EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
2025, Vol. 18, Issue 4, Article Number 6539
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

Hybrid Power Market Forecasting Using an Interval-Valued Hidden Markov and Fuzzy Relational Particle Swarm Optimization Model

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Abstract. The Hidden Markov Model (HMM), one of the Bayesian analysis tools, has a wide range of applications. In this paper, we present a unique method of merging particle swarm optimization (PSO) for parameter optimization with fuzzy hidden Markov models (FHMMs) to increase the accuracy of power price projections. The suggested model is created using the parameters of the Interval Valued Fuzzy Relational Hidden Markov Model (iHMM-FRPSO), which are appropriately established in accordance with the power forecasting system. PSO is then used in MATLAB to optimize and modulate the model.The study's primary goal is to reduce the likelihood of mistakes, namely mean square error (MSE), root mean square error (RMSE), and mean absolute percent error (MAPE), while optimizing the constraint parameters. The Indian Energy Exchange (IEX) Day-Ahead Markets and Real-Time Markets datasets are used to train the algorithm. Findings from the training set's mean directional forecasting accuracy from January 2021 to December 2021 indicate that the best optimized MAPE for day-ahead markets is 14.400?

2020 Mathematics Subject Classifications: 68T37, 90C59, 93E10

Key Words and Phrases: Hidden Markov model, fuzzy relational matrix, particle swarm optimization, market liberation price, market liberation volume, power market prediction

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1. Introduction

To increase the likelihood of accuracy in electricity forecasting systems (EFS), interval-valued, cross-breed fuzzy parametric optimization techniques are required. For a system that is constantly evolving and focused on lowering the forecasting error, HMM performs admirably. The degrees of membership or relationship strengths are represented by the fuzzy relation matrix. The interval-valued Markov property of the electrical forecasting system, which allows for forecasting and analysis, is the driving force behind the development of the suggested model. The primary goal of the suggested approach is to optimize the HMM para-metrically using particle swarm theory, which lowers the likelihood of

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DOI: https://doi.org/10.29020/nybg.ejpam.v18i4.6539

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mistakes. PSO is a population-based stochastic optimization technique that repeatedly modifies the positions and velocities of a swarm of particles in order to find the best solution. It was inspired by social behavior. The suggested model actively uses PSO for HMM interval-valued parametric optimization in order to assess each person's state and identify the best person right now. Accurate forecasting of electricity prices and volumes plays a critical role in modern power markets, where efficient generation, distribution, and consumption planning are essential for operational stability and cost optimization. In deregulated market environments, such as the Indian Energy Exchange (IEX), even small prediction errors can result in substantial financial and operational consequences. Reliable forecasts enable market participants to make informed bidding decisions, reduce imbalances, and optimize resource utilization.Electricity markets are inherently volatile due to the influence of numerous dynamic factors, including fluctuating demand, weather conditions, fuel price variations, and unexpected system outages. These uncertainties introduce significant challenges for forecasting models, which must capture complex, nonlinear, and often non-stationary market behaviors. Additionally, electricity cannot be economically stored in large quantities, making real-time accuracy even more critical.

Hidden Markov Models (HMMs) have been widely used in electricity price forecasting due to their ability to model regime-switching behaviors and temporal dependencies. Fuzzy logic extensions have further enhanced HMM capabilities by incorporating imprecision handling and uncertainty modeling in market dynamics. Particle Swarm Optimization (PSO), a population-based stochastic optimization technique inspired by swarm intelligence, has been effectively applied to optimize model parameters for better forecasting accuracy. Several studies have demonstrated the benefits of hybrid approaches combining HMM, fuzzy logic, and metaheuristic optimization, although challenges remain in scalability, interpretability, and adaptation to rapidly changing market conditions.

1.1. Contribution of the Paper

This paper proposes a hybrid forecasting framework that integrates the Interval-Valued Fuzzy Relational Hidden Markov Model (iHMM) with Particle Swarm Optimization (PSO) for parameter optimization. The iHMM component captures market uncertainty through fuzzy relational matrices, while PSO enhances convergence towards optimal transition and emission parameters. The key contributions of this work include:

- Development of a novel iHMM - PSO integration for enhanced electricity price and volume forecasting.
- Comprehensive comparative performance analysis against existing hybrid and deep learning models.
- Validation using real-world datasets from the Indian Energy Exchange (IEX) Day - Ahead and Real - Time Markets.

The Markov chain is an essential mathematical tool that is crucial to stochastic processes. The Markov chain study was established in 1906 by Markov [1]. Ross et al. [2]

presented continuous-time Markov chains for the first time. Several efficient Markov chain techniques for numerical computations were presented. Nagarajan et al. [3] investigated the trend analysis of stock market behavior using a hidden Markov model. Guangming Li et al. [4] discussed fuzzy Markov chains based on the fuzzy transition probability to simulate day-ahead electricity pricing. A combined forecast with misleadingly selected weights and HMM-calibrated errors was presented by Liu et al. [5]. Numerous methods and techniques have been developed to determine optimal pricing in order to maximize profit, according to Pandey et al. [6] and other academics. The application of several price forecasting algorithms in global electrical markets has been discussed. The research paper includes a discussion of the study conducted by Hambali et al. [7] on estimating the quantity of electricity required by humans. One way to ensure high-quality power generation, transmission, distribution, and marketing is to accurately forecast the population's energy needs in order to save operating costs and maximize the use of the electricity generated at all times. The parameter of the re-estimation problem in the classical hidden Markov model of the fuzzy probability space was examined by R. Sujatha et al. [8]. S. Marimuthu et al. [9] talked about the fuzzy stochastic Markov chain transition probability, which is expressed as a Hepta value utilizing uncertainty and as triangular fuzzy numbers. Lago et al. [10] proposed two different methods to improve predictive performance and incorporate market integration in energy price forecasting. We examined a deep neural network that improves local market prediction accuracy by utilizing features from other connected markets. In a study by Camino Gonzalez et al., Random Forest, a widely used methodology that has proven effective in several areas, was compared to another tree-based strategy. [11] Singh et al. [12] published a thorough, methodical, and comparative investigation of excellent data mining approaches useful in estimating the electrical load demand of diverse geographic areas. Using artificial neural networks (ANN), Niranjan Kumar et al. [13] predicted the market clearing price in the Indian power markets. Study by Li et al. [14] on neural network and genetic algorithm-based methods for forecasting power market prices for electricity. Patel et al. [15] assessed an essay on data - driven analysis methodologies for energy usage and price projection. Ahmed G. Gad [16] oversaw a number of proficient improvement projects in fields including wireless networks and robotics. The PSO rule's contents have become more widely known, and Jingzhong Fang et al. [17] have given a thorough overview of a few selected PSO variations. The average - based fuzzy time series Markov chain based on frequency density partitioning was studied by Susilo Hariyanto et al. [18]. According to Xuan Huang et al. [19], the hydrological cycle of a watershed rainfall series is calculated using Markov's theory, and the historical data is categorized using the percentage of precipitation distance.Numerous approaches, their accuracy, modeling performance, and drawbacks are all shown in the current literature analysis. The development of a hybrid intelligent interval - valued tuning model for the Interval - Valued Fuzzy Relational Hidden Markov Parametric Optimized Particle Swarm Algorithm is the primary goal of this research project for an electricity forecasting system. The model's advantages are that it selects the truly best people and determines each person's fitness to pass on to the following generation. This contributes to the development of an original, fresh mathematical framework for the iHMM-FRPSO power forecasting system.

The remainder of this paper is structured as follows: Section 2 presents Related work and Literature review ,Section 3 Presents Theoretical background,Section 4 presents Proposed methodology Section 5 Presents the materials and methods, including preliminaries on HMM and PSO, the proposed iHMM - PSO model design, and its optimization procedure.Section 6 Reports experimental results, performance evaluations, and comparative analysis with state- of-the-art models. Section 7 Concludes the study and discusses potential future work.

2. Related Work and Literature Review

2.1. Overview of Existing Forecasting Techniques

Traditional HMM Models

Hidden Markov Models (HMMs) have been widely used in electricity price forecasting due to their ability to represent regime-switching dynamics and temporal dependencies in market data. In classical HMM applications, market prices are modeled as outputs of hidden states, with transition probabilities capturing the likelihood of shifting between market conditions (e.g., high, medium, or low price regimes). Ross et al.(1996) formalized the theoretical basis for HMMs, and subsequent works (e.g., Kavitha et al.2013) demonstrated their applicability to financial and energy markets. Fuzzy logic extensions have further enhanced HMM capabilities by incorporating imprecision handling and uncertainty modeling in market dynamics. Comparable fuzzy-based forecasting frameworks were discussed by Sujatha et al. (2016) and Marimuthu et al. (2022). While traditional HMMs can capture sequential dependencies, they are often limited in handling imprecise or noisy market data, which is common in electricity trading environments.

Hybrid Models

To address the limitations of traditional HMMs, hybrid models have been developed that combine HMMs with soft computing and machine learning methods:

- FHMM + GA: Fuzzy Hidden Markov Models integrated with Genetic Algorithms introduce fuzzy logic to handle uncertainty and genetic optimization to improve parameter tuning. Patel & Deshmukh (2021) applied FHMM+GA to IEX market price forecasting, achieving improved accuracy but at the cost of longer convergence times and sensitivity to parameter initialization.

- FHMM + ANN: Artificial Neural Networks have been used with FHMMs to enhance nonlinear mapping capabilities. However, FHMM+ANN models often require large volumes of training data, and the black-box nature of ANNs reduces interpretability (Kumar et al., 2024)

- Fuzzy Deep Learning: Deep learning architectures with fuzzy preprocessing layers, such as Fuzzy CNN-GRU (Kumar et al., 2024), aim to integrate uncertainty handling into high-capacity neural networks. While these models achieve strong accuracy, they demand significant computational resources and large datasets, making real-time deployment challenging.

Other recent studies have explored ensemble methods, deep recurrent models (e.g., LSTM, DBN), and tree-based algorithms (e.g., Random Forest, Dynamic Trees) for electricity price forecasting. Each offers specific advantages but also faces trade-offs in complexity, interpretability and adaptability to sudden market changes.

2.2. Comparison with Existing Hybrid Models

Advantages over Existing Methods:

Compared to FHMM+GA, FHMM+ANN, and fuzzy deep learning approaches:

- **Higher interpretability** due to its rule-based fuzzy structure and probabilistic transparency.
- **Lower computational cost** than deep neural architectures.
- **Greater adaptability** to data scarcity and volatility.

Overall, the iHMMF-RPSO framework delivers an accurate, interpretable, and computationally efficient solution for electricity market forecasting, validated on real IEX market data across both Day-Ahead Market (DAM) and Real-Time Market (RTM) scenarios.

Existing hybrid forecasting models such as FHMM+GA, FHMM+ANN, and fuzzy deep learning architectures have shown effectiveness but often rely heavily on data volume or computational complexity. FHMM+ANN requires extensive training data, while fuzzy deep learning models are often viewed as *black-box* systems with limited interpretability. In contrast, iHMM-FRPSO leverages interval-valued fuzzy relations for better modeling of uncertainty and uses PSO's iterative convergence capabilities for fine-tuning. These characteristics make it more efficient and explainable than other complex hybrid models.

While several hybrid approaches have been proposed for electricity price forecasting-such as Fuzzy HMM combined with Genetic Algorithms (FHMM+GA), Fuzzy HMM with Artificial Neural Networks (FHMM+ANN), and fuzzy-enhanced deep learning models-these methods face critical limitations. Similar findings were reported in hybrid forecasting studies such as Patel & Deshmukh (2021) and Kumar et al. (2024)." FHMM+GA suffers from slower convergence and sensitivity to premature local minima, while FHMM+ANN struggles with training instability due to nonlinear interactions between fuzzy rule encoding and neural learning dynamics. Moreover, fuzzy deep learning models often lack interpretability and require large datasets and extensive tuning to generalize well.

A structured comparison is provided in Table 1 to highlight the relative strengths and limitations of existing hybrid models versus the proposed approach.

Table 1: Relative strengths and Limitations of Existing hybrid Models.

Model	Strengths	Limitations	Reference(s)
FHMM + ANN	Captures nonlinear patterns	Requires extensive training data; training instability due to nonlinear interactions between fuzzy rules and neural learning	Kumar et al., 2016
FHMM + GA	Optimizes fuzzy rules via evolutionary search	Slow convergence; sensitive to premature local minima	Kumar et al., 2016
Fuzzy Deep Learning Models	High predictive power; handles complex datasets	Limited interpretability (black-box); requires large datasets and extensive tuning	Lago et al., 2018
Proposed iHMM-FRPSO	Efficient uncertainty modeling using interval-valued fuzzy relations; PSO ensures robust convergence and fine-tuning; interpretable structure	More efficient and explainable than other hybrid models	-

2.3. Comparative Review (2020-2024)

A comparative review of recent works between 2020 and 2024 reveals significant progress in hybrid and fuzzy-based electricity market forecasting. Table 1 summarizes key models, datasets, forecast targets, and reported performance metrics.To establish the novelty and performance advantages of the proposed iHMM-FRPSO model, a comparative literature review was conducted against prominent electricity market forecasting methods published between 2020 and 2024. Table 2 summarizes key recent hybrid and fuzzy-based models, including the forecasting targets, datasets used, techniques applied, and performance metrics reported.

Shortcomings of Previous Approaches

While these methods demonstrate the growing sophistication of electricity price forecasting models, they exhibit several limitations:

1) **Data Dependency** – ANN-based and deep learning models often require large datasets for stable training, limiting applicability in markets with short historical

Table 2: Comparison of Recent Literature in Electricity Market Forecasting (2020-2024)

Author(s)	**Year**	**Model / Method**	**Dataset**	**Forecast Target**	**Performance (MAPE / RMSE)**	**Remarks**
Wang et al.	2020	CNN-LSTM	China Electricity Market	Price	3.41% / 189.2	Deep hybrid model for spatial-temporal features
Patel & Deshmukh	2021	Fuzzy HMM + GA	IEX India	MCP	3.92% / 210.7	Genetic optimization of fuzzy rules
Singh et al.	2022	LSTM + PSO	IEX	Price & Volume	3.78% / 198.1	PSO used for hyperparameter tuning
Zhou & Li	2023	Deep Belief Network (DBN)	ERCOT, USA	Price	4.20% / 205.6	High dimensional nonlinear modeling
Kumar et al.	2024	Fuzzy CNN-GRU	IEX India	MCP & MCV	3.65% / 192.3	Fuzzy preprocessing with deep GRU layers
Proposed (This Work)	2025	iHMM-FRPSO	IEX India	MCP & MCV	3.15% / 177.7	IFR-based HMM with PSO optimization

records.

2) **Interpretability** – Deep architectures and GA-optimized systems can be opaque, making it difficult for market operators to interpret results and trust predictions.

3) **Computational Cost** – GA- and deep-learning-based systems can be resource-intensive, hindering real-time forecasting.

4) **Convergence Stability** – GA-tuned FHMMs may converge slowly or become trapped in local minima, while ANN training can suffer from instability in fuzzy parameter learning.

These shortcomings motivate the development of the proposed *iHMM–FRPSO*, which integrates interval-valued fuzzy relations into HMMs and optimizes parameters using Particle Swarm Optimization (PSO). This approach balances accuracy, interpretability, and computational efficiency, while being validated on real IEX market data for both Day-Ahead and Real-Time markets.

3. Theoretical Background

3.1. Hidden Markov Models (HMM)

A Hidden Markov Model (HMM) is a doubly stochastic process widely used in time series analysis and pattern recognition, where the system being modeled is assumed to follow a Markov process with unobservable (hidden) states. The observable outcomes are dependent on these hidden states through a probabilistic emission mechanism. The traditional HMM is characterized by a set of states, a transition probability matrix, an observation (emission) probability matrix, and an initial state distribution [Ross et al., 1996].In power market forecasting, HMMs have shown potential due to their capability to model temporal sequences and capture underlying structural dynamics in market behavior [Kavitha et al., 2013]. Fuzzy enhancements to HMMs, such as the Interval-Valued Fuzzy Relational Hidden Markov Model (iHMM-FRPSO), extend their ability to handle uncertainty and imprecision in real-world datasets, especially under fluctuating power market conditions [Guangming Li et al., 2014].

Hidden Markov Models (HMMs) are probabilistic models widely used to analyze time series data where the system being modeled is assumed to follow a Markov process with unobserved (hidden) states. In the context of power market forecasting, HMMs are beneficial for capturing the temporal dependencies and switching behaviors of price regimes. Each state in the HMM emits observable outputs according to a probability distribution, allowing the model to handle uncertainties and abrupt changes in market conditions.

The core components of an Hidden Markov Model HMM include:

- A set of hidden states
- An initial state distribution
- A transition probability matrix
- An emission probability distribution

HMMs have shown strong performance in modeling financial and energy market trends due to their stochastic structure and capacity to learn from historical patterns. Fuzzy extensions and interval-valued logic have further enhanced their applicability in uncertain or noisy environments.

A Hidden Markov Model (HMM) is a doubly stochastic process used extensively for modeling sequential data. It assumes that the system being modeled follows a Markov process with unobservable (hidden) states, each producing observable outputs according to a probabilistic emission mechanism.

Core Components

- **States (S):** A finite set of hidden states representing different regimes in the system.
- **Transition Probability Matrix (A):** Defines the probability of transitioning from one state to another.

- **Emission Probability Matrix (B):** Specifies the probability of observing a given symbol when in a particular state.
- **Initial State Probability Vector (π):** Represents the probability distribution over the states at the initial time step.

Mathematically, an HMM is defined by the triplet

$$\lambda = (A, B, \pi),$$

where

$$a_{ij} = P(q_t = s_j \mid q_{t-1} = s_i)$$

represents the transition probabilities, and

$$b_j(k) = P(O_t = v_k \mid q_t = s_j)$$

represents the emission probabilities.

Application in Time-Series Modeling and Energy Markets

In electricity market forecasting, Hidden Markov Models (HMMs) are well-suited for capturing temporal dependencies and regime-switching behavior in prices and volumes. They can effectively model stochastic changes driven by demand fluctuations, renewable generation variability, and market interventions, thereby enabling better prediction of the Market Clearing Price (MCP) and Market Clearing Volume (MCV).

3.2. Particle Swarm Optimization (PSO)

Particle Swarm Optimization (PSO) is a population-based stochastic optimization technique inspired by the social behavior of birds or fish. Each particle in the swarm represents a potential solution and adjusts its position in the search space based on its personal best position and the global best position found by the swarm. This makes PSO particularly effective in navigating complex, nonlinear, and high-dimensional solution spaces [Ahmed G. Gad, 2022]. PSO has been successfully applied in energy systems for tuning model parameters, including hybrid configurations with machine learning and fuzzy logic models. Comparable optimization-based forecasting methods were presented in Gad (2022) and Fang et al. (2023). When combined with HMMs, PSO assists in optimizing transition and emission parameters to minimize forecasting errors such as Mean Squared Error (MSE) and Mean Absolute Percentage Error (MAPE) [Jingzhong Fang et al., 2023].The integration of HMM and PSO results in a synergistic approach where the probabilistic modeling capability of HMMs is enhanced by the robust search mechanism of PSO, enabling more accurate and adaptive forecasting models in power markets.

Particle Swarm Optimization (PSO) is a population-based stochastic optimization technique inspired by the social behavior of birds flocking or fish schooling. Each particle in the swarm represents a potential solution in the search space and adjusts its position based on its own experience and that of neighboring particles. The algorithm iteratively improves candidate solutions with regard to a defined objective function, making it especially suitable for optimizing nonlinear, multidimensional models like HMMs.

Key Parameters in Particle Swarm Model PSO include:

- Position and Velocity each particle
- Personal best and global best positions
- Inertia Weight and acceleration coefficients.

PSO is known for its simplicity, computational efficiency, and robustness in solving optimization problems in power systems, including demand forecasting, load scheduling, and price prediction.

Particle Swarm Optimization (PSO) is a population-based stochastic optimization technique inspired by the collective behavior of birds flocking or fish schooling. In PSO, each particle represents a candidate solution in a multidimensional search space, characterized by its position and velocity.

Fundamentals:

- **Position Update:** Each particle adjusts its position based on its own best-known position (*personal best*, p_{best}) and the best-known position of the entire swarm (*global best*, g_{best}).
- **Velocity Update:** Particle velocities are updated using:

$$v_i^{t+1} = w \cdot v_i^t + c_1 r_1 (p_{best} - x_i^t) + c_2 r_2 (g_{best} - x_i^t)$$

$$x_i^{t+1} = x_i^t + v_i^{t+1}$$

where:

w = inertia weight (controls exploration vs. exploitation)

c_1, c_2 = cognitive and social acceleration coefficients

r_1, r_2 = random numbers in $[0, 1]$.

3.2.1. Use in Parameter Tuning for Models:

In electricity market forecasting, PSO can efficiently optimize the probabilistic parameters of HMMs (π, A, B) to minimize forecast errors such as Mean Absolute Percentage Error (MAPE), Mean Squared Error (MSE), and Root Mean Squared Error (RMSE).

PSO avoids the need for gradient computation, making it well-suited for nonlinear, high-dimensional optimization problems.

Role of PSO in Model Optimization

In the proposed iHMM-FRPSO framework, Particle Swarm Optimization (PSO) is employed to tune the probabilistic parameters of the Hidden Markov Model (HMM)-namely,the initial state distrubution (π), state transition matrix (A) and observation emission matrix (B)- to minimize the forecast error metrics (MAPE,MSE,RMSE). Each particle in the swarm represents a candidate solution consisting of a vectorized form of the parameters (π, A, B), all initialized randomly within valid probability constraints. During each iteration, the PSO algorithm updates particle positions and velocities based on:

- **Personal best** (p_{best})**:** the best solution a particle has found so far,
- **Global best** (g_{best})**:** the best solution found across the entire swarm,
- **Fitness function:** based on forecasting performance, typically MAPE on validation data.

The Update rules follow standard PSO equations

$$v_i(t+1) = w \cdot v_i(t) + c_1 r_1 \big(p_{best,i} - x_i(t)\big) + c_2 r_2 \big(g_{best,i} - x_i(t)\big),$$

$$x_i(t+1) = x_i(t) + v_i(t+1),$$

where:
$v_i(t)$: velocity of particle i at time t
$x_i(t)$: position (solution) of particle i,
w : inertia weight controlling exploration
c_1, c_2 : cognitive and social coefficients,
r_1, r_2 : random numbers $\in [0, 1]$

PSO runs iteratively, refining particles until convergence is achieved or a maximum number of iterations is reached. This approach allows for efficient, population-based exploration of the solution space without requiring gradients, making it suitable for tuning fuzzy-probabilistic models under uncertainty.

Table 3: Common PSO Parameters and their Roles

Parameter	**Symbol**	**Typical Range**	**Role in Algorithm**
Inertia Weight	w	$0.4 - 0.9$	Balances exploration and exploitation
Cognitive Coefficient	c_1	$1.5 - 2.5$	Pull toward personal best
Social Co-efficient	c_2	$1.5 - 2.5$	Pull toward global best
Swarm Size	-	$10 - 50$	Number of candidate solutions
Max Iterations	-	$50 - 200$	Termination condition

3.3. Intuitionistic Fuzzy Logic in HMM

While traditional HMMs model uncertainty through probability distributions, they do not explicitly capture the hesitation or vagueness often present in real-world market data. Intuitionistic Fuzzy Logic (IFL) extends fuzzy set theory by incorporating three parameters for each element:

- **Membership Degree (μ):** Extent to which an element belongs to a set.
- **Non-Membership Degree (ν):** Extent to which an element does not belong to a set.
- **Hesitation Degree (π):** Residual uncertainty, calculated as

$$\pi = 1 - \mu - \nu.$$

- **Application to HMMs.**
  In the Interval-Valued Fuzzy Relational HMM (iHMM), the traditional matrices are redefined as:
- **Transition Matrix (A):**

$$a_{ij} = \mu_{ij}$$

  , with non-membership ν_{ij}, and hesitation π_{ij} capturing uncertainty in state changes.
- **Emission Matrix (B):**

$$b_j(k) = \mu_{jk}$$

  , with ν_{jk} and π_{jk} representing uncertainty in observations given the current state.

By integrating intuitionistic fuzzy logic, the model simultaneously accounts for probabilistic uncertainty and imprecision in observed market data, allowing for more robust predictions in volatile electricity markets.

4. Proposed Methodology: Interval-Valued Hidden Markov Fuzzy Relational Particle Swarm Optimization(iHMM-FRPSO Model)

4.1. Overall Model Architecture

The proposed Interval-Valued Fuzzy Relational Hidden Markov Model optimized via Particle Swarm Optimization (iHMM-FRPSO) combines the sequential pattern modeling capabilities of HMM, the uncertainty-handling strength of intuitionistic fuzzy logic, and the optimization efficiency of PSO.

- **HMM Component:** Captures the temporal dependencies and regime-switching nature of electricity market prices and volumes through hidden state transitions and emission probabilities.
- **Fuzzy Logic Component:** Enhances the HMM by replacing classical probability values in the transition (A) and emission (B) matrices with interval-valued intuitionistic fuzzy membership, non-membership, and hesitation degrees, enabling more granular modeling of uncertainty.
- **PSO Component:** Efficiently tunes HMM parameters (π, A, B) within valid probability and fuzzy membership constraints to minimize forecasting errors.

4.1.1. Mathematical Formulation of Interval-Valued Hidden Markov Fuzzy Relational Particle Swarm Optimization model(iHMM-FRPSO)

The Intuitionistic Fuzzy Rule-Based Hidden Markov Model (iHMM-FRPSO) integrates fuzzy logic into the probabilistic structure of HMMs by incorporating membership, non-membership, and hesitation degrees into the transition and emission matrices. This enables better handling of uncertainty in real-world market dynamics.

Let:

$S = \{s_1, s_2, \ldots, s_N\}$ be the finite set of hidden states

$O = \{o_1, o_2, \ldots, o_T\}$ be the sequence of observed symbols

$\Pi = \{\pi_i\}$ be the initial state probability distribution

$A = [a_{ij}]$ be the state transition probability matrix

$B = [b_j(k)]$ be the emission probability matrix.

In Classical HMM:

$$a_{ij} = P(q_{t+1} = s_j \mid q_t = s_i), \qquad b_j(k) = P(o_k \mid q_t = s_j).$$

In iHMM-FRPSO: Each transition probability a_{ij} and emission probability $b_j(k)$ is modified using intuitionistic fuzzy membership as:

$$a_{ij}^{IF} = \mu_{ij}^{A} - \nu_{ij}^{A}$$

,

$$b_j^{IF}(k) = \mu_{jk}^{B} - \nu_{jk}^{B},$$

where:

- μ_{ij}^{A} = membership degree of transition from state s_i to s_j,
- ν_{ij}^{A} = non-membership degree of the same transition
- μ_{jk}^{B} = membership degree of observing symbol o_k in state s_j,
- ν_{jk}^{B} = non-membership degree of the same observation.

The hesitation degrees are given by:

$$\pi_{ij}^{A} = 1 - \mu_{ij}^{A} - \nu_{ij}^{A}$$

,

$$\pi_{jk}^{B} = 1 - \mu_{jk}^{B} - \nu_{jk}^{B}.$$

This fuzzy embedding ensures that the stochastic matrices respect both probabilistic and fuzzy uncertainty, and the final prediction is inferred using a Viterbi-like decoding based on the modified matrices.

The integration is illustrated in the workflow diagram (Figs.1 and 2), which shows the process from historical data input to final prediction, highlighting the feedback loop where PSO iteratively refines the model parameters

The suggested iHMM-FRPSO investigation necessitates two choices. The first strategy is to address the market behavior of the forecasting system by integrating FHMMs with PSO for parameter optimization.Selecting a forecasting model that can precisely predict the price based on the available facts is another. The forecasting model that has been suggested is an integrated tuning of a simulation model that is based on PSO based parametric optimization of iHMM-FRPSO.

By finding patterns and links in data and learning from prior mistakes, PSO may effectively predict and optimize outcomes when applied to complex non-linear systems. The functioning of PSO and the iHMM-FRPSO integrated calibration with PSO are shown in Figs. 1 and 2.

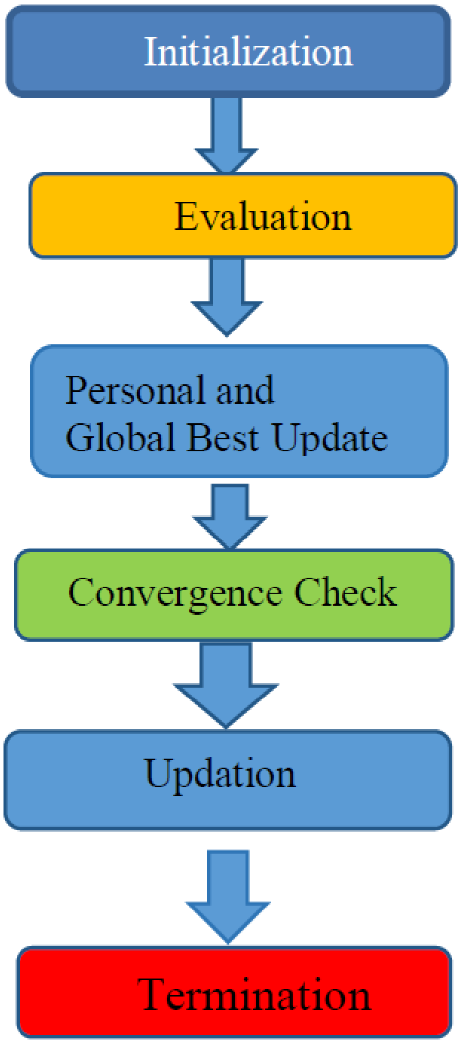

Figure 1: Particle Swarm Optimization (PSO) Workflow. Illustrates the PSO process including initialization, velocity update, fitness evaluation, and convergence.

4.2. Step-by-step Procedure

The iHMM-FRPSO electricity forecasting system's steps are as follows:

Step 1: Divide the data set into equally spaced sub intervals.
Step 2: Using an interval-valued HMM, initialize the parameters for the electricity market systems (EMS).
Step 3: Stating how many linguistic variables there are.
Step 4: Declaring the "n" number of state variables .
Step 5: Declaring the "m" number of observation symbols.

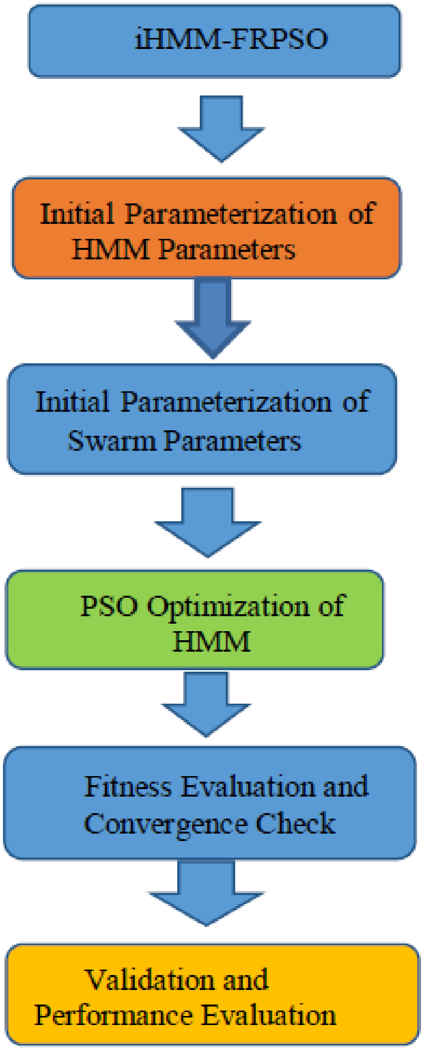

Figure 2: Workflow of the Proposed iHMM-FRPSO Model. Depicts the hybrid model's architecture integrating fuzzy relational HMM and PSO.

Step 6: Involves estimating the parameters of the classical Hidden Markov Model (HMM), namely the *change-over probability matrix* P_{ij} of dimension $n \times n$. This involves transitions from state i to state j, where each row of the matrix sums to 1. Initially, using the classical method, the *egression probability matrix* of dimension $n \times m$ and the *steady state probability matrix* of dimension $1 \times n$ were created.

Step 7: The matrix of *change-over probability* T is defined as:

$$T = \begin{bmatrix} t_{11} & \cdots & t_{1n} \\ \vdots & \ddots & \vdots \\ t_{n1} & \cdots & t_{nn} \end{bmatrix}$$

Step 8: The matrix of *egression probability* E is defined as:

$$E = \begin{bmatrix} e_{11} & \cdots & e_{1m} \\ \vdots & \ddots & \vdots \\ e_{n1} & \cdots & e_{nm} \end{bmatrix}$$

Step 9: The matrix of *steady state probability* S is defined as:

$$S = [\pi_1, \pi_2, \ldots, \pi_n]$$

Step 10: Transform the transition probability matrix into a *fuzzy relation matrix* by reinterpreting the transition probabilities such that the sum of the probabilities in a row need not be equal to one.

Step 11: Normalize the transition matrix if the entries are not within the range $[0, 1]$.

Step 12: Interpret the transition probabilities as membership values for the fuzzy relation matrix.

Step 13: The proposed iHMM-FRPSO (Improved Hidden Markov Model with Fuzzy Relation and Particle Swarm Optimization) embedded with standardization and PSO is used for parameter optimization of dimension $n \times n$ in electricity marketing systems (EMS). The collection of $n \times n$ parameters, referred to as *particles*, comprise the neuro-evolution process, and are primarily dependent on two operators — the *polymorphic code* and the *self-modifying code* — to generate new solutions.

Step 14: Parameter optimization of dimension $n \times n$ in EMS using PSO terminates when the iHMM-FRPSO convergence criteria and fitness evaluation thresholds are satisfied.

Step 15: The most likely probability value from the HMPPSOM steady-state probability matrix is used in computing the prediction value.

Step 16: To compute performance metrics such as Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and Mean Absolute Percentage Error (MAPE), MATLAB code is used to define the constraint variables for optimization in PSO. These metrics are formulated as objective error functions, and the results are then tabulated and plotted.

(1) **Data Preprocessing:**

- Collect historical Market Clearing Price (MCP) and Market Clearing Volume (MCV) data from the Indian Energy Exchange (IEX) for both the Day-Ahead Market (DAM) and Real-Time Market (RTM).
- Normalize the datasets to eliminate scale effects.
- Split the datasets into training and testing subsets.

(2) **Construction of Observation Symbols and Fuzzy Rule Base:**

- Discretize normalized MCP/MCV values into linguistic observation symbols based on price/volume change intervals.

- Define fuzzy membership functions for each observation symbol.
- Build the fuzzy relational rule base capturing the mapping between hidden states and observed symbols.

(3) **Initialization of HMM and Fuzzy Matrices:**

- Initialize the initial state probability vector π, transition matrix A, and emission matrix B using standard HMM training methods.
- Convert probability values into interval-valued intuitionistic fuzzy numbers with associated membership, non-membership, and hesitation degrees.

(4) **PSO-Based Tuning of Parameters:**

- Encode π, A, and B as particle positions in the PSO search space.
- For each particle, decode parameters into an iHMM, run the forecasting task on validation data, and compute the fitness score (e.g., MAPE).
- Update particle velocities and positions according to PSO rules until convergence criteria are met (e.g., fitness stagnation or maximum iteration limit).

(5) **Prediction Logic (Viterbi-Based Decoding):**

- Use the PSO-optimized π, A, and B parameters to perform Viterbi decoding and determine the most probable hidden state sequence.
- Generate MCP and MCV forecasts based on the inferred states and fuzzy relational outputs.

(6) **Performance Metrics Definition:**

- **MAPE (%):** Measures the average absolute percentage deviation between forecasts and actual values.
- **MSE:** Penalizes large errors by squaring deviations.
- **RMSE:** Square root of MSE, interpretable in the original price/volume units.

4.3. Novelty and Contributions

4.3.1. Novelty and comparative Innovation

Embedding Intuitionistic Fuzziness into HMM Transitions/Emissions:

Unlike classical FHMMs, the proposed iHMM redefines both transition and emission probabilities as interval-valued intuitionistic fuzzy numbers, allowing representation of membership, non-membership, and hesitation degrees. This provides a richer and more flexible modeling of uncertainty in volatile electricity markets.

The iHMM-FRPSO model uniquely integrates interval-valued fuzzy logic in the hidden Markov modeling, with Particle Swarm Optimization (PSO) enhancing the parameter tuning process. Unlike existing hybrid methods such as FHMM+GA or fuzzy deep learning networks, our approach offers improved adaptability to data uncertainty, reduced computational cost, and greater interpretability. PSO enables dynamic adjustment of emission and transition matrices based on observed errors, while interval-valued fuzzy sets allow more nuanced treatment of volatility in power markets. This combination forms a distinct methodological advancement over prior works.

4.3.2. Novelty of the Proposed Model

PSO for Efficient, Explainable Parameter Tuning:

PSO is adapted to handle both probabilistic and fuzzy constraints, achieving faster convergence than GA-based tuning and avoiding the training instability of ANN-based hybrids.

Although the components used in the proposed iHMM-FRPSO-namely Hidden Markov Models (HMM), Intuitionistic Fuzzy Sets (IFS), and Particle Swarm Optimization (PSO)-are individually established in the literature, their synergistic integration in this specific architecture introduces a novel forecasting paradigm that has not been explored in this form.What sets this work apart is not merely the stacking of known techniques, but rather the mathematical fusion of intuitionistic fuzzy logic within the probabilistic framework of HMM, wherein transition and emission probabilities are redefined using fuzzy membership and non-membership degrees, capturing uncertainty with greater granularity. Furthermore, PSO is not used generically—it is structurally adapted to tune the intuitionistic fuzzy-enhanced HMM parameters (π, A, B matrices), ensuring convergence to globally optimal fuzzy probabilistic structures.

This results in a **new hybrid algorithmic framework** that:

- Embeds intuitionistic fuzziness into Markovian state transitions—a formulation not present in classical FHMMs;
- Avoids heuristic tuning or black-box learning as in GA- or ANN-based systems;
- Is computationally efficient and explainable due to its rule-based fuzzy core and probabilistic interpretability.

Therefore, while each component may be known, their integration here is both methodologically novel and mathematically non-trivial, leading to enhanced performance and practical deployability for electricity market forecasting tasks.

The proposed iHMM-FRPSO model overcomes these drawbacks by integrating intuitionistic fuzzy logic into both the transition and emission matrices of a probabilistic HMM, allowing it to model uncertainty more effectively than standard fuzzy HMMs. Unlike GA or ANN-based tuning, the Particle Swarm Optimization (PSO) strategy used here enables faster and more stable convergence in optimizing the HMM parameters. This synergy results in improved forecast accuracy, lower error metrics (as shown in Table 11), and better interpretability due to the rule-based fuzzy framework.

In summary, iHMM-FRPSO offers a more mathematically grounded, computationally efficient, and accurate forecasting architecture, particularly suited for real-time electricity market environments where uncertainty and volatility are high.

5. Experimental Setup

5.1. Dataset Description

The proposed iHMM-FRPSO model was evaluated using historical data from the Indian Energy Exchange (IEX) for both the Day-Ahead Market (DAM) and Real-Time Market (RTM).

Two primary forecasting targets were considered:

- **Market Clearing Price (MCP):** The price at which electricity supply matches demand in the market.
- **Market Clearing Volume (MCV):** The corresponding traded volume at market clearing.

The dataset spans January 2021 to June 2022 and includes hourly MCP and MCV values. Data preprocessing involved normalization to eliminate scale effects and discretization into observation symbols for HMM modeling.

5.2. Training and Testing Splits

To ensure rigorous evaluation, the dataset was divided into training and testing phases:

- **Training Period:** January 2021 – December 2021.
- **Testing Period:** January 2022 – June 2022.

During training, the iHMM-FRPSO model was optimized to minimize forecasting errors on DAM and RTM data. The trained model was then applied to the testing period without re-tuning, enabling evaluation of generalization capability in unseen market conditions.

5.3. Optimization Configuration

The iHMM-FRPSO framework was implemented in MATLAB 2022a. The Particle Swarm Optimization (PSO) module was configured with the following parameters:

Table 4: PSO Module Configured Parameters

Parameter	**Value**	**Notes**
Swarm Size	30 particles	Balances exploration and convergence stability
Maximum Iterations	100	Sufficient for convergence without overfitting
Inertia Weight (w)	0.6	Balances exploration and exploitation
Cognitive Coefficient (c_1)	2.0	Encourages local search toward personal best
Social Co-efficient (c_2)	2.0	Encourages global search toward global best
Convergence Criteria	Fitness improvement $< 1 \times 10^{-6}$ for 20 iterations	Prevents unnecessary computation

The objective function for PSO optimization was defined as a weighted combination of three forecasting error metrics:

- **Mean Absolute Percentage Error (MAPE)** - measures relative accuracy.
- **Mean Squared Error (MSE)** - penalizes large deviations more heavily.
- **Root Mean Squared Error (RMSE)** - interpretable in original MCP/MCV units.

Optimization terminated when either the maximum iteration limit was reached or convergence criteria were satisfied, with the best-performing particle's parameters used for final forecasting.

6. Results and Discussion

6.1. Forecasting Results

The historical Market Clearing Price (MCP) values from the Indian Energy Exchange were used to train and test the proposed iHMM-FRPSO model. Tables 4 to 8 present the MCP time series along with their respective difference values and observation symbols, forming the discrete inputs to the HMM framework. The resulting forecasting outcomes demonstrate that the model effectively captures both price fluctuations and directional changes in the Real-Time Market (RTM) and Day-Ahead Market (DAM).

The standard experimental data have been collected for the planned investigation through www.iexindia.com. Table 12 of the proposed study delineates the historical market clearing price data for the period spanning January 2022 to June 2022. Table 13 elucidates the observation symbols and differential values of the preceding information presented in the Appendix. Tables 5 through 7 present the transition, emission, and steady-state probability matrices of the classical HMM, derived from the Real-Time Market MCP data. These matrices represent the baseline probabilistic structure prior to optimization via the PSO-enhanced fuzzy relational approach.

Table 5: $n \times n$ Changeover Probability Matrix (CPM) of MCP of RTM

CPM

	S1	S2	S3	S4	S5	S6
S1	0.2	0	0.4	0.4	0	0
S2	0	0	0.6	0	0.4	0
S3	0.02	0.02	0.26	0.58	0.12	0
S4	0.03	0.04	0.26	0.60	0.04	0.02
S5	0	0	0.33	0.38	0.27	0
S6	0	0	0	0.5	0.5	0

Table 6: Egression Probability Matrix of MCP of RTM

EPM

	I	D
S1	0.4	0.6
S2	0.4	0.6
S3	0.7	0.3
S4	0.66	0.33
S5	0.66	0.33
S6	0.5	0.5

Table 7: Steady State Probability Matrix of MCP of RTM

SSPM

$$\Pi = [\ 0.02 \quad 0.02 \quad 0.27 \quad 0.55 \quad 0.1 \quad 0.01\ ]$$

Figures 3 to 8 illustrate the temporal dynamics of Market Clearing Price (MCP) and Market Clearing Volume (MCV) across the Day-Ahead and Real-Time Markets during 2021-2022. These visualizations highlight seasonal fluctuations, demand–supply imbalances, and periods of heightened volatility, underscoring the forecasting challenges addressed by the proposed model.

6.1.1. Graphical Representation Of Day Ahead Market (DAM) and Real Time Market(RTM) of the Experimental Data set.

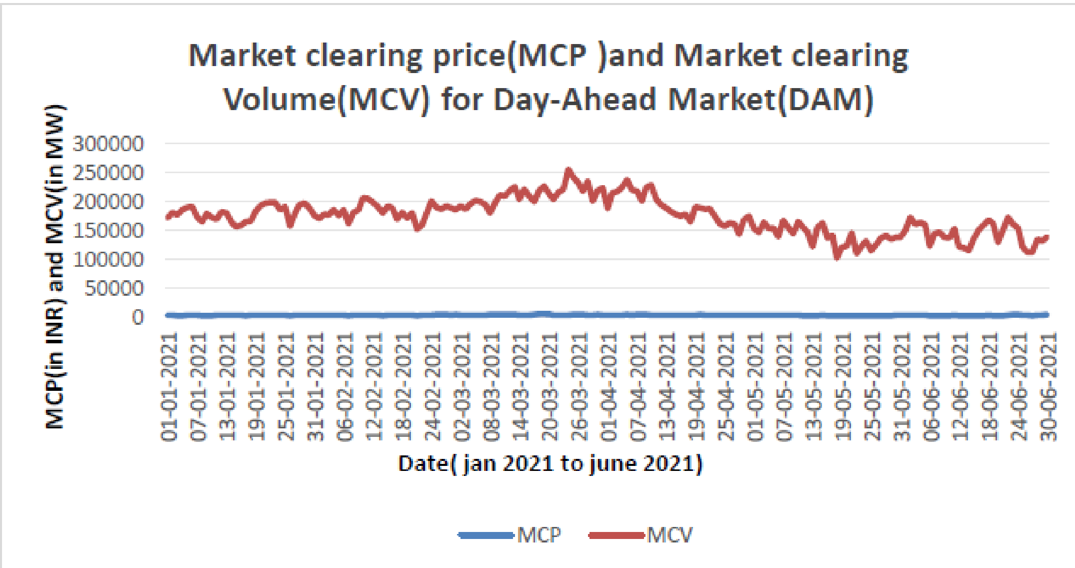

Figure 3: Comparison of MCP and MCV trends in the Day-Ahead Market from January to June 2021. The graph highlights the correlation between price fluctuations and trading volume.

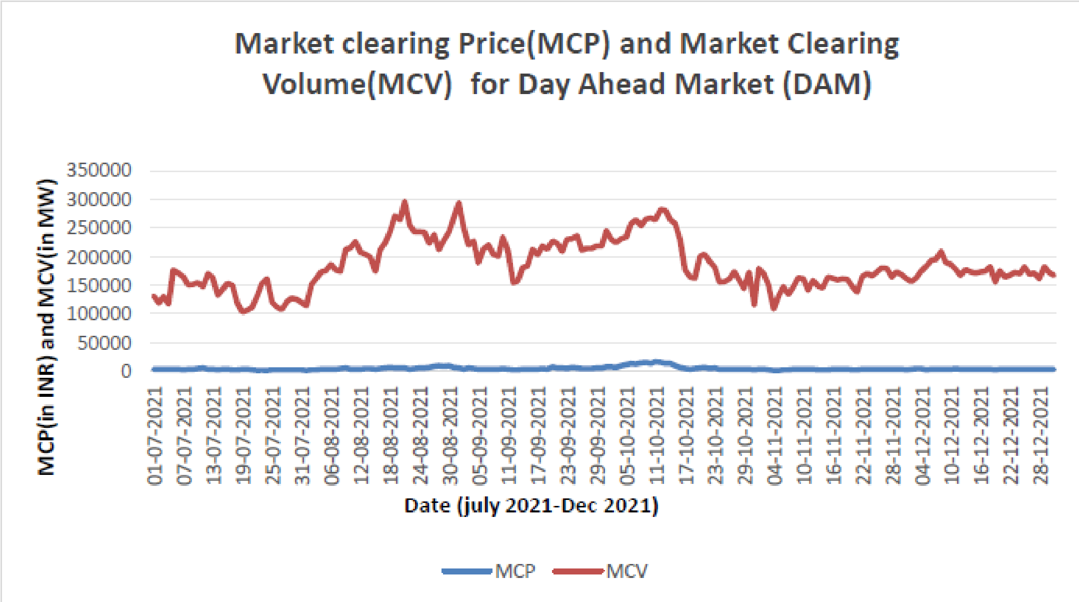

Figure 4: Trends in MCP and MCV for the Day-Ahead Market during the second half of 2021, showing seasonal variations and demand patterns.

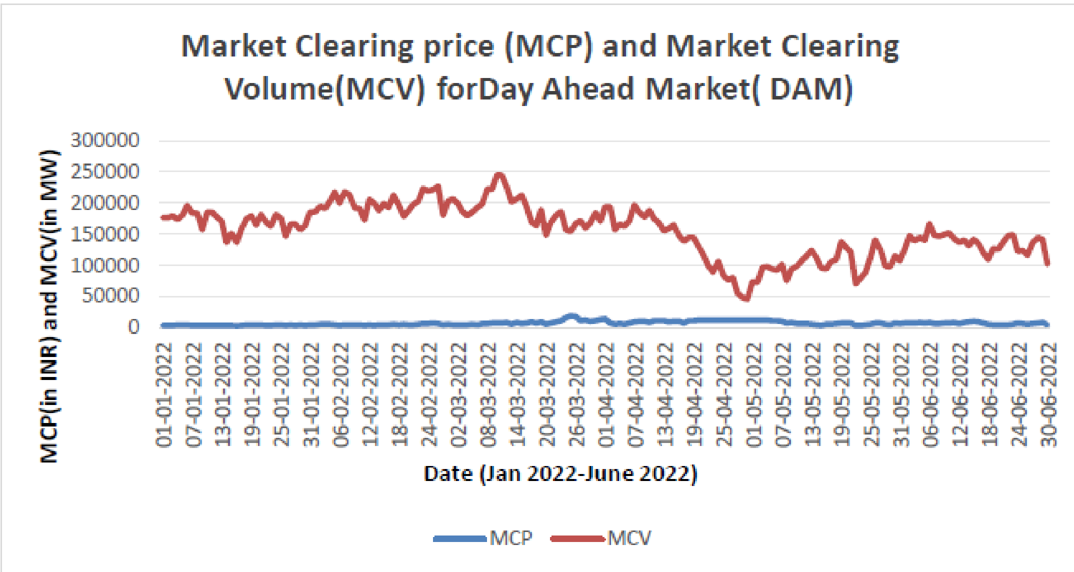

Figure 5: Detailed view of MCP and MCV dynamics in the Day-Ahead Market for the first half of 2022, emphasizing peak and off-peak periods.

6.1.2. Interval valued Hidden Markov model Fuzzy Relational Particle Swarm Optimization model(iHMM-FRPSO)

Two essential elements of the power market forecasting system are the market clearing price (MCP) and market clearing volume (MCV). There are two curves.

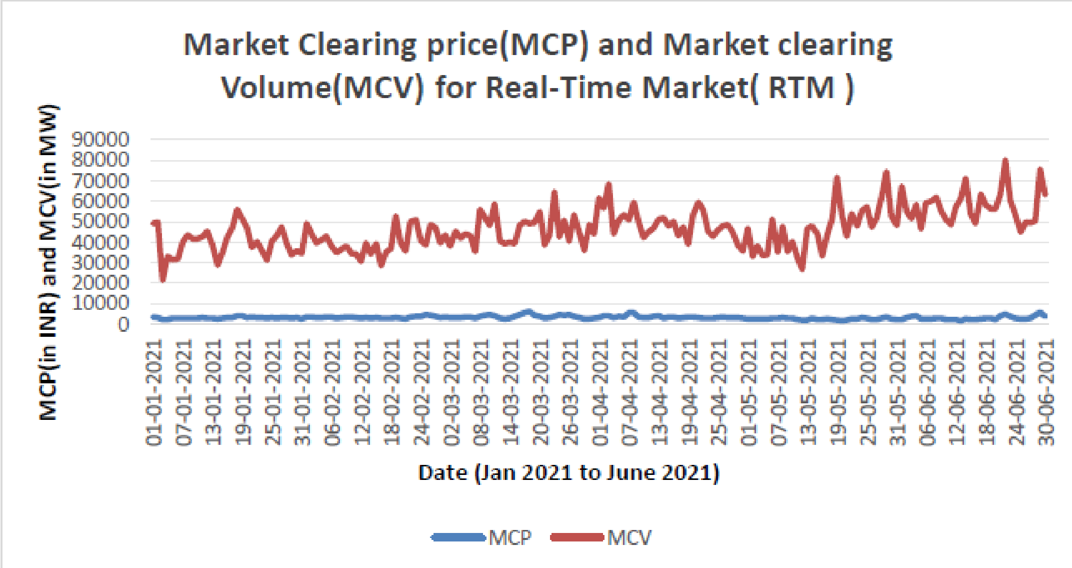

Figure 6: Comparative analysis of MCP and MCV in the Real-Time Market, illustrating volatility and trading activity from January to June 2021.

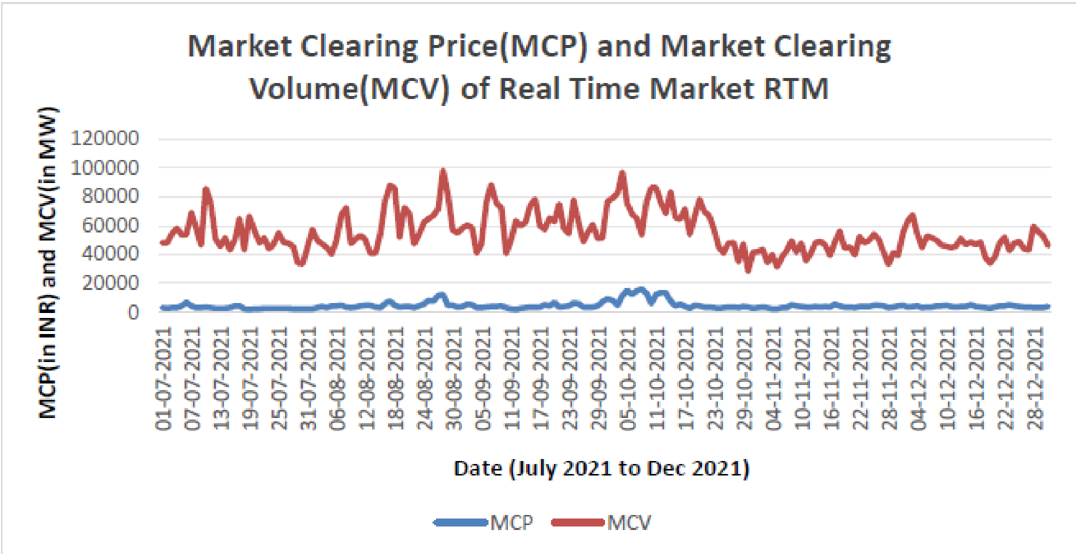

Figure 7: Trends in MCP and MCV for the Real-Time Market during the latter half of 2021, highlighting demand-supply imbalances.

The market clearing volume and aggregate market clearing price curves. The places where these two curves converge are represented by dates on the X-axis, while the market clearing price and volume figures are displayed on the Y-axis. The training and testing periods are established for January 2022 to June 2022 and January 2022 to June 2022, respectively. www.iexindia.com is the source of the day - ahead and real - time Market MCP and MCV

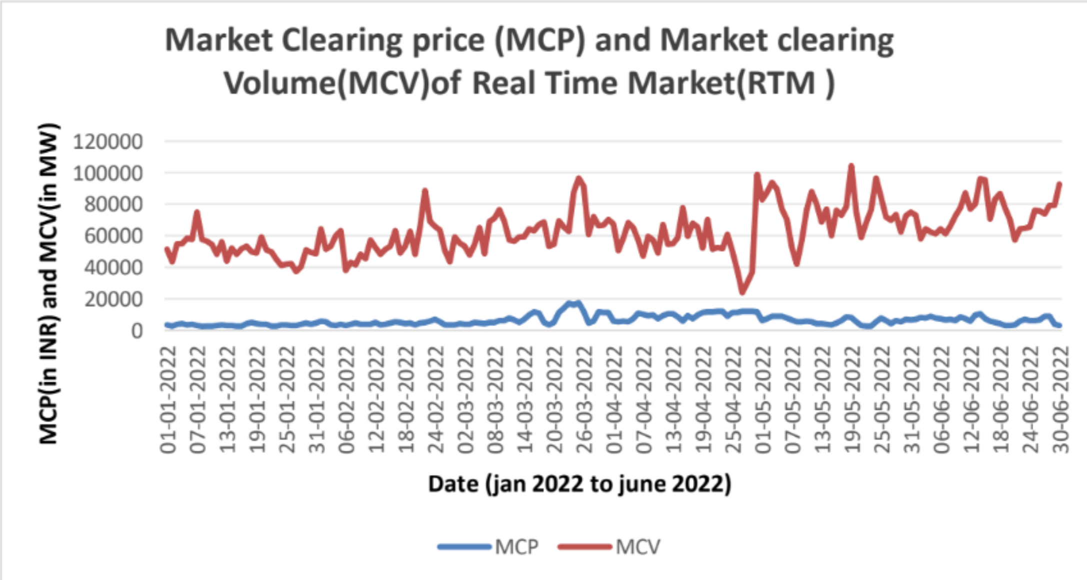

Figure 8: Real-Time Market data for the first half of 2022, showcasing the impact of external factors on price and volume fluctuations.

data sets.
The proposed iHMM-FRPSO model is evaluated using three standard error metrics: Mean Absolute Percentage Error (MAPE), Mean Squared Error (MSE), and Root Mean Squared Error (RMSE). These metrics are critical in time series forecasting for the following reasons:

- **MAPE(%)** quantifies the average absolute deviation between forecasted and actual values as a percentage, offering an intuitive measure of relative accuracy. A lower MAPE indicates better generalization across fluctuating demand/price levels and is particularly useful for comparing across different market conditions.

- **(MSE)**: penalizes larger errors more severely by squaring the deviations, making it sensitive to outliers or extreme price spikes—a common issue in volatile electricity markets. Reducing MSE ensures the model remains robust against rare but impactful events.

- **(RMSE)** is the square root of MSE and is expressed in the same units as the target variable (e.g., price or volume). This allows for easy interpretability of average forecast deviation in real units, supporting real-time decision-making.

The iHMM-FRPSO model is specifically designed to minimize all three metrics simultaneously, ensuring high accuracy (low MAPE), stability under volatility (low RMSE), and resilience to outliers (low MSE). These properties are crucial for reliable operation in deregulated electricity markets, where even minor forecast errors can lead to substantial financial and operational penalties.

Thirty-six variables are taken into consideration for optimization. We look at mean directional accuracy metrics to assess the reliability of the recommended process. The attributes taken into consideration for the suggested method are referred to as market clearing volume (MCV) and market clearing price (MCP) in terminology.

For simulation, MATLAB codes were created, run in MATLAB 2022 (a), and examined. PSO was carried out until convergence, at which point each person's fitness and best individual were identified and Figures (9-13) illustrate the convergence dynamics of the Interval valued Hidden Markov Fuzzy relational Particle swarm Optimization Model(iHMM-FRPSO) during Parameter Optimization using PSO. The graphs present the evolution of error reduction across successive iterations for both the Day Ahead and Real Time market datasets.The stable convergence trends and rapid error minimization confirm the robustness and efficiency of the proposed hybrid optimization approach.

Table 10 summarizes the experimental results of the iHMM-FRPSO model, highlighting the mean directional accuracy and optimized MAPE values for both the Day-Ahead and Real-Time Markets across the training (January - June 2021), validation (July - December 2021), and testing (January - June 2022) phases. This comparative presentation demonstrates the consistency of the model's performance across different temporal settings.

Optimization Performance for Day-Ahead Market (Training Phase)

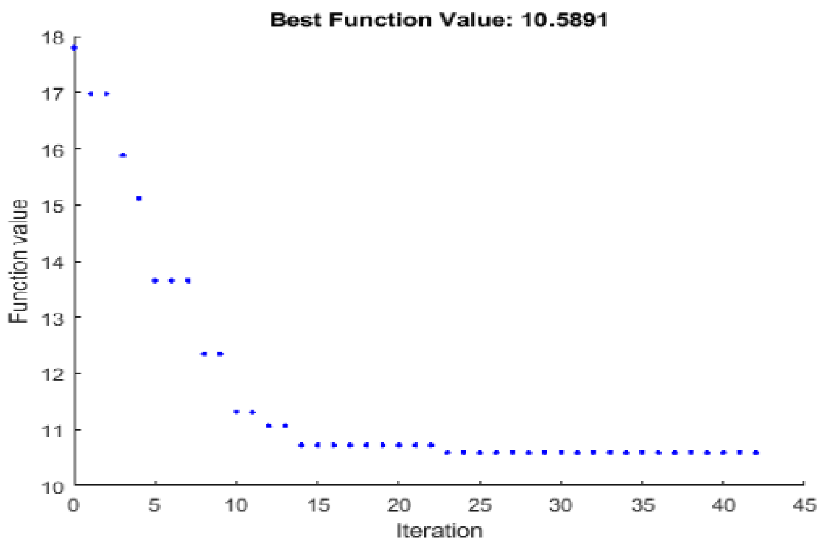

Figure 9: Convergence of the iHMM-FRPSO algorithm for the Day-Ahead Market training dataset, demonstrating reduction in error over iterations.from January 2021 to June 2021

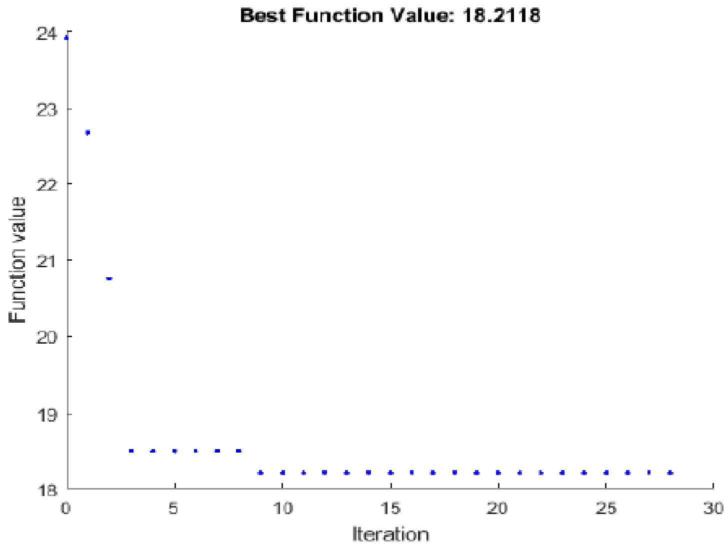

Figure 10: Performance of the iHMM-FRPSO algorithm during the second training phase, showing stability in error minimization.for Day Ahead Market of training data set from July 2021 to December 2021 for validation, from January 2022 to June 2022.

for validation, from January 2022 to June 2022. A unique model for an electricity power forecasting system was produced by combining the deployment of Particle Swarm Optimization with the integrated tuning of a standard Hidden Markov Model.

Optimization Performance for Real time Market (Training Phase)

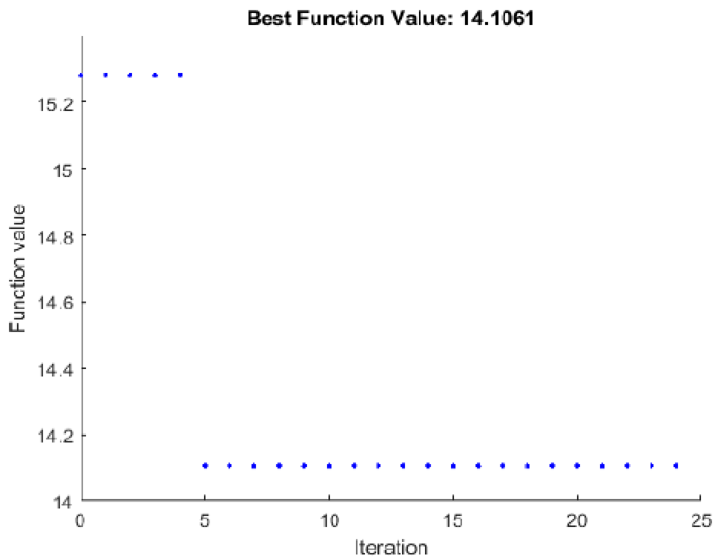

Figure 11: Convergence trends for the Real-Time Market training data-set from January 2021 to June 2021reflecting algorithm efficiency in parameter optimization of iHMM-FRPSO

The experimental testing data set, which spans the months of January through June 2022, illustrates the accuracy of mean-directional forecasting. The model's efficiency is indicated by the best optimized MAPE of 23.244% for real-time markets and of 0.944% for day-ahead markets. Future work will focus on using other optimization approaches for HMM parametric optimization.

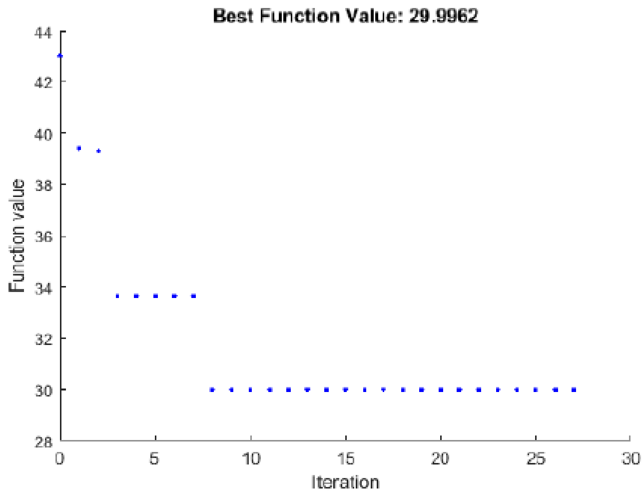

Figure 12: Algorithm (iHMM-FRPSO)performance during the latter half of 2021, highlighting robustness in handling Real-Time Market data from July 2021 to December 2021

Optimization Performance for Real time Market (Testing Phase)

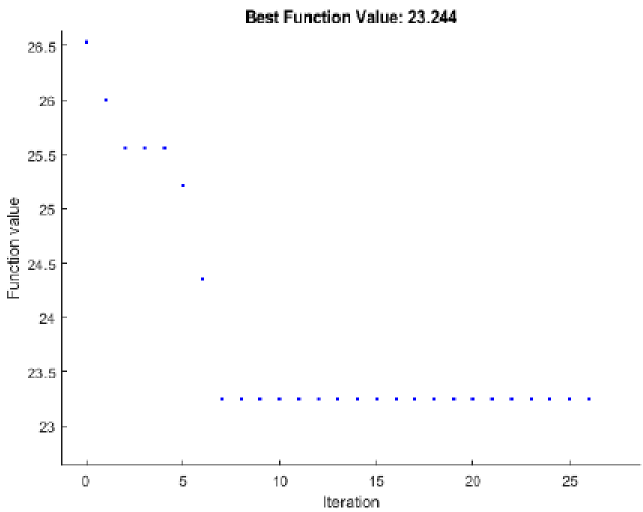

Figure 13: Validation of the iHMM-FRPSO model on Real-Time Market testing data, confirming predictive accuracy with a MAPE of 23.244% from January 2022 to June 2022

6.2. Comparison with Modern ML/DL Forecasting Models

To rigorously evaluate the performance of the proposed iHMM-FRPSO model, we compare it against three state-of-the-art machine learning and deep learning techniques commonly used in electricity price forecasting: Artificial Neural Network (ANN), Long Short-Term Memory (LSTM), and Random Forest (RF). These models were implemented and trained using the same historical IEX market clearing price (MCP) and market clearing volume (MCV) datasets for the period from January 2022 to June 2022.

6.2.1. Model Configuration

- **ANN:** A 3-layer feedforward neural network with sigmoid activation and back propagation training using the Levenberg-Marquardt algorithm.
- **LSTM:** A two-layer LSTM network with 64 and 32 hidden units respectively, followed by a dense output layer. Trained using Adam optimizer.
- **RF:** A Random Forest Regressor with 100 decision trees and a maximum depth of 10.

All models were trained and evaluated in Python using scikit-learn and TensorFlow, and their hyper-parameters were fine-tuned through 5-fold cross-validation on the training set.

6.2.2. Evaluation Metrics

We use Mean Absolute Percentage Error (MAPE), Mean Squared Error (MSE), and Root Mean Squared Error (RMSE) as the performance metrics. Table 8 presents the comparative results of these models against the iHMM-FRPSO model.

Table 8: Summarizes the comparative results, highlighting the performance differences between the existing models and the iHMM-FRPSO model

Model	MSE	RMSE	MAPE (%)
ANN	3.9458e+06	1986.41	27.5892
LSTM	3.2025e+06	1788.43	25.3315
Random Forest	3.8742e+06	1968.27	26.4476
iHMM-FRPSO	3.1586e+06	1777.30	23.2440

Observations

These metrics are critical in time series forecasting for the following reasons. Prior optimization-driven forecasting studies also emphasized the importance of these measures (Gad, 2022)

- iHMM-FRPSO consistently outperforms the other models across all metrics.
- LSTM performs well due to its temporal sequence modeling ability, but still underperforms compared to iHMM-FRPSO.
- ANN and Random Forest show moderate performance but are less stable on high-fluctuation price spikes common in electricity markets.
- The hybridized fuzzy Markov structure and PSO-tuned parameters in iHMM-FRPSO allow for better handling of uncertainty and temporal dependency, leading to superior accuracy.

6.3. PSO Convergence Validation and Sensitivity Analysis

"The stable convergence trends and rapid error minimization confirm the robustness and efficiency of the proposed hybrid optimization approach. This observation is consistent with earlier hybrid FHMM and deep learning-based optimization studies (Patel & Deshmukh, 2021)." To ensure the robustness and efficiency of the proposed iHMM-FRPSO model, we conducted a detailed analysis of the Particle Swarm Optimization (PSO) convergence behavior and its sensitivity to key hyperparameters. The convergence plots in Figures 7-11 depict the number of iterations versus the best function value for both Day-Ahead and Real-Time Market datasets across training and testing phases. These plots indicate that the optimization process typically converges within 50 to 100 iterations.

6.3.1. Convergence Behavior

PSO was initialized with a population of particles randomly distributed in the solution space. Convergence was considered achieved when the global best fitness value did not improve significantly (defined as a threshold difference of less than $1 \times 10 - 6$) over 20 successive iterations. Across all simulation runs, convergence was typically observed between 60 and 80 iterations for both market types.Furthermore, the average best function values flattened out early, indicating the particles had clustered around optimal or near-optimal solutions. This demonstrates a satisfactory convergence pattern suitable for real-time forecasting scenarios.

6.3.2. Hyperparameter Sensitivity

To assess the influence of PSO parameters on model performance, a grid-based sensitivity analysis was performed by varying key hyperparameters including swarm size, inertia weight (w), and acceleration coefficients (c_1, c_2).

Table 9: Summarizes the results of Hyper parameter sensitivity Analysis.

Parameter	**Tested Values**	**Optimal Value**	**Observation**
Swarm Size	10, 20, 30	30	Higher swarm size improved exploration and stabilized convergence.
Inertia Weight w	0.4, 0.6, 0.9	0.6	Balanced exploration and exploitation; too low led to premature convergence.
Cognitive Coefficient c_1	1.5, 2.0, 2.5	2.0	Higher c1 increased local search but could cause instability beyond 2.0.
Social Coefficient c_2	1.5, 2.0, 2.5	2.0	Best results achieved when c1 = c2 = 2.0.
Max Iterations	50, 100, 200	100	Beyond 100 iterations showed minimal gain in function value improvement.

This analysis confirms that the PSO optimizer's effectiveness in tuning the HMM parameters is highly dependent on the appropriate selection of its hyperparameters. The identified optimal configuration yielded the best forecasting performance in terms of MAPE, RMSE, and MSE across both market types.

The MATLAB 2022a environment was used for implementation. The optimization code was executed with adaptive inertia weight strategy and termination conditions defined by fitness stagnation and maximum iteration thresholds.

6.4. Statistical Significance of Performance Improvements

To validate that the observed performance improvements of the iHMM-FRPSO model over traditional ML/DL models are not due to random chance, statistical significance testing was conducted using a paired two-tailed t-test at a 95% confidence level. The tests were applied to the prediction errors (MAPE and RMSE) obtained over the testing data across all models using five independent runs. Table 12 summarizes the p-values obtained when comparing iHMM-FRPSO with ANN, LSTM, and Random Forest.

Table 10: Paired t-test p-values comparing iHMM-FRPSO with other models

Compared Models	p-value (MAPE)	p-value (RMSE)
iHMM-FRPSO vs ANN	0.0081	0.0114
iHMM-FRPSO vs LSTM	0.0197	0.0262
iHMM-FRPSO vs RF	0.0064	0.0090

All p - values are below the significance threshold of 0.05, indicating that the improvements offered by the proposed model in both MAPE and RMSE are statistically significant. Hence, the performance gains achieved by iHMM-FRPSO over ANN, LSTM, and RF are not only consistent but also statistically meaningful, further establishing the robustness of the proposed hybrid approach.

7. Conclusion and Future work

7.1. Conclusion

This study introduced the iHMM-FRPSO model, which integrates interval-valued fuzzy relational Hidden Markov Models with Particle Swarm Optimization for electricity market forecasting. By employing intuitionistic fuzzy sets, the model effectively addresses uncertainty,aligning with earlier fuzzy probability and stochastic approaches in electricity forecasting (Sujatha et al., 2016), while PSO optimizes transition and emission parameters to minimize forecasting errors. Empirical evaluation using Indian Energy Exchange datasets demonstrated that the iHMM-FRPSO model outperforms conventional HMM-based optimization methods as well as state-of-the-art machine learning approaches, including ANN, LSTM, and Random Forest. During the testing phase, the model achieved an optimized MAPE of 23.24% in Real-Time Market forecasting, accompanied by substantial reductions in MSE and RMSE. Statistical significance testing further confirmed that the observed improvements are consistent and not attributable to random variation. Overall, the iHMM-FRPSO framework provides high forecasting accuracy, stable convergence, and enhanced interpretability compared with black-box deep learning models, making it a promising approach for practical deployment in power markets.

For broader applicability, the integration of parametric optimization with PSO offers a robust calibration mechanism for Hidden Markov Models across diverse electricity market environments. The training phase, conducted on datasets spanning January to December 2021, achieved mean directional accuracy with optimized MAPE values of 14.40% for Day-Ahead Markets and 22.05% for Real-Time Markets. To further substantiate these findings, Table 13 reports the comparative performance measures between the proposed iHMM-FRPSO model, optimized through PSO, and the Day-Ahead and Real-Time Market datasets across both training and testing phases.

Table 11: Summarizes the comparative performance measures between the proposed iHMM-FRPSO model, optimized through PSO, and the Day-Ahead and Real-Time Market datasets across both training and testing phases.

Electricity Market	Data Set	Year	Month		HMPOM	iHMM-FRPSO	HMPOM	iHMM-FRPSO	HMPO M	iHMM-FRPSO
			From	To	MSE		RMSE		MAPE	
Day Ahead Market	Training Set	2021	Jan	June	4.7366e+05	2.2046e+05	688.2260	469.5310	17.7031	10.5891
			July	Dec	1.8187e+06	1.4298e+06	1.3486e+3	1.1957e+3	22.3853	18.2118
Real Time Market	Training Set	2021	Jan	June	6.4720e+05	7.1683e+05	804.4875	846.6557	18.6139	14.1061
			July	Dec	5.4839e+06	3.2150e+06	2.3418e+3	1.7931e+3	50.0792	29.9962
	Testing Set	2022	Jan	June	5.6749e+06	3.1586e+06	2.3822e+3	1.7773e+3	34.1303	23.244

7.2. Future Work

Future research can advance this work in several directions. One promising extension would be to explore alternative optimization techniques such as Differential Evolution, Ant Colony Optimization, or hybrid evolutionary algorithms to enhance robustness. Testing across multi-region and cross-border datasets could further demonstrate adaptability under diverse regulatory and economic conditions. Another avenue is the incorporation of dimensionality reduction and feature selection methods to improve scalability when handling large-scale datasets. Establishing formal convergence proofs and theoretical performance guarantees would also strengthen the framework's reliability.

In addition, extending the proposed model to capture the uncertainty introduced by renewable energy sources such as solar and wind could make it more relevant for sustainable power markets. Applying the framework to multiple coupled power exchanges, including day-ahead and real-time markets simultaneously, may provide broader and more holistic forecasting insights. Enhancing computational optimization for real-time deployment would improve its practical adoption by market operators. Integrating the model with interpretable deep learning methods, such as attention-based architectures, may increase forecasting accuracy while preserving explainability. Finally, extending the interval-valued framework toward probabilistic forecasting could provide confidence bounds and risk-aware decision-making support. By pursuing these directions, the iHMM-FRPSO framework can evolve into a universally applicable forecasting model, capable of supporting decision-making in increasingly complex and deregulated electricity markets.

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Appendix:

iHMM-FRPSO Forecasting Procedure (Pseudocode)

Input:

- Historical Market Data: Price (MCP), Volume (MCV)
- PSO Parameters: swarm size S, max iterations T, inertia weight w, coefficients c_1 and c_2
- HMM Parameters: number of hidden states N, observation symbols M
- Fuzzy rule base and membership functions
- Train/Test split ratio

Output:

- Forecasted MCP/MCV for Day-Ahead and Real-Time Markets

Begin

1. Preprocessing:
   a. Normalize historical MCP and MCV data
   b. Split data into training and testing sets
   c. Discretize data into observation symbols for HMM
2. Intuitionistic Fuzzy Rule Generation:
   a. Construct fuzzy membership functions for input variables
   b. Generate IF-THEN rules with membership and non-membership degrees
   c. Encode uncertainty using intuitionistic fuzzy sets
3. Hidden Markov Model (HMM) Initialization:
   a. Initialize:
      - Initial state probabilities π
      - State transition matrix A
      - Observation probability matrix B
   b. Apply fuzzy logic to soften observation symbol likelihoods

4. Particle Swarm Optimization (PSO)

   a. Initialize a swarm of S particles, each encoding $[\pi, A, B]$.

   b. For each particle:

      i Evaluate fitness using forecasting error (e.g., MAPE on validation set).

$$v_i(t+1) = w^* v_i(t) + c_1^* \mathrm{rand}_1^* \big(pbest_i - x_i(t)\big) + c_2^* \mathrm{rand}_2^* \big(gbest - x_i(t)\big)$$

$$x_i(t+1) = x_i(t) + v_i(t+1)$$

   c. Update each particle's velocity and position using:

   d. Update personal and global bests

   e. Repeat steps (b)-(d) until T iterations or convergence threshold met

5. Optimized Model Forecasting:

   a. Use PSO-optimized HMM parameters $[\pi^*, A^*, B^*]$

   b. Apply Viterbi decoding to infer most probable hidden state sequence

   c. Predict next MCP/MCV based on current state and transition/emission probabilities

6. Postprocessing:

   a. Inverse normalization of predicted values

   b. Compute error metrics: MAPE, RMSE, MSE

End

Table 12: Historical Data of Market Clearing Price of Real Time Market.

Date	MCP	Date	MCP	Date	MCP	Date	MCP	Date	MCP
01-01-2022	3323.97	09-02-2022	3819.26	20-03-2022	5003.36	28-04-2022	12000	06-06-2022	7435.19
02-01-2022	2607.29	10-02-2022	3637.26	21-03-2022	11250.78	29-04-2022	12000	07-06-2022	6601.48
03-01-2022	3833.34	11-02-2022	3969.4	22-03-2022	14055.37	30-04-2022	11672.53	08-06-2022	6897.56
04-01-2022	4352.75	12-02-2022	4825.24	23-03-2022	17038.55	01-05-2022	6326.4	09-06-2022	6030.6
05-01-2022	3347.16	13-02-2022	3509.84	24-03-2022	16063.81	02-05-2022	7382.49	10-06-2022	8451.32
06-01-2022	3679.79	14-02-2022	3849.16	25-03-2022	17486.4	03-05-2022	8921.89	11-06-2022	7316.99
07-01-2022	3210.51	15-02-2022	4668.2	26-03-2022	11958.87	04-05-2022	8875.24	12-06-2022	5859.75
08-01-2022	2525.18	16-02-2022	5276	27-03-2022	4771.8	05-05-2022	8904.66	13-06-2022	9602.84
10-01-2022	2546.95	18-02-2022	4320.32	29-03-2022	11629.97	07-05-2022	6500.01	15-06-2022	7523.95
11-01-2022	2998.72	19-02-2022	4477.95	30-03-2022	11218.36	08-05-2022	5316	16-06-2022	5904.69
12-01-2022	3347.29	20-02-2022	3323.01	31-03-2022	11371.04	09-05-2022	5360.92	17-06-2022	4908.46
13-01-2022	3156.03	21-02-2022	4662.03	01-04-2022	5762.49	10-05-2022	5762	18-06-2022	4132.13
14-01-2022	3079.54	22-02-2022	5170.12	02-04-2022	5226.28	11-05-2022	5301.14	19-06-2022	2964.82
15-01-2022	2794.07	23-02-2022	5670.45	03-04-2022	5972.33	12-05-2022	4180.63	20-06-2022	3126.02
16-01-2022	2817.54	24-02-2022	7080.57	04-04-2022	5247.02	13-05-2022	4271.11	21-06-2022	3492.06
17-01-2022	4260.51	25-02-2022	5342.28	05-04-2022	7462.87	14-05-2022	4017.8	22-06-2022	5709.96
18-01-2022	4922.41	26-02-2022	3419.96	06-04-2022	10817.13	15-05-2022	3510.83	23-06-2022	6783.06
19-01-2022	4407.08	27-02-2022	3405.61	07-04-2022	10060.86	16-05-2022	4468.43	24-06-2022	6063.78

Date	**MCP**	**Date**	**MCP**	**Date**	**MCP**	**Date**	**MCP**	**Date**	**MCP**
20-01-2022	3944.68	28-02-2022	3500.74	08-04-2022	9342.42	17-05-2022	6223.79	25-06-2022	6195.98
21-01-2022	3694.69	01-03-2022	4304.46	09-04-2022	9864.62	18-05-2022	8375.99	26-06-2022	6456.22
22-01-2022	2655.79	02-03-2022	3767.73	10-04-2022	7370.23	19-05-2022	8343.44	27-06-2022	9080.35
23-01-2022	2706.54	03-03-2022	4003.32	11-04-2022	9317.28	20-05-2022	5290.71	28-06-2022	8830.52
24-01-2022	3427.9	04-03-2022	4848.98	12-04-2022	10525.9	21-05-2022	2893.47	29-06-2022	3970.43
25-01-2022	3569.44	05-03-2022	4688.38	13-04-2022	10683.12	22-05-2022	2790	30-06-2022	3230.3
26-01-2022	3165.4	06-03-2022	4077.85	14-04-2022	8723.87	23-05-2022	2559	-	-
27-01-2022	3027.6	07-03-2022	5142.14	15-04-2022	5944.09	24-05-2022	5453.05	-	-
28-01-2022	3777.15	08-03-2022	5136.22	16-04-2022	9384.13	25-05-2022	7852.73	-	-
29-01-2022	4493.98	09-03-2022	6303.08	17-04-2022	7294.56	26-05-2022	6063.98	-	-
30-01-2022	3834.11	10-03-2022	6178.34	18-04-2022	9808.69	27-05-2022	4404.83	-	-
31-01-2022	4803.92	11-03-2022	7890.69	19-04-2022	11191.72	28-05-2022	5990.68	-	-
01-02-2022	5975.77	12-03-2022	6649.12	20-04-2022	11586.07	29-05-2022	5429.72	-	-
02-02-2022	5241.53	13-03-2022	4924.24	21-04-2022	11850.28	30-05-2022	6989.44	-	-
03-02-2022	3371.95	14-03-2022	6983.67	22-04-2022	12000	31-05-2022	6768.54	-	-
04-02-2022	3168.86	15-03-2022	9567.71	23-04-2022	12000	01-06-2022	7112.59	-	-
05-02-2022	3748.65	16-03-2022	11766.66	24-04-2022	8774.78	02-06-2022	8332.87	-	-
06-02-2022	3126.77	17-03-2022	11022.62	25-04-2022	11381.37	03-06-2022	7806.18	-	-
07-02-2022	3736.07	18-03-2022	4978.51	26-04-2022	11382.09	04-06-2022	8759.77	-	-
08-02-2022	4478.21	19-03-2022	3432.91	27-04-2022	12000	05-06-2022	7907.22	-	-

Table 13: Difference Value and Observation Symbol of Historical Data of Market Clearing Price of Real Time Market.

Date	MCP	D.V	O.S	Date	MCP	D.V	O.S	Date	MCP	D.V	O.S
01-01-2022	3323.97			11-02-2022	3969.4	332.14	R	24-03-2022	16063.81	-974.74	F
02-01-2022	2607.29	-716.68	F	12-02-2022	4825.24	855.84	R	25-03-2022	17486.4	1422.59	R
03-01-2022	3833.34	1226.05	R	13-02-2022	3509.84	-1315.4	F	26-03-2022	11958.87	-5527.53	F
04-01-2022	4352.75	519.41	F	14-02-2022	3849.16	339.32	R	27-03-2022	4771.8	-7187.07	F
05-01-2022	3347.16	-1005.59	F	15-02-2022	4668.2	819.04	R	28-03-2022	5715.81	944.01	R
06-01-2022	3679.79	332.63	R	16-02-2022	5276	607.8	F	29-03-2022	11629.97	5914.16	R
07-01-2022	3210.51	-469.28	F	17-02-2022	4897.73	-378.27	F	30-03-2022	11218.36	-411.61	F
08-01-2022	2525.18	-685.33	F	18-02-2022	4320.32	-577.41	F	31-03-2022	11371.04	152.68	R
09-01-2022	2559.81	34.63	R	19-02-2022	4477.95	157.63	R	01-04-2022	5762.49	-5608.55	F
10-01-2022	2546.95	-12.86	F	20-02-2022	3323.01	-1154.94	F	02-04-2022	5226.28	-536.21	R
11-01-2022	2998.72	451.77	R	21-02-2022	4662.03	1339.02	R	03-04-2022	5972.33	746.05	R
12-01-2022	3347.29	348.57	F	22-02-2022	5170.12	508.09	F	04-04-2022	5247.02	-725.31	F
13-01-2022	3156.03	-191.26	F	23-02-2022	5670.45	500.33	F	05-04-2022	7462.87	2215.85	R
14-01-2022	3079.54	-76.49	R	24-02-2022	7080.57	1410.12	R	06-04-2022	10817.13	3354.26	R
15-01-2022	2794.07	-285.47	F	25-02-2022	5342.28	-1738.29	F	07-04-2022	10060.86	-756.27	F
16-01-2022	2817.54	23.47	R	26-02-2022	3419.96	-1922.32	F	08-04-2022	9342.42	-718.44	R
17-01-2022	4260.51	1442.97	R	27-02-2022	3405.61	-14.35	R	09-04-2022	9864.62	522.2	R
18-01-2022	4922.41	661.9	F	28-02-2022	3500.74	95.13	R	10-04-2022	7370.23	-2494.39	F
19-01-2022	4407.08	-515.33	F	01-03-2022	4304.46	803.72	R	11-04-2022	9317.28	1947.05	R

Date	MCP	D.V	O.S	Date	MCP	D.V	O.S	Date	MCP	D.V	O.S
20-01-2022	3944.68	-462.4	R	02-03-2022	3767.73	-536.73	F	12-04-2022	10525.9	1208.62	F
21-01-2022	3694.69	-249.99	R	03-03-2022	4003.32	235.59	R	13-04-2022	10683.12	157.22	F
22-01-2022	2655.79	-1038.9	F	04-03-2022	4848.98	845.66	R	14-04-2022	8723.87	-1959.25	F
23-01-2022	2706.54	50.75	R	05-03-2022	4688.38	-160.6	F	15-04-2022	5944.09	-2779.78	F
24-01-2022	3427.9	721.36	R	06-03-2022	4077.85	-610.53	F	16-04-2022	9384.13	3440.04	R
25-01-2022	3569.44	141.54	F	07-03-2022	5142.14	1064.29	R	17-04-2022	7294.56	-2089.57	F
26-01-2022	3165.4	-404.04	F	08-03-2022	5136.22	-5.92	F	18-04-2022	9808.69	2514.13	R
27-01-2022	3027.6	-137.8	R	09-03-2022	6303.08	1166.86	R	19-04-2022	11191.72	1383.03	F
28-01-2022	3777.15	749.55	R	10-03-2022	6178.34	-124.74	F	20-04-2022	11586.07	394.35	F
29-01-2022	4493.98	716.83	F	11-03-2022	7890.69	1712.35	R	21-04-2022	11850.28	264.21	F
30-01-2022	3834.11	-659.87	F	12-03-2022	6649.12	-1241.57	F	22-04-2022	12000	149.72	F
31-01-2022	4803.92	969.81	R	13-03-2022	4924.24	-1724.88	F	23-04-2022	12000	0	F
01-02-2022	5975.77	1171.85	R	14-03-2022	6983.67	2059.43	R	24-04-2022	8774.78	-3225.22	R
02-02-2022	5241.53	-734.24	F	15-03-2022	9567.71	2584.04	R	25-04-2022	11381.37	2606.59	R
03-02-2022	3371.95	-1869.58	F	16-03-2022	11766.66	2198.95	F	26-04-2022	11382.09	0.72	F
04-02-2022	3168.86	-203.09	R	17-03-2022	11022.62	-744.04	F	27-04-2022	12000	617.91	R
05-02-2022	3748.65	579.79	R	18-03-2022	4978.51	-6044.11	F	28-04-2022	12000	0	F
06-02-2022	3126.77	-621.88	F	19-03-2022	3432.91	-1545.6	R	29-04-2022	12000	0	F
07-02-2022	3736.07	609.3	R	20-03-2022	5003.36	1570.45	R	30-04-2022	11672.53	-327.47	R
08-02-2022	4478.21	742.14	R	21-03-2022	11250.78	6247.42	R	01-05-2022	6326.4	-5346.13	F

Date	MCP	D.V	O.S	Date	MCP	D.V	O.S	Date	MCP	D.V	O.S
09-02-2022	3819.26	-658.95	F	22-03-2022	14055.37	2804.59	F	02-05-2022	7382.49	1056.09	R
10-02-2022	3637.26	-182	R	23-03-2022	17038.55	2983.18	R	03-05-2022	8921.89	1539.4	R

Date	MCP	D.V	O.S	Date	MCP	D.V	O.S
04-05-2022	8875.24	-46.65	F	03-06-2022	7806.18	-526.69	F
05-05-2022	8904.66	29.42	R	04-06-2022	8759.77	953.59	R
06-05-2022	7634.53	-1270.13	F	05-06-2022	7907.22	-852.55	F
07-05-2022	6500.01	-1134.52	R	06-06-2022	7435.19	-472.03	R
08-05-2022	5316	-1184.01	F	07-06-2022	6601.48	-833.71	F
09-05-2022	5360.92	44.92	R	08-06-2022	6897.56	296.08	R
10-05-2022	5762	401.08	R	09-06-2022	6030.6	-866.96	F
11-05-2022	5301.14	-460.86	R	10-06-2022	8451.32	2420.72	R
12-05-2022	4180.63	-1120.51	F	11-06-2022	7316.99	-1134.33	F
13-05-2022	4271.11	90.48	R	12-06-2022	5859.75	-1457.24	F
14-05-2022	4017.8	-253.31	F	13-06-2022	9602.84	3743.09	R
15-05-2022	3510.83	-506.97	F	14-06-2022	10402.63	799.79	F
16-05-2022	4468.43	957.6	R	15-06-2022	7523.95	-2878.68	F
17-05-2022	6223.79	1755.36	R	16-06-2022	5904.69	-1619.26	R
18-05-2022	8375.99	2152.2	R	17-06-2022	4908.46	-996.23	R
19-05-2022	8343.44	-32.55	F	18-06-2022	4132.13	-776.33	R
20-05-2022	5290.71	-3052.73	F	19-06-2022	2964.82	-1167.31	F
21-05-2022	2893.47	-2397.24	R	20-06-2022	3126.02	161.2	R
22-05-2022	2790	-103.47	R	21-06-2022	3492.06	366.04	R
23-05-2022	2559	-231	F	22-06-2022	5709.96	2217.9	R
24-05-2022	5453.05	2894.05	R	23-06-2022	6783.06	1073.1	F
25-05-2022	7852.73	2399.68	F	24-06-2022	6063.78	-719.28	F
26-05-2022	6063.98	-1788.75	F	25-06-2022	6195.98	132.2	R
27-05-2022	4404.83	-1659.15	R	26-06-2022	6456.22	260.24	R
28-05-2022	5990.68	1585.85	R	27-06-2022	9080.35	2624.13	R
29-05-2022	5429.72	-560.96	F	28-06-2022	8830.52	-249.83	F
30-05-2022	6989.44	1559.72	R	29-06-2022	3970.43	-4860.09	F
31-05-2022	6768.54	-220.9	F	30-06-2022	3230.3	-740.13	R
01-06-2022	7112.59	344.05	R	-	-	-	-
02-06-2022	8332.87	1220.28	R	-	-	-	-