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An Enhanced Class of Estimators for the Population Mean Using Neutrosophic Statistics: A Case Study of the Islamabad Stock Exchange

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Abstract. Point estimates have their limitations in survey sampling due to the fact that they provide just a single value for the parameter under study, which may vary between samples due to sampling errors. By producing interval estimates of the expected position of the parameter, the neutrosophic approach serves as a viable alternative in sampling theory. The neutrosophic approach optimizes the traditional strategy for effectively handling ambiguous data. To find the mean of the population using neutrosophic information we introduce a new family of estimators that incorporate additional information. Discovering the bias and mean square error is performed up to the first-order approximation. These estimators are ideal for data which is logical, confused, or ambiguous. This estimator is designed to make neutrosophic statistics (NS) in basic random sampling easier to understand. To better show the range of possible values for our population parameter, we show numerical findings for these estimators as intervals rather than single points. To further assess the efficiency of our proposed neutrosophic estimator, we utilize interval data and simulation derived from the Islamabad Stock Exchange (ISE).

2020 Mathematics Subject Classifications: 62D05, 62f10

Key Words and Phrases: Neutrosophic statistics, simulation, visualization, mean estimation, bias, mean squared error (MSE), percentage relative efficiency (PRE)

1. Introduction

In conventional statistics, data are already established and are represented by precise figures. The estimation of population means using auxiliary data has been extensively studied and developed by numerous scholars in traditional statistics. Typically, a specific value is assigned to a study variable for each population unit before sample selection. A high correlation between the study and auxiliary variables significantly lowers the ratio's sampling

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error when the auxiliary variable is also considered. This impacts the smallest necessary sample size for ratio estimation methods, or how much these methods can reduce sample size without losing precision. Classical statistics focus on fixed data, where all possible measurement errors are preemptively addressed, leading to a scenario with no measurement error margin. Consequently, new approaches are required to handle indeterminate data. Fuzzy logic (FL) is one approach for managing data when precise measurements of the variable of interest are unavailable. Fuzzy statistics allow for the analysis of data with uncertain, ambiguous, or undefined characteristics, although this method does not account for the level of data indeterminacy. Conversely, neutrosophic logic extends FL, enabling the assessment of both conclusive and inconclusive elements of data under vague or uncertain conditions. The utilization of uncertain logic in decision-making processes has surged Ali and Mahmood [3] and Jan et al. [9], accompanied by an enhancement in the complexity of its methodologies. Following the concept of fuzzy sets, the development of a complex neutrosophic set Liu et al. [11] represents an advanced stage of evolution in fuzzy collections. These collections and their broader applications are detailed in Li et al. [10], which includes thorough discussions of their characteristics and functionalities, and also examines interval-valued neutrosophic sets. When fuzzy sets are inadequate for addressing ambiguities in decision-making contexts, neutrosophic sets emerge as a superior alternative. Neutrosophic sets are categorized into diverse types. Chakraborty et al. [14] has introduced a trapezoidal bipolar neutrosophic quantity, specifically designed for decision-making applications. Additionally, generalized spherical fuzzy numbers and their analytical frameworks were discussed in a publication by Haque et al.

Different research support the use of a mobile application to handle geometric and mathematical operations on pentagonal neutrosophic numbers Chakraborty et al. [13]. The obsession with neutrosophic numbers intense with time; for example, a technique has been devised for the cylindrical neutrosophic environment Chakraborty et al. [12]. Neutrosophic statistics provide a solution for issues involving ambiguous data display. It is imperative to use neutrosophic statistics instead of traditional statistics when data or samples show neutrosophic features. Statistically, neutrosophic approaches Smarandache [16] are used where the data from populations or samples are vague, obscure, or not well-defined. This discussion covers several uses of neutrosophic logic: by addressing uncertainties in the opinions of survey participants, neutrosophic logic enhances survey sampling techniques. It adopts a more intricate approach to data gathering and analysis by accepting ambiguity, conflicting information, and indeterminate details. This approach leads to a deeper understanding of diverse viewpoints, thereby improving the accuracy and reliability of survey results.

Neutrosophic data pertains to datasets characterized by inherent uncertainties, analyzed through neutrosophic numerical methods. In the realm of neutrosophic statistics, the exact sample size may remain undefined Smarandache [16]. It has been established that neutrosophic statistics provide substantial benefits and should be integrated into current models dealing with uncertainty Aslam [23] and Smarandache [24]. Rock engineers utilize neutrosophic numbers to investigate the effects of gauge and anisotropy on the total roughness coefficient, which results in reduced loss of information and the generation of an adequate number of appropriate functions Chen et al. [25]. A novel method for conducting neutrosophic analysis of variance has been introduced within the framework of neutrosophic data Aslam [30]. Contributions from Singh et al. [32] and Alsharif et al. [33] have discussed diverse kerosene-based nano-fluids in a dual-zone vertical annulus with electro-osmosis. Pioneers in the area of neutrosophic interval statistics, Aslam [26], Aslam [27], Aslam [28] and Aslam [29] have led developments in their fields. Further information on neutrosophic data is available in references Tahir et al. [17], Yadav and Smarandache [18], Bouze and Subzar [21], Aslam [22], Das et al [36], Granados et al. [34].

In real-world applications, a variety of factors can be examined, yet data collection can be exceedingly costly, particularly when the data is not clear. This situation makes it expensive and risky to accurately determine a population's value using methods designed for uncertain data. When both primary and auxiliary variables are neutrosophic, it becomes unfeasible to describe the issue. Reviews of existing literature reveal that there has been no substantial research on the use of enhanced generalized class estimators for estimating the unknown mean of a population with auxiliary variables in survey sampling. This observation is based on current studies in published works. The field of statistics is yet to see significant pioneering research to be deemed complete. This research begins to address this gap.

Neutrosophic analysis is advantageous for dealing with data that is either incomplete or vague. This approach also accommodates multiple viewpoints. During data collection using specific instruments, it is conceivable that some data may fall into an undefined category. The failure of traditional analytical methods due to statistical ambiguity necessitates the adoption of neutrosophic statistics, which acts as both a substitute and a supplement to traditional statistics in uncertain conditions. Recent progress in neutrosophic survey sampling is significant, however, the wider field of estimation is still largely uncharted and requires meticulous attention due to the uncertainties involved in the data framework. Products with minor flaws in measurements or manufacturing defects are acceptable within a certain tolerance, such as in the production of nuts or bolts. Employing traditional statistical methods that yield a single result could result in the discarding of otherwise usable items. Neutrosophic statistics offer the most accurate evaluation of interval results with the minimal mean squared error, effectively tackling these issues Kumar et al. [19], Raghav [20] and Granado et al. [35].

1.1. Research gap

Previously researchers who studied survey sampling only looked at data that was particular, clear, and certain. The one thing that all of these methods can reliably give you is not always free of mistakes, overstatements, or underestimates. Using these methods only leads to one correct result. Neutrosophic data, on the other hand, are present in many circumstances and under certain conditions. In this case, neutrosophic statistics are used even when more typical statistical approaches didn't work. Neutrosophic data includes observations that aren't clear, reasoning that isn't clear, and fuzzy interval values. This means that data from trials or populations can be used as neutrosophic numbers with interval values. One of the hypotheses said that the genuine observation, whose value could not be known when the data was being collected, would fall within that range. In the real world, there is a lot more data that is hard to define than data that is easy to define. This shows that neutrosophic research needs more statistical methods to be devised. In the actual world, it might be hard and expensive to get information on all the possible study parts, especially when the proof isn't clear. It is impossible to explain the problem when a study and its extra variables are neutrosophic. A literature review indicated that enhanced generalized class estimators have not been studied while trying to use regression cum-exponential estimation methods to find a way to estimate the unknown population mean with extra variables in neutrosophic data. This is the conclusion that comes from the review's results. Therefore, there is a demand for additional statistical methods tailored to neutrosophic research. This is clear from a thorough look of Alomair and Ahmad [1]. More research has to be done in this area of statistics. This study gives a basic overview of the topic.

Table 1: LIST OF ACRONYMS

MSE	Mean Square Error
PRE	Percentage Relative Efficiency
SRSWOR	Simple Random Sampling Without Replacement
ISE	Islamabad Stock Exchange
NS	Neutrosophic statistics
FL	Fuzzy Logic

2. Methods and materials

Choose a neutrosophic sample of size $n_N\in (n_L,\ n_U)$, from a population of N_N units $(\Omega_1,\ \Omega_2,\ \ldots,\ \Omega_N)$ with the help of *SRSWOR*. Allow y_{Ni} be the i^{th} sample observation of neutrosophic data, of the form $y_N \in (y_L,\ y_U)$ and similarly for auxiliary variable $x_N\in (x_L,\ x_U)$. As $\overline{Y}_N\in (Y_L, Y_U)$, and $\overline{X}_N\in (X_L, X_U)$ X_U is the population mean of neutrosophic study and auxiliary variable. The neutrosophic coefficient of variation for y_N and x_N are denoted by $C_{yN}\in (C_{yNL},\ C_{yNU})$, and $C_{xN}\in (C_{xNL},\ C_{xNU})$. Let $s^2_{yN} = \frac{\sum_{i=1}^{n}(y_{iN}-\overline{y}_N)^2}{n_N-1}$, $s^2_{xN} = \frac{\sum_{i=1}^{n}(x_{iN}-\overline{x}_N)^2}{n_N-1}$, be the unbiased sample variances conforming to population variances $S^2_{yN} = \frac{\sum_{i=1}^{N}(y_{iN}-\overline{Y}_N)^2}{N_N-1}$, $S^2_{xN} = \frac{\sum_{i=1}^{N}\left(x_{iN}-\overline{X}_N\right)^2}{N_N-1}$, of Y_N and X_N respectively.

Let C_{yN} and C_{xN}, denote the population coefficient of variation of y_N and x_N, where $C_{yN}=\frac{S_{yN}}{\overline{Y}}$, and $C_{XN}=\frac{S_{xN}}{\overline{X}}$.

Where $S_{yN} = \sqrt{\frac{\sum_{i=1}^{N}(y_{iN}-\overline{Y}_N)^2}{N_N-1}}$ and $S_{xN}= \sqrt{\frac{\sum_{i=1}^{N}\left(x_{iN}-\overline{X}_N\right)^2}{N_N-1}}$. The correlation between

y_N and x_N is denoted by ρ_{yxN}. i.e. $\rho_{yxN}= \frac{S_{yxN}}{S_{yN}S_{xN}}$,

$$S_{yx} = \frac{\sum_{i=1}^{N} \left(y_{iN} - \overline{Y}_N\right)\left(x_{iN} - \overline{X}_N\right)}{N_N - 1}, \lambda_N = \left(\frac{1}{n_N} - \frac{1}{N_N}\right), \lambda_N \in (\xi_L, \xi_U).$$

3. Review of existing estimators

In this section, we have presented some prevailing counterparts.

(i) The conventional estimator is given in equation (1):

$$\widehat{\overline{Y}}_{UN} = \overline{y}_N \tag{1}$$

The variance of $\widehat{\overline{Y}}_{UN}$is given in equation (2):

$$V(\widehat{\overline{Y}}_{UN}) = \lambda_N \overline{Y}_N^2 C_{yN}^2 \tag{2}$$

(ii) The adopted ratio estimator recommended by Cochran [42] is given in equation (3):

$$\widehat{\overline{Y}}_{RN} = \overline{y}_N \left(\frac{\overline{X}_N}{\overline{x}_N}\right) \tag{3}$$

The bias of $\widehat{\overline{Y}}_{RN}$ is given by:

Bias($\widehat{\overline{Y}}_{RN}$) $\cong \lambda_N \overline{Y}_N \left(C_{yN}^2 - \rho_{yxN} C_{yN} C_{xN}\right)$,

The MSE of $\widehat{\overline{Y}}_{RN}$ is given in equation (4):

$$MSE(\widehat{\overline{Y}}_{RN}) \cong \lambda_N \overline{Y}_N^2 \ \left(C_{yN}^2 + C_{xN}^2 - 2\rho_{yxN} C_{yN} C_{xN}\right) \tag{4}$$

(iii) The product estimator suggested by Murthy [7], given in equation (5):

$$\widehat{\overline{Y}}_{PN} = \overline{y}_N \left(\frac{\overline{x}_N}{\overline{X}_N}\right) \tag{5}$$

The bias of $\widehat{\overline{Y}}_{PN}$ is given by:

Bias ($\widehat{\overline{Y}}_{PN}$) $\cong \lambda_N \overline{Y}_N \ \rho_{yxN} C_{yN} C_{xN}$,

The MSE of $\widehat{\overline{Y}}_{PN}$ is given in equation (6):

$$MSE(\widehat{\overline{Y}}_{PN}) \cong \lambda_N \overline{Y}_N^2 \ \left(C_{yN}^2 + C_{xN}^2 + 2\rho_{yxN} C_{yN} C_{xN}\right) \tag{6}$$

(iv) The usual difference estimator is given in equation (7):

$$\widehat{\widehat{Y}}_{DN} = \overline{y}_N + Q\left(\overline{X}_N - \overline{x}_N\right), \tag{7}$$

where Q is constant.

$Q= \rho_{yxN}\left(\frac{S_{yN}}{S_{xN}}\right)$

The variance of $\widehat{\widehat{Y}}_{DN}$ is given in equation (8):

$$\mathrm{Var}(\widehat{\widehat{Y}}_{DN})_{\ min} \cong \lambda_N \overline{Y}_N^2 C_{yN}^2 (1-\rho_{yxN}^2) = \ \mathrm{MSE}(\hat{P}_{DN})_{\ min} \tag{8}$$

(v) Bahl and Tuteja [6] recommended the following two estimators, which are given in Equation (9) and Equation (10):

$$\widehat{\widehat{Y}}_{BTRN} = \overline{y}_N exp\left(\frac{\overline{X}_N - \overline{x}_N}{\overline{X}_N + \overline{x}_N}\right) \tag{9}$$

$$\widehat{\widehat{Y}}_{BTPN} = \overline{y}_N exp\left(\frac{\overline{x}_N - \overline{X}_N}{\overline{X}_N + \overline{x}_N}\right) \tag{10}$$

The bias of $\widehat{\widehat{Y}}_{BTN}$ is given by:

$\mathrm{Bias}\left(\widehat{\widehat{Y}}_{BTRN}\right) \cong \ \lambda_N \ \overline{Y}_N \left[\frac{3}{8}\ C_{xN}^2 - \frac{1}{2}\rho_{yxN}C_{yN}C_{xN}\right],$

The MSE of $\widehat{\widehat{Y}}_{BTRN}$ is given in equation (11):

$$MSE\left(\widehat{\widehat{Y}}_{BTRN}\right) \cong \lambda_N \overline{Y}_N^2 \left[C_{yN}^2 + \frac{1}{4}C_{xN}^2 - \rho_{yxN}C_{yN}C_{xN}\right] \tag{11}$$

The bias of $\widehat{\widehat{Y}}_{BTPN}$ is given by:

$\mathrm{Bias}\left(\widehat{\widehat{Y}}_{BTPN}\right) \cong \ \lambda_N \ \overline{Y}_N \left[\frac{3}{8}\ C_{xN}^2 + \frac{1}{2}\rho_{yxN}C_{yN}C_{xN}\right],$

The MSE of $\widehat{\widehat{Y}}_{BTPN}$ is given in equation (12):

$$MSE\left(\widehat{\widehat{Y}}_{BTPN}\right) \cong \lambda_N \overline{Y}_N^2 \left[C_{yN}^2 + \frac{1}{4}C_{xN}^2 + \rho_{yxN}C_{yN}C_{xN}\right] \tag{12}$$

(vi) Singh et al. [31] recommended a generalized exponential-type estimator, which is given in equation (13):

$$\widehat{\widehat{Y}}_{SN} = \overline{y}_N exp\left(\frac{a\left(\overline{X}_N - \overline{x}_N\right)}{a\left(\overline{X}_N + \overline{x}_N\right) + 2b}\right) \tag{13}$$

$\text{Bias}\left(\widehat{\overline{Y}}_{SN}\right) \cong \lambda_N \, \overline{Y}_N \left(\frac{3}{8}\, \theta_N^2 C_{xN}^2 - \frac{1}{2}\theta_N \rho_{yxN} C_{yN} C_{xN}\right),$

The mean square error of $\widehat{\overline{Y}}_{SN}$ is given in equation (14):

$$MSE\left(\widehat{\overline{Y}}_{SN}\right) = \lambda_N \overline{Y}_N^2 \left(C_{yN}^2 + \frac{1}{4}\theta_N^2 C_{xN}^2 + \theta_N \rho_{yxN} C_{yN} C_{xN}\right), \tag{14}$$

where $\theta_N = \frac{a\overline{X}_N}{a\overline{X}_N + b}$.

4. Suggested estimator:

By taking Inspiration from Alomair and Ahmad [1], Ahmad et al. [2] and Ahmad et al. [4], we propose enhanced generalized neutrosophic estimators for calculating the population mean with simple random sampling and auxiliary data. These neutrosophic estimators could outperform traditional estimators when the observations of the study variable are nondeterministic, but might not be as effective when the observations are deterministic. The proposed estimator is presented in equation (15):

$$\widehat{\overline{Y}}^{(*)}_{Prop,\ N} = \left[\ Q_3 \overline{y}_N + \ \ Q_4 \left(\overline{X}_N - \overline{x}_N\right)\right] exp\left(\frac{a\left(\overline{X}_N - \overline{x}_N\right)}{a\left(\overline{X}_N + \overline{x}_N\right) + 2b}\right) \tag{15}$$

where $\theta_N = \frac{a\overline{X}_N}{a\overline{X}_N + b}$

Some members of the generalized class of estimators are given in Table 2.

Table 2: Some estimators of $\widehat{\overline{Y}}^{(*)a,b}_{Prop,\ N}$

a	b	$\widehat{\overline{Y}}^{(*)}_{Prop,\ N}$
1	C_{xN}	$\widehat{\overline{Y}}^{(1)}_{Prop,\ N}$
1	$\beta_{\mathbf{2}(xN)}$	$\widehat{\overline{Y}}^{(2)}_{Prop,\ N}$
$\beta_{\mathbf{2}(xN)}$	C_{xN}	$\widehat{\overline{Y}}^{(3)}_{Prop,\ N}$
C_{xN}	$\beta_{\mathbf{2}(xN)}$	$\widehat{\overline{Y}}^{(4)}_{Prop,\ N}$
1	ρ_{yxN}	$\widehat{\overline{Y}}^{(5)}_{Prop,\ N}$
C_{xN}	ρ_{yxN}	$\widehat{\overline{Y}}^{(6)}_{Prop,\ N}$
ρ_{yxN}	C_{xN}	$\widehat{\overline{Y}}^{(7)}_{Prop,\ N}$
$\beta_{\mathbf{2}(xN)}$	ρ_{yxN}	$\widehat{\overline{Y}}^{(8)}_{Prop,\ N}$
ρ_{yxN}	$\beta_{\mathbf{2}(xN)}$	$\widehat{\overline{Y}}^{(9)}_{Prop,\ N}$
1	$N_N \overline{X}_N$	$\widehat{\overline{Y}}^{(10)}_{Prop,\ N}$

Simplifying equation (15), we got:

$$\widehat{\overline{Y}}^{(*)}_{Prop,\ N} = \left\{\ Q_3\overline{Y}_N\left(1+\xi_0\right) -\ Q_4\overline{X}_N\xi_1\right\} exp\left(1-\frac{\theta_N\xi_1}{2}+\frac{3\theta_N^2\xi_1^2}{8}+\ldots\right) \quad (16)$$

By expanding equation (16):

$$\left(\widehat{\overline{Y}}^{(*)}_{Prop,\ N} - \overline{Y}_N\right) = \begin{bmatrix} -\overline{Y}_N + \overline{Y}_N Q_3 + \overline{Y}_N\xi_0\ Q_3 - \dfrac{1}{2}\ \overline{Y}_N\ \theta_N\xi_1\ Q_3 - \overline{X}_N\xi_1\ Q_4 \\ -\dfrac{1}{2}\ \overline{Y}_N\ \theta_N\xi_0\xi_1\ Q_3 + \dfrac{3}{8}\overline{Y}_N\theta_N^2\xi_1^2\ Q_3 + \dfrac{1}{2}\overline{X}_N\ \theta_N\xi_1^2\ Q_4 \end{bmatrix} \quad (17)$$

From equation (17), the bias and MSE of $\widehat{\overline{Y}}^{(*)}_{Prop,\ N}$, are given by:

$\text{Bias}\left(\widehat{\overline{Y}}^{(*)}_{Prop,\ N}\right) \cong \frac{1}{8}\left[-8\overline{Y}_N + 4\lambda_N\theta_N C_{xN}(\overline{X}_N C_{xN}\ Q_4) + \overline{Y}_N\ Q_3\left\{8+\lambda_N\theta_N C_{xN}\left(3\theta_N C_{xN} - 4C_{yN}\rho_{yxN}\right)\right\}\right]$

The MSE of $\widehat{\overline{Y}}^{(*)}_{Prop,\ N}$, is given in equation (18):

$$MSE\left(\widehat{\overline{Y}}^{(*)}_{Prop,\ N}\right) \cong \begin{bmatrix} \overline{Y}_N^2 + \lambda_N\overline{X}_N C_{xN}^2\ Q_4\left(-\overline{Y}_N\theta_N + \overline{X}_N\ Q_4\right) \\ +\overline{Y}_N^2 Q_4^2\left[1+\lambda_N\left\{C_{yN}^2 + \theta_N C_{xN}\left(\theta_N - 2C_u\rho_{yxN}\right)\right\}\right] \\ +\dfrac{1}{4}\overline{Y}_N\ Q_3\left[-8\overline{Y}_N + C_{xN}\left\{\theta_N C_{xN}\left(-3\overline{Y}_N\theta_N + 8\overline{X}_N\ \ Q_4\right)\right\}\right] \\ \dfrac{1}{4}\overline{Y}_N\ Q_3\left[+4C_{yN}\left(\overline{Y}_N\theta_N - 2\overline{X}_N\ Q_4\right)\rho_{yxN}\right] \end{bmatrix} \quad (18)$$

The values of Q_3 and Q_4 are:

$$Q_{3(opt)} = \frac{8-\lambda_N\theta_N^2 C_{xN}^2}{8\left\{1+\lambda_N C_{yN}^2\left(1-\rho_{yxN}\right)\right\}},$$

and

$$Q_{4(opt)} = \frac{\overline{Y}_N\left[\lambda_N\theta_N^2 C_{xN}^3 + 8C_{yN}\rho_{yxN} - \lambda_N\theta_N^2 C_{xN}^2 C_{yN}\rho_{yxN} - 4\boldsymbol{\theta_N} C_{xN}(1-\lambda_N C_{yN}^2\{1-\rho_{yxN}^2\})\right]}{8\overline{X}_N C_{xN}\{1-\lambda_N C_{yN}^2\{1-\rho_{yxN}^2\}\}}$$

Using values of $Q_{3(opt)}$, and $Q_{4(opt)}$ in equation (18), we got the minimum mean square error, which is given in equation (19):

$$\text{MSE}\left(\widehat{\overline{Y}}^{(*)}_{Prop,\ N}\right)_{min} \cong \frac{\lambda_N\overline{Y}_N^2\left(64C_{yN}^2\left(1-\rho_{yxN}^2\right) - \lambda_N\theta_N^4 C_{xN}^4 - 16\lambda_N\theta_N^2 C_{xN}^2 C_{yN}^2\left(1-\rho_{yxN}^2\right)\right)}{64\left\{1+\lambda_N C_{yN}^2\left(1-\rho_{yxN}^2\right)\right\}} \quad (19)$$

Figure 1:

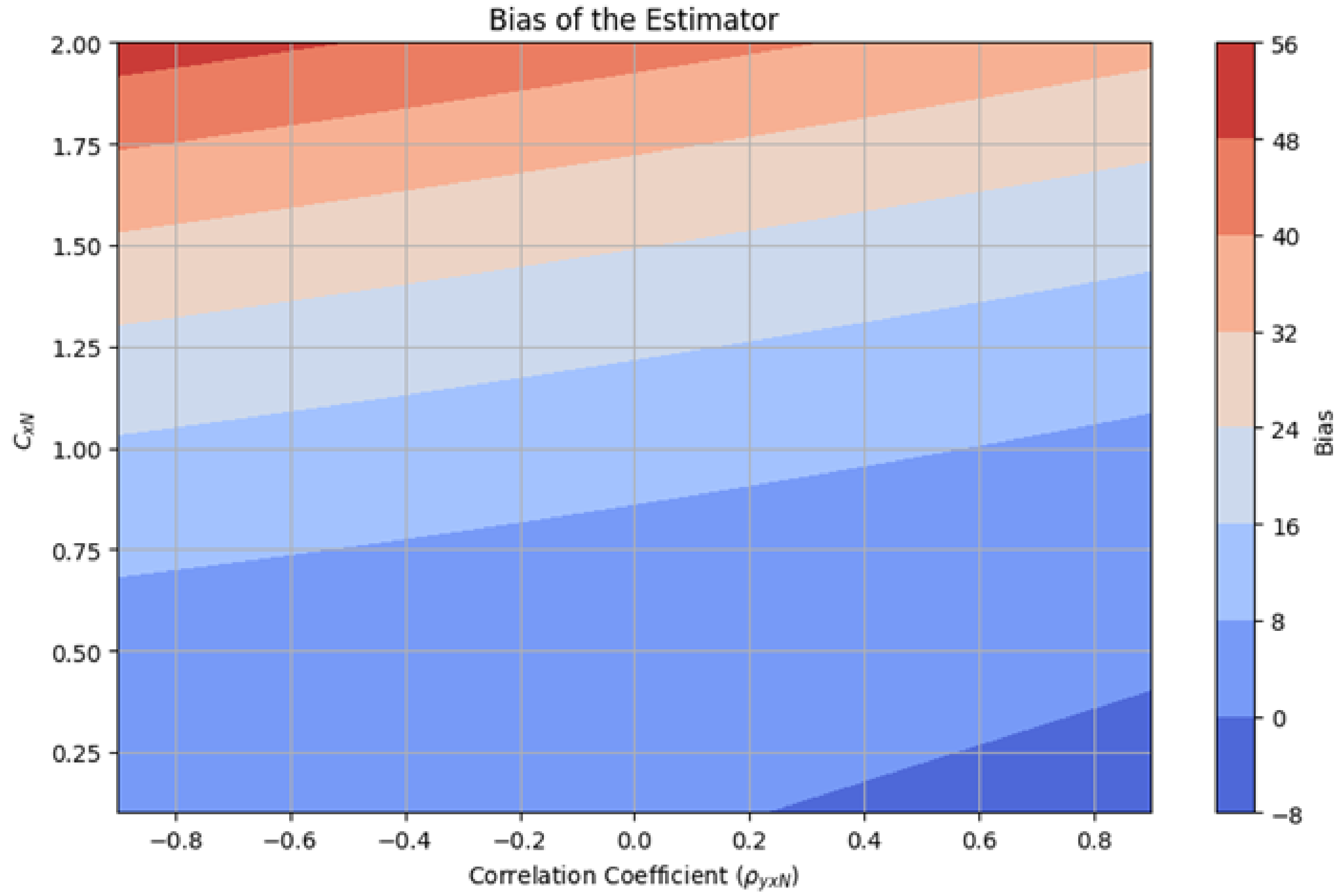

Figure 1: Showing Bias of Estimator

5. Numerical study

In this section, we utilize real data to compare the proposed and current estimators numerically. A numerical study was performed using interval data containing uncertain values from the ISE, focusing on United Bank Limited (UBL), taken from Alomair and Ahmad [1]. In Table 2 we see the data sets and their explanations. Estimators are compared with one another based on their PRE. The PRE of $\widehat{\overline{Y}}_{UN}$ with due regard to $\widehat{\overline{Y}}_{UN}$ is expressed below:

PRE= $\frac{Var(\widehat{\overline{Y}}_{UN})}{MSE\left(\widehat{\overline{Y}}_{i,NN}\right)}$ $\times 100$,

where $i = \widehat{\overline{Y}}_{RN}, \widehat{\overline{Y}}_{PN}, \widehat{\overline{Y}}_{DN}, \widehat{\overline{Y}}_{RDN}, \widehat{\overline{Y}}_{BTRN}, \widehat{\overline{Y}}_{BTPN}, \widehat{\overline{Y}}_{SN}$,
$\widehat{\overline{Y}}^{(*)}_{Prop,\ N}$ $(* = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10)$.

Table 3: Summary statistics using actual data

Parameter	Values	Parameter	Values
N_N	[239,239]	$\overline{X}_N$	[149.6231,153.5848]
n_N	[35,35]	S^2_{yN}	[2968.468,3131.635]
λ_N	[0.02438733, 0.02438733]	S^2_{xN}	[3110.931,3156.542]
$\overline{Y}_N$	[131.5651,135.6154]	ρ_{yxN}	[0.8680465,0.5235659]

Table 4: Biases using actual data

Estimator	Bias Values
$\widehat{\overline{Y}}_{UN}$	-
$\widehat{\overline{Y}}_{RN}$	[-0.002514721, -0.001244049]
$\widehat{\overline{Y}}_{PN}$	[0.003125356, 0.001836447]
$\widehat{\overline{Y}}_{DN}$	-
$\widehat{\overline{Y}}_{BTRN}$	[0.006398227, 0.006869877]
$\widehat{\overline{Y}}_{BTPN}$	[0.009523583, 0.008706324]
$\widehat{\overline{Y}}_{BTPN}$	[0.006390427, 0.00686232]
$\widehat{\overline{Y}}^{(*)}_{Prop,\ N}$	[-0.00273157, -0.0175342]

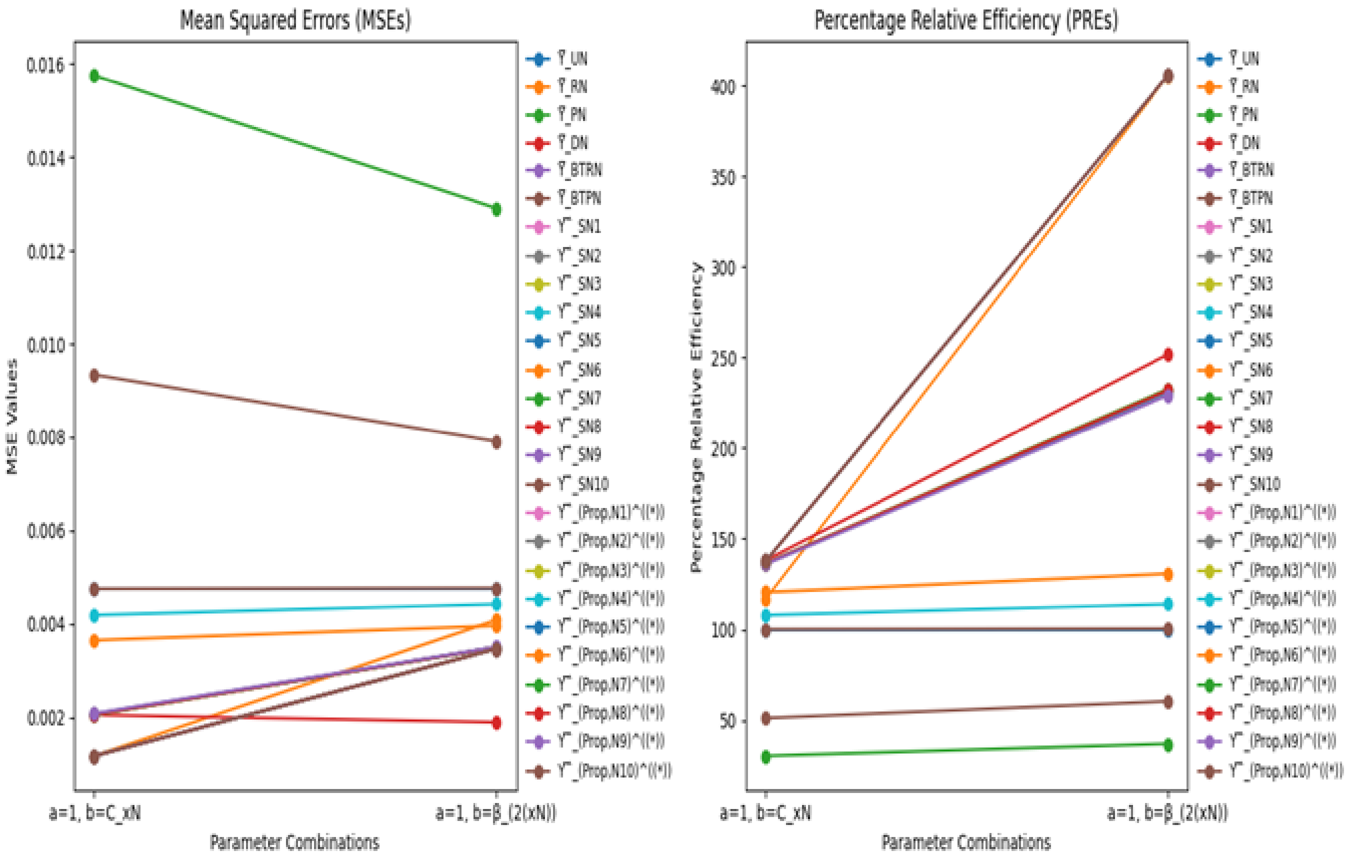

Figure 2: Showing MSEs and PREs of estimators based on real data set

Table 5: MSE using actual data

Estimator	Values	For values of a and b	$\widehat{\overline{Y}}_{SN}$	$\widehat{\overline{Y}}^{(*)}_{Prop,\ N}$
$\widehat{\overline{Y}}_{UN}$	[0.004771434, 0.004771434]	[For a=1, b=C_{xN}]	[0.002051822, 0.003495624]	[0.001176135, 0.003463482]
$\widehat{\overline{Y}}_{RN}$	[0.001176741, 0.004079918]	[For a=1, b=$\beta_{\mathbf{2}(xN)}$]	[0.002077862, 0.003500653]	[0.001176136, 0.003463481]
$\widehat{\overline{Y}}_{PN}$	[0.01574454, 0.01290342]	[For a=$\beta_{\mathbf{2}(xN)}$, b=C_{xN}]	[0.002051802, 0.00349562]	[0.001176135, 0.003463482]
$\widehat{\overline{Y}}_{DN}$	[0.002050780, 0.001895328]	[For a=C_{xN}, b=ρ_{yxN}]	[0.004184703, 0.004426577]	[0.001176135, 0.003463482]
$\widehat{\overline{Y}}_{BTRN}$	[0.002051786, 0.003495617]	[For a=1, b=ρ_{yxN}]	[0.002062184, 0.003496803]	[0.001176135, 0.003463482]
$\widehat{\overline{Y}}_{BTPN}$	[0.009335684, 0.007907368]	[For a=C_{xN}, b=ρ_{yxN}]	[0.003647159, 0.003956309]	[0.001176135, 0.003463482]
		[For a=ρ_{yxN}, b=C_{xN}]	[0.002051827, 0.00349563]	[0.001176135, 0.003463482]
		[For a=$\beta_{\mathbf{2}(xN)}$, b=ρ_{yxN}]	[0.002056549, 0.003496158]	[0.001176135, 0.003463482]
		[For a=ρ_{yxN}, b=$\beta_{\mathbf{2}(xN)}$]	[0.002081793, 0.003505421]	[0.001176135, 0.003463482]
		[For a=1, b=$N_N\overline{X}_N$]	[0.004756275, 0.004771342]	[0.001176135, 0.003463482]

5.1. Theoretical Analysis of MSE and PRE Using Actual Data

Mean Squared Errors (MSEs)

(i) The MSE gauges the average squared difference between the actual and projected values. Better estimate performance is shown by minimum MSE values.

(ii) Table 5 shows changing MSE values for several estimators and parameter combina-

Table 6: PRE using actual data

Estimator	**Values**	For values of a and b	$\widehat{\overline{\boldsymbol{Y}}}_{\boldsymbol{SN}}$	$\widehat{\overline{\boldsymbol{Y}}}^{(*)}_{\boldsymbol{Prop,\ N}}$
$\widehat{\overline{Y}}_{UN}$	[100,100]	[For a=1, b=C_{xN}]	[136.4973, 232.5462]	[137.7641, 405.6875]
$\widehat{\overline{Y}}_{RN}$	[116.9493, 405.4787]	[For a=1, b=$\beta_{\mathbf{2}(xN)}$]	[136.3012, 229.6319]	[137.7642, 405.6875]
$\widehat{\overline{Y}}_{PN}$	[30.30533, 36.97806]	[For a=$\beta_{\mathbf{2}(xN)}$, b=C_{xN}]	[136.4975, 232.5484]	[137.7631, 405.6865]
$\widehat{\overline{Y}}_{DN}$	[138.28, 251.7471]	[For a=C_{xN}, b=ρ_{yxN}]	[107.7906, 114.0208]	[137.7631, 405.6864]
$\widehat{\overline{Y}}_{BTRN}$	[136.4976, 232.5503]	[For a=1, b=ρ_{yxN}]	[136.4513, 231.3777]	[137.7661, 405.6895]
$\widehat{\overline{Y}}_{BTPN}$	[51.10963, 60.34162]	[For a=C_{xN}, b=ρ_{yxN}]	[120.6031, 130.8261]	[137.7641, 405.6874]
		[For a=ρ_{yxN}, b=C_{xN}]	[136.4971, 232.5456]	[137.7641, 405.6875]
		[For a=$\beta_{\mathbf{2}(xN)}$, b=ρ_{yxN}]	[136.4765, 232.0116]	[137.7641, 405.6875]
		[For a=ρ_{yxN}, b=$\beta_{\mathbf{2}(xN)}$]	[136.1159, 229.1983]	[137.7641, 405.6875]
		[For a=1, b=$N_N\overline{X}_N$]	[100.0019, 100.3187]	[137.7641, 405.6874]

tions. Estimator $\widehat{\overline{Y}}_{ARN}$, for instance, indicates that it offers reliable estimates with less variability by showing somewhat low MSE values compared to other estimators. With somewhat high MSE values, estimator $\widehat{\overline{Y}}_{PN}$ indicates poorer accuracy than other estimators.

(iii) The choice of parameters (a and b) influences the performance of the estimators.

For instance, estimators with $a{=}b{=}C_{xN}$ tend to have lower MSE values compared to other parameter combinations. This suggests that considering the mean of X values (C_{xN}) as both a and b yields more accurate estimates.

(iv) To minimize MSE, it is advisable to choose parameter combinations that lead to lower MSE values, such as $a{=}b{=}C_{xN}$.

Percentage Relative Efficiency (PREs)

(i) The PRE measures the relative prediction error compared to the actual values. Higher PRE values indicate greater prediction errors.

(ii) Table 6 shows different PRE values for several estimators and parameter combinations. for instance: With rather high PRE values, estimator $\widehat{\overline{Y}}_{ARN}$ indicates more prediction errors than other estimators. Relatively accurate predictions are shown by estimator $\widehat{\overline{Y}}^{(*)}_{Prop,\ N}$ whose PRE values cross to 100.

(iii) As compared to all existing considered estimators, the performance of $\widehat{\overline{Y}}^{(*)}_{Prop,\ N}$ is more accurate in terms of minimum MSE and higher percentage relative efficiency.

(iv) Similar to MSE, the choice of parameters (a and b) influences the PRE. Parameter combinations that involve the mean of X values (C_{xN}.) tend to yield lower PRE values, suggesting better prediction accuracy.

(v) To minimize prediction errors, it is advisable to choose parameter combinations that lead to lower PRE values, such as $a{=}b{=}C_{xN}$.

Overall Analysis

The suggested class of estimators $\widehat{\overline{Y}}^{(*)}_{Prop,\ N}$generally performs better in terms of minimum MSE and higher PRE compared to other estimators across different parameter combinations. Choosing parameter combinations where both a and b are equal to the mean of X values (C_{xN}) tends to lead to more accurate estimates and predictions.

6. Simulation study

This section illustrates the algebraic procedure for comparing $\boldsymbol{\widehat{\overline{Y}}^{(*)}_{Prop,\ N}}$ to its existing estimators, $\widehat{\overline{Y}}_{UN}$, $\widehat{\overline{Y}}_{RN}$, $\widehat{\overline{Y}}_{PN}$, $\widehat{\overline{Y}}_{DN}$, $\widehat{\overline{Y}}_{BTRN}$, $\widehat{\overline{Y}}_{BTPN}$, and $\widehat{\overline{Y}}_{SN}$.
Numerical results are validated by means of a simulation research. We simulated neutrosophic data and performed MSE and PRE based comparison. Neutrosophic random variables, X_N and Y_N follow the neutrosophic normal distribution. With mean μ_{XN} and variance σ^2_{XN} exhibits a neutrosophic normal distribution. $Y_N$likewise has mean μ_{YN} and variance σ^2_{YN}. When $X_N \in (X_L,\ X_U)$ and $Y_N \in (Y_L,\ Y_U)$. Comparatively mean and variance of neutrosophic random variables X_N and Y_N have lower and upper limits. We

have produced 5000 random normal variate from the neutrosophic normal distribution, i.e., $Y_N \sim N_N$ ((5, 8),((0.9)^2, (1.2)^2) and $X_N \sim N_N$ ((15, 18),((0.5)^2, (0.7)^2)).
The simulation study algorithm, which demonstrates the performance of different $\widehat{\overline{Y}}_{UN}$ estimators, is as follows:

(i) The Average mean square error of the estimators is defined by:

$$Average\ MSE\left(\widehat{\overline{Y}}_{UN}\right) = \frac{1}{500}\sum_{i=1}^{500} E(\widehat{\overline{Y}}_i - \widehat{\overline{Y}}_{UN})^2$$

(ii) The PRE of estimators as compared to the usual estimator $\widehat{\overline{Y}}_{UN}$ is defined by:

$$PRE\,(i) = \frac{Var(\widehat{\overline{Y}}_{UN})}{MSE(\widehat{\overline{Y}}_i)} \times 100$$

(iii) The Average percentage relative efficiency of the estimators is given by:

$$Average\ PRE\,(i) = \frac{1}{500}\sum_{i=1}^{500} PRE(\widehat{\overline{Y}}_i)$$

Table 7: Biases using simulated data

Estimator	Bias Values
$\widehat{\overline{Y}}_{UN}$	-
$\widehat{\overline{Y}}_{RN}$	[-0.001244049, -0.001244049]
$\widehat{\overline{Y}}_{PN}$	[0.001836447, 0.002023558]
$\widehat{\overline{Y}}_{DN}$	-
$\widehat{\overline{Y}}_{BTRN}$	[0.006869877, 0.006934565]
$\widehat{\overline{Y}}_{BTPN}$	[0.008706324, 0.009213423]
$\widehat{\overline{Y}}_{BTPN}$	[0.00686232, 0.00694341]
$\widehat{\overline{Y}}^{(*)}_{Prop,\ N}$	[-0.0175342,-0.0163563]

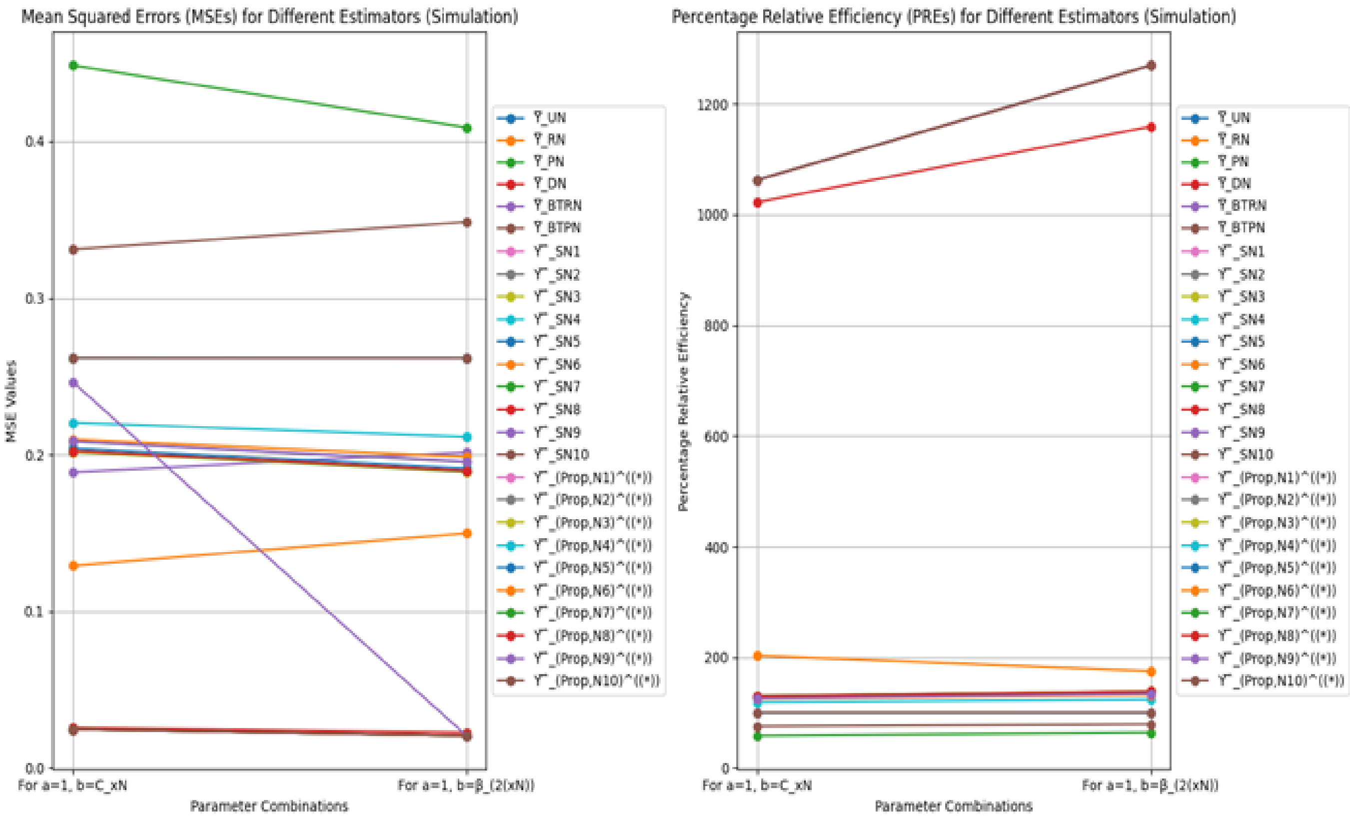

Figure 3: Showing MSEs and PREs of estimators based on simulation study

Table 8: MSE using a simulation study

Estimator	**Values**	For values of a and b	$\widehat{\overline{\boldsymbol{Y}}}_{\boldsymbol{SN}}$	$\widehat{\overline{\boldsymbol{Y}}}^{(*)}_{\boldsymbol{Prop,\ N}}$
$\widehat{\overline{Y}}_{UN}$	[0.2619458, 0.2619458]	[For a=1, b=C_{xN}]	[0.2022682, 0.1893483]	[0.02465215, 0.02063134]
$\widehat{\overline{Y}}_{RN}$	[0.1290996, 0.1497684]	[For a=1, b=$\beta_{\mathbf{2}(xN)}$]	[0.2082852, 0.1952096]	[0.02465263, 0.02063211]
$\widehat{\overline{Y}}_{PN}$	[0.448552, 0.4089126]	[For a=$\beta_{\mathbf{2}(xN)}$, b=C_{xN}]	[0.2017871, 0.1890021]	[0.02465212, 0.02063127]
$\widehat{\overline{Y}}_{DN}$	[0.02566218, 0.02260200]	[For a=C_{xN}, b=ρ_{yxN}]	[0.2204352¸ 0.2113867]	[0.02465366, 0.02063334]
$\widehat{\overline{Y}}_{BTRN}$	[0.1888027, 0.2015085]	[For a=1, b=ρ_{yxN}]	[0.2040597, 0.1911544]	[0.0246523, 0.02063158]
$\widehat{\overline{Y}}_{BTPN}$	[0.3310805, 0.3485289]	[For a=C_{xN}, b=ρ_{yxN}]	[0.2098011, 0.1985855]	[0.02465288, 0.02063229]
		[For a=ρ_{yxN}, b=C_{xN}]	[0.2022997, 0.1893758]	[0.02465215, 0.02063134]
		[For a=$\beta_{\mathbf{2}(xN)}$, b=ρ_{yxN}]	[0.202462, 0.1896759]	[0.02465218, 0.02063136]
		[For a=ρ_{yxN}, b=$\beta_{\mathbf{2}(xN)}$]	[0.2085363, 0.1955049]	[0.2465265, 0.02063214]
		[For a=1, b=$N_N\overline{X}_N$]	[0.2618358, 0.2617713]	[0.02465485, 0.02063499]

6.1. Theoretical Analysis of MSE and PRE Using Simulation Data

Mean Squared Errors (MSEs)

(i) The MSE indicates the average squared difference between the actual and projected values. Greater MSE values point to more estimate mistakes.

(ii) Table 8 shows changing MSE values for several estimators and parameter combina-

Table 9: PRE using simulation

Estimator	**Values**	For values of a and b	$\widehat{\overline{\boldsymbol{Y}}}_{\boldsymbol{SN}}$	$\widehat{\overline{\boldsymbol{Y}}}^{(*)}_{\boldsymbol{Prop,\ N}}$
$\widehat{\overline{Y}}_{UN}$	[100,100]	[For a=1, b=C_{xN}]	[129.5042, 138.3407]	[1062.568, 1269.65]
$\widehat{\overline{Y}}_{RN}$	[202.9022, 174.9006]	[For a=1, b=$\beta_{\mathbf{2}(xN)}$]	[125.7631, 134.1869]	[1062.547, 1269.603]
$\widehat{\overline{Y}}_{PN}$	[58.3981, 64.05912]	[For a=$\beta_{\mathbf{2}(xN)}$, b=C_{xN}]	[129.813, 138.5941]	[1062.569, 1269.654]
$\widehat{\overline{Y}}_{DN}$	[1022.347, 1158.95]	[For a=C_{xN}, b=ρ_{yxN}]	[118.8312, 123.9178]	[1062.503, 1269.527]
$\widehat{\overline{Y}}_{BTRN}$	[129.9925, 138.7405]	[For a=1, b=ρ_{yxN}]	[128.3673, 137.0336]	[1062.561, 1269.635]
$\widehat{\overline{Y}}_{BTPN}$	[75.15756, 79.11846]	[For a=C_{xN}, b=ρ_{yxN}]	[124.8543, 131.9058]	[1062.537, 1269.592]
		[For a=ρ_{yxN}, b=C_{xN}]	[129.4841, 138.3207]	[1062.568, 1269.65]
		[For a=$\beta_{\mathbf{2}(xN)}$, b=ρ_{yxN}]	[129.3802, 138.1018]	[1062.567, 1269.649]
		[For a=ρ_{yxN}, b=$\beta_{\mathbf{2}(xN)}$]	[125.6116, 133.9843]	[1062.546,1269.601]
		[For a=1, b=$N_N\overline{X}_N$]	[100.042, 100.0667]	[1062.451, 1269.426]

tions. As one illustration: With somewhat high MSE values, estimator $\widehat{\overline{Y}}_{PN}$indicates more estimating error than other estimators. With rather low MSE values, estimator $\widehat{\overline{Y}}_{DN}$ suggests improved accuracy performance.

(iii) The performance of the estimators depends on the parameters a and b selected.

For example, parameter combinations including the mean of X values C_{xN}or other pertinent statistics usually produce smaller MSE values, hence suggesting improved estimate accuracy.

(iv) To minimize MSE, it is advisable to choose parameter combinations that lead to lower MSE values, such as those involving relevant statistics like C_{xN} or $\beta_{\mathbf{2}(xN)}$.

Percentage Relative Efficiency (PREs)

(i) The PRE measures the relative prediction error compared to the actual values. Higher PRE values indicate larger prediction errors.

(ii) Table 9 shows us different PRE values for several estimators and parameter combinations. As an instance: With rather high PRE values, Estimator $\widehat{\overline{Y}}_{RN}$ indicates more prediction errors than other estimators. With PRE values crossing to 100, estimator $\widehat{\overline{Y}}^{(*)}_{Prop,\ N}$ suggests rather accurate predictions.

(iii) Similar to MSE, the choice of parameters (a and b) influences the PRE. Parameter combinations that involve relevant statistics like C_{xN} or $\beta_{\mathbf{2}(xN)}$ tend to yield lower PRE values, suggesting better prediction accuracy.

(iv) To minimize prediction errors, choosing parameter combinations that lead to lower PRE values, such as those involving relevant statistics like C_{xN} or $\beta_{\mathbf{2}(xN)}$ is advisable.

Overall Analysis

(i) The suggested class of estimaotrs $\widehat{\overline{Y}}^{(*)}_{Prop,\ N}$ generally have better MSE and PRE than other estimators over several parameter settings.

(ii) The performance of estimators is significantly influenced by the parameters chosen; so, combinations of appropriate statistics produce superior accuracy in both estimation and prediction.

(iii) Theoretical study indicates that choosing the best parameter combinations depends on knowing the link between parameters and their influence on estimator performance.

7. Discussion

In table 1, we presented a list of acronyms. In Table 2, we present the generalized class of estimators. Table 3 consist the overall statistics for the real data sets. Table 4 shows the detailed bias from estimators used on the real data set. Tables 5 and Table 6 record the results from the real data set. Table 5 details the mean square errors for both existing and proposed estimators, while Table 6 shows the percentage relative efficiency results for all considered estimators in this analysis. By varying the parameters 'a' and 'b', the proposed generalized class of estimators achieves different minimum mean squared errors, as shown in Tables 5 and 8. Based on the numerical result it is found that the MSE of our generalized class of estimators is minimal as compared to all considered estimators in this paper. The Biases of estimators using simulated data sets are given in Table 7. Similarly, the MSE and PRE using the simulated data sets are given in Table 8 and Table 9. Data presented in Table 6 and Table 9 show that changing a and b changes the PREs for any variety of estimators. Furthermore, it is possible to notice that PREs are higher when $a = 1$ and $b = \rho_{yxN}$, $a = \rho_{yxN}$ and $b = C_{xN}$, $a = \rho_{yxN}$ and b = $\beta_{\mathbf{2}(xN)}$, and somewhat lower when $a = 1$ and $b = N_N\overline{X}_N$. The numerical results are shown with the help of graphs. The Figure 1, shown the biases of all considered estimators. Figure 2 and Figure 3, shown the MSEs and PREs for the real data set and a simulation study. It is demonstrated that the suggested estimators achieve minimum MSE demonstrating improvement in efficiency as compared to existing estimators.

8. Conclusion

Point estimates in survey sampling are limited by providing only a single value for the parameter being studied, which can differ across samples due to sampling error. Conversely, parameter estimates in sampling theory are significantly enhanced by the neutrosophic approach, which offers credible interval estimates with a high probability of encompassing the parameter. Thus, the neutrosophic strategy, an advancement of the traditional method, addresses ambiguous, indeterminate, or uncertain elements. This paper proposes a new general class of estimators for the population mean using neutrosophic data and simple random sampling. By varying the values of 'a' and 'b', this new class of estimators yields different minimum values for each case. The numerical analysis shows that the Mean Squared Error (MSE) of these new estimators is the lowest when compared to all other estimators discussed in this study. The results confirm that these estimators have the lowest MSE, indicating enhanced efficiency over several other modified estimators. This study paves the way for future research in creating more accurate estimators suitable for a broad range of neutrosophic data and sampling techniques, including stratified ranked set sampling, systematic sampling, and predictive methods in simple random sampling.

Data availability
All the data are available within the article.
Conflict of interest
The authors declare no conflict of interest.
Authors contribution
Sanaa Al-Marzouki: Wrote the main manuscript, analysis and visualization
Sohaib Ahmad: Writing original draft and supervision

References

[1] Alomair, A.M.; Ahmad, S. *New comprehensive mean estimation using regression-cum-exponential type estimator: Application with neutrosophic data.* Kuwait J. Sci. **2025**, *52*, 100346.

[2] Ahmad, S.; Masood, S.; Alomair, A.M.; Alomair, M.A. *Improved modified estimator for estimation of median using auxiliary information under simple random sampling.* Sci. Rep. **2024**, *14*, 16504.

[3] Ali, Z.; Mahmood, T. *Complex neutrosophic generalized dice similarity measures and their application to decision making.* CAAI Trans. Intell. Technol. **2020**, *5*, 78–87.

[4] Ahmad, S.; Hussain, S.; Shabbir, J.; Aamir, M.; El-Morshedy, M.; Ahmad, Z.; Al-rajhi, S. *Improved generalized family of estimators in estimating the finite population mean using two auxiliary variables under two-stage sampling.* AIMS Math. **2022**, *7*, 10609–10624.

[5] Azeem, M.; Hanif, M. *The joint influence of measurement error and non-response on estimation of population mean.* Commun. Stat. Theory Methods **2017**, *46*, 1679–1693.

[6] Bahl, S.; Tuteja, R. *Ratio and product type exponential estimators.* J. Inf. Optim. Sci. **1991**, *12*, 159–164.

[7] Murthy, M.N. *Product method of estimation.* Sankhya Indian J. Stat. Ser. A **1964**, *26*, 69–74.

[8] Singh, H.P.; Kumar, S. *Combination of regression and ratio estimate in the presence of nonresponse.* Braz. J. Probab. Stat. **2011**, *25*, 205–217.

[9] Jan, N.; Zedam, L.; Mahmood, T.; Ullah, K.; Ali, Z. *Multiple attribute decision-making method under linguistic cubic information.* J. Intell. Fuzzy Syst. **2019**, *36*, 253–269.

[10] Li, D.F.; Mahmood, T.; Ali, Z.; Dong, Y. *Decision-making based on interval-valued complex single-valued neutrosophic hesitant fuzzy generalized hybrid weighted averaging operators.* J. Intell. Fuzzy Syst. **2020**, *38*, 4359–4401.

[11] Liu, P.; Ali, Z.; Mahmood, T. *The distance measures and cross-entropy based on complex fuzzy sets and their application in decision-making.* J. Intell. Fuzzy Syst. **2020**, *39*, 3351–3374.

[12] Chakraborty, A.; Mondal, S.P.; Alam, S.; Mahata, A. *Cylindrical neutrosophic single-valued number and its application in networking problems, multi-criterion group decision-making problems, and graph theory.* CAAI Trans. Intell. Technol. **2020**, *5*, 68–77.

[13] Chakraborty, A.; Banik, B.; Mondal, S.P.; Alam, S. *Arithmetic and geometric operators of pentagonal neutrosophic number and its application in mobile communication service based MCGDM problem.* Neutrosophic Sets Syst. **2020**, *32*, 61–79.

[14] Chakraborty, A.; Mondal, S.P.; Alam, S.; Dey, A. *Classification of trapezoidal bipolar neutrosophic number, de-polarization technique and its execution in cloud service-based MCGDM problem.* Complex Intell. Syst. **2021**, *7*, 145–162.

[15] Haque, T.S.; Chakraborty, A.; Mondal, S.P.; Alam, S. *Approach to solve multi-criteria group decision-making problems by exponential operational law in generalized spherical*

fuzzy environment. CAAI Trans. Intell. Technol. **2020**, *5*, 106–114.

[16] Smarandache, F. *Neutrosophy: Neutrosophic Probability, Set, and Logic: Analytic Synthesis & Synthetic Analysis*; American Research Press: Rehoboth, NM, USA, **1998**.

[17] Tahir, Z.; Khan, H.; Aslam, M.; Shabbir, J.; Mahmood, Y.; Smarandache, F. *Neutrosophic ratio-type estimators for estimating the population mean*. Complex Intell. Syst. **2021**, *7*, 2991–3001.

[18] Yadav, S.K.; Smarandache, F. *Generalized neutrosophic sampling strategy for elevated estimation of population mean*. Neutrosophic Sets Syst. **2023**, *53*, 14.

[19] Kumar, S.; Kour, S.P.; Choudhary, M.; Sharma, V. *Determination of population mean using neutrosophic, exponential-type estimator*. Lobachevskii J. Math. **2022**, *43*, 3359–3367.

[20] Raghav, Y.S. *Neutrosophic generalized exponential robust ratio type estimators*. Int. J. Anal. Appl. **2023**, *21*, 41.

[21] Bouza-Herrera, C.N.; Subzar, M. *Estimating the ratio of a crisp variable and a neutrosophic variable*. Int. J. Neutrosophic Sci. **2020**, *11*, 9–21.

[22] Aslam, M. *A variable acceptance sampling plan under the neutrosophic statistical interval method*. Symmetry **2019**, *11*, 114.

[23] Aslam, M. *A new sampling plan using neutrosophic process loss consideration*. Symmetry **2018**, *10*, 132.

[24] Smarandache, F. *Introduction to Neutrosophic Statistics*; Infinite Study: Columbus, OH, USA, **2014**.

[25] Chen, J.; Ye, J.; Du, S. *Scale effect and anisotropy analyzed for neutrosophic numbers of rock joint roughness coefficient based on neutrosophic statistics*. Symmetry **2017**, *9*, 208.

[26] Aslam, M. *Monitoring the road traffic crashes using NEWMA chart and repetitive sampling*. Int. J. Inj. Control Saf. Promot. **2020**, *28*, 39–45.

[27] Aslam, M. *A new goodness of fit test in the presence of uncertain parameters*. Complex Intell. Syst. **2021**, *7*, 359–365.

[28] Aslam, M. *A study on skewness and kurtosis estimators of wind speed distribution under indeterminacy*. Theor. Appl. Climatol. **2021**, *143*, 1227–1234.

[29] Aslam, M.; Arif, O.H.; Sherwani, R.A.K. *New diagnosis test under the neutrosophic statistics: An application to diabetic patients*. BioMed Res. Int. **2020**, *2020*, 2083167.

[30] Aslam, M. *Neutrosophic analysis of variance: Application to university students*. Complex Intell. Syst. **2019**, *5*, 403–407.

[31] Singh, R.; Chauhan, P.; Sawan, N.; Smarandache, F. *Improvement in estimating the population mean using exponential estimator in simple random sampling*. In *Auxiliary Information and a priori Values in Construction of Improved Estimators*; Infinite Study: Columbus, OH, USA, **2007**; p. 33.

[32] Singh, A.; Kulkarni, H.; Smarandache, F.; Vishwakarma, G.K. *Computation of separate ratio and regression estimator under neutrosophic stratified sampling: An application to climate data*. J. Fuzzy Ext. Appl. **2024**, *5*, 605–621.

[33] Alsharif, A.M.; Abdellateef, A.I.; Elmaboud, Y.A.; Abdelsalam, S.I. *Performance*

enhancement of a DC-operated micropump with electroosmosis in a hybrid nanofluid: Fractional Cattaneo heat flux problem. Appl. Math. Mech. **2022**, *43*, 931–944.

[34] Granados, C.; Das, A.K.; Das, B. *Some continuous neutrosophic distributions with neutrosophic parameters based on neutrosophic random variables.* Adv. Theory Nonlinear Anal. Appl. **2022**, *6*, 380–389.

[35] Granados, C.; Das, A.K.; Osu, B.O. *Weighted neutrosophic soft multiset and its application to decision making.* YUJOR **2023**, *33*, 293–308.

[36] Das, A.K.; Das, R.; Das, S.; Debnath, B.K.; Granados, C.; Shil, B.; Das, R. *A comprehensive study of neutrosophic superhyper BCI-semigroups and their algebraic significance.* Trans. Fuzzy Sets Syst. **2025**, *8*, 80.