EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
2025, Vol. 18, Issue 3, Article Number 6407
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

Various Types of Supra ϵ-Separation Axioms and Relationships

M. Aldawood [1], Alaa M. Abd El-latif[2], Khaled A. Aldwoah[3], A. A. Azzam[1,4], Abdelhalim Hasnaoui[2,*], M. I. Elashiry[2], Husham M. Attaalfadeel[2], Enas H. Elkordy[1,5]

[1] *Department of Mathematics, Faculty of Science and Humanities, Prince Sattam Bin Abdulaziz University, Alkharj 11942, Saudi Arabia*
[2] *Department of Mathematics, College of Science, Northern Border University, Arar 91431, Saudi Arabia*
[3] *Department of Mathematics, Faculty of Science, Islamic University of Madinah, Medinah, Saudi Arabia*
[4] *Department of Mathematics, Faculty of Science, New Valley University, Elkharga 72511, Egypt*
[5] *Mathematics and Computer Science Department, Faculty of Science, Beni-Suef University, Beni Suef, Egypt*

Abstract. This manuscript presents a new weaker version of supra septarian axioms based on supra ϵ-open sets, along with its essential features, which are called supra-ϵ-T_j-space, $j = 0, 1, 2$, in the framework of supra topological spaces (or STSs). We give comprehensive explanations of each type of them, backed up by several examples and counterexamples that highlight the significance of our original approaches. We also provide a diagram that outlines these relationships. Additionally, we present the supra ϵ-symmetric property and supra difference property and study their effects on these version of supra-ϵ-septarian axioms. In especial, we show that the two concepts of supra-ϵ-T_0-space and supra-ϵ-T_1-space are the same for any STS that fulfills supra ϵ-symmetric property. Finally, we study the supra topological and supra hereditary properties for each of the previously discussed approaches. In particular, we show that the property of being a supra-ϵ-T_j-space, where $j = 0, 1, 2$, is a supra-hereditary (topological) property.

2020 Mathematics Subject Classifications: 54A05, 54C10, 54D10

Key Words and Phrases: Supra ϵ-separation axioms, Supra-ϵ-Hausdorff-space, Supra ϵ-symmetric property

---

*Corresponding author.
DOI: https://doi.org/10.29020/nybg.ejpam.v18i3.6407

Email addresses: m.aldawood@psau.edu.sa (M. Aldawood), alaa.ali@nbu.edu.sa, alaa_8560@yahoo.com (A. M. Abd El-latif), aldwoah@yahoo.com (K. A. Aldwoah), aa.azzam@psau.edu.sa (A. A. Azzam), Abdllhalim.hasanawa@nbu.edu.sa (A. Hasnaoui), Mustafa.elashiry@nbu.edu.sa (M. I. Elashiry), Husham.Alhassan@nbu.edu.sa (H. M. Attaalfadeel), e.elkordy@psau.edu.sa (E. H. Elkordy)

1. Introduction

The investigation of different kinds of generalized open, supra open, and soft open sets and their fundamental features has played a significant role in topological, supra topological, and soft topological research over the past few decades.

In 1963, Levine [1] first proposed semi-open sets (continuity). After two years, Njasta [2] presented the concept of α-open sets. In 1982, Mashhour et al. [3] provided the concept of pre open sets (pre continuity). The notion of β-open sets (β-continuity) was presented by Abd-El-Monsef et al. [4] in 1983. The definition of b-open sets was studied in detail in [5, 6] in 1966. Based on [7], Piotrowski [8] presented the notion of somewhat open sets (continuity). The approach of somewhere dense sets (or sd-sets) was proposed in [9, 10]. Additional facets of this idea were examined in [11]. Alqahtani introduced the approach of F-open [12]. The $\mathcal{N}$-open sets approach was presented by Alqahtani and Abd El-latif [13] in 2024, and it is generalized almost all of the earlier concepts. Alghamdi et al. provided new types of operators in context of primal topological spaces [14].

The concept of supra open sets, which take into account the fundamental components of supra topology (or STS), was introduced by Mashhour et al. [15]. Among the basic topological concepts they developed were the separation axioms, continuity, and closure (interior) operators. Along with their key characteristics, the notions of supra semi- [16] (R- [17], β- [18], b- [19], pre- [20], and α [21]) open sets have been presented. Abd El-latif et al. [22] proposed the concept of supra ϵ-open sets in STSs. He and his coauthors [23] used this concept to investigate new forms of supra continuity.

The field of broadly applicable soft open sets [24, 25], soft semi-open sets [26, 27], various kinds of soft continuity [28, 29], soft sd-sets [30, 31], and nearly soft β-open sets [32] has produced a variety of soft open sets and soft continuity. More studies on soft continuity were conducted later [33, 34]. In [35], the idea of the soft ideal was first introduced. After thet, Fatouh et al. [36] used soft semi-open sets to generalize this idea. Soft compactness [37], soft connectedness [38], soft generalized open sets [39], soft open sets via soft ideals [40, 41], soft separation axioms [42], generalized soft rough sets [43, 44], and congruence representations via soft ideals [45] are some of the topological characteristics that this concept is then used to generalize. Certain applications of soft δ-closed sets [46] and certain lower soft separation axioms [47] were recently introduced.

The definition of supra soft topological space was introduced by El-Sheikh et al. [48]. Later research has examined several kinds of generalized supra soft operators using supra soft-b-open sets [49], supra generalized closed soft sets in terms of soft ideals [50, 51], supra soft sw-open sets [52], supra soft δ_i-open sets [53, 54], and soft separation axioms [55]. Recently, Abd El-latif et al. used the notion of supra soft sd-sets [56, 57] to present novel kinds of soft connectedness [58] and several types of compactness and connectedness [59, 60]. Alqahtani et al. presented the notion of soft nodec spaces [61, 62].

Our aim of this work is to provide new types of generalized separation axioms. In special, we present three new types of separation axioms inspired by supra ϵ-open sets named supra-ϵ-T_0-space, supra-ϵ-T_1-space, and supra-ϵ-T_2-space. We give in-depth explanations of each of them, backed up by several examples and counterexamples that highlight the significance of our original ideas. Additionally, as shown in figure 1, we provide a diagram that summarizes their links and connections to earlier research.

$$\begin{array}{ccccc} \text{supra-}T_2\text{-space} & \Longrightarrow & \text{supra-}T_1\text{-space} & \Longrightarrow & \text{supra-}T_0\text{-space} \\ \Downarrow & & \Downarrow & & \Downarrow \\ \text{supra-}\epsilon\text{-}T_2\text{-space} & \Longrightarrow & \text{supra-}\epsilon\text{-}T_1\text{-space} & \Longrightarrow & \text{supra-}\epsilon\text{-}T_0\text{-space} \end{array}$$

Diagram 1. The connections among various types of separation axioms in the context of STSs inspired by supra ϵ-open sets

Finally, we assess the supra topological and supra hereditary properties for each of the concepts. In particular, we show that the property of being a supra-ϵ-T_j-space, where $j = 0, 1, 2$, is a supra-hereditary (topological) property.

2. Preliminaries and background

Definition 1. *[15] Supra topology (or STS) on λ is the family $\vartheta \subseteq 2^{\lambda}$ which contains λ and $\emptyset$ and closed under arbitrary union. Additionally, H and H^c are referred to as supra open and supra closed sets, respectively, if $H \in \vartheta$. Also, $SO(\lambda)$ will also be used to indicate the class of all supra open sets. Furthermore, ϑ is referred to as an associated STS with σ if $\sigma \subset \vartheta$ for a given topology σ.*

Definition 2. *[15] The $int^s(H)$ ($cl^s(H)$, $fr^s(H)$) will indicate the supra interior (closure, boundary) for a subset H of an STS (λ, ϑ), where*

$$int^s(H) = \cup\{C : C \in \vartheta \text{ and } C \subseteq H\},\ cl^s(H) = \cap\{D : D \in \vartheta^c \text{ and } H \subseteq D\} \text{ and }$$
$$fr^s(H) = cl^s(H) \backslash int^s(H).$$

Definition 3. *[17] Let E be a subset of an STS (λ, ϑ). If $int^s(cl^s(E)) \neq \emptyset$, then $E \in SRO(\lambda)$. Also, if $int^s(cl^s(E)) = \emptyset$, then $E \in SND(\lambda)$.*

Definition 4. *[22] Regarding the subset S of an STS (λ, ϑ), the family*

$$\vartheta_S = \{S \cap J : J \in \vartheta\}$$

defines an STS on S, which is referred to as a supra subspace of (λ, ϑ).

Definition 5. *[22] A subset E of an STS (λ, ϑ) is referred to as supra ϵ-open set if either $E = \emptyset$ or*

$$E \subseteq \begin{cases} fr^s(E) \cup int^s(cl^s(E)), & E \in SRO(\lambda), \\ fr^s(E), & E \in SND(\lambda) \text{ and } fr^s(E) \text{ is } infinite. \end{cases}$$

Additionally, E^c is referred to as supra ϵ-closed-set. Furthermore, all supra ϵ-open (respectively, supra ϵ-closed) sets will be classified by $SO_{\epsilon}(\lambda)$ (respectively, $SC_{\epsilon}(\lambda)$).

Definition 6. *[22] The $int^s_{\epsilon}(S)$ ($cl^s_{\epsilon}(S)$) will indicate the supra ϵ-interior (closure) of S for a subset S of an STS (λ, ϑ), where*

$$int^s_\epsilon(S) = \cup\{J : J \in SO_\epsilon(\lambda) \text{ and } J \subseteq S\} \text{ and } cl^s_\epsilon(S) = \cap\{N : N \in SC_\epsilon(\lambda) \text{ and } S \subseteq N\}$$

Theorem 1. *[22] If we consider a subset T of an STS (λ, ϑ) with $\sigma \subset \vartheta$, we have that*

(1) $cl^s_\epsilon(T^c) = [int^s_\epsilon(T)]^c$.

(2) $int^s_\epsilon(T^c) = [cl^s_\epsilon(T)]^c$.

(3) $int(T) \subseteq int^s(T) \subseteq int^s_\epsilon(T)$, *where $int(T)$ refers the interior of T w.r.t σ.*

(4) $cl^s_\epsilon(T) \subseteq cl^s(T) \subseteq cl(T)$, *where $cl(T)$ refers the closure of T w.r.t σ.*

Definition 7. *[22] Let T be a subset of an STS (λ, ϑ) with an arbitrary point $s \in \lambda$. If each supra ϵ-open set J_s containing s, we have that*

$$[T\backslash\{s\}] \cap J_s \neq \emptyset,$$

then s is referred to as a supra ϵ-accumulation point of T. The notation $acc_\epsilon(T)$ will represent the set of all supra ϵ-accumulation points of T.

3. Separation axioms inspired by supra ϵ-open sets and relationships

In this section, we present three new types of separation axioms inspired by supra ϵ-open sets named supra-ϵ-T_0-space, supra-ϵ-T_1-space, and supra-ϵ-Hausdorff-space. We provide thorough descriptions of each of them. Specifically, we explore sufficient conditions for several analogous linkages between them and generally illustrate their key characteristics. Moreover, we propose a diagram [see diagram 1] that summarizes their relationships. Furthermore, we introduce the supra ϵ-symmetric property and demonstrate that, for any STS that satisfies it, the two approaches of supra-ϵ-T_0-space and supra-ϵ-T_1-space are identical.

Definition 8. *An STS (λ, ϑ) is said to be*

(1) *Supra-ϵ-T_0-space if for each two distinct points there is a supra-ϵ-open set including one but excluding the other.*

(2) *Supra-ϵ-T_1-space if for each two distinct points $\nu_1, \nu_2 \in \lambda$, then there are two supra-ϵ-open subsets μ_1 and μ_2 of λ, such that $\nu_1 \in \mu_1$, $\nu_2 \notin \mu_1$, and $\nu_1 \notin \mu_2$, $\nu_2 \in \mu_2$.*

(3) *Supra-ϵ-T_2-space "supra-ϵ-Hausdorff space" if for each two distinct points $\nu_1, \nu_2 \in \lambda$, then there are two disjoint supra-ϵ-open subsets μ_1 and μ_2 of λ, such that $\nu_1 \in \mu_1$ and $\nu_2 \in \mu_2$.*

Theorem 2. **(1)** *Any supra-ϵ-T_j-space is supra-ϵ-T_{j-1}, $j = 1, 2$.*

(2) *Any supra-T_j-space is supra-ϵ-T_j, $j = 1, 2$.*

Proof. It is clear from Definition 8 and form the fact that every supra open set is supra ϵ-open.

Remark 1. *The converse of Theorem 2 is not hold as the upcoming examples will demonstrate.*

Example 1. **(1)** *Let $\vartheta = \{\lambda, \emptyset, \{b\}\}$ be an STS on $\lambda = \{v, b, n\}$. Then we have that $SO_\epsilon(\lambda) = \{\lambda, \emptyset, \{v, b\}, \{b, n\}, \{b\}\}$. It easy to check that, λ is supra-ϵ-T_0-space, however λ is not supra-ϵ-T_1, since $v \neq n \in \lambda$, however there are not two supra-ϵ-open subsets of λ separate them.*

(2) *Let $\vartheta = \{\lambda, \emptyset, \{a, s\}, \{a, s, d\}, \{a, d\}, \{s, d\}\}$ be an STS on $\lambda = \{a, s, d, f\}$. Then we have that $SO_\epsilon(\lambda) = \{\lambda, \emptyset, \{a, s\}, \{a, d\}, \{s, d\}, \{a, s, d\}, \{a, s, f\}, \{a, d, f\}, \{s, d, f\}\}$. It follows that, λ is supra-ϵ-T_1-space, however λ is not supra-ϵ-T_2, since $d \neq f \in \lambda$, however there are not two disjoint supra-ϵ-open subsets of λ separates them. .*

(3) *In (1), we have that λ is supra-ϵ-T_0-space, however λ is not supra-T_0, since $v \neq n \in \lambda$, however there are not two supra open subsets of λ separate them.*

(4) *Let $\vartheta = \{\lambda, \emptyset, \{a, s\}, \{d, f\}, \{a, d\}, \{s, f\}, \{s, d\}, \{a, s, d\}, \{a, s, f\}, \{a, d, f\}\}$ be an STS on $\lambda = \{a, s, d, f\}$. Then we have that $SO_\epsilon(\lambda) = \vartheta$. It follows that, λ is supra-ϵ-T_1-space, however λ is not supra-T_1, since $\{a\} \notin \vartheta^c$.*

Proposition 1. *For an STS (λ, ϑ), the following implications are held, which are not reversible, depending on the previously mentioned results.*

$$\begin{array}{ccccc} \text{supra-}T_2\text{-space} & \Longrightarrow & \text{supra-}T_1\text{-space} & \Longrightarrow & \text{supra-}T_0\text{-space} \\ \Downarrow & & \Downarrow & & \Downarrow \\ \text{supra-}\epsilon\text{-}T_2\text{-space} & \Longrightarrow & \text{supra-}\epsilon\text{-}T_1\text{-space} & \Longrightarrow & \text{supra-}\epsilon\text{-}T_0\text{-space} \end{array}$$

Diagram 1. *The connections among various types of separation axioms in the context of STSs inspired by supra ϵ-open sets*

Theorem 3. *Any STS (λ, ϑ) is supra-ϵ-T_0-space.*

Proof. Let λ be an STS and let $\nu_1 \neq \nu_2$ in λ. This follows that, either $\lambda \setminus \{\nu_1\} \in SRO(\lambda)$ or $\lambda \setminus \{\nu_1\} \in SND(\lambda)$. If $\lambda \setminus \{\nu_1\} \in SND(\lambda)$, then $\overline{\lambda \setminus \{\nu_1\}} \neq \lambda$, and so $\lambda \setminus \{\nu_1\} \in \vartheta^c$; and hence $\{\nu_1\} \in \vartheta$. Therefore, $\{\nu_1\}$ is a supra ϵ-open set containing ν_1, but not ν_2.
In addition, if $\lambda \setminus \{\nu_1\} \in SRO(\lambda)$, then $\lambda \setminus \{\nu_1\} \subseteq \overline{\lambda \setminus \{\nu_1\}}^{\circ} \cup fr^s(\lambda \setminus \{\nu_1\}) = \overline{\lambda \setminus \{\nu_1\}}$ is a supra ϵ-open set containing ν_2, but not ν_1. Consequently, λ is supra-ϵ-T_0-space.

Lemma 1. *Any infinite subset of an STS (λ, ϑ) is supra-ϵ-open.*

Proof. Let H be any infinite subset of an STS (λ, ϑ). Then, H is either $H \in SRO(\lambda)$ or $H \in SND(\lambda)$. If $H \in SND(\lambda)$, then $H \subseteq \overline{H}^{\circ} \cup fr^s(H) = fr^s(H)$, $fr^s(H)$ is infinite. Hence, $H \in SO_\epsilon(\lambda)$. Also, if $H \in SRO(\lambda)$, then $H \subseteq \overline{H}^{\circ} \cup fr^s(H) = \overline{H} \in SO_\epsilon(\lambda)$.

Theorem 4. *Any supra-T_1-space is supra-ϵ-T_2.*

Proof. Let (λ, ϑ) be a supra-T_1-space and let $\nu_1 \neq \nu_2$ in v. If λ is finite, then λ is the discrete space, hence λ is supra-ϵ-T_2-space. Now, assume that λ is infinite. Then, there exist two infinite disjoint K and H subsets of λ, containing ν_1 and ν_2, respectively. By Lemma 1, K and H are supra-ϵ-open sets. Therefore, λ is supra-ϵ-T_2-space.

Corollary 1. *Any infinite STS (λ, ϑ) is supra-ϵ-T_2.*

Proof. It follows from Lemma 1.

Definition 9. *[63] If $H \in \psi$ implies that $H \backslash \{\nu\} \in \psi$ for $\nu \in \lambda$, then a subfamily $\psi \subseteq 2^\lambda$ in a nonempty set λ is considered to have the difference property.*

Proposition 2. *Any STS (λ, ϑ) has the difference property for the category $SO_\epsilon(\lambda)$ is supra-ϵ-T_1-space.*

Proof. Let $\nu_1 \neq \nu_2 \in \lambda$. Given the difference property for $\lambda \in SO_\epsilon(\lambda)$, we have that $\lambda \backslash \{\nu_1\} \in SO_\epsilon(\lambda)$ and $\lambda \backslash \{\nu_2\} \in SO_\epsilon(\lambda)$ which separate ν_1 and ν_2. Consequently, λ is supra-ϵ-T_1-space.

Theorem 5. *For any STS (λ, ϑ), the following are equivalent:*

(1) *λ is supra-ϵ-T_0-space;*

(2) *For each $\nu_1 \neq \nu_2 \in \lambda$, $cl^s_\epsilon(\{\nu_1\}) \neq cl^s_\epsilon(\{\nu_2\})$;*

(3) *For each $\nu \in \lambda$, $acc_\epsilon(\{\nu\}) = \cup\{G : G \in SC_\epsilon(\lambda)\}$.*

Proof.

(1) $\Rightarrow$ (2) Let $\nu_1 \neq \nu_2 \in \lambda$. Given (1), there is a supra-ϵ-open D set including one point (say ν_1) but excluding the other. This follows that, $\nu_1 \in D$ and $D \cap \{\nu_2\} = \emptyset$. Hence, $\nu_1 \notin cl^s_\epsilon(\{\nu_2\})$, however $\nu_1 \in cl^s_\epsilon(\{\nu_1\})$. Thus, $cl^s_\epsilon(\{\nu_1\}) \neq cl^s_\epsilon(\{\nu_2\})$.

(2) $\Rightarrow$ (3) Let $\omega \in acc_\epsilon(\{\nu\})$, then $\omega \neq \nu$ and $\omega \in acc_\epsilon(\{\nu\}) \cup \{\nu\} = cl^s_\epsilon(\{\nu\})$. Hence, $\omega \in cl^s_\epsilon\{\omega\} \subseteq cl^s_\epsilon(\{\nu\}) = acc_\epsilon(\{\nu\}) \cup \{\nu\}$. Since $\omega \notin \{\nu\}$, $\omega \in acc_\epsilon(\{\nu\})$. Therefore, $\omega \in cl^s_\epsilon\{\omega\} \subseteq acc_\epsilon(\{\nu\})$, and consequently $acc_\epsilon(\{\nu\}) = \cup\{cl^s_\epsilon\{\omega\} : \omega \in acc_\epsilon(\{\nu\})\}$.

(3) $\Rightarrow$ (1) Let $\nu_1 \neq \nu_2 \in \lambda$. Then, either $\nu_2 \in acc_\epsilon(\{\nu_1\})$ or $\nu_2 \notin acc_\epsilon(\{\nu_1\})$. If $\nu_2 \in acc_\epsilon(\{\nu_1\})$, then there is $H \in SC_\epsilon(\lambda)$ such that $\nu_2 \in H \subseteq acc_\epsilon(\{\nu_1\})$. Since $\nu_1 \notin acc_\epsilon(\{\nu_1\}$, $\nu_1 \notin H$, and so $\nu_1 \in H^c$ and $\nu_2 \notin H^c$, $H^c \in SO_\epsilon(\lambda)$. Thus, λ is supra-ϵ-T_0-space.
Additionally, if $\nu_2 \notin acc_\epsilon(\{\nu_1\})$, then there is $K \in SC_\epsilon(\lambda)$ such that $\nu_2 \in K$ and $\nu_1 \notin K$. Thus, λ is supra-ϵ-T_0-space.

Definition 10. *A subset S of an STS (λ, ϑ) is called supra ϵ-dense if $cl^s_\epsilon(S) = \lambda$.*

Corollary 2. *If (λ, ϑ) is a supra-ϵ-T_0-space, then there is at most a supra ϵ-dense singleton set in λ.*

Proof. It is follows from Theorem 5.

Theorem 6. *For any STS (λ, ϑ), the following are equivalent:*

(1) λ *is supra-ϵ-T_1-space;*

(2) *For each $\nu \in \lambda$, $\{\nu\} \in SC_\epsilon(\lambda)$;*

(3) $\cap\{H : H \in SO_\epsilon(\lambda)$ *and* $C \subseteq H\} = C$;

(4) *For each $\nu \in \lambda$, $acc_\epsilon(\{\nu\}) = \emptyset$.*

Proof.

(1) $\Rightarrow$ (2) Let $\nu \in \lambda$. We prove that $\{\nu\}^c \in SO_\epsilon(\lambda)$, so let $\omega \in \{\nu\}^c$. Then, $\omega \neq \nu$. Given (1), there is $H_\omega \in SO_\epsilon(\lambda)$ such that $\omega \in H_\omega$ and $\nu \notin H_\omega$. Hence, $\omega \in H_\omega \subseteq \{\nu\}^c$, and consequently $\{\nu\}^c \in SO_\epsilon(\lambda)$. Therefore, we get the desired outcome.

(2) $\Rightarrow$ (3) Let $K \subseteq \lambda$. Given (2), for all $\nu \in K^c$ we have that $\{\nu\} \in SC_\epsilon(\lambda)$. Then, $\{\nu\}^c \in SO_\epsilon(\lambda)$. Hence, $K \subseteq \{H : H \in SO_\epsilon(\lambda) \textit{ and } K \subseteq H\} \subseteq \{\{\nu\}^c : \nu \in K^c\} \subseteq K$. Therefore, $K = \{H : H \in SO_\epsilon(\lambda) \textit{ and } K \subseteq H\}$.

(3) $\Rightarrow$ (4) Assume the contrary that $acc_\epsilon(\{\nu\}) \neq \emptyset$ for some $\nu \in \lambda$, then there is $\omega \neq \nu$ such that $\omega \in acc_\epsilon(\{\nu\})$. Hence, for every supra ϵ-open set G_ω containing ω we have that

$$[G_\omega \backslash \{\omega\}] \cap \{\nu\} \neq \emptyset \text{ and so}$$

$\nu \in G_\omega \backslash \{\omega\}$. This means that, every supra ϵ-open set G_ω containing ω also contains ν. Therefore, $\cap\{G_\omega : G_\omega \in SO_\epsilon(\lambda) \textit{ and } \{\omega\} \subseteq G_\omega\} \neq \{\omega\}$, which contradicts (3). Thus, for each $\nu \in \lambda$, $acc_\epsilon(\{\nu\}) = \emptyset$.

(4) $\Rightarrow$ (1) Let $\nu_1 \neq \nu_2 \in \lambda$. Given (4), $acc_\epsilon(\{\nu_1\}) = \emptyset$ and $acc_\epsilon(\{\nu_2\}) = \emptyset$ which follows $cl_\epsilon(\{\nu_1\}) = \{\nu_1\} \cup acc_\epsilon(\{\nu_1\}) = \{\nu_1\}$ and $cl_\epsilon(\{\nu_2\}) = \{\nu_2\} \cup acc_\epsilon(\{\nu_2\}) = \{\nu_2\}$. Hence, $\{\nu_1\}^c$ *and* $\{\nu_2\}^c \in SO_\epsilon(\lambda)$ which separate ν_1 and ν_2. Consequently, λ is supra-ϵ-T_1-space.

Definition 11. *The space (λ, ϑ) is referred to as supra ϵ-symmetric if $\nu_1 \in cl_\epsilon(\{\nu_2\})$ demonstrates that $\nu_2 \in cl_\epsilon(\{\nu_1\})$ for $\nu_1 \neq \nu_2 \in \lambda$.*

Theorem 7. *Every supra ϵ-symmetric and supra-ϵ-T_0-space (λ, ϑ) is supra-ϵ-T_1.*

Proof. Let $\nu_1 \neq \nu_2 \in \lambda$. Since λ is supra-ϵ-T_0-space, there is a supra-ϵ-open set H including one point (say ν_1) but excluding the other and so $\nu_1 \notin cl_\epsilon(\{\nu_2\})$. Given λ is supra ϵ-symmetric, $\nu_2 \notin cl_\epsilon(\{\nu_1\})$. Therefore, $[cl_\epsilon(\{\nu_1\})]^c$ and $[cl_\epsilon(\{\nu_2\})]^c$ are two supra-ϵ-open sets which separate ν_1 and ν_2. Thus, λ is supra-ϵ-T_1-space.

Corollary 3. *Every supra ϵ-symmetric space is supra-ϵ-T_0-space if and only if it is supra-ϵ-T_1.*

Proof. It is direct from Theorem 2 and Theorem 7.

Theorem 8. *For any STS (λ, ϑ), the following are equivalent:*

(1) *λ is supra-ϵ-T_2-space;*

(2) *$\psi = \{(\nu, \nu) : \nu \in \lambda\}$ is supra-ϵ-closed subset of $\lambda \times \lambda$;*

(3) *$\cap\{H_\nu : H_\nu \in SC_\epsilon(\lambda)\} = \{\nu\}$, for each $\nu \in \lambda$.*

Proof.

(1) $\Rightarrow$ (2) Let $(\nu_1, \nu_2) \in \lambda \times \lambda \backslash \psi$, then $\nu_1 \neq \nu_2$. Given (1), there are two disjoint supra-ϵ-open subsets μ_1 and μ_2 of λ, such that $\nu_1 \in \mu_1$ and $\nu_2 \in \mu_2$. Hence, $(\nu_1, \nu_2) \in \mu_1 \times \mu_1 \subseteq \lambda \times \lambda \backslash \psi$, and thus $\lambda \times \lambda \backslash \psi$ is a supra supra-ϵ-neighborhood for each of its points. Therefore, ψ is supra-ϵ-closed subset of $\lambda \times \lambda$.

(2) $\Rightarrow$ (1) Assume that $\psi = \{(\nu, \nu) : \nu \in \lambda\}$ is supra-ϵ-closed subset of $\lambda \times \lambda$ and $\nu_1 \neq \nu_2 \in \lambda$, then $\lambda \times \lambda \backslash \psi$ is supra-ϵ-open set including (ν_1, ν_2). Hence, there are $A, B \in SO_\epsilon(\lambda)$ such that $(\nu_1, \nu_2) \in A \times B \subseteq \lambda \times \lambda \backslash \psi$. Therefore, $A, B \in SO_\epsilon(\lambda)$ separate ν_1 and ν_2 with $A \cap B = \emptyset$. Thus, λ is supra-ϵ-T_2-space.

(1) $\Rightarrow$ (3) Let λ be a supra-ϵ-T_2-space. Then, for any $\nu_1 \neq \nu_2 \in \lambda$, there are two disjoint supra-ϵ-open subsets μ_1 and μ_2 of λ, such that $\nu_1 \in \mu_1$ and $\nu_2 \in \mu_2$. This implies that $\nu_1 \in cl_\epsilon(\mu_1) \subseteq \mu_2^c$, $\mu_2^c \in SC_\epsilon(\lambda)$ which containing ν_1 but not ν_2. Hence, $\cap\{\mu_2^c : \nu_1 \in \mu_2^c \in SC_\epsilon(\lambda)\} = \{\nu_1\}$.

(3) $\Rightarrow$ (1) Let $\nu_1 \neq \nu_2 \in \lambda$. Given (3), $\cap\{H_{\nu_1} : H_{\nu_1} \in SC_\epsilon(\lambda)\} = \{\nu_1\}$. This means that, there is $H_{\nu_1} \in SC_\epsilon(\lambda)$ including ν_1 but not ν_2. Hence, there is $O_{\nu_1} \in SO_\epsilon(\lambda)$ such that $\nu_1 \in cl_\epsilon(O_{\nu_1}) \subseteq H_{\nu_1}$, and consequently O_{ν_1}, $[cl_\epsilon(O_{\nu_1})]^c \in SO_\epsilon(\lambda)$ separate ν_1 and ν_2 with $[cl_\epsilon(O_{\nu_1})]^c \cap O_{\nu_1} = \emptyset$. Therefore, λ is supra-ϵ-T_2-space.

4. More features of supra ϵ-separation axioms

Herein, we study the supra hereditary property and supra topological property for each aforementioned notion. In special, we show that he property of being a supra-ϵ-T_j-space, $j = 0, 1, 2$, is a supra hereditary property. Moreover, we show that he property of being a supra-ϵ-T_j-space, $j = 0, 1, 2$, is a supra topological property under special types of supra-ϵ-functions.

Definition 12. *For the subset L of an STS (λ, ν), the class*

$$\nu_L = \{L \cap O : O \in SO_\epsilon(\lambda)\}$$

defines an STS on L, and it is called a supra-ϵ-subspace of (λ, ν).

Since Definition 12 provides a clear evidence for the next two propositions, their proofs are excluded.

Proposition 3. *Let (U, ν_U) be an supra ϵ-subspace of an STS (λ, ν) and V be a subset of λ. Then, $(cl_\epsilon(V))_{\nu_U} = U \cap cl_\epsilon(V)$.*

Proposition 4. *Let (U, ν_U) be an supra ϵ-subspace of an STS (λ, ν) and V be a subset of λ. Then, $V \in SC_\epsilon(U)$ if and only if there is $N \in SC_\epsilon(\lambda)$ such that $V = U \cap N$.*

Theorem 9. *Every supra-ϵ-subspace of supra-ϵ-T_j-space is supra-ϵ-T_j, $j = 0, 1, 2$.*

Proof. The other cases are evidently contained in the case of $j = 2$, which we prove. Assume that (χ, ϑ_χ) is a supra subspace of supra-ϵ-T_2-space (λ, ϑ) and $\nu_1 \neq \nu_2 \in \chi \subseteq \lambda$. Given λ is supra-ϵ-T_2, then there are two disjoint supra-ϵ-open subsets μ_1 and μ_2 of λ, such that $\nu_1 \in \mu_1$ and $\nu_2 \in \mu_2$. Hence, $\nu_1 \in \mu_1 \cap \chi$ and $\nu_2 \in \mu_2 \cap \chi$ such that $[\mu_1 \cap \chi] \cap [\mu_2 \cap \chi] = \chi \cap [\mu_1 \cap \mu_1] = \chi \cap \emptyset = \emptyset$ and $\mu_1 \cap \chi, \mu_2 \cap \chi \in \vartheta_\chi$. Therefore, χ is supra-ϵ-T_2-space.

Definition 13. *A function $\Lambda_\epsilon : (\lambda_1, \sigma_1) \rightarrow (\lambda_2, \sigma_2)$ with ϑ_1 as an associated STS with σ_1 is said to be a supra ϵ-continuous (abbreviate: supra ϵ-cts) if $\Lambda_\epsilon^{-1}(G) \in SO_\epsilon(\lambda_1)$ for each $G \in \sigma_2$.*

Theorem 10. *If $\Lambda_\epsilon : (\lambda_1, \sigma_1) \rightarrow (\lambda_2, \sigma_2)$ is an injective supra ϵ-cts function with ϑ_1 as an associated STS with σ_1 such that (λ_2, σ_2) is T_j-space, then (λ_1, ϑ_1) is a supra-ϵ-$T_j, j = 0, 1, 2$.*

Proof. The other cases are evidently contained in the case of $j = 2$, which we prove. Let $\nu_1 \neq \nu_2 \in \lambda_1$. Since Λ_ϵ is injective, there are $\zeta_1 \neq \zeta_2 \in \lambda_2$ such that $\Lambda_\epsilon(\nu_1) = \zeta_1$ and $\Lambda_\epsilon(\nu_2) = \zeta_2$. Since (λ_2, σ_2) is T_2-space, there are two disjoint open subsets μ_1 and μ_2 of λ_2, such that $\zeta_1 \in \mu_1$ and $\zeta_2 \in \mu_2$. Given Λ_ϵ is supra ϵ-cts, $\Lambda_\epsilon^{-1}(\mu_1)$ and $\Lambda_\epsilon^{-1}(\mu_2)$ are two disjoint supra-ϵ-open subsets of λ_1 containing ν_1, ν_2, respectively. Therefore, (λ_1, ϑ_1) is supra-ϵ-T_2.

Definition 14. *A function $\Lambda_\epsilon : (\sigma_1, \nu_1) \rightarrow (\sigma_2, \nu_2)$ with ϑ_1, ϑ_2 associated STSs with σ_1, σ_2, respectively, is said to be supra ϵ-irresolute if $\Lambda_\epsilon^{-1}(D) \in SO_\epsilon(\lambda_1)$ for each $D \in SO_\epsilon(\lambda_2)$.*

Proposition 5. *If $\Lambda_\epsilon : (\lambda_1, \sigma_1) \rightarrow (\lambda_2, \sigma_2)$ is an injective supra ϵ-irresolute function with ϑ_1, ϑ_2 associated STSs with σ_1, σ_2 respectively, such that (λ_2, σ_2) is supra-ϵ-T_j-space, then (λ_1, ϑ_1) is a supra-ϵ-$T_j, j = 0, 1, 2$.*

Proof. The other cases are evidently contained in the case of $j = 2$, which we prove. Let $\nu_1 \neq \nu_2 \in \lambda_1$. Since Λ_ϵ is injective, there are $\zeta_1 \neq \zeta_2 \in \lambda_2$ such that $\Lambda_\epsilon(\nu_1) = \zeta_1$ and $\Lambda_\epsilon(\nu_2) = \zeta_2$. Since (λ_2, σ_2) is supra-ϵ-T_2, there are two disjoint supra-ϵ-open subsets μ_1 and μ_2 of λ_2, such that $\zeta_1 \in \mu_1$ and $\zeta_2 \in \mu_2$. Given Λ_ϵ is supra ϵ-irresolute, $\Lambda_\epsilon^{-1}(\mu_1)$ and $\Lambda_\epsilon^{-1}(\mu_2)$ are two disjoint supra-ϵ-open subsets of λ_1 containing ν_1, ν_2, respectively. Therefore, (λ_1, ϑ_1) is supra-ϵ-T_2.

Definition 15. *A function $\Lambda_\epsilon : (\lambda_1, \sigma_1) \to (\lambda_2, \sigma_2)$ with ϑ_1, ϑ_2 associated STSs with σ_1, σ_2 respectively, is said to be supra ϵ (ϵ^*)-open if $\Lambda_\epsilon(U) \in SO_\epsilon(\lambda_2)$ for each $U \in \sigma_1$ ($U \in SO_\epsilon(\lambda_1)$).*

Theorem 11. *Let $\Lambda_\epsilon : (\lambda_1, \sigma_1) \to (\lambda_2, \sigma_2)$ be a function with ϑ_2 as an associated STS with σ_2 and $\alpha \subseteq \lambda_1$, then*

$$\Lambda_\epsilon \text{ is supra } \epsilon\text{-open if and only if } \Lambda_\epsilon(int(\alpha)) \subseteq int^s_\epsilon[\Lambda_\epsilon(\alpha)] \; \forall \; \alpha \subseteq \lambda_1.$$

Proof. " $\Rightarrow$ " Let Λ_ϵ be a supra ϵ-open function and $\alpha \subseteq \lambda_1$. Since $int(\alpha) \subseteq \alpha$, $\Lambda_\epsilon(int(\alpha)) \subseteq \Lambda_\epsilon((\alpha))$, which leads to

$$\Lambda_\epsilon(int(\alpha)) = int^s_\epsilon[\Lambda_\epsilon(int(\alpha))] \subseteq int^s_\epsilon[\Lambda_\epsilon((\alpha))], \text{ given } \Lambda_\epsilon \text{ is supra } \epsilon\text{-open.}$$

" $\Leftarrow$ " Suppose that $\alpha \in \sigma_1$. Considering the condition,

$$\Lambda_\epsilon(\alpha) = \Lambda_\epsilon(int(\alpha)) \subseteq int^s_\epsilon[\Lambda_\epsilon(\alpha)] \; .$$

However, we have that

$$int^s_\epsilon[\Lambda_\epsilon(\alpha)] \subseteq \Lambda_\epsilon(\alpha).$$

Hence,

$$int^s_\epsilon[\Lambda_\epsilon(\alpha)] = \Lambda_\epsilon(\alpha).$$

Therefore,

$$\Lambda_\epsilon(\alpha) \in SO_\epsilon(\lambda_2), \text{ and consequently } \Lambda_\epsilon \text{ is a supra } \epsilon\text{-open function.}$$

Proposition 6. *Let $\Lambda_\epsilon : (\lambda_1, \sigma_1) \to (\lambda_2, \sigma_2)$ be a function with ϑ_1, ϑ_2 associated STSs with σ_1, σ_2 respectively, and $\alpha \subseteq \lambda_1$, then*

$$\Lambda_\epsilon \text{ is supra } \epsilon^*\text{-open if and only if } \Lambda_\epsilon(int^s_\epsilon(\alpha)) \subseteq int^s_\epsilon[\Lambda_\epsilon(\alpha)] \; \forall \; \alpha \subseteq \lambda_1.$$

Proof. It is similar to the proof of Theorem 11.

Theorem 12. *The image of each T_j-space is a supra-ϵ-T_j under a bijective supra ϵ-open function, $j = 0, 1, 2$.*

Proof. The other cases are evidently contained in the case of $j = 2$, which we prove. Let $\Lambda_\epsilon : (\lambda_1, \sigma_1) \to (\lambda_2, \sigma_2)$ with ϑ_1, ϑ_2 associated STSs with σ_1, σ_2 respectively, be a bijective supra ϵ-open function such that (λ_1, ϑ_1) is T_2-space.

Let $\theta_1 \neq \theta_2 \in \lambda_2$. Since Λ_ϵ is bijective, there are $\xi_1 \neq \xi_2 \in \lambda_1$ such that $\Lambda_\epsilon(\xi_1) = \theta_1$ and $\Lambda_\epsilon(\xi_2) = \theta_2$. Since (λ_1, σ_1) is T_2-space, there are two disjoint open subsets ρ_1 and ρ_2 of λ_2, such that $\xi_1 \in \rho_1$ and $\xi_2 \in \rho_2$. Given Λ_ϵ is supra ϵ-open, $\Lambda_\epsilon(\rho_1)$ and $\Lambda_\epsilon(\rho_2)$ are two disjoint supra-ϵ-open subsets of λ_2 containing θ_1, θ_2, respectively. Therefore, (λ_2, ϑ_2) is supra-ϵ-T_2.

Corollary 4. *The image of each supra-ϵ-T_j-space is a supra-ϵ-T_j under a bijective supra ϵ^*-open function, $j = 0, 1, 2$.*

Proof. It is deduced using a similar procedure to that of Theorem 12.

Definition 16. *A function $\Lambda_\epsilon : (\lambda_1, \sigma_1) \rightarrow (\lambda_2, \sigma_2)$ with ϑ_1, ϑ_2 associated STSs with σ_1, σ_2 respectively, is said to be supra-ϵ^*-homeomorphism if it is bijective supra ϵ^*-open and supra ϵ^*-cts*

Lemma 2. *If $\Lambda_\epsilon : (\lambda_1, \sigma_1) \rightarrow (\lambda_2, \sigma_2)$ is supra-ϵ^*-homeomorphism function with ϑ_1, ϑ_2 associated STSs with σ_1, σ_2 respectively, then (λ_2, σ_2) is supra-ϵ-T_j-space if and only if (λ_1, ϑ_1) is supra-ϵ-$T_j, j = 0, 1, 2$.*

Proof. It is follows from Proposition 5 and Corollary 4.

5. Conclusion

A new weaker version of supra septarian axioms based on supra ϵ-open sets is presented in this manuscript, along with its key characteristics named supra-ϵ-T_j-space, $j = 0, 1, 2$. In detail, we present three new types of separation axioms inspired by supra ϵ-open sets named supra-ϵ-T_0-space, supra-ϵ-T_1-space, and supra-ϵ-Hausdorff-space. We provide thorough descriptions of each of them supported with several examples and counterexamples that demonstrate the importance of our novel concepts. Specifically, we explore sufficient conditions for several analogous linkages between them and generally illustrate their key characteristics. Furthermore, we propose a diagram that encapsulates their connections [see figure 1]. Additionally, we present the supra ϵ-symmetric property and show that the two notions of supra-ϵ-T_0-space and supra-ϵ-T_1-space are the same for any STS that fulfills it. Finally, for each of the previously described concepts, we examine the supra topological and supra hereditary properties. Specifically, we demonstrate that the property of being a supra-ϵ-T_j-space, where $j = 0, 1, 2$, is a supra-hereditary (topological) property. From the specific approaches described in this paper, additional research on the theoretical aspects of these generalized concepts could be carried out by looking at the following subjects:

- Introducing more types of septarian axioms based on supra ϵ-open sets, like supra-ϵ-completely space, supra-ϵ-regular-space, supra-ϵ-completely regular space, supra-ϵ-normal-space, supra-ϵ-T_3-space, and supra-ϵ-T_4-space.
- Considering whether information systems can benefit from the use of these kinds of separation axioms.
- Introducing theses concepts to fuzzy supra soft topological spaces [64, 65].
- Applying our new notions to rough approximations based on relations with decision making applications [66].

Conflicts of interest

In relation to the publication of this work, the authors declare that they have no competing interests.

Author contributions

Each author's contribution was equal.

Acknowledgements

The authors extend their appreciation to the Deanship of Scientific Research at Northern Border University, Arar, KSA for funding this research work through the project number "NBU-FFR-2025-2941-02". Also, this study is supported via funding from Prince Sattam bin Abdulaziz University project number (PSAU/2025/R/1446).

References

[1] N. Levine. Semi-open sets and semi-continuity in topological spaces. *American Mathematical Monthly*, 70(1):36–41, 1963.

[2] O. Njastad. On some classes of nearly open sets. *Pacific Journal of Mathematics*, 15(3):961–970, 1965.

[3] A. Mashhour, M. Abd El-Monsef, and S. El-Deeb. On precontinuous and weak precontinuous mappings. *Proceedings of the Mathematical and Physical Society*, 53:47–53, 1982.

[4] M. Abd El-Monsef, S. El-Deeb, and R. Mahmoud. β-open sets and β-continuous mappings. *Bulletin of the Faculty of Science, Assiut University*, 12(1):77–90, 1983.

[5] D. Andrijevic. On b-open sets. *Matematicki Vesnik*, 48:59–64, 1996.

[6] J. Dontchev and M. Przemski. On the various decompositions of continuous and some weakly continuous functions. *Acta Mathematica Hungarica*, 71(1-2):109–120, 1996.

[7] K. R. Gentry and H. B. Hoyle, III. Somewhat continuous functions. *Czechoslovak Mathematical Journal*, 21:5–12, 1971.

[8] Z. Piotrowski. A survey of results concerning generalized continuity on topological spaces. *Acta Mathematica Universitatis Comenianae*, 52:91–110, 1987.

[9] O. Njastad. On some classes of nearly open sets. *Pacific Journal of Mathematics*, 15:961–970, 1965.

[10] C. C. Pugh. *Real Mathematical Analysis*. Springer Science and Business Media, 2003.

[11] T. M. Al-Shami. Somewhere dense sets and st_1 spaces. *Punjab University Journal of Mathematics*, 49(2):101–111, 2017.

[12] M. H. Alqahtani. F-open and f-closed sets in topological spaces. *European Journal of Pure and Applied Mathematics*, 16:819–832, 2023.

[13] M. H. Alqahtani and A. M. Abd El-latif. Separation axioms via novel operators in the frame of topological spaces and applications. *AIMS Mathematics*, 9(6):14213–14227, 2024.

[14] O. Alghamdi, Ahmad Al-Omari, and M. H. Alqahtani. Novel operators in the frame of primal topological spaces. *AIMS Mathematics*, 9(9):25792–25808, 2024.

[15] A. S. Mashhour, A. A. Allam, F. S. Mahmoud, and F. H. Kheder. On supra topological spaces. *Indian Journal of Pure and Applied Mathematics*, pages 502–510, 1983.

[16] T. Al-Shami. On supra semi open sets and some applications on topological spaces. *Journal of Advanced Studies in Topology*, 8(2):144–153, 2017.

[17] M. E. El-Shafei, M. Abo-Elhamayel, and T. M. Al-Shami. On supra r-open sets and some applications on topological spaces. *Journal of Progressive Research in Mathematics*, 8:1237–1248, 2016.

[18] S. Jafari and S. Tahiliani. Supra β-open sets and supra β-continuity on topological spaces. *Annales Universitatis Scientiarum Budapestinensis*, 56:1–9, 2013.

[19] O. R. Sayed and T. Noiri. On supra b-open sets and supra b-continuity on topological spaces. *European Journal of Pure and Applied Mathematics*, 3:295–302, 2010.

[20] O. R. Sayed. Supra pre-open sets and supra pre-continuous on topological spaces. *Series Mathematics and Information*, 20:79–88, 2010.

[21] R. Devi, S. Sampathkumar, and M. Caldas. On α-open sets and sa-continuous maps. *General Mathematics*, 16:77–84, 2008.

[22] A. M. Abd El-latif, Radwan Abu-Gdairi, A. A. Azzam, Husham M. Attaalfadeel, Shaaban M. Shaaban, M. Aldawood, and Khaled A. Aldwoah. Supra ϵ-open sets: Features, operators and applications. *European Journal of Pure and Applied Mathematics*, 18(2):5969, 2025.

[23] A. M. Abd El-latif, Radwan Abu-Gdairi, Abd elfattah Azzam, Fatouh Gharib, Husham Mohammed Alhassan Attaalfadeel, Walid Abdelfattah, Shaaban M. Shaaban, and M. Aldawood. Novel types of supra functions inspired by supra ϵ-open sets. *European Journal of Pure and Applied Mathematics*, 18(2):6020, 2025.

[24] M. Shabir and M. Naz. On soft topological spaces. *Computers and Mathematics with Applications*, 61:1786–1799, 2011.

[25] Zanyar A. Ameen and S. Al Ghour. Cluster soft sets and cluster soft topologies. *Computational and Applied Mathematics*, 42:337, 2023.

[26] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh, and A. M. Abd El-latif. Soft semi separation axioms and irresolute soft functions. *Annals of Fuzzy Mathematics and Informatics*, 8(2):305–318, 2014.

[27] Tareq M. Al-shami, Abdelwaheb Mhemdi, and Radwan Abu-Gdairi. A novel framework for generalizations of soft open sets and its applications via soft topologies. *Mathematics*, 11(4):840, 2023.

[28] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh, and A. M. Abd El-latif. γ-operation and decompositions of some forms of soft continuity in soft topological spaces. *Annals of Fuzzy Mathematics and Informatics*, 7(2):181–196, 2014.

[29] S. A. El-Sheikh and A. M. Abd El-latif. Characterization of b-open soft sets in soft topological spaces. *Journal of New Theory*, 2:8–18, 2015.

[30] T. M. Al-shami. Soft somewhere dense sets on soft topological spaces. *Communications of the Korean Mathematical Society*, 33(2):1341–1356, 2018.

[31] A. M. Abd El-latif, A. A. Azzam, R. Abu-Gdairi, M. H. Alqahtani, and G. M. Abd-Elhamed. Applications on soft somewhere dense sets. *Journal of Interdisciplinary Mathematics*, 27:1679–1699, 2024.

[32] Radwan Abu-Gdairi, A. A. Azzam, and Ibrahim Noaman. Nearly soft β-open sets via soft ditopological spaces. *European Journal of Pure and Applied Mathematics*,

15(1):126–134, 2022.

[33] S. Al Ghour. Soft ω-continuity and soft ω_s-continuity in soft topological spaces. *International Journal of Fuzzy Logic and Intelligent Systems*, 22(2):183–192, 2022.

[34] S. Al Ghour and B. Irshidat. On θ_ω continuity. *Heliyon*, 6(2):e03349, 2020.

[35] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh, and A. M. Abd El-latif. Soft ideal theory, soft local function and generated soft topological spaces. *Applied Mathematics and Information Sciences*, 8(4):1595–1603, 2014.

[36] F. Gharib and A. M. Abd El-latif. Soft semi local functions in soft ideal topological spaces. *European Journal of Pure and Applied Mathematics*, 12(3):857–869, 2019.

[37] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh, and A. M. Abd El-latif. Soft semi compactness via soft ideals. *Applied Mathematics and Information Sciences*, 8(5):2297–2306, 2014.

[38] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh, and A. M. Abd El-latif. Soft connectedness via soft ideals. *Journal of New Results in Science*, 4:90–108, 2014.

[39] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh, and A. M. Abd El-latif. γ-operation and decompositions of some forms of soft continuity of soft topological spaces via soft ideal. *Annals of Fuzzy Mathematics and Informatics*, 9(3):385–402, 2015.

[40] M. Akdag and F. Erol. Soft i-sets and soft i-continuity of functions. *Gazi University Journal of Science*, 27:923–932, 2014.

[41] A. A. Nasef, M. Parimala, R. Jeevitha, and M. K. El-Sayed. Soft ideal theory and applications. *International Journal of Nonlinear Analysis and Applications*, 13(2):1335–1342, 2022.

[42] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh, and A. M. Abd El-latif. Soft semi (quasi) hausdorff spaces via soft ideals. *South Asian Journal of Mathematics*, 4(6):265–284, 2014.

[43] A. M. Abd El-latif. Generalized soft rough sets and generated soft ideal rough topological spaces. *Journal of Intelligent and Fuzzy Systems*, 34:517–524, 2018.

[44] A. M. Abd El-latif. New generalized fuzzy soft rough approximations applied to fuzzy topological spaces. *Journal of Intelligent and Fuzzy Systems*, 35:2123–2136, 2018.

[45] Mesfer H. Alqahtani and Zanyar A. Ameen. Congruence representations via soft ideals in soft topological spaces. *Axioms*, 12:1015, 2023.

[46] S. Saleh, Laith R. Flaih, and Khaled F. Jasim. Some applications of soft δ-closed sets in soft closure spaces. *Communications in Mathematics and Applications*, 14(2):481–492, 2023.

[47] S. Saleh and K. Hur. On some lower soft separation axioms. *Annals of Fuzzy Mathematics and Informatics*, 19(1):61–72, 2020.

[48] S. A. El-Sheikh and A. M. Abd El-latif. Decompositions of some types of supra soft sets and soft continuity. *International Journal of Mathematical Trends and Technology*, 9(1):37–56, 2014.

[49] A. M. Abd El-latif and S. Karataş. Supra b-open soft sets and supra b-soft continuity on soft topological spaces. *Journal of Mathematics and Computer Applications Research*, 5(1):1–18, 2015.

[50] A. M. Abd El-latif. Soft supra strongly generalized closed sets. *Journal of Intelligent*

and Fuzzy Systems, 31(3):1311–1317, 2016.

[51] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh, and A. M. Abd El-latif. Supra generalized closed soft sets with respect to an soft ideal in supra soft topological spaces. *Applied Mathematics and Information Sciences*, 8(4):1731–1740, 2014.

[52] A. M. Abd El-latif, Radwan Abu-Gdairi, A. A. Azzam, F. A. Gharib, and Khaled A. Aldwoah. Supra soft somewhat open sets: Characterizations and continuity. *European Journal of Pure and Applied Mathematics*, 18(1):5863, 2025.

[53] A. M. Abd El-latif and Mesfer H. Alqahtani. New soft operators related to supra soft δ_i-open sets and applications. *AIMS Mathematics*, 9(2):3076–3096, 2024.

[54] Alaa M. Abd El-latif, Mesfer H. Alqahtani, and F. A. Gharib. Strictly wider class of soft sets via supra soft δ-closure operator. *International Journal of Analysis and Applications*, 22:47, 2024.

[55] S. Saleh, T. Al-Shami, Laith R. Flaiha, Murad Arar, and Radwan Abu-Gdairi. r_i-separation axioms via supra soft topological spaces. *Journal of Mathematics and Computer Science*, 32:263–274, 2024.

[56] A. M. Abd El-latif. Novel types of supra soft operators via supra soft sd-sets and applications. *AIMS Mathematics*, 9(3):6586–6602, 2024.

[57] A. M. Abd El-latif and M. H. Alqahtani. Novel categories of supra soft continuous maps via new soft operators. *AIMS Mathematics*, 9:7449–7470, 2024.

[58] A. M. Abd El-latif, A. A. Azzam, Radwan Abu-Gdairi, M. Aldawood, and Mesfer H. Alqahtani. New versions of maps and connected spaces via supra soft sd-operators. *PloS One*, 19(10):e0304042, 2024.

[59] A. M. Abd El-latif, Radwan Abu Gdairi, A. A. Azzam, Khaled A. Aldwoah, M. Aldawood, and Shaaban M. Shaaban. Applications of the supra soft sd-closure operator to soft connectedness and compactness. *European Journal of Pure and Applied Mathematics*, 18(2):5896, 2025.

[60] A. M. Abd El-latif. Specific types of lindelofness and compactness based on novel supra soft operator. *AIMS Mathematics*, 10(4):8144–8164, 2025.

[61] Mesfer H. Alqahtani and Zanyar A. Ameen. Soft nodec spaces. *AIMS Mathematics*, 9(2):3289–3302, 2024.

[62] M. H. Alqahtani, O. F. Alghamdi, and Z. A. Ameen. Nodecness of soft generalized topological spaces. *International Journal of Analysis and Applications*, 22:149, 2024.

[63] T. M. Al-shami, E. A. Abo-Tabl, B. A. Asaad, and M. A. Arahet. Limit points and separation axioms with respect to supra semi-open sets. *European Journal of Pure and Applied Mathematics*, 13(3):427–443, 2020.

[64] A. M. Abd El-latif. Some properties of fuzzy supra soft topological spaces. *European Journal of Pure and Applied Mathematics*, 12(3):999–1017, 2019.

[65] A. M. Abd El-Latif. Results on fuzzy supra soft topological spaces. *Journal of Interdisciplinary Mathematics*, 22(8):1311–1323, 2019.

[66] Rani Sumaira Kanwal, Saqib Mazher Qurashi, Rizwan Gul, Alaa M. Abd El-latif, Tareq M. Al-shami, and Faiza Tufail. New insights into rough approximations of a fuzzy set inspired by soft relations with decision making applications. *AIMS Mathematics*, 10(4):9637–9673, 2025.