EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
2025, Vol. 18, Issue 3, Article Number 6372
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

Planarity of Intuitionistic Pythagorean Fuzzy Graphs and Its Application in Decision Making

Obbu Ramesh[1], Nainaru Tarakaramu[2,*], G. YuvaRoopa Lakshmi[3], Sharief Basha S[4], Sarvar Iskandarov[5], Akbar Toyirov[6], Jushkinbek Yuldoshev[7], Sardor Sabirov[8], K. K. Prashanth[9]

[1] *Department of Mathematics, Presidency University, Bengaluru-560064, Karnataka, India*
[2] *Department of Mathematics, School of Liberal Arts and Sciences, Mohan Babu University, Sree Sainath Nagar, Tirupati-517102, Andhra Pradesh, India*
[3] *Department of Mathematics, Govt. Degree College for Women, Wanaparthy (Nandi Hills), 509103, Telangana, India*
[4] *Department of Analytics, School of Computer Science and Engineering, Vellore Institute of Technology, Vellore 632014, Tamil Nadu, India*
[5] *Department of Mathematical Analyze, Urgench State University, Urgench City, Uzbekistan*
[6] *Department of Information Technology and Exact Sciences, Termez University of Economics and Service, Termez, Uzbekistan*
[7] *Department of Pedagogy and Primary Education Methodology, Urgench Innovation University, Urgench, Uzbekistan*
[8] *Department of General Professional Sciences, Mamun University, Khiva, Uzbekistan*
[9] *Department of Mathematics, New Horizon College of Engineering, Bangalore-560103, Karnataka, India*

Abstract. This paper introduces the concept of Intuitionistic Pythagorean Fuzzy Graphs (IPFGs) and outlines their key characteristics as a means of addressing such uncertain scenarios by examining Pythagorean uncertain planarity values using weak, strong, and significant edges. An Intuitionistic Pythagorean Fuzzy Graph is a generalization of a traditional graph that can solve certain problems beyond the capabilities of both classical graph theory and fuzzy graph theory. The approach based on Pythagorean fuzzy graphs offers greater flexibility in handling human judgment data compared to other fuzzy models.
This paper defines the concept of planarity in an Intuitionistic Pythagorean Fuzzy Graph using the notions of intersecting value and Intuitionistic Pythagorean Fuzzy Planarity Value. The upper and lower bounds of this planarity value are then established through a series of interrelated theorems. Finally, the weak and strong planarity of an Intuitionistic Pythagorean Fuzzy Graph is delineated, and the results of the analysis are discussed. The paper poses a challenge that demonstrates the application of the proposed concept.

2020 Mathematics Subject Classifications: 05C10, 03E72, 68T37
Key Words and Phrases: Intuitionistic Pythagorean fuzzy graph, Crossing point, Strong Planarity and Weak Planarity

---

*Corresponding author.
DOI: https://doi.org/10.29020/nybg.ejpam.v18i3.6372

Email addresses: nainaru143@gmail.com (N. Tarakaramu)

Nomenclature

PFPGs - Pythagorean Fuzzy Planar Graphs

IPFGs - Intuitionistic Pythagorean Fuzzy Graphs

IFGs - Intuitionistic Fuzzy Graphs

IFSs - Intuitionistic Fuzzy Sets

IFPR - Intuitionistic Fuzzy Preference Relations

PFDGs - Pythagorean Fuzzy Dual Graphs

ζ_{rs} - The strength of the Pythagorean fuzzy edge rs

ζ_P - The crossing rate at the point

ξ - Intuitionistic Pythagorean uncertain planarity rate

1. Introduction

The concept of graphs is recognized as one of the branches of mathematics that has undergone tremendous development in recent years due to its numerous applications. Because of its applications in a wide range of fields, including physics, electrical engineering, biology, and operations research, graph theory is rapidly becoming central to mathematics.[1] Euler's polyhedral formula, which relates the faces, edges, and vertices of a polyhedron, serves as the foundation for the theory of planar graphs.[2] New concept of Planar graphs are used to clearly layout and shape complicated radio electronic circuits, planetary gearboxes, chemical molecules and railway maps inside the modern-day technology. Subway tunnels, railway lines, Pipelines, metro traces and electric powered transmission traces are vital while modelling an urban town. Crossing is advantageous because it saves space and is inexpensive, but it also has some disadvantages. Although crossing such lines poses a significant threat to human life, it is possible with the right precautions. When compared to crossing between two crowded routes, the crossing between the uncrowded route and crowded route poses fewer dangers. The relations "uncrowded route" and "crowded route" are mentioned to as "weak edge" and "strong edge" in fuzzy graphs, respectively. Fuzzy planar graph theory is made possible by allowing for crossings of this kind [3].
However, the modern fuzzy set concept began with Zadeh's 1965 definition of the fuzzy subset of a crisp set as a generalization of the standard subset [4]. Allowing to the traditional idea, any component has whichever a membership degree of one if it is related to the subset being studied or a membership degree of zero if it is not. However, the fuzzy idea states that any element with association degree among zero and one is a member of the studied fuzzy subset, which is nearer to human thought due to the fact that our thoughts work with the estimated type and do not know everything exactly. This can, for instance, recognize the gray gradients of the white and black colours. The indistinct idea

has involved a lot of attention due to its numerous presentations in industries, communications, and artificial intelligence, despite the fact that it is considered a new mathematical field. Consequently, numerous scientists and researchers have focused on improving this concept and implementing it across all math disciplines. One of these fields where the fuzzy idea can be put into practice is graph theory.[5] Rosenfeld first proposed this theory in 1975 when he studied fuzzy relations on fuzzy sets and established the structure of fuzzy graphs by extending several graph-theoretical concepts through analogous conclusions. He also discussed concepts such as cut vertices, trees, forests, paths, connectedness, and bridges. Additionally, Mordeson introduced the concepts of fuzzy diagrams and fuzzy hyper diagrams.[6] Rakesh Jaiswal explored the application of fuzzy graph coloring to the traffic light problem in another study.[7] Lavanya also examined the concept of fuzzy total coloring and its application in job scheduling. [8] Eslahchi discovered the vertex strength of fuzzy graphs in a different study. Deep structure was added to fuzzy graph theory in recent years that are a study on fuzzy labelling graphs [9], a graph associated to a poly group with respect to an automorphism [10], Sustainable goals in combating human trafficking [11], Fuzzy graph theory with applications to human trafficking [12], Fuzzy graph theory [13] and Product of bipolar fuzzy graphs and their degree [14]. Then referred about intuitionistic fuzzy graphs, their operations [15], and their Signless laplacian energies [16].Naz et al. [17] based on Akram and Davvaz's IFGs [18] proposed PFGs and their applications. In, some PFG-related findings were discussed [19]. Naz and Akram conducted research on the Pythagorean fuzzy graph energy [20]. IFGs2k was dealt with by Dhavudh and Srinivasan [21]. Verma and co. and Akram and others [22] suggested a few PFG operations. In recent times, Akram et al. [23] introduced a few graphs in an environment that was Pythagorean fuzzy. Sahoo et al did their research on intuitionistic fuzzy competition graphs [24], Product of intuitionistic fuzzy graphs, Colouring of Mixed Fuzzy Graph [25], Certain Types of Edge Irregular Intuitionistic Fuzzy Graphs [26] and Colouring of Mixed Fuzzy Graph and its Application in COVID19 [27]. Shaik Noorjahan and Sharief Basha Shaik proposed using correlation coefficient measurements between each option and the optimal choice to calculate the ranking order of the alternatives in the study [28]. Shaik Noorjahan and Shaik Sharief Basha investigated the use of the intuitionistic fuzzy rough preference relation method for processing the intuitionistic fuzzy rough weighted average [29]. Naveen Kumar Akula and Shaik Sharief Basha proposed a novel method for calculating the comparative position loads of establishments by manipulating the undecided corroboration of IFPR and the correlation coefficient between one IFPR and other items [30]. Noorjahan S and Sharief Basha S proposed a novel method for calculating the relative position loads of establishments by varying the correlation coefficient between the intuitionistic fuzzy rough preference relation of one entity and the other items, as well as the undecided corroboration within the intuitionistic fuzzy rough preference relation [31]. [32] Areej Almuhaimeed presents the concepts of a normal HX-subgroup and a Pythagorean fuzzy HX-Subgroup. [33] Using the Pythagorean fuzzy soft interior operator, A.A. Azzam and M. Aldawood present the idea of Pythagorean fuzzy soft somewhat open sets and expand its use to Pythagorean fuzzy soft topological spaces. This paper extends graph theoretical results within the intuitionistic Pythagorean fuzzy environment. The structure and applicability of planar graphs are fascinating. For instance, when designing complex radio electronic circuits, components can be arranged so that the conductors connecting them do not cross, and the concept of planar graphs can be applied to address this issue. The precise structuring of a street or communication network is made possible through the concepts of intuitionistic Pythagorean fuzzy graphs (IPFGs), Pythagorean fuzzy planar graphs (PFPGs), and Pythagorean fuzzy dual graphs (PFDGs), which are discussed in this study. By utilizing these diagrams, some real-world problems can be analysed and designed effectively. Planarity is a significant property that is central to this research. Non-planar IPFGs are also critically analysed. Intuitionistic Pythagorean fuzzy planar graphs and Pythagorean fuzzy dual graphs are found to be closely related. By introducing the concepts of crossing rate and intuitionistic Pythagorean fuzzy planarity rate, this paper defines the terms intuitionistic Pythagorean fuzzy planar graphs and planarity of intuitionistic Pythagorean fuzzy planar graphs, and examines related results. Applications of IPFPGs are also discussed.

2. Preliminaries

This section provides a brief discussion of intuitionistic fuzzy sets, intuitionistic fuzzy diagrams, Pythagorean fuzzy sets, and graph theory.

2.1. Intuitionistic fuzzy graph

Sets whose elements have varying degrees of membership and non-membership are known as intuitionistic fuzzy sets. The concept of intuitionistic fuzzy sets was introduced by Krassimir Atanassov in 1983 as an extension of Lotfi Zadeh's fuzzy set theory, which itself extends the classical notion of a set.

Definition 1 ([34])**.** *Let us take X is a non-empty set. An IFS A is defined in X is $A = \{x, \mu_A(x), \gamma_A(x) : x \in X\}$, where $\mu_A : X \to [0,1]$ and $\gamma_A : X \to [0,1]$ represents the membership and non-membership functions of A and it satisfies the condition $0 \leq \mu_A(x) + \gamma_A(x) \leq 1, \forall x \in X$ with the degree of indeterminacy given by $\pi_A(x) = 1 - \mu_A(x) - \gamma_A(x)$*

2.2. Intuitionistic Fuzzy graph

Intuitionistic fuzzy relations and intuitionistic fuzzy graphs (IFGs) were first introduced by Atanassov. The field of mathematics and its applications has seen exponential growth in research on the theory of intuitionistic fuzzy sets (IFSs). This includes developments in information sciences as well as conventional mathematics.

Definition 2 ([35])**.** *An IFG is well-defined as $G = (V, E, \mu, \gamma)$,where V denotes the set of vertices, E represents the set of edges, μ is a fuzzy relationship function demarcated on $V \times V$, and γ is a fuzzy non-relationship function expressed as $\mu(v_i, v_j)$ by μ_{ij} and $\gamma(v_i, v_j)$ by γ_{ij}, such that*

(i) $0 \leq \mu_{ij} + \gamma_{ij} \leq 1, (i, j = 1, 2, 3, \ldots, n)$

(ii) $0 \leq \mu_{ij}, \gamma_{ij}, \pi_{ij} \leq 1$, *here* $\pi_{ij} = 1 - \mu_{ij} - \gamma_{ij}, (i, j = 1, 2, 3, \ldots, n)$

2.3. Pythagorean Fuzzy Set

Definition 3 ([36]). *A definition of the Pythagorean fuzzy set R in a finite discourse universe* $S = (s_1, s_2, \ldots, s_n)$ *is given by*

$$R = (x, \mu_R(s), \gamma_R(s)/s \in S),$$

Where $\mu_R(s) : S \to [0,1]$ *and* $\gamma_R(s) :\to [0,1]$ *and* $0 \leq \mu_R^2(s) + \gamma_R^2(s) \leq 1, \forall s \in S$, *where the numbers* $\mu_R(s)$ *and* $\gamma_R(s)$ *represent the ranking of membership and the ranking of non-membership, respectively, of* $s \in S$ *in R.*

Additionally, the hesitation degree, also known as the Pythagorean index, can be defined as for each Pythagorean fuzzy set R in S $\eta_R(s) = \sqrt{(1 - \mu_R^2(s) - \gamma_R^2(s))}$

Definition 4 ([20]). *A PFG on a non-empty set X is a pair G= (P, Q), where P is a PFS on X and Q is a PFR on X such that* $\mu_R(u,v) \leq min(\mu_R(u), \mu_R(v))$ *and* $\gamma_R(u,v) \leq max(\gamma_R(u), \gamma_R(v))$, *and* $0 \leq \mu_R^2(u,v) + \gamma_R^2(u,v) \leq 1, \forall (u,v) \in X$.

A Pythagorean fuzzy graph (PFG) specifies the degree of membership of an element in a given set, where the degree of non-membership is simply one minus the degree of membership. By contrast, an intuitionistic Pythagorean fuzzy graph (IPFG) allows for both a degree of membership and a degree of non-membership, which are largely independent; the only condition is that their sum does not exceed one.

Pythagorean fuzzy graphs, as a new extension of Euler's graph theory, offer an improved way to represent complex graphical problems. Unlike intuitionistic Pythagorean fuzzy graphs, which require that the sum of the membership and non-membership degrees for any edge or vertex does not exceed one, this approach provides additional advantages. [37] R. R. Yager provided a comparison between the fuzzy set and the intuitionistic fuzzy set through Figure 1.

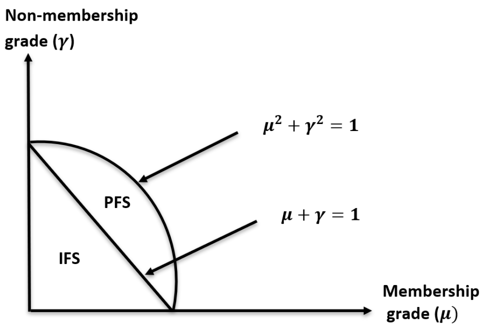

Figure 1: Comparison between IFS and PFS

3. Planarity of Intuitionistic Pythagorean Fuzzy Graphs

In this segment, the concept of planarity of intuitionistic Pythagorean fuzzy diagrams is outlined, based on several crucial interrelated classifications such as crossing rate and Pythagorean fuzzy planarity rate. Let G = (M,N) be an intuitionistic Pythagorean fuzzy graph and for an assured geometric representation and it takes dual edges$((rs), \mu_N((rs))_i, \gamma_N((rs))_i)$ and $((uv), \mu_N((uv))_j, \gamma_N((uv))_j)$ which intersect at a particular point P, wherever i,j are permanent integers. By fuzzy logic, if one at smallest of $\mu_N(rs)_i$ and $\mu_N(uv)_j$ is nearby to 0 in that case, the crossing won't matter much in the representation. But then if together of $\mu_N(rs)_i$ and $\mu_N(uv)_j$ are closed to 1, in that case, the crossing is matter much in the illustration.

Based on this concept, the crossover rate and the intuitionistic Pythagorean fuzzy planarity rate are explained.

Definition 5. *The strength of the Pythagorean fuzzy edge rs is defined as*

$$\zeta_{rs} = (\mu_{rs}, \gamma_{rs}) = (\frac{\mu_N(rs)_i}{\mu_M(r) \wedge \mu_M(s)}, \frac{\gamma_N(rs)_i}{\gamma_M(r) \wedge \gamma_M(s)})$$

Definition 6. *Let G= (M,N) be an intuitionistic Pythagorean fuzzy diagram through a convinced symmetrical picture which takes the crossing argument 'P' among the intuitionistic Pythagorean an edge rs of IPFG is known as Strong if $\mu_{rs} \geq 0.5$ and $\gamma_{rs} \leq 0.5$ otherwise known as weak.uncertain edges $((rs), \mu_N(rs)_i, \gamma_N(rs)_i)$ and $((uv), \mu_N(uv)_j, \gamma_N(uv)_j)$. The crossing rate at the point is then defined as*

$$\zeta_p = (\mu_p, \gamma_p) = (\frac{\mu_{rs} + \mu_{uv}}{2}, \frac{\gamma_{rs} + \gamma_{uv}}{2})$$

If the total of crossing arguments in an intuitionistic Pythagorean uncertain graph upsurge, then the planarity value of IPFG declines. Hence the rate of ζ_P upsurges, then the planarity value declines. So that ζ_P is proportionate inversely to planarity of IPFG. What defines an intuitionistic Pythagorean fuzzy graph is the knowledge of its planarity value.

Definition 7. *Let G= (M, N) be an intuitionistic Pythagorean fuzzy graph with a sure geometric picture which have the crossing points $P_1, P_2, \ldots, P_n$ and now express the intuitionistic Pythagorean uncertain planarity rate ξ of this symmetrical representation of 'G' by*

$$\zeta = (\zeta_\mu, \zeta_\gamma) = \left(\frac{1}{1 + \mu_{\xi_1} + \mu_{\xi_2} + - - + \mu_{\xi_n}}, \frac{1}{1 + \gamma_{\xi_1} + \gamma_{\xi_2} + - - + \gamma_{\xi_n}}\right)$$

Every symmetrical representation of an intuitionistic Pythagorean uncertain graph is planar with a specific planarity rate, because it is obvious that $0 \leq \xi \leq 1$.

Example 1. *Let G= (M, N) be an intuitionistic Pythagorean fuzzy graph*

In the above Figure 2, there two crossing point 'P1 and p2'.
The crossing value at the point P1 is

$$\zeta_{p_1} = (\frac{0.7 + 0.4}{2}, \frac{0.3 + 0.3}{2}) = (0.55, 0.3)$$

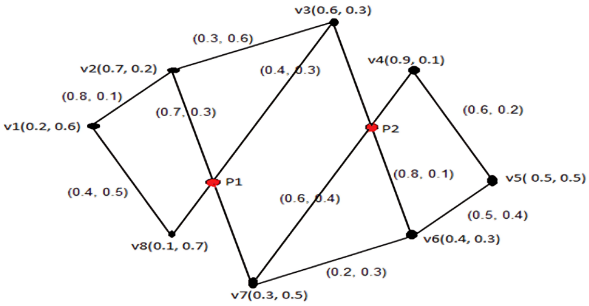

Figure 2: IPFG

and the crossing value at the point p2 is :

$$\zeta_{p_2} = (\frac{0.6+0.8}{2}, \frac{0.4+0.1}{2}) = (0.7, 0.25)$$

so, the inutionistic Pythagorean uncertain planarity rate of this geometric representation of G

$$\xi = (\xi_\mu, \xi_\gamma) = (\frac{1}{1+0.55+0.7}, \frac{1}{1+0.3+0.25})$$
$$\xi = (0.44, 0.65)$$

Theorem 1. *For any intuitionistic Pythagorean fuzzy graph with geometric representation which has 'k' and 'l' crossing points of membership and non-membership standards are:*

$$\left(\frac{1}{1+k} \leq \xi_\mu \leq \frac{1}{1+k_\mu}, \frac{1}{1+l} \leq \xi \leq \frac{1}{1+l\Omega}\right)$$

where $\eta = min\{\mu_{\sigma(r)} : r \in V\}$ *and* $\Omega = min\{\gamma_{\sigma(r)} : r \in V\}$

Proof. *by the definition 6, we have*

$$\xi = (\xi_\mu, \xi_\gamma) = \left(\frac{1}{1+\mu_{\zeta_1}+\mu_{\zeta_2}+\cdots+\mu_{\zeta_k}}, \frac{1}{1+\gamma_{\zeta_1}+\gamma_{\zeta_2}+\cdots+\gamma_{\zeta_l}}\right)$$

and definition 5, then

$$\zeta_P = (\mu_P, \gamma_P) = \left(\frac{\mu_{rs}+\mu_{uv}}{2}, \frac{\gamma_{rs}+\gamma_{uv}}{2}\right)$$

Since $(\mu_{rs}, \gamma_{rs}) \leq (1,1)$, *for any intuitionistic Pythagorean fuzzy graph edge* $((rs), \mu_N(rs)_i, \gamma_N(rs)_i)$ *in G, then*

$$\zeta_{P(i,j)} \leq \left(\frac{1+1}{2}, \frac{1+1}{2}\right) = (1,1)$$

For $\{i = 1, 2, \ldots, k\}$ *and* $\{j = 1, 2, \ldots, 1\}$, *given by*

$$(1+\mu_{\zeta_1}+\mu_{\zeta_2}+\cdots+\mu_{\zeta_k},\ 1+\gamma_{\zeta_1}+\gamma_{\zeta_2}+\cdots+\gamma_{\zeta_l}) \leq (k, l)$$

So,

$$\xi = \left(\frac{1}{1+\mu_{\zeta_1}+\mu_{\zeta_2}+\cdots+\mu_{\zeta_k}},\ \frac{1}{1+\gamma_{\zeta_1}+\gamma_{\zeta_2}+\cdots+\gamma_{\zeta_l}} \right) \geq \left(\frac{1}{1+k},\ \frac{1}{1+l} \right)$$

On the otherhand, $(\mu_{rg}, \gamma_{rs}) \geq \sigma(r) \wedge \sigma(s)$, *for any intuitionistic Pythagorean fuzzy two edges in* G.

$$((rs),\ \mu_N((rs))_i,\ \gamma_N((rs))_i) \quad and \quad ((uv),\ \mu_N((uv))_j,\ \gamma_N((uv))_j)$$

Since $\eta = \min\left\{\mu_{\sigma(r)} : r \in V,\ \gamma_{\sigma(r)} : r \in V\right\}$, *then* $(\mu_{rs}, \gamma_{rs}) \geq (\eta, \Omega)$

$$\zeta_{P(i,j)} \geq \left(\frac{\eta+\eta}{2},\ \frac{\Omega+\Omega}{2} \right) = (\eta, \Omega)$$

It has $\mu_{\zeta_1}+\mu_{\zeta_2}+\cdots+\mu_{\zeta_n} \geq k\eta$ *and* $\gamma_{\zeta_1}+\gamma_{\zeta_2}+\cdots+\gamma_{\zeta_l} \geq l\Omega$ *Thus, for* $\{i = 1, 2, \ldots, k\}$ *and* $\{j = 1, 2, \ldots, 1\}$, *given by*

$$\xi = \left(\frac{1}{1+\mu_{\zeta_1}+\mu_{\zeta_2}+\cdots+\mu_{\zeta_k}},\ \frac{1}{1+\gamma_{\zeta_1}+\gamma_{\zeta_2}+\cdots+\gamma_{\zeta_l}} \right) \leq \left(\frac{1}{1+k\eta},\ \frac{1}{1+l\Omega} \right)$$

Theorem 2. *For any intuitionistic Pythagorean fuzzy graph G with symmetrical representation which takes (k, l) crossing points and which are edges steady.*

$$\xi = \left(\frac{1}{1+k\alpha},\ \frac{1}{1+l\beta} \right)$$

Where $\alpha = \mu_{rs}$ *and* $\beta = \gamma_{rs}$, *for any intuitionistic Pythagorean fuzzy two edges*

$$((rs),\ \mu_N(rs)_i,\ \gamma_N(rs)_i) \quad and \quad ((uv),\ \mu_N(uv)_j,\ \gamma_N(uv)_j) \ \ in\ G.$$

Proof. According to definition 5, we have

$$\zeta_P = (\mu_P, \gamma_P) = \left(\frac{\mu_{rs}+\mu_{uv}}{2},\ \frac{\gamma_{rs}+\gamma_{uv}}{2} \right)$$

Since G *is edge stable, then* $\mu_{rs} = \mu_{uv} = \alpha, \gamma_{rs} = \gamma_{uv} = \beta$, *for any intuitionistic Pythagorean fuzzy two edges* $((rs), \mu_N(rs)_i, \gamma_N(rs)_i$ *and* $((uv), \mu_N(uv)_i, \gamma_N(uv)_i$ *in* G

$$\zeta_{P(i,j)} = \left(\frac{\alpha+\alpha}{2},\ \frac{\beta+\beta}{2} \right) = (\alpha, \beta)$$

For $\{i = 1, 2, \ldots, k\}$ *and* $j = \{j = 0, 1, 2, \ldots, 1\}$, *given by*

$$\xi = \left(\frac{1}{1+\mu_{\zeta_1}+\mu_{\zeta_2}+\cdots+\mu_{\zeta_k}},\ \frac{1}{1+\gamma_{\zeta_1}+\gamma_{\zeta_2}+\cdots+\gamma_{\zeta_l}} \right)$$

$$\xi = \left(\frac{1}{1+k\alpha},\ \frac{1}{1+l\beta} \right)$$

Theorem 3. *For any intuitionistic Pythagorean fuzzy graph G with geometric representation which has (k, l) crossing points, all of them in between the Pythagorean fuzzy edges, which are ineffective.*

$$\xi > \left(\frac{1}{1+k\alpha}, \frac{1}{1+l\beta}\right)$$

Where $\alpha = maxi\{\mu_{\sigma(r)} : r \in V\}$ *and also* $\beta = maxi\{\gamma_{\sigma(r)} : r \in V\}$

Proof. According to the defintion 2, we have

$$\zeta_P = (\mu_P, \gamma_P) = \left(\frac{\mu_{rs} + \mu_{uv}}{2}, \frac{\gamma_{rs} + \gamma_{uv}}{2}\right)$$

On account that each n crossing arguments in G are among non-effective Pythagorean uncertain edges, then

$$\mu_{rs} < \max\{\sigma(r), \sigma(s)\}, \mu_{uv} < \max\{\sigma(u), \sigma(v)\}$$

$$And\ \gamma_{rs} < \max\{\sigma(r), \sigma(s)\}, \gamma_{uv} < \max\{\sigma(u), \sigma(v)\}$$

MEanwhile $\alpha = maxi\{\mu_{\sigma(r)} : x \in V\}$ *and also* $\beta = maxi\{\gamma_{\sigma(r)} : x \in V\}$

$$\zeta_P = (\mu_P, \gamma_P) = \left(\frac{\mu_{rs} + \mu_{uv}}{2}, \frac{\gamma_{rs} + \gamma_{uv}}{2}\right)$$

$$\zeta_P < \left(\frac{\alpha + \alpha}{2}, \frac{\beta + \beta}{2}\right) = (\alpha, \beta)$$

For $\{i = 1, 2, \ldots, k\}$ *and* $j = \{j = 0, 1, 2, \ldots, 1\}$, *given by*

$$\xi = \left(\frac{1}{1 + \mu_{\zeta_1} + \mu_{\zeta_2} + \cdots + \mu_{\zeta_k}}, \frac{1}{1 + \gamma_{\zeta_1} + \gamma_{\zeta_2} + \cdots + \gamma_{\zeta_l}}\right)$$

$$\xi > \left(\frac{1}{1+k\alpha}, \frac{1}{1+l\beta}\right)$$

Definition 8. *Let G = (M, N) be an intuitionistic Pythagorean fuzzy graph with a sure geometric picture which have the crossing points* $P_1, P_2, \ldots, P_n$ *so that m of these crossing points gratifies that* $\zeta_P = (\mu_P, \gamma_P) \geq 0.5$, *for* $i = (0, 1, 2, \ldots, k)$ *and* $j = (0, 1, 2, \ldots, l)$ *then said geometric picture of G intuitionistic Pythagorean fuzzy diagram if*

$$\xi = (\xi_\mu, \xi_\gamma) > (0.5,\ 0.5) \quad \& \quad m < \frac{n}{2}$$

This indicates that an intuitionistic Pythagorean fuzzy graph is the appropriate geometric representation of G if the crossing points that satisfy $zeta_P \geq 0.5$ *are fewer than half of the entire number of crossing arguments n and the intuitionistic Pythagorean fuzzy graph planarity rate is greater than 0.5. In this case, this symmetrical representation of G is referred to as an intuitionistic Pythagorean uncertain weak planar diagram.*

Example 2. *Let $G = (M, N)$ be an intuitionistic Pythagorean fuzzy diagram denoted in a symmetrical representation presented in Figure 3 below*

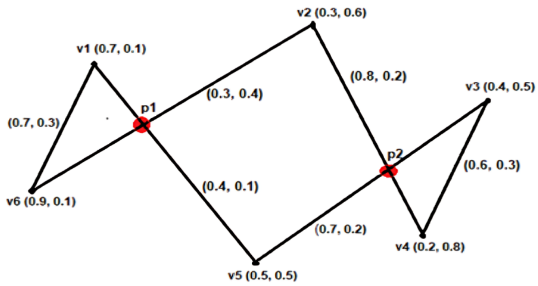

Figure 3: IPFG

In the figure 3, there two crossing point 'P1 and p2'and Their crossing values are given below :

$$\zeta_{p_1} = (\frac{0.4+0.3}{2}, \frac{0.4+0.1}{2}) = (0.35, 0.25)$$

$$and\ \zeta_{p_2} = (\frac{0.8+0.7}{2}, \frac{0.2+0.2}{2}) = (0.75, 0.2)$$

so, the inutionistic Pythagorean uncertain planarity rate of this geometric representation of G

$$\xi = (\xi_\mu, \xi_\gamma) = (\frac{1}{1+0.35+0.75}, \frac{1}{1+0.25+0.2})$$
$$\xi = (0.48, 0.69)$$

Therefore, the weak planar from the geometric representation G^I based on membership planarity value and non-membership planarity value. Already, if $\xi_\gamma(=0.69) > 0.5$, so the geometric illustration of G is weak planar since the formula $m < \frac{n}{2}$ is not satisfied.

Theorem 4. *If G be an intuitionistic Pythagorean fuzzy graph with geometric illustration which have n crossing points satisfy that $\zeta_{Pi} < 0.5$, for all values of $i = 1, 2, \ldots, n$, then*

$$\xi = (\xi_\mu, \xi_\gamma) > \left(\frac{2}{2+n}, \frac{2}{2+n}\right)$$

Proof. *Meanwhile $\zeta_{pi} < 0.5$ for $i = 1, 2, \ldots, n$*
It is given by $1 + \mu_{\zeta_1} + \mu_{\zeta_2} + \cdots + \mu_{\zeta_n} < (0.5)n$ and $1 + \gamma_{\zeta_1} + \gamma_{\zeta_2} + \cdots + \gamma_{\zeta_n} < (0.5)n$

$$\xi = \left(\frac{1}{1+\mu_{\zeta_1}+\mu_{\zeta_2}+\cdots+\mu_{\zeta_n}}, \frac{1}{1+\gamma_{\zeta_1}+\gamma_{\zeta_2}+\cdots+\gamma_{\zeta_n}}\right)$$

$$\xi = (\xi_\mu, \xi_\gamma) > \left(\frac{1}{1+0.5n}, \frac{1}{1+0.5n}\right)$$

Multiplying and dividing by '2' on the right hand side, we get

$$\xi = (\xi_\mu, \xi_\gamma) > \left(\frac{1}{1+\frac{n}{2}}, \ \frac{1}{1+\frac{n}{2}} \right)$$

Corollary 1. *If G be an intuitionistic Pythagorean fuzzy graph with geometric illustration which have 1 or 2 crossing points gives that $\zeta_{Pi} < 0.5$ for all crossing points, then this geometric illustration of G is an intuitionistic Pythagorean fuzzy strong planar graph.*

Proof. Redering to Theorem 4, then

$$\xi = (\xi_\mu, \xi_\gamma) > \left(\frac{2}{2+n}, \ \frac{2}{2+n} \right)$$

For n=1,

$$\xi = (\xi_\mu, \xi_\gamma) > \left(\frac{2}{2+1}, \ \frac{2}{2+1} \right)$$

$$\xi = (\xi_\mu, \xi_\gamma) > \left(\frac{2}{3}, \ \frac{2}{3} \right)$$

$$\xi = (\xi_\mu, \xi_\gamma) = (0.66, \ 0.66)$$

For n=2,

$$\xi = (\xi_\mu, \xi_\gamma) > \left(\frac{2}{2+2}, \ \frac{2}{2+2} \right)$$

$$\xi = (\xi_\mu, \xi_\gamma) > \left(\frac{2}{4}, \ \frac{2}{4} \right)$$

$$\xi = (\xi_\mu, \xi_\gamma) = (0.5, \ 0.5)$$

So, this geometric representation of G is an intuitionistic Pythagorean fuzzy strong planar graph.

4. Application

Water from the river reaches people's homes through sturdy pipelines that twist, turn, and cross various regions within cities or village streets, carrying water under high pressure. To reduce this high-pressure water force, a device known as a water tank is used. Dispensing water from the tank is straightfor-ward, as it connects to different houses through smaller pipes. However, when connecting houses in different areas, crossings between these small pipes may occur. Sometimes, such crossings are benefi-cial because they save space and reduce costs. However, pipe crossings can also cause problems, as they may lead to pipe breakage and water leakage, resulting in water loss and failure to deliver water to its intended destination. To overcome this issue, crossings between pipes should be minimized, and high-quality pipes should be used for installation. The real-world methodology of intuitionistic Pythag-orean fuzzy graphs can be applied to model and manage such situations, helping to reduce the risk of damage and associated costs.

Take into account a water tank wherein unique regions are related as proven in figure. Every region v1, v2,. . . , v8 is signified by using a vertex and every pipe connection between exclusive areas through small pipe is signified by edges. The membership score of a vertex characterizes the likelihood of a breakdown, whereas the non-membership score suggests the probability of no breakdown occurring in the connections between various areas. Similarly, the mem-bership score of an edge indicates the degree of risk associated with a pipe connection between areas, while the non-membership score denotes the absence of such a connection risk.

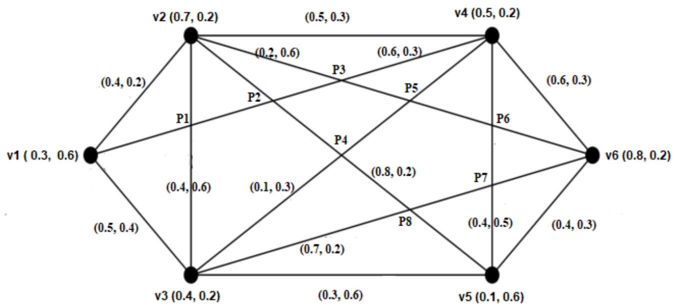

Figure 4: IPFG

Because the quantity of crossings increases, the charge of destruction will increase. Therefore, the size of the planarity cost is vital. In the figure 4, there are eight crossings points' P1, P2, P3, P4, P5, P6, P7 and P8 between pair of pipe connections $(v_1v_4, v_2v_3), (v_1v_4, v_2v_5), (v_1v_4, v_2v_6),$ and (v_2v_5, v_3v_6) espectively.

The Strength of pipes are

$$v_1v_4 = \left(\frac{0.6}{0.3}, \frac{0.3}{0.6}\right) = (2,\ 0.5); \quad v_2v_3 = \left(\frac{0.4}{0.4}, \frac{0.6}{0.2}\right) = (1,\ 2)$$

$$v_2v_5 = \left(\frac{0.8}{0.1}, \frac{0.2}{0.6}\right) = (8,\ 0.33); \quad v_2v_6 = \left(\frac{0.2}{0.7}, \frac{0.6}{0.2}\right) = (0.28,\ 3)$$

$$v_3v_4 = \left(\frac{0.1}{0.4}, \frac{0.3}{0.2}\right) = (0.25,\ 1.5); \quad v_4v_5 = \left(\frac{0.4}{0.1}, \frac{0.5}{0.6}\right) = (4,\ 0.83)$$

$$v_3v_6 = \left(\frac{0.7}{0.4}, \frac{0.2}{0.2}\right) = (1.75,\ 1)$$

Thus, the intuitionistic Pythagorean uncertain planarity rate is $\xi = (0.055, 0.098)$
Because the planarity rate is at a lowest, it designates the opportunity of excessive damage, to reduce the planarity, it can trade the graphical illustration as proven in the under determine.

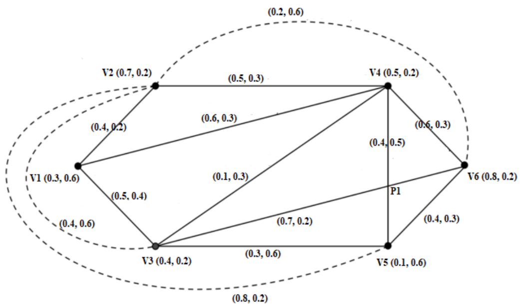

Figure 5: IPFG

Hence that the number of crossover factors is inversely proportional to planarity. Meanwhile the number of crossing factors reduction. In figure 5, there is only one crossings point P1 between pair of pipe connections (v_3v_6, v_4v_5) respectively.
The Strength of pipes are

$$v_4v_5 = \left(\frac{0.4}{0.1}, \frac{0.5}{0.6}\right) = (4,\ 0.83)$$

$$v_3v_6 = \left(\frac{0.7}{0.4}, \frac{0.2}{0.2}\right) = (1.75,\ 1)$$

Thus, the Pythagorean uncertain planarity rate is $\xi = (0.15,\ 0.35)$ increase and score of ruin reductions. In addition, the illustration in the preceding figure 5 reveals that P1 is the only crossing left that cannot be decreased; however, the likelihood of a pipe connection and the rate of destruction caused by it can be reduced by using high-quality pipes between v1 and v2, v4 and v6. For that reason, the crossing could be less dangerous. As a result, finish that Pythagorean uncertain linking model may be used for tracing and identifying the rate of damage. Through probing and taking greater unique security procedures, the share of damage may be decreased and plenty of human problems can be minimized.

Methodology: Reducing crossings and maintaining planarity between the connections of the wate tank pipes.

INPUT: A discrete set of units $U = \{U_1, U_2, U_3, \ldots, U_n\}$, a set of water tank connections $E = \{E_1, E_2, E_3, \ldots, E_m\}$ between the pipes and a set of point of intersections $C = \{C_1, C_2, C_3, \ldots, C_n\}$.
OUTPUT: Minimized crossing and increased planarity value.

1. Begin

2. Compute the strength of the edge E_i, where $i = 1, 2, \ldots, n$ by using the formula

$$\zeta_{rs} = (\mu_{rs}, \gamma_{rs}) = \left(\frac{\mu_N(rs)_j}{\mu_M(r) \wedge \mu_M(s)}, \frac{\gamma_N(rs)_j}{\gamma_M(r) \wedge \gamma_M(s)}\right)$$

3. 3. Determine the intersecting point between edges by using the formula

$$\zeta_P = (\mu_P, \gamma_P) = \left(\frac{\mu_{rs} + \mu_{uv}}{2}, \ \frac{\gamma_{rs} + \gamma_{uv}}{2} \right)$$

4. 4. Find the intuitionistic Pythagorean fuzzy planarity value defined as

$$\xi = (\xi_\mu, \xi_\gamma) = \left(\frac{1}{1 + \mu_{\zeta_1} + \mu_{\zeta_2} + \cdots + \mu_{\zeta_n}}, \ \frac{1}{1 + \gamma_{\zeta_1} + \gamma_{\zeta_2} + \cdots + \gamma_{\zeta_n}} \right)$$

5. Keep the graphical representation of the edges $E_j$and E_k, if $\xi_\mu > 0.5$ and $\zeta_\gamma < 0.86$ otherwise change the graphical representation.

6. If there are no new crossings in the graphical representation of the edges E_j and E_k, then modify it,if not, maintain the prior depiction.

7. The crossing and planarity values will be reduced and raised, respectively, by altering the graphical depiction of the edges E_j and E_k.

8. end

5. Comparative Analysis

Pythagorean fuzzy graphs, a widely used extension of fuzzy and intuitionistic fuzzy graphs, effectively represent structural relationships among multiple objects when the connections between them are uncertain.

Therefore, the method developed in this article is more comprehensive than existing approaches based on fuzzy and intuitionistic fuzzy graphs. The results clearly demonstrate that the proposed technique, which utilizes the intersection points between edges and the intuitionistic Pythagorean fuzzy planarity value, provides a more refined mechanism for handling higher levels of uncertainty and hesitation.

Pythagorean Fuzzy Graphs (PFGs) represent a more generic model capable of managing a wider range of uncertainty due to the Pythagorean property. Although both Intuitionistic Fuzzy Graphs (IFGs) and PFGs are employed to model uncertainty in graph topologies, PFGs are better suited to depict scenarios in which the sum of the degrees of membership and non-membership exceeds 1. In contrast, IFGs cannot effectively represent such cases, even if the squares of these degrees satisfy the required constraints. Compared to fuzzy sets and intuitionistic fuzzy sets (IFSs), Pythagorean fuzzy sets and PFGs offer enhanced flexibility for managing human judgment data. PFGs are viewed as more powerful and ap-plicable than fuzzy graphs and IFGs when it comes to managing uncertainty and incomplete data.

6. Conclusion

A fuzzy graph can effectively represent the uncertainty present in various types of networks. Com-pared to classical, fuzzy, and intuitionistic fuzzy models, Pythagorean fuzzy models offer greater pre-cision, flexibility, and compatibility for such systems. In this paper, we introduce the concept of intui-tionistic Pythagorean fuzzy graphs (IPFGs). We have developed a set of operational laws for IPFGs and examined their related theorems in detail. We also determine the planarity of IPFGs formed through certain operations. The intersecting value and the Pythagorean fuzzy planarity value are then used to define the planarity of a Pythagorean fuzzy graph. In addition, we demonstrate that certain theorems establish bounds for both general and specific planarity values of a Pythagorean fuzzy graph. Subsequently, related theorems and applications of strong and weak planarity are presented and de-fined. Finally, the applicability of the proposed generalization of fuzzy graph theory is demonstrated through an application based on intuitionistic Pythagorean fuzzy preference graphs (IPFGs). Our future research will focus on: (1) interval-valued intuitionistic Pythagorean fuzzy graphs, (2) simplified inter-val-valued intuitionistic Pythagorean fuzzy graphs, and (3) hesitant intuitionistic Pythagorean fuzzy graphs. Further studies should also explore planarity, expressions, and dual structures in Pythagorean fuzzy graphs.

Author Declarations

Conflict of interest: The authors have no conflicts to disclose. **Author Contributions:Obbu Ramesh:** Mathematical Analysis, Investigation (equal). **Nainaru Tarakaramu:** Editing review, Super vision (equal). **YuvaRoopa Lakshmi:** Developed the theory and performed the computations (equal). **Sharief Basha S:** Super vision (equal). **Sarvar Iskandarov:** Investigation (equal). **Akbar Toyirov:** Verified the Methodology (equal). **Jushkinbek Yuldoshev:** Formulations, Verified the Methodology (equal). **Sardor. Sabirov:** Conceived of the presented idea (equal).**K. K. Prashanth:** Review editing, Developed the theory and performed the computations (equal).

References

[1] A. Pal, S. Samanta, and M. Pal. Concept of fuzzy planar graphs. In *Proceedings of the Science and Information Conference*, pages 557–563, London, UK.

[2] A. Pal, S. Samanta, and M. Pal. New concept of fuzzy planar graphs. *International Journal of Advanced Research in Artificial Intelligence*, 3:52–59, 2014.

[3] T. Pramanik, S. Samanta, and M. Pal. Special planar fuzzy graph configurations. *International Journal of Scientific Research Publications*, 2:1–4, 2012.

[4] L. A. Zadeh. Fuzzy sets. *Information and Control*, 8:338–353, 1965.

[5] A. Rosenfeld. Fuzzy graphs. In L. A. Zadeh, K. S. Fu, and M. Shimura, editors, *Fuzzy Sets and Their Applications*, pages 77–95. Academic Press, New York, 1975.

[6] R. Jaiswal and S. Rai. Application of fuzzy graph coloring in traffic light problem. *International Journal of Innovative Research in Science, Engineering and Technology*, 5:6950–6955, 2016.

[7] S. Lavanya and R. Sattanathan. Fuzzy total coloring of fuzzy graphs. *International Journal of Information Technology and Knowledge Management*, 2:37–39, 1998.

[8] Ch. Eslahchi and B. N. Onagh. Vertex strength of fuzzy graph. *International Journal of Mathematics and Mathematical Sciences*, pages Article ID 43614, 1–9, 2006.

[9] R. A. Borzooei, H. Rashmanlou, S. Samanta, and M. Pal. A study on fuzzy labelling graphs. *Journal of Intelligent and Fuzzy Systems*, 30:3349–3355, 2016.

[10] E. Mohammad Zadeh. A graph associated to a poly group with respect to an automorphism. *Journal of Algebraic Hyperstructures and Logical Algebras*, 2:99–112, 2021.

[11] J. N. Mordeson and S. Mathew. Sustainable goals in combating human trafficking: Analysis by mathematics of uncertainty. *Journal of Algebraic Hyperstructures and Logical Algebras*, 1(1):49–59, 2020.

[12] J. N. Mordeson, S. Mathew, and D. S. Malik. *Fuzzy graph theory with applications to human trafficking*. Springer, 2018.

[13] J. N. Mordeson, S. Mathew, and D. S. Malik. *Fuzzy graph theory*. Springer, 2018.

[14] H. Rashmanlou, S. Samanta, and R. A. Borzooei. Product of bipolar fuzzy graphs and their degree. *International Journal of General Systems*, 45(1):1–14, 2016.

[15] Obbu Ramesh and S. Sharief Basha. An intuitionistic fuzzy graph's signless laplacian energy. *International Journal of Engineering & Technology*, 7(4.10):892–895, 2018.

[16] Obbu Ramesh and S. Sharief Basha. Signless laplacian energy of operations on intuitionistic fuzzy graphs. *International Journal of Engineering & Technology*, 7(4.10):888–891, 2018.

[17] S. Naz, S. Ashraf, and M. Akram. A novel approach to decision-making with pythagorean fuzzy information. *Mathematics*, 6:1–28, 2018.

[18] M. Akram and B. Davvaz. Strong intuitionistic fuzzy graphs. *Filomat*, 26:177–196, 2012.

[19] R. Verma, J. M. Merigo, and M. Sahni. Pythagorean fuzzy graphs: Some results. *arXiv preprint*, 2018.

[20] M. Akram and S. Naz. Energy of pythagorean fuzzy graphs with applications. *Mathematics*, 6:136, 2018.

[21] S. S. Dhavudh and R. Srinivasan. Properties of intuitionistic fuzzy graphs of second type. *International Journal of Computational and Applied Mathematics*, 12:815–823, 2017.

[22] M. Akram, A. Habib, F. Ilyas, and J. M. Dar. Specific types of pythagorean fuzzy graphs and application to decision-making. *Mathematics and Computers in Applications*, 23:42, 2018.

[23] M. Akram, J. M. Dar, and S. Naz. Certain graphs under pythagorean fuzzy environment. *Complex Intelligent Systems*. in press.

[24] S. Sahoo and M. Pal. Intuitionistic fuzzy competition graphs. *Journal of Applied Mathematics and Computation*, 52:37–57, 2016.

[25] Sankar Sahoo and Madhumangal Pal. Product of intuitionistic fuzzy graphs and degree. *Journal Unknown*, pages 1059–1067, 2017.

[26] Sankar Sahoo and Madhumangal Pal. Certain types of edge irregular intuitionistic fuzzy graphs. *Journal Unknown*, pages 295–305, 2018.

[27] Sankar Sahoo. Colouring of mixed fuzzy graph and its application in covid19. *Journal of Multiple Valued Logic and Soft Computing*, 35:473–489, 2020.

[28] S. Noorjahan and S. B. Shaik. Decision-making using the correlation coefficient measures of intuitionistic fuzzy rough graph. *Hacettepe Journal of Mathematics and Statistics*, 53(6):1774–1797, 2024.

[29] S. Noorjahan and S. S. Basha. The laplacian energy of an intuitionistic fuzzy rough graph and its utilisation in decision-making. *Operations Research and Decisions*, 35(1):81–107, 2025.

[30] N. K. Akula and S. B. Shaik. Correlation coefficient measure of intuitionistic fuzzy graphs with application in money investing schemes. *Computing and Informatics*, 42(2):436–456, 2023.

[31] S. Noorjahan and S. Sharief Basha. Developing an intuitionistic fuzzy rough new correlation coefficient approach for enhancing robotic vacuum cleaner. *Science Progress*, 107(3), 2024.

[32] Pythagorean fuzzy hx-subgroups and their applications. *European Journal of Pure and Applied Mathematics*, 18(2):5768, 2025.

[33] Pythagorean fuzzy soft somewhat continuous functions. *European Journal of Pure and Applied Mathematics*, 17(4):4147–4163, 2024.

[34] K. Atanassov. Intuitionistic fuzzy sets. 1983.

[35] R. Parvathy and M. G. Karunambigai. Intuitionistic fuzzy graphs. In *Computational Intelligence, Theory and Applications, International Conference*, Germany.

[36] P. A. Ejegwa. Pythagorean fuzzy set and its application in career placements based on academic performance using max–min–max composition. *Complex Intelligent Systems*, 5:165–175, 2019.

[37] R. R. Yager. Pythagorean membership grades in multicriteria decision making. *IEEE Transactions on Fuzzy Systems*, 22(4):958–965, 2014.