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Novel Results On D-Soft Compact Spaces

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Abstract. This paper introduces and investigates D-soft compact spaces, a novel generalization of compactness in soft topological spaces using D-soft covers. We establish fundamental properties, characterizations, and relationships between D-soft compactness and other forms of soft compactness. Key results include the hereditary nature of D-soft compactness under specific conditions, the equivalence between D-soft and soft compactness in soft locally indiscrete spaces, and preservation under continuous mappings. The study provides new perspectives on compactness conditions in soft topology with potential applications in decision-making and uncertainty modeling. Several examples and counterexamples illustrate the theoretical developments, demonstrating that D-soft compactness is generally stronger than soft compactness.

2020 Mathematics Subject Classifications: AMS 54D30, 54E99, 54D10

Key Words and Phrases: Soft sets, D-Soft set, soft cover, D-soft cover, Soft set theory, Soft topology, Soft compact space, D-soft compact space.

Notation

Throughout this paper, we use the following notation:

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- S denotes a universe set
- A denotes a set of parameters
- $P(S)$ denotes the power set of S
- (T, E) denotes a soft set where $T : E \to P(S)$ and $E \subseteq A$
- (S, Υ, A) denotes a soft topological space
- (U, A) denotes a soft open set when clear from context
- ϕ denotes the null soft set

1. Introduction

The theory of soft sets, introduced by Molodtsov [1] in 1999, has emerged as a powerful mathematical tool for handling uncertainty and vagueness in various fields including engineering, medicine, economics, and environmental sciences. Unlike traditional approaches such as probability theory, fuzzy sets, and rough sets, soft set theory provides a parameterized family of subsets of the universe, offering greater flexibility in dealing with uncertainty.

Following Molodtsov's pioneering work, numerous researchers have contributed to the development and application of soft set theory. Maji et al. [2] introduced basic operations on soft sets, while Çağman and Enginoğlu [3, 4] developed soft set theory for decision-making and soft matrix theory. The algebraic structures of soft sets were studied by Aktaş and Çağman [5], leading to various applications in different mathematical structures. Abbas et al. [6] further generalized operations in soft set theory via relaxed conditions on parameters.

Recent developments in soft set applications include the work of Al-Sharqi et al. [7] on interval complex neutrosophic soft sets, Gulistan et al. [8] on neutrosophic cubic soft matrices for decision making, Khan and Zhu [9] on parameter reduction algorithms, and Muhiuddin et al. [10] on generalized ideals based on fuzzy soft set theory. Ulucay [11] introduced soft representation of soft groups, while practical applications were explored by Xiao [12] in medical diagnosis using hybrid fuzzy soft sets, Voskoglou [13] in decision making with TFNs and soft sets, and Zhang et al. [14] with N-soft rough sets.

The introduction of soft topology by Shabir and Naz [15] in 2011 marked a significant milestone in extending topological concepts to the soft set framework. This development opened new avenues for research in generalized topological spaces, building upon classical topology foundations established by Willard [16] and Engelking [17]. Subsequently, various topological properties have been investigated in the soft setting, including the work of Aygünoğlu and Aygün [18] on notes about soft topological spaces and Arockiarani and Selvi [19] on soft slightly πg-continuous functions.

Compactness, being one of the most fundamental concepts in topology, has naturally attracted attention in soft topology. The notion of soft compactness was introduced and studied in the context of soft topological spaces as an extension of classical compactness. However, despite these efforts, the theory of soft compactness remains relatively underdeveloped compared to its classical counterpart, particularly regarding covering properties and their applications.

The concept of D-sets in classical topology, introduced by Tong [20], provides a generalization of open sets that has proven useful in studying various topological properties. Qoqazeh et al. [21] investigated D-compact topological spaces, while Mustafa and Qoqazeh [22] studied supra D-sets and associated separation axioms. The soft version of D-sets was introduced by Mahmood [23], leading to new perspectives on soft topological structures and weak soft $d\tilde{n}$-sets.

Oudetallah et al. have made significant contributions to the theory of D-sets and related concepts. Oudetallah et al. [24] investigated D-metacompactness in topological spaces, establishing fundamental properties and relationships with other covering properties. This work was extended to r-compactness in both topological and bitopological spaces [25], providing new insights into generalized compactness conditions. Further investigations by Oudetallah [26] on nearly metacompact spaces in bitopological settings, and Oudetallah and AL-Hawari [27] on other generalizations of pairwise expandable spaces, have enriched the theory of covering properties in generalized topological spaces.

Recent advances (2023-2025) in soft topological spaces have focused on refining compactness conditions and exploring new types of coverings. Notable contributions include the work of Al-shami et al. [28] on compactness and connectedness via soft somewhat open sets, Mhemdi [29] on novel types of soft compact and connected spaces inspired by soft q-sets, and Alqahtani and Ameen [30] on soft nodec spaces. These studies have revealed the rich structure of soft topological spaces and motivated further investigations into specialized forms of compactness.

More recent developments include Alghamdi et al. [31] on soft submaximal and soft door spaces, Saleh and Salih [32] on c-continuity and c-compact spaces via soft sets, Al-Mufarrij and Al-Ghour [33] on regular-closed functions between soft topological spaces, and Alshammari et al. [34] on r-fuzzy soft δ-open sets with applications. These works demonstrate the continuing evolution and diversification of soft topology research.

Beyond topological considerations, the mathematical structures underlying our work have connections to geometric and functional analytic concepts. Oudetallah and Abualigah [35] investigated h-convexity in metric linear spaces, which shares structural similarities with compactness properties in terms of covering and separation conditions. Oudetallah [36] recently studied novel results on near Lindelöfness in topological spaces, which provides relevant insights for extending covering properties to soft settings.

Recent developments in approximation spaces and their applications have shown promising connections to soft topological methods. Studies on κ-neighborhoods [38], innovative rough set approaches for medical diagnosis of COVID-19 variants using novel initial-neighborhood systems [37], accurate diagnosis for COVID-19 variants using nearly initial-rough sets [38], enhancing rheumatic fever analysis via tritopological approximation spaces [39], and exploring β-basic rough sets and their applications in medicine [40] have demonstrated the practical relevance of these theoretical frameworks in real-world medical applications.

This study aims to introduce and investigate D-soft compact spaces, a new class of soft topological spaces that generalizes both soft compactness and D-compactness. By introducing D-soft covers—a novel type of covering in soft topology—we provide a fresh perspective on compactness conditions. Our approach builds upon the extensive work on D-sets and related concepts by Oudetallah and collaborators [24–27, 35, 36], adapting these ideas to the soft topological setting. This addresses existing gaps in the literature and offers a more nuanced understanding of covering properties in soft topological spaces.

The significance of this research is multifaceted. Theoretically, it extends the foundational understanding of compactness by introducing adjustable D-soft parameters that can be tailored to different topological structures. Practically, it provides tools for modeling uncertainty in complex systems where traditional compactness conditions may be too restrictive. The framework developed here has potential applications in decision-support systems, pattern recognition, and data analysis where soft computing approaches are beneficial, as demonstrated by recent applications in medical diagnosis [37–40].

The main contributions of this paper are:

(i) Introduction of D-soft covers and D-soft compact spaces with their fundamental properties

(ii) Establishment of relationships between D-soft compactness and existing forms of soft compactness

(iii) Characterization theorems for D-soft compact spaces under various conditions

(iv) Investigation of hereditary properties and preservation under mappings

(v) Illustrative examples demonstrating the distinctions between different types of soft compactness

The organization of the paper is as follows: In Section 2, we review essential concepts from soft set theory and soft topology, providing the foundation for our work. In Section 3, we present the main definitions of D-soft covers and D-soft compactness concepts; we then develop the theory through several theorems and results, supported by illustrative figures and examples. Section 4 concludes the paper with a summary of our findings and directions for future research.

2. Basic Principles and Primary Notions

In this section, we present the main concepts and results that we employed in our study in the theory of soft sets in soft topological spaces.

Definition 1. *[1] Let S be a universe set and A be a set of parameters. Suppose that the power set of S is represented by $P(S)$ and $E \subset A$. A pair (T, E) is called a soft set over S if $T : E \longrightarrow P(S)$; for any $e \in E$, we have $T(e) \in P(S)$.*

Definition 2. *[15] A soft topological space is a triplet (S, Υ, A) where S is a nonempty set and Υ is a collection of soft sets over S with parameter set A such that Υ satisfies the following axioms:*

(i) Both S and $\emptyset$ belong to Υ.

(ii) Any union of soft sets in Υ belongs to Υ.

(iii) Any finite intersection of soft sets in Υ belongs to Υ.

The soft topological space is denoted by (S, Υ, A) where A is a parameter set and such a collection, Υ, over S is called a soft topology. The elements of Υ are called soft open sets, and their complements are called soft closed sets.

Table 1: Comparison between Classical and Soft Topological Concepts

Classical Topology	**Soft Topology**
Set X	Soft set (F, A)
Open set U	Soft open set (U, A)
Topology τ	Soft topology Υ
Open cover	Soft open cover
Compact space	Soft compact space
D-set	D-soft set
D-cover	D-soft cover
D-compact	D-soft compact

Definition 3. *[18] The soft-set (T, A) is said to be covered by a family σ of soft-sets if $(T, A) \subseteq \cup\{(T_i, A); (T_i, A) \in \sigma; i \in I\}$, where σ is referred to as a soft-open cover if all of its elements are soft-open sets.*

Definition 4. *[3] A soft topological space (S, Υ, A) is called soft-compact if every soft-open cover has a finite soft-subcover. This generalizes the classical notion of compactness to the soft set setting.*

Definition 5. *[16, 17] A topological space (S, Υ) is compact if every open cover of S has a finite subcover. This means that whenever $S = \bigcup_{i \in I} U_i$ where each U_i is open, there exists a finite subset $J \subseteq I$ such that $S = \bigcup_{i \in J} U_i$.*

Definition 6. *[20] Let (S, Υ) be a topological space. A subset $S_1 \subseteq S$ is called a D-set if there exist two open sets U_1 and U_2 in Υ such that $U_1 \neq S$ and $S_1 = U_1 - U_2$. We say that S_1 is the D-set generated by U_1 and U_2.*

Definition 7. *[23] A soft subset (F, A) of a soft topological space (S, Υ, A) is called a D-soft set if there exist two soft open sets (U, A) and (V, A) in Υ such that $U \neq S$ and $F \subset (U, A) - (V, A)$.*

Remark 1. *[23] Every soft-open set is a D-soft set.*

However, as the following example shows, the converse of Remark 2.1 is not true in general.

Definition 8. *[21] Let (S, Υ) be a topological space. A cover $U = \{U_\gamma : \gamma \in \Gamma\}$ of S is called a D-cover if every U_γ is a D-set for all $\gamma \in \Gamma$.*

Definition 9. *[21] A topological space (S, Υ) is called D-compact if every D-cover has a finite subcover.*

Definition 10. *[22] The topological space (S, Υ) is called locally indiscrete if every open set in Υ is a clopen set.*

Definition 11. *[19] A soft topological space (S, Υ, A) is called soft locally indiscrete if every soft open set in (Υ, A) is a soft clopen set.*

3. Main Results

The notion of D-soft compactness in soft topological spaces is presented in this section along with some of its properties and its relations with other spaces. Our approach builds upon the extensive work on D-compactness and related concepts in classical topology [24, 25], adapting these ideas to the soft topological framework.

Example 1. *Let $S = \{4, 5, 7\}$ and $A = \{a_1, a_2\}$ such that $a_1 =$"even number", $a_2 =$"odd numbers". Suppose that $T(a_1) = \{4\}$, $T(a_2) = \{5, 7\}$, then the collection $(T, E) = \{(a_1, \{4\}), (a_2, \{5, 7\})\}$ is a soft set over S.*

Example 2. *Let $S = \mathbb{N}$, $A = \{a\}$ and $\Upsilon = \{S, \phi, (U, A)\}$ be a soft topology on S, where the soft set (U, A) is defined by $U(a) = \{1\}$. Then (U, A) is a D-soft set but not a soft open set. To see this, take the soft open sets (W, A) and (V, A) where $W(a) = S$ and $V(a) = S - \{1\}$. Then $(U, A) = (W, A) - (V, A)$, showing it is a D-soft set. However, $(U, A) \notin \Upsilon$, so it is not soft open.*

Definition 12. *A soft cover $(U, A) = \{(U_\gamma, A) : \gamma \in \Gamma\}$ of a soft topological space (S, Υ, A) is called a D-soft cover if (U_γ, A) is a D-soft set for every $\gamma \in \Gamma$.*

Every soft-open cover is clearly a D-soft cover, but the converse is not true in general. For example, in the soft topological space $(\mathbb{R}, \Upsilon_{cof}, A = \{a\})$, the collection $(U, A) = \{(a, \{n\}) : n \in \mathbb{R}\}$ is a D-soft cover that is not a soft open cover.

Definition 13. *The soft topological space (S, Υ, A) is called D-soft compact if every D-soft cover of S has a finite soft-subcover.*

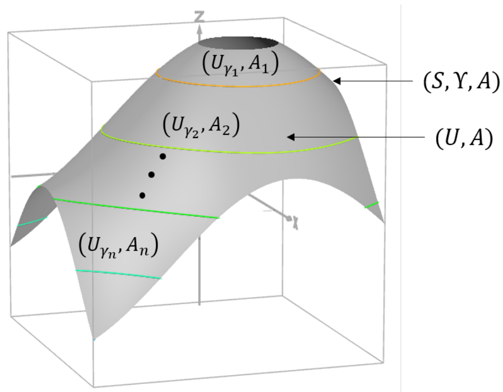

Figure 1: The space of D-soft compact.

Figure 1 shows the D-soft compact space for which $(U, A) = \{(U_\gamma, A) : \gamma \in \Gamma\}$ is D-soft cover of the soft topological space (S, Υ, A) possesses a finite soft-subcovers $\{(U_{\gamma_1}, A_1), (U_{\gamma_2}, A_2), \ldots, (U_{\gamma_n}, A_n)\}$, in which each (U_γ, A) is a D-soft set for all $\gamma \in \Gamma$.

Example 3. *Consider the following examples of D-soft compact and non-D-soft compact spaces:*

- *Let $S = \{1, 2, 3\}$, $A = \{a_1, a_2, a_3\}$, and $\Upsilon = \{\phi, S, (U_1, A), (U_2, A)\}$ where (U_1, A) is defined by $U_1(a_i) = \{1, 2\}$ for all i, and (U_2, A) is defined by $U_2(a_i) = \{2, 3\}$ for all i. Then (S, Υ, A) is D-soft compact because S is finite.*
- *Let $A = \{a\}$ and $(\mathbb{R}, \Upsilon, A)$ be the soft usual topological space. Then $(\mathbb{R}, \Upsilon, A)$ is not D-soft compact. To see this, consider the D-soft cover $(U, A) = \{((n-1, n+1), A) : n \in \mathbb{Z}\}$. This cover has no finite subcover, hence $(\mathbb{R}, \Upsilon, A)$ is not D-soft compact.*

Lemma 1. *In a soft locally indiscrete space (S, Υ, A), the union of any collection of D-soft sets is a D-soft set.*

Proof. *Let $\{(F_i, A) : i \in I\}$ be a collection of D-soft sets in a soft locally indiscrete space. For each i, there exist soft open sets (U_i, A) and (V_i, A) such that $U_i \neq S$ and $(F_i, A) \subseteq (U_i, A) - (V_i, A)$. Since the space is soft locally indiscrete, all soft open sets are soft clopen. Let $(U, A) = \bigcup_{i \in I}(U_i, A)$ and $(V, A) = \bigcup_{i \in I}(V_i, A)$. Then (U, A) and (V, A) are soft open (hence soft clopen), and $\bigcup_{i \in I}(F_i, A) \subseteq (U, A) - (V, A)$. Since at least one $U_i \neq S$, we have $U \neq S$ (as S is soft clopen). Therefore, $\bigcup_{i \in I}(F_i, A)$ is a D-soft set.*

Theorem 1. *For every soft topology on S, the space (S, Υ, A) is D-soft compact if S is a nonempty finite set.*

Proof. *Suppose $S = \{s_1, s_2, \ldots, s_n\}$ is a finite set and $(U, A) = \{(U_\gamma, A) : \gamma \in \Gamma\}$ is a D-soft cover of S. Since (U, A) covers S, for each $s_i \in S$ there exists some $\gamma_i \in \Gamma$ such that $s_i \in (U_{\gamma_i}, A)$. Therefore, the collection $(U, A)^* = \{(U_{\gamma_1}, A), (U_{\gamma_2}, A), \ldots, (U_{\gamma_n}, A)\}$ is a finite soft subcover of (U, A) for S. Thus, (S, Υ, A) is a D-soft compact space.*

Corollary 1. *In a soft locally indiscrete space, every D-soft set is soft clopen.*

Proof. Let (U, A) be a D-soft set in a soft locally indiscrete space. Then $(U, A) = (W, A) - (Z, A)$ for some soft open sets (W, A) and (Z, A). Since the space is soft locally indiscrete, both (W, A) and (Z, A) are soft clopen. Therefore, (U, A) is the difference of two soft clopen sets, which makes it soft clopen.

Corollary 2. *In a soft locally indiscrete space, the union of any collection of D-soft sets is a D-soft set.*

Proof. This follows directly from Lemma 3.1.

Theorem 2. *Every D-soft compact space is soft compact.*
Proof. Let (S, Υ, A) be a D-soft compact space and let $(U, A) = \{(U_\gamma, A) : \gamma \in \Gamma\}$ be any soft open cover of S. Since every soft open set is a D-soft set (Remark 2.1), (U, A) is also a D-soft cover. By D-soft compactness, (U, A) has a finite soft subcover. Therefore, (S, Υ, A) is soft compact.

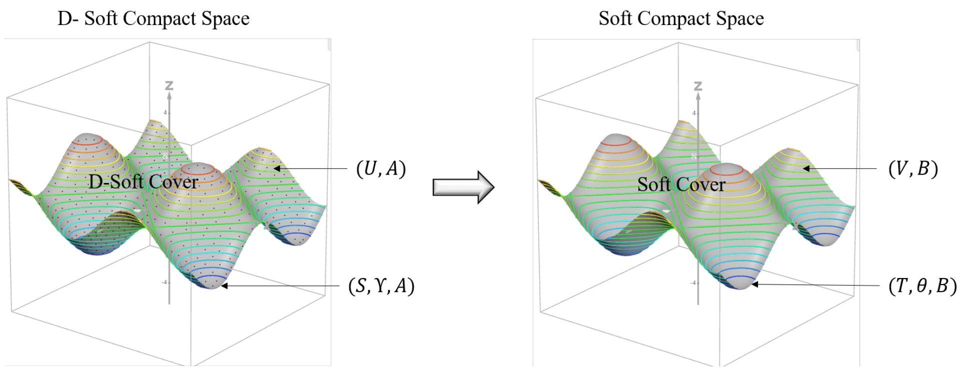

Figure 2: The relation of D-soft compact and soft compact spaces.

The primary connection between D-soft compact and soft compact spaces is depicted in Figure 2. Any D-soft compact space should be soft compact, according to the important claim that $(U, A) = \{(U_\gamma, A) : \gamma \in \Gamma\}$ is a D-soft cover of the soft topological space (S, Υ, A) with a finite soft-subcover, and $(V, B) = \{(V_\gamma, B) : \gamma \in \Gamma\}$ is a soft-cover of the soft topological space (T, θ, A).

We demonstrate in the following example that the opposite of Theorem 3.2 does not necessarily have to be true.

Example 4. *The soft-cofinite topological space $(\mathbb{R}, \Upsilon, A)$, where $A = \{a_1, a_2\}$, is soft-compact but not a D-soft compact. In a soft topological space $(\mathbb{R}, \Upsilon, A)$, any soft-set of the form $(a_1, \mathbb{R} - \{y\})$ or $(a_2, \mathbb{R} - \{y\})$; $y \in \mathbb{R}$ is a soft-open set. Let $(W, A) = (a_1, \mathbb{R} - \{y_1\})$ and $(Z, A) = (a_2, \mathbb{R} - \{y_2\})$ stand now. In such case, $(U, A) = (W, A) - (Z, A) =$*

$(a_1, \{y_2\})$ is an inaccessible D-soft set. With no finite soft-subcover, the collection $(U, A) = (a_{i=1,2}, \{y\}) : y \in \mathbb{R}\}$ is a D-soft cover of $(\mathbb{R}, \Upsilon, A)$. If $(U, A) = \{(a_{i=1,2}, \{y\}) : y \in \mathbb{R}\}$ has a finite soft-subcover $\{(a_{i=1,2}, \{y_1\}), (a_{i=1,2}, \{y_2\}), \ldots, (a_{i=1,2}, \{y_n\})\}$, then $\mathbb{R} \subseteq \bigcup_{k=1}^{n}(a_{i=1,2}, \{y_k\})$, i.e., $\mathbb{R}$ is a finite soft-set, which is contradictory.

In the next example, we show that the contrapositive of Theorem 3.2. is true in general.

Example 5. *Since A is an arbitrary parameter, the soft topological space $(\mathbb{R}, \Upsilon_{l.r}, A)$ is not soft-compact and thus not D-soft compact.*

In the next example presents that the converse of Theorem 3.2. may be true with extra conditions.

Theorem 3. *If the soft compact space (S, Υ, A) is a soft locally indiscrete, then it is D-soft compact.*

Proof. *The collection (U_γ, A) is a soft-clopen set for every $\gamma \in \Gamma$. Let $(U, A) = \{(U_\gamma, A) : \gamma \in \Gamma\}$ be a D-soft cover of (S, Υ, A). (U, A) is a soft-open cover of (S, Υ, A), in this case. Now, (U, A) has a finite soft-subcover since the space (S, Υ, A) is soft compact. Thus, we have what is needed.*

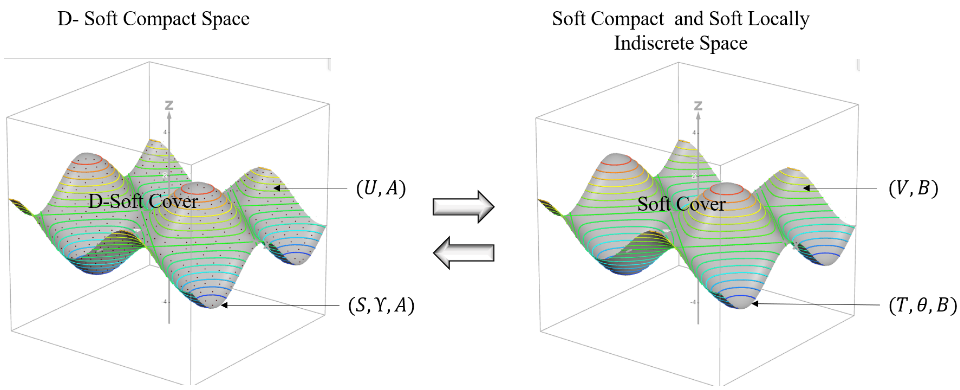

Figure 3: The relation of D-soft compact and soft compact spaces with soft locally indiscreet condition.

Figure 3 illustrates the complex relationship between D-soft compact and soft compact spaces with addition condition. It presents a curial assertion that any D-soft compact space should be soft compact, while the opposite is true if the soft compact space satisfies the soft locally indiscreet condition, such that $(U, A) = \{(U_\gamma, A) : \gamma \in \Gamma\}$ is D-soft cover of the soft topological space (S, Υ, A) has a finite soft-subcover, and $(V, B) = \{(V_\gamma, B) : \gamma \in \Gamma\}$ is soft-cover of the soft topological space (T, θ, A) has a finite-subcover.

Example 6. *(1) Let $S = \mathbb{R}$, $A = \{a\}$ and Υ_{ind} be a soft indiscrete topology on S. Then the soft space (S, Υ_{ind}, A) is D-soft compact, because it is locally indiscrete and soft compact. (2) Let $S = \mathbb{R}$, $A = \{a_1, a_2, a_3, a_4, a_5, a_6\}$ and $\Upsilon = \{\phi, \mathbb{R}, \mathbb{R} - \{7\}, \{7\}\}$ be soft topology on S. Then the space (S, Υ, A) is soft locally indiscrete and soft compact, so it is D-soft compact.*

Theorem 4. *Let (S, Υ_s, A_s) be a soft topological space and $Q \subseteq S$, then (Q, Υ_q, A_q) is D-soft compact iff every each soft-cover of Q by D-soft sets in S has a finite soft-subcover.*

Proof. $\Longrightarrow$) *Let (Q, Υ_q, A_q) be D-soft compact and $(U, A_s) = \{(U_\gamma, A_s) : \gamma \in \Gamma\}$ be D-soft cover of Q by D-soft sets in S. Now, for all $\gamma \in \Gamma$ we have that the collection $(U^*_\gamma, A^*_q) = (U_\gamma, A_s) \cap Q$ is a D-soft sets in Q. Then $(U^*, A^*_q) = \{(U^*_\gamma, A^*_q) : \gamma \in \Gamma\}$ is D-soft cover of Q by D-soft sets in Q. Since the space (Q, Υ_q, A_q) is a is D-soft compact, then (U^*, A^*_q) has a finite soft-subcover $\{(U^*_{\gamma_1}, A^*_{q_1}), (U^*_{\gamma_2}, A^*_{q_2}), \ldots, (U^*_{\gamma_n}, A^*_{q_n})\}$ for Q. Hence we get that the family $\{(U_{\gamma_1}, A_{s_1}), (U_{\gamma_2}, A_{s_2}), \ldots, (U_{\gamma_n}, A_{s_n})\}$ is a finite soft-subcover of (U, A_s) in S for Q, where $(U^*_{\gamma_i}, A^*_{q_i}) = (U_{\gamma_i}, A_{s_i}) \cap Q$ for all $i = 1, 2, \ldots, n$ and $\gamma \in \Gamma$. Hence we have the required.*

$\Longleftarrow$) *Let Q have a finite soft-subcover for every D-soft cover of Q by D-soft sets in S. It is assumed that $(V, A_q) = \{(V_\gamma, A_q) : \gamma \in \Gamma\}$ is a D-soft cover of Q by D-soft sets in Q. Then, for any $\gamma \in \Gamma$, there are D-soft sets (U_γ, A_s) in S such that $(V_\gamma, A_q) = (U_\gamma, A_s) \cap Q$. As we now know, $(U, A_s) = \{(U_\gamma, A_s) : \gamma \in \Gamma\}$ is the D-soft cover of Q by D-soft sets in S. then by assumption the collection (U, A_s) has a finite-subcover $\{(U_{\gamma_1}, A_{s_1}), (U_{\gamma_2}, A_{s_2}), \ldots, (U_{\gamma_n}, A_{s_n})\}$. Hence we get that the family $\{(V_{\gamma_1}, A_{q_1}), (V_{\gamma_2}, A_{q_2}), \ldots, (V_{\gamma_n}, A_{q_n})\}$ is a finite soft-subcover of (V, A_q) for Q, because $V_{\gamma_i} \subseteq U_{\gamma_i}$, for all $\gamma \in \Gamma$ and $i = 1, 2, \ldots, n$. Hence we have the required.*

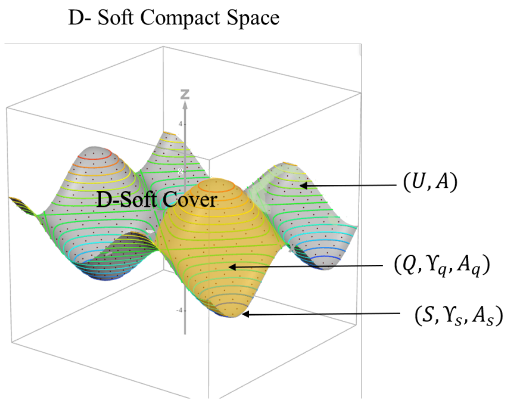

Figure 4: The soft subspace topology of D-soft compact space.

A key distinction between the soft topological space and its soft subspace is depicted in Figure 4, in which the soft topological subspace (Q, Υ_q, A_q) of the D-soft compact space (S, Υ_s, A_s) must be D-soft compact space such that (U, A) is D-soft cover of the soft topological subspace (Q, Υ_q, A_q) has a finite soft-subcover, for any D-soft sets in the space (S, Υ_s, A_s).

Corollary 3. *If (Q, Υ_q, A_q) is a D-soft compact space, then any soft-open cover of Q by soft-open sets in S has a finite soft-subcover.*

Theorem 5. *Let (S, Υ_1, A_1) and (S, Υ_2, A_2) be two soft topological spaces. If (S, Υ_2, A_2) is D-soft compact and $(\Upsilon_1, A_1) \subseteq (\Upsilon_2, A_2)$, then (S, Υ_1, A_1) is a D-soft compact space.*

Proof. *Let* $(U, A_1) = \{(U_\gamma, A_1) : \gamma \in \Gamma\}$ *be a D-soft cover of* (S, Υ_1, A_1)*. Then we get that* (U, A_1) *is also D-soft cover of* (S, Υ_2, A_2)*, because* $(\Upsilon_1, A_1) \subseteq (\Upsilon_2, A_2)$*, so it has finite soft-subcover, Hence we have the required that the space* (S, Υ_1, A_1) *is D-soft compact.*

Theorem 6. *All of the D-soft compact subspaces are closed soft spaces of D-soft compact spaces.*

Proof. Assume that S is a D-soft compact space and $Q \subset S$ is a closed set. Let $(U, A) = ((A_\gamma, A) : \gamma \in \Gamma)$. Given a D-soft cover of Q by D-soft sets of S. When S is D soft compact (which we denote by writing S^c as a D soft cover), so is its collection $(U, A) \cap \{S - Q\}$, and we will have a finite soft sub cover of that family, or, we have a D soft cover to the problem, that is $(U, A)^*$, of finite size. Now the collection $(U, A)^* - \{S - Q\}$ is a finite soft subcover of (U, A) for Q. Thus, we have what is needed.

4. Conclusion

In this paper, we have introduced and investigated the concept of D-soft compact spaces, providing a new perspective on compactness in soft topological spaces. Using the novel concept of D-soft cover, we have established a framework that generalizes classical compactness while addressing limitations in existing soft topology literature. Our main contributions include:

- The introduction of D-soft covers as a new type of covering in soft topology
- The definition and characterization of D-soft compact spaces
- Proof that D-soft compactness implies soft compactness, with the converse holding in soft locally indiscrete spaces
- Investigation of hereditary properties and behavior under continuous mappings
- Illustrative examples distinguishing D-soft compactness from soft compactness

The results obtained show that D-soft compact spaces not only enhance the theoretical foundation of soft set theory but also open new avenues for applications in fields requiring uncertainty modeling, such as computer science and decision-support systems. The framework developed here provides an efficient and innovative contribution to the evolving field of soft topology and encourages further research into the structural characteristics and interrelationships of soft compact spaces.

The study of D-soft compact spaces opens several promising research directions:

(i) **Extension to other covering properties**: Future studies could focus on expanding D-soft compactness into more complex soft topological frameworks, such as D-soft Lindelofness and D-soft metacompactness, building upon the work on near Lindelofness [36] and D-metacompactness [24].

(ii) **Bitopological generalizations**: The concepts developed here could be extended to soft bitopological spaces, following the approach used for nearly metacompact spaces [26] and pairwise expandable spaces [27].

(iii) **Connections with approximation spaces**: Building on the work of Nawar et al. (2022) on $\theta\beta$-approximation spaces and El-Bably et al. (2025) on their theoretical refinements, there is potential for integrating D-soft compactness with ideal-related topological frameworks. These connections could enhance both theoretical understanding and practical applications.

(iv) **Applications in decision-making**: The framework developed here could be applied to real-world problems in medical diagnosis, pattern recognition, and data analysis, following the successful applications of rough set approaches demonstrated in recent literature.

(v) **Relationships with fuzzy and rough set theories**: Investigating how D-soft compact spaces connect with fuzzy topological spaces and rough set models could yield new hybrid approaches for handling uncertainty.

(vi) **Geometric connections**: The relationship between D-soft compactness and convexity properties in soft settings, inspired by work on h-convexity [35], presents an intriguing avenue for future research.

We emphasize that any extensions should maintain fidelity to the original formulations, particularly regarding the $\theta\beta$-concepts established by Nawar et al. (2022) and refined by El-Bably et al. (2025), as these provide the accurate foundation for ideal-related topological structures.

We hope that this study will encourage researchers to explore this field and discover additional connections with other areas.

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