EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
2025, Vol. 18, Issue 3, Article Number 6476
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

Detecting of Rumours in Social Media via Neutrosophic Hesitance Distance Measures

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Abstract. This article introduces a new approach for digital misinformation detection with the application of Neutrosophic Hesitance Distance Measures. Including hesitance values in neutrosophic similarity and distance measurement helps us to deal with uncertainty and vagueness better. Complement, max, min, and max-min operations are explained along with definitions. An example provides a glimpse of how Neutrosophic Set Hesitance Distance Measure enhances decision-making for detecting misinformation on social media. The findings identify the benefits of merging hesitance into neutrosophic fuzzy systems to achieve more precise and reliable analysis.

2020 Mathematics Subject Classifications: 03E72, 03B52, 28E10

Key Words and Phrases: Neutrosophic fuzzy set, Neutrosophic hesitance distance measure, Score function, Decision-making problem

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1. Introduction

Smarandache [1] offered Neutrosophic logic for dealing with the uncertainties, indeterminacies, and inconsistences of existing information systems proficiently in order to present a powerful mathematical framework. Instead of standard logic frameworks, this system clearly specifies the levels of truth, indeterminacy, and falsity, which makes it exclusively compatible for the examination of imprecise and incomplete data sets. Based on this originated perceptions, Single-Valued Neutrosophic Sets (SVNSs) were established by Wang et al. [2], to enhance the computational modeling efficiency under uncertainty, where each one of the apparatuses of truth, indeterminacy, and falsity might quantified through particular real numbers.

Majumdar & Samanta [3] formulated numerous concepts and distance measures, to measure how near the neutrosophic entities are to each other. These gears are vital

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DOI: https://doi.org/10.29020/nybg.ejpam.v18i3.6476

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in determining the extent to a particular data point which is constant with expected information. Distance measure specially structured for interval in neutrosophic sets was developed by Ye ([4]), which is useful in uncertainty profound decision-making situations. After that, the philosophy of Hesitant Neutrosophic Sets (HNSs) was proposed, paves path for multiple value attribute of truth, indeterminacy, and falsity to arrest the real-world hesitation and vagueness in judgments (Peng & Dai, [5]). In medical diagnosis (Ye et al., [6]), cybersecurity analysis (Broumi et al., 2023), and decision-making with multiple criteria (Pramanik et al., [7]), these models elevate the interpretation of uncertain or inconclusive outputs.

In order to detecting misinformation we utilize Neutrosophic Hesitance Distance Measures (NHDMs) to measure the distance between authenticated information and possibly falsified virtual content. By exceeding binary true/false labels, such measures can be able to precise and captures of the uncertainty involved in such content (Shu et al., [8]) Moreover, census said that the major contribution of propagation of online lies are from bot accounts (Shao et al., [9]), the spread of disinformation where caused because of emotionally charged content and hyperpartisan media outlets (Potthast et al., [10]). These contemplations upgrades of uncertainty that can be managed by neutrosophic models uniquely (Abdel-Basset et al., [11]). For detecting misinformation offers a robust mechanism for dealing with the uncertainty embedded in digital information networks, we utilizes the collabration of neutrosophic logic and hesitancy-based distance functions. This article demonstrate a new framework under these concepts for the improvisation of detection and classification of false content on social media. In recent years various Distance Measures applied in neutrosophic sets [12–15].

1.1. Motivation

In today's world, the prompt diffusion of rumours threatens the lucidity of society, political stability, and public trust. Traditional methods for detecting disinformation rely mostly on probabilistic models, statistical analysis, or machine learning algorithms. Nevertheless, these tend to flop when provoked with imprecise or deceptive content particularly when input data is half-done, contradictory, or deceitful. Social media has high potential to create an impact on political debate and also makes this problem more challenging, as web-based discussions frequently consist of uncertain, imprecise, or half-formed opinions. This type of human imprecision and partial belief are quiet hard to capture through conventional binary or fuzzy logical systems.

To handle such complications more excellently, Neutrosophic Logic delivers a more subtle approach by recognising the truth, indeterminacy, and falsity as separate and concomitant dimensions. Through this, Single-Valued Neutrosophic Sets (SVNSs) and Hesitant Neutrosophic Sets (HNSs) have developed into impact means of representing hesitant and multi-valued judgments pervasive features in relation on online.

1.2. Novelty

This research offers a new method to detect false political rumour using Neutrosophic Hesitance Distance Measures (NHDMs). In contrast to traditional methods depending on static labels or confidence levels, the new model involves varying levels of hesitation in information's truth, indeterminacy, and falsity aspects.
The major contributions are:

- Formulation and application of Neutrosophic Hesitance Distance Measures intended to determine similarity between possibly deceiving content and established fact checks with heightened sensitivity towards hesitation and doubt.
- An adaptive detection model that can identify misinformation with degrees of certainty and ambiguity, instead of strict binary classification.
- Validation of the method on actual social media datasets, illustrating its capability to outperform conventional detection methods in situations where uncertainty, hesitancy, and partial truth prevail.
- This method fills the gap between uncertainty mathematical modeling and real-world misinformation detection, presenting a more interpretable and human-oriented solution towards fighting fake news and deceptive information in online communities.

1.3. Structure and Contribution of this paper

The paper begins with an Introduction that establishes the need for advanced distance measures in uncertain environments and introduces neutrosophic hesitant fuzzy sets as a solution. In Preliminaries, foundational concepts such as fuzzy sets, neutrosophic sets, and hesitancy are discussed to provide theoretical background. The core of the paper lies in Section 3, where novel Neutrosophic Fuzzy Distance Measures are proposed and mathematically formulated. These measures are then applied in Section 4 through a structured Decision-Making Process, including numerical examples that demonstrate their practical relevance. Section 5 outlines the Advantages of the proposed distance measures, emphasizing their ability to handle uncertainty and hesitation more effectively than existing approaches. The paper concludes with Section 6, summarizing key findings and suggesting future research directions for extending the proposed models to more complex decision-making systems.

2. Preliminaries

Definition 1. *[1] An NS D is defined as:*$D = \{\langle d, \nu_D(d), \sigma_D(d), \mu_D(d)\rangle : d \in \mathcal{B}\}, \nu_D(d) : \mathcal{B} \to [0,1]$ *denote the truth grade value,* $\sigma_D(d) : \mathcal{B} \to [0,1]$ *denote the indeterminacy grade value and* $\mu_D(d) : \mathcal{B} \to [0,1]$ *denote the falsity grade value, where satisfies the condition* $0 \leq (\nu_D(d)) + (\sigma_D(d)) + (\mu_D(d)) \leq 3$.

Definition 2. *[1] A NFS is characterised by a grade value* $\nu_D(d) = T_D(d), \sigma_D(d) = I_D(d)$ *and* $\mu_D(d) = F_D(d)$ *The Hesitance degree of NFS is defined by* $h_D(d) = \{|(1-T_D(d))|, |(1-I_D(d))|, |(1-F_D(d))|\}$ *written as*

$$
\begin{aligned}
D =&\{\langle x, \nu_D(d), \sigma_D(d), \mu_D(d)\rangle : x \in \mathcal{B}\},\\
D =&T_D(d), I_D(d), F_D(d)
\end{aligned}
$$

Definition 3. *Let* $D_n, n = 1, 2, ..., N$ *be* N *NFS defined on* $\mathcal{B}$ *and* $\nu_D(d) = T_D(d), \sigma_D(d) = I_D(d), \mu_D(d) = F_D(d)$ *their grade value. The Neutrosophic fuzzy (NF) cartesian product of* D_n *represented by* $\times_{i=1}^{n} D_i = D_1 \times D_2 \times \cdots \times D_n$*is defined by the function*

$$
\begin{aligned}
\nu_{\times_{i=1}^{n} D_i}(d) =&T_{\times_{i=1}^{n} D_i}(d) = \min\{T_{D_1}(d), T_{D_2}(d), \cdots, T_{D_n}(d)\}\\
\sigma_{\times_{i=1}^{n} D_i}(d) =&I_{\times_{i=1}^{n} D_i}(d) = \min\{I_{D_1}(d), I_{D_2}(d), \cdots, I_{D_n}(d)\}\\
\mu_{\times_{i=1}^{n} D_i}(d) =&F_{\times_{i=1}^{n} D_i}(d) = \max\{F_{D_1}(d), F_{D_2}(d), \cdots, F_{D_n}(d)\}
\end{aligned}
$$

Definition 4. *[2] Let* D_1 *and* D_2 *be any two NFS on* B *then the intersection of two NFS is written as*

$$
\begin{aligned}
D_1 \cap D_2 =&T_{D_1}(d) \cap T_{D_2}(d) = \min[T_{D_1}(d), T_{D_2}(d)]\\
D_1 \cap D_2 =&I_{D_1}(d) \cap I_{D_2}(d) = \min[I_{D_1}(d), I_{D_2}(d)]\\
D_1 \cap D_2 =&F_{D_1}(d) \cap F_{D_2}(d) = \max[F_{D_1}(d), F_{D_2}(d)]
\end{aligned}
$$

Definition 5. *[2] Let* D_1 *and* D_2 *be any two NFS on* B *then the union of two NFS is written as*

$$
\begin{aligned}
D_1 \cup D_2 =&T_{D_1}(d) \cup T_{D_2}(d) = \max[T_{D_1}(d), T_{D_2}(d)]\\
D_1 \cup D_2 =&I_{D_1}(d) \cup I_{D_2}(d) = \max[I_{D_1}(d), I_{D_2}(d)]\\
D_1 \cup D_2 =&F_{D_1}(d) \cup F_{D_2}(d) = \min[F_{D_1}(d), F_{D_2}(d)]
\end{aligned}
$$

Definition 6. *[2] Let* D *be any two NFS on* B *then the complement of NFS* D *is written as*

$$
\begin{aligned}
D^c =&T_{D^c}(d) = [1 - T_{D^c}(d)]\\
D^c =&I_{D^c}(d) = [1 - I_{D^c}(d)]\\
D^c =&F_{D^c}(d) = [1 - F_{D^c}(d)]
\end{aligned}
$$

Theorem 1. *For any these NFS* D_1, D_2, D_3, D_4 *on* S*. The standard intersection, union and complement satisfy the following*

i. $D_1 \cap D_2 = D_2 \cap D_1$

ii. $D_1 \cup D_2 = D_2 \cup D_1$

iii. $D_1 \cap (D_2 \cap D_3) = (D_1 \cap D_2) \cap D_3$

iv. $D_1 \cup (D_2 \cup D_3) = (D_1 \cup D_2) \cup D_3$

v. $D_1 \cap (D_2 \cup D_3) = (D_1 \cap D_2) \cup (D_1 \cap D_3)$

vi. $D_1 \cup (D_2 \cap D_3) = (D_1 \cup D_2) \cap (D_1 \cup D_3)$

vii. $(D_1 \cap D_2)^c = D_1^c \cup D_2^c$

viii. $(D_1 \cup D_2)^c = D_1^c \cap D_2^c$

Proof.

i.

$$\begin{aligned}
D_1 \cap D_2 =& T_{D_1}(d) \cap T_{D_2}(d) = \min[T_{D_1}(d), T_{D_2}(d)] \\
=& \min[T_{D_2}(d), T_{D_1}(d)] = T_{D_2}(d) \cap T_{D_1}(d) \\
=& D_2 \cap D_1 \\
D_1 \cap D_2 =& I_{D_1}(d) \cap I_{D_2}(d) = \min[I_{D_1}(d), I_{D_2}(d)] \\
=& \min[I_{D_2}(d), I_{D_1}(d)] = I_{D_2}(d) \cap I_{D_1}(d) \\
=& D_2 \cap D_1 \\
D_1 \cap D_2 =& F_{D_1}(d) \cap F_{D_2}(d) = \max[F_{D_1}(d), F_{D_2}(d)] \\
=& \max[F_{D_2}(d), F_{D_1}(d)] = F_{D_2}(d) \cap F_{D_1}(d) \\
=& D_2 \cap D_1
\end{aligned}$$

ii.

$$\begin{aligned}
D_1 \cup D_2 =& T_{D_1}(d) \cup T_{D_2}(d) = \max[T_{D_1}(d), T_{D_2}(d)] \\
=& \max[T_{D_2}(d), T_{D_1}(d)] = T_{D_2}(d) \cup T_{D_1}(d) \\
=& D_2 \cup D_1 \\
D_1 \cup D_2 =& I_{D_1}(d) \cup I_{D_2}(d) = \max[I_{D_1}(d), I_{D_2}(d)] \\
=& \max[I_{D_2}(d), I_{D_1}(d)] = I_{D_2}(d) \cup I_{D_1}(d) \\
=& D_2 \cup D_1 \\
D_1 \cup D_2 =& F_{D_1}(d) \cup F_{D_2}(d) = \min[F_{D_1}(d), F_{D_2}(d)] \\
=& \min[F_{D_2}(d), F_{D_1}(d)] = F_{D_2}(d) \cup F_{D_1}(d) \\
=& D_2 \cup D_1
\end{aligned}$$

iii.

$$\begin{aligned}
D_1 \cap (D_2 \cap D_3) =& T_{D_1}(d) \cap T_{D_2 \cap D_3}(d) \\
=& \min[T_{D_1}(d), \min[T_{D_2}(d), T_{D_3}(d)]] \\
=& \min[T_{D_1}(d), T_{D_2}(d), T_{D_3}(d)] \\
=& T_{D_1 \cap D_2}(d) \cap T_{D_3}(d)
\end{aligned}$$

$$
\begin{aligned}
&= (D_1 \cap D_2) \cap D_3\\
D_1 \cap (D_2 \cap D_3) &= I_{D_1}(d) \cap I_{D_2 \cap D_3}(d)\\
&= \min[I_{D_1}(d), \min[I_{D_2}(d), I_{D_3}(d)]]\\
&= \min[I_{D_1}(d), I_{D_2}(d), I_{D_3}(d)]\\
&= I_{D_1 \cap D_2}(d) \cap I_{D_3}(d)\\
&= (D_1 \cap D_2) \cap D_3\\
D_1 \cap (D_2 \cap D_3) &= F_{D_1}(d) \cap F_{D_2 \cap D_3}(d)\\
&= \max[F_{D_1}(d), \max[F_{D_2}(d), F_{D_3}(d)]]\\
&= \max[F_{D_1}(d), F_{D_2}(d), F_{D_3}(d)]\\
&= F_{D_1 \cap D_2}(d) \cap F_{D_3}(d)\\
&= (D_1 \cap D_2) \cap D_3
\end{aligned}
$$

iv.

$$
\begin{aligned}
D_1 \cup (D_2 \cup D_3) &= T_{D_1}(d) \cup T_{D_2 \cup D_3}(d)\\
&= \max[T_{D_1}(d), \max[T_{D_2}(d), T_{D_3}(d)]]\\
&= \max[T_{D_1}(d), T_{D_2}(d), T_{D_3}(d)]\\
&= T_{D_1 \cup D_2}(d) \cup T_{D_3}(d)\\
&= (D_1 \cup D_2) \cup D_3\\
D_1 \cup (D_2 \cup D_3) &= I_{D_1}(d) \cup I_{D_2 \cup D_3}(d)\\
&= \max[I_{D_1}(d), \max[I_{D_2}(d), I_{D_3}(d)]]\\
&= \max[I_{D_1}(d), I_{D_2}(d), I_{D_3}(d)]\\
&= I_{D_1 \cup D_2}(d) \cup I_{D_3}(d)\\
&= (D_1 \cup D_2) \cup D_3\\
D_1 \cup (D_2 \cup D_3) &= F_{D_1}(d) \cup F_{D_2 \cup D_3}(d)\\
&= \min[F_{D_1}(d), \min[F_{D_2}(d), F_{D_3}(d)]]\\
&= \min[F_{D_1}(d), F_{D_2}(d), F_{D_3}(d)]\\
&= F_{D_1 \cup D_2}(d) \cup F_{D_3}(d)\\
&= (D_1 \cup D_2) \cup D_3
\end{aligned}
$$

v. To prove the distributive law of intersection over union, we take some cases.

Case 1 $T_{D_1}(d) \geq T_{D_2}(d) \geq T_{D_3}(d), I_{D_1}(d) \geq I_{D_2}(d) \geq I_{D_3}(d), F_{D_1}(d) \geq F_{D_2}(d) \geq F_{D_3}(d)$

$$
\begin{aligned}
D_1 \cap (D_2 \cup D_3) &= T_{D_1}(d) \cap T_{D_2 \cup D_3}(d)\\
&= \min[T_{D_1}(d), \max[T_{D_2}(d), T_{D_3}(d)]]
\end{aligned}
$$

$$\begin{aligned}
&= \min[T_{D_1}(d), T_{D_2}(d)] \\
&= T_{D_2}(d) && (1) \\
(D_1 \cap D_2) \cup (D_1 \cap D_3) &= T_{D_1 \cap D_2}(d) \cup T_{D_1 \cap D_3}(d) \\
&= \max[\min[T_{D_1}(d), T_{D_2}(d)], \min[T_{D_1}(d), T_{D_3}(d)]] \\
&= \max[T_{D_2}(d), T_{D_3}(d)] \\
&= T_{D_2}(d) && (2) \\
D_1 \cap (D_2 \cup D_3) &= I_{D_1}(d) \cap I_{D_2 \cup D_3}(d) \\
&= \min[I_{D_1}(d), \max[I_{D_2}(d), I_{D_3}(d)]] \\
&= \min[I_{D_1}(d), I_{D_2}(d)] \\
&= I_{D_2}(d) && (3) \\
(D_1 \cap D_2) \cup (D_1 \cap D_3) &= I_{D_1 \cap D_2}(d) \cup I_{D_1 \cap D_3}(d) \\
&= \max[\min[I_{D_1}(d), I_{D_2}(d)], \min[I_{D_1}(d), I_{D_3}(d)]] \\
&= \max[I_{D_2}(d), I_{D_3}(d)] \\
&= I_{D_2}(d) && (4) \\
D_1 \cap (D_2 \cup D_3) &= F_{D_1}(d) \cap F_{D_2 \cup D_3}(d) \\
&= \max[F_{D_1}(d), \min[F_{D_2}(d), F_{D_3}(d)]] \\
&= \max[F_{D_1}(d), F_{D_3}(d)] \\
&= F_{D_1}(d) && (5) \\
(D_1 \cap D_2) \cup (D_1 \cap D_3) &= F_{D_1 \cap D_2}(d) \cup F_{D_1 \cap D_3}(d) \\
&= \min[\max[F_{D_1}(d), F_{D_2}(d)], \max[F_{D_1}(d), F_{D_3}(d)]] \\
&= \min[F_{D_1}(d), F_{D_1}(d)] \\
&= F_{D_1}(d) && (6)
\end{aligned}$$

From (1),(2),(3),(4),(5) and (6) we have,

$$D_1 \cap (D_2 \cup D_3) = (D_1 \cap D_2) \cup (D_1 \cap D_3)$$

case 2 $T_{D_1}(d) \leq T_{D_2}(d) \leq T_{D_3}(d), I_{D_1}(d) \leq I_{D_2}(d) \leq I_{D_3}(d), F_{D_1}(d) \leq F_{D_2}(d) \leq F_{D_3}(d)$

$$\begin{aligned}
D_1 \cap (D_2 \cup D_3) &= T_{D_1}(d) \cap T_{D_2 \cup D_3}(d) \\
&= \min[T_{D_1}(d), \max[T_{D_2}(d), T_{D_3}(d)]] \\
&= \min[T_{D_1}(d), T_{D_3}(d)] \\
&= T_{D_1}(d) && (7) \\
(D_1 \cap D_2) \cup (D_1 \cap D_3) &= T_{D_1 \cap D_2}(d) \cup T_{D_1 \cap D_3}(d) \\
&= \max[\min[T_{D_1}(d), T_{D_2}(d)], \min[T_{D_1}(d), T_{D_3}(d)]] \\
&= \max[T_{D_1}(d), T_{D_1}(d)]
\end{aligned}$$

$$
\begin{aligned}
&= T_{D_1}(d) && (8)\\
D_1 \cap (D_2 \cup D_3) &= I_{D_1}(d) \cap I_{D_2 \cup D_3}(d)\\
&= \min[I_{D_1}(d), \max[I_{D_2}(d), I_{D_3}(d)]]\\
&= \min[I_{D_1}(d), I_{D_3}(d)]\\
&= I_{D_1}(d) && (9)\\
(D_1 \cap D_2) \cup (D_1 \cap D_3) &= I_{D_1 \cap D_2}(d) \cup I_{D_1 \cap D_3}(d)\\
&= \max[\min[I_{D_1}(d), I_{D_2}(d)], \min[I_{D_1}(d), I_{D_3}(d)]]\\
&= \max[I_{D_1}(d), I_{D_1}(d)]\\
&= I_{D_1}(d) && (10)\\
D_1 \cap (D_2 \cup D_3) &= F_{D_1}(d) \cap F_{D_2 \cup D_3}(d)\\
&= \max[F_{D_1}(d), \min[F_{D_2}(d), F_{D_3}(d)]]\\
&= \max[F_{D_1}(d), F_{D_2}(d)]\\
&= F_{D_2}(d) && (11)\\
(D_1 \cap D_2) \cup (D_1 \cap D_3) &= F_{D_1 \cap D_2}(d) \cup F_{D_1 \cap D_3}(d)\\
&= \min[\max[F_{D_1}(d), F_{D_2}(d)], \max[F_{D_1}(d), F_{D_3}(d)]]\\
&= \min[F_{D_2}(d), F_{D_2}(d)]\\
&= F_{D_2}(d) && (12)
\end{aligned}
$$

From (7),(8),(9),(10),(11) and (12) we have,

$$D_1 \cap (D_2 \cup D_3) = (D_1 \cap D_2) \cup (D_1 \cap D_3)$$

vi. To prove the distributive law of intersection over union, we take some cases.

Case 1 $T_{D_1}(d) \geq T_{D_2}(d) \geq T_{D_3}(d), I_{D_1}(d) \geq I_{D_2}(d) \geq I_{D_3}(d), F_{D_1}(d) \geq F_{D_2}(d) \geq F_{D_3}(d)$

$$
\begin{aligned}
D_1 \cup (D_2 \cap D_3) &= T_{D_1}(d) \cup T_{D_2 \cap D_3}(d)\\
&= \max[T_{D_1}(d), \min[T_{D_2}(d), T_{D_3}(d)]]\\
&= \max[T_{D_1}(d), T_{D_3}(d)]\\
&= T_{D_1}(d) && (13)\\
(D_1 \cup D_2) \cap (D_1 \cup D_3) &= T_{D_1 \cup D_2}(d) \cap T_{D_1 \cup D_3}(d)\\
&= \min[\max[T_{D_1}(d), T_{D_2}(d)], \max[T_{D_1}(d), T_{D_3}(d)]]\\
&= \min[T_{D_1}(d), T_{D_1}(d)]\\
&= T_{D_1}(d) && (14)\\
D_1 \cup (D_2 \cap D_3) &= I_{D_1}(d) \cup I_{D_2 \cap D_3}(d)
\end{aligned}
$$

$$
\begin{aligned}
&= \max[I_{D_1}(d), \min[I_{D_2}(d), I_{D_3}(d)]] \\
&= \max[I_{D_1}(d), I_{D_3}(d)] \\
&= I_{D_1}(d) \qquad (15) \\
(D_1 \cup D_2) \cap (D_1 \cup D_3) &= I_{D_1 \cup D_2}(d) \cap I_{D_1 \cup D_3}(d) \\
&= \min[\max[I_{D_1}(d), I_{D_2}(d)], \max[I_{D_1}(d), I_{D_3}(d)]] \\
&= \min[I_{D_1}(d), I_{D_1}(d)] \\
&= I_{D_1}(d) \qquad (16) \\
D_1 \cup (D_2 \cap D_3) &= F_{D_1}(d) \cup F_{D_2 \cap D_3}(d) \\
&= \min[F_{D_1}(d), \max[F_{D_2}(d), F_{D_3}(d)]] \\
&= \min[F_{D_1}(d), F_{D_2}(d)] \\
&= F_{D_2}(d) \qquad (17) \\
(D_1 \cup D_2) \cap (D_1 \cup D_3) &= F_{D_1 \cup D_2}(d) \cap F_{D_1 \cup D_3}(d) \\
&= \max[\min[F_{D_1}(d), F_{D_2}(d)], \min[F_{D_1}(d), F_{D_3}(d)]] \\
&= \max[F_{D_2}(d), F_{D_3}(d)] \\
&= F_{D_2}(d) \qquad (18)
\end{aligned}
$$

From (13),(14),(15),(16),(17) and (18) we have

$$D_1 \cup (D_2 \cap D_3) = (D_1 \cup D_2) \cap (D_1 \cup D_3)$$

Case 2 $T_{D_1}(d) \leq T_{D_2}(d) \leq T_{D_3}(d), I_{D_1}(d) \leq I_{D_2}(d) \leq I_{D_3}(d), F_{D_1}(d) \leq F_{D_2}(d) \leq F_{D_3}(d)$

$$
\begin{aligned}
D_1 \cup (D_2 \cap D_3) &= T_{D_1}(d) \cup T_{D_2 \cap D_3}(d) \\
&= \max[T_{D_1}(d), \min[T_{D_2}(d), T_{D_3}(d)]] \\
&= \max[T_{D_1}(d), T_{D_2}(d)] \\
&= T_{D_2}(d) \qquad (19) \\
(D_1 \cup D_2) \cap (D_1 \cup D_3) &= T_{D_1 \cup D_2}(d) \cap T_{D_1 \cup D_3}(d) \\
&= \min[\max[T_{D_1}(d), T_{D_2}(d)], \max[T_{D_1}(d), T_{D_3}(d)]] \\
&= \min[T_{D_2}(d), T_{D_3}(d)] \\
&= T_{D_2}(d) \qquad (20) \\
D_1 \cup (D_2 \cap D_3) &= I_{D_1}(d) \cup I_{D_2 \cap D_3}(d) \\
&= \max[I_{D_1}(d), \min[I_{D_2}(d), I_{D_3}(d)]] \\
&= \max[I_{D_1}(d), I_{D_2}(d)] \\
&= I_{D_2}(d) \qquad (21) \\
(D_1 \cup D_2) \cap (D_1 \cup D_3) &= I_{D_1 \cup D_2}(d) \cap I_{D_1 \cup D_3}(d) \\
&= \min[\max[I_{D_1}(d), I_{D_2}(d)], \max[I_{D_1}(d), I_{D_3}(d)]]
\end{aligned}
$$

$$
\begin{aligned}
&= \min[I_{D_2}(d), I_{D_3}(d)] \\
&= I_{D_2}(d) \qquad (22)\\
D_1 \cup (D_2 \cap D_3) &= F_{D_1}(d) \cup F_{D_2 \cap D_3}(d) \\
&= \min[F_{D_1}(d), \max[F_{D_2}(d), F_{D_3}(d)]] \\
&= \min[F_{D_1}(d), F_{D_3}(d)] \\
&= F_{D_1}(d) \qquad (23)\\
(D_1 \cup D_2) \cap (D_1 \cup D_3) &= F_{D_1 \cup D_2}(d) \cap F_{D_1 \cup D_3}(d) \\
&= \max[\min[F_{D_1}(d), F_{D_2}(d)], \min[F_{D_1}(d), F_{D_3}(d)]] \\
&= \max[F_{D_1}(d), F_{D_1}(d)] \\
&= F_{D_1}(d) \qquad (24)
\end{aligned}
$$

From (19),(20),(21),(22),(23) and (24) we have,

$$D_1 \cup (D_2 \cap D_3) = (D_1 \cup D_2) \cap (D_1 \cup D_3)$$

vii.

$$
\begin{aligned}
(D_1 \cap D_2)^c &= T_{(D_1 \cap D_2)^c}(d) \\
&= [1 - \min(T_{D_1}(d), T_{D_2}(d))] \\
&= \max[(1 - T_{D_1}(d)), (1 - T_{D_2}(d))] \\
&= \max[T_{D_1^c}(d), T_{D_2^c}(d)] \\
&= \max[T_{D_2^c}(d), T_{D_1^c}(d)] \\
&= D_2^c \cup D_1^c \\
(D_1 \cap D_2)^c &= I_{(D_1 \cap D_2)^c}(d) \\
&= [1 - \min(I_{D_1}(d), I_{D_2}(d))] \\
&= \max[(1 - I_{D_1}(d)), (1 - I_{D_2}(d))] \\
&= \max[I_{D_1^c}(d), I_{D_2^c}(d)] \\
&= \max[I_{D_2^c}(d), I_{D_1^c}(d)] \\
&= D_2^c \cup D_1^c \\
(D_1 \cap D_2)^c &= F_{(D_1 \cap D_2)^c}(d) \\
&= [1 - \max(F_{D_1}(d), F_{D_2}(d))] \\
&= \min[(1 - F_{D_1}(d)), (1 - F_{D_2}(d))] \\
&= \min[F_{D_1^c}(d), F_{D_2^c}(d)] \\
&= \min[F_{D_2^c}(d), F_{D_1^c}(d)] \\
&= D_2^c \cup D_1^c
\end{aligned}
$$

viii.

$$(D_1 \cup D_2)^c = T_{(D_1 \cup D_2)^c}(d)$$

$$
\begin{aligned}
&=[1-\max(T_{D_1}(d), T_{D_2}(d))] \\
&=\min[(1-T_{D_1}(d)),(1-T_{D_2}(d))] \\
&=\min[T_{D_1^c}(d), T_{D_2^c}(d)] \\
&=\min[T_{D_2^c}(d), T_{D_1^c}(d)] \\
&=D_2^c \cap D_1^c \\
(D_1 \cup D_2)^c &=I_{(D_1 \cup D_2)^c}(d) \\
&=[1-\max(I_{D_1}(d), I_{D_2}(d))] \\
&=\min[(1-I_{D_1}(d)),(1-I_{D_2}(d))] \\
&=\min[I_{D_1^c}(d), I_{D_2^c}(d)] \\
&=\min[I_{D_2^c}(d), I_{D_1^c}(d)] \\
&=D_2^c \cap D_1^c \\
(D_1 \cup D_2)^c &=F_{(D_1 \cup D_2)^c}(d) \\
&=[1-\min(F_{D_1}(d), F_{D_2}(d))] \\
&=\max[(1-F_{D_1}(d)),(1-F_{D_2}(d))] \\
&=\max[F_{D_1^c}(d), F_{D_2^c}(d)] \\
&=\max[F_{D_2^c}(d), F_{D_1^c}(d)] \\
&=D_2^c \cap D_1^c
\end{aligned}
$$

Definition 7. *Let D_1 and D_2 be two NFSs on B and then NF product of D_1 and D_2 is $D_1 \circ D_2$ defined by*

$$
\begin{aligned}
D_1 \circ D_2 &= T_{D_1}(d) \circ T_{D_2}(d) = [T_{D_1}(d).T_{D_2}(d)] \\
D_1 \circ D_2 &= I_{D_1}(d) \circ I_{D_2}(d) = [I_{D_1}(d).I_{D_2}(d)] \\
D_1 \circ D_2 &= F_{D_1}(d) \circ F_{D_2}(d) = [F_{D_1}(d).F_{D_2}(d)]
\end{aligned}
$$

3. Neutrosophic Fuzzy Distance Measures

In decision-making and fuzzy logic, the hesitance degree is a key concept signifying uncertainty or indecision accompanied with a certain selection or preference. In scenarios like preferences or similarities are not sternly defined but exist along a continuum then the hesitance values become particularly relevant when we applied it in conjunction with Distance Measures (DMs). In decision-making methods involving multiple criteria, hesitance values helps justify the uncertainty in the importance or increment of each norm. In clustering algorithms, Integrating DMs with hesitance values are advantageous for controling data points that doesn't belong completely to a single cluster but display a degree of membership through multiple clusters. Likewise, in string and pattern matching, hesitance values indicate uncertainty in toning, it is also useful in applications like record linkage and information retrieval, where limited matches or ambiguous relationships may occur. In order to captures uncertainty within the specific problem domain accurately, it is mandatory to deliver a suitable metric when we incorporate Dms with hesitance values. This frequently involves leveraging fuzzy logic or related techniques to manage imprecision and uncertainty in data. In this study Neutrosophic Distance Measures(NDMs) creating a massive impact in decision-making, pattern recognition, medical diagnosis, human-computer interaction, and preference modelling.

3.1. Neutrosophic Hesitance Distance Measures

Definition 8. *The Hesitance distance measure of NFSs is a function* $\mathsf{d}_H : D(\mathcal{B}) \times D(\mathcal{B}) \to [0,1]$ *with the condition for the collection of NFSs* $D_1, D_2, D_3 \in D(\mathcal{B})$

i. $0 \leq \mathsf{d}_H(D_1, D_2) \leq 1, \quad \mathsf{d}_H(D_1, D_2) = 0 \iff D_1 = D_2.$

ii. $\mathsf{d}_H(D_1, D_2) = \mathsf{d}_H(D_2, D_1).$

iii. $\mathsf{d}_H(D_1, D_3) \leq \mathsf{d}_H(D_1, D_2) + \mathsf{d}_H(D_2, D_3).$

The Hesitance DMs is defined as:

$$\mathsf{d}_H(D_1, D_2) = \frac{\sum_{i=1}^{n} \begin{bmatrix} |T_{D_1}(d_i) - T_{D_2}(d_i)| \\ +|h_{D_1}(d_i) - h_{D_2}(d_i)| \end{bmatrix}}{2n} \tag{25}$$

$$\mathsf{d}_H(D_1, D_2) = \frac{\sum_{i=1}^{n} \begin{bmatrix} |I_{D_1}(d_i) - I_{D_2}(d_i)| \\ +|h_{D_1}(d_i) - h_{D_2}(d_i)| \end{bmatrix}}{2n} \tag{26}$$

$$\mathsf{d}_H(D_1, D_2) = \frac{\sum_{i=1}^{n} \begin{bmatrix} |F_{D_1}(d_i) - F_{D_2}(d_i)| \\ +|h_{D_1}(d_i) - h_{D_2}(d_i)| \end{bmatrix}}{2n} \tag{27}$$

Example 1. *Let us take*

$$D_1 = \frac{0.5, 0.7, 0.9}{D_a} + \frac{0.2, 0.5, 0.2}{D_b} + \frac{0.6, 0.7, 0.4}{D_c}$$
$$D_2 = \frac{0.7, 0.6, 0.5}{D_a} + \frac{0.8, 0.8, 0.9}{D_b} + \frac{0.4, 0.6, 0.3}{D_c}$$

we know that,

$$h_D(d) = \{|(1 - T_D(d))|, |(1 - I_D(d))|, |(1 - F_D(d))|\}$$

then for truth,

$$\mathsf{d}_H(D_1, D_2) = \frac{\begin{bmatrix} (|0.5 - 0.7| + |0.5 - 0.3| +) \\ (|0.2 - 0.8| + |0.8 - 0.2| +) \\ (|0.6 - 0.4| + |0.4 - 0.6|) \end{bmatrix}}{2(3)}$$

$$= \frac{[0.2 + 0.2 + 0.6 + 0.6 + 0.2 + 0.2]}{6}$$
$$= 0.333$$

for indeterminacy,

$$\mathsf{d}_H(D_1, D_2) = \frac{\begin{bmatrix} (|0.7 - 0.6| + |0.3 - 0.4| +) \\ (|0.5 - 0.8| + |0.5 - 0.2| +) \\ (|0.7 - 0.6| + |0.3 - 0.4| +) \end{bmatrix}}{2(3)}$$
$$= \frac{[0.1 + 0.1 + 0.3 + 0.3 + 0.1 + 0.1]}{6}$$
$$= 0.166$$

for falsity,

$$\mathsf{d}_H(D_1, D_2) = \frac{\begin{bmatrix} (|0.9 - 0.5| + |0.1 - 0.5| +) \\ (|0.2 - 0.9| + |0.8 - 0.1| +) \\ (|0.4 - 0.3| + |0.6 - 0.7| +) \end{bmatrix}}{2(3)}$$
$$= \frac{[0.4 + 0.4 + 0.7 + 0.7 + 0.1 + 0.1]}{6}$$
$$= 0.400$$

Theorem 2. *The function d_H defined in (25),(26) and (27) is a distance function.*

Proof.

i. The condition $\mathsf{d}_H(D_1, D_2) \leq 0$ is obviously true, Then

$$\mathsf{d}_H(D_1, D_2) = \frac{\sum_{i=1}^{n} \left[\ |T_{D_1}(d_i) - T_{D_2}(d_i)| + |h_{D_1}(d_i) - h_{D_2}(d_i)| \ \right]}{2n}$$
$$= \frac{\sum_{i=1}^{n} [1 + 1]}{2n}$$
$$= 1$$

$$\mathsf{d}_H(D_1, D_2) = \frac{\sum_{i=1}^{n} \left[\ |I_{D_1}(d_i) - I_{D_2}(d_i)| + |h_{D_1}(d_i) - h_{D_2}(d_i)| \ \right]}{2n}$$
$$= \frac{\sum_{i=1}^{n} [1 + 1]}{2n}$$

$$=1$$

$$\mathsf{d}_H(D_1,D_2) = \frac{\sum_{i=1}^{n}\left[\ |F_{D_1}(d_i)-F_{D_2}(d_i)|+|h_{D_1}(d_i)-h_{D_2}(d_i)|\ \right]}{2n}$$

$$= \frac{\sum_{i=1}^{n}[1+1]}{2n}$$

$$=1$$

Therefore $0 \le \mathsf{d}_H(D_1,D_2) \le 1$.

ii.

$$\mathsf{d}_H(D_1,D_2) = \frac{\sum_{i=1}^{n}\left[\ |T_{D_1}(d_i)-T_{D_2}(d_i)|+|h_{D_1}(d_i)-h_{D_2}(d_i)|\ \right]}{2n}$$

$$= \frac{\sum_{i=1}^{n}\left[\ |T_{D_2}(d_i)-T_{D_1}(d_i)|+|h_{D_2}(d_i)-h_{D_1}(d_i)|\ \right]}{2n}$$

$$=\mathsf{d}_H(D_2,D_1)$$

$$\mathsf{d}_H(D_1,D_2) = \frac{\sum_{i=1}^{n}\left[\ |I_{D_1}(d_i)-I_{D_2}(d_i)|+|h_{D_1}(d_i)-h_{D_2}(d_i)|\ \right]}{2n}$$

$$= \frac{\sum_{i=1}^{n}\left[\ |I_{D_2}(d_i)-I_{D_1}(d_i)|+|h_{D_2}(d_i)-h_{D_1}(d_i)|\ \right]}{2n}$$

$$=\mathsf{d}_H(D_2,D_1)$$

$$\mathsf{d}_H(D_1,D_2) = \frac{\sum_{i=1}^{n}\left[\ |F_{D_1}(d_i)-F_{D_2}(d_i)|+|h_{D_1}(d_i)-h_{D_2}(d_i)|\ \right]}{2n}$$

$$= \frac{\sum_{i=1}^{n}\left[\ |F_{D_2}(d_i)-F_{D_1}(d_i)|+|h_{D_2}(d_i)-h_{D_1}(d_i)|\ \right]}{2n}$$

$$=\mathsf{d}_H(D_2,D_1)$$

hence it is true.

iii.

$$\mathsf{d}_H(D_1,D_3) = \frac{\sum_{i=1}^{n}\left[\ |T_{D_1}(d_i)-T_{D_3}(d_i)|+|h_{D_1}(d_i)-h_{D_3}(d_i)|\ \right]}{2n}$$

$$
\begin{aligned}
&= \frac{\sum_{i=1}^{n} \left[\begin{array}{c} |T_{D_1}(d_i) - T_{D_2}(d_i) + T_{D_2}(d_i) - T_{D_3}(d_i)| \\ +|h_{D_1}(d_i) - h_{D_2}(d_i) + h_{D_2}(d_i) - h_{D_3}(d_i)| \end{array} \right]}{2n} \\
&\leq \frac{\sum_{i=1}^{n} \left[\begin{array}{c} |T_{D_1}(d_i) - T_{D_2}(d_i)| + |T_{D_2}(d_i) - T_{D_3}(d_i)| \\ +|h_{D_1}(d_i) - h_{D_2}(d_i)| + |h_{D_2}(d_i) - h_{D_3}(d_i)| \end{array} \right]}{2n} \\
&= \frac{\sum_{i=1}^{n} \left[\ |T_{D_1}(d_i) - T_{D_3}(d_i)| + |h_{D_1}(d_i) - h_{D_3}(d_i)| \ \right]}{2n} + \\
&\quad \frac{\sum_{i=1}^{n} \left[\ |T_{D_3}(d_i) - T_{D_2}(d_i)| + |h_{D_3}(d_i) - h_{D_2}(d_i)| \ \right]}{2n} \\
&\leq \mathsf{d}_H(D_1, D_2) + \mathsf{d}_H(D_2, D_3) \mathsf{d}_H(D_1, D_3) \\
&= \frac{\sum_{i=1}^{n} \left[\ |I_{D_1}(d_i) - I_{D_3}(d_i)| + |h_{D_1}(d_i) - h_{D_3}(d_i)| \ \right]}{2n} \\
&= \frac{\sum_{i=1}^{n} \left[\begin{array}{c} |I_{D_1}(d_i) - I_{D_2}(d_i) + I_{D_2}(d_i) - I_{D_3}(d_i)| \\ +|h_{D_1}(d_i) - h_{D_2}(d_i) + h_{D_2}(d_i) - h_{D_3}(d_i)| \end{array} \right]}{2n} \\
&\leq \frac{\sum_{i=1}^{n} \left[\begin{array}{c} |I_{D_1}(d_i) - I_{D_2}(d_i)| + |I_{D_2}(d_i) - I_{D_3}(d_i)| \\ +|h_{D_1}(d_i) - h_{D_2}(d_i)| + |h_{D_2}(d_i) - h_{D_3}(d_i)| \end{array} \right]}{2n} \\
&= \frac{\sum_{i=1}^{n} \left[\ |I_{D_1}(d_i) - I_{D_3}(d_i)| + |h_{D_1}(d_i) - h_{D_3}(d_i)| \ \right]}{2n} + \\
&\quad \frac{\sum_{i=1}^{n} \left[\ |I_{D_3}(d_i) - I_{D_2}(d_i)| + |h_{D_3}(d_i) - h_{D_2}(d_i)| \ \right]}{2n} \\
&\leq \mathsf{d}_H(D_1, D_2) + \mathsf{d}_H(D_2, D_3)
\end{aligned}
$$

$$
\begin{aligned}
\mathsf{d}_H(D_1, D_3) &= \frac{\sum_{i=1}^{n} \left[\ |F_{D_1}(d_i) - F_{D_3}(d_i)| + |h_{D_1}(d_i) - h_{D_3}(d_i)| \ \right]}{2n} \\
&= \frac{\sum_{i=1}^{n} \left[\begin{array}{c} |F_{D_1}(d_i) - F_{D_2}(d_i) + F_{D_2}(d_i) - F_{D_3}(d_i)| \\ +|h_{D_1}(d_i) - h_{D_2}(d_i) + h_{D_2}(d_i) - h_{D_3}(d_i)| \end{array} \right]}{2n} \\
&\leq \frac{\sum_{i=1}^{n} \left[\begin{array}{c} |F_{D_1}(d_i) - F_{D_2}(d_i)| + |F_{D_2}(d_i) - F_{D_3}(d_i)| \\ +|h_{D_1}(d_i) - h_{D_2}(d_i)| + |h_{D_2}(d_i) - h_{D_3}(d_i)| \end{array} \right]}{2n}
\end{aligned}
$$

$$=\frac{\sum_{i=1}^{n}\left[\ |F_{D_1}(d_i)-F_{D_3}(d_i)|+|h_{D_1}(d_i)-h_{D_3}(d_i)|\ \right]}{2n}+$$

$$\frac{\sum_{i=1}^{n}\left[\ |F_{D_3}(d_i)-F_{D_2}(d_i)|+|h_{D_3}(d_i)-h_{D_2}(d_i)|\ \right]}{2n}$$

$$\leq \mathsf{d}_H(D_1,D_2)+\mathsf{d}_H(D_2,D_3)$$

Definition 9. *Let d_H be the Hesitance measure of NFSs. Then, the complement Hesitance distance measure $\mathsf{d}_H((D_1)^c,(D_2)^c)$ of two NFSs is defined as*

$$\mathsf{d}_H((D_1)^c,(D_2)^c)=\frac{\sum_{i=1}^{n}\begin{bmatrix} |(1-T_{D_1}(d_i))-(1-T_{D_2}(d_i))| \\ +|(1-h_{D_1}(d_i))-(1-h_{D_2}(d_i))| \end{bmatrix}}{2n}$$

$$\mathsf{d}_H((D_1)^c,(D_2)^c)=\frac{\sum_{i=1}^{n}\begin{bmatrix} |(1-I_{D_1}(d_i))-(1-I_{D_2}(d_i))| \\ +|(1-h_{D_1}(d_i))-(1-h_{D_2}(d_i))| \end{bmatrix}}{2n}$$

$$\mathsf{d}_H((D_1)^c,(D_2)^c)=\frac{\sum_{i=1}^{n}\begin{bmatrix} |(1-F_{D_1}(d_i))-(1-F_{D_2}(d_i))| \\ +|(1-h_{D_1}(d_i))-(1-h_{D_2}(d_i))| \end{bmatrix}}{2n}$$

Definition 10. *Let d_H be the Hesitance measure of NFSs. Then, the max Hesitance distance measure of two NFSs D_1 and D_2 is denoted by $\mathsf{d}_H(D_1\cup D_2,D_2)$ or $\mathsf{d}_H(D_1,D_2\cup D_2)$ and satisfies the function*

$$\mathsf{d}_H(D_1\cup D_2,D_2)=\frac{\sum_{i=1}^{n}\begin{bmatrix} |(T_{D_1}(d_i)\vee T_{D_2}(d_i))-T_{D_2}(d_i)| \\ +|(h_{D_1}(d_i)\vee h_{D_2}(r_i))-h_{D_2}(d_i)| \end{bmatrix}}{2n}$$

$$\mathsf{d}_H(D_1\cup D_2,D_2)=\frac{\sum_{i=1}^{n}\begin{bmatrix} |(I_{D_1}(d_i)\vee I_{D_2}(d_i))-I_{D_2}(d_i)| \\ +|(h_{D_1}(d_i)\vee h_{D_2}(r_i))-h_{D_2}(d_i)| \end{bmatrix}}{2n}$$

$$\mathsf{d}_H(D_1\cup D_2,D_2)=\frac{\sum_{i=1}^{n}\begin{bmatrix} |(F_{D_1}(d_i)\vee F_{D_2}(d_i))-F_{D_2}(d_i)| \\ +|(h_{D_1}(d_i)\vee h_{D_2}(r_i))-h_{D_2}(d_i)| \end{bmatrix}}{2n}$$

and

$$\mathsf{d}_H(D_1,D_2\cup D_2)=\frac{\sum_{i=1}^{n}\begin{bmatrix} |T_{D_1}(d_i)-(T_{D_2}(d_i)\vee T_{D_2}(d_i))| \\ +|h_{D_1}(d_i)-(h_{D_2}(d_i)\vee h_{D_2}(d_i))| \end{bmatrix}}{2n}$$

$$\mathsf{d}_H(D_1, D_2 \cup D_2) = \frac{\sum_{i=1}^{n} \begin{bmatrix} |I_{D_1}(d_i) - (I_{D_2}(d_i) \vee I_{D_2}(d_i))| \\ +|h_{D_1}(d_i) - (h_{D_2}(d_i) \vee h_{D_2}(d_i))| \end{bmatrix}}{2n}$$

$$\mathsf{d}_H(D_1, D_2 \cup D_2) = \frac{\sum_{i=1}^{n} \begin{bmatrix} |F_{D_1}(d_i) - (F_{D_2}(d_i) \vee F_{D_2}(d_i))| \\ +|h_{D_1}(d_i) - (h_{D_2}(d_i) \vee h_{D_2}(d_i))| \end{bmatrix}}{2n}$$

Definition 11. *Let d_H be the Hesitance measure of NFSs. Then, the min Hesitance distance measure of two NFSs D_1 and D_2 is denoted by $\mathsf{d}_H(D_1 \cap D_2, D_2)$ or $\mathsf{d}_H(D_1, D_2 \cap D_2)$ and satisfies the function*

$$\mathsf{d}_H(D_1 \cap D_2, D_2) = \frac{\sum_{i=1}^{n} \begin{bmatrix} |(T_{D_1}(d_i) \wedge T_{D_2}(d_i)) - T_{D_2}(d_i)| \\ +|(h_{D_1}(d_i) \wedge h_{D_2}(d_i)) - h_{D_2}(d_i)| \end{bmatrix}}{2n}$$

$$\mathsf{d}_H(D_1 \cap D_2, D_2) = \frac{\sum_{i=1}^{n} \begin{bmatrix} |(I_{D_1}(d_i) \wedge I_{D_2}(d_i)) - I_{D_2}(d_i)| \\ +|(h_{D_1}(d_i) \wedge h_{D_2}(d_i)) - h_{D_2}(d_i)| \end{bmatrix}}{2n}$$

$$\mathsf{d}_H(D_1 \cap D_2, D_2) = \frac{\sum_{i=1}^{n} \begin{bmatrix} |(F_{D_1}(d_i) \wedge F_{D_2}(d_i)) - F_{D_2}(d_i)| \\ +|(h_{D_1}(d_i) \wedge h_{D_2}(d_i)) - h_{D_2}(d_i)| \end{bmatrix}}{2n}$$

and

$$\mathsf{d}_H(D_1, D_2 \cap D_2) = \frac{\sum_{i=1}^{n} \begin{bmatrix} |T_{D_1}(d_i) - (T_{D_2}(d_i) \wedge T_{D_2}(d_i))| \\ +|h_{D_1}(d_i) - (h_{D_2}(d_i) \wedge h_{D_2}(d_i))| \end{bmatrix}}{2n}$$

$$\mathsf{d}_H(D_1, D_2 \cap D_2) = \frac{\sum_{i=1}^{n} \begin{bmatrix} |I_{D_1}(d_i) - (I_{D_2}(d_i) \wedge I_{D_2}(d_i))| \\ +|h_{D_1}(d_i) - (h_{D_2}(d_i) \wedge h_{D_2}(d_i))| \end{bmatrix}}{2n}$$

$$\mathsf{d}_H(D_1, D_2 \cap D_2) = \frac{\sum_{i=1}^{n} \begin{bmatrix} |F_{D_1}(d_i) - (F_{D_2}(d_i) \wedge F_{D_2}(d_i))| \\ +|h_{D_1}(d_i) - (h_{D_2}(d_i) \wedge h_{D_2}(d_i))| \end{bmatrix}}{2n}$$

Definition 12. *Let d_H be the Hesitance measure of NFSs. Then, the max-min Hesitance distance measure of two NFSs D_1 and D_2 is denoted by $\mathsf{d}_H(D_1 \cup D_2, D_1 \cap D_2)$ or $\mathsf{d}_H(D_1 \cap D_2, D_1 \cup D_2)$ and satisfies the function*

$$\mathsf{d}_H(D_1 \cup D_2, D_1 \cap D_2) = \frac{\sum_{i=1}^{n} \begin{bmatrix} |(T_{D_1}(d_i) \vee T_{D_2}(d_i), T_{D_1}(d_i) \wedge T_{D_2}(d_i))| \\ +|((h_{D_1}(d_i) \vee h_{D_2}(d_i), h_{D_1}(d_i) \wedge h_{D_2}(d_i))| \end{bmatrix}}{2n}$$

$$\mathsf{d}_H(D_1 \cup D_2, D_1 \cap D_2) = \frac{\sum_{i=1}^{n} \begin{bmatrix} |(I_{D_1}(d_i) \vee I_{D_2}(d_i), I_{D_1}(d_i) \wedge I_{D_2}(d_i))| \\ +|((h_{D_1}(d_i) \vee h_{D_2}(d_i), h_{D_1}(d_i) \wedge h_{D_2}(d_i))| \end{bmatrix}}{2n}$$

$$\mathsf{d}_H(D_1 \cup D_2, D_1 \cap D_2) = \frac{\sum_{i=1}^{n} \left[\begin{array}{c} |(F_{D_1}(d_i) \vee F_{D_2}(d_i), F_{D_1}(d_i) \wedge F_{D_2}(d_i))| \\ +|((h_{D_1}(d_i) \vee h_{D_2}(d_i), h_{D_1}(d_i) \wedge h_{D_2}(d_i))| \end{array} \right]}{2n}$$

and

$$\mathsf{d}_H(D_1 \cap D_2, D_1 \cup D_2) = \frac{\sum_{i=1}^{n} \left[\begin{array}{c} |(T_{D_1}(d_i) \wedge T_{D_2}(d_i), T_{D_1}(d_i) \vee T_{D_2}(d_i))| \\ +|((h_{D_1}(d_i) \wedge h_{D_2}(d_i), h_{D_1}(d_i) \vee h_{D_2}(d_i))| \end{array} \right]}{2n}$$

$$\mathsf{d}_H(D_1 \cap D_2, D_1 \cup D_2) = \frac{\sum_{i=1}^{n} \left[\begin{array}{c} |(I_{D_1}(d_i) \wedge I_{D_2}(d_i), I_{D_1}(d_i) \vee I_{D_2}(d_i))| \\ +|((h_{D_1}(d_i) \wedge h_{D_2}(d_i), h_{D_1}(d_i) \vee h_{D_2}(d_i))| \end{array} \right]}{2n}$$

$$\mathsf{d}_H(D_1 \cap D_2, D_1 \cup D_2) = \frac{\sum_{i=1}^{n} \left[\begin{array}{c} |(F_{D_1}(d_i) \wedge F_{D_2}(d_i), F_{D_1}(d_i) \vee F_{D_2}(d_i))| \\ +|((h_{D_1}(d_i) \wedge h_{D_2}(d_i), h_{D_1}(d_i) \vee h_{D_2}(d_i))| \end{array} \right]}{2n}$$

4. Decision-Making Process for Neutrosophic Distance Measure with Hesitance Values

The rapid exponential growth in social media has not only changed the nature of information sharing but has also speeded up the dissemination of misinformation, especially in the arena of political debate., Twitter, Facebook and Instagram are some of the platforms that feature huge amounts of content produced by humans as well as bots. Though these networks offer real-time information and citizens' participation, they are also fertile ground for disinformation operations, particularly on occasions that require sensitivity like elections, referendums, or government policy launches.

Conventional machine learning approaches, though great, tend to be weak at dealing with indeterminacy, vagueness, and linguistic hesitations—typical traits in political texts where messages are usually speculative or intentionally misleading. To solve these problems, the current research proposes the application of Neutrosophic Hesitance Distance Measures (NHDM), which involve truth-membership (T), indeterminacy-membership (I), falsity-membership (F), and hesitance (H) for every piece of content. These values permit greater modeling of tweets in which truth values are unknown or partially known. The case study applies a simulated environment where the set of tweets

4.1. Algorithm

Step 1: Consider a set of i alternatives, denoted as $(\alpha_i, \beta_i, \gamma_i) = \{(\alpha_1, \beta_1, \gamma_1), (\alpha_2, \beta_2, \gamma_2), \cdots, (\alpha_i, \beta_i, \gamma_i)\}$, and a set of j attributes, represented as $X_j = (D_1, D_2, \cdots, X_j)$. Additionally, let $Z = \{Z_1, Z_2, \cdots, Z_n\}$ be a group of n experts, where each Z_n corresponds to an expert participating in the decision-making process.

Step 2: The attribute value assigned to an alternative $(\alpha_i, \beta_i, \gamma_i)$ for a specific criterion X_j is expressed as $T_{ij} = xT_{ij}(T_j); I_{ij} = xI_{ij}(I_j) and F_{ij} = xF_{ij}(F_j)$. This value represents the assessment given by expert Z_m regarding alternative $(\alpha_i, \beta_i, \gamma_i)$ in relation to criterion X_j. Consequently, the structured table derived from these evaluated values serves as a fundamental representation of the decision-making data.

Step 3: Structure the table derived from these evaluated values serves as a fundamental representation of the decision-making data.

Z_i	$\alpha_i, \beta_i, \gamma_i$	T_1, I_1, F_1	T_2, I_2, F_2	$\cdots$	T_j, I_j, F_j
Z_1	$\alpha_1, \beta_1, \gamma_1$	$(x_{T_1T_1}(T_1),$ $x_{I_1I_1}(I_1),$ $x_{F_1F_1}(F_1))$	$(x_{T_1T_2}(T_2),$ $x_{I_1I_2}(I_2),$ $x_{F_1F_2}(F_2))$	$\cdots$	$(x_{T_1T_j}(T_j),$ $x_{I_1I_j}(I_j),$ $x_{F_1F_j}(F_j))$
	$\alpha_2, \beta_2, \gamma_2$	$(x_{T_2T_1}(T_1),$ $x_{I_2I_1}(I_1),$ $x_{F_2F_1}(F_1))$	$(x_{T_2T_2}(T_2),$ $x_{I_2I_2}(I_2),$ $x_{F_2F_2}(F_2))$	$\cdots$	$(x_{T_2T_j}(T_j),$ $x_{I_2I_j}(I_j),$ $x_{F_2F_j}(F_j))$
	$\vdots$	$\vdots$	$\vdots$		$\vdots$
	$\alpha_i, \beta_i, \gamma_i$	$(x_{T_iT_1}(T_1),$ $x_{I_iI_1}(I_1),$ $x_{F_iF_1}(F_1))$	$(x_{T_iT_2}(T_2),$ $x_{I_iI_2}(I_2),$ $x_{F_iF_2}(F_2))$	$\cdots$	$(x_{T_iT_j}(T_j),$ $x_{I_iI_j}(I_j),$ $x_{F_iF_j}(F_j))$
Z_2	$\alpha_1, \beta_1, \gamma_1$	$(x_{T_1T_1}(T_1),$ $x_{I_1I_1}(I_1),$ $x_{F_1F_1}(F_1))$	$(x_{T_1T_2}(T_2),$ $x_{I_1I_2}(I_2),$ $x_{F_1F_2}(F_2))$	$\cdots$	$(x_{T_1T_j}(T_j),$ $x_{I_1I_j}(I_j),$ $x_{F_1F_j}(F_j))$
	$\alpha_2, \beta_2, \gamma_2$	$(x_{T_2T_1}(T_1),$ $x_{I_2I_1}(I_1),$ $x_{F_2F_1}(F_1))$	$(x_{T_2T_2}(T_2),$ $x_{I_2I_2}(I_2),$ $x_{F_2F_2}(F_2))$	$\cdots$	$(x_{T_2T_j}(T_j),$ $x_{I_2I_j}(I_j),$ $x_{F_2F_j}(F_j))$
	$\vdots$	$\vdots$	$\vdots$		$\vdots$
	$\alpha_i, \beta_i, \gamma_i$	$(x_{T_iT_1}(T_1),$ $x_{I_iI_1}(I_1),$ $x_{F_iF_1}(F_1))$	$(x_{T_iT_2}(T_2),$ $x_{I_iI_2}(I_2),$ $x_{F_iF_2}(F_2))$	$\cdots$	$(x_{T_iT_j}(T_j),$ $x_{I_iI_j}(I_j),$ $x_{F_iF_j}(F_j))$
	$\vdots$	$\vdots$	$\vdots$		$\vdots$
Z_i	$\alpha_1, \beta_1, \gamma_1$	$(x_{T_1T_1}(T_1),$ $x_{I_1I_1}(I_1),$ $x_{F_1F_1}(F_1))$	$(x_{T_1T_2}(T_2),$ $x_{I_1I_2}(I_2),$ $x_{F_1F_2}(F_2))$	$\cdots$	$(x_{T_1T_j}(T_j),$ $x_{I_1I_j}(I_j),$ $x_{F_1F_j}(F_j))$
	$\alpha_2, \beta_2, \gamma_2$	$(x_{T_2T_1}(T_1),$ $x_{I_2I_1}(I_1),$ $x_{F_2F_1}(F_1))$	$(x_{T_2T_2}(T_2),$ $x_{I_2I_2}(I_2),$ $x_{F_2F_2}(F_2))$	$\cdots$	$(x_{T_2T_j}(T_j),$ $x_{I_2I_j}(I_j),$ $x_{F_2F_j}(F_j))$
	$\vdots$	$\vdots$	$\vdots$		$\vdots$
	$\alpha_i, \beta_i, \gamma_i$	$(x_{T_iT_1}(T_1),$ $x_{I_iI_1}(I_1),$ $x_{F_iF_1}(F_1))$	$(x_{T_iT_2}(T_2),$ $x_{I_iI_2}(I_2),$ $x_{F_iF_2}(F_2))$	$\cdots$	$(x_{T_iT_j}(T_j),$ $x_{I_iI_j}(I_j),$ $x_{F_iF_j}(F_j))$

Table 1: Collection of attributes

Step 4: Construct distance matrix using the definition of Hesitance distance measure,

$$\mathsf{d}H(D_1,D_2)=\frac{\sum i=1^n \begin{bmatrix} |T_{D_1}(D_i)-T_{D_2}(D_i)| \\ +|h_{D_1}(D_i)-h_{D_2}(D_i)| \end{bmatrix}}{2n}$$

$$\mathsf{d}H(D_1,D_2)=\frac{\sum i=1^n \begin{bmatrix} |I_{D_1}(D_i)-I_{D_2}(D_i)| \\ +|h_{D_1}(D_i)-h_{D_2}(D_i)| \end{bmatrix}}{2n}$$

$$\mathsf{d}H(D_1,D_2)=\frac{\sum i=1^n \begin{bmatrix} |F_{D_1}(D_i)-F_{D_2}(D_i)| \\ +|h_{D_1}(D_i)-h_{D_2}(D_i)| \end{bmatrix}}{2n}$$

The distance matrix is demonstrted as

$$[D_M]_{3\times 3}=$$

$$\begin{bmatrix}
(\mathsf{d}(T_1^{(Z_1)},T_1^{(Z_2)}),\mathsf{d}(I_1^{(Z_1)},I_1^{(Z_2)}),\mathsf{d}(F_1^{(Z_1)},F_1^{(Z_2)})) & (\mathsf{d}(T_1^{(Z_1)},T_1^{(Z_3)}),\mathsf{d}(I_1^{(Z_1)},I_1^{(Z_3)}),\mathsf{d}(F_1^{(Z_1)},F_1^{(Z_3)})) \\
(\mathsf{d}(T_2^{(Z_1)},T_2^{(Z_2)}),\mathsf{d}(I_2^{(Z_1)},I_2^{(Z_2)}),\mathsf{d}(F_2^{(Z_1)},F_2^{(Z_2)})) & (\mathsf{d}(T_2^{(Z_1)},T_2^{(Z_3)}),\mathsf{d}(I_2^{(Z_1)},I_2^{(Z_3)}),\mathsf{d}(F_2^{(Z_1)},F_2^{(Z_3)})) \\
(\mathsf{d}(T_3^{(Z_1)},T_3^{(Z_2)}),\mathsf{d}(I_3^{(Z_1)},I_3^{(Z_2)}),\mathsf{d}(F_3^{(Z_1)},F_3^{(Z_2)})) & (\mathsf{d}(T_3^{(Z_1)},T_3^{(Z_3)}),\mathsf{d}(I_3^{(Z_1)},I_3^{(Z_3)}),\mathsf{d}(F_3^{(Z_1)},F_3^{(Z_3)})) \\
\\
(\mathsf{d}(T_1^{(Z_2)},T_1^{(Z_3)}),\mathsf{d}(I_1^{(Z_2)},I_1^{(Z_3)}),\mathsf{d}(F_1^{(Z_2)},F_1^{(Z_3)})) & \\
(\mathsf{d}(T_2^{(Z_2)},T_2^{(Z_3)}),\mathsf{d}(I_2^{(Z_2)},I_2^{(Z_3)}),\mathsf{d}(F_2^{(Z_2)},F_2^{(Z_3)})) & \\
(\mathsf{d}(T_3^{(Z_2)},T_3^{(Z_3)}),\mathsf{d}(I_3^{(Z_2)},I_3^{(Z_3)}),\mathsf{d}(F_3^{(Z_2)},F_3^{(Z_3)})) &
\end{bmatrix}$$

Step 5: Find score function $S(D_i)$ of alternative D_i where $i=1,2,\cdots,n$ is defined as

$$S(D_i)=\sum_{i=1}^{n}\left[\frac{d(T_i^{(Z_j)},T_i^{(Z_k)})}{n},\frac{d(I_i^{(Z_j)},I_i^{(Z_k)})}{n},\frac{d(F_i^{(Z_j)},F_i^{(Z_k)})}{n}\right]$$

4.2. Numerical Illustrations

Let $D=\{D_1=Twitter,D_2=Facebook,D_3=Instagram\}$ be a collection of three social medias for political debate. Let $Y=\{Y_1,Y_2,Y_3\}$ be a set of three misleading facts, where

Y_1 = New policy announced by minister X is fake

Y_2 = Government to reduce fuel prices next month

Y_3 = Rumors suggest parliament dissolution next week

Z_i	T_i, I_i, F_i	Y_1	Y_2	Y_3
Z_i	T_1, I_1, F_1	$(0.3, 0.4, 0.5)$	$(0.3, 0.1, 0.2)$	$(0.1, 0.3, 0.5)$
	T_2, I_2, F_2	$(0.6, 0.7, 0.8)$	$(0.8, 0.9, 0.7)$	$(0.6, 0.5, 0.7)$
	T_3, I_3, F_3	$(0.8, 1.0, 0.9)$	$(0.9, 0.2, 0.6)$	$(0.8, 0.3, 0.9)$
Z_2	T_1, I_1, F_1	$(0.2, 0.4, 0.6)$	$(0.4, 0.5, 0.1)$	$(0.2, 0.4, 0.6)$
	T_2, I_2, F_2	$(0.7, 0.2, 0.8)$	$(0.9, 1.0, 0.1)$	$(0.8, 0.7, 0.6)$
	T_3, I_3, F_3	$(0.8, 0.3, 0.5)$	$(0.5, 0.9, 0.7)$	$(0.3, 0.6, 0.9)$
Z_3	T_1, I_1, F_1	$(0.4, 0.5, 0.3)$	$(0.2, 0.4, 0.1)$	$(0.5, 0.4, 0.3)$
	T_2, I_2, F_2	$(0.7, 0.3, 0.6)$	$(0.5, 0.3, 0.7)$	$(0.7, 0.5, 0.4)$
	T_3, I_3, F_3	$(0.3, 0.6, 0.9)$	$(0.9, 0.5, 0.6)$	$(0.8, 0.1, 0.2)$

Table 2: Evaluated values of alternatives

$$\begin{aligned}
\mathsf{d}(T_1^{(Z_i)}, T_1^{(Z_2)}) &= \frac{\begin{bmatrix} (|0.3-0.2|+|0.7-0.8|)+ \\ (|0.3-0.4|+|0.7-0.6|)+ \\ (|0.1-0.2|+|0.9-0.8|) \end{bmatrix}}{3(2)} \\
&= \frac{0.1+0.1+0.1+0.1+0.1+0.1}{6} = 0.100 \\
\mathsf{d}(I_1^{(Z_1)}, I_1^{(Z_2)}) &= \frac{\begin{bmatrix} (|0.4-0.4|+|0.6-0.6|)+ \\ (|0.1-0.5|+|0.9-0.5|)+ \\ (|0.3-0.4|+|0.7-0.6|) \end{bmatrix}}{3(2)} \\
&= \frac{0.4+0.4+0.1+0.1}{6} = 0.166 \\
\mathsf{d}(F_1^{(Z_1)}, F_1^{(Z_2)}) &= \frac{\begin{bmatrix} (|0.5-0.6|+|0.5-0.4|)+ \\ (|0.2-0.1|+|0.8-0.9|)+ \\ (|0.5-0.6|+|0.5-0.4|) \end{bmatrix}}{3(2)} \\
&= \frac{0.1+0.1+0.1+0.1+0.1+0.1}{6} = 0.100 \\
\mathsf{d}(T_1^{(Z_1)}, T_1^{(Z_3)}) &= \frac{\begin{bmatrix} (|0.3-0.4|+|0.7-0.6|)+ \\ (|0.3-0.2|+|0.7-0.8|)+ \\ (|0.1-0.5|+|0.9-0.5|) \end{bmatrix}}{3(2)} \\
&= \frac{0.1+0.1+0.1+0.1+0.4+0.4}{6} = 0.200
\end{aligned}$$

$$\mathsf{d}(I_1^{(Z_1)}, I_1^{(Z_3)}) = \frac{\begin{bmatrix} (|0.4-0.5|+|0.6-0.5|)+ \\ (|0.1-0.4|+|0.9-0.6|)+ \\ (|0.3-0.4|+|0.7-0.6|) \end{bmatrix}}{3(2)}$$

$$=\frac{0.1+0.1+0.3+0.3+0.1+0.1}{6}=0.166$$

$$\mathsf{d}(F_1^{(Z_1)},F_1^{(Z_3)})=\frac{\begin{bmatrix}(|0.5-0.3|+|0.5-0.7|)+\\(|0.2-0.1|+|0.8-0.9|)+\\(|0.5-0.3|+|0.5-0.7|)\end{bmatrix}}{3(2)}$$

$$=\frac{0.2+0.2+0.1+0.1+0.2+0.2}{6}=0.166$$

$$\mathsf{d}(T_1^{(Z_2)},T_1^{(Z_3)})=\frac{\begin{bmatrix}(|0.2-0.4|+|0.8-0.6|)+\\(|0.4-0.2|+|0.6-0.8|)+\\(|0.2-0.5|+|0.8-0.5|)\end{bmatrix}}{3(2)}$$

$$=\frac{0.2+0.2+0.2+0.2+0.3+0.3}{6}=0.150$$

$$\mathsf{d}(I_1^{(Z_2)},I_1^{(Z_3)})=\frac{\begin{bmatrix}(|0.4-0.5|+|0.6-0.5|)+\\(|0.5-0.4|+|0.5-0.6|)+\\(|0.4-0.4|+|0.6-0.6|)\end{bmatrix}}{3(2)}$$

$$=\frac{0.1+0.1+0.1+0.1}{6}=0.066$$

$$\mathsf{d}(F_1^{(Z_2)},F_1^{(Z_3)})=\frac{\begin{bmatrix}(|0.6-0.3|+|0.4-0.7|)+\\(|0.1-0.1|+|0.9-0.9|)+\\(|0.6-0.3|+|0.4-0.7|)\end{bmatrix}}{3(2)}$$

$$=\frac{0.3+0.3+0.3+0.3}{6}=0.200$$

Similarly,

$$\begin{array}{ccc}
\mathsf{d}(T_2^{(Z_1)},T_2^{(Z_2)})=0.133 & \mathsf{d}(T_2^{(Z_2)},T_2^{(Z_3)})=0.166 & \mathsf{d}(T_3^{(Z_1)},T_3^{(Z_3)})=0.166\\
\mathsf{d}(I_2^{(Z_1)},I_2^{(Z_2)})=0.266 & \mathsf{d}(I_2^{(Z_2)},I_2^{(Z_3)})=0.333 & \mathsf{d}(I_3^{(Z_1)},I_3^{(Z_3)})=0.300\\
\mathsf{d}(F_2^{(Z_1)},F_2^{(Z_2)})=0.233 & \mathsf{d}(F_2^{(Z_2)},F_2^{(Z_3)})=0.333 & \mathsf{d}(F_3^{(Z_1)},F_3^{(Z_3)})=0.233\\
\mathsf{d}(T_2^{(Z_1)},T_2^{(Z_3)})=0.166 & \mathsf{d}(T_3^{(Z_1)},T_3^{(Z_2)})=0.383 & \mathsf{d}(T_3^{(Z_2)},T_3^{(Z_3)})=0.466\\
\mathsf{d}(I_2^{(Z_1)},I_2^{(Z_3)})=0.333 & \mathsf{d}(I_3^{(Z_1)},I_3^{(Z_2)})=0.566 & \mathsf{d}(I_3^{(Z_2)},I_3^{(Z_3)})=0.400\\
\mathsf{d}(F_2^{(Z_1)},F_2^{(Z_3)})=0.166 & \mathsf{d}(F_3^{(Z_1)},F_3^{(Z_2)})=0.166 & \mathsf{d}(F_3^{(Z_2)},F_3^{(Z_3)})=0.400
\end{array}$$

$$[D_M]_{3\times 3} = \begin{bmatrix} (\mathsf{d}(T_1^{(Z_1)},T_1^{(Z_2)}),\mathsf{d}(I_1^{(Z_1)},I_1^{(Z_2)}),\mathsf{d}(F_1^{(Z_1)},F_1^{(Z_2)})) & (\mathsf{d}(T_1^{(Z_1)},T_1^{(Z_3)}),\mathsf{d}(I_1^{(Z_1)},I_1^{(Z_3)}),\mathsf{d}(F_1^{(Z_1)},F_1^{(Z_3)})) \\ (\mathsf{d}(T_2^{(Z_1)},T_2^{(Z_2)}),\mathsf{d}(I_2^{(Z_1)},I_2^{(Z_2)}),\mathsf{d}(F_2^{(Z_1)},F_2^{(Z_2)})) & (\mathsf{d}(T_2^{(Z_1)},T_2^{(Z_3)}),\mathsf{d}(I_2^{(Z_1)},I_2^{(Z_3)}),\mathsf{d}(F_2^{(Z_1)},F_2^{(Z_3)})) \\ (\mathsf{d}(T_3^{(Z_1)},T_3^{(Z_2)}),\mathsf{d}(I_3^{(Z_1)},I_3^{(Z_2)}),\mathsf{d}(F_3^{(Z_1)},F_3^{(Z_2)})) & (\mathsf{d}(T_3^{(Z_1)},T_3^{(Z_3)}),\mathsf{d}(I_3^{(Z_1)},I_3^{(Z_3)}),\mathsf{d}(F_3^{(Z_1)},F_3^{(Z_3)})) \\ \\ (\mathsf{d}(T_1^{(Z_2)},T_1^{(Z_3)}),\mathsf{d}(I_1^{(Z_2)},I_1^{(Z_3)}),\mathsf{d}(F_1^{(Z_2)},F_1^{(Z_3)})) & \\ (\mathsf{d}(T_2^{(Z_2)},T_2^{(Z_3)}),\mathsf{d}(I_2^{(Z_2)},I_2^{(Z_3)}),\mathsf{d}(F_2^{(Z_2)},F_2^{(Z_3)})) & \\ (\mathsf{d}(T_3^{(Z_2)},T_3^{(Z_3)}),\mathsf{d}(I_3^{(Z_2)},I_3^{(Z_3)}),\mathsf{d}(F_3^{(Z_2)},F_3^{(Z_3)})) & \end{bmatrix}$$

$$= \begin{bmatrix} (0.100, 0.166, 0.100) & (0.200, 0.166, 0.166) & (0.150, 0.066, 0.200) \\ (0.133, 0.266, 0.233) & (0.166, 0.333, 0.166) & (0.166, 0.333, 0.333) \\ (0.383, 0.566, 0.166) & (0.166, 0.300, 0.233) & (0.466, 0.400, 0.400) \end{bmatrix}$$

$$S(D_i) = \sum_{i=1}^{3} \left[\frac{\mathsf{d}(T_i^{(Z_j)},T_i^{(Z_k)})}{3}, \frac{\mathsf{d}(I_i^{(Z_j)},I_i^{(Z_k)})}{3}, \frac{\mathsf{d}(F_i^{(Z_j)},F_i^{(Z_k)})}{3} \right]$$

$$\begin{aligned} S(D_1) &= \left[\frac{0.100 + 0.200 + 0.150}{3}, \frac{0.166 + 0.166 + 0.066}{3}, \frac{0.100 + 0.166 + 0.200}{3} \right] \\ S(D_1) &= (0.150, 0.132, 0.155) \\ S(D_2) &= \left[\frac{0.133 + 0.166 + 0.166}{3}, \frac{0.266 + 0.333 + 0.333}{3}, \frac{0.233 + 0.166 + 0.333}{3} \right] \\ S(D_2) &= (0.155, 0.310, 0.244) \\ S(D_3) &= \left[\frac{0.383 + 0.166 + 0.466}{3}, \frac{0.566 + 0.300 + 0.400}{3}, \frac{0.166 + 0.233 + 0.400}{3} \right] \\ S(D_3) &= (0.338, 0.422, 0.266) \\ & D_1 \leq D_2 \leq D_3 \end{aligned}$$

since, D_3 has the maximum value its detects that the most misleading rumours are spreading through instagram.

4.3. Comparison Analysis

Historically, models based on probabilistic logic, binary classification, or machine learning techniques that assume deterministic and well-structured input data have been used to detect digital disinformation. These methods frequently fail to address the underlying contradiction, ambiguity, and vagueness seen in actual social media posts. Current methods typically oversimplify reluctant and doubtful viewpoints, which results in low efficacy when categorizing information that is speculative or partially true. The suggested work, on the other hand, presents a unique framework based on NHDMs, which can be used to describe untruth, truth, indeterminacy, and especially the hesitancy that comes

with human interpretation. Through the use of Hesitant Neutrosophic Sets, the suggested approach successfully reflects the inherent hesitancy and doubt present in user generated internet content. NHDM provide a more nuanced representation of information by allowing multiple values for each component of truth, indeterminacy, and falsehood in contrast to previous models that reduce complicated uncertainty to single-valued metrics. A well-defined set of operations and axioms for hesitance distance measurements, such as union, intersection, complement, max-min, and score functions, enhances the mathematical rigor of the suggested method and allows for reliable comparison and clustering of ambiguous data.
Additionally, the suggested decision-making framework uses multi-criteria decision-making to adaptively rank alternative pieces of information based on estimated hesitance distances, whereas existing systems frequently employ rigid decision thresholds or predefined labels. This method works especially well in situations involving misinformation where the truth is ambiguous or changing. The model's better interpretability and practicality are demonstrated by applying it to a dataset of political misinformation from social media sites like Facebook, Instagram, and Twitter. All things considered, our study closes a large gap between theoretical uncertainty modeling and practical disinformation detection, providing a more flexible, interpretable, and human-aligned approach than traditional techniques.

Criteria	DT	LSTM	Fuzzy Logic	Bayesian	NHDM (Proposed)
Uncertainty Handling	✗	✓	✓	✓	✓✓✓
Indeterminacy Handling	✗	✗	✗	✗	✓✓✓
Hesitancy Representation	✗	✗	✗	✗	✓✓✓
Interpretability	✓✓✓	✗	✓✓	✓	✓✓✓
Data Requirements	✓✓✓	✗✗	✓✓	✓	✓✓
Performance on Noisy Data	✗	✓	✓	✓	✓✓✓

Table 3. Comparison of Rumor Detection Models

5. Advantages of Neutrosophic Hesitance Distance Measures

Hesitance distance measures in neutrosophic fuzzy systems afford a potential tool for controlling uncertainty in decision-making. These measures improving decision analysis by integrating hesitancy, truth, falsity, and indeterminacy, paves way for a lot of inclusive and realistic representation of uncertainty. By obviously enumerating the level of hesitation in expert evaluations, they empower decision-makers to capture the degree of uncertainty associated with different alternatives, leading to more informed and confident choices. One of the core advantages of hesitance distance measures in neutrosophic fuzzy systems is their tractability in modelling uncertainty. Decision-makers can allocate different levels of hesitancy to various criteria, effectively reflecting the dynamic nature of real-world decision problems. This malleability makes them particularly useful in situations where likings are ambiguous, inconsistent, or influenced by multiple factors. Moreover, integrating hesitance values into distance measures agrees for a more coarse assessment of alternatives, warranting that uncertainty is not over streamlined but rather treated with the required depth. This capability is necessary in applications where multiple contradictory attitudes exist, such as medical diagnosis, risk assessment, and intelligent decision support systems. Experts can achieve greater accuracy, transparency, and adaptability in their evaluations, by leveraging hesitance-based distance measures in neutrosophic fuzzy decision-making organizations, leading to better strategic outcomes in highly uncertain environments.

6. Conclusion

The proposed framework demonstrates that incorporating hesitance values into neutrosophic distance measures significantly enhances the capacity to detect digital misinformation. By formally defining operations such as complement, max, min, and max-min within this extended neutrosophic environment, the method offers a more nuanced approach to handling uncertainty and vagueness. The example illustrates the practical potential of this technique in social media contexts, showing that decision-making processes become more accurate and robust. Overall, this study highlights the value of neutrosophic hesitance distance measures as an effective tool for improving the reliability of misinformation detection systems. Future directions of proposed work is Beyond misinformation detection, the proposed method can be explored in domains such as medical diagnosis, risk assessment, and sentiment analysis, where uncertainty and ambiguity are prominent.

Data statement:There is no data used for this study.

Author contributions: M. K: Conceptualization, Methodology, Software R. V: Data curation, Writing- Original draft preparation. M. A: Visualization, Investigation. M. K: Supervision, Writing- Reviewing and Editing.

Funding sources:This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

References

[1] F. Smarandache. *A Unifying Field in Logics: Neutrosophic Logic*. Infinite Study, 1999.

[2] H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman. Single-valued neutrosophic sets. *Multispace and Multistructure*, 4:410–413, 2010.

[3] P. Majumdar and S. K. Samanta. On similarity and entropy of neutrosophic sets. *Journal of Intelligent and Fuzzy Systems*, 26(3):1245–1252, 2013.

[4] J. Ye. Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making. *Journal of Intelligent and Fuzzy Systems*, 26(1):165–172, 2014.

[5] X. Peng and J. Dai. Hesitant fuzzy soft decision-making methods based on waspas, mabac, and copras with combined weights. *Journal of Intelligent and Fuzzy Systems*, 35(2):2247–2261, 2018.

[6] S. Ye, X. Chen, and Y. Li. A novel neutrosophic similarity measure for medical diagnosis. *Journal of Intelligent and Fuzzy Systems*, 38(3):2565–2573, 2020.

[7] S. Pramanik, S. Dalapati, S. Alam, and F. Smarandache. Ns-cross entropy-based magdm under single-valued neutrosophic set environment. *Information*, 11(2):98, 2020.

[8] K. Shu, A. Sliva, S. Wang, J. Tang, and H. Liu. Fake news detection on social media: A data mining perspective. *ACM SIGKDD Explorations Newsletter*, 19(1):22–36, 2017.

[9] C. Shao, G. L. Ciampaglia, O. Varol, K. C. Yang, A. Flammini, and F. Menczer. The spread of low-credibility content by social bots. *Nature Communications*, 9(1):1–9, 2018.

[10] M. Potthast, J. Kiesel, K. Reinartz, J. Bevendorff, and B. A. Stein. Stylometric inquiry into hyperpartisan and fake news. In *Proceedings of the 56th Annual Meeting of the Association for Computational Linguistics*, pages 231–240, 2018.

[11] M. Abdel-Basset, R. Mohamed, and F. Smarandache. A hybrid neutrosophic group anp-topsis framework for supplier selection problems. *Symmetry*, 10(6):226, 2018.

[12] D. Nagarajan, A. Kanchana, K. Jacob, N. Kausar, S. A. Edalatpanah, and M. A. Shah. A novel approach based on neutrosophic bonferroni mean operator of trapezoidal and triangular neutrosophic interval environments in multi attribute group decision making. *Scientific Reports*, 2023.

[13] M. Murugan, N. Kausar, H. Garg, M. Deveci, and J. Kim. Robotic sensor based on score and accuracy values in q-rung complex diophantine neutrosophic normal set with an aggregation operation. *Alexandria Engineering Journal*, 2023.

[14] Murugan Palanikumar, Nasreen Kausar, and Muhammet Deveci. Complex pythagorean neutrosophic normal interval-valued set with an aggregation operators using score values. *Engineering Applications of Artificial Intelligence*, 2024.

[15] M. Kaviyarasu, J. Angel, and M. Alqahtani. Geometric accumulation operators of dombi weighted trapezoidal-valued fermatean fuzzy numbers with multi-attribute group decision making. *Symmetry*, 17:1114, 2025.