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Neutrosophic Bi-Ideals in INK-Algebras

Mounikalakshmi Remala[1], Eswarlal Tamma[1,*], Yella Bhargavi[1]

[1] *Department of Engineering Mathematics, College of Engineering, Koneru Lakshmaiah Education Foundation, Vaddeswaram, Andhra Pradesh, India*

Abstract. The main aim of this research article is to identify the bi-ideals in INK-algebra. This paper introduces the notion of neutrosophic bi-ideals in INK-algebra and discusses basic operations such as order-reversing and order-preserving properties. It is shown that the intersection and union (with containment) of two neutrosophic bi-ideals result in another neutrosophic bi-ideal. Further, the paper explores homomorphisms and epimorphisms between neutrosophic bi-ideals in INK-algebras. It is demonstrated that the direct product of two neutrosophic sets in an INK-subalgebra remains within the same structure. Observations regarding the direct product of neutrosophic bi-ideals in INK-algebras are provided. Finally, an application related to neutrosophic bi-ideals is discussed.

2020 Mathematics Subject Classifications: 03E72, 08A72, 06F25

Key Words and Phrases: INK-algebra, neutrosophic set, neutrosophic INK-algebra, neutrosophic bi-ideal, homomorphism of neutrosophic bi-ideal, direct product of neutrosophic bi-ideal

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1. Introduction

Iseki and Tanaka [1, 2] have worked on the concept of BCK and BCI algebras to pick-up their characteristics and applications. There exists an immense area of empirical applications in fuzzy sets, Intuitionistic fuzzy sets and Neutrosophic sets. The literature works of Fuzzy sub algebras and fuzzy K-ideals in INK-algebras, fuzzy p-ideal in INK-algebra, Fuzzy translation of INK-ideal of INK-algebras also they proposed On intuitionistic fuzzy INK-ideals of INK-algebras, Direct product of intuitionistic fuzzy K-ideals of INK-algebras, Intuitionistic fuzzy translation on INK-algebra and also they have discussed neutosophic set in INK-algebra, neutrosophic h-ideal in INK-algebra have discussed by Kaviyarasu and Indhira, [3–7] and homomorphism and anti-homomorphism of neutrosophic INK-algebras have done by Mounikalakshmi, Eswarlal, Venkata Kalyani and Aiyred Iampan [8]. Recently, Kaviyarasu and Rajeshwari [9] discussed Translation of neutrosophic INK-algebras and Mounikalakshmi, Eswarlal [10, 11] worked on bipolar fuzzy INK subalgebras of INK

*Corresponding author.
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Email addresses: mouninaidu0521@gmail.com (M. Remala), eswarlal@kluniversity.in (E. Tamma)

algebras and Implicative ideals and Positive Implicative INK-Ideals in Neutrosophic INK-algebra. Moreover, the work of INK-algebra has been explored in different aspects of fuzzy sets, bipolar fuzzy sets, intuitionistic fuzzy sets and also in neutrosophic sets.

Zadeh, traversed the notion of fuzzy sets in 1965, which is an extension of classical set theory that deals with vagueness and uncertainty in the given data. Classical set theory asserts that an element is either a member of a set or not. Fuzzy set theory discourses this by introducing membership values scaling from 0 to 1, which have been used to represent the degree to which an element is affiliated with a set. Fuzzy set theory has innumerable implementations in diverse fields, comprises of artificial intelligence, decision-making and control systems that allow the both modeling and handling of vague and imprecise information. Moreover, Kuroki [12, 13] discussed his work on fuzzy bi-ideals in semigroups and Fuzzy generalized bi-ideals in semigroups. Yiarayong [14] have done results on fuzzy bi-ideal theory applied on semi-groups. Eventually, Atanassov proposed the generalization of fuzzy set, which is Intuitionistic Fuzzy Set(IFS) in the year 1980, which gives information about a fresh parameter known as "non-membership degree," where fuzzy tells us about the membership degree but in IFS gives information about uncertainty and vagueness regarding membership degrees and non-membership degrees. Intuitionistic fuzzy sets have applications in decision-making where uncertainty plays a significant role. Various approaches of IFS include expert systems, risk assessment and medical diagnosis, where precise decisions are difficult to maintain on vague or uncertain data or information. Young Bae Jun and Kyung Ho Kim [15] have done work on Intuitionistic fuzzy ideals of BCK-algebra. Few authors, Kim and Lee [16, 17] have discussed results of On intuitionistic fuzzy bi-ideals of semi-groups, Interval valued intuitionistic fuzzy bi-ideals of semigroups. Also, Bhargavi [18, 19] and Raagamayi [20] have discussed the results on Vague bi-ideals. Sindhu and Himaya Jaleela Begun [21] have worked on Intuitionistic fuzzy bi-ideals of BCK-algebras. Sequentially, Vinnela and Raagamayi [22] have worked on Bipolar fuzzy bi-ideals of gamma near ring. Sub-sequentially few more authors have completed work on bi-ideals [7, 23–25] like bi-ideals in semi-groups, fuzzy bi-ideals and generalized fuzzy bi-ideals in semigroups also [14, 26, 27].

Later, the neutrosophic sets were introduced by smarandache in 1990, which is the generalization of classical sets, fuzzy sets and Intuitionistic fuzzy sets by involving a new parameter called "indeterminancy." Neutrosophic sets are particularly useful in situations where uncertainty exist also holds the basic components of truth, falsehood and indeterminacy are simultaneously considered. Neutrosophic sets have plenty of approaches in many fields like decision making, expert systems, image processing and fuzzy logic thus enables us to do effective modeling and analysis in situations where classical set theory falls short. Some authors De Gruyter [28] have discussed few results on MBJ-neutrosophic ideals of BCK/BCI-algebras. Muhuuddin and Young Bae Jun [29] have done further results of neutrosophic subalgebras in BCK/BCI-algebras based on neutrosophic points. Now in this article we look over the concept of bi-ideals in INK-algebra in terms of neutrosophic sets.

2. Preliminaries

In this segment, we have used the some of the definitions which are being utilized for this work.

Definition 2.1. [30] An INK-algebra ($\ddot{\ddot{U}}$, $\bullet$, 0) is said to be an INK-algebra if it satisfies the following conditions for any ε, з, ɚ $\in \ddot{\ddot{U}}$:

- **INK-1:** ((ε $\bullet$ з) $\bullet$ (ε $\bullet$ ɚ)) $\bullet$ (ɚ $\bullet$ з) $= 0$.
- **INK-2:** ((ε $\bullet$ ɚ) $\bullet$ (з $\bullet$ ɚ)) $\bullet$ (ε $\bullet$ з) $= 0$.
- **INK-3:** ε $\bullet$ $0 =$ ε.
- **INK-4:** ε $\bullet$ з $= 0$ and з $\bullet$ ε $= 0$ imply ε $=$ з.

Note. In $\ddot{\ddot{U}}$ we can interpret $\leq$ by ε $\leq$ з if and only if ε $\bullet$ з $= 0$.

Definition 2.2. [30] Let Y be a non-empty subset of a INK-algebra $\ddot{\ddot{U}}$, then Y is said to be an INK-sub-algebra of $\ddot{\ddot{U}}$, if ε$\bullet$ з$\in$ Y where ε, з $\in \ddot{\ddot{U}}$.

Definition 2.3. [3] A fuzzy set g in a INK-algebra $\ddot{\ddot{U}}$ is known as FINK-subalgebra of $\ddot{\ddot{U}}$ if g(ε$\bullet$ з) $\geq$ min{g(ε), g(з)} $\forall$ ε, з$\in \ddot{\ddot{U}}$.

Definition 2.4. [3] Let fuzzy set g in INK-algebra $\ddot{\ddot{U}}$ is known as fuzzy-ideal, if it satisfies:
FID-1: g(0) $\geq$ g(ε)
FID-2: g(ε) $\geq$ min{g(ε$\bullet$ з), g(з)} $\in \ddot{\ddot{U}}$.

Definition 2.5. [3] Let F be a non-empty subset of a INK-algebra $\ddot{\ddot{U}}$. Then F is defined as INK-ideal of $\ddot{\ddot{U}}$ if

(i) $0 \in F$,

(ii) ((ɚ $\bullet$ ε) $\bullet$ (ɚ $\bullet$ з)) $\in F$ and з $\in F$ imply ε $\in F$ for all ε, з, ɚ $\in \ddot{\ddot{U}}$.

Definition 2.6. [6] A neutrosophic set g=(g_T, g_I, g_F) in X is called a neutrosophic INK sub-algebra of $\ddot{\ddot{U}}$ if it satisfies the following condition, for all ε, з, ɚ$\in \ddot{\ddot{U}}$.

(i) g_T(ε$\bullet$ з) $\geq$ min {g_T (ε), g_T (з)}

(ii) g_F(ε$\bullet$ з) $\leq$ max {g_F (ε), g_F (з)}

(iii) g_F(ε$\bullet$ з) $\leq$ max {g_F (ε), g_F (з)}

Example 2.1. Consider the INK-algebra $\ddot{\ddot{U}} = \{0, a, b\}$ with the following Cayley table.

$\bullet$	0	a	b
0	0	b	a
a	a	0	b
b	b	a	0

A neutrosophic set $g = (g_T, g_I, g_F)$ on $\ddot{\ddot{U}}$ is defined by

	0	a	b
g_T	0.2	0.5	0.6
g_I	0.9	0.8	0.8
g_F	0.8	0.5	0.4

Then g=(g_T, g_I, g_F) be a neutrosophic sub-algebra. Let us take (randomly) for truth-membership degree ε= a & з= b.
So, g_T(a • b) $\geq$ min {g_T (a), g_T (b)}
g_T(b) $\geq$ min {g_T (a), g_T (b)} = 0.6 > 0.5
Also, g_I(a • b) $\leq$ max {g_I (a), g_I (b)}
g_I(b) $\leq$ max {g_I (a), g_I (b)} = 0.8 = 0.8
Similarly, g_F(a • b) $\leq$ max {g_F (a), g_F (b)}
g_F(b) $\leq$ max {g_F (a), g_F (b)} = 0.4 < 0.5
Likely, for all outcomes the above condition satisfied.

Definition 2.7. [6] A neutrosophic set $g = (g_T, g_I, g_F)$ in $\ddot{\ddot{U}}$ is termed as neutrosophic ideal of $\ddot{\ddot{U}}$ if it satisfies the following condition, for all $ε, з \in \ddot{\ddot{U}}$:

(i) $g_T(0) \geq g_T(ε)$, $g_I(0) \leq g_I(ε)$, $g_F(0) \leq g_F(ε)$.

(ii) $g_T(ε) \geq \min\{\, g_T(ε \bullet з),\, g_T(з) \,\}$.

(iii) $g_I(ε) \leq \max\{\, g_I(ε \bullet з),\, g_I(з) \,\}$.

(iv) $g_F(ε) \leq \max\{\, g_F(ε \bullet з),\, g_F(з) \,\}$.

Definition 2.8. [6] A neutrosophic set $g = (g_T, g_I, g_F)$ in X is termed as neutrosophic INK-ideal of $\ddot{\ddot{U}}$ if it satisfies the following condition, for all $ε, з, ɚ \in \ddot{\ddot{U}}$:

(i) $g_T(0) \geq g_T(ε)$, $g_I(0) \leq g_I(ε)$, $g_F(0) \leq g_F(ε)$.

(ii) $g_T(ε) \geq \min\{\, g_T((ɚ \bullet ε) \bullet (ɚ \bullet з)),\, g_T(з) \,\}$.

(iii) $g_I(ε) \leq \max\{\, g_I((ɚ \bullet ε) \bullet (ɚ \bullet з)),\, g_I(з) \,\}$.

(iv) $g_F(ε) \leq \max\{\, g_F((ɚ \bullet ε) \bullet (ɚ \bullet з)),\, g_F(з) \,\}$.

Definition 2.9. [6] Let g=(g_T, g_I, g_F) and h=(h_T, h_I, h_F) be two neutrosophic sets in $\ddot{\ddot{U}}$, then the union and intersection are defined by

(i) g ∪ h (ε) = {< ε, max{T_g(ε), T_h(ε)}, min {I $_g$(ε), I_h(ε)}, min {F $_g$(ε), F_h(ε)}}

(ii) g ∩ h (ε) = {< ε, min{T_g(ε), T_h(ε)}, max {I $_g$(ε), I_h(ε)}, max {F $_g$(ε), F_h(ε)}}.

Definition 2.10. [8] A Mapping $¢ : \ddot{\ddot{U}} \to \breve{\ddot{\ddot{U}}}$ of INK-algebras is known as homomorphism if ¢(ε• з) = ¢(ε) • ¢(з) for all ε, з∈ $\ddot{\ddot{U}}$. If $¢ : \ddot{\ddot{U}} \to \breve{\ddot{\ddot{U}}}$ is a homomorphism then ¢(0)=0.

3. Intersection and Union on Neutrosophic Bi-ideals

Definition 3.1. An INK-algebra $\ddot{\ddot{U}}$ is said to be associative INK algebra if it satisfies (ε• з)•ɚ= ε• (з•ɚ)

Definition 3.2. Let F be a non-empty subset of a INK algebra $\ddot{\ddot{U}}$. Then F is defined as Bi-ideal of INK-algebra $\ddot{\ddot{U}}$ if

(i) 0 ∈ F,

(ii) ε• з• ɚ∈ F and ɚ∈ F imply that ε• з∈ F for all ε, з, ɚ∈ $\ddot{\ddot{U}}$.

Definition 3.3. A Neutrosophic set g=(g_T, g_I, g_F) is called a neutrosophic Bi-ideal of INK-algebra $\ddot{\ddot{U}}$ if it satisfies

(i) $g_T(0) \geq g_T$(ε), $g_I(0) \leq g_I$(ε), $g_F(0) \leq g_F$(ε)

(ii) g_T(ε• з) ≥ min {g_T(ε• з• ɚ), g_T(ɚ)}.

(iii) g_I(ε• з) ≤ max {g_I(ε• з• ɚ), g_I(ɚ)}.

(iv) g_F(ε• з) ≤ max {g_F(ε• з• ɚ), g_F(ɚ)} for all ε, з, ɚ∈ $\ddot{\ddot{U}}$.

Example 3.1. Consider INK-algebra $\ddot{\ddot{U}}$={0, 2, 4, 6} with Cayley table

•	0	2	4	6
0	0	0	0	0
2	2	0	0	2
4	4	2	0	4
6	6	6	6	0

•	0	2	4	6
g_T	0.8	0.7	0.6	0.5
g_I	0.7	0.8	0.8	0.9
g_F	0.4	0.5	0.5	0.8

Let us take (randomly) ε= 0, з= 4 , ɚ= 6. So, $g_T(0 \bullet 4) \geq \min \{g_T(0 \bullet 4 \bullet 6), g_T(6)\}$
$g_T(0) \geq \min \{g_T(0), g_T(6)\} = 0.8 > 0.5$
Also, $g_I(0 \bullet 4) \leq \max \{g_I(0 \bullet 4 \bullet 6), g_I(6)\}$
$g_I(0) \leq \max \{g_I(0), g_I(6)\} = 0.7 < 0.9$
Similarly, $g_F(0 \bullet 4) \leq \max \{g_F(0 \bullet 4 \bullet 6), g_F(6)\}$
$g_F(0) \leq \max \{g_F(0), g_F(6)\} = 0.4 < 0.8$
Likely, for all outcomes neytrosophic bi-ideal condition satisfied.

Then g=(g_T, g_I, g_F) be neutrosophic bi-ideal of INK-algebra of $\ddot{\ddot{U}}$.

Lemma 3.1. Let neutrosophic set g=(g_T, g_I, g_F) in INK-algebra $\ddot{\ddot{U}}$ is an neutrosophic bi-ideal of $\ddot{\ddot{U}}$. If the inequality ε• з≤ ɚholds in $\ddot{\ddot{U}}$, then

(i) g_T(ε• з) $\geq$ min {g_T (ε), g_T (ɚ)}.

(ii) g_I(ε• з) $\leq$ max {g_I (ε), g_I (ɚ)}.

(iii) g_F(ε• з) $\leq$ max {g_F (ε), g_F (ɚ)}.

Proof. Let ε, з, ɚ$\in \overset{::}{U}$ be such that ε• з$\leq$ ɚ. By the definition of partial order in INK algebra. We have ε• з$\leq$ ɚ$\Longrightarrow$ (ε• з) • ɚ=0 $\Longrightarrow$ ε• з• ɚ=0. Since g=(g_T, g_I, g_F) is an neutrosophic Bi-ideal of INK algebra $\overset{::}{U}$, by def 3.3.1 it satisfies the following conditions for all ε, з, ɚ$\in \overset{::}{U}$
Now

$$g_T(\text{ε} \bullet \text{з}) \geq \min\{g_T(\text{ε} \bullet \text{з} \bullet \text{ɚ}),\ g_T(\text{ɚ})\} \tag{3.1}$$

Since

$$g_T(\text{ε} \bullet \text{з}) \geq \min\{g_T(0),\ g_T(\text{ɚ})\} \tag{3.2}$$

Now, Using the property of Neutrosophic bi-ideal that g_T (0) $\geq$ g_T(ε), we obtain

$$\min\{g_T(0),\ g_T(\text{ɚ})\} \geq \min\{g_T(\text{ε}),\ g_T(\text{ɚ})\} \tag{3.3}$$

substitute (3.3) into (3.2) then we get, g_T(ε• з) $\geq$ min{ g_T(ε), g_T(ɚ)}. Therefore

$$g_I(\text{ε} \bullet \text{з}) \leq \max\{g_I(\text{ε} \bullet \text{з} \bullet \text{ɚ}),\ g_I(\text{ɚ})\} \tag{3.4}$$

Since ε• з• ɚ=0, it follows (3.1)

$$g_I(\text{ε} \bullet \text{з}) \leq \max\{g_I(0),\ g_I(\text{ɚ})\} \tag{3.5}$$

Now, Using the property of Neutrosophic bi-ideal that g_I (0) $\leq$ g_I(ε), we obtain

$$\max\{g_I(0),\ g_I(\text{ɚ})\} \leq \max\{g_I(\text{ε}),\ g_I(\text{ɚ})\} \tag{3.6}$$

substitute (3.6) into (3.5) then we get, g_I(ε• з) $\leq$ max{ g_I(ε), g_T(ɚ)}. Also (iii)

$$g_F(\text{ε} \bullet \text{з}) \leq \max\{g_F(\text{ε} \bullet \text{з} \bullet \text{ɚ}),\ g_F(\text{ɚ})\} \tag{3.7}$$

Since ε• з• ɚ=0, it follows (3.1)

$$g_F(\text{ε} \bullet \text{з}) \leq \max\{g_F(0),\ g_F(\text{ɚ})\} \tag{3.8}$$

Now, Using the property of Neutrosophic bi-ideal that g_F (0) $\leq$ g_F(ε), we obtain

$$\max\{g_F(0),\ g_F(\text{ɚ})\} \leq \max\{g_F(\text{ε}),\ g_F(\text{ɚ})\} \tag{3.9}$$

substitute (3.9) into (3.8) then we get, g_F(ε• з) $\leq$ max{ g_F(ε), g_F(ɚ)}
Hence Proved.
Lemma 3.2. Let NS g=(g_T, g_I, g_F) be a Neutrosophic Bi-ideal of $\overset{::}{U}$. If the inequality ε• з$\leq$ ɚ holds in $\overset{::}{U}$ then g_T(ε• з) $\geq$ g_T(ɚ), g_I(ε• з) $\leq$ g_I(ɚ) and g_F(ε• з) $\leq$ g_F(ɚ) that g_T is order reversing and g_I, g_F are order preserving.
Proof. By using INK ordering, $\text{ε} \leq \text{з} \Leftrightarrow \text{ε} \bullet \text{з} = 0$. So the condition $\text{ε} \bullet \text{з} \leq \text{ɚ}$ implies $(\text{ε}\bullet\text{з})\bullet\text{ɚ} = 0 \Rightarrow \text{ε}\bullet\text{з}\bullet\text{ɚ} = 0$. Let $g = (g_T, g_I, g_F)$ be a Neutrosophic Bi-ideal of INK-algebra $\overset{::}{U}$ for all $\text{ε}, \text{з}, \text{ɚ} \in \overset{::}{U}$.

(i) $g_T(\varepsilon \bullet з) \geq \min\{g_T(\varepsilon \bullet з \bullet ɚ),\ g_T(ɚ)\}$. Since $g_T(\varepsilon \bullet з \bullet ɚ) = g_T(0)$ and $g_T(0) \geq g_T(ɚ)$ (from the definition), we get $g_T(\varepsilon \bullet з) \geq \min\{g_T(0), g_T(ɚ)\} = g_T(ɚ)$.

(ii) $g_I(\varepsilon \bullet з) \leq \max\{g_I(\varepsilon \bullet з \bullet ɚ),\ g_I(ɚ)\}$. Since $g_I(\varepsilon \bullet з \bullet ɚ) = g_I(0)$ and $g_I(0) \leq g_I(ɚ)$ (from the definition), we obtain $g_I(\varepsilon \bullet з) \leq \max\{g_I(0), g_I(ɚ)\} = g_I(ɚ)$.

(iii) $g_F(\varepsilon \bullet з) \leq \max\{g_F(\varepsilon \bullet з \bullet ɚ),\ g_F(ɚ)\}$. Since $g_F(\varepsilon \bullet з \bullet ɚ) = g_F(0)$ and $g_F(0) \leq g_F(ɚ)$ (from the definition), we get $g_F(\varepsilon \bullet з) \leq \max\{g_F(0), g_F(ɚ)\} = g_F(ɚ)$.

Hence proved.

Theorem 3.1. Every neutrosophic bi-ideal of $\ddot{U}$ is an neutrosophic sub-algebra of $\ddot{U}$.

Proof. Let neutrosophic set g=(g_T, g_I, g_F) be a neutrosophic bi-ideal of $\ddot{U}$. Since $\varepsilon \bullet з \leq \varepsilon$ for all $\varepsilon, з, ɚ \in \ddot{U}$. Consider $\varepsilon \bullet з \bullet з \leq \varepsilon$ for all $\varepsilon, з, ɚ \in \ddot{U}$,

$$(\varepsilon \bullet (з \bullet з) \bullet \varepsilon) = 0 \Rightarrow (\varepsilon \bullet 0 \bullet \varepsilon) = ((\varepsilon \bullet 0) \bullet \varepsilon) = 0.$$

It follows that, $g_T(\varepsilon \bullet з \bullet з) \geq g_T(\varepsilon)$, $g_I(\varepsilon \bullet з \bullet з) \leq g_I(\varepsilon)$ and $g_F(\varepsilon \bullet з \bullet з) \leq g_F(\varepsilon)$. By Considering the Bi-ideal conditions, and substitute $ɚ = з$ for all $\varepsilon, з, ɚ \in \ddot{U}$.

(i) $g_T(\varepsilon \bullet з) \geq \min\ \{\ g_T(\varepsilon \bullet з \bullet ɚ), g_T(ɚ)\}$
$= \min\{g_T(\varepsilon \bullet з \bullet з),\ g_T(з)\}$

(ii) $g_I(\varepsilon \bullet з) \leq \max\ \{\ g_I(\varepsilon \bullet з \bullet ɚ), g_I(ɚ)\}$
$= \max\{g_I(\varepsilon \bullet з \bullet з),\ g_I(з)\}$

(iii) Therefore, $g_F(\varepsilon \bullet з) \leq \max\ \{g_F\ (\varepsilon \bullet з \bullet ɚ), g_F(ɚ)\}$
$= \max\{g_F(\varepsilon \bullet з \bullet з),\ g_F(з)\}$.

Theorem 3.2. Let g=(g_T, g_I, g_F) and h=(h_T, h_I, h_F) be two neutrosophic bi-ideals of INK-algebra $\ddot{U}$. Then g $\cap$ h = ($T_{g\cap h}(\varepsilon)$, $I_{g\cap h}(\varepsilon)$, $F_{g\cap h}(\varepsilon)$) is a neutrosophic bi-ideal of INK sub-algebra of $\ddot{U}$.

Proof. Let g and h be two neutrosophic bi ideals of INK-algebra $\ddot{U}$.

(i) Truth membership

$$\begin{aligned} T_{g\cap h}(\varepsilon\bullet\ з) &= \min\{g_T(\varepsilon\bullet\ з), h_T(\varepsilon\bullet\ з)\} \\ &\geq \min\ \{\min\{g_T(\varepsilon\bullet\ з\bullet\ ɚ), g_T(ɚ)\}, \min\{h_T(\varepsilon\bullet\ з\bullet\ ɚ), h_T(ɚ)\} \\ &= \min\{\min\{\ g_T(\varepsilon\bullet\ з\bullet\ ɚ), h_T(\varepsilon\bullet\ з\bullet\ ɚ)\}, \min\{g_T(ɚ), h_T(ɚ)\} \\ &\geq \min\{T_{g\cap h}\ (\varepsilon\bullet\ з\bullet\ ɚ)\ , T_{g\cap h}\ (ɚ)\} \end{aligned}$$

(ii) Indeterminacy membership

$$\begin{aligned} I_{g\cap h}(\varepsilon\bullet\ з) &= \max\{g_I(\varepsilon\bullet\ з), h_I(\varepsilon\bullet\ з)\} \\ &\leq \max\ \{\max\{g_I(\varepsilon\bullet\ з\bullet\ ɚ), g_I(ɚ)\}, \max\{h_I(\varepsilon\bullet\ з\bullet\ ɚ), h_I(ɚ)\} \\ &= \max\{\max\{\ g_I(\varepsilon\bullet\ з\bullet\ ɚ), h_I(\varepsilon\bullet\ з\bullet\ ɚ)\}, \max\{g_I(ɚ), h_I(ɚ)\} \\ &\leq \max\{I_{g\cap h}\ (\varepsilon\bullet\ з\bullet\ ɚ)\ , I_{g\cap h}\ (ɚ)\} \end{aligned}$$

(iii) (Falsehood membership)

$$\begin{aligned} F_{g\cap h}(\varepsilon\bullet \mathfrak{z}) &= \max\{g_F(\varepsilon\bullet \mathfrak{z}),\ h_F(\varepsilon\bullet \mathfrak{z})\} \\ &\leq \max\ \{\max\{g_F(\varepsilon\bullet \mathfrak{z}\bullet \mathfrak{r}),\ g_F(\mathfrak{r})\},\ \max\{h_F(\varepsilon\bullet \mathfrak{z}\bullet \mathfrak{r}),\ h_F(\mathfrak{r})\} \\ &= \max\{\max\{\ g_F(\varepsilon\bullet \mathfrak{z}\bullet \mathfrak{r}),\ h_F(\varepsilon\bullet \mathfrak{z}\bullet \mathfrak{r})\},\ \max\{g_F(\mathfrak{r}),\ h_F(\mathfrak{r})\} \\ &\leq \max\{F_{g\cap h}\ (\varepsilon\bullet \mathfrak{z}\bullet \mathfrak{r})\ ,\ F_{g\cap h}\ (\mathfrak{r})\} \end{aligned}$$

Remark 3.1. The union of neutrosophic bi-ideal of INK-sub algebra of INK-algebra $\ddot{\ddot{U}}$ need not be a union of neutrosophic bi ideal.

Example 3.2. Consider INK sub algebra $\ddot{\ddot{U}} = \{0, 2, 4, 6\}$ with the following Cayley table.

$\bullet$	0	2	4	6
0	0	0	0	0
2	2	0	0	2
4	4	2	0	4
6	6	6	6	0

Define Neutrosophic bi-ideal $g = (g_T, g_I, g_F)$ by the component memberships below.

$\bullet$	0	2	4	6
g_T	0.4	0.6	0.3	0.5
g_I	0.3	0.8	0.5	0.6
g_F	0.4	0.6	0.4	0.3

Define Neutrosophic bi-ideal $h = (h_T, h_I, h_F)$ by the component memberships below.

$\bullet$	0	2	4	6
h_T	0.6	0.4	0.4	0.4
h_I	0.7	0.8	0.5	0.4
h_F	0.8	0.2	0.4	0.5

Clearly, g and h are two Neutrosophic bi-ideals of INK sub-algebras. Here $T_{g\cup h}(2 \bullet 4)$ = 0.4 but it is not greater than or equal to i.e., 0.5= min $\{T_{g\cup h}(2 \bullet 4 \bullet 6),\ T_{g\cup h}(6)\}$. Similarly, for $I_{g\cup h}(2 \bullet 4) = 0.7$ but it is not less than or equal to i.e., 0.4= max $\{I_{g\cup h}(2 \bullet 4 \bullet 6),\ T_{g\cup h}(6)\}$. Also for, $I_{g\cup h}(2 \bullet 4) = 0.8$ but it is not less than or equal to i.e., 0.4= max $\{I_{g\cup h}(2 \bullet 4 \bullet 6),\ T_{g\cup h}(6)\}$. Therefore, g ∪ h = $(T_{g\cup h},\ I_{g\cup h},\ F_{g\cup h})$ is not a neutrosophic bi-ideal of INK sub-algebra. Thus, union of neutrosophic bi-ideal of INK-sub algebras is not a neutrosophic bi-ideal. In particular, that follows

Theorem 3.3. Let $g = (g_T, g_I, g_F)$ and $h = (h_T, h_I, h_F)$ be two neutrosophic bi-ideals of INK sub algebras of INK algebra $\ddot{\ddot{U}}$, then $g\cup h$ is a neutrosophic bi-ideal of INK sub-algebra only if $g \subseteq h$ or $h \subseteq g$.

Proof. Suppose $g \subseteq h$. Let $\varepsilon, \mathfrak{z} \in \ddot{\ddot{U}}$.

(i) Truth membership

$$T_{g\cup h}(\varepsilon \bullet з) = \max\{\, g_T(\varepsilon \bullet з),\, h_T(\varepsilon \bullet з)\,\} = h_T(\varepsilon \bullet з) \geq \min\{\, h_T(\varepsilon \bullet з \bullet \eth),\, h_T(\eth)\,\}$$
$$\geq \min\Big\{ \max\{g_T(\varepsilon \bullet з \bullet \eth),\, h_T(\varepsilon \bullet з \bullet \eth)\} \bullet \max\{g_T(\eth),\, h_T(\eth)\}\Big\}$$
$$= \max\{\, T_{g\cup h}(\varepsilon \bullet з \bullet \eth),\, T_{g\cup h}(\eth)\,\}.$$

(ii) Indeterminacy membership

$$I_{g\cup h}(\varepsilon \bullet з) = \min\{\, g_I(\varepsilon \bullet з),\, h_I(\varepsilon \bullet з)\,\} = h_I(\varepsilon \bullet з) \leq \max\{\, h_I(\varepsilon \bullet з \bullet \eth),\, h_I(\eth)\,\}$$
$$\leq \max\Big\{ \min\{g_I(\varepsilon \bullet з \bullet \eth),\, h_I(\varepsilon \bullet з \bullet \eth)\} \bullet \min\{g_I(\eth),\, h_I(\eth)\}\Big\}$$
$$= \min\{\, I_{g\cup h}(\varepsilon \bullet з \bullet \eth),\, I_{g\cup h}(\eth)\,\}.$$

(iii) Falsehood membership

$$F_{g\cup h}(\varepsilon \bullet з) = \min\{\, g_F(\varepsilon \bullet з),\, h_F(\varepsilon \bullet з)\,\} = h_F(\varepsilon \bullet з) \leq \max\{\, h_F(\varepsilon \bullet з \bullet \eth),\, h_F(\eth)\,\}$$
$$\leq \max\Big\{ \min\{g_F(\varepsilon \bullet з \bullet \eth),\, h_F(\varepsilon \bullet з \bullet \eth)\} \bullet \min\{g_F(\eth),\, h_F(\eth)\}\Big\}$$
$$= \min\{\, F_{g\cup h}(\varepsilon \bullet з \bullet \eth),\, F_{g\cup h}(\eth)\,\}.$$

This completes the verification under the assumption $g \subseteq h$.

4. Homomorphism of Neutrosophic Bi-ideal of INK sub-algebra

Definition 4.1. Let $¢ : \ddot{U} \to \ddot{\breve{U}}$ be a homomorphism of INK-algebra and $g = (g_T, g_I, g_F)$ be a neutrosophic set in $\breve{X}$, then the neutrosophic set $g[¢] = (g_T[¢], g_I[¢], g_F[¢])$ in $\ddot{U}$ is defined by neutrosophic set such that for every $\varepsilon \in \ddot{U}$ is called pre-image of g under $¢$.

$$g_T[¢] : \ddot{U} \to [0,1], \quad g_T[¢](\varepsilon) = g_T(¢(\varepsilon)).$$
$$g_I[¢] : \ddot{U} \to [0,1], \quad g_I[¢](\varepsilon) = g_I(¢(\varepsilon)).$$
$$g_F[¢] : \ddot{U} \to [0,1], \quad g_F[¢](\varepsilon) = g_F(¢(\varepsilon)).$$

Proof. Let $¢ : \ddot{U} \to \ddot{\breve{U}}$ be a homomorphism of INK-algebra. If $g = (g_T, g_I, g_F)$ be a neutrosophic bi-ideal in INK-algebra Y, and $g[¢] = (g_T[¢], g_I[¢], g_F[¢])$ be the pre-image of g under $¢$ is defined by

We first have that
$$g_T[¢](\varepsilon \bullet з) = g_T(¢(\varepsilon \bullet з)) \geq g_T(0) = g_T(¢(0)),$$
$$g_I[¢](\varepsilon \bullet з) = g_I(¢(\varepsilon \bullet з)) \leq g_I(0) = g_I(¢(0)),$$
$$g_F[¢](\varepsilon \bullet з) = g_F(¢(\varepsilon \bullet з)) \leq g_F(0) = g_F(¢(0)) \quad \text{for all } \varepsilon \in \ddot{U}.$$

$$
\begin{aligned}
\text{Consider } g_T[¢](ε \bullet з) &= g_T(¢(ε \bullet з)) \\
&\geq \min\{\, g_T[¢](ε \bullet з \bullet ɚ),\ g_T[¢](ɚ) \,\} \\
&\geq \min\{\, g_T(¢(ε \bullet з \bullet ɚ)),\ g_T(¢(ɚ)) \,\} \\
&\geq \min\{\, g_T(¢((ε \bullet з) \bullet ɚ)),\ g_T(¢(ɚ)) \,\} \\
&\geq \min\{\, g_T((¢(ε) \bullet ¢(з)) \bullet (¢(ɚ))),\ g_T(¢(ɚ)) \,\} \\
&\geq \min\{\, g_T(¢(ε) \bullet (¢(з))),\ g_T(¢(ɚ)),\ g_T(¢(ɚ)) \,\} \\
&\geq \min\{\, g_T(¢(ε \bullet з)) \,\}.
\end{aligned}
$$

$$
\begin{aligned}
\text{Also for, } g_I[¢](ε \bullet з) &= g_I(¢(ε \bullet з)) \\
&\leq \max\{\, g_I[¢](ε \bullet з \bullet ɚ),\ g_I[¢](ɚ) \,\} \\
&\leq \max\{\, g_I(¢(ε \bullet з \bullet ɚ)),\ g_I(¢(ɚ)) \,\} \\
&\leq \max\{\, g_I(¢((ε \bullet з) \bullet ɚ))),\ g_I(¢(ɚ)) \,\} \\
&\leq \max\{\, g_I((¢(ε) \bullet ¢(з)) \bullet (¢(ɚ))),\ g_I(¢(ɚ)) \,\} \\
&\leq \max\{\, g_I(¢(ε) \bullet ¢(з)),\ g_I(¢(ɚ)),\ g_I(¢(ɚ)) \,\} \\
&\leq \max\{\, g_I(¢(ε \bullet з)) \,\}.
\end{aligned}
$$

$$
\begin{aligned}
\text{Similarly, } g_F[¢](ε \bullet з) &= g_F(¢(ε \bullet з)) \\
&\leq \max\{\, g_F[¢](ε \bullet з \bullet ɚ),\ g_F[¢](ɚ) \,\} \\
&\leq \max\{\, g_F(¢(ε \bullet з \bullet ɚ)),\ g_F(¢(ɚ)) \,\} \\
&\leq \max\{\, g_F(¢((ε \bullet з) \bullet ɚ))),\ g_F(¢(ɚ)) \,\} \\
&\leq \max\{\, g_F((¢(ε) \bullet ¢(з)) \bullet (¢(ɚ))),\ g_F(¢(ɚ)) \,\} \\
&\leq \max\{\, g_F(¢(ε) \bullet ¢(з)),\ g_F(¢(ɚ)),\ g_F(¢(ɚ)) \,\} \\
&\leq \max\{\, g_F(¢(ε \bullet з)) \,\}.
\end{aligned}
$$

Theorem 4.1. Let $¢ : \ddot{\ddot{U}} \to \ddot{\breve{U}}$ be an epimorphism of INK-algebra. If $g[¢] = (g_T[¢], g_I[¢], g_F[¢])$ is an neutrosophic bi-ideal of INK sub-algebra of INK-algebra $\ddot{\ddot{U}}$, then $g = (g_T, g_I, g_F)$ is an neutrosophic bi-ideal of INK sub-algebra of INK-algebra $\ddot{\ddot{U}}$.

Proof. Let $ε, з \in \ddot{\ddot{U}}$, there exist $ε, з \in X$ such that $¢(ε \bullet з) = ε \bullet з$. Then

$g_T(ε \bullet з) = g_T(¢)(ε \bullet з), g_T[¢](ε \bullet з) \geq g_T = g_T(¢(0)) = g_T(0)$,

$g_I(ε \bullet з) = g_I(¢)(ε \bullet з), g_I[¢](ε \bullet з) \leq g_I = g_I(¢(0)) = g_I(0)$,

$g_F(ε \bullet з) = g_F(¢)(ε \bullet з), g_F[¢](ε \bullet з) \leq g_F = g_F(¢(0)) = g_F(0)$.

$$
\begin{aligned}
\text{Consider } g_T(ε \bullet з) = g_T(¢)(ε \bullet з) &= g_T[¢](ε \bullet з) \\
&\geq \min\{\, g_T([¢](ε \bullet з) \bullet [¢](ɚ)),\ g_T[¢](ɚ) \,\} \\
&\geq \min\{\, g_T([¢](ε \bullet з \bullet ɚ)),\ g_T[¢](ɚ) \,\} \\
&\geq \min\{\, g_T((¢(ε \bullet з \bullet ɚ)),\ g_T(¢(ɚ)) \,\} \\
&\geq \min\{\, g_T(ε \bullet з \bullet ɚ),\ g_T(ɚ) \,\}.
\end{aligned}
$$

$$\begin{aligned}
\text{Also for } g_I(\varepsilon \bullet \text{з}) &= g_I(\text{¢})(\varepsilon \bullet \text{з}) = g_I[\text{¢}](\varepsilon \bullet \text{з}) \\
&\leq \max\{\, g_I([\text{¢}](\varepsilon \bullet \text{з}) \bullet [\text{¢}](\text{ɚ})),\ g_I[\text{¢}](\text{ɚ}) \,\} \\
&\leq \max\{\, g_I([\text{¢}](\varepsilon \bullet \text{з} \bullet \text{ɚ})),\ g_I[\text{¢}](\text{ɚ}) \,\} \\
&\leq \max\{\, g_I((\text{¢}(\varepsilon \bullet \text{з} \bullet \text{ɚ})),\ g_I(\text{¢}(\text{ɚ})) \,\} \\
&\leq \max\{\, g_I(\varepsilon \bullet \text{з} \bullet \text{ɚ}),\ g_I(\text{ɚ}) \,\}.
\end{aligned}$$

$$\begin{aligned}
\text{Similarly, } g_F(\varepsilon \bullet \text{з}) &= g_F(\text{¢})(\varepsilon \bullet \text{з}) = g_F[\text{¢}](\varepsilon \bullet \text{з}) \\
&\leq \max\{\, g_I([\text{¢}](\varepsilon \bullet \text{з}) \bullet [\text{¢}](\text{ɚ})),\ g_F[\text{¢}](\text{ɚ}) \,\} \\
&\leq \max\{\, g_F([\text{¢}](\varepsilon \bullet \text{з} \bullet \text{ɚ})),\ g_F[\text{¢}](\text{ɚ}) \,\} \\
&\leq \max\{\, g_F((\text{¢}(\varepsilon \bullet \text{з} \bullet \text{ɚ})),\ g_F(\text{¢}(\text{ɚ})) \,\} \\
&\leq \max\{\, g_F(\varepsilon \bullet \text{з} \bullet \text{ɚ}),\ g_F(\text{ɚ}) \,\}.
\end{aligned}$$

5. Direct Product of Neutrosophic Bi-ideal of INK sub-algebra

Definition 5.1. Let g=(g_T, g_I, g_F) and h=(h_T, h_I, h_F) be two neutrosophic sets of INK algebras $\ddot{\ddot{U}}_1$ and $\ddot{\ddot{U}}_2$ respectively. Then the direct product of neutrosophic set g and h is given by g × h= ($g_{T(g\times h)}$, $g_{I(g\times h)}$, $g_{F(g\times h)}$) with

(i) $g_{T(g\times h)}$ (ε_1,з_1) = min {$g_{T(g)}(\varepsilon_1)$, $g_{T(h)}(\text{з}_1)$}

(ii) $g_{I(g\times h)}$ (ε_1,з_1) = max {$g_{I(g)}(\varepsilon_1)$, $g_{I(h)}(\text{з}_1)$}

(iii) $g_{F(g\times h)}$ (ε_1,з_1) = max {$g_{F(g)}(\varepsilon_1)$, $g_{F(h)}(\text{з}_1)$} for all $\varepsilon_1,\text{з}_1 \in \ddot{\ddot{U}}_1 \times \ddot{\ddot{U}}_2$.

Definition 5.2. Let g × h= ($g_{T(g\times h)}$, $g_{I(g\times h)}$, $g_{F(g\times h)}$) be neutrosophic set in INK-algebra $\ddot{\ddot{U}}_1$ and $\ddot{\ddot{U}}_2$ respectively. Then the direct product of neutrosophic INK algebra of $\ddot{\ddot{U}}_1 \times \ddot{\ddot{U}}_2$.

(i) $g_{T(g\times h)}$ (ε_1 ,з_1)• (ε_2 ,з_2)) ≥ min{ $g_{T(g\times h)}$ (ε_1 ,з_1), $g_{T(g\times h)}$ (ε_2 ,з_2)}

(ii) $g_{I(g\times h)}$ (ε_1 ,з_1)• (ε_2 ,з_2)) ≤ max{ $g_{I(g\times h)}$ (ε_1 ,з_1), $g_{I(g\times h)}$ (ε_2 ,з_2)}

(iii) $g_{F(g\times h)}$ (ε_1 ,з_1)• (ε_2 ,з_2)) ≤ max{ $g_{F(g\times h)}$ (ε_1 ,з_1), $g_{F(g\times h)}$ (ε_2 ,з_2)} for all (ε_1, ε_2, ε_3) and (з_1, з_2, з_3) ∈ $\ddot{\ddot{U}}_1 \times \ddot{\ddot{U}}_2$. $\ddot{\ddot{U}}_2$.

Theorem 5.1. Let g=(g_T, g_I, g_F) and h=(h_T, h_I, h_F) be two neutrosophic INK algebras $\ddot{\ddot{U}}_1$ and $\ddot{\ddot{U}}_2$ respectively. Then g × h = ($g_{T(g\times h)}$, $g_{I(g\times h)}$, $g_{F(g\times h)}$) is also neutrosphic sub algebra of INK-algebra of $\ddot{\ddot{U}}_1 \times \ddot{\ddot{U}}_2$.

Proof. For any $(\varepsilon_1, \mathfrak{z}_1), (\varepsilon_2, \mathfrak{z}_2) \in \ddot{\ddot{U}}_1 \times \ddot{\ddot{U}}_2$. Now,

$$\begin{aligned}
g_{T(g\times h)}(\varepsilon_1, \mathfrak{z}_1) \bullet (\varepsilon_2, \mathfrak{z}_2) &= g_{T(g\times h)}(\varepsilon_1 \bullet \varepsilon_2),\ (\mathfrak{z}_1 \bullet \mathfrak{z}_2) \\
&= \min\{\, g_{T(g)}(\varepsilon_1 \bullet \varepsilon_2),\ g_{T(h)}(\mathfrak{z}_1 \bullet \mathfrak{z}_2) \,\} \\
&= \min\{\, g_{T(g)}(\varepsilon_1 \bullet \varepsilon_2),\ g_{T(h)}(\mathfrak{z}_1 \bullet \mathfrak{z}_2) \,\} \\
&\geq \min\{\, \min\{g_{T(g)}(\varepsilon_1), g_{T(g)}(\varepsilon_2)\},\ \min\{g_{T(h)}(\mathfrak{z}_1), g_{T(h)}(\mathfrak{z}_2)\} \,\} \\
&\geq \min\{\, g_{T(g\times h)}(\varepsilon_1, \varepsilon_2),\ g_{T(g\times h)}(\mathfrak{z}_1, \mathfrak{z}_2) \,\}.
\end{aligned}$$

Then,

$$\begin{aligned}
g_{I(g\times h)}(\varepsilon_1, \mathfrak{z}_1) \bullet (\varepsilon_2, \mathfrak{z}_2) &= g_{I(g\times h)}(\varepsilon_1 \bullet \varepsilon_2),\ (\mathfrak{z}_1 \bullet \mathfrak{z}_2) \\
&= \max\{\, g_{I(g)}(\varepsilon_1 \bullet \varepsilon_2),\ g_{I(h)}(\mathfrak{z}_1 \bullet \mathfrak{z}_2) \,\} \\
&= \max\{\, g_{I(g)}(\varepsilon_1 \bullet \varepsilon_2),\ g_{I(h)}(\mathfrak{z}_1 \bullet \mathfrak{z}_2) \,\} \\
&\leq \max\{\, \max\{g_{I(g)}(\varepsilon_1), g_{I(g)}(\varepsilon_2)\},\ \max\{g_{I(h)}(\mathfrak{z}_1), g_{I(h)}(\mathfrak{z}_2)\} \,\} \\
&\leq \max\{\, g_{I(g\times h)}(\varepsilon_1, \varepsilon_2),\ g_{I(g\times h)}(\mathfrak{z}_1, \mathfrak{z}_2) \,\}.
\end{aligned}$$

Also,

$$\begin{aligned}
g_{F(g\times h)}(\varepsilon_1, \mathfrak{z}_1) \bullet (\varepsilon_2, \mathfrak{z}_2) &= g_{F(g\times h)}(\varepsilon_1 \bullet \varepsilon_2),\ (\mathfrak{z}_1 \bullet \mathfrak{z}_2) \\
&= \max\{\, g_{F(g)}(\varepsilon_1 \bullet \varepsilon_2),\ g_{F(h)}(\mathfrak{z}_1 \bullet \mathfrak{z}_2) \,\} \\
&= \max\{\, g_{F(g)}(\varepsilon_1 \bullet \varepsilon_2),\ g_{F(h)}(\mathfrak{z}_1 \bullet \mathfrak{z}_2) \,\} \\
&\leq \max\{\, \max\{g_{F(g)}(\varepsilon_1), g_{F(h)}(\varepsilon_2)\},\ \max\{g_{F(g)}(\mathfrak{z}_1), g_{F(h)}(\mathfrak{z}_2)\} \,\} \\
&\leq \max\{\, g_{F(g\times h)}(\varepsilon_1, \varepsilon_2),\ g_{F(g\times h)}(\mathfrak{z}_1, \mathfrak{z}_2) \,\}.
\end{aligned}$$

Theorem 5.2. Let $g = (g_T, g_I, g_F)$ and $h = (h_T, h_I, h_F)$ be two neutrosophic INK algebras $\ddot{\ddot{U}}_1$ and $\ddot{\ddot{U}}_2$ respectively. Then

(i) $g_{T(g\times h)}(0,0) = g_{T(g\times h)}(\varepsilon_1, \mathfrak{z}_1)$,

(ii) $g_{I(g\times h)}(0,0) = g_{I(g\times h)}(\varepsilon_1, \mathfrak{z}_1)$,

(iii) $g_{I(g\times h)}(0,0) = g_{I(g\times h)}(\varepsilon_1, \mathfrak{z}_1)$ for all $(\varepsilon_1, \mathfrak{z}_1) \in \ddot{\ddot{U}}_1 \times \ddot{\ddot{U}}_2$.

Proof. By definition,

$$\begin{aligned}
g_{T(g\times h)}(0,0) &= g_{T(g\times h)}\big((\varepsilon_1, \mathfrak{z}_1) \bullet (\varepsilon_1, \mathfrak{z}_1)\big) \\
&= \min\big\{\, g_{T(g\times h)}\big((\varepsilon_1, \mathfrak{z}_1) \bullet (\varepsilon_1, \mathfrak{z}_1)\big),\ g_{T(g\times h)}(\varepsilon_1, \mathfrak{z}_1) \,\big\} \\
&\geq g_{T(g\times h)}(\varepsilon_1, \mathfrak{z}_1).
\end{aligned}$$

$$\begin{aligned}
g_{I(g\times h)}(0,0) &= g_{I(g\times h)}\big((\varepsilon_1, \mathfrak{z}_1) \bullet (\varepsilon_1, \mathfrak{z}_1)\big) \\
&= \max\big\{\, g_{I(g\times h)}\big((\varepsilon_1, \mathfrak{z}_1) \bullet (\varepsilon_1, \mathfrak{z}_1)\big),\ g_{I(g\times h)}(\varepsilon_1, \mathfrak{z}_1) \,\big\} \\
&\leq g_{I(g\times h)}(\varepsilon_1, \mathfrak{z}_1).
\end{aligned}$$

$$\begin{aligned} g_{F(g\times h)}(0,0) &= g_{F(g\times h)}\big((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_1,\mathfrak{z}_1)\big)\\ &= \max\{\, g_{F(g\times h)}\big((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_1,\mathfrak{z}_1)\big),\ g_{F(g\times h)}(\varepsilon_1,\mathfrak{z}_1)\,\}\\ &\le g_{F(g\times h)}(\varepsilon_1,\mathfrak{z}_1). \end{aligned}$$

Definition 5.3. Let $g\times h = \big(g_{T(g\times h)}, g_{I(g\times h)}, g_{F(g\times h)}\big)$ of $\ddot{\ddot{U}}_1$ and $\ddot{\ddot{U}}_2$ be the direct product of neutrosophic bi-ideals of $\ddot{\ddot{U}}_1\times\ddot{\ddot{U}}_2$ if

(i) $g_{T(g\times h)}(0,0)\ge g_{T(g\times h)}(\varepsilon_1,\mathfrak{z}_1)$,

(ii) $g_{I(g\times h)}(0,0)\le g_{I(g\times h)}(\varepsilon_1,\mathfrak{z}_1)$,

(iii) $g_{F(g\times h)}(0,0)\le g_{F(g\times h)}(\varepsilon_1,\mathfrak{z}_1)$,

(iv) $g_{T(g\times h)}\big((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_2,\mathfrak{z}_2)\big)\ge \min\{\, g_{T(g\times h)}\big((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_2,\mathfrak{z}_2)\bullet(\varepsilon_3,\mathfrak{z}_3)\big),\ g_{T(g\times h)}(\varepsilon_3,\mathfrak{z}_3)\,\}$,

(v) $g_{I(g\times h)}\big((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_2,\mathfrak{z}_2)\big)\le \max\{\, g_{I(g\times h)}\big((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_2,\mathfrak{z}_2)\bullet(\varepsilon_3,\mathfrak{z}_3)\big),\ g_{I(g\times h)}(\varepsilon_3,\mathfrak{z}_3)\,\}$,

(vi) $g_{F(g\times h)}\big((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_2,\mathfrak{z}_2)\big)\le \max\{\, g_{F(g\times h)}\big((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_2,\mathfrak{z}_2)\bullet(\varepsilon_3,\mathfrak{z}_3)\big),\ g_{F(g\times h)}(\varepsilon_3,\mathfrak{z}_3)\,\}$.

Theorem 5.3. Let $g=(g_T,g_I,g_F)$ and $h=(h_T,h_I,h_F)$ be two neutrosophic bi-ideals of INK algebras $\ddot{\ddot{U}}_1$ and $\ddot{\ddot{U}}_2$ respectively. Then the direct product of neutrosophic bi-ideals of INK-algebra g and h is given by $g\times h=\big(g_{T(g\times h)},\, g_{I(g\times h)},\, g_{F(g\times h)}\big)$.

Proof. For any $(\varepsilon_1,\varepsilon_2,\varepsilon_3)$ and $(\mathfrak{z}_1,\mathfrak{z}_2,\mathfrak{z}_3)\in g\times h$.

$$\begin{aligned} \text{Then}\quad g_{T(g\times h)}(0,0) &= \min\{\, g_{T(g)}(0),\ g_{T(h)}(0)\,\}\\ &\ge \{\, g_{T(g)}(\varepsilon_1),\ g_{T(h)}(\mathfrak{z}_1)\,\}\\ &\ge g_{T(g\times h)}(\varepsilon_1,\mathfrak{z}_1). \end{aligned}$$

$$\begin{aligned} \text{Now}\quad g_{I(g\times h)}(0,0) &= \max\{\, g_{T(g)}(0),\ g_{I(h)}(0)\,\}\\ &\le \{\, g_{I(g)}(\varepsilon_1),\ g_{I(h)}(\mathfrak{z}_1)\,\}\\ &\le g_{I(g\times h)}(\varepsilon_1,\mathfrak{z}_1). \end{aligned}$$

$$\begin{aligned} \text{Also}\quad g_{I(g\times h)}(0,0) &= \max\{\, g_{T(g)}(0),\ g_{I(h)}(0)\,\}\\ &\le \{\, g_{I(g)}(\varepsilon_1),\ g_{I(h)}(\mathfrak{z}_1)\,\}\\ &\le g_{I(g\times h)}(\varepsilon_1,\mathfrak{z}_1). \end{aligned}$$

$$\begin{aligned} \text{Now}\quad g_{T(g\times h)}\big((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_2,\mathfrak{z}_2)\big) &= g_{T(g\times h)}\big(\,\varepsilon_1\bullet\mathfrak{z}_1,\ \varepsilon_2\bullet\mathfrak{z}_2\,\big)\\ &= \min\{\, g_{T(g)}\big(\,\varepsilon_1\bullet\mathfrak{z}_1,\ \varepsilon_2\bullet\mathfrak{z}_2\,\big)\,\}\\ &= \min\{\, \min\{g_{T(g)}(\varepsilon_1\bullet\varepsilon_2\bullet\varepsilon_3),\ g_{T(g)}(\varepsilon_3)\},\\ &\qquad \min\{g_{T(h)}(\mathfrak{z}_1\bullet\mathfrak{z}_2\bullet\mathfrak{z}_3),\ g_{T(h)}(\mathfrak{z}_3)\}\,\}\\ &\ge \min\{\, g_{T(g\times h)}\big((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_2,\mathfrak{z}_2)\bullet(\varepsilon_3,\mathfrak{z}_3)\big),\\ &\qquad g_{T(g\times h)}(\varepsilon_3,\mathfrak{z}_3)\,\}. \end{aligned}$$

$$
\begin{aligned}
\text{Then,}\quad g_{I(g\times h)}\big((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_2,\mathfrak{z}_2)\big) &= g_{I(g\times h)}\big(\,\varepsilon_1\bullet\mathfrak{z}_1,\ \varepsilon_2\bullet\mathfrak{z}_2\,\big)\\
&= \max\big\{\,g_{I(g)}\big(\,\varepsilon_1\bullet\mathfrak{z}_1,\ \varepsilon_2\bullet\mathfrak{z}_2\,\big)\,\big\}\\
&= \max\big\{\,\max\{g_{I(g)}(\varepsilon_1\bullet\varepsilon_2\bullet\varepsilon_3),\ g_{I(g)}(\varepsilon_3)\},\\
&\qquad\quad \max\{g_{I(h)}(\mathfrak{z}_1\bullet\mathfrak{z}_2\bullet\mathfrak{z}_3),\ g_{I(h)}(\mathfrak{z}_3)\}\,\big\}\\
&\le \max\big\{\,g_{I(g\times h)}\big((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_2,\mathfrak{z}_2)\bullet(\varepsilon_3,\mathfrak{z}_3)\big),\\
&\qquad g_{I(g\times h)}(\varepsilon_3,\mathfrak{z}_3)\,\big\}.
\end{aligned}
$$

$$
\begin{aligned}
\text{Also,}\quad g_{F(g\times h)}\big((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_2,\mathfrak{z}_2)\big) &= g_{F(g\times h)}\big(\,\varepsilon_1\bullet\mathfrak{z}_1,\ \varepsilon_2\bullet\mathfrak{z}_2\,\big)\\
&= \max\big\{\,g_{F(g)}\big(\,\varepsilon_1\bullet\mathfrak{z}_1,\ \varepsilon_2\bullet\mathfrak{z}_2\,\big)\,\big\}\\
&= \max\big\{\,\max\{g_{F(g)}(\varepsilon_1\bullet\varepsilon_2\bullet\varepsilon_3),\ g_{F(g)}(\varepsilon_3)\},\\
&\qquad\quad \max\{g_{F(h)}(\mathfrak{z}_1\bullet\mathfrak{z}_2\bullet\mathfrak{z}_3),\ g_{F(h)}(\mathfrak{z}_3)\}\,\big\}\\
&\le \max\big\{\,g_{F(g\times h)}\big((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_2,\mathfrak{z}_2)\bullet(\varepsilon_3,\mathfrak{z}_3)\big),\\
&\qquad g_{F(g\times h)}(\varepsilon_3,\mathfrak{z}_3)\,\big\}.
\end{aligned}
$$

Theorem 5.4. Let g × h = $(\mathrm{g}_{T(g\times h)},\ \mathrm{g}_{I(g\times h)},\ \mathrm{g}_{F(g\times h)})$ and i × j = $(\mathrm{g}_{\,T(i\times j)},\ \mathrm{g}_{I(i\times j)},\ \mathrm{g}_{F(i\times j)})$ is a neutrosophic bi-ideal of INK-algebra $\ddot{\ddot{U}}_1$ and $\ddot{\ddot{U}}_2$. Then (g × h) ∩ (i × j) = $(\mathrm{g}_{T(g\times h)\,\cap\,(i\times j)},\ \mathrm{g}_{I(g\times h)\,\cap\,(i\times j)},\ \mathrm{g}_{F(g\times h)\,\cap\,(i\times j)})$

Proof. For any $(\varepsilon_1,\varepsilon_2,\varepsilon_3)$ and $(\mathfrak{z}_1,\mathfrak{z}_2,\mathfrak{z}_3)\in\ddot{\ddot{U}}_1\times\ddot{\ddot{U}}_2$.

Consider $g_{T(g\times h)}(0,0)\ \ge\ \min\big\{\,g_{T(g\times h)}(\varepsilon_1,\mathfrak{z}_1)\,\big\}$ and $g_{T(i\times j)}(0,0)\ \ge\ \min\big\{\,g_{T(i\times j)}(\varepsilon_1,\mathfrak{z}_1)\,\big\}$.
$\big\{\,g_{T(g\times h)}(0,0),\ g_{T(i\times j)}(0,0)\,\big\}\ \ge\ \big\{\,g_{T(g\times h)}(\varepsilon_1,\mathfrak{z}_1),\ g_{T(i\times j)}(\varepsilon_1,\mathfrak{z}_1)\,\big\}$,
$\min\big\{\,g_{T(g\times h)}(0,0),\ g_{T(i\times j)}(0,0)\,\big\}\ \ge\ \min\big\{\,g_{T(i\times j)}(\varepsilon_1,\mathfrak{z}_1),\ g_{T(i\times j)}(\varepsilon_1,\mathfrak{z}_1)\,\big\}$.

$$g_{T\big((g\times h)\cap(i\times j)\big)}(0,0)\ \ge\ g_{T\big((g\times h)\cap(i\times j)\big)}(\varepsilon_1,\mathfrak{z}_1).$$

$g_{I(g\times h)}(0,0)\ \le\ \max\big\{\,g_{I(g\times h)}(\varepsilon_1,\mathfrak{z}_1)\,\big\},\qquad g_{I(i\times j)}(0,0)\ \le\ \max\big\{\,g_{I(i\times j)}(\varepsilon_1,\mathfrak{z}_1)\,\big\}$,
$\big\{\,g_{I(g\times h)}(0,0),\ g_{I(i\times j)}(0,0)\,\big\}\ \le\ \big\{\,g_{I(g\times h)}(\varepsilon_1,\mathfrak{z}_1),\ g_{I(i\times j)}(\varepsilon_1,\mathfrak{z}_1)\,\big\}$,
$\max\big\{\,g_{I(g\times h)}(0,0),\ g_{I(i\times j)}(0,0)\,\big\}\ \le\ \max\big\{\,g_{I(i\times j)}(\varepsilon_1,\mathfrak{z}_1),\ g_{I(i\times j)}(\varepsilon_1,\mathfrak{z}_1)\,\big\}$.

$$g_{F\big((g\times h)\cap(i\times j)\big)}(0,0)\ \le\ g_{F\big((g\times h)\cap(i\times j)\big)}(\varepsilon_1,\mathfrak{z}_1).$$

$g_{F(g\times h)}(0,0)\ \le\ \max\big\{\,g_{F(g\times h)}(\varepsilon_1,\mathfrak{z}_1)\,\big\},\qquad g_{F(i\times j)}(0,0)\ \le\ \max\big\{\,g_{F(i\times j)}(\varepsilon_1,\mathfrak{z}_1)\,\big\}$,
$\big\{\,g_{F(g\times h)}(0,0),\ g_{F(i\times j)}(0,0)\,\big\}\ \le\ \big\{\,g_{F(g\times h)}(\varepsilon_1,\mathfrak{z}_1),\ g_{F(i\times j)}(\varepsilon_1,\mathfrak{z}_1)\,\big\}$,
$\max\big\{\,g_{F(g\times h)}(0,0),\ g_{F(i\times j)}(0,0)\,\big\}\ \le\ \max\big\{\,g_{F(i\times j)}(\varepsilon_1,\mathfrak{z}_1),\ g_{F(i\times j)}(\varepsilon_1,\mathfrak{z}_1)\,\big\}$.

$$g_{F\left((g\times h)\cap(i\times j)\right)}(0,0) \ \leq \ g_{F\left((g\times h)\cap(i\times j)\right)}(\varepsilon_1,\mathfrak{z}_1).$$

Now $(\varepsilon_1,\mathfrak{z}_1,\mathfrak{r}_1)$, $(\varepsilon_2,\mathfrak{z}_2,\mathfrak{r}_2)\in X_1\times X_2$.

Consider

$$g_{T(g\times h)}(\varepsilon_1,\mathfrak{z}_1)=\min\Big\{\, g_{T(g\times h)}\big((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_2,\mathfrak{z}_2)\bullet(\varepsilon_3,\mathfrak{z}_3)\big),\ g_{T(g\times h)}(\varepsilon_3,\mathfrak{z}_3)\,\Big\},$$

$$g_{T(i\times j)}(\varepsilon_1,\mathfrak{z}_1)=\min\Big\{\, g_{T(i\times j)}\big((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_2,\mathfrak{z}_2)\bullet(\varepsilon_3,\mathfrak{z}_3)\big),\ g_{T(i\times j)}(\varepsilon_3,\mathfrak{z}_3)\,\Big\}.$$

$$\begin{aligned}
&g_{T(g\times h)}(\varepsilon_1,\mathfrak{z}_1),\ g_{T(i\times j)}(\varepsilon_1,\mathfrak{z}_1)\\
&\quad\geq \min\Big\{\min\{\, g_{T(g\times h)}((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_2,\mathfrak{z}_2)\bullet(\varepsilon_3,\mathfrak{z}_3)),\ g_{T(g\times h)}(\varepsilon_3,\mathfrak{z}_3)\,\},\\
&\qquad\qquad \min\{\, g_{T(i\times j)}((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_2,\mathfrak{z}_2)\bullet(\varepsilon_3,\mathfrak{z}_3)),\ g_{T(i\times j)}(\varepsilon_3,\mathfrak{z}_3)\,\}\Big\}\\
&\quad\geq \min\Big\{\min\{\, g_{T(g\times h)}((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_2,\mathfrak{z}_2)\bullet(\varepsilon_3,\mathfrak{z}_3)),\ g_{T(i\times j)}((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_2,\mathfrak{z}_2)\bullet(\varepsilon_3,\mathfrak{z}_3))\,\},\\
&\qquad\qquad \min\{\, g_{T(g\times h)}(\varepsilon_3,\mathfrak{z}_3),\ g_{T(i\times j)}(\varepsilon_3,\mathfrak{z}_3)\,\}\Big\}.
\end{aligned}$$

$$g_{T\left((g\times h)\cap(i\times j)\right)}(\varepsilon_1,\mathfrak{z}_1) \ \geq \ \Big\{\, g_{T\left((g\times h)\cap(i\times j)\right)}\big((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_2,\mathfrak{z}_2)\bullet(\varepsilon_3,\mathfrak{z}_3)\big),\ g_{T\left((g\times h)\cap(i\times j)\right)}(\varepsilon_3,\mathfrak{z}_3)\,\Big\}.$$

Also for

$$g_{I(g\times h)}(\varepsilon_1,\mathfrak{z}_1)=\max\Big\{\, g_{I(g\times h)}\big((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_2,\mathfrak{z}_2)\bullet(\varepsilon_3,\mathfrak{z}_3)\big),\ g_{I(g\times h)}(\varepsilon_3,\mathfrak{z}_3)\,\Big\},$$

$$g_{I(i\times j)}(\varepsilon_1,\mathfrak{z}_1)=\max\Big\{\, g_{I(i\times j)}\big((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_2,\mathfrak{z}_2)\bullet(\varepsilon_3,\mathfrak{z}_3)\big),\ g_{I(i\times j)}(\varepsilon_3,\mathfrak{z}_3)\,\Big\},$$

$$\begin{aligned}
&g_{I(g\times h)}(\varepsilon_1,\mathfrak{z}_1),\ g_{I(i\times j)}(\varepsilon_1,\mathfrak{z}_1)\\
&\quad\leq \max\Big\{\max\{\, g_{I(g\times h)}((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_2,\mathfrak{z}_2)\bullet(\varepsilon_3,\mathfrak{z}_3)),\ g_{I(g\times h)}(\varepsilon_3,\mathfrak{z}_3)\,\},\\
&\qquad\qquad \max\{\, g_{I(i\times j)}((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_2,\mathfrak{z}_2)\bullet(\varepsilon_3,\mathfrak{z}_3)),\ g_{I(i\times j)}(\varepsilon_3,\mathfrak{z}_3)\,\}\Big\}\\
&\quad\leq \max\Big\{\max\{\, g_{I(g\times h)}((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_2,\mathfrak{z}_2)\bullet(\varepsilon_3,\mathfrak{z}_3)),\ g_{I(i\times j)}((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_2,\mathfrak{z}_2)\bullet(\varepsilon_3,\mathfrak{z}_3))\,\},\\
&\qquad\qquad \max\{\, g_{I(g\times h)}(\varepsilon_3,\mathfrak{z}_3),\ g_{I(i\times j)}(\varepsilon_3,\mathfrak{z}_3)\,\}\Big\}.
\end{aligned}$$

$$g_{I\left((g\times h)\cap(i\times j)\right)}(\varepsilon_1,\mathfrak{z}_1) \ \leq \ \Big\{\, g_{I\left((g\times h)\cap(i\times j)\right)}\big((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_2,\mathfrak{z}_2)\bullet(\varepsilon_3,\mathfrak{z}_3)\big),\ g_{I\left((g\times h)\cap(i\times j)\right)}(\varepsilon_3,\mathfrak{z}_3)\,\Big\}.$$

Similarly for

$$g_{F(g\times h)}(\varepsilon_1,\mathfrak{z}_1)=\max\Big\{\, g_{F(g\times h)}\big((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_2,\mathfrak{z}_2)\bullet(\varepsilon_3,\mathfrak{z}_3)\big),\ g_{F(g\times h)}(\varepsilon_3,\mathfrak{z}_3)\,\Big\},$$

$$g_{F(i\times j)}(\varepsilon_1,\mathfrak{z}_1)=\max\Big\{\, g_{F(i\times j)}\big((\varepsilon_1,\mathfrak{z}_1)\bullet(\varepsilon_2,\mathfrak{z}_2)\bullet(\varepsilon_3,\mathfrak{z}_3)\big),\ g_{F(i\times j)}(\varepsilon_3,\mathfrak{z}_3)\,\Big\},$$

$$
\begin{aligned}
&g_{F(g\times h)}(\varepsilon_1, \text{з}_1),\ g_{F(i\times j)}(\varepsilon_1, \text{з}_1)\\
&\quad\le \max\Big\{\max\{\, g_{F(g\times h)}((\varepsilon_1, \text{з}_1)\bullet(\varepsilon_2, \text{з}_2)\bullet(\varepsilon_3, \text{з}_3)),\ g_{F(g\times h)}(\varepsilon_3, \text{з}_3)\,\},\\
&\qquad\qquad \max\{\, g_{F(i\times j)}((\varepsilon_1, \text{з}_1)\bullet(\varepsilon_2, \text{з}_2)\bullet(\varepsilon_3, \text{з}_3)),\ g_{F(i\times j)}(\varepsilon_3, \text{з}_3)\,\}\Big\}\\
&\quad\le \max\Big\{\max\{\, g_{F(g\times h)}((\varepsilon_1, \text{з}_1)\bullet(\varepsilon_2, \text{з}_2)\bullet(\varepsilon_3, \text{з}_3)),\ g_{F(i\times j)}((\varepsilon_1, \text{з}_1)\bullet(\varepsilon_2, \text{з}_2)\bullet(\varepsilon_3, \text{з}_3))\,\},\\
&\qquad\qquad \max\{\, g_{F(g\times h)}(\varepsilon_3, \text{з}_3),\ g_{F(i\times j)}(\varepsilon_3, \text{з}_3)\,\}\Big\}.
\end{aligned}
$$

$$
g_{F\left((g\times h)\cap(i\times j)\right)}(\varepsilon_1, \text{з}_1) \le \Big\{ g_{F\left((g\times h)\cap(i\times j)\right)}\big((\varepsilon_1, \text{з}_1)\bullet(\varepsilon_2, \text{з}_2)\bullet(\varepsilon_3, \text{з}_3)\big),\ g_{F\left((g\times h)\cap(i\times j)\right)}(\varepsilon_3, \text{з}_3) \Big\}.
$$

This completes the required formulation for $(g\times h)\cap(i\times j) = \big(g_{T(g\times h)}\cap g_{T(i\times j)},\ g_{I(g\times h)}\cap g_{I(i\times j)},\ g_{F(g\times h)}\cap g_{F(i\times j)}\big)$.

6. Application of Bi-ideal in INK-algebra

Flowchart for Neutrosophic Bi-ideal in INK-algebra

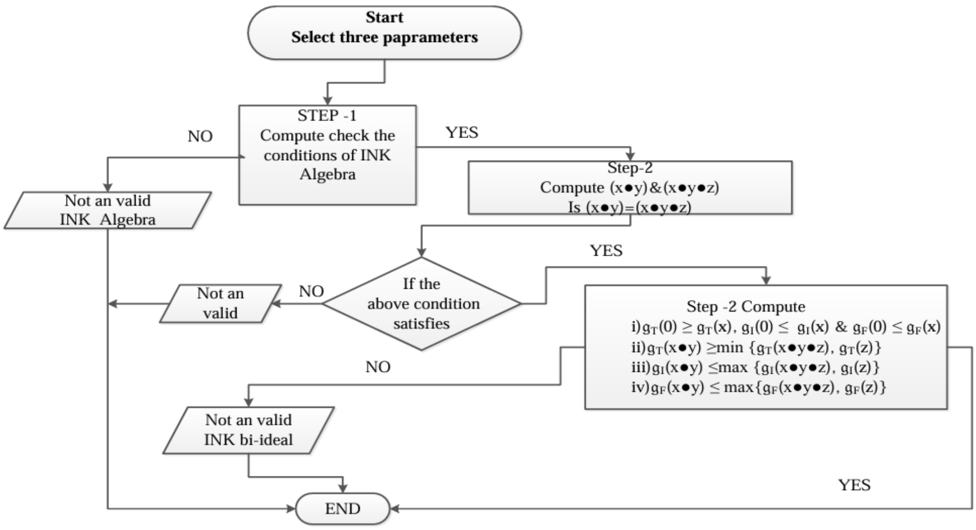

Figure 1: Flowchart for Neutrosophic Bi-ideal in INK-algebra

7. Selection of Research Guide/Supervisor by using Neutrosophic Bi-ideals in INK-algebra

In academic research, selecting the right guide is necessary a critical decision that can be effect and shape scholars entire research experience. The choice or consideration of selection of guide depends on multiple parameters. To model this selection process logically, we apply the concept of neutrosophic Bi-ideal in INK-algebra, which allows us to evaluate whether removing a less important factor still results in a valid and trustworthy decision.

We define a set of elements in our INK-algebra $\ddot{\ddot{U}}$={0, A, G, C} where each and every parameter represents guide's selection. Here, A= Alignment / P = Proposal idea, G=Guide's Academic and research experience, C=Availability of communication with guide.

The element "0" represents an ideal guide selection. In this context, the term $\varepsilon_1 \bullet \mathfrak{z}_1$ interpreted as the how much mismatch is still there when choosing a guide based on those two factors in that order. This is taken for a better experience, a mismatch in any of them may not invalidate the overall decision if the core match is strong.

To evaluate this system, we construct a cayley table of INK-algebra such as associativity –like behaviour, and conditions of INK-algebras also we have to define membership values for each parameter.

Table: 1 Cayley Table on $\ddot{\ddot{U}}$

$\bullet$	0	A	G	C
0	0	A	G	C
A	A	0	C	G
G	G	C	0	A
C	C	G	A	0

Table: 2 Neutrosophic Membership Degrees

$\bullet$	0	A	G	C
g_T	1.0	0.9	0.9	0.6
g_I	0.0	0.2	0.5	0.2
g_F	0.0	0.3	0.1	0.4

{If $\varepsilon_1 \bullet \mathfrak{z}_1 \bullet \mathfrak{r}_1 = \varepsilon_1 \bullet \mathfrak{z}_1$, and one factor (eg = $\mathfrak{r}_1$) is trusted, then does the core result $\varepsilon_1 \bullet \mathfrak{z}_1$ hold up on its own.}

If the Researcher consider the criteria like Guide academic & research experience, followed by Alignment/Proposal idea and then Communication of avialbility of Guide.

Let's break down

(i) $g_T(0) \geq g_T(G) \Rightarrow 1.0 > 0.9$,

(ii) $g_I(0) \leq g_I(G) \Rightarrow 0.0 < 0.5$,

(iii) $g_F(0) \leq g_F(G) \Rightarrow 0.0 < 0.1$.

(iv) $g_T(G \bullet A) \geq \min\{g_T(G \bullet A \bullet C),\ g_T(C)\}$,
$g_T(G) \geq \min\{g_T(G),\ g_T(C)\}$
$0.9 \geq \min\{0.9,\ 0.8\}$
$0.9 > 0.8$.

(v) $g_I(G \bullet A) \leq \max\{g_I(G \bullet A \bullet C),\ g_I(C)\}$,
$g_I(G) \leq \max\{g_I(G),\ g_T(C)\}$
$0.5 \leq \max\{0.5,\ 0.1\}$
$0.5 = 0.5$.

(vi) $g_F(G \bullet A) \leq \max\{g_F(G \bullet A \bullet C),\ g_F(C)\}$,
$g_F(G) \leq \max\{g_F(G),\ g_F(C)\}$
$0.1 \leq \max\{0.1,\ 0.2\}$
$0.1 < 0.2$.

Based on the result as long as starting with guide is experienced. Then, it is sufficient to make reliable descion. This allows us to ignore whether Alignment(A) is perfect or Communcation(C) is ideal.

8. Conclusion

Our article investigates into the thought of neutrosophic bi-ideals within the framework of INK-algebras. We begin by starting neutrosophic bi-ideals of INK-algebras. We then explore their properties, including how intersection of neutrosophic bi-ideals works, and how containment relationships define them union of neutrosophic bi-ideals by one containing the other., Also, we investigate homomorphisms and epimorphisms of neutrosophic bi-ideals. Subsequently, we discuss the direct product of neutrosophic sets and demonstrate that the intersection of direct products of neutrosophic bi-ideals remains a neutrosophic bi-ideal. Finally, we look over the application of bi-ideal in INK-algebra by considering parameters of selecting guide by a research scholar by using neutrosophic bi-ideals in INK-algebra.

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