + n \leq 1\}$, *be a picture fuzzy set then the associated spherical picture fuzzy* S_r, *may be expressed as:*

$$S_r = \{\langle y, C_r(\xi_S(y), \psi_S(y), \rho_S(y))\rangle | y \in Y\},$$

where C_r *is a function depicts a sphere with a radius of* γ *and a center of* $(\xi_S(y), \psi_S(y), \rho_S(y))$,

$$C_r(\xi_S(y), \psi_S(y), \rho_S(y)) = \Bigg\{\langle l, m, n\rangle | l, m, n \in [0,1] \text{ and }$$
$$\sqrt{(\xi_S(y) - l)^2 + (\psi_S(y) - m)^2 + (\rho_S(y) - n)^2} \leq \gamma\Bigg\} \bigcap P^*$$
$$= \left\{\langle l, m, n\rangle | \sqrt{(\xi_S(y) - l)^2 + (\psi_S(y) - m)^2 + (\rho_S(y) - n)^2} \leq \gamma : l + m + n \leq 1\right\}.$$

Definition 8. *From a given collection of* $(PFSs)$

$$\{\langle m_{i,1}, n_{i,1}, o_{i,1}\rangle, \langle m_{i,2}, n_{i,2}, o_{i,2}\rangle, \langle m_{i,3}, n_{i,3}, o_{i,3}\rangle, ...\}$$

$(SPFS)$ *is calculated as follows:*

$$\langle \xi(S_i), \psi(S_i), \rho(S_i)\rangle = \left\langle \frac{\sum_{j=1}^{k_i} m_{i,j}}{k_i}, \frac{\sum_{j=1}^{k_i} n_{i,j}}{k_i}, \frac{\sum_{j=1}^{k_i} o_{i,j}}{k_i} \right\rangle, \tag{1}$$

where k_i denote the number of decision makers.

$$\gamma_i = \max_{1\leq j\leq k_i} \sqrt{(\xi(S_i) - m_{i,j})^2 + (\psi(S_i) - n_{i,j})^2 + (\rho(S_i) - o_{i,j})^2}. \tag{2}$$

Some fundamental algorithms and operations for spherical picture fuzzy sets are provided here to facilitate their application in multi-attribute decision making.

Definition 9. *Let $S_1 = \{\langle x, \xi_1(y), \psi_1(y), \rho_1(y); \gamma_1\rangle | y \in Y\}$ and $S_2 = \{\langle x, \xi_2(y), \psi_2(y), \rho_2(y); \gamma_2\rangle | y \in Y\}$, be two $SPFSs$, then operation between them can be defined as follows:*

$$\begin{aligned}
S_1 \oplus_{\min} S_2 &= \{\langle y, \xi_1(y) + \xi_2(y) - \xi_1(y)\xi_2(y), \psi_1(y)\psi_2(y), \rho_1(y)\rho_2(y);\\
&\quad \min(\gamma_1, \gamma_2)\rangle | y \in Y\}.\\
S_1 \oplus_{\max} S_2 &= \{\langle y, \xi_1(y) + \xi_2(y) - \xi_1(y)\xi_2(y), \psi_1(y)\psi_2(y), \rho_1(y)\rho_2(y);\\
&\quad \max(\gamma_1, \gamma_2)\rangle | y \in Y\}.\\
S_1 \otimes_{\min} S_2 &= \{\langle y, \xi_1(y)\xi_2(y), \psi_1(y) + \psi_2(y) - \psi_1(y)\psi_2(y), \rho_1(y) + \rho_2(y)\\
&\quad - \rho_1(y)\rho_2(y); \min(\gamma_1, \gamma_2)\rangle | y \in Y\}.\\
S_1 \otimes_{\max} S_2 &= \{\langle y, \xi_1(y)\xi_2(y), \psi_1(y) + \psi_2(y) - \psi_1(y)\psi_2(y), \rho_1(y) + \rho_2(y)\\
&\quad - \rho_1(y)\rho_2(y); \max(\gamma_1, \gamma_2)\rangle | y \in Y\}.\\
S_1 \odot S_2 &= \{\langle y, \xi_1(y)\xi_2(y), \psi_1(y) + \psi_2(y) - \psi_1(y)\psi_2(y), \rho_1(y) + \rho_2(y)\\
&\quad - \rho_1(y)\rho_2(y); \frac{\gamma_1 + \gamma_2}{2}\rangle | y \in Y\}.
\end{aligned} \tag{3}$$

Definition 10. *Let $S = \{\langle y, \xi_s(y), \psi_s(y), \rho_1(y); \gamma_1\rangle | y \in Y\}$ be a spherical picture fuzzy set. Subsequently, the score function of the spherical picture fuzzy set is*

$$S_c(S) = \frac{1}{3}(\xi_s - \psi_s - \rho_s + \sqrt{2\gamma}(2\lambda - 1)),$$

$\lambda = 0$ and $\lambda = 1$ respectively show complete pessimism and optimism, while $\lambda = 0.5$ is indicator for nonchalance on behalf of the decision maker.

4. Distance analysis of spherical picture fuzzy set

This paper examines the incorporation of hesitation degree into the spherical picture fuzzy set distance by assigning it to partial affirmation, impartial affirmation, and partial negation. This approach indirectly integrates hesitation degree into the distance metric, aiming to improve the handling of imprecise information in practice. Below is the initial definition of the assignment of $\mathfrak{HD}$ to $\mathfrak{MD}$, $\mathfrak{N_tMD}$, and $\mathfrak{NMD}$.

Definition 11. *Let $S_1 = \{\langle y, \xi_{S_1}(y), \psi_{S_1}(y), \rho_{S_1}(y), \gamma_{S_1}\rangle | y_i \in Y\}$ be a $SPFS$ on Y along with the distribution of $\mathfrak{MD}$, $\mathfrak{NMD}$ and $\mathfrak{N_tMD}$ by $\mathfrak{HD}$ $\phi_A(y_i)$ is defined as:*

$$CD^{S_1}_{\phi\to\xi}(y) = \frac{1}{2}[\phi_{S_1}(y) + 2\xi_{S_1}(y)],$$

$$CD^{S_1}_{\phi\to\psi}(y) = \frac{1}{2}[\phi_{S_1}(y) + 2\psi_{S_1}(y)],$$
$$CD^{S_1}_{\phi\to\rho}(y) = \frac{1}{2}[\phi_{S_1}(y) + 2\rho_{S_1}(y)].$$

The degree of hesitation can be seen as the decision maker's hesitancy to accept, reject or support an uncertain object, as it reflects the unknown degree of knowledge. Since there may be a degree of partial negation as well as a degree of partial affirmation, and impartial affirmation in the hesitation degree, so the hesitation degree is divided equally into $\mathfrak{MD}$, $\mathfrak{NMD}$, and $\mathfrak{N_tMD}$.

In $SPFS$ radius is obtained from the calculation of $\mathfrak{MD}$, $\mathfrak{NMD}$, and $\mathfrak{N_tMD}$, Hence, if changes occur in these degrees, the radius will also change. On the basis of this, the radius of the $SPFS$ is defined according to the newly assigned hesitation degree.

Definition 12. *Let there is a set of $PFSs$ pairs*

$$\{\langle m_{i,1}, n_{i,1}, o_{i,1}\rangle, \langle m_{i,2}, n_{i,2}, o_{i,2}\rangle, \langle m_{i,3}, n_{i,3}, o_{i,3}\rangle, ...\}$$

then the radius of the $SPFS$

$$\langle \xi(S_i), \psi(Si), \rho(Si)\rangle = \left\langle \frac{\sum_{j=1}^{k_i} m_{i,j}}{k_i}, \frac{\sum_{j=1}^{k_i} n_{i,j}}{k_i}, \frac{\sum_{j=1}^{k_i} o_{i,j}}{k_i} \right\rangle$$

is calculated as

$$\gamma_{ij} = \max_{1\le j\le k_i} \left| \sqrt{\left(D\xi_i - Dm_i\right)^2 + \left(D\psi_i - Dn_i\right)^2 + \left(D\rho_i - Do_i\right)^2} \right| \tag{4}$$

where

$$D\xi_i = \left(\xi(S_i) + \frac{1}{2}\phi(S_i)\right), Dm_i = \left(m_i + \frac{1}{2}l_i\right)$$
$$D\psi_i = \left(\psi(S_i) + \frac{1}{2}\phi(S_i)\right), Dn_i = \left(n_i + \frac{1}{2}l_i\right)$$
$$D\rho_i = \left(\rho(S_i) + \frac{1}{2}\phi(S_i)\right), Do_i = \left(o_i + \frac{1}{2}l_i\right)$$
$$\phi(S_i) = 1 - \xi(S_i) - \psi(S_i) - \rho(S_i), l_{ij} = 1 - m_i - n_i - o_i,$$

next we define distance metric of $SPFS$ under hesitancy degree. The degree is assigned based on the motivation of the distance metric for C-IFS given in literature [21].

Definition 13. *Let* $S_1 = \{\langle y, \xi_{S_1}(y), \psi_{S_1}(y), \rho_{S_1}(y); \gamma_{S_1}\rangle | y \in Y\}$, *and* $S_2 = \{\langle y, \xi_{S_2}(y), \psi_{S_2}(y), \rho_{S_2}(y); \gamma_{S_2}\rangle | y \in Y\}$, *be two spherical picture fuzzy sets on the universe set* Y *then the distance metric is defined as:*

$$\partial_c(S_1, S_2) = \frac{|\gamma_{S_1} - \gamma_{S_2}|}{\sqrt{2}} + \frac{1}{2}\left((\Delta_{\xi}^{S_1S_2})^2 + (\Delta_{\psi}^{S_1S_2})^2 + (\Delta_{\rho}^{S_1S_2})^2 + (\Delta_{\phi\to\xi}^{S_1S_2})^2 + (\Delta_{\phi\to\psi}^{S_1S_2})^2 + (\Delta_{\phi\to\rho}^{S_1S_2})^2\right)^{\frac{1}{2}} \quad (5)$$

where

$$\begin{aligned}
\Delta_{\xi}^{S_1S_2} =& |\xi_{S_1}(y) - \xi_{S_2}(y)|, \\
\Delta_{\psi}^{S_1S_2} =& |\psi_{S_1}(y) - \psi_{S_2}(y)|, \\
\Delta_{\rho}^{S_1S_2} =& |\rho_{S_1}(y) - \rho_{S_2}(y)|, \\
\Delta_{\phi\to\xi}^{S_1S_2} =& |SD_{\phi\to\xi}^{S_1} - SD_{\phi\to\xi}^{S_2}|, \\
\Delta_{\phi\to\psi}^{S_1S_2} =& |SD_{\phi\to\psi}^{S_1} - SD_{\phi\to\psi}^{S_2}|, \\
\Delta_{\phi\to\rho}^{S_1S_2} =& |SD_{\phi\to\rho}^{S_1} - SD_{\phi\to\rho}^{S_2}|.
\end{aligned}$$

The distance metric $\partial_c(A, B)$ is constructed in two main steps: first, the radius difference between the two spherical picture fuzzy sets ($SPFS$) is calculated; second, the concept of distance of Euclidean in real space is applied. The formula for distance in a six-dimensional real space is constructed by examining $(\xi_{S_1}(y), \psi_{S_1}(y), \rho_{S_1}(y), SD_{\phi\to\xi}^{S_1}, SD_{\phi\to\psi}^{S_1}, SD_{\phi\to\rho}^{S_1})$, and $(\xi_{S_2}(y), \psi_{S_2}(y), \rho_{S_2}(y), SD_{\phi\to\xi}^{S_2}, SD_{\phi\to\psi}^{S_2}, SD_{\phi\to\rho}^{S_2})$, the coordinates of two points. Therefore the following theorem can be obtained.

Theorem 1. *Let*

$$S_1 = \{\langle y, \xi_{S_1}(y), \psi_{S_1}(y), \rho_{S_1}(y); \gamma_{S_1}\rangle | \ \ y \in Y\},$$

$$S_2 = \{\langle y, \xi_{S_2}(y), \psi_{S_2}(y), \rho_{S_2}(y); \gamma_{S_2}\rangle | \ \ y \in Y\},$$

and

$$S_3 = \{\langle y, \xi_{S_3}(y), \psi_{S_3}(y), \rho_{S_3}(y); \gamma_{S_3}\rangle | \ \ y \in Y\},$$

be three $SPFSs$ *on universe of discourse* Y. *Then*

a) $\partial_c(S_1, S_2) \geq 0, \partial_c(S_1, S_2) = 0$ *if and only if* $S_1 = S_2$.

b) $\partial_c(S_1, S_2) = \partial_c(S_2, S_1)$.

c) $\partial_c(S_1, S_2) + \partial_c(S_2, S_3) \geq \partial_c(S_1, S_3)$.

Proof. To show that $\partial_c(S_1, S_2)$ is a distance metric, it will suffice to prove that it satisfies the three conditions given in Definition 13.

a) Obviously $\partial_c(S_1, S_2) \geq 0$. If $S_1 = S_2$, then
$\Delta_{\xi}^{S_1S_2} = \Delta_{\psi}^{S_1S_2} = \Delta_{\rho}^{S_1S_2} = 0$,
$\Delta_{\xi\to\phi}^{S_1S_2} = \Delta_{\psi\to\phi}^{S_1S_2} = \Delta_{\rho\to\phi}^{S_1S_2} = 0,$

and $\gamma_{S_1} = \gamma_{S_2} = 0$. Thus $\partial_c(S_1, S_2) = 0$.
Conversely, $\partial_c(S_1, S_2) = 0$ implies that $\Delta_{\xi}^{S_1S_2} = \Delta_{\psi}^{S_1S_2} = \Delta_{\rho}^{S_1S_2} = 0$ and

$$\begin{aligned} \xi_{S_1}(y) &= \xi_{S_2}(y), \\ \psi_{S_1}(y) &= \psi_{S_2}(y), \\ \rho_{S_1}(y) &= \rho_{S_2}(y). \end{aligned}$$

Therefore $S_1 = S_2$.

b) $\partial_c(S_1, S_2) = \partial_c(S_2, S_1)$ is obvious.

c) Consider

$$|\gamma_{S_3} - \gamma_{S_1}| = |\gamma_{S_3} - \gamma_{S_2} + \gamma_{S_2} - \gamma_{S_1}| \leq |\gamma_{S_3} - \gamma_{S_2}| + |\gamma_{S_2} - \gamma_{S_1}|,$$

which implies

$$\frac{|\gamma_{S_3} - \gamma_{S_1}|}{\sqrt{2}} \leq \frac{|\gamma_{S_1} - \gamma_{S_2}|}{\sqrt{2}} + \frac{|\gamma_{S_2} - \gamma_{S_3}|}{\sqrt{2}}. \tag{6}$$

Also it can be noticed that

$$D_c(S_1, S_2) = \left(\frac{1}{4} \left[(\Delta_{\xi}^{S_1S_2})^2 + (\Delta_{\psi}^{S_1S_2})^2 + (\Delta_{\rho}^{S_1S_2})^2 + (\Delta_{\phi\to\xi}^{S_1S_2})^2 + (\Delta_{\phi\to\psi}^{S_1S_2})^2 + (\Delta_{\phi\to\rho}^{S_1S_2})^2 \right] \right)^{\frac{1}{2}}$$

is Euclidean distance between the two points
$(\xi_{S_1}(y), \psi_{S_1}(y), \rho_{S_1}(y), SD_{\phi\to\xi}^{S_1}, SD_{\phi\to\psi}^{S_1}, SD_{\phi\to\rho}^{S_1})$
and $(\xi_{S_2}(y), \psi_{S_2}(y), \rho_{S_2}(y), SD_{\phi\to\xi}^{S_2}, SD_{\phi\to\psi}^{S_2}, SD_{\phi\to\rho}^{S_2})$ in six-dimensional space divided by 2, therefor

$$D_c(S_1, S_3) \leq D_c(S_1, S_2) + D_c(S_2, S_3) \tag{7}$$

Adding (6) and (7), we have

$$\begin{aligned} &\frac{|\gamma_{S_3} - \gamma_{S_1}|}{\sqrt{2}} + \left(\frac{1}{4} \left[(\Delta_{\xi}^{S_1S_3})^2 + (\Delta_{\psi}^{S_1S_3})^2 + (\Delta_{\rho}^{S_1S_3})^2 + (\Delta_{\phi\to\xi}^{S_1S_3})^2 + (\Delta_{\phi\to\psi}^{S_1S_3})^2 + (\Delta_{\phi\to\rho}^{S_1S_3})^2 \right] \right)^{\frac{1}{2}} \\ &\leq \frac{|\gamma_{S_1} - \gamma_{S_2}|}{\sqrt{2}} + \left(\frac{1}{4} \left[(\Delta_{\xi}^{S_1S_2})^2 + (\Delta_{\psi}^{S_1S_2})^2 + (\Delta_{\rho}^{S_1S_2})^2 + (\Delta_{\phi\to\xi}^{S_1S_2})^2 + (\Delta_{\phi\to\psi}^{S_1S_2})^2 + (\Delta_{\phi\to\rho}^{S_1S_2})^2 \right] \right)^{\frac{1}{2}} \\ &+ \frac{|\gamma_{S_2} - \gamma_{S_3}|}{\sqrt{2}} + \left(\frac{1}{4} \left[(\Delta_{\xi}^{S_2S_3})^2 + (\Delta_{\psi}^{S_2S_3})^2 + (\Delta_{\rho}^{S_2S_3})^2 + (\Delta_{\phi\to\xi}^{S_2S_3})^2 + (\Delta_{\phi\to\psi}^{S_2S_3})^2 + (\Delta_{\phi\to\rho}^{S_2S_3})^2 \right] \right)^{\frac{1}{2}}, \end{aligned}$$

thus $\partial_c(S_1, S_2)$ satisfies the triangular inequality.

Definition 14. *let S_1, S_2 be two random SPFSs on the universe $Y = \{x_1, x_2, \cdots, x_n\}$ then the normalized SPFS distance between S_1 and S_2 is defined as*

$$N\partial_c(S_1, S_2) = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{|\Delta_{\gamma}^{S_1S_2}(i)|}{\sqrt{2}} + \left[\frac{1}{4} (\Delta_{\xi}^{S_1S_2}(i))^2 + (\Delta_{\psi}^{S_1S_2}(i))^2 + (\Delta_{\rho}^{S_1S_2}(i))^2 + \right.\right.$$

$$(\Delta_{\phi\to\xi}^{S_1S_2}(i))^2 + (\Delta_{\phi\to\psi}^{S_1S_2}(i))^2 + (\Delta_{\phi\to\rho}^{S_1S_2}(i))^2\Bigg]^{\frac{1}{2}}\Bigg). \quad (8)$$

Where

$$\Delta_{\gamma}^{S_1S_2}(i) = |\gamma_{S_1}(x_i) - \gamma_{S_2}(x_i)|$$

$$\Delta_{\xi}^{S_1S_2}(i) = |\xi_{S_1}(x_i) - \xi_{S_2}(x_i)|,$$

$$\Delta_{\psi}^{S_1S_2}(i) = |\psi_{S_1}(x_i) - \psi_{S_2}(x_i)|,$$

$$\Delta_{\rho}^{S_1S_2}(i) = |\rho_{S_1}(x_i) - \rho_{S_2}(x_i)|,$$

$$\Delta_{\phi\to\xi}^{S_1S_2}(i) = |SD_{\phi\to\xi}^{S_1}(i) - SD_{\phi\to\xi}^{S_2}(i)|,$$

$$\Delta_{\phi\to\psi}^{S_1S_2}(i) = |SD_{\phi\to\psi}^{S_1}(i) - SD_{\phi\to\psi}^{S_2}(i)|,$$

$$\Delta_{\phi\to\rho}^{S_1S_2}(i) = |SD_{\phi\to\rho}^{S_1}(i) - SD_{\phi\to\rho}^{S_2}(i)|.$$

To evaluate the feasibility and effectiveness of the newly proposed spherical picture fuzzy set ($SPFS$) distance measure, we apply it to a real-world multi-criteria decision-making ($MCDM$) problem.

5. Multi-Criteria Decision Making via Spherical Picture Fuzzy Sets

$TOPSIS$ is a well-established and effective approach for solving multi-attribute decision-making problems. In this paper, we introduce a $SPFS-$based $TOPSIS$ method that incorporates the newly developed distance measure. This method enhances traditional approaches and improves the decision-making process by addressing uncertainties more comprehensively through spherical picture fuzzy sets.

Step 1. Let us define the solution set for the $TOPSIS$ problem as follows:

$$\sigma_i = \{\sigma_1, \sigma_2, ...\sigma_m\}, i = 1, 2, ..., m;$$

where σ_i represents the alternatives in the decision problem, and m represents number of available alternatives. The decision criteria set is defined as

$$C_j = \{C_1, C_2, ..., C_n\}, j = 1, 2, ..., n;$$

where C_j represents the criteria by which the alternatives are evaluated, and n is the total number of criteria. Let

$$W = \{W_1, W_2, ..., W_n\},$$

be the weight vector associated with the criteria group, where

$$W_j \geq 0, \sum_{j=1}^{n} W_i = 1;$$

which ensures that the total importance of the criteria adds up to 1. The decision maker (DM), with the assistance of experts from various fields, evaluates the alternatives based on the criteria. The experts provide judgments regarding the solutions and decision criteria in order to facilitate the process of decision making.

Step 2. Analyze the perspectives of decision-makers, create an expert linguistic decision matrix, and use Table 1 to immediately convert qualitative data into picture fuzzy numbers (PFNs).

Table 1: Picture fuzzy numbers semantic quantization table.

Linguistic Value	**PFNs**
Certainly high value (CHV)	$\langle 0.8, 0.1, 0.0\rangle$
Very high value (VHV)	$\langle 0.4, 0.2, 0.3\rangle$
High value (HV)	$\langle 0.5, 0.3, 0.0\rangle$
Above average value (AAV)	$\langle 0.3, 0.3, 0.2\rangle$
Average value (AV)	$\langle 0.7, 0.1, 0.1\rangle$
Under average value (UAV)	$\langle 0.4, 0.3, 0.2\rangle$
Low value (LV)	$\langle 0.3, 0.4, 0.1\rangle$
Very low value (VLV)	$\langle 0.6, 0.2, 0.1\rangle$
Certainly low value (CLV)	$\langle 0.4, 0.3, 0.1\rangle$

Step 3. Create aggregated picture fuzzy numbers from the picture fuzzy pairs representing the perspectives of several decision-makers for the same choice and decision criteria in the decision matrix. $\langle \xi(C_i), \psi(C_i), \rho(C_i)\rangle$ using equation(1). Then use equation(4) to determine the matching radius length in order to build a fuzzy decision matrix $M = (x_{ij})_{nm}$, with a spherical picture, where $x_{ij} = \{\langle \xi_{ij}, \psi_{ij}, \rho_{ij}; \gamma_{ij}\rangle\}$ denotes the spherical picture fuzzy number of the alternative with respect to the criteria.

Step 4. To obtain the weighted sum table of picture fuzzy conditions, quantify the weight information from Table 3 and determine the weights of the various Conditions. Next, to create the spherical picture fuzzy set criteria weight matrix $W = (\omega_j)_{1\times n}$, where $\omega_j = \{\langle \xi_j, \psi_j, \rho_j; \gamma_j\rangle\}$, the maximum radius r is determined using equation(4).

Step 5. Create the decision matrix $G = (g_{ij})m \times n$, which is weighted. Each element $g_{ij} = \langle \xi_{ij}, \psi_{ij}, \rho_{ij}, \gamma_{ij}\rangle$ is calculated using the weight matrix W acquired in step 4, the spherical picture fuzzy set decision matrix M, and equation(3).

Step 6. Determine the best negative solution, G^-, and the positive ideal solution, G^+ for the choice matrix.

$$G^+ = \{\langle(\max_i g_{ij}|j \in \Im_1), (\min_i g_{ij}|j \in \Im_2), (\min_i g_{ij}|j \in \Im_3)\rangle|j = 1, 2, 3, ...n\} \tag{9}$$

$$G^- = \{\langle(\min_i g_{ij}|j \in \Im_1), (\max_i g_{ij}|j \in \Im_2), (\max_i g_{ij}|j \in \Im_3)\rangle|j = 1, 2, 3, ...n\} \tag{10}$$

where $g_j^+ = \{\langle\xi_j^+, \psi_j^+, \rho_j^+; \gamma_j^+\rangle\}$,$g_j^- = \{\langle\xi_j^-, \psi_j^-, \rho_j^-; \gamma_j^-\rangle\}$ represents the $SPFS$ with highest and lowest membership degrees among j criteria. $\Im_1, \Im_2, \Im_3$ represent the beneficial criteria and the cost criteria.

Table 2: Quantization table of weight information.

Linguistic Value	**PFNs**
Certainly high importance (CHI)	$\langle 0.8, 0.1, 0.0\rangle$
Very high importance (VHI)	$\langle 0.4, 0.2, 0.3\rangle$
High importance (HI)	$\langle 0.5, 0.3, 0.0\rangle$
Above average importance (AAI)	$\langle 0.3, 0.3, 0.2\rangle$
Average importance (AI)	$\langle 0.7, 0.1, 0.1\rangle$
Under average importance (UAI)	$\langle 0.4, 0.3, 0.2\rangle$
Low importance (LI)	$\langle 0.3, 0.4, 0.1\rangle$
Very low importance (VLI)	$\langle 0.6, 0.2, 0.1\rangle$
Certainly low importance (CLI)	$\langle 0.4, 0.3, 0.1\rangle$

Step 7. Determine the distance between each option and the ideal solutions. The positive ideal solution, $N\partial_c^+(\sigma_i)$, and the negative ideal solution, $N\partial_c^-(\sigma_i)$, using the new distance equation(5) proposed in this paper.

Step 8. Calculate the relative closeness coefficient $R_{CC}(\sigma_i)$ using normalized $SPFS$ distances (8), rank the alternatives based on the $R_{CC}(\sigma_i)$, and finally select the best option.

$$R_{CC}(\sigma_i) = \frac{N\partial_c^-(\sigma_i)}{N\partial_c^-(\sigma_i) + N\partial_c^+(\sigma_i)} \tag{11}$$

6. An Application to Pandemic Hospital Site Selection

Using a hospital site example from the literature [22], the distance metric presented in this work is tested for validity below. For the public health systems, mass infectious diseases have always been an extremely challenging issue. Not only does it present a risk to events related to the lives of individuals, national and international public health even has the ability to trigger social unrest and have a serious detrimental effect on economic growth. Many scholars are focusing their research on how to address widespread public health events in an effective manner. The issue of allocating medical resources will be discussed in this paper starting with the locations of Istanbul's hospitals.

Step 1. Firstly, seven locations for hospitals were determined, σ_1-Bakirköy, σ_2-Sancaktepe, σ_3-Eyüp, σ_4-Esenyurt, σ_5-Çatalca, σ_6-Tuzla, σ_7-Ataşehir, which are randomly placed in different location of Istanbul. There are seven characteristics that must be taken into account during the selection process: ς_1(Cost), ς_2(Demographics), ς_3(Environmental Factors), ς_4(Transportation opportunities), ς_5(Healthcare and medical practices), ς_6(Infrastructure), ς_7(Spread of the virus). This leads to the event set $\sigma = \{\sigma_1, \sigma_2, ..., \sigma_7\}$ and the criteria set $C = \{\varsigma_1, \varsigma_2, ..., \varsigma_7\}$. Additionally, three fuzzy multi-criteria decision-making experts, designated as $DM1$, $DM2$, and $DM3$, were chosen to serve as decision makers.

Step 2. Based on their knowledge and the actual scenario, the decision makers (DMs) evaluated each proposal, and the resulting expert decision matrix is displayed in Table 3. Next, by quantifying semantic information, the qualitative evaluation data is converted into fuzzy sets (Table 1)

Table 3: Expert decision sheet.

Criterion	**DMs**	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7
	DM1	HV	LV	AV	VLV	AV	AV	AAV
ς_1	DM2	HV	UAV	AV	LV	UAV	AV	HV
	DM3	AAV	LV	AAV	LV	UAV	AV	HV
	DM1	AAV	VHV	VHV	VHV	UAV	AAV	HV
ς_2	DM2	AAV	HV	HV	VHV	LV	AV	HV
	DM3	AV	VHV	HV	CHV	LV	AAV	VHV
	DM1	LV	AV	UAV	VLV	HV	AV	UAV
ς_3	DM2	LV	AV	UAV	LV	AAV	UAV	AV
	DM3	UAV	AAV	LV	LV	HV	AV	AV
	DM1	HV	AV	AAV	UAV	UAV	UAV	AAV
ς_4	DM2	VHV	AV	HV	UAV	UAV	LV	AAV
	DM3	CHV	AV	AAV	AV	LV	UAV	HV
	DM1	AAV	AV	AAV	AV	UAV	LV	UAV
ς_5	DM2	AAV	AV	AAV	AV	LV	UAV	UAV
	DM3	HV	UAV	HV	AAV	AV	AV	AV
	DM1	UAV	AV	AAV	CLV	VHV	AV	LV
ς_6	DM2	UAV	AAV	AAV	CLV	CHV	AAV	LV
	DM3	LV	HV	HV	VLV	CHV	AAV	VLV
	DM1	HV	HV	HV	VHV	VLV	LV	HV
ς_7	DM2	VHV	AAV	AAV	CHV	CLV	LV	HV
	DM3	VHV	HV	HV	CHV	VLV	UAV	VHV

Step 3. The spherical picture fuzzy decision matrix displayed in Table 4 is created by combining the picture fuzzy evaluation data provided by the DMs in accordance with equations (2) and (4).

Step 4. Table 2 was used to measure the data using the weight assessment criteria from Table 6 in the literature [23]. The weight data was then merged using equations (2) and (4) to produce the criteria weight matrix (Table 6).

Step 5. As shown in Table 7, determine the spherical picture fuzzy decision matrix for each criterion $\mathfrak{C}_{\mathfrak{r}}j$ after applying the weights in accordance with equation (3).

Step 6 and 7. Equations (9) and (10) are used to calculate the positive and negative ideal solutions for various criteria once the weighted decision matrix has been obtained.

$G^{+} = \{\langle 0.284, 0.361, 0.278, 0.277\rangle, \langle 0.144, 0.515, 0.306, 0.165\rangle, \langle 0.159, 0.533, 0.222, 0.150\rangle, \langle 0.171, 0.511, 0.250, 0.165\rangle, \langle 0.184, 0.489, 0.306, 0.180\rangle, \langle 0.245, 0.422, 0.334, 0.200\rangle, \langle 0.133, 0.578, 0.220, 0.175\rangle\}$,
$G^{-} = \{\langle 0.245, 0.417, 0.191, 0.246\rangle, \langle 0.202, 0.438, 0.280, 0.219\rangle, \langle 0.188, 0.510, 0.130, 0.188\rangle, \langle 0.327, 0.340, 0.190, 0.120\rangle, \langle 0.208, 0.444, 0.278, 0.207\rangle, \langle 0.184, 0.488, 0.250, 0.178\rangle, \langle 0.173, 0.533, 0.160, 0.213\rangle\}$.

Table 4: Spherical picture fuzzy set decision matrix.

Criterion	σ_1	σ_2
$\varsigma 1$	$\langle 0.433, 0.3, 0.067, 0.189\rangle$	$\langle 0.333, 0.367, 0.133, 0.058\rangle$
$\varsigma 2$	$\langle 0.433, 0.233, 0.166, 0.304\rangle$	$\langle 0.433, 0.233, 0.2, 0.219\rangle$
$\varsigma 3$	$\langle 0.333, 0.367, 0.133, 0.111\rangle$	$\langle 0.567, 0.167, 0.133, 0.304\rangle$
$\varsigma 4$	$\langle 0.567, 0.2, 0.1, 0.272\rangle$	$\langle 0.7, 0.1, 0.1, 0.0000\rangle$
$\varsigma 5$	$\langle 0.367, 0.3, 0.133, 0.188\rangle$	$\langle 0.6, 0.167, 0.133, 0.249\rangle$
$\varsigma 6$	$\langle 0.367, 0.333, 0.167, 0.111\rangle$	$\langle 0.5, 0.233, 0.1, 0.245\rangle$
$\varsigma 7$	$\langle 0.433, 0.233, 0.2, 0.218\rangle$	$\langle 0.433, 0.3, 0.067, 0.188\rangle$
	σ_3	σ_4
$\varsigma 1$	$\langle 0.567, 0.167, 0.133, 0.303\rangle$	$\langle 0.4, 0.333, 0.1, 0.238\rangle$
$\varsigma 2$	$\langle 0.467, 0.267, 0.1, 0.219\rangle$	$\langle 0.333, 0.167, 0.2, 0.340\rangle$
$\varsigma 3$	$\langle 0.367, 0.333, 0.167, 0.111\rangle$	$\langle 0.4, 0.333, 0.1, 0.238\rangle$
$\varsigma 4$	$\langle 0.367, 0.3, 0.133, 0.207\rangle$	$\langle 0.5, 0.233, 0.167, 0.249\rangle$
$\varsigma 5$	$\langle 0.367, 0.3, 0.133, 0.188\rangle$	$\langle 0.567, 0.167, 0.133, 0.303\rangle$
$\varsigma 6$	$\langle 0.367, 0.3, 0.133, 0.465\rangle$	$\langle 0.467, 0.267, 0.1, 0.145\rangle$
$\varsigma 7$	$\langle 0.433, 0.3, 0.067, 0.188\rangle$	$\langle 0.667, 0.133, 0.1, 0.381\rangle$
	σ_5	σ_6
$\varsigma 1$	$\langle 0.5, 0.233, 0.167, 0.249\rangle$	$\langle 0.7, 0.1, 0.1, 0.0000\rangle$
$\varsigma 2$	$\langle 0.333, 0.367, 0.133, 0.111\rangle$	$\langle 0.433, 0.233, 0.166, 0.304\rangle$
$\varsigma 3$	$\langle 0.433, 0.3, 0.067, 0.188\rangle$	$\langle 0.6, 0.167, 0.133, 0.335\rangle$
$\varsigma 4$	$\langle 0.367, 0.333, 0.167, 0.111\rangle$	$\langle 0.367, 0.333, 0.167, 0.111\rangle$
$\varsigma 5$	$\langle 0.467, 0.267, 0.133, 0.288\rangle$	$\langle 0.467, 0.267, 0.133, 0.288\rangle$
$\varsigma 6$	$\langle 0.667, 0.133, 0.2, 0.289\rangle$	$\langle 0.433, 0.233, 0.167, 0.304\rangle$
$\varsigma 7$	$\langle 0.533, 0.233, 0.1, 0.145\rangle$	$\langle 0.333, 0.367, 0.133, 0.111\rangle$
	σ_7	
$\varsigma 1$	$\langle 0.433,\ 0.3,\ 0.066,\ 0.189\rangle$	
$\varsigma 2$	$\langle 0.467,\ 0.267,\ 0.1,\ 0.218\rangle$	
$\varsigma 3$	$\langle 0.6,\ 0.167,\ 0.133,\ 0.335\rangle$	
$\varsigma 4$	$\langle 0.367,\ 0.3,\ 0.133,\ 0.188\rangle$	
$\varsigma 5$	$\langle 0.5,\ 0.233,\ 0.167,\ 0.249\rangle$	
$\varsigma 6$	$\langle 0.4,\ 0.333,\ 0.1,\ 0.259\rangle$	
$\varsigma 7$	$\langle 0.467,\ 0.267,\ 0.1,\ 0.218\rangle$	

Table 5: Criteria weighting evaluation table.

Criterion	**DM1**	**DM2**	**DM3**	**Cost**	**Benefit**
ς_1	AI	AI	AAI	✓	
ς_2	VHI	VHI	HI		✓
ς_3	AAI	HI	HI		✓
ς_4	HI	HI	VHI		✓
ς_5	LI	UAI	UAI		✓
ς_6	LI	UAI	UAI		✓
ς_7	VLI	LI	LI		✓

Table 6: Criteria weight sheet

Criterion	**Criteria Weight**
ς_1	$\langle 0.567, 0.167, 0.133, 0.304\rangle$
ς_2	$\langle 0.433, 0.233, 0.200, 0.219\rangle$
ς_3	$\langle 0.433, 0.300, 0.067, 0.188\rangle$
ς_4	$\langle 0.467, 0.267, 0.100, 0.219\rangle$
ς_5	$\langle 0.367, 0.333, 0.167, 0.111\rangle$
ς_6	$\langle 0.367, 0.333, 0.167, 0.111\rangle$
ς_7	$\langle 0.400, 0.333, 0.100, 0.238\rangle$

Table 7: Weighted decision matrix

Criterion	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7
ς_1	$\langle 0.245, 0.417, 0.191, 0.246\rangle$	$\langle 0.189, 0.473, 0.248, 0.058\rangle$	$\langle 0.322, 0.306, 0.248, 0.304\rangle$	$\langle 0.227, 0.444, 0.220, 0.271\rangle$	$\langle 0.284, 0.361, 0.278, 0.277\rangle$	$\langle 0.397, 0.250, 0.220, 0.152\rangle$	$\langle 0.246, 0.417, 0.190, 0.247\rangle$
ς_2	$\langle 0.188, 0.412, 0.333, 0.262\rangle$	$\langle 0.188, 0.412, 0.360, 0.219\rangle$	$\langle 0.202, 0.438, 0.280, 0.219\rangle$	$\langle 0.231, 0.361, 0.360, 0.280\rangle$	$\langle 0.144, 0.515, 0.306, 0.165\rangle$	$\langle 0.188, 0.412, 0.333, 0.262\rangle$	$\langle 0.202, 0.438, 0.280, 0.219\rangle$
ς_3	$\langle 0.144, 0.557, 0.191, 0.150\rangle$	$\langle 0.246, 0.417, 0.191, 0.246\rangle$	$\langle 0.159, 0.533, 0.222, 0.150\rangle$	$\langle 0.173, 0.533, 0.160, 0.213\rangle$	$\langle 0.188, 0.510, 0.130, 0.188\rangle$	$\langle 0.260, 0.417, 0.191, 0.262\rangle$	$\langle 0.260, 0.417, 0.191, 0.262\rangle$
ς_4	$\langle 0.265, 0.414, 0.190, 0.246\rangle$	$\langle 0.327, 0.340, 0.190, 0.120\rangle$	$\langle 0.171, 0.487, 0.220, 0.213\rangle$	$\langle 0.234, 0.438, 0.250, 0.234\rangle$	$\langle 0.171, 0.511, 0.250, 0.165\rangle$	$\langle 0.171, 0.511, 0.250, 0.165\rangle$	$\langle 0.171, 0.487, 0.220, 0.203\rangle$
ς_5	$\langle 0.135, 0.533, 0.278, 0.450\rangle$	$\langle 0.220, 0.444, 0.278, 0.180\rangle$	$\langle 0.135, 0.533, 0.278, 0.207\rangle$	$\langle 0.208, 0.444, 0.278, 0.207\rangle$	$\langle 0.171, 0.511, 0.278, 0.200\rangle$	$\langle 0.171, 0.511, 0.278, 0.200\rangle$	$\langle 0.184, 0.489, 0.306, 0.180\rangle$
ς_6	$\langle 0.135, 0.555, 0.306, 0.111\rangle$	$\langle 0.184, 0.488, 0.250, 0.178\rangle$	$\langle 0.135, 0.533, 0.278, 0.288\rangle$	$\langle 0.171, 0.511, 0.250, 0.128\rangle$	$\langle 0.245, 0.422, 0.334, 0.200\rangle$	$\langle 0.159, 0.488, 0.306, 0.208\rangle$	$\langle 0.147, 0.555, 0.250, 0.185\rangle$
ς_7	$\langle 0.227, 0.444, 0.220, 0.271\rangle$	$\langle 0.173, 0.533, 0.160, 0.213\rangle$	$\langle 0.173, 0.533, 0.160, 0.213\rangle$	$\langle 0.267, 0.422, 0.190, 0.310\rangle$	$\langle 0.213, 0.488, 0.190, 0.192\rangle$	$\langle 0.133, 0.578, 0.220, 0.175\rangle$	$\langle 0.187, 0.511, 0.190, 0.228\rangle$

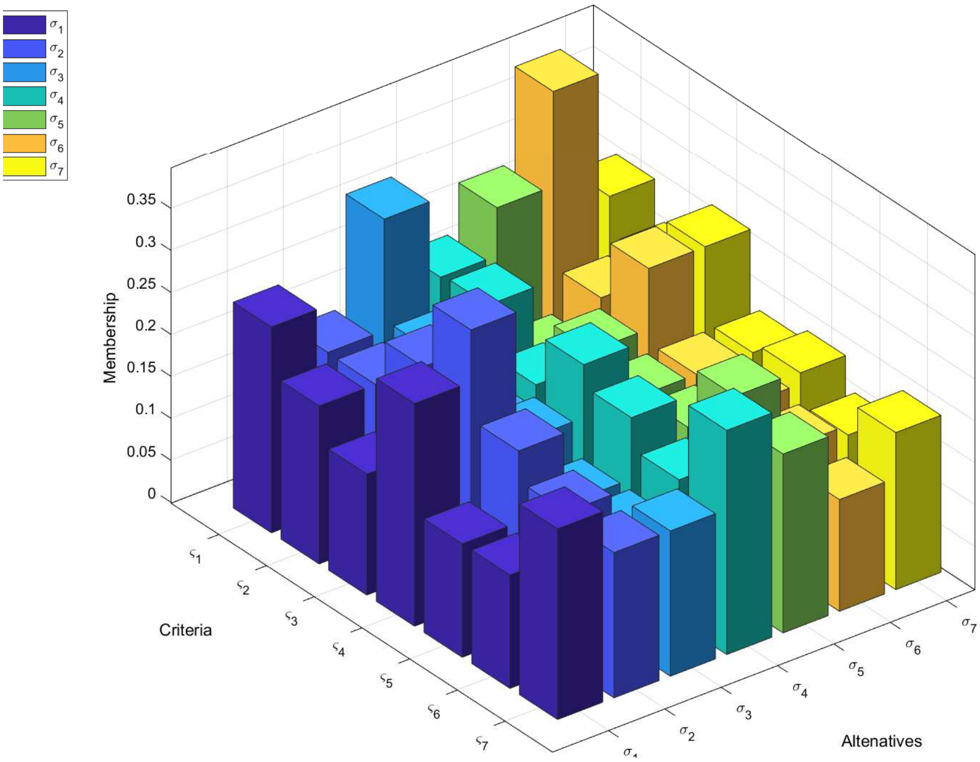

Figure 3: Representation of alternatives on the basis of membership degrees

The distances between each alternative and the positive and negative ideal solutions are then computed using the spherical picture fuzzy set distance measure, equation(5), as shown in Table 8.

Table 8: The distance of each alternative to the positive and negative ideal solutions.

Distance	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7
$N\partial_c^+(\sigma_i)$	0.125	0.134	0.077	0.122	0.042	0.081	0.097
$N\partial_c^-(\sigma_i)$	0.093	0.057	0.088	0.079	0.097	0.119	0.068

Step 8. From the distance data obtained for different alternatives in Table 8, the relative closeness coefficient of each alternative to the positive ideal solution is calculated using Eq (11) and ranked. The results of the relative closeness coefficient calculation are shown in Table 9.

Table 9: Relative closeness coefficients of alternatives to positive and negative ideal solutions.

Alternatives	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7
$R_{CC}(\sigma_i)$	0.427	0.298	0.533	0.393	0.698	0.595	0.412

The ranking of the different candidates according to the principle of maximum proximity $R_{CC}(\sigma_i)$ is

$$\sigma_5 \succ \sigma_6 \succ \sigma_3 \succ \sigma_1 \succ \sigma_7 \succ \sigma_4 \succ \sigma_2.$$

Consequently optimal hospital site is σ_5.

7. Conclusion

Spherical picture fuzzy set is a relatively broad class of fuzzy set which unifies the concepts of picture fuzzy set and spherical fuzzy set. Spherical picture fuzzy set has a stronger ability to express uncertain information and can better reflect the essential characteristics of the objective world. In this paper, a new distance measure on the basis of spherical picture fuzzy sets is presented. Structural properties of $SPFS$ along with some basic set-theoretical operations and aggregation operators are discussed. A numerical case study of pandemic hospital site selection is used to illustrate the effectiveness and rationality of $SPFS$-$TOPSIS$ method in this paper. This method not only considers the three factors of membership degree, non- membership degree and radius, but also considers the potential association between hesitation degree, membership degree and non- membership degree.

The limitation of this manuscript is that the distribution ratio about the hesitation parameter in the distance metric can have more forms, so the distribution ratio of hesitation can be a direction for future research. The distance metric proposed in this paper can further be considered to apply it to different multi-criteria decision models, and then solve a wider range of multi-criteria decision problems. In addition, the distance metric proposed in this paper can be used to solve problems related to pattern recognition and medical diagnosis. Hopefully the newly introduced concept of spherical picture fuzzy set and related proposed tools may provide a gateway for researchers to further dive in it.

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