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A Characterization of Diagonal Solutions for a Class of Linear Matrix Inequality

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Abstract. This paper investigates the existence of positive diagonal solutions for a class of linear matrix inequalities (LMIs) involving a triple of real $n \times n$ matrices (A_1, A_2, A_3). We provide equivalent conditions linking the negative definiteness of a structured block matrix to properties of positive semidefinite test matrices and P-matrices under Hadamard transformations. Our results extend classical stability results to multi-matrix settings with applications to control theory and network dynamics.

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Key Words and Phrases: Diagonal matrices, P-matrix, positive definite matrix, matrix inequality, matrix stability

1. Introduction

Linear matrix inequalities (LMIs) play a fundamental role in various fields of applied mathematics, including control theory [1–3], optimization [4], and stability analysis of dynamical systems [5]. These inequalities often arise in the study of matrix stability properties, such as Lyapunov stability, and are instrumental in characterizing the behavior of complex systems. A particularly interesting and challenging problem within this domain is the identification of diagonal solutions—specifically, positive diagonal matrices—that satisfy certain LMIs. Such solutions are not only mathematically intriguing but also have practical implications in areas such as network control [6, 7] and evolutionary dynamics [8].

Lyapunov stability is a cornerstone of dynamical systems theory, providing a framework to assess the stability of equilibrium points without explicitly solving differential equations [5, 9]. For a linear system $\dot{x} = Ax$, where $A \in \mathbb{R}^{n\times n}$, the system is said to be Lyapunov stable if there exists a symmetric positive definite matrix $P \succ 0$ such that the

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Lyapunov inequality $A^T P + PA \prec 0$ holds, ensuring that the quadratic form $V(x) = x^T Px$ serves as a Lyapunov function whose time derivative is negative definite along system trajectories [1]. This condition guarantees asymptotic stability, meaning solutions converge to the origin as time progresses. A related and more specific concept, Lyapunov diagonal stability, applies when such a P can be chosen as a positive diagonal matrix [10]. This property is particularly significant in the context of linear matrix inequalities (LMIs), as it imposes a structured constraint on the solution space, linking stability to the existence of diagonal matrices that satisfy inequalities like those studied in this work. For a matrix A to be Lyapunov diagonally stable, $-A$ must often exhibit properties such as being a P-matrix [11, 12], a connection that underpins our analysis of diagonal solutions for the triple (A_1, A_2, A_3).

The importance of Lyapunov diagonal stability extends beyond theoretical elegance; it arises naturally in applications where system matrices possess inherent structural properties, such as in networked control systems [6, 13] or compartmental models in biology [14]. By restricting P to be diagonal, we enforce a decoupling of variables that simplifies stability analysis and computation, a feature exploited in our characterizations of positive diagonal solutions [15]. This paper builds on these concepts to explore conditions under which a structured block matrix, involving multiple matrices A_1, A_2, A_3, admits such diagonal solutions, thereby generalizing classical stability results to multi-matrix settings with practical implications in control and dynamics.

In this paper, we focus on a specific class of LMIs involving a triple of real $n \times n$ matrices (A_1, A_2, A_3). Our objective is to characterize the conditions under which this triple admits a *positive diagonal solution*, defined as a set of positive diagonal matrices $P_1, P_2, P_3 \in \mathbb{R}^{n\times n}$ such that the block matrix

$$B = \begin{bmatrix} A_1^T P_1 + P_1 A_1 + P_2 + P_3 & P_1 A_2 & P_1 A_3 \\ A_2^T P_1 & -P_2 & 0 \\ A_3^T P_1 & 0 & -P_3 \end{bmatrix}$$

is negative definite. This formulation generalizes classical stability problems and introduces additional complexity due to the interplay between the matrices A_1, A_2, A_3 and the diagonal structure of P_1, P_2, P_3.

2. Background

To establish a foundation for our results, we introduce several key concepts. All matrices considered in this work are real. Additionally, a matrix is positive (negative, respectively) definite if all its eigenvalues are positive (negative, respectively); meanwhile, we say it is positive (negative, respectively) semidefinite if all its eigenvalues are nonnegative (nonpositive, respectively). We shall adopt the notation $X \succ 0$ ($X \prec 0$, respectively) to indicate that a matrix $X \in \mathbb{R}^{n\times n}$ is positive definite (negative definite, respectively). Similarly, we denote a positive semidefinite (negative semidefinite, respectively) matrix by $X \succeq 0$ ($X \preceq 0$, respectively). Unless stated otherwise, a positive or negative definite or semidefinite matrix is assumed to be symmetric. If X is a diagonal positive definite matrix, we write positive diagonal matrix since it is clear that a diagonal positive definite matrix has all its diagonal elements positive.

Let $X \in \mathbb{R}^{n\times n}$ and $u, v \in \mathbb{R}^n$. The trace of X is denoted by $\operatorname{tr}(X)$, and $\operatorname{diag}(X)$ represents the vector in $\mathbb{R}^n$ whose j-th component corresponds to the j-th diagonal element of X. We shall employ the notation $u \geq v$ to indicate that $u_j \geq v_j$ for all $j = 1, \ldots, n$, and we use $u \geqslant v$ if $u_j \geqslant v_j$ for all $j = 1, \ldots, n$.

We denote the Hadamard product of two matrices X and Y, both in $\mathbb{R}^{n\times n}$, as $X \circ Y$, where $(X \circ Y)_{ij} = x_{ij}y_{ij}$. A matrix $X \in \mathbb{R}^{n\times n}$ is a P-matrix if every principal minor of X is positive. It is a well-established result (see Theorem 6.2.3 of [11]) that this condition is equivalent to the requirement that, for every non-zero $u \in \mathbb{R}^n$, there exists an index i such that $u_i(Xu)_i > 0$.

For a block matrix $H \in \mathbb{R}^{3n\times 3n}$, partitioned as

$$H = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{12}^T & H_{22} & H_{23} \\ H_{13}^T & H_{23}^T & H_{33} \end{bmatrix},$$

we assume each block $H_{ij} \in \mathbb{R}^{n\times n}$, and denote the (i, j)-th entry of H_{11} as $h_{ij}^{11} = (H_{11})_{ij}$.

Our analysis builds on several foundational results. For instance, it is well-known that the Hadamard product of two positive semidefinite matrices is positive semidefinite, and if both are positive definite, their product remains positive definite. Furthermore, if a matrix is Lyapunov diagonally stable—meaning there exists a positive diagonal matrix P such that $A^TP + PA$ is negative definite—then its negative is a P-matrix. These properties underpin our exploration of diagonal solutions and their stability characteristics.

Lemma 1 ([16, 17])**.** *Suppose A and B are $n \times n$ positive semidefinite matrices. Then the Hadamard product $A \circ B$ is also positive semidefinite. Moreover, if A and B are both positive definite, their Hadamard product $A \circ B$ will be positive definite.*

Lemma 2 ([10, 12])**.** *If a matrix $A \in \mathbb{R}^{n\times n}$ is Lyapunov diagonally stable, then $-A$ is a P-matrix.*

Let $A_1, A_2, A_3 \in \mathbb{R}^{n\times n}$. We say that the triple (A_1, A_2, A_3) admits a positive diagonal solution if there exist positive diagonal matrices $P_1, P_2, P_3 \in \mathbb{R}^{n\times n}$ such that the block matrix

$$\begin{bmatrix} A_1^TP_1 + P_1A_1 + P_2 + P_3 & P_1A_2 & P_1A_3 \\ A_2^TP_1 & -P_2 & 0 \\ A_3^TP_1 & 0 & -P_3 \end{bmatrix} \tag{1}$$

is negative definite. Furthermore, we say that (P_1, P_2, P_3) is a positive diagonal solution for the triple (A_1, A_2, A_3).

Lemma 3 ([18])**.** *Let $A_1, A_2,$ and A_3 be real $n \times n$ matrices. If the triple (A_1, A_2, A_3) admits a positive diagonal solution, then $A_1 + A_2 + A_3$ is a Lyapunov diagonally stable matrix.*

3. Main Results

This section presents our key findings on positive diagonal solutions for the LMI defined by the triple (A_1, A_2, A_3) and the block matrix B. Extending classical stability criteria, we establish equivalent conditions linking B negative definiteness to test matrices and P-matrix properties under Hadamard transformations. We begin with preliminary equivalences (Theorems 2.1 and 2.2), followed by new characterizations (Theorems 2.3 and 2.4), offering tools for stability analysis in multi-matrix systems.

3.1. Preliminaries

Theorem 1 ([18]). *Let $A_1, A_2, A_3 \in \mathbb{R}^{n\times n}$. Then, the following statements are equivalent:*

(i) There are positive diagonal matrices P_1, P_2, and P_3 such that

$$B = \begin{bmatrix} A_1^T P_1 + P_1 A_1 + P_2 + P_3 & P_1 A_2 & P_1 A_3 \\ A_2^T P_1 & -P_2 & 0 \\ A_3^T P_1 & 0 & -P_3 \end{bmatrix} \prec 0. \tag{2}$$

(ii) For any nonzero positive semidefinite matrix $H \in \mathbb{R}^{3n\times 3n}$, partitioned into 3×3 block matrices with each block in $\mathbb{R}^{n\times n}$ as

$$H = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{12}^T & H_{22} & H_{23} \\ H_{13}^T & H_{23}^T & H_{33} \end{bmatrix}, \tag{3}$$

satisfying the conditions

$$\operatorname{diag}(H_{11}) \geq \operatorname{diag}(H_{22}) \quad \textit{and} \quad \operatorname{diag}(H_{11}) \geq \operatorname{diag}(H_{33}),$$

at least one diagonal element of the matrix

$$A_1 H_{11} + A_2 H_{12}^T + A_3 H_{13}^T$$

is negative.

Theorem 2 ([18]). *Let $A_1, A_2, A_3 \in \mathbb{R}^{n\times n}$. Then, the following statements are equivalent:*

(i) There are positive diagonal matrices P_1, P_2, and P_3 such that

$$B = \begin{bmatrix} A_1^T P_1 + P_1 A_1 + P_2 + P_3 & P_1 A_2 & P_1 A_3 \\ A_2^T P_1 & -P_2 & 0 \\ A_3^T P_1 & 0 & -P_3 \end{bmatrix} \prec 0.$$

(ii) For any nonzero positive semidefinite matrix $H \in \mathbb{R}^{3n\times 3n}$, partitioned into 3×3 block matrices with each block in $\mathbb{R}^{n\times n}$ as

$$H = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{12}^T & H_{22} & H_{23} \\ H_{13}^T & H_{23}^T & H_{33} \end{bmatrix},$$

satisfying the conditions

$$\mathrm{diag}(H_{11}) = \mathrm{diag}(H_{22}) = \mathrm{diag}(H_{33}),$$

at least one diagonal element of the matrix

$$A_1H_{11} + A_2H_{12}^T + A_3H_{13}^T$$

is negative.

3.2. New characterizations

Theorem 3. *Let $A_1, A_2, A_3 \in \mathbb{R}^{n\times n}$. The triple (A_1, A_2, A_3) admits a positive diagonal solution with respect to* (1) *if and only if the triple $(A_1 \circ S_{11}, A_2 \circ S_{12}, A_3 \circ S_{13})$ also admits a positive diagonal solution with respect to* (1) *for any positive semidefinite matrix $S \in \mathbb{R}^{3n\times 3n}$ of the form*

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12}^T & S_{22} & S_{23} \\ S_{13}^T & S_{23}^T & S_{33} \end{bmatrix},$$

where S_{11}, S_{22}, S_{33} satisfy the condition $\mathrm{diag}(S_{11}) = \mathrm{diag}(S_{22}) = \mathrm{diag}(S_{33}) \gg 0$.

Proof. Necessity: Suppose that H is a nonzero positive semidefinite matrix in $\mathbb{R}^{3n\times 3n}$ in the form

$$H = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{12}^T & H_{22} & H_{23} \\ H_{13}^T & H_{23}^T & H_{33} \end{bmatrix},$$

with $\mathrm{diag}(H_{11}) = \mathrm{diag}(H_{22}) = \mathrm{diag}(H_{33})$. Now, define the matrix $K = S \circ H$. According to Lemma 1, K is a positive semidefinite matrix. Furthermore, as the diagonal entries of S are strictly positive and H is a nonzero matrix, then K must be a nonzero matrix. Additionally, by the conditions $\mathrm{diag}(H_{11}) = \mathrm{diag}(H_{22}) = \mathrm{diag}(H_{33})$ and $\mathrm{diag}(S_{11}) = \mathrm{diag}(S_{22}) = \mathrm{diag}(S_{33})$, it is clear that the same condition holds for K as well, i.e.,

$$\mathrm{diag}(K_{11}) = \mathrm{diag}(K_{22}) = \mathrm{diag}(K_{33}).$$

Since, by assumption, the triple (A_1, A_2, A_3) admits a positive diagonal solution with respect to (1), then, by Theorem 1, the matrix

$$A_1K_{11} + A_2K_{12}^T + A_3K_{13}^T$$

has at least one diagonal element that is less than zero. Next, observe that $K_{11} = S_{11} \circ H_{11}$, $K_{12} = S_{12} \circ H_{12}$, and $K_{13} = S_{13} \circ H_{13}$. Moreover, for every $k \in \{1, \ldots, n\}$, we have

$$\begin{aligned}(A_1 K_{11})_{kk} &= (A_1(S_{11} \circ H_{11}))_{kk} \\ &= \sum_{i=1}^{n} a_{ki}^1 (S_{11} \circ H_{11})_{ik} \\ &= \sum_{i=1}^{n} a_{ki}^1 s_{ik}^{11} h_{ik}^{11} \\ &= \sum_{i=1}^{n} a_{ki}^1 s_{ki}^{11} h_{ik}^{11} \\ &= \sum_{i=1}^{n} (A_1 \circ S_{11})_{ki} h_{ik}^{11} \\ &= ((A_1 \circ S_{11}) H_{11})_{kk}.\end{aligned}$$

Similarly, we have

$$\begin{aligned}(A_2 K_{12}^T)_{kk} &= (A_2(S_{12}^T \circ H_{12}^T))_{kk} \\ &= \sum_{i=1}^{n} a_{ki}^2 (S_{12}^T \circ H_{12}^T)_{ik} \\ &= \sum_{i=1}^{n} a_{ki}^2 s_{ki}^{12} h_{ki}^{12} \\ &= \sum_{i=1}^{n} (A_2 \circ S_{12})_{ki} h_{ki}^{12} \\ &= ((A_2 \circ S_{12}) H_{12}^T)_{kk},\end{aligned}$$

and using the same computations, we obtain that

$$(A_3 K_{13}^T)_{kk} = ((A_3 \circ S_{13}) H_{13}^T)_{kk}.$$

Since

$$A_1 K_{11} + A_2 K_{12}^T + A_3 K_{13}^T$$

has at least one diagonal element, i.e., there is some $k \in \{1, \ldots, n\}$ such that

$$(A_1 K_{11} + A_2 K_{12}^T + A_3 K_{13}^T)_{kk} < 0,$$

hence, it follows that

$$((A_1 \circ S_{11}) H_{11})_{kk} + ((A_2 \circ S_{12}) H_{12}^T)_{kk} + ((A_3 \circ S_{13}) H_{13}^T)_{kk} < 0.$$

Finally, using Theorem 2, this implies that the triple $(A_1 \circ S_{11}, A_2 \circ S_{12}, A_3 \circ S_{13})$ admits a positive diagonal solution with respect to (1).

Sufficiency: Let S be the $3n \times 3n$ matrix of all ones, which is positive semidefinite with $\operatorname{diag}(S_{11}) = \operatorname{diag}(S_{22}) = \operatorname{diag}(S_{33}) = \mathbf{1} \gg 0$. Then, for each i, $A_i \circ S_{ii} = A_i$, and the condition reduces to the original triple (A_1, A_2, A_3), which holds by assumption. Now, the proof is complete.

Theorem 4. *Let $A_1, A_2, A_3 \in \mathbb{R}^{n\times n}$. The triple (A_1, A_2, A_3) admits a positive diagonal solution with respect to (1) if and only if $-(A_1 \circ S_{11} + A_2 \circ S_{12} + A_3 \circ S_{13})$ is a P-matrix for any positive semidefinite matrix $S \in \mathbb{R}^{3n\times 3n}$ of the form*

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12}^T & S_{22} & S_{23} \\ S_{13}^T & S_{23}^T & S_{33} \end{bmatrix},$$

where S_{11}, S_{22}, S_{33} satisfy the condition $diag(S_{11}) = diag(S_{22}) = diag(S_{33}) \gg 0$.

Proof. Necessity: By assuming that $A_1, A_2, A_3 \in \mathbb{R}^{n\times n}$ admits a positive diagonal solution, it follows, by Theorem 3, that the triple $(A_1 \circ S_{11}, A_2 \circ S_{12}, A_3 \circ S_{13})$ has a positive diagonal solution as well. Thus, by Lemma 3, the matrix

$$A_1 \circ S_{11} + A_2 \circ S_{12} + A_3 \circ S_{13}$$

is Lyapunov diagonally stable. Finally, according to Lemma 2, this means that

$$-(A_1 \circ S_{11} + A_2 \circ S_{12} + A_3 \circ S_{13})$$

is a P-matrix.

Sufficiency: Suppose that H is a nonzero positive semidefinite matrix in $\mathbb{R}^{3n\times 3n}$ partitioned as the following

$$H = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{12}^T & H_{22} & H_{23} \\ H_{13}^T & H_{23}^T & H_{33} \end{bmatrix},$$

and satisfying the condition $\operatorname{diag}(H_{11}) = \operatorname{diag}(H_{22}) = \operatorname{diag}(H_{33})$. Next, construct a matrix $S \in \mathbb{R}^{3n\times 3n}$ to be such that

$$s_{ij} = \begin{cases} h_{ij} & \text{if } i \neq j \\ h_{ii} & \text{if } i = j \text{ and } h_{ii} > 0 \\ 1 & \text{if } i = j \text{ and } h_{ii} = 0. \end{cases}$$

Clearly, S is positive semidefinite with strictly positive diagonal entries. Furthermore, it satisfies the condition $\operatorname{diag}(S_{11}) = \operatorname{diag}(S_{22}) = \operatorname{diag}(S_{33})$ because $\operatorname{diag}(H_{11}) = \operatorname{diag}(H_{22}) = \operatorname{diag}(H_{33})$. Therefore, we have

$$-(A_1 \circ S_{11} + A_2 \circ S_{12} + A_3 \circ S_{13})$$

is a P-matrix. Now, construct $v \in \mathbb{R}^n$ to be such that $v_k = 1$ when $h_{kk} > 0$ and $v_k = 0$ when $h_{kk} = 0$ for $k \in \{1, \ldots, n\}$. Hence, using the P-matrix properties, there is an index k such that

$$v_k[(A_1 \circ S_{11} + A_2 \circ S_{12} + A_3 \circ S_{13})v]_k < 0.$$

This inequality means that

$$\sum_{i=1}^{n} a_{ki}^1 h_{ki}^{11} + \sum_{i=1}^{n} a_{ki}^2 h_{ki}^{12} + \sum_{i=1}^{n} a_{ki}^3 h_{ki}^{13} < 0,$$

which implies that

$$(A_1 H_{11} + A_2 H_{12}^T + A_3 H_{13}^T)_{kk} < 0.$$

It follows, from Theorem 2, that the triple (A_1, A_2, A_3) admits a positive diagonal solution. Now, the proof is complete.

4. Conclusions

We have characterized the existence of positive diagonal solutions for a class of LMIs, providing equivalent conditions via test matrices (Theorems 1, 2) and Hadamard product transformations (Theorems 3, 4). These results generalize classical stability criteria and offer tools for analyzing multi-matrix systems in control and dynamics.

5. Conclusions

In this paper, we have investigated the existence of positive diagonal solutions for a class of linear matrix inequalities involving a structured block matrix defined by a triple of matrices (A_1, A_2, A_3). We established several equivalent conditions that characterize such solutions, linking the negative definiteness of the block matrix to properties of positive semidefinite test matrices and to the P-matrix property of transformed matrices under Hadamard products.

These results generalize classical Lyapunov diagonal stability conditions to a multi-matrix setting, offering new insights and tools for the stability analysis of complex systems. In particular, the characterizations provided in this paper contribute to the understanding of how structural constraints and matrix interactions affect stability, with potential applications in control theory, networked systems, and biological models.

Future work may explore numerical algorithms to compute such diagonal solutions efficiently and investigate extensions to nonlinear systems or systems with time-varying parameters. The approach developed here opens avenues for further research on structured stability criteria in high-dimensional and interconnected dynamical systems.

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