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Developing a Generalized Class of Estimators for Estimation of Population Mean Using Neutrosophic Approach

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Abstract. Preceding studies have only been conducted with clear, determinate data, as the classical statistics approach is unable to accommodate ambiguity and uncertainty. As a generalization and alternative to classical statistics for such indeterminate or uncertain data, neutrosophic statistics is concerned with dealing such ambiguity in data. Making use of neutrosophic data, we propose a generalized class of estimators for the population mean under simple random sampling. Based on the numerical outcome it is presented that the suggested estimators achieved well in terms of minimum MSE. In order to more accurately represent the range of values throughout which our population parameter occurs, the results of these estimators are provided as intervals rather than a single value. We use simulation and interval data from the Islamabad Stock Exchange, focused on the UBL, to further investigate the efficiency of the suggested neutrosophic estimator. The numerical results confirm the suggested generalized neutrosophic estimators are superior to the existing methods. The mean square error (MSE) and percentage relative efficiency (PRE) performance measures demonstrate that the developed neutrosophic regression type estimator is always better than the conventional neutrosophic ratio estimator, neutrosophic product estimators, and the neutrosophic exponential ratio estimator. This paper addresses concerning how the neutrosophic regression estimator can make estimates more accurate when working with data that is unclear or uncertain and has a wide range of correlation between the study and the auxiliary variables that are being examined.

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Key Words and Phrases: Neutrosophic statistics, generalized estimator, mean estimation, bias, MSE, PRE

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1. Introduction

In conventional statistics, the information being analyzed is known and measurable. Many authors in the field of classical statistics spent considerable time investigating and developing estimators of the mean of a population in the presence of auxiliary information. Research reveals that the sampling error for ratio is greatly decreased when the auxiliary variable is incorporated in addition to the research variable when there is a high degree of correlation between the study variable and the auxiliary variable. This has significant implications for the minimum sample size required by the ratio estimation method, or the degree to which the sample size can be reduced by the approach without compromising accuracy. However, there are situations in which the available data is not random but rather indeterminate, hazy, confusing, or imprecise, and for which classical statistics and its methodologies are inadequate. Estimation based on classical statistical approaches performs poorly under these conditions. One approach to such a problem is to use fuzzy logic [31, 32], however this still doesn't account for uncertainty. Neutrosophic approaches are significantly more trustworthy in such circumstances. They have to contend with uncertainty as well as randomness. Unlike estimate problems in classical random sampling schemes, where the data is predetermined, the subject of neutrosophic estimating is new and consequently unexplored [34], and [35]. Its broad applicability, however, has given it a higher profile than classical statistical methods, leading to its use in areas like decision making [36]. In classical statistics, the emphasis is on determined data, which is obtained after all possible measurement errors have been ruled out. This holds true when there is zero margin for error in the measurements being made. Therefore, we require cutting-edge approaches to make sense of the unknown information. When precise bounds on the variable of interest are unavailable, fuzzy logic can be used as a means of disseminating such knowledge.

However, neutrosophic logic can be viewed of as a generalization of fuzzy logic; it allows for the evaluation of inconclusive alongside a definite part of the observations, and it can be applied to analysis performed under imprecise or uncertain circumstances. Recently, there has been a meteoric rise in the sophistication of the approach surrounding the application of uncertain logic in decision building [3, 9]. Following on from fuzzy collections is the next stage of development known as a complicated neutrosophic set [11]. In [10], we find a thorough presentation of fuzzy collections and its generalizations, including an analysis of their attributes and operations and a look into interval valued neutrosophic sets. When the fuzzy set is inadequate to resolve the ambiguity of a given decision-making situation, the neutrosophic set should be considered. A great deal of differentiation exists between neutrosophic cliques. In a study [14], researchers devised a trapezoidal bipolar neutrosophic quantity and a classification scheme for it to be used in decision-making contexts. In a single paper [15], the theory behind generalized spherical fuzzy numbers was laid forth alongside a thorough analytic scheme and other methods. To execute mathematical and geometric operations on pentagonal neutrosophic numbers, another study suggests using a mobile communication application [13]. Neutrosophic numbers are gaining more and more attention from scientists in recent years; for instance, a method was devised for application within the cylindrical neutrosophic domain [12]. When the data are ambiguous, statisticians turn to the neutral approach known as neutrosophic statistics. In contrast to traditional statistics, neutrosophic statistics are appropriate when working with data or a sample that includes neutrosophic. Statisticians use neutrosophic approaches [16] when there is a lack of clarity in either the population or the sample observations. The term "neutrosophic data" is used to describe a set of information where the exact values are unknown; neutrosophic numerical processes are used to analyze this type of information. The sample size may not be presented as an expected value [16] in neutrosophic statistics. Incorporating neutrosophic statistics into the current uncertainty framework was found to be highly beneficial by the group [23, 24]. By using neutrosophic numbers, rock engineers can investigate the gauge consequence and anisotropy of the combined roughness coefficient. Because of this, there is less data loss and more fitted functions are generated than would otherwise be possible [25]. A novel method of analyzing neutrosophic data via analysis of variance was introduced [30]. These authors [26-29] were the pioneers in their fields when it comes to neutrosophic interval statistics. Researchers interested in neutrosophic data might learn more by consulting the sources [17-22].

1.1. Research gap

Previous research on survey sampling has solely focused on gathering facts and figures that may be defined as accurate, definite, and unambiguous. A single correct answer is obtained by employing such methods; nonetheless, the result may be incorrect, overstated, or understated. However, in many cases and under particular conditions, the data are of a neutrosophic kind; on this occasion, neutrosophic statistics is applied, but more typical statistical processes were unsuccessful. Data with a neutrosophic flavor feature murky interval values, confusing logic, and ambiguous findings. In this way, data from experiments or populations might represent interval values in neutrosophic numbers. Although its precise value was unknown at the time of data collection, it was assumed that the true observation would lie within that range. There is more uncertain data than definite data available in the real world. Therefore, more statistical approaches specific to neutrosophic study are required. Due to the complete amount of potential study variables in the actual world, data collection can become prohibitively expensive, particularly in cases when the confirmation is vague. As a result, attempting to keep the population under tight control using techniques not optimized for ambiguous data will be both risky and costly. It's impossible to provide an explanation when the study design and any relevant covariates are neutrosophic. In order to estimate the anonymous population mean in the occurrence of auxiliary variables via survey sampling, enhanced generalized class estimators have not been investigated previously, according to a literature review. This finding is a product of academics poring over past studies that have already been published. There are not enough promising articles out there to satisfy the demand in the field of statistics just yet. Beginning with this research, we can now go further in this area.

1.2. Scope of the suggested idea

When working with imperfect or unclear data sets, neutrosophic analysis might be helpful. Furthermore, this method allows for competing viewpoints to coexist. Certain observations may be given in a range of unknown values if certain instruments are used to collect the data. Because of statistical uncertainty, traditional methods of investigation are now worthless. Survey sampling based on neutrosophic has come a long way in recent years, but the generalized class of estimating is still novel and must be approached with great caution due to uncertainty in the underlying data source. Products with insignificant measurement errors or manufacturing mistakes will be accepted if they fall inside a particular measurement range, such as when a machine produces nuts or bolts and we measure those things. Using outdated statistical approaches that provide only a single value increases the likelihood of discarding perfectly serviceable things. Neutrosophic statistics delivers precise assessment of interval outcomes.

Neutrosophic statistics is a new way of looking at things that can work with datasets that aren't completely clear or have missing information. This strategy lets people have different beliefs and works with a range of uncertain figures that could stand for some observations, even a precise measurement. On the other hand, standard statistics don't work well when there is ambiguity. This is where neutrosophic statistics come in, giving us a new way to look at data analysis. In actual life, we often lack knowledge of the population parameters. In these situations, statistical inference methods may need to be more useful. Instead, reasonable estimates are utilized to solve the problem of an unknown parameter value by guessing its values. This practical technique gives the statistician confidence that the resulting data are not clear but still valuable. Neutrosophic statistics are a solid way to solve these problems since they can find the optimal interval value with the least mean square error. Interested authors may read [37-44] to learn about neutrosophic approach based on different sampling design.

2. Terminlogy

Consider a neutrosophic sample of size $n_N \in (n_L, n_U)$, which is chosen from a population of N_N units $(\Omega_1,\ \Omega_2,\ \ldots,\ \Omega_N)$. Let y_{Ni} is the i^{th} sample observation of neutrosophic data, which is of the form $y_N \in (y_L, y_U)$ and similarly for auxiliary variable $x_N \in (x_L, x_U)$, and $z_N \in (z_L, z_U)$. As $\overline{Y}_N \in (Y_L, Y_U)$ and $\overline{X}_N \in (X_L, X_U)$, $\overline{Z}_N \in (Z_L, Z_U)$ be the population mean of neutrosophic study and auxiliary variables. The neutrosophic coefficient of variation for y_N, x_N z_N are denoted by $C_{yN} \in (C_{yNL}, C_{yNU})$, $C_{xN} \in (C_{xNL}, C_{xNU})$ and $C_{zN} \in (C_{zNL}, C_{zNU})$. Let $s^2_{yN} = \frac{\sum_{i=1}^{n}(y_{iN}-\overline{y}_N)^2}{n_N-1}$, $s^2_{xN} = \frac{\sum_{i=1}^{n}(x_{iN}-\overline{x}_N)^2}{n_N-1}$ and $s^2_{zN} = \frac{\sum_{i=1}^{n}(z_{iN}-\overline{z}_N)^2}{n_N-1}$, be the unbiased sample variances conforming to population variances $S^2_{yN} = \frac{\sum_{i=1}^{N}(y_{iN}-\overline{Y}_N)^2}{N_N-1}$, $S^2_{xN} = \frac{\sum_{i=1}^{N}\left(x_{iN}-\overline{X}_N\right)^2}{N_N-1}$, and $S^2_{zN} = \frac{\sum_{i=1}^{N}\left(x_{iN}-\overline{X}_N\right)^2}{N_N-1}$ of Y_N, X_N, and Z_N correspondingly.

Let C_{yN}, C_{xN} and C_{zN}, denotes the population coefficient of variation of y_N, x_N and z_N, where $C_{yN}=\frac{S_{yN}}{\overline{Y}}$, $C_{XN}=\frac{S_{xN}}{\overline{X}}$ and $C_{ZN}=\frac{S_{zN}}{\overline{Z}}$.

$$\rho_{yxN} = \frac{S_{yxN}}{S_{yN}S_{xN}}, \rho_{yzN} = \frac{S_{yzN}}{S_{yN}S_{zN}}, \rho_{xzN} = \frac{S_{xzN}}{S_{xN}S_{zN}}$$

Where $S_{yN} = \sqrt{\frac{\sum_{i=1}^{N}\left(y_{iN}-\overline{Y}_N\right)^2}{N_N-1}}$, $S_{xN}=\sqrt{\frac{\sum_{i=1}^{N}\left(x_{iN}-\overline{X}_N\right)^2}{N_N-1}}$ and $S_{zN}=\sqrt{\frac{\sum_{i=1}^{N}\left(z_{iN}-\overline{Z}_N\right)^2}{N_N-1}}$

$$S_{yx} = \frac{\sum_{i=1}^{N}\left(y_{iN}-\overline{Y}_N\right)\left(x_{iN}-\overline{X}_N\right)}{N_N-1}, S_{yx} = \frac{\sum_{i=1}^{N}\left(y_{iN}-\overline{Y}_N\right)\left(z_{iN}-\overline{Z}_N\right)}{N_N-1}, S_{xz} = \frac{\sum_{i=1}^{N}\left(x_{iN}-\overline{X}_N\right)\left(z_{iN}-\overline{Z}_N\right)}{N_N-1}$$

$$\lambda_N = \left(\frac{1}{n_N} - \frac{1}{N_N}\right).$$

3. Existing estimators

In this section, we have presented some existing counterparts.

(i) The conventional estimator for population mean based on neutrosophic statistics, is given by:

$$\widehat{\overline{Y}}_{UN} = \overline{y}_N \tag{1}$$

The variance of $\widehat{\overline{Y}}_{UN}$, is given by:

$$Var(\widehat{\overline{Y}}_{UN}) = \lambda_N \overline{Y}_N^2 C_{yN}^2 \tag{2}$$

$$\widehat{\overline{Y}}_{RN} = \overline{y}_N\left(\frac{\overline{X}_N}{\overline{x}_N}\right) \tag{3}$$

$$\widehat{\overline{Y}}_{RN} = \overline{Y}_N\left(1+e_o\right)\left(\frac{\overline{X}_N}{\overline{X}_N\left(1+e_1\right)}\right)$$

$$\widehat{\overline{Y}}_{RN} = \overline{Y}_N\left(1+e_o\right)\left(1+e_1\right)^{-1}$$

$$\widehat{\overline{Y}}_{RN} = \overline{Y}_N\left(1+e_o\right)\left(1-e_1+e_1^2\right)$$

$$\widehat{\overline{Y}}_{RN} = \overline{Y}_N\left(1-e_1+e_1^2+e_o-e_oe_1-e_oe_1^2\right) \tag{4}$$

$$\widehat{\overline{Y}}_{RN} - \overline{Y}_N = \overline{Y}_N\left(1-e_1+e_1^2+e_o-e_oe_1-e_oe_1^2\right) \tag{5}$$

Apply expectation both sides of the equation(5), we have:

$$E\left(\widehat{\overline{Y}}_{RN} - \overline{Y}_N\right) = \overline{Y}_N E\left(1-e_1+e_1^2+e_o-e_oe_1-e_oe_1^2\right) \tag{6}$$

$$E\left(\widehat{\overline{Y}}_{RN} - \overline{Y}_N\right) = \overline{Y}_N E\left(-e_1+e_1^2+e_o-e_oe_1-e_oe_1^2\right) \tag{7}$$

Bias$\left(\widehat{\overline{Y}}_{RN}\right)$=$\overline{Y}_N \left(E\left(e_1\right)+E\left(e_1^2\right)+E\left(e_o\right)-E\left(e_oe_1\right)-E\left(e_oe_1^2\right)\right)$

Where

$$E\left(e_i\right)=0$$

Ignore higher order approximation, we have:

Bias$\left(\widehat{\overline{Y}}_{RN}\right)$=$\overline{Y}_N \left(E\left(e_1^2\right)+E\left(e_oe_1\right)\right)$

Apply expectation, we have:

The bias of $\widehat{\overline{Y}}_{RN}$, are given by:

Bias($\widehat{\overline{Y}}_{RN}$) $\cong$?$_N\overline{Y}_N \left(C_{yN}^2-\rho_{yxN}C_{yN}C_{xN}\right)$,

Now

Squaring and apply expectation of equation (6), we have

E$\left(\widehat{\overline{Y}}_{RN}-\overline{Y}_N\right)^2$=$\overline{Y}_N E\left(-e_1+e_1^2+e_o-e_oe_1-e_oe_1^2\right)$

Ignore higher order, we have:

MSE$\left(\widehat{\overline{Y}}_{RN}\right)$= $\overline{Y}_N^2$E$(e_o-e_1)^2$

MSE$\left(\widehat{\overline{Y}}_{RN}\right)$= $\overline{Y}_N^2$E$\left(e_0^2+e_1^2-2e_oe_1\right)$

Apply expectation, we have:

MSE$\left(\widehat{\overline{Y}}_{RN}\right)$= $\overline{Y}_N^2$E$\left(E\left(e_0^2\right)+E\left(e_1^2\right)-2E\left(e_oe_1\right)\right)$

Finally after applying the expected values, we got the mean squared error of $\widehat{\overline{Y}}_{RN}$:

$$MSE(\widehat{\overline{Y}}_{RN}) \cong \lambda_N\overline{Y}_N^2 \left(C_{yN}^2+C_{xN}^2-2\rho_{yxN}C_{yN}C_{xN}\right) \tag{8}$$

(ii) The product estimator, given by:

$$\widehat{\overline{Y}}_{PN}=\overline{y}_N\left(\frac{\overline{x}_N}{\overline{X}_N}\right) \tag{9}$$

The properties of $\widehat{\overline{Y}}_{PN}$, are given by:

Bias($\widehat{\overline{Y}}_{PN}$) $\cong \lambda_N\overline{Y}_N\ \rho_{yxN}C_{yN}C_{xN}$,

and

$$MSE(\widehat{\overline{Y}}_{PN}) \cong \lambda_N\overline{Y}_N^2 \left(C_{yN}^2+C_{xN}^2+2\rho_{yxN}C_{yN}C_{xN}\right) \tag{10}$$

(iii) The adopted difference estimator is given by:

$$\hat{\overline{Y}}_{DN} = \overline{y}_N + Q\left(\overline{X}_N - \overline{x}_N\right), \tag{11}$$

where Q is constant.

$Q_{optimum} = \rho_{yxN}\left(\frac{S_{yN}}{S_{xN}}\right)$

The variance of $\hat{\overline{Y}}_{DN}$, at the optimum value of $Q_{optimum}$ is given by:

$$\text{Var}(\hat{\overline{Y}}_{DN})_{\ min} \cong \lambda_N \overline{Y}_N^2 C_{yN}^2 (1-\rho_{yxN}^2) \tag{12}$$

(iv) The [6] recommended as:

$$\hat{\overline{Y}}_{BTRN} = \overline{y}_N exp\left(\frac{\overline{X}_N - \overline{x}_N}{\overline{X}_N + \overline{x}_N}\right), \tag{13}$$

$$\hat{\overline{Y}}_{BTPN} = \overline{y}_N exp\left(\frac{\overline{x}_N - \overline{X}_N}{\overline{X}_N + \overline{x}_N}\right) \tag{14}$$

The properties of $\hat{\overline{Y}}_{BTN}$, are given by:

$\text{Bias}\left(\hat{\overline{Y}}_{BTRN}\right) \cong\ \lambda_N\ \overline{Y}_N \left(\frac{3}{8}\ C_{xN}^2 - \frac{1}{2}\rho_{yxN} C_{yN} C_{xN}\right)$,

$$MSE\left(\hat{\overline{Y}}_{BTRN}\right) \cong \lambda_N \overline{Y}_N^2 \left(C_{yN}^2 + \frac{1}{4}C_{xN}^2 - \rho_{yxN} C_{yN} C_{xN}\right) \tag{15}$$

$\text{Bias}\left(\hat{\overline{Y}}_{BTPN}\right) \cong\ \lambda_N\ \overline{Y}_N \left(\frac{3}{8}\ C_{xN}^2 - \frac{1}{2}\rho_{yxN} C_{yN} C_{xN}\right)$,

$$MSE\left(\hat{\overline{Y}}_{BTPN}\right) \cong \lambda_N \overline{Y}_N^2 \left(C_{yN}^2 + \frac{1}{4}C_{xN}^2 + \rho_{yxN} C_{yN} C_{xN}\right) \tag{16}$$

(v) The exponential estimator developed by [33] given by:

$$\hat{\overline{Y}}_{SN} = \overline{y}_N exp\left(\frac{a\left(\overline{X}_N - \overline{x}_N\right)}{a\left(\overline{X}_N + \overline{x}_N\right) + 2b}\right) \tag{17}$$

$\text{Bias}\left(\hat{\overline{Y}}_{SN}\right) \cong\ \lambda_N\ \overline{Y}_N \left(\frac{3}{8}\ \theta_N^2 C_{xN}^2 - \frac{1}{2}\theta_N \rho_{yxN} C_{yN} C_{xN}\right)$,

$$MSE\left(\hat{\overline{Y}}_{SN}\right) = \lambda_N \overline{Y}_N^2 \left(C_{yN}^2 + \frac{1}{4}\theta_N^2 C_{xN}^2 + \theta_N \rho_{yxN} C_{yN} C_{xN}\right), \tag{18}$$

where $\theta_N = \frac{a\overline{X}_N}{a\overline{X}_N + b}$.

4. Suggested estimator:

Classical statistics can only be utilized on investigations with clear, unambiguous data. It fails to apply with data that is hazy or uncertain. Neurosophic statistics is a way to handle data that is ambiguous or not certain. It is both an alternative to and a generalization of classical statistics for this specific type of data. Taking motivation from [4], we recommended an improved generalized neutrosophic estimators under simple random sampling using twofold auxiliary information. The neutrosophic estimators may be superior to classical estimators in situations where the study variable's observations are nondeterministic; they may perform inadequately in situations where the observations are deterministic. The suggested neutrosophic estimators have been shown to be more trustworthy than the traditional method of estimating. Simulation studies and medical uses in neutrosophic settings have shown that the suggested estimators are even more accurate. The recommended class of estimators is given by:

$$\widehat{\overline{Y}}^{(*)a,b}_{Prop,\ N} = \left[\psi_{15}\overline{y}_N + \psi_{16}\left(\overline{X}_N - \overline{x}_N\right) + \psi_{17}\left(\overline{Z}_N - \overline{z}_N\right)\right] exp\left(\frac{a\left(\overline{X}_N - \overline{x}_N\right)}{a\left(\overline{X}_N + \overline{x}_N\right) + (a-1)\, a\overline{X}_N(a+1)\mathrm{b}}\right) \tag{19}$$

where ψ_{15}, ψ_{16}, and ψ_{17} are constants. Some members of the suggested class of estimators are given in Table 1.

Table 1: Some estimators of $\widehat{\overline{Y}}^{(*)a,b}_{Prop,\ N}$

a	b	$\widehat{\overline{Y}}^{(*)}_{Prop,\ N}$
1	C_{xN}	$\widehat{\overline{Y}}^{1}_{Prop,\ N}$
1	$\beta_{\mathbf{2}(xN)}$	$\widehat{\overline{Y}}^{2}_{Prop,\ N}$
$\beta_{\mathbf{2}(xN)}$	C_{xN}	$\widehat{\overline{Y}}^{3}_{Prop,\ N}$
C_{xN}	$\beta_{\mathbf{2}(xN)}$	$\widehat{\overline{Y}}^{4}_{Prop,\ N}$
1	ρ_{yxN}	$\widehat{\overline{Y}}^{5}_{Prop,\ N}$
C_{xN}	ρ_{yxN}	$\widehat{\overline{Y}}^{6}_{Prop,\ N}$
ρ_{yxN}	C_{xN}	$\widehat{\overline{Y}}^{7}_{Prop,\ N}$
$\beta_{\mathbf{2}(xN)}$	ρ_{yxN}	$\widehat{\overline{Y}}^{8}_{Prop,\ N}$
ρ_{yxN}	$\beta_{\mathbf{2}(xN)}$	$\widehat{\overline{Y}}^{9}_{Prop,\ N}$
1	$N_N\overline{X}_N$	$\widehat{\overline{Y}}^{10}_{Prop,\ N}$

Solving equation (19), we have:

$$\widehat{\overline{Y}}^{(*)a,b}_{Prop,\ N} = \left[\psi_{15}\overline{Y}_N\left(1+\xi_0\right) - \psi_{16}\overline{X}_N\xi_2 - \psi_{17}\overline{Z}_N\xi_3\right]\left[1 - \theta\xi_1 + \frac{3}{8}\theta^2\xi_1^2 + \dots\right] \tag{20}$$

where $\theta = \frac{\alpha \overline{X}_N}{[(\alpha+1)(a\overline{X}_N+\beta)]}$, is a known quantity.

$$\widehat{\overline{Y}}^{(*)a,b}_{Prop,\ N} - \overline{Y}_N = \left[-\overline{Y}_N + \overline{Y}_N\psi_{15} + \overline{Y}_N\psi_{15}\xi_0 + \frac{5}{8}\overline{Y}_N\psi_{15}\xi_1^2 - \psi_{16}\overline{Z}_N\xi_2 + \overline{X}_N\psi_{17}\theta^2\xi_1^2 \right] \tag{21}$$

Taking expectations of equation (21), we have:

$$Bias(\widehat{\overline{Y}}^{(*)a,b}_{Prop,\ N}) = \overline{Y}_N \left[\psi_{15} - 1 + \frac{5}{8}\lambda\psi_{15}C_{xN}^2 + \lambda\psi_{17}\overline{Z}_N\theta C_{xN}^2 \right] \tag{22}$$

Squaring equation (21), we have:

$$\left(\widehat{\overline{Y}}^{(*)a,b}_{Prop,\ N} - \overline{Y}_N \right)^2 = \overline{Y}_N^2 + \overline{Y}_N^2\psi_{15}^2 + \overline{Y}_N^2\psi_{15}^2\xi_0^2 + \frac{5}{4}\overline{Y}_N^2\psi_{15}^2\xi_1^2 + \overline{Z}_N^2\psi_{16}^2\xi_2^2 + \overline{X}_N^2\psi_{17}^2\xi_1^2 - 2\overline{Y}_N\overline{Z}_N\psi_{15}\psi_{16}\xi_0\xi_2$$
$$+2\,\overline{Y}_N\overline{X}_N\psi_{15}\psi_{17}\theta\xi_1^2 - 2\,\overline{Y}_N\overline{X}_N\psi_{15}\psi_{17}\xi_0\xi_1 + 2\,\overline{Y}_N\overline{Z}_N\psi_{16}\psi_{17}\xi_1\xi_2 - 2\,\overline{Y}_N^2\psi_{15} - \frac{5}{4}\overline{Y}_N^2\xi_1^2$$
$$-2\,\overline{Y}_N\overline{X}_N\psi_{17}\theta\xi_1^2 \tag{23}$$

$$MSE(\widehat{\overline{Y}}^{(*)a,b}_{Prop,\ N}) = \overline{Y}_N^2 \begin{bmatrix} 1 + \psi_{15}^2\left\{ 1 + \lambda\left(C_{yN}^2 + \frac{5}{4}C_{xN}^2 \right) + \lambda\overline{Z}_N^2\psi_{16}^2C_{zN}^2 - \lambda R_1\psi_{17}C_{xN}^2\left(2\theta + R_1\psi_{17}\right) \right\} \\ -2R_2\psi_{15}\lambda_{16}\rho_{C_{yN}C_{zN}}C_{yN}C_{zN} - 2\lambda R_1\psi_{15}\psi_{17}\left(\rho_{C_{yN}C_{xN}}C_{yN}C_{xN} - \theta C_{xN}^2\right) \\ +2\lambda R_1R_2\psi_{16}\psi_{17}\rho_{C_{yN}C_{zN}}C_{yN}C_{zN} \end{bmatrix} \tag{24}$$

Where $R_1 = \frac{\overline{X}_N}{\overline{Y}_N}$, $R_2 = \frac{\overline{Z}_N}{\overline{Y}_N}$.
Differentiate equation (24) w.r.t ψ_{15}, we have:

$$\frac{\Delta MSE}{\Delta\lambda_{15}} = 2\overline{Y}_N^2\left[\lambda_{15}\ \lambda C_{yN}^2 - R_2\lambda_{16}\rho_{C_{yN}C_{zN}}C_{yN}C_{zN} - \lambda R_1\lambda_{17}\rho_{C_{yN}C_{xN}}C_{yN}C_{xN} \right] = 0$$

Differentiate equation (24) w.r.t ψ_{16}, we have:

$$\frac{\Delta MSE}{\Delta\lambda_{16}} = 2\overline{Y}_N^2\left[\lambda_{15}\ \lambda\ \overline{Z}_N^2\psi_{16}^2C_{zN}^2 - R_2\lambda_{15}\rho_{C_{yN}C_{zN}}C_{yN}C_{zN} - \lambda R_1R_2\lambda_{17}\rho_{C_{yN}C_{zN}}C_{yN}C_{zN} \right] = 0$$

Differentiate equation (24) w.r.t ψ_{17}, we have:

$$\frac{\Delta MSE}{\Delta\lambda_{17}} = 2\overline{Y}_N^2\left[-\lambda R_1C_{xN}^2\lambda + R_1 - \lambda R_1\lambda_{15}\rho_{C_{yN}C_{xN}}C_{yN}C_{xN} + \lambda R_1R_2\lambda_{16}\rho_{C_{yN}C_{zN}}C_{yN}C_{zN} \right] = 0$$

This is done by taking partial derivatives of equation (24) with respect to ψ_{15}, ψ_{16} and ψ_{17} and equate to zero, and solving the resulting system of equation to find their optimal values The optimal values of ψ_{15}, ψ_{16} and ψ_{17}, are given by:

$$\psi_{15} = \left(\frac{1 - \frac{1}{2}\theta^2C_{xN}^2}{1 + \lambda C_{yN}^2(1 - Q_{yN.xNzN}^2)} \right),$$

$$\psi_{16} = \left(\frac{\overline{Y}_N \left[\lambda\theta^3 \left(-1 + \rho^2_{xNzN} - C_{yN} \left(1 - \frac{1}{2}\lambda\theta^2 C^2_{xN} \right) \left(\rho_{yNxN} - \rho_{xNzN}\rho_{yNzN} \right) + \theta C_{xN} \left\{ -1 + \rho^2_{xNzN} \right\} \right) \left(-1 + \lambda C^2_{xN} \left\{ 1 - Q^2_{yN.xNzN} \right\} \right) \right]}{\overline{X}_N C_{xN} \left\{ -1 + \rho^2_{xNzN} \right\} \left(1 + \lambda C^2_{xN} \left\{ 1 - Q^2_{yN.xNzN} \right\} \right)} \right),$$

$$\psi_{17} = \left(\frac{\overline{Y}_N \left(1 - \frac{1}{2}\lambda\theta^2 C^2_{xN} \right) C_{yN} \left(\rho_{yNzN}\rho_{yNxN} - \rho_{xNzN} \right)}{\overline{Z}_N C_{zN} \left\{ -1 + \rho^2_{xNzN} \right\} \left[1 + \lambda C^2_{xN} \left\{ 1 - Q^2_{yN.xNzN} \right\} \right]} \right)$$

Substituting these optimal values into equation (24), which gives us minimized MSE as shown in equation (25).

$$MSE\left(\widehat{\widehat{Y}}^{(*)a,b}_{Prop,\ N}\right) = \frac{\lambda \overline{Y}^2_N \left[C^2_{yN} \left\{ 1 - Q^2_{yN.xNzN} \right\} - \frac{1}{2}\lambda\theta^4 C^4_{xN} - \lambda\theta^2 C^2_{xN} \left\{ 1 - Q^2_{yN.xNzN} \right\} \right]}{1 + \lambda C^2_{yN} \left\{ 1 - Q^2_{yN.xNzN} \right\}}, \tag{25}$$

where

$$Q^2_{yN.xNzN} = \frac{\rho^2_{yNxN+}\rho^2_{yNzN} - 2\rho_{yNxN}\rho_{yNzN}\rho_{xNzN}}{1 - \rho^2_{xNzN}}.$$

5. Numerical study

In this section, we consider neutrosophic data for numercial comparisions of the suggested and exisitng estimators. In Table 2, we see the data sets and their description.

$PRE = \frac{Var(\widehat{\widehat{Y}}_{UN})}{MSE\left(\widehat{\widehat{Y}}_{i,NN}\right)} \times 100$,

where $i = \widehat{\widehat{Y}}_{RN}, \widehat{\widehat{Y}}_{PN}, \widehat{\widehat{Y}}_{DN}, , \widehat{\widehat{Y}}_{BTRN}, \widehat{\widehat{Y}}_{BTPN}, \widehat{\widehat{Y}}_{SN}, \widehat{\widehat{Y}}^{(*)}_{Prop,\ N}$ $(* = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10)$.
We performed a numerical analysis of UBL data from the Islamabad Stock Exchange, which included interval data with uncertain values. The data were taken from [1] and [2]. The summary statistics of the data are presented in Table 2.

Table 2: Summary statistics using neutrosophic data

Parameter	Values	Parameter	Values
N_N	[239,239]	S^2_{yN}	[2968.468,3131.635]
n_N	[35,35]	S^2_{xN}	[3110.931,3156.542]
λ_N	[0.02438733, 0.02438733]	S^2_{zN}	[4779.904, 4783.409]
$\overline{Y}_N$	[131.5651,135.6154]	ρ_{yxN}	[0.8680465,0.5235659]
$\overline{X}_N$	[149.6231,153.5848]	ρ_{yzN}	[0.1637699, 0.1761911]
$\overline{Z}_N$	[119.6402, 119.4603]	ρ_{xzN}	[0.3753044, 0.3712835]

Table 3: MSE using neutrosophic data

Estimator	**MSE Values of existing work**	$\hat{\bar{\bar{Y}}}_{SN}$ **(Singh estimator)**	$\hat{\bar{\bar{Y}}}_{propN}$**(Proposed work)**
$\hat{\bar{\bar{Y}}}_{UN}$	[0.004330113, 0.004330113]	[0.003172306, 0.001862044]	[0.003087266, 0.0009190622]
$\hat{\bar{\bar{Y}}}_{RN}$	[0.003702557, 0.001067901]	[0.00317687, 0.001885676]	[0.003088775, 0.0009195191]
$\hat{\bar{\bar{Y}}}_{PN}$	[0.01170995, 0.01428829]	[0.003172302, 0.001862026]	[0.003087265, 0.0009190619]
$\hat{\bar{\bar{Y}}}_{DN}$	[0.003143137, 0.001067352]	[0.004017153, 0.003797651]	[0.003139938, 0.0009346095]
$\hat{\bar{\bar{Y}}}_{BTRN}$	[0.0031723, 0.001862012]	[0.003173376, 0.001871448]	[0.003087632, 0.0009192456]
$\hat{\bar{\bar{Y}}}_{BTPN}$	[0.008472206, 0.007175997]	[0.003590381, 0.003309825]	[0.003130105, 0.0009332394]
		[0.003172311, 0.001862049] [0.00317279, 0.001866334] [0.003181196, 0.001889243] [0.00433003, 0.004329979]	[0.003087268, 0.0009190623] [0.003087433, 0.0009191462] [0.003090096, 0.000919587] [0.003141473, 0.0009350626]

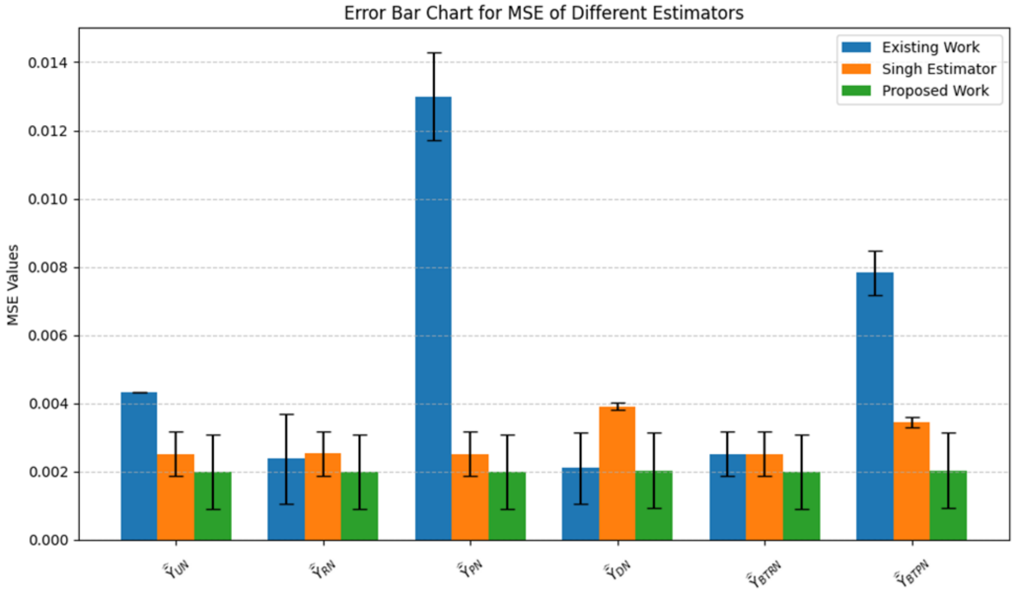

Figure 1: Showing mean squared error of all considered estimators based on real data set

Table 4: PRE using neutrosophic data

Estimator	**PRE Values of existing work**	$\widehat{\bar{\bar{Y}}}_{SN}$ **(Singh estimator)**	$\widehat{\bar{\bar{Y}}}_{propN}$**(Proposed work)**
$\widehat{\bar{\bar{Y}}}_{UN}$	[100, 100]	[136.4973, 232.5462]	[140.2572, 471.1447]
$\widehat{\bar{\bar{Y}}}_{RN}$	[116.9493, 405.4787,]	[136.3012, 229.6319]	[140.1887, 470.9106]
$\widehat{\bar{\bar{Y}}}_{PN}$	[30.30533, 36.97806]	[136.4975, 232.5484]	[140.2573, 471.1449]
$\widehat{\bar{\bar{Y}}}_{DN}$	[137.764, 405.6874]	[107.7906, 114.0208]	[137.9044¸ 463.3072]
$\widehat{\bar{\bar{Y}}}_{BTRN}$	[136.4976, 232.5503]	[136.4513, 231.3777]	[140.2406, 471.0507]
$\widehat{\bar{\bar{Y}}}_{BTPN}$	[51.10963, 60.34162]	[120.6031, 130.8261]	[138.3376, 463.9874]
		[136.4971, 232.5456] [136.4765¸232.0116] [136.1159, 229.1983] [100.0019, 100.0031]	[140.2571, 471.1447] [140.2496, 471.1017] [140.1288, 470.8759] [137.837, 463.0827]

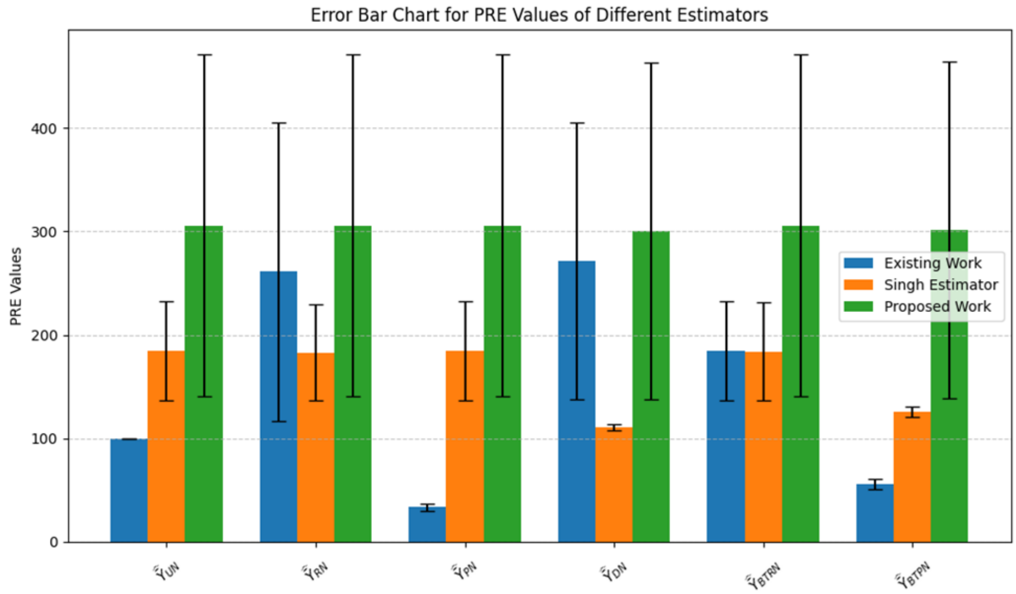

Figure 2: Showing percentage relative efficiency of all considered estimators based on real data set

6. Simulation study

In this section, a simulation study is conducted for the validation of numerical results. We used simulated neutrosophic data and comparison is done on the basis of MSE and PRE. Y_N, X_N and Z_N are neutrosophic random variables, following the neutrosophic normal distribution. The Y_N has mean μ_{YN} and variance σ^2_{YN}. The X_N has neutrosophic normal distribution with mean μ_{XN} and variance σ^2_{XN}. The Z_N has neutrosophic normal distribution with mean μ_{ZN} and variance σ^2_{ZN}. Where $Y_N \in (Y_L,\, Y_U)$, $X_N \in (X_L,\, X_U)$ and $Z_N \in (z_L,\, z_U)$. We have generated 5000 random normal variate by using neutrosophic normal distribution, i.e. $Y_N \sim N_N$ ((5, 8),((0.9)^2 , (1.2)^2) and $X_N \sim N_N$ ((15, 18),((0.5)^2 , (0.7)^2)).

A neutrosophic normal distribution is typically denoted as:

$X_N \sim \mathrm{N}\left[(\mu_L,\ \mu_U),\quad (\sigma^2_L,\quad \sigma^2_U\ )\right]$

where

The mean of the distribution lies in the interval $(\mu_L,\ \mu_U)$

The variance lies in the interval $(\sigma^2_L,\quad \sigma^2_U)$

The random variable $X_N \in (X_L,\ X_U)$, derived from the above parameters.

For simulation purpose we generate 5000 random samples from neutrosophic normal distributions defined as:

$$Y_N \sim N_N((5,8),((0.9)^2,\ (1.2)^2)$$

$$X_N \sim N_N((15,18),(\ (0.5)^2,\ (0.7)^2)).$$

For each simulated value, first randomly sample a mean and variance within the specified intervals.

For $\boldsymbol{Y_N}$
$Y_N \sim$ U (5, 8)
$\sigma_Y^2 \sim$ U(0.81, 1.44)
For $\boldsymbol{X_N}$
$X_N \sim$ U (15, 18)
$\sigma_X^2 \sim$ U(0.25, 0.49)
By utilizing μ and σ^2 to generate values for standard normal distribution.
$x_i \sim N_N\left(\mu_X,\ \sigma_X^2\right)$ and $y_i \sim N_N\left(\mu_Y,\ \sigma_Y^2\right)$
Repeat the above steps for 5000 iteration to generate 5000 simulated values for each of X_N and Y_N.

Table 5: MSE using simulated neutrosophic data

Estimator	**MSE Values of existing work**	$\widehat{\overline{\boldsymbol{Y}}}_{\boldsymbol{SN}}$ **(Singh estimator)**	$\widehat{\overline{\boldsymbol{Y}}}_{\boldsymbol{propN}}$ **(Proposed work)**
$\widehat{\overline{Y}}_{UN}$	[0.2619458, 0.2619458]	[0.2022682, 0.1893483]	[0.02465215, 0.02063134]
$\widehat{\overline{Y}}_{RN}$	[0.1290996, 0.1497684]	[0.2082852, 0.1952096]	[0.02465263, 0.02063211]
$\widehat{\overline{Y}}_{PN}$	[0.448552, 0.4089126]	[0.2017871, 0.1890021]	[0.02465212, 0.02063127]
$\widehat{\overline{Y}}_{DN}$	[0.02065194, 0.02466429]	[0.2204352¸ 0.2113867]	[0.02465366, 0.02063334]
$\widehat{\overline{Y}}_{BTRN}$	[0.1888027, 0.2015085]	[0.2040597, 0.1911544]	[0.0246523, 0.02063158]
$\widehat{\overline{Y}}_{BTPN}$	[0.3310805, 0.3485289]	[0.2098011, 0.1985855]	[0.02465288, 0.02063229]
		[0.2022997, 0.1893758] [0.202462, 0.1896759] [0.2085363, 0.1955049] [0.2618358¸0.2617713]	[0.02465215, 0.02063134] [0.02465218, 0.02063136] [0.2465265, 0.02063214] [0.02465485, 0.02063499]

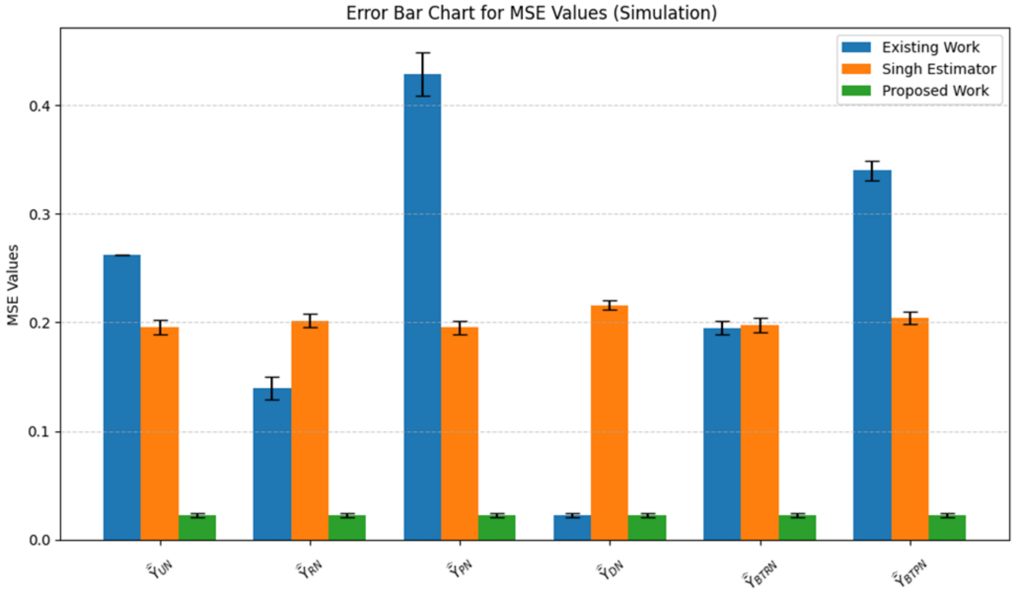

Figure 3: Showing mean squared error of all considered estimators based on simulation study

Table 6: PRE using simulated neutrosophic data

Estimator	**PRE Values of existing work**	$\widehat{\overline{\boldsymbol{Y}}}_{\boldsymbol{SN}}$ **(Singh estimator)**	$\widehat{\overline{\boldsymbol{Y}}}_{\boldsymbol{propN}}$ **(Proposed work)**
$\widehat{\overline{Y}}_{UN}$	[100,100]	[129.5042, 138.3407]	[1062.568, 1269.65]
$\widehat{\overline{Y}}_{RN}$	[202.9022, 174.9006]	[125.7631, 134.1869]	[1062.547, 1269.603]
$\widehat{\overline{Y}}_{PN}$	[58.3981, 64.05912]	[129.813, 138.5941]	[1062.569, 1269.654]
$\widehat{\overline{Y}}_{DN}$	[1062.045, 1268.383]	[118.8312, 123.9178]	[1062.503, 1269.527]
$\widehat{\overline{Y}}_{BTRN}$	[129.9925, 138.7405]	[128.3673, 137.0336]	[1062.561, 1269.635]
$\widehat{\overline{Y}}_{BTPN}$	[75.15756, 79.11846]	[124.8543, 131.9058]	[1062.537, 1269.592]
		[129.4841, 138.3207]	[1062.568, 1269.65]
		[129.3802, 138.1018]	[1062.567, 1269.649]
		[125.6116, 133.9843]	[1062.546,1269.601]
		[100.042, 100.0667]	[1062.451, 1269.426]

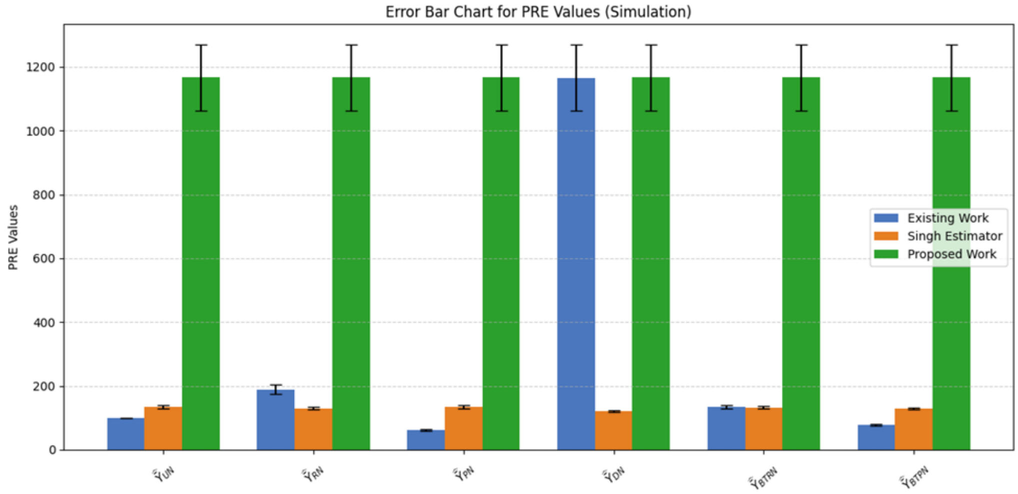

Figure 4: Showing percentage relative efficiency of all considered estimators based on simulation study

7. Conclusion

Point estimates in survey sampling have the limitation of providing only a single value for the parameter in controversy, which can vary between samples due to sampling error. Therefore, ambiguous, indeterminate, or uncertain facts are dealt with using the neutrosophic strategy, which is an extension of the classical approach. In this article, we have recommended neutrosophic general class of estimators for estimating population mean under simple random sampling. The suggested estimators are checked with both actual UBL data and simulated data to determine how effectively they work. The result based on the neutrosophic data for the mean squared error and percentage relative efficiency are given in Tables 3 and 4. On the way the mean square error and PRE based on simulated data are given in Tables 5 and 6. It is established that the proposed estimators achieve better than a number of alternately modified estimators. This research provides the way for future work in developing more precise estimators for use with a wide variety of neutrosophic data and sampling strategies. This study adds to its creativity by incorporating pre-existing estimators into the neutrosophic framework. This shows how adaptable and versatile it is. The results imply that neutrosophic statistics are a strong way to look at data that is not certain, which makes it easier to make decisions in a number of situations. We suggest using these sophisticated estimators in the future and pressure the need for more study to make them work better with different types of neutrosophic data and ways of sampling. Also, future studies will include more than one sample design, like systematic, sequential, and double sampling.

Data availability

All the data are available within the manuscript.

Conflict of interest

The authors declare no conflict of interest.

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