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A Fractional Smoking Transmission Model Utilizing the Caputo-Fabrizio Operator: A Comprehensive Analysis and Simulation

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Abstract. Over the past few years, several novel formulations of fractional derivatives have emerged, facilitating the construction of mathematical models that can address a broad spectrum of real-world applications. This study presents and investigates a mathematical model for smoking behavior incorporating the Caputo–Fabrizio fractional differential operator, which is particularly valued for its non-singular kernel and ability to capture memory effects in dynamic systems without the complexities of singularities. Here, the population is divided into six distinct compartments: mainly susceptible, snuffing, irregular, habitual, regular, and quit. To explore the model's dynamics, the Laplace Adomian Decomposition Method (LADM) and the Aboodh Adomian Decomposition Method (AADM) are employed. A comparative analysis of LADM and AADM is carried out using MATLAB software, with numerical simulations conducted for various fractional orders to assess the accuracy and efficiency of both techniques. Both methodologies demonstrated a high level of accuracy and consistency in their results, highlighting their reliability for modeling complex systems.

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1. Introduction

Mathematical modeling is an effective tool in this era to analyze dynamic processes, simulate real-world situations, identify important variables, test hypotheses, increase efficiency, evaluate risks, create cutting-edge technologies and an advance knowledge across

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disciplines. Generally, mathematical modeling is based on differential equations, but in recent years, the introduction of fractional-order derivatives has enhanced these models by capturing memory effects and nonlocal behaviors. This advancement has led to more accurate and realistic representations of complex systems where traditional models often fall short in depicting long-term dependencies and heterogeneous dynamics [1]. Mathematical models supported by numerical simulations are increasingly used by researchers to investigate the transmission mechanisms and control strategies of communicable diseases [2]. Almost every branch of science and engineering has been interested in fractional order models [3]. In biomedical engineering, infectious diseases are one of the main area that have caught the attention of researchers in recent years, especially after Covid 19 pandemic. Fractional order analysis has garnered increasing attention due to its effectiveness in the study of biological models [4]. The dynamics of infectious diseases are more accurately and precisely represented with the use of Fractional Calculus [5] [6]. Fractional differential equations (FDEs) are differential equations that have derivatives of fractional order. They give a more extensive and precise mathematical tool for simulating intricate physical processes with memory effects, like biological systems, viscoelastic materials, and anomalous diffusion [3] [4].

The smoking model serves as a vital tool for analyzing one of the most widespread and troublesome social issues faced globally. These models (theoretical or mathematical) are used to explain tobacco usage dynamics, smoking behaviour, and its impacts on humans population. They have the ability to analyze and forecast pattern of smoking initiation, cessation, and the health effects of tobacco use in a variety of disciplines, including epidemiology, public health, economics, and behavioural science [7]. A smoker has a 70 percent higher risk of having a heart attack than a nonsmoker. Similar to this, smokers have an incidence rate of lung cancer that is 10 percent higher than nonsmokers.[8] According to the World Health Organization, the effects of smoking on various body organs cause around 5 million deaths worldwide, and by 2030, that number might rise to up to 8 million annually [9]. To safeguard human health, numerous researchers, mathematicians, and medical professionals are working to curb smoking. As a result, many studies have focused on analyzing smoking behaviour through a variety of mathematical modeling techniques.

The pioneering work in the theory of epidemics is presented by Kermack and Mckendrick (1927). They created the compartmental mathematical model, that explains the evolution of infectious diseases [10]. The n-order model possesses the capacity to capture the transmission dynamics of contagious illnesses with greater depth and flexibility. By incorporating memory effects and nonlocal behavior—especially through fractional derivatives like Caputo-Fabrizio—it enables more realistic and accurate predictions of disease spread over time. In Mathematical modeling, researchers typically divide the population into three distinct compartments: Susceptible (S), Infected (I), and Removed (R), which collectively capture the dynamics of disease transmission.

The basic compartment smoking mathematical model was developed in 1997 by O Sharomi et. al. [11] which gives the qualitative analysis of the dynamics of smoking and its impact on public health. The model was then extended to the variability of smoking habits and related illnesses. In addition, in 2007, Ham conducted a survey in Korea that documented the different phases and smoking habits of the students cited [12]. In the same era integer-order model with a dynamic interaction is described in the literature that explains the behaviour of the smokers who smoke infrequently. Several additional researchers have contributed to the literature by developing smoking models formulated in both fractional and integer orders, enriching the understanding of behavioral dynamics and intervention strategies.[13] [14]. Numerical techniques are used by researchers to develope a smoking model [13]. Many researchers studied smoking model using various Numerical techniques like LADM (Laplace-Adomian Decomposition method) [15], HPM (Homotopy Perturbation technique) [9], Reduced Diferential Transform technique [16], Atangana–Toufik Method [17], and Fractional Differential Transformation technique [18]. With these successful attempts, there is a scope of research in this area. Some parameters like “Habitual” has not been given much attention by the researchers.

In this paper, an expanded smoking model is presented with added compartments, including susceptible, snuff, irregular, regular, and quit to represent more enduring behaviour and outside factors. The special inclusion of a ’Habitual’ section enables us to differentiate between habitual (deep-rooted) smokers and occasional or light smokers based on the frequency and intensity of their smoking behaviour. This refinement captures the effects of long-term dependence and behavioural reinforcement by incorporating fractional-order derivatives, enabling a more precise simulation of memory effects and gradual transitions. The creation of more successful stage-specific intervention tactics is supported by these extensions, which offer a deeper understanding of the course of tobacco use. The smoking population is generally categorized as a single group in many theoretical frameworks. But in reality, over time, some smokers gradually become more devoted to the habit and more consistent with it. A compartment ”Habitual” is included in this model to appropriately capture this distinction. This particular category of smokers developed a strong, long-lasting habit that is difficult to break. Habitual smokers have developed a physical and psychological need for nicotine. The inclusion of this compartment enhances the ability of the presented model to better understand long-term smoking behaviour and is helpful in designing more effective interventions to support individuals in their efforts to quit.

2. Preliminaries

2.1. Fundamental concepts

Before delving into the main result, first recall some essential definitions and theoretical preliminaries.

Definition 1. *Let* $r(u) \in \mathcal{H}^1(0, \alpha)$*,* $\alpha > 0$*, and let* $q \in (0, 1)$

then, Caputo-Fabrizio fractional derivative of function $r(u)$ is given by,[19] [20]

$$ {}^{\mathcal{CF}}D_u^q[r(u)] = \frac{M(q)}{1-q}\int_0^u r'(y)\,\exp\Big[-\frac{q\,(u-y)}{1-q}\Big]\,dy \tag{1} $$

where $M(q)$ is the normalized constants satisfying $M(0)=M(1)=1$.
The Sobolev space $\mathcal{H}^1(0,\alpha)$ consists of the functions on $(0,\alpha)$ that are square integrable and have square integrable first (weak) derivatives.

Definition 2. *Caputo-Fabrizio fractional integral with $q \in (0,1)$ for function $r(u)$ is defined by,[9]*

$$ {}^{\mathcal{CF}}I_u^q[r(u)] = \frac{1-q}{N(q)}\,r(u)] + \frac{q}{N(q)}\int_0^u r(y)dy, \quad u \geq 0. \tag{2} $$

Definition 3. *The Caputo-Fabrizio fractional derivative in general formula for Laplace Transform is as follows,[21]*

$$ \mathcal{L}\big\{{}^{\mathcal{CF}}D_u^q r(u)\big\}\,(s) = \frac{s\,\mathcal{L}[r(u)] - r(0)}{s+q(1-s)}, \quad s \geq 0. \tag{3} $$

Definition 4. *The Caputo-Fabrizio fractional derivative in general formula for Aboodh Transform is given by,[22][23]*

$$ \mathcal{A}\big\{{}^{\mathcal{CF}}D_u^q r(u)\big\} = \frac{s\,\mathcal{A}[l(s)] - \frac{l(0)}{s}}{s+q(1-s)} \tag{4} $$

Definition 5. *The Aboodh Transform of the function is defined as,[24][22][23]*

$$ \mathcal{A}[r(u)] = \mathcal{A}[l(s)] = \frac{1}{s}\int_0^\infty r(u)e^{-su}\,du, \quad u \geq 0, \quad q_1 \leq s \leq q_2, \quad q_1, q_2 > 0. \tag{5} $$

3. Model Formulation

In this section, A a comprehensive framework is provided, which is required for the execution of the proposed methodology in the smoking model and in its fractional representation. The application of mathematical modeling can serve as an effective strategy to reduce the proliferation of tobacco consumption, as it has been recognized as a critical instrument for understanding pandemics in recent years. The flow chart given in Figure 1 is designed to depict the structure and flow of the model, which gives a clear interpretation of the functional behavior. The node points are the compartments used in the model, and the parameters are represented by lines.

A generic model is the susceptible-exposed-infected-recovered(SEIR) so, in order to achieve a more profound comprehension of the qualitative assessment and the numerical

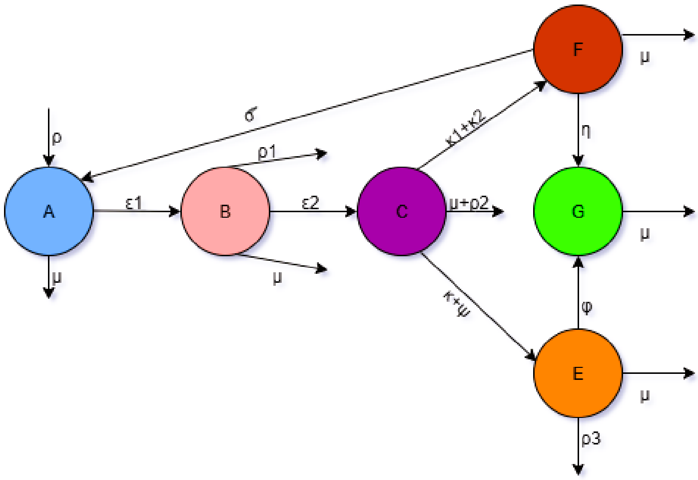

Figure 1: Flow diagram of the smoking model

iterative methodologies employed for its resolution, the revised iteration of smoking model will be examined as follows,

$$
\begin{aligned}
\frac{dA(u)}{du} &= \rho - \epsilon_1 A(u)B(u) + \sigma F(u) - \mu A(u),\\
\frac{dB(u)}{du} &= \epsilon_1 A(u)B(u) - \epsilon_2 B(u)C(u) - \rho_1 B(u) - \mu B(u),\\
\frac{dC(u)}{du} &= \epsilon_2 B(u)C(u) - (\kappa + \rho_2 + \mu + \kappa_1 + \kappa_2 + \psi)\, C(u),\\
\frac{dE(u)}{du} &= (\kappa + \psi)\, C(u) - (\mu + \phi + \rho_3)\, E(u),\\
\frac{dF(u)}{du} &= (\kappa_1 + \kappa_2)\, C(u) - (\eta + \mu + \sigma)\, F(u),\\
\frac{dG(u)}{du} &= \eta F(u) + \phi E(u) - \mu G(u),
\end{aligned}
\tag{6}
$$

The population is segregated into six classes as a description of model compartments, which is given in the TABLE I. TABLE II, shows the description of model parameters with different age groups like 18-24, 25-44, and 45 and above.

Table 1: Description of model compartments

variable	Description
$A(u)$	Susceptible smokers
$B(u)$	Snuffing class
$C(u)$	Irregular smokers
$E(u)$	Habitual smokers
$F(u)$	Regular smokers
$G(u)$	Quit smokers

Table 2: Description of model parameters

Parameter	Description
ρ	Recruitment frequency (birth or migration)
ρ_1	Snuffing class death rate because of smoking
ρ_2	Death rate due to smoking
ρ_3	Death rate due to Habitual smoking
ϵ_1	vulnerable population transition rate for age group 18-24 to Snuffing class
ϵ_2	Snuffing rate becomes Irregular smokers for age group 18-24
κ	Irregular smokers rate become Habitual smokers for age group 18-24
κ_1	Irregular smokers rate turning to Regular smokers for age group 18-24
κ_2	Irregular smokers rate become Regular smokers for age group 25-44
η	Departing rate of active smokers in 45 and above age group
μ	natural death rate
σ	Recovery rate in 25-44 age group
ϕ	Habitual smokers rate to quit smokers with support system for 45 and above age group
ψ	Irregular smokers rate becomes Habitual smokers for 25-44 age group

4. Model Solution

In the context of Caputo-Fabrizio fractional derivative, the updated smoking model is examined and explained as follows,

$$
\begin{aligned}
{}^{\mathcal{CF}}D_u^q(A)(u) &= \rho - \epsilon_1 A(u)B(u) + \sigma F(u) - \mu A(u),\\
{}^{\mathcal{CF}}D_u^q(B)(u) &= \epsilon_1 A(u)B(u) - \epsilon_2 B(u)C(u) - \rho_1 B(u) - \mu B(u),\\
{}^{\mathcal{CF}}D_u^q(C)(u) &= \epsilon_2 B(u)C(u) - (\kappa + \rho_2 + \mu + \kappa_1 + \kappa_2 + \psi)\, C(u),\\
{}^{\mathcal{CF}}D_u^q(E)(u) &= (\kappa + \psi)\, C(u) - (\mu + \phi + \rho_3)\, E(u),\\
{}^{\mathcal{CF}}D_u^q(F)(u) &= (\kappa_1 + \kappa_2)\, C(u) - (\eta + \mu + \sigma)\, F(u),\\
{}^{\mathcal{CF}}D_u^q(G)(u) &= \eta F(u) + \phi E(u) - \mu G(u),
\end{aligned}
\tag{7}
$$

with initial Conditions

$$
A(0) = n_1, \quad B(0) = n_2, \quad C(0) = n_3, \quad E(0) = n_4, \quad F(0) = n_5, \quad G(0) = n_6. \tag{8}
$$

In the following subsection, we found the solution for the smoking model represented by equation (7) with the initial conditions. The techniques used to solve these equation by Laplace Adomain Decomposition Method and Aboodh Adomain Decomposition Method.

4.1. Laplace Adomain Decomposition method

Now, applying Laplace transform on equation (7) with the initial condition and the normalized constant
$M(\rho) = 1$
so, The obtained equation will be

$$\begin{aligned}
\mathcal{L}\Big[{}^{\mathcal{CF}}D_u^q(A)(u)\Big] &= \mathcal{L}\Big[\rho - \epsilon_1 A(u)B(u) + \sigma F(u) - \mu A(u)\Big],\\
\mathcal{L}\Big[{}^{\mathcal{CF}}D_u^q(B)(u)\Big] &= \mathcal{L}\Big[\epsilon_1 A(u)B(u) - \epsilon_2 B(u)C(u) - \rho_1 B(u) - \mu B(u)\Big],\\
\mathcal{L}\Big[{}^{\mathcal{CF}}D_u^q(C)(u)\Big] &= \mathcal{L}\Big[\epsilon_2 B(u)C(u) - (\kappa + \rho_2 + \mu + \kappa_1 + \kappa_2 + \psi)\, C(u)\Big],\\
\mathcal{L}\Big[{}^{\mathcal{CF}}D_u^q(E)(u)\Big] &= \mathcal{L}\Big[(\kappa + \psi)\, C(u) - (\mu + \phi + \rho_3)\, E(u)\Big],\\
\mathcal{L}\Big[{}^{\mathcal{CF}}D_u^q(F)(u)\Big] &= \mathcal{L}\Big[(\kappa_1 + \kappa_2)\, C(u) - (\eta + \mu + \sigma)\, F(u)\Big],\\
\mathcal{L}\Big[{}^{\mathcal{CF}}D_u^q(G)(u)\Big] &= \mathcal{L}\Big[\eta F(t) + \phi E(u) - \mu G(u)\Big],
\end{aligned} \tag{9}$$

with the initial conditions,

$$\begin{aligned}
\mathcal{L}\Big(A(u)\Big) &= \frac{A(0)}{s} + \frac{s + q(1-s)}{s}\,\mathcal{L}\Big(\rho - \epsilon_1 A(u)B(u) + \sigma F(u) - \mu A(u)\Big),\\
\mathcal{L}\Big(B(u)\Big) &= \frac{B(0)}{s} + \frac{s + q(1-s)}{s}\,\mathcal{L}\Big(\epsilon_1 A(u)B(u) - \epsilon_2 B(u)C(u) - \rho_1 B(u) - \mu B(u)\Big),\\
\mathcal{L}\Big(C(u)\Big) &= \frac{C(0)}{s} + \frac{s + q(1-s)}{s}\,\mathcal{L}\Big(\epsilon_2 B(u)C(u) - (\kappa + \rho_2 + \mu + \kappa_1 + \kappa_2 + \psi)\, C(u)\Big),\\
\mathcal{L}\Big(E(u)\Big) &= \frac{E(0)}{s} + \frac{s + q(1-s)}{s}\,\mathcal{L}\Big((\kappa + \psi)\, C(u) - (\mu + \phi + \rho_3)\, E(u)\Big),\\
\mathcal{L}\Big(F(u)\Big) &= \frac{F(0)}{s} + \frac{s + q(1-s)}{s}\,\mathcal{L}\Big((\kappa_1 + \kappa_2)\, C(u) - (\eta + \mu + \sigma)\, F(u)\Big),\\
\mathcal{L}\Big(G(u)\Big) &= \frac{G(0)}{s} + \frac{s + q(1-s)}{s}\,\mathcal{L}\Big(\eta F(u) + \phi E(u) - \mu G(u)\Big),
\end{aligned} \tag{10}$$

Consider the solution for the compartments A(u), B(u), C(u), E(u), F(u), G(u) in the form of infinite series,

$$A(u) = \sum_{z=0}^{\infty} A_z(u), \quad B(u) = \sum_{z=0}^{\infty} B_z(u), \quad C(u) = \sum_{z=0}^{\infty} C_z(u), \quad E(u) = \sum_{z=0}^{\infty} E_z(u), \quad F(u) = \sum_{z=0}^{\infty} F_z(u),$$

$$G(u) = \sum_{z=0}^{\infty} G_z(u), \quad A(u)B(u) = \sum_{z=0}^{\infty} T_z(u), \quad B(u)C(u) = \sum_{z=0}^{\infty} X_z(u). \tag{11}$$

Nonlinear terms are decomposed using Adomian polynomials and is defined as, [25]

$$T_z(u) = \frac{1}{n!}\frac{d^n}{d\rho^n}\Big[\sum_{z=0}^{n} \rho^z A_z(u) \sum_{z=0}^{n} \rho^z B_z(u)\Big]_{\rho=0}, \quad X_z(u) = \frac{1}{n!}\frac{d^n}{d\rho^n}\Big[\sum_{z=0}^{n} \rho^z B_z(u) \sum_{z=0}^{n} \rho^z C_z(u)\Big]_{\rho=0}.$$

Now applying the equation (11) in equation (10), The obtained equations are,

$$\begin{aligned}
\mathcal{L}\Big[\sum_{z=0}^{\infty} A_z(u)\Big] &= \frac{A(0)}{s} + \frac{s+q(1-s)}{s}\mathcal{L}\Big[\rho - \epsilon_1 \sum_{z=0}^{\infty} T_z(u) + \sigma \sum_{z=0}^{\infty} F_z(u) - \mu \sum_{z=0}^{\infty} A_z(u)\Big],\\
\mathcal{L}\Big[\sum_{z=0}^{\infty} B_z(u)\Big] &= \frac{B(0)}{s} + \frac{s+q(1-s)}{s}\mathcal{L}\Big[\epsilon_1 \sum_{z=0}^{\infty} T_z(u) - \epsilon_2 \sum_{z=0}^{\infty} X_z(u) - \rho_1 \sum_{z=0}^{\infty} B_z(u) - \mu \sum_{z=0}^{\infty} B_z(u)\Big],\\
\mathcal{L}\Big[\sum_{z=0}^{\infty} C_z(u)\Big] &= \frac{C(0)}{s} + \frac{s+q(1-s)}{s}\mathcal{L}\Big[\epsilon_2 \sum_{z=0}^{\infty} X_z(u) - (\kappa + \rho_2 + \mu + \kappa_1 + \kappa_2 + \psi) \sum_{z=0}^{\infty} C_z(u)\Big],\\
\mathcal{L}\Big[\sum_{z=0}^{\infty} E_z(u)\Big] &= \frac{E(0)}{s} + \frac{s+q(1-s)}{s}\mathcal{L}\Big[(\kappa + \psi) \sum_{z=0}^{\infty} C_z(u) - (\mu + \phi + \rho_3) \sum_{z=0}^{\infty} E_z(u)\Big],\\
\mathcal{L}\Big[\sum_{z=0}^{\infty} F_z(u)\Big] &= \frac{F(0)}{s} + \frac{s+q(1-s)}{s}\mathcal{L}\Big[(\kappa_1 + \kappa_2) \sum_{z=0}^{\infty} C_z(u) - (\eta + \mu + \sigma) \sum_{z=0}^{\infty} F_z(u)\Big],\\
\mathcal{L}\Big[\sum_{z=0}^{\infty} G_z(u)\Big] &= \frac{G(0)}{s} + \frac{s+q(1-s)}{s}\mathcal{L}\Big[\eta \sum_{z=0}^{\infty} F_z(u) + \phi \sum_{z=0}^{\infty} E_z(u) - \mu \sum_{z=0}^{\infty} G_z(u)\Big].
\end{aligned} \tag{12}$$

Equating both sides of the above equations, the follwing is obtained.

$$\mathcal{L}[A_0(u)] = \frac{n_1}{s}, \quad \mathcal{L}[B_0(u)] = \frac{n_2}{s}, \quad \mathcal{L}[C_0(u)] = \frac{n_3}{s}, \quad \mathcal{L}[E_0(u)] = \frac{n_4}{s}, \quad \mathcal{L}[F_0(u)] = \frac{n_5}{s}, \quad \mathcal{L}[G_0(u)] = \frac{n_6}{s},$$

$$\begin{aligned}
\mathcal{L}\Big(A_1(u)\Big) &= \frac{s+q(1-s)}{s}\mathcal{L}\Big(\rho - \epsilon_1 T_0(u) + \sigma F_0(u) - \mu A_0(u)\Big),\\
\mathcal{L}\Big(B_1(u)\Big) &= \frac{s+q(1-s)}{s}\mathcal{L}\Big(\epsilon_1 T_0(u) - \epsilon_2 X_0(u) - \rho_1 B_0(u) - \mu B_0(u)\Big),\\
\mathcal{L}\Big(C_1(u)\Big) &= \frac{s+q(1-s)}{s}\mathcal{L}\Big(\epsilon_2 X_0(u) - (\kappa + \rho_2 + \mu + \kappa_1 + \kappa_2 + \psi)\, C_0(u)\Big),\\
\mathcal{L}\Big(E_1(u)\Big) &= \frac{s+q(1-s)}{s}\mathcal{L}\Big((\kappa + \psi)\, C_0(u) - (\mu + \phi + \rho_3)\, E_0(u)\Big),\\
\mathcal{L}\Big(F_1(u)\Big) &= \frac{s+q(1-s)}{s}\mathcal{L}\Big((\kappa_1 + \kappa_2)\, C_0(u) - (\eta + \mu + \sigma)\, F_0(u)\Big),\\
\mathcal{L}\Big(G_1(u)\Big) &= \frac{s+q(1-s)}{s}\mathcal{L}\Big(\eta F_0(u) + \phi E_0(u) - \mu G_0(u)\Big),
\end{aligned}$$

$$\mathcal{L}\Big(A_2(u)\Big) = \frac{s+q(1-s)}{s}\mathcal{L}\Big(\rho - \epsilon_1 T_1(u) + \sigma F_1(u) - \mu A_1(u)\Big),$$
$$\mathcal{L}\Big(B_2(u)\Big) = \frac{s+q(1-s)}{s}\mathcal{L}\Big(\epsilon_1 T_1(u) - \epsilon_2 X_1(u) - \rho_1 B_1(u) - \mu B_1(u)\Big),$$
$$\mathcal{L}\Big(C_2(u)\Big) = \frac{s+q(1-s)}{s}\mathcal{L}\Big(\epsilon_2 X_1(u) - (\kappa + \rho_2 + \mu + \kappa_1 + \kappa_2 + \psi)\, C_1(u)\Big),$$
$$\mathcal{L}\Big(E_2(u)\Big) = \frac{s+q(1-s)}{s}\mathcal{L}\Big((\kappa + \psi)\, C_1(u) - (\mu + \phi + \rho_3)\, E_1(u)\Big),$$
$$\mathcal{L}\Big(F_2(u)\Big) = \frac{s+q(1-s)}{s}\mathcal{L}\Big[(\kappa_1 + \kappa_2)\, C_1(u) - (\eta + \mu + \sigma)\, F_1(u)\Big),$$
$$\mathcal{L}\Big(G_2(u)\Big) = \frac{s+q(1-s)}{s}\mathcal{L}\Big(\eta F_1(u) + \phi E_1(u) - \mu G_1(u)\Big), \tag{13}$$

and so on ....

$$\mathcal{L}\Big(A_{z+1}(u)\Big) = \frac{s+q(1-s)}{s}\mathcal{L}\Big(\rho - \epsilon_1 T_z(u) + \sigma F_z(u) - \mu A_z(u)\Big),$$
$$\mathcal{L}\Big(B_{z+1}(u)\Big) = \frac{s+q(1-s)}{s}\mathcal{L}\Big(\epsilon_1 T_z(u) - \epsilon_2 X_z(u) - \rho_1 B_z(u) - \mu B_z(u)\Big),$$
$$\mathcal{L}\Big(C_{z+1}(u)\Big) = \frac{s+q(1-s)}{s}\mathcal{L}\Big(\epsilon_2 X_z(u) - (\kappa + \rho_2 + \mu + \kappa_1 + \kappa_2 + \psi)\, C_z(u)\Big),$$
$$\mathcal{L}\Big(E_{z+1}(u)\Big) = \frac{s+q(1-s)}{s}\mathcal{L}\Big((\kappa + \psi)\, C_z(u) - (\mu + \phi + \rho_3)\, E_z(u)\Big),$$
$$\mathcal{L}\Big(F_{z+1}(u)\Big) = \frac{s+q(1-s)}{s}\mathcal{L}\Big((\kappa_1 + \kappa_2)\, C_z(u) - (\eta + \mu + \sigma)\, F_z(u)\Big),$$
$$\mathcal{L}\Big(G_{z+1}(u)\Big) = \frac{s+q(1-s)}{s}\mathcal{L}\Big(\eta F_z(u) + \phi E_z(u) - \mu G_z(u)\Big). \tag{14}$$

Now taking inverse Laplace transform on both sides,the obtained equations are,
$A_0 = n_1,\ B_0 = n_2,\ C_0 = n_3,\ E_0 = n_4,\ F_0 = n_5,\ G_0 = n_6,$

$$A_1(u) = [\rho - \epsilon_1 n_1 n_2 + \sigma n_5 - \mu n_1] \quad [1 + q(u-1)],$$
$$B_1(u) = [\epsilon_1 n_1 n_2 - \epsilon_2 n_2 n_3 - \rho_1 n_2 - \mu n_2] \quad [1 + q(u-1)],$$
$$C_1(u) = [\epsilon_2 n_2 n_3 - (\kappa + \rho_2 + \mu + \kappa_1 + \kappa_2 + \psi) n_3] \quad [1 + q(u-1)],$$
$$E_1(u) = [(\kappa + \psi) n_3 - (\mu + \phi + \rho_3) n_4] \quad [1 + q(u-1)],$$
$$F_1(u) = [(\kappa_1 + \kappa_2)\, n_3 - (\eta + \mu + \sigma) n_5] \quad [1 + q(u-1)],$$
$$G_1(u) = [\eta n_5 + \phi n_4 - \mu n_6] \quad [1 + q(u-1)],$$
$$A_2(u) = [\rho][1 + q(u-1)] - [\epsilon_1(a_{11} n_2 + n_1 b_{11}) + \sigma f_{11} - \mu a_{11}]\Big[\frac{q^2 u^2}{2!} - 2q^2 u + 2qu + (q-1)^2\Big],$$
$$B_2(u) = [\epsilon_1(a_{11} n_2 + n_1 b_{22}) - \epsilon_2(c_{11} n_2 + n_3 b_{11}) + \rho_1 b_{11} + \mu b_{11}]\Big[\frac{q^2 u^2}{2!} - 2q^2 u + 2qu + (q-1)^2\Big],$$

$$\begin{aligned}
C_2(u) &= [\epsilon_2(c_{11}n_2 + n_3b_{11}) - (\kappa + \rho_2 + \mu + \kappa_1 + \kappa_2 + \psi)c_{11}]\Big[\frac{q^2u^2}{2!} - 2q^2u + 2qu + (q-1)^2\Big],\\
E_2(u) &= [(\kappa + \psi)c_{11} - (\mu + \phi + \rho_3)e_{11}]\Big[\frac{q^2u^2}{2!} - 2q^2u + 2qu + (q-1)^2\Big],\\
F_2(u) &= [(\kappa_1 + \kappa_2)c_{11} - (\eta + \mu + \sigma)f_{11}]\Big[\frac{q^2u^2}{2!} - 2q^2u + 2qu + (q-1)^2\Big],\\
G_2(u) &= [\eta f_{11} + \phi e_{11} - \mu g_{11}]\Big[\frac{q^2u^2}{2!} - 2q^2u + 2qu + (q-1)^2\Big].
\end{aligned} \tag{15}$$

Moreover, the remaining variables may be derived in an analogous manner correspondingly. Thus the indeterminate quantities in the preceding equations are delineated as follows:

$$\begin{aligned}
a_{11}(u) &= [\rho - \epsilon_1 n_1 n_2 + \sigma n_5 - \mu n_1] \quad [1 + q(u-1)],\\
b_{11}(u) &= [\epsilon_1 n_1 n_2 - \epsilon_2 n_2 n_3 - \rho_1 n_2 - \mu n_2] \quad [1 + q(u-1)],\\
c_{11}(u) &= [\epsilon_2 n_2 n_3 - (\kappa + \rho_2 + \mu + \kappa_1 + \kappa_2 + \psi)n_3] \quad [1 + q(u-1)],\\
e_{11}(u) &= [(\kappa + \psi)n_3 - (\mu + \phi + \rho_3)n_4] \quad [1 + q(u-1)],\\
f_{11}(u) &= [(\kappa_1 + \kappa_2)\, n_3 - (\eta + \mu + \sigma)n_5] \quad [1 + q(u-1)],\\
g_{11}(u) &= [\eta n_5 + \phi n_4 - \mu n_6] \quad [1 + q(u-1)].
\end{aligned} \tag{16}$$

4.2. Aboodh Adomain Decomposition Method

Now, applying the Aboodh Transform in Equation (7) with the initial conditions and the normalized constant,
$M(\rho) = 1$
The obtained equation will be ,

$$\begin{aligned}
\mathcal{A}\Big[{}^{\mathcal{CF}}D_u^q(A)(u)\Big] &= \mathcal{A}\Big[\rho - \epsilon_1 A(u)B(u) + \sigma F(u) - \mu A(u)\Big],\\
\mathcal{A}\Big[{}^{\mathcal{CF}}D_u^q(B)(u)\Big] &= \mathcal{A}\Big[\epsilon_1 A(u)B(u) - \epsilon_2 B(u)C(u) - \rho_1 B(u) - \mu B(u)\Big],\\
\mathcal{A}\Big[{}^{\mathcal{CF}}D_u^q(C)(u)\Big] &= \mathcal{A}\Big[\epsilon_2 B(u)C(u) - (\kappa + \rho_2 + \mu + \kappa_1 + \kappa_2 + \psi)\, C(u)\Big],\\
\mathcal{A}\Big[{}^{\mathcal{CF}}D_u^q(F)(u)\Big] &= \mathcal{A}\Big[(\kappa + \psi)\, C(u) - (\mu + \phi + \rho_3)\, E(u)\Big],\\
\mathcal{A}\Big[{}^{\mathcal{CF}}D_u^q(E)(u)\Big] &= \mathcal{A}\Big[(\kappa_1 + \kappa_2)\, C(u) - (\eta + \mu + \sigma)\, F(u)\Big],\\
\mathcal{A}\Big[{}^{\mathcal{CF}}D_u^q(G)(u)\Big] &= \mathcal{A}\Big[\eta F(u) + \phi E(u) - \mu G(u)\Big],
\end{aligned} \tag{17}$$

with the initial conditions,

$$\begin{aligned}
\mathcal{A}\Big(A(u)\Big) &= \frac{A(0)}{s^2} + \frac{s+q(1-s)}{s}\,\mathcal{A}\Big(\rho - \epsilon_1 A(u)B(u) + \sigma F(u) - \mu A(u)\Big),\\
\mathcal{A}\Big(B(u)\Big) &= \frac{B(0)}{s^2} + \frac{s+q(1-s)}{s}\,\mathcal{A}\Big(\epsilon_1 A(u)B(u) - \epsilon_2 B(u)C(u) - \rho_1 B(u) - \mu B(u)\Big),\\
\mathcal{A}\Big(C(u)\Big) &= \frac{C(0)}{s^2} + \frac{s+q(1-s)}{s}\,\mathcal{A}\Big(\epsilon_2 B(u)C(u) - (\kappa + \rho_2 + \mu + \kappa_1 + \kappa_2 + \psi)\,C(u)\Big),\\
\mathcal{A}\Big(E(u)\Big) &= \frac{E(0)}{s^2} + \frac{s+q(1-s)}{s}\,\mathcal{A}\Big((\kappa + \psi)\,C(u) - (\mu + \phi + \rho_3)\,E(u)\Big),\\
\mathcal{A}\Big(F(u)\Big) &= \frac{F(0)}{s^2} + \frac{s+q(1-s)}{s}\,\mathcal{A}\Big((\kappa_1 + \kappa_2)\,C(u) - (\eta + \mu + \sigma)\,F(u)\Big),\\
\mathcal{A}\Big(G(u)\Big) &= \frac{G(0)}{s^2} + \frac{s+q(1-s)}{s}\,\mathcal{A}\Big(\eta F(u) + \phi E(u) - \mu G(u)\Big), \qquad (18)
\end{aligned}$$

$$A(u) = \sum_{z=0}^{\infty} A_z(u), \quad B(u) = \sum_{z=0}^{\infty} B_z(u), \quad C(u) = \sum_{z=0}^{\infty} C_z(u), \quad E(u) = \sum_{z=0}^{\infty} E_z(u), \quad F(u) = \sum_{z=0}^{\infty} F_z(u),$$

$$G(u) = \sum_{z=0}^{\infty} G_z(u), \quad A(u)B(u) = \sum_{z=0}^{\infty} T_z(u), \quad B(u)C(u) = \sum_{z=0}^{\infty} X_z(u). \qquad (19)$$

Nonlinear terms are decomposed using Adomian polynomials and is defined as,

$$T_z(u) = \frac{1}{n!}\frac{d^n}{d\rho^n}\Big[\sum_{z=0}^{n} \rho^z A_z(u) \sum_{z=0}^{n} \rho^z B_z(u)\Big]_{\rho=0}, \quad X_z(u) = \frac{1}{n!}\frac{d^n}{d\rho^n}\Big[\sum_{z=0}^{n} \rho^z B_z(u) \sum_{z=0}^{n} \rho^z C_z(u)\Big]_{\rho=0}.$$

Now applying the equation(19) in equation (18), the follwing is obtained.

$$\begin{aligned}
\mathcal{A}\Big[\sum_{z=0}^{\infty} A_z(u)\Big] &= \frac{A(0)}{s^2} + \frac{s+q(1-s)}{s}\mathcal{A}\Big[\rho - \epsilon_1 \sum_{z=0}^{\infty} T_z(u) + \sigma \sum_{z=0}^{\infty} F_z(u) - \mu \sum_{z=0}^{\infty} A_z(u)\Big],\\
\mathcal{A}\Big[\sum_{z=0}^{\infty} B_z(u)\Big] &= \frac{B(0)}{s^2} + \frac{s+q(1-s)}{s}\mathcal{A}\Big[\epsilon_1 \sum_{z=0}^{\infty} T_z(u) - \epsilon_2 \sum_{z=0}^{\infty} X_z(u) - \rho_1 \sum_{z=0}^{\infty} B_z(u) - \mu \sum_{z=0}^{\infty} B_z(u)\Big],\\
\mathcal{A}\Big[\sum_{z=0}^{\infty} C_z(u)\Big] &= \frac{C(0)}{s^2} + \frac{s+q(1-s)}{s}\mathcal{A}\Big[\epsilon_2 \sum_{z=0}^{\infty} X_z(u) - (\kappa + \rho_2 + \mu + \kappa_1 + \kappa_2 + \psi) \sum_{z=0}^{\infty} C_z(u)\Big],\\
\mathcal{A}\Big[\sum_{z=0}^{\infty} E_z(u)\Big] &= \frac{E(0)}{s^2} + \frac{s+q(1-s)}{s}\mathcal{L}\Big[(\kappa + \psi) \sum_{z=0}^{\infty} C_z(u) - (\mu + \phi + \rho_3) \sum_{z=0}^{\infty} E_z(u)\Big],\\
\mathcal{A}\Big[\sum_{z=0}^{\infty} F_z(u)\Big] &= \frac{F(0)}{s^2} + \frac{s+q(1-s)}{s}\mathcal{L}\Big[(\kappa_1 + \kappa_2) \sum_{z=0}^{\infty} C_z(u) - (\eta + \mu + \sigma) \sum_{z=0}^{\infty} F_z(u)\Big],\\
\mathcal{A}\Big[\sum_{z=0}^{\infty} G_z(u)\Big] &= \frac{G(0)}{s^2} + \frac{s+q(1-s)}{s}\mathcal{A}\Big[\eta \sum_{z=0}^{\infty} F_z(u) + \phi \sum_{z=0}^{\infty} E_z(u) - \mu \sum_{z=0}^{\infty} G_z(u)\Big]. \qquad (20)
\end{aligned}$$

Equating both sides of the above equations, obtained equation will be,

$$\mathcal{A}[A_0(u)] = \frac{n_1}{s^2}, \quad \mathcal{A}[B_0(u)] = \frac{n_2}{s^2}, \quad \mathcal{A}[C_0(u)] = \frac{n_3}{s^2}, \quad \mathcal{A}[E_0(u)] = \frac{n_4}{s^2}, \quad \mathcal{A}[F_0(u)] = \frac{n_5}{s^2}, \quad \mathcal{A}[G_0(u)] = \frac{n_6}{s^2},$$

$$\begin{aligned}
\mathcal{A}\Big[A_1(u)\Big] &= \frac{s+q(1-s)}{s}\mathcal{A}\Big[\rho - \epsilon_1 T_0(u) + \sigma F_0(u) - \mu A_0(u)\Big],\\
\mathcal{A}\Big[B_1(u)\Big] &= \frac{s+q(1-s)}{s}\mathcal{A}\Big[\epsilon_1 T_0(u) - \epsilon_2 X_0(u) - \rho_1 B_0(u) - \mu B_0(u)\Big],\\
\mathcal{A}\Big[C_1(u)\Big] &= \frac{s+q(1-s)}{s}\mathcal{A}\Big[\epsilon_2 X_0(u) - (\kappa + \rho_2 + \mu + \kappa_1 + \kappa_2 + \psi)\, C_0(u)\Big],\\
\mathcal{A}\Big[E_1(u)\Big] &= \frac{s+q(1-s)}{s}\mathcal{A}\Big[(\kappa + \psi)\, C_0(u) - (\mu + \phi + \rho_3)\, E_0(u)\Big],\\
\mathcal{A}\Big[F_1(u)\Big] &= \frac{s+q(1-s)}{s}\mathcal{A}\Big[(\kappa_1 + \kappa_2)\, C_0(u) - (\eta + \mu + \sigma)\, F_0(u)\Big],\\
\mathcal{A}\Big[G_1(u)\Big] &= \frac{s+q(1-s)}{s}\mathcal{A}\Big[\eta F_0(u) + \phi E_0(u) - \mu G_0(u)\Big],\\
\mathcal{A}\Big[A_2(u)\Big] &= \frac{s+q(1-s)}{s}\mathcal{A}\Big[\rho - \epsilon_1 T_1(u) + \sigma F_1(u) - \mu A_1(u)\Big],\\
\mathcal{A}\Big[B_2(u)\Big] &= \frac{s+q(1-s)}{s}\mathcal{A}\Big[\epsilon_1 T_1(u) - \epsilon_2 X_1(u) - \rho_1 B_1(u) - \mu B_1(u)\Big],\\
\mathcal{A}\Big[C_2(u)\Big] &= \frac{s+q(1-s)}{s}\mathcal{A}\Big[\epsilon_2 X_1(u) - (\kappa + \rho_2 + \mu + \kappa_1 + \kappa_2 + \psi)\, C_1(u)\Big],\\
\mathcal{A}\Big[E_2(u)\Big] &= \frac{s+q(1-s)}{s}\mathcal{A}\Big[(\kappa + \psi)\, C_1(u) - (\mu + \phi + \rho_3)\, E_1(u)\Big],\\
\mathcal{A}\Big[F_2(u)\Big] &= \frac{s+q(1-s)}{s}\mathcal{A}\Big[(\kappa_1 + \kappa_2)\, C_1(u) - (\eta + \mu + \sigma)\, F_1(u)\Big],\\
\mathcal{A}\Big[G_2(u)\Big] &= \frac{s+q(1-s)}{s}\mathcal{A}\Big[\eta F_1(u) + \phi E_1(u) - \mu G_1(u)\Big],
\end{aligned} \tag{21}$$

and so on ....

$$\begin{aligned}
\mathcal{A}\Big[A_{z+1}(u)\Big] &= \frac{s+q(1-s)}{s}\mathcal{A}\Big[\rho - \epsilon_1 T_z(u) + \sigma F_z(u) - \mu A_z(u)\Big],\\
\mathcal{A}\Big[B_{z+1}(u)\Big] &= \frac{s+q(1-s)}{s}\mathcal{A}\Big[\epsilon_1 T_z(u) - \epsilon_2 X_z(u) - \rho_1 B_z(u) - \mu B_z(u)\Big],\\
\mathcal{A}\Big[C_{z+1}(u)\Big] &= \frac{s+q(1-s)}{s}\mathcal{A}\Big[\epsilon_2 X_z(u) - (\kappa + \rho_2 + \mu + \kappa_1 + \kappa_2 + \psi)\, C_z(u)\Big],\\
\mathcal{A}\Big[E_{z+1}(u)\Big] &= \frac{s+q(1-s)}{s}\mathcal{A}\Big[(\kappa + \psi)\, C_z(u) - (\mu + \phi + \rho_3)\, E_z(u)\Big],\\
\mathcal{A}\Big[F_{z+1}(u)\Big] &= \frac{s+q(1-s)}{s}\mathcal{A}\Big[(\kappa_1 + \kappa_2)\, C_z(u) - (\eta + \mu + \sigma)\, F_z(u)\Big],\\
\mathcal{A}\Big[G_{z+1}(u)\Big] &= \frac{s+q(1-s)}{s}\mathcal{A}\Big[\eta F_z(u) + \phi E_z(u) - \mu G_z(u)\Big].
\end{aligned} \tag{22}$$

Now applying the Inverse Aboodh Transform on both sides, The obtained equation will be,
$A_0 = n_1$, $B_0 = n_2$, $C_0 = n_3$, $E_0 = n_4$, $F_0 = n_5$, $G_0 = n_6$,

$$
\begin{aligned}
A_1(u) &= [\rho - \epsilon_1 n_1 n_2 + \sigma n_5 - \mu n_1] \quad [1 + q(u-1)],\\
B_1(u) &= [\epsilon_1 n_1 n_2 - \epsilon_2 n_2 n_3 - \rho_1 n_2 - \mu n_2] \quad [1 + q(u-1)],\\
C_1(u) &= [\epsilon_2 n_2 n_3 - (\kappa + \rho_2 + \mu + \kappa_1 + \kappa_2 + \psi) n_3] \quad [1 + q(u-1)],\\
E_1(u) &= [(\kappa + \psi) n_3 - (\mu + \phi + \rho_3) n_4] \quad [1 + q(u-1)],\\
F_1(u) &= [(\kappa_1 + \kappa_2)\, n_3 - (\eta + \mu + \sigma) n_5] \quad [1 + q(u-1)],\\
G_1(u) &= [\eta n_5 + \phi n_4 - \mu n_6] \quad [1 + q(u-1)],\\
A_2(u) &= [\rho][1 + q(u-1)] - [\epsilon_1(a_{11} n_2 + n_1 b_{11}) + \sigma f_{11} - \mu a_{11}]\Big[\frac{q^2u^2}{2!} - 2q^2u + 2qu + (q-1)^2\Big],\\
B_2(u) &= [\epsilon_1(a_{11} n_2 + n_1 b_{22}) - \epsilon_2(c_{11} n_2 + n_3 b_{11}) + \rho_1 b_{11} + \mu b_{11}]\Big[\frac{q^2u^2}{2!} - 2q^2u + 2qu + (q-1)^2\Big],\\
C_2(u) &= [\epsilon_2(c_{11} n_2 + n_3 b_{11}) - (\kappa + \rho_2 + \mu + \kappa_1 + \kappa_2 + \psi) c_{11}]\Big[\frac{q^2u^2}{2!} - 2q^2u + 2qu + (q-1)^2\Big],\\
E_2(u) &= [(\kappa + \psi) c_{11} - (\mu + \phi + \rho_3) e_{11}]\Big[\frac{q^2u^2}{2!} - 2q^2u + 2qu + (q-1)^2\Big],\\
F_2(u) &= [(\kappa_1 + \kappa_2) c_{11} - (\eta + \mu + \sigma) f_{11}]\Big[\frac{q^2u^2}{2!} - 2q^2u + 2qu + (q-1)^2\Big],\\
G_2(u) &= [\eta f_{11} + \phi e_{11} - \mu g_{11}]\Big[\frac{q^2u^2}{2!} - 2q^2u + 2qu + (q-1)^2\Big].
\end{aligned}
\tag{23}
$$

The remaining variables can be obtained in a similar way. The indeterminate quantities in the earlier equations are described as follows:

$$
\begin{aligned}
a_{11}(u) &= [\rho - \epsilon_1 n_1 n_2 + \sigma n_5 - \mu n_1] \quad [1 + q(u-1)],\\
b_{11}(u) &= [\epsilon_1 n_1 n_2 - \epsilon_2 n_2 n_3 - \rho_1 n_2 - \mu n_2] \quad [1 + q(u-1)],\\
c_{11}(u) &= [\epsilon_2 n_2 n_3 - (\kappa + \rho_2 + \mu + \kappa_1 + \kappa_2 + \psi) n_3] \quad [1 + q(u-1)],\\
e_{11}(u) &= [(\kappa + \psi) n_3 - (\mu + \phi + \rho_3) n_4] \quad [1 + q(u-1)],\\
f_{11}(u) &= [(\kappa_1 + \kappa_2)\, n_3 - (\eta + \mu + \sigma) n_5] \quad [1 + q(u-1)],\\
g_{11}(u) &= [\eta n_5 + \phi n_4 - \mu n_6] \quad [1 + q(u-1)].
\end{aligned}
\tag{24}
$$

It is observed that both LADM and AADM yield identical terms, demonstrating their consistency in solving nonlinear fractional-order differential equations (FODEs). This alignment confirms the accuracy and reliability of each method in handling such complex systems. The agreement in results underscores the effectiveness of both approaches and supports the validity of the proposed fractional smoking model. Furthermore, by preserving the fundamental dynamics of the system, these consistent results highlight the robustness of LADM and AADM as validation tools for fractional models. The use of fractional calculus further strengthens the applicability of these techniques, thus increasing confidence in both analytical and numerical solutions to real-world problems.

4.3. Discussion of Numerical Simulation

In this section, hypothetical values are allocated to the specific parameter and these values are used in the model (7) by

$$n_1 = 70, \quad n_2 = 45, \quad n_3 = 35, \quad n_4 = 24, \quad n_5 = 10, \quad n_6 = 8,$$
$$\rho = 0.1, \quad \rho_1 = 0.002, \quad \rho_2 = 0.0025, \quad \rho_3 = 0.003, \quad \sigma = 0.003, \quad \mu = 0.0015,$$
$$\kappa = 0.04, \quad \kappa_1 = 0.003, \quad \kappa_2 = 0.002, \quad \phi = 0.005, \quad \psi = 0.001, \quad \eta = 0.03, \quad \epsilon_1 = 0.003, \quad \epsilon_2 = 0.002.$$

Using parametric values in proposed model (7), the following is obtained,

$$A_0 = 70, \quad B_0 = 45, \quad C_0 = 35, \quad E_0 = 24, \quad F_0 = 10, \quad G_0 = 8,$$
$$A_1(u) = [-9.425] \quad [1 + q(u-1)],$$
$$B_1(u) = [6.1425] \quad [1 + q(u-1)],$$
$$C_1(u) = [1.4] \quad [1 + q(u-1)],$$
$$E_1(u) = [1.207] \quad [1 + q(u-1)],$$
$$F_1(u) = [-0.17] \quad [1 + q(u-1)],$$
$$G_1(u) = [0.408] \quad [1 + q(u-1)],$$

$$A_2(u) = [0.1][1 + q(u-1)] - [0.0039]\Big[\frac{q^2u^2}{2!} - 2q^2u + 2qu + (q-1)^2\Big],$$
$$B_2(u) = [-0.517]\Big[\frac{q^2u^2}{2!} - 2q^2u + 2qu + (q-1)^2\Big],$$
$$C_2(u) = [0.486]\Big[\frac{q^2u^2}{2!} - 2q^2u + 2qu + (q-1)^2\Big],$$
$$E_2(u) = [0.0459]\Big[\frac{q^2u^2}{2!} - 2q^2u + 2qu + (q-1)^2\Big],$$
$$F_2(u) = [0.0129]\Big[\frac{q^2u^2}{2!} - 2q^2u + 2qu + (q-1)^2\Big],$$
$$G_2(u) = [0.0003]\Big[\frac{q^2u^2}{2!} - 2q^2u + 2qu + (q-1)^2\Big].$$

Further, the solutions for first few terms can be found as,

$$A_(u) = 70 - [9.325] \quad [1 + q(u-1)] - [0.0039]\Big[\frac{q^2u^2}{2!} - 2q^2u + 2qu + (q-1)^2\Big],$$

$$B_(u) = 45 + [6.1425] \quad [1 + q(u-1)] - [0.517]\Big[\frac{q^2u^2}{2!} - 2q^2u + 2qu + (q-1)^2\Big],$$

$$C_(u) = 35 + [1.4] \quad [1 + q(u-1)] + [0.486]\Big[\frac{q^2u^2}{2!} - 2q^2u + 2qu + (q-1)^2\Big],$$

$$E_(u) = 24 + [1.207] \quad [1 + q(u-1)] + [0.0459]\Big[\frac{q^2u^2}{2!} - 2q^2u + 2qu + (q-1)^2\Big],$$

$$F_(u) = 10 - [0.17] \quad [1 + q(u-1)] + [0.0129]\Big[\frac{q^2u^2}{2!} - 2q^2u + 2qu + (q-1)^2\Big],$$

$$G_(u) = 8 + [0.408] \quad [1 + q(u-1)] - [0.0003]\Big[\frac{q^2u^2}{2!} - 2q^2u + 2qu + (q-1)^2\Big].$$

5. Analysis of Model

The simulation is done using MATLAB and graphs are plotted. It gives the idea about the interrelation between the parameters.

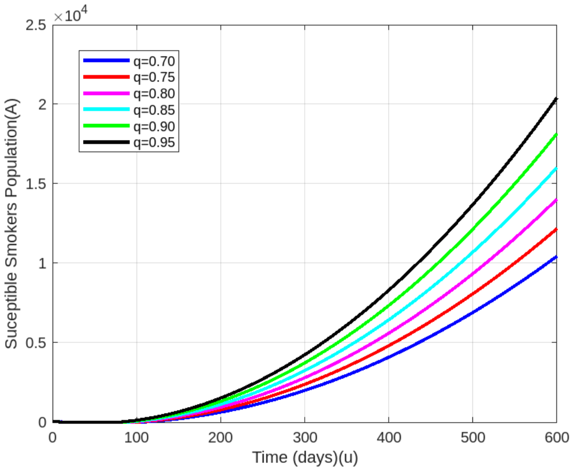

Figure 2: Susceptible smokers(A) vs time(u) by Laplace Trans.

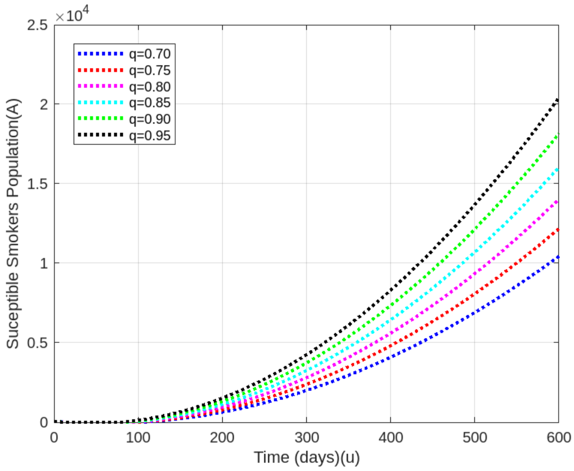

Figure 3: Susceptible smokers(A) vs time(u) by Aboodh Trans.

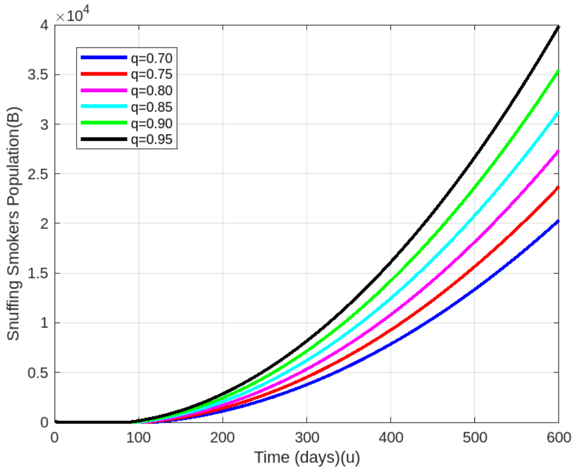

Figure 4: Snuffing smokers(B) vs time(u) by Laplace Trans.

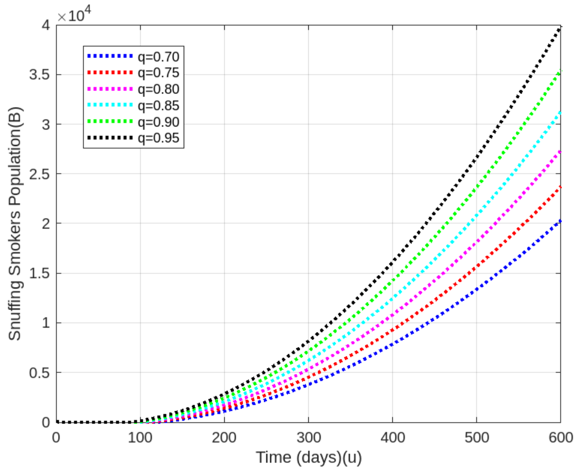

Figure 5: Snuffing smokers(B) vs time(u) by Aboodh Trans.

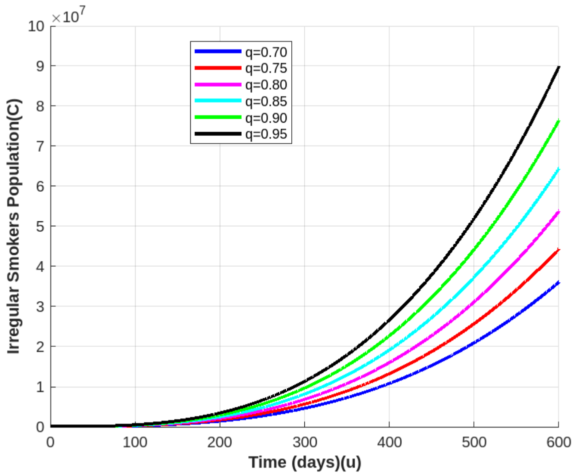

Figure 6: Irregular smokers(C) vs time(u) by Laplace Trans.

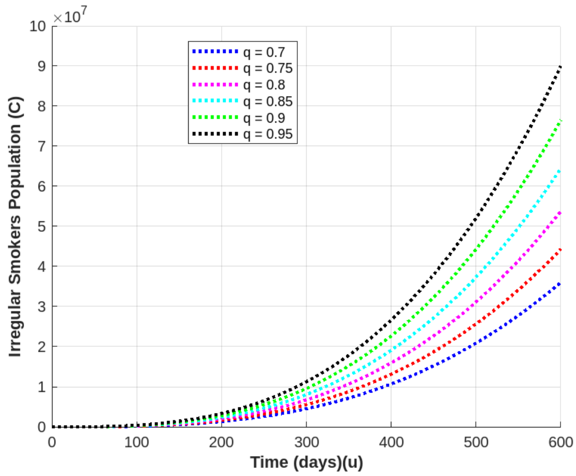

Figure 7: Irregular smokers(C) vs time(u) by Aboodh Trans.

The graphs show how different groups of people move through smoking behaviors over time using different fractional orders in the model. Figures 2–13 present solutions versus various fractional-order q. As shown clearly in figures 2–13, $A(u)$,$B(u)$,$C(u)$,$E(u)$, $F(u)$,$G(u)$, increases fast for lower fractional values of $q = 0.70$ and increases slowly for higher fractional values of $q = 0.95$. The Caputo-Fabrizio operator-based fractional-order smoking model illustrates the temporal evolution of each compartment with increasing trends across the system. The progression begins with the Susceptible group, representing individuals at risk of tobacco exposure. Over time, the transition to the Snuffing stage reflects early experimentation, followed by an upward shift in the Habitual compartment, indicating developing patterns of smoking behavior.

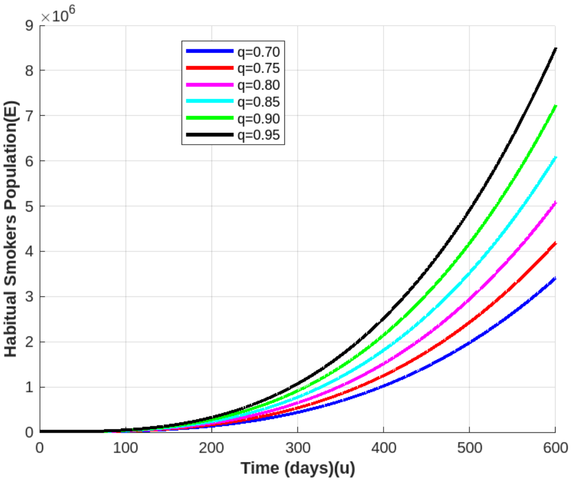

Figure 8: Habitual smokers(E) vs time(u) by Laplace Trans.

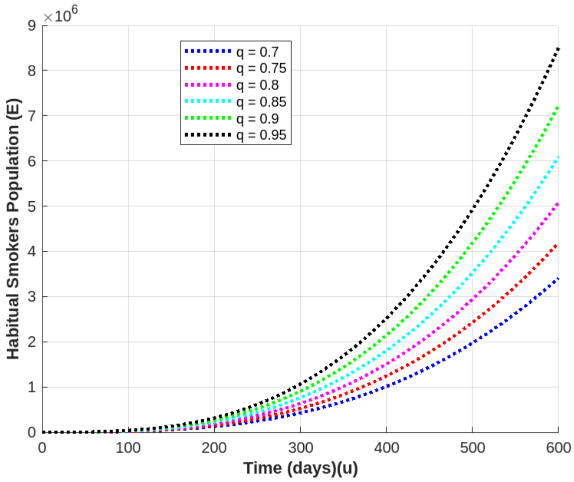

Figure 9: Habitual smokers(E) vs time(u) by Aboodh Trans.

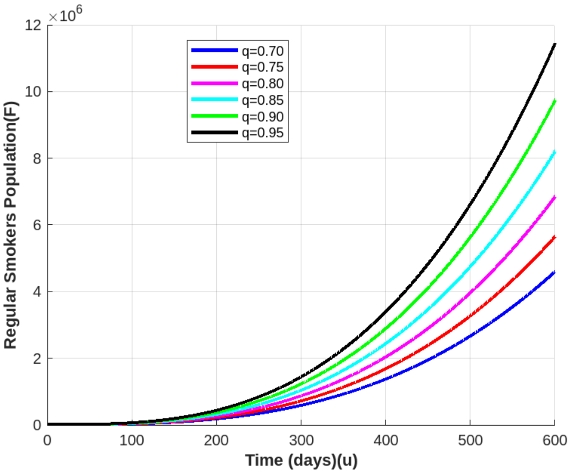

Figure 10: Regular smokers(F) vs time(u) by Laplace Trans.

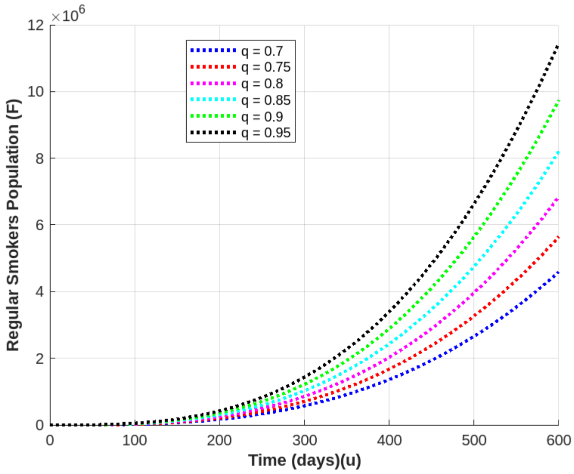

Figure 11: Regular smokers(F)vs time(u) by Aboodh Trans.

The Irregular and Regular smoker populations also exhibit rising curves, showing further reinforcement of the habit. Eventually, the model depicts a steady increase in the Quit compartment, as individuals gradually succeed in cessation. The growth patterns depict how individuals move through increasingly dependent smoking stages before attempting cessation. The smooth flow between compartments underscores the memory effect embedded in the Caputo-Fabrizio fractional derivative, allowing a more realistic representation of smoking dynamics and behavioral relapse.

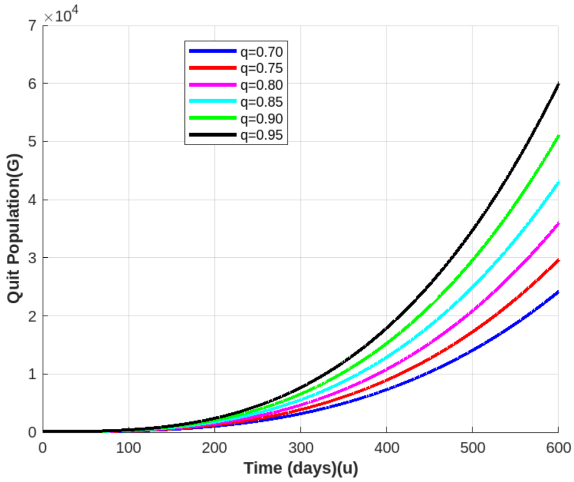

Figure 12: Quit smokers(G) vs time(u) by Laplace Trans.

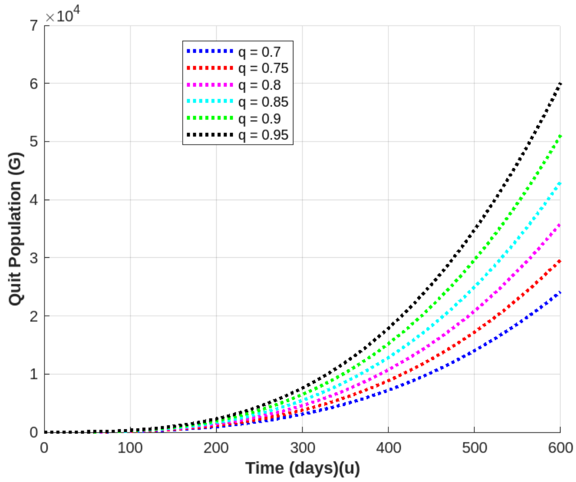

Figure 13: Quit smokers(G) vs time(u) by Aboodh Trans.

6. Conclusion

This research introduces an innovative methodology for the representation of smoking behavior through the implementation of a fractional-order mathematical framework utilizing the Caputo-Fabrizio derivative. In comparison with the traditional integer-order models, the application of fractional calculus facilitates a more comprehensive. Especially when the parameters like memory and hereditary influences are involved, which affect crucially in the dynamics of human behaviour particularly when it comes to addiction and relapse. According to the concept, smoking behavior develops gradually, starting with those who are susceptible and moving through snuffing, frequent use, intermittent smoking, and finally regular smoking. The cohort of regular smokers remains dominant throughout the experiment, even if the number of people quitting smoking keeps rising. This phenomenon demonstrates how difficult it is to break smoking habits once they have been formed. The increasing number of people quitting smoking is a sign that intervention efforts are working, but it is not enough to reverse the overall trend. This discordance is largely caused by strong behavioral habits and a tendency to relapse. To strengthen the impact of smoking cessation programs, efforts should emphasize prompt engagement and continuous assistance for individuals in advanced stages of tobacco dependency.

In this study, the interconnection between the parameters involved in the smoking models is presented. Numerical methodologies utilizing two specific transforms, LADM-Laplace Adomian Decomposition Method and AADM-Aboodh Adomian Decomposition Method, are used to solve the model. Both methodologies exhibited a high degree of consistency and precision in their outcomes. This strong agreement shows that these methods can be trusted or relied upon when used to model complex systems, such as nonlinear fractional-order differential equations. Numerical simulation is performed using

MATLAB for the various compartments (such as susceptible smokers, snuffing class, irregular smokers, habitual smokers, regular smokers & quit smokers) and the graphs are plotted. The model highlights a deeper understanding of the model development and possible control using fractional-order methods.

In summary, the study explains the significant advantages of fractional-order models for behavioural epidemiology research, particularly those that contain the Caputo-Fabrizio operator. Future studies are also encouraged by the work to investigate different kinds of fractional operators or broaden the model by including more social or psychological parameters. Overall, the study shows that memory (represented by the fractional order) greatly affects smoking behaviour, especially in helping or delaying the move toward quitting. Strong public health efforts are needed, especially for Regular, Irregular, and Habitual smokers, to overcome the influence of long-term habits.

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