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Perfect 2-Distance Zero Forcing in Graphs

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Abstract. Let G be a graph. Then a 2-distance color change rule is defined as follows: If a vertex $x \in V(G)$ is colored and has exactly one hop neighbor y is uncolored, then y will become colored. Moreover, let $u, v, w \in V(G)$. If u 2-forces v and v 2-forces w, then we say that v and w are perfectly 2-forced by u, and this process can extend to a chain of 2-forcing initiated by a single vertex. In addition, a subset S of a vertex-set $V(G)$ of G is called a *perfect 2-distance zero forcing set* of G if there exists $s \in S$ such that s perfectly 2-forces all other vertices outside S. The minimum cardinality of a perfect 2-distance zero forcing set of G, denoted by $Z_p^2(G)$, is called the *perfect 2-distance zero forcing number* of G. In this paper, this new parameter is introduced and initially investigated on some classes of graphs and on the join of two graphs. A particular variant of zero forcing called perfect co-zero forcing is defined to study the behavior of the perfect 2-distance zero forcing sets in the join of graphs. Characterizations of perfect 2-distance zero forcing sets are formulated and subsequently used to obtain some formulas for solving the perfect 2-distance zero forcing numbers of the join of some graphs.

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Key Words and Phrases: Perfect 2-distance zero forcing set, perfect 2-distance zero forcing number, perfect co-zero forcing

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1. Introduction

The concept of zero forcing set was initially introduced in [1] as a bound for the minimum rank problem. A zero forcing set in a graph is a subset of vertices with a particular dynamic propagation property. The zero forcing process starts with a set of initially colored vertices, typically with one color representing "active" and another "inactive" or

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"unassigned." Then, using certain propagation rules, the active vertices force neighboring inactive vertices to become active. The goal is to determine the minimum size of a zero forcing set required to force all vertices in the graph to become active. The concept of zero forcing was further studied by many researchers, and these studies can be found in [1–10].

In 2024, J. Hassan et al. [11], introduced another variant of zero forcing by changing the distance to two (2) for a certain vertex to force another vertex in a graph. The said parameter was investigated on some classes of graphs as well as examined its relationships with other parameters such as standard zero forcing and hop domination.

In this paper, we introduce another variant of 2-distance zero forcing by adding a certain property wherein the reinforcement is not allowed, and we call it a perfect 2-distance zero forcing. That is, only one vertex in a set is needed to force all other vertices outside the considered set, if any. This is far different compared to the standard 2-distance zero forcing wherein the reinforcement is allowed. We believe, this new parameter and its results would serve as reference to future researchers who will study on variants of zero forcing, and would lead to an interesting topics of research in the future.

2. Terminology and Notation

Let $G = (V(G), E(G))$ be a simple and undirected graph. The *distance* $d_G(u, v)$ in G of two vertices u, v is the length of a shortest *u-v path* in G. The greatest distance between any two vertices in G, denoted by $diam(G)$, is called the *diameter* of G. A vertex of a in G is a *hop neighbor* of a vertex b in G if $d_G(a, b) = 2$.

Let G be a graph and let $x, y \in V(G)$. Then the 2-distance color change rule is if x is colored (active) vertex and exactly one hop neighbor y of x is uncolored (inactive) , then y will become colored (active). A 2-distance zero forcing set N of G is a subset of vertices of G such that when the vertices in N are colored (active) and the remaining vertices are uncolored(inactive) initially, repeated application of the 2-distance color change rule all vertices of G will become colored (active). The minimum cardinality of a 2-distance zero forcing set of G, denoted by $Z^2(G)$, is called the 2-distance zero forcing number of G.

Let G and H be any two graphs. The *join* of G and H, denoted by $G+H$ is the graph with vertex set $V(G + H) = V(G) \cup V(H)$ and edge set

$$E(G + H) = E(G) \cup E(H) \cup \{uv : u \in V(G), v \in V(H)\}.$$

3. Results

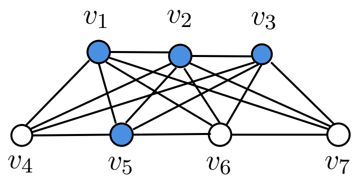
We begin this section by introducing the concept of perfect 2-distance zero forcing in a graph.

Definition 1. Let G be a graph. Then a 2-distance color change rule is defined as follows: If a vertex $x \in V(G)$ is colored and has exactly one hop neighbor y that is uncolored, then y will become colored. In this case, we say that a vertex y is 2-forced by a vertex x in G.

Moreover, let $u, v, w \in V(G)$. If u 2-forces v and v 2-forces w, then we say that v and w are perfectly 2-forced by u, and this process can extend to a chain of 2-forcing initiated by a single vertex. In addition, a subset S of a vertex-set $V(G)$ of G is called a *perfect 2-distance zero forcing set* of G if there exists $s \in S$ such that s perfectly 2-forces all other vertices outside S. The minimum cardinality of a perfect 2-distance zero forcing set of G, denoted by $Z_p^2(G)$, is called the *perfect 2-distance zero forcing number* of G.

Example 1. Consider the graph $P_3 + P_4$ below. Let $S = \{v_1, v_2, v_3, v_5\}$. Then vertex v_7 is 2-forced by vertex v_5, vertex v_4 is 2-forced by vertex v_7, and v_6 is 2-forced by vertex v_4. It follows that vertices v_4, v_6 and v_7 are perfectly 2-forced by vertex v_5. Therefore, S is a perfect 2-distance zero forcing set of $P_3 + P_4$. It can easily be verified that a perfect 2-distance zero forcing number of $P_3 + P_4$ is 4, that is, $Z_p^2(P_3 + P_4) = 4$.

$P_3 + P_4$:



Theorem 1. *Let G be a graph. Then each of the following holds:*

(i) $Z^2(G) \leq Z_P^2(G) \leq |V(G)|$.

(ii) Let G be a non-trivial graph. If G has a dominating vertex, then $Z_p^2(G) \geq 2$.

(iii) If every vertex of G is a dominating vertex, then $Z_P^2(G) = |V(G)|$.

Proof. (i). Let G be a graph, and R be a minimum perfect 2-distance zero forcing set of G. Then $|R| = Z_P^2(G)$ and R is a 2-distance zero forcing set of G. Thus, $Z^2(G) \leq |R| = Z_P^2(G)$. The upper bound is clear since every perfect 2-distance zero forcing set of G is always a subset of $V(G)$.

(ii). Let $x \in V(G)$ be a dominating vertex of G. Then $d_G(x, u) = 1$ for all $u \in V(G) \setminus \{x\}$. Suppose that $x \notin S$, where S is a minimum perfect 2-distance zero forcing set of G. Then there must be a vertex $w \in V(G) \setminus \{x\}$ such that $d_G(w, x) = 2$, which is a contradiction. Thus, $x \in S$. Since G is a non-trivial and x is a dominating vertex of G, there exists $y \in V(G) \setminus \{x\}$ such that $y \in S$. Therefore, $Z_P^2(G) \geq 2$.

(iii). Let $V(G) = \{v_1, v_2, \cdots, v_k\}$. Since v_1 is a dominating vertex of G, $d_G(v_1, v_i) = 1$ for all $i \in \{2, 3, \cdots, k\}$. Applying the same argument in the proof of (ii), $v_1 \in Q$, where Q is a minimum perfect 2-distance zero forcing set of G. Now, since $v_2 \in V(G) \setminus \{v_1\}$ is a dominating vertex, v_2 must be also in Q. Continuing in this manner, $V(G) \subseteq Q$, that is, $Q = V(G)$. Hence, $Z_p^2(G) = |V(G)|$. □

Theorem 2. *Let n be a positive integer.*

$$Z_p^2(P_n) = \begin{cases} \frac{n}{2}+1, \text{if } n \text{ is even} \\ \frac{n+1}{2}, \text{if } n \text{ is odd} \end{cases}$$

Proof. Clearly, $Z_p^2(P_1) = 1$ and $Z_p^2(P_2) = 2 = Z_p^2(P_3)$. Suppose that $n \geq 4$ and even, Let $P_n = [v_1, v_2, ..., v_n]$, and consider $B = \{v_1, v_2, v_4, v_6, \ldots, v_n\}$. Then vertices $v_3, v_5, \ldots, v_{n-1}$ are perfectly 2-forced by v_1. It follows that B is perfect 2-distance zero forcing set of P_n. Notice that, if we remove v_i from B for some $i \in \{2, 4, \ldots, n\}$, then v_i will not be perfectly 2-forced by v_1. Thus, B is a minimum perfect 2-distance zero forcing set of P_n. Hence, $Z_p^2(P_n) = \frac{n}{2} + 1$ for all even integers $n \geq 2$.

Now, assume that $n \geq 5$ and odd. Let $A = \{v_1, v_2, v_4, v_6, \ldots, v_{n-1}\}$. Then vertices $v_3, v_5, \ldots, v_n$ are perfectly 2-forced by v_1. Thus, A is a perfect 2-distance zero forcing set of P_n. Moreover, if we remove v_j from A for some $j \in \{2, 4, \ldots, n-1\}$, then v_j will not be perfectly 2- forced by v_1. This means that A is a minimum perfect 2-distance zero forcing set of P_n. Hence, $Z_p^2(P_n) = \frac{n+1}{2}$ for all odd integers $n \geq 5$. □

To study the behavior of perfect 2-distance zero forcing sets in the join of any two graphs, the following concept shall be defined:

Definition 2. *Let G be a graph. Then a co-color change rule is defined as follows: If a vertex $x \in V(G)$ is colored black and has exactly one non-neighbor y that is colored white, then y will become black. In this case, we say that a vertex y is co-forced by a vertex x in G. Moreover, let $u, v, w \in V(G)$. If u co-forces v and v co-forces w, then we say that v and w are perfectly co-forced by u, and this process can extend to a chain of co-forcing initiated by a single vertex. In addition, a subset B of a vertex-set $V(G)$ of G is called a perfect co-zero forcing set of G if there exists $u \in B$ such that u perfectly co-forces all vertices outside B. The minimum cardinality of a perfect co-zero forcing set of G, denoted by $Z_{pco}(G)$, is called the perfect co-zero forcing number of G.*

Lemma 1. *Let G be a non-trivial graph. If G has a dominating vertex, then $Z_{pco}(G) \geq 2$.*

Proof. Let $v \in V(G)$ be a dominating vertex of G, and let T be a minimum perfect co-zero forcing set of G. Assume that $v \notin T$. Then there must be a vertex $w \in T \setminus \{v\}$ such that w co-forces v, that is, $d_G(v, w) \geq 2$. Since v is a dominating vertex of G, it follows that $d_G(v, u) = 1$ for all $u \in V(G) \setminus \{v\}$. Note that $w \in V(G) \setminus \{v\}$, a contradiction. Therefore, $v \in T$.

Moreover, since $d_G(v, u) = 1$ for all $u \in V(G) \setminus \{v\}$, it follows that v cannot co-force any vertex $u \in V(G) \setminus \{v\}$. Thus, there must be another vertex $t \in T$ such that t co-forces u (possible $t = u$). Thus, T has at least two elements, that is, $Z_{pco}(G) \geq 2$.

Theorem 3. *Let m be a positive integer. Then*

$$Z_{pco}(C_m) = \begin{cases} 3 \text{ , } if\ m = 3, 4 \\ m\text{-}3,\ if\ m \geq 5. \end{cases}$$

Proof. Since C_3 has dominating vertex, it follows that $Z_{pco}(C_3) \geq 2$ by Lemma 1. Assume that $Z_{pco}(C_3) = 2$, say, $Q = \{a, b\}$ is a minimum perfect co-zero forcing set of C_3, where $C_3 = [a, b, c, a]$. Then either vertex a or b must co-forced vertex c. That is, $d_{C_3}(a, c) \geq 2$ or $d_{C_3}(b, c) \geq 2$. However, this is a contradiction to the fact that each pair of vertices in C_3 are adjacent. Therefore, $Z_{pco}(C_3) = 2$ is not possible. Since $V(C_3)$ is a perfect co-zero forcing set of C_3, it follows that $Z_{pco}(C_3) = 3$.

For $m = 4$, let $C_4 = [v_1, v_2, v_3, v_4, v_1]$, and consider $Q' = \{v_1, v_2, v_3\}$. Then Q' is a perfect co-zero forcing set of C_4. Hence, $Z_{pco}(C_4) \leq 3$. Assume that $Z_{pco}(C_4) = 2$, say, a minimum perfect co zero forcing set of C_4 is $R = \{v_i, v_j\}$, where $i, j \in \{1, 2, 3, 4\}$. If v_i and v_j are adjacent, then neither v_i nor v_j can co-force all the remaining vertices outside R, a contradiction. If v_i and v_j are non-adjacent, then the remaining two vertices outside R are both adjacent to v_i and v_j. That is, neither v_i nor v_j can co-force these two vertices. Hence, $Z_{pco}(C_4) = 2$ is not possible. Since $\{v_1, v_2, v_3\}$ is a perfect co-zero forcing set of C_4, we have $Z_{pco}(C_4) = 3$.

Now, let $n \geq 5$ and $C_n = [x_1, x_2, \cdots, x_m, x_1]$. Consider $X = \{x_1, x_3, x_4, \cdots, x_{m-2}\}$. Then vertices x_{m-1}, x_2 and x_m are perfectly co-forced by vertices x_1. It follows that X is a perfect co-zero forcing set of C_m. Thus, $Z_{pco}(C_m) \leq m - 3$ for all $m \geq 5$. Assume that $Z_{pco}(C_m) \leq m - 4$. Then there are at least four vertices $x_i, x_j, x_k, x_l \in V(C_m) \setminus N$, where N is a minimum perfect co-zero forcing set of C_m. Since the graph is cycle, at least two vertices in $\{x_i, x_j, x_k, x_l\}$ have distance of at least two to any vertex in N. That is, none of the vertices in N can perfectly co-forces these vertices, which is a contradiction. In this case, $Z_{pco}(C_m) \leq m - 4$ is not possible. Therefore, $Z_{pco}(C_m) = m - 3$ for all $m \geq 5$. □

Theorem 4. *Let n be a natural number. Then*

$$Z_{pco}(P_n) = \begin{cases} n \text{ , } if\ n = 1, 2 \\ 2,\ if\ n = 3. \\ n\text{-}3,\ if\ n \geq 4. \end{cases}$$

Proof. Clearly $Z_{pco}(P_1) = 1$ and $Z_{pco}(P_2) = 2$. For $n = 3$, let $P_3 = [a_1, a_2, a_3]$. Consider $S = \{a_1, a_2\}$. Then a_3 is co-forced by a vertex a_1. Thus, S is a perfect co-zero forcing set of P_3, and so $Z_{pco}(P_3) \leq 2$. Since a_2 is a dominating vertex, if follows that $Z_{pco}(P_3) = 2$ by Lemma 1.

Let $n = 4$, and suppose that $P_4 = [a_1, a_2, a_3, a_4]$. Consider $B = \{a_2\}$. Then a_4, a_1 and a_3 are perfectly co-forced by vertex a_2. Thus, B is a perfect co-zero forcing set of P_4.

Hence, $Z_{pco}(P_4) = 1$.

Now, for $n \geq 5$, let $P_n = [a_1, a_2, \dots, a_n]$. Let $B' = \{a_2, a_5, a_6, \dots, a_n\}$. Then vertices a_4, a_1 and a_3 are perfectly co-forced by vertex a_2. Thus, B' is a perfect co-zero forcing set of P_n, and so $Z_{pco}(P_n) \leq n-3$. Suppose that $Z_{pco}(P_n) \leq n-4$. Then there exist at least four vertices a_i, a_j, a_k and a_l in $V(G)$ such that $a_i, a_j, a_k, a_l \notin Q$, where Q is minimum perfect co-zero forcing set of P_n. Since the graph is a path graph, at least two vertices in $\{a_i, a_j, a_k, a_l\}$ have distance two to every vertex in Q. Thus, none of the vertices in Q can perfectly co-forces vertices outside Q, a contradiction to the fact that Q is a perfect co-zero forcing set of P_n. In this case, $Z_{pco}(P_n) \leq n-4$ is not possible. Hence, $Z_{pco}(P_n) = n-3$ for all $n \geq 5$. □

We shall now characterize the perfect 2-distance zero forcing sets in the join of two graphs as follows:

Theorem 5. Let G and H be any two graphs. Then $P \subseteq V(G+H)$ is a perfect 2-distance zero forcing if and only if one of the following conditions hold:

(i) $P = P_G \cup V(H)$, where P_G is a perfect co-zero forcing set of G.

(ii) $P = V(G) \cup P_H$ such that P_H is a perfect co-zero forcing set of H.

Proof. Suppose that P is a perfect 2-distance zero forcing set of $G+H$. Then $P = P_G \cup P_H$, where $P_G \subseteq V(G)$ and $P_H \subseteq V(H)$. If $P_G = \varnothing$, then $P = P_H$. However, P_H cannot 2-force any vertex in $V(G)$, a contradiction. The same assertion follows when we let $P_H = \varnothing$. Thus, $P_G \neq \varnothing$ and $P_H \neq \varnothing$.

Now, if $P_G \subset V(G)$ and $P_H \subset V(H)$, that is, P_G and P_H are proper subsets of $V(G)$ and $V(H)$, respectively. Then there exist $x \in V(G) \setminus P_G$ and $y \in V(H) \setminus P_H$. Notice that, vertex x can only be 2-forced by a vertex in $P_G \subseteq V(G)$, and vertex y can only be 2-forced by a vertex in $P_H \subseteq V(H)$. Thus, none of the vertices in P can perfectly 2-forces both x and y, a contradiction to our assumption that P is a perfect 2-distance zero forcing set of $G+H$. Hence, the remaining cases are either $P = P_G \cup V(H)$ or $P = V(G) \cup P_H$ where $P_G \subseteq V(G)$ and $P_H \subseteq V(H)$.

Assume that $P = P_G \cup V(H)$. Suppose on the contrary that P_G is not a perfect co-zero forcing set of G. Then there exists $y \in V(G) \setminus P_G$ such that y cannot be perfectly co-forced by any vertex in P_G under the graph G. This means that y cannot be perfectly 2- forced by any vertex in P_G under the graph $G+H$. However, this is a contradiction to our assumption that P is a 2-distance zero forcing set of $G+H$. Hence, P_G is a perfect co-zero forcing set in G, and so (i) holds.

Similarly, P_H is a perfect co-zero forcing set in H when $P = V(G) \cup P_H$. That is, (ii) holds.

Conversely, suppose that $P = P_G \cup V(H)$, where P_G is a perfect co-zero forcing set of G. Then there exists $x \in P_G$ such that x perfectly co-forces other vertices outside P_G in G. This means that x can perfectly 2-forces other vertices outside P in $G+H$. It follows that $P = P_G \cup V(H)$ is a perfect 2-distance zero forcing set of $G+H$. Similarly, the same assertion follows when (ii) holds. □

Theorem 6. *Let G and H be any graphs. Then*

$$Z_p^2(G+H) = min\{Z_{pco}(G) + |V(H)|, |V(G)| + Z_{pco}(H)\}.$$

Proof. Let P be a minimum perfect 2-distance zero forcing set of $G+H$. Then by Theorem 4, either $P = P_G \cup V(H)$ or $P = V(G) \cup P_H$ where P_G and P_H are perfect co-zero forcing sets of G and H, respectively. Hence, either

$$Z_p^2(G+H) = |P| \geq Z_{pco}(G) + |V(H)| \text{ or } Z_p^2(G+H) = |P| \geq |V(G)| + Z_{pco}(H).$$

Now, suppose that either $P = P_G \cup V(H)$ or $P = V(G) \cup P_H$, where P_G and P_H are minimum perfect co-zero forcing sets of G and H, respectively. Then P is a perfect 2-distance zero forcing set of $G+H$ by Theorem 4. Thus, either

$$Z_p^2(G+H) \leq |P| = Z_{pco}(G) + |V(H)| \text{ or } Z_p^2(G+H) \leq |P| = Z_{pco}(H) + |V(G)|.$$

Hence,

$$Z_p^2(G+H) = Z_{pco}(G) + |V(H)| \text{ or } Z_p^2(G+H) = Z_{pco}(H) + |V(G)|.$$

Consequently,

$$Z_p^2(G+H) = min\{Z_{pco}(G) + |V(H)|, |V(G)| + Z_{pco}(H)\}.$$

□

Corollary 1. *Let n and m be a natural numbers. Then each of the following holds:*

(i) $Z_p^2(S_n) = Z_p^2(K_1 + \bar{K}_n) = \begin{cases} n+1\ , \textit{if } n = 1 \\ n, \textit{if } n \geq 2. \end{cases}$

(ii) $Z_p^2(W_n) = Z_p^2(K_1 + C_n) = \begin{cases} 4\ , \textit{if } n = 3, 4 \\ n-2, \textit{if } n \geq 5. \end{cases}$

(iii) $Z_p^2(F_n) = Z_p^2(K_1 + P_n) = \begin{cases} n+1 \ , \textit{if}\ n = 1, 2 \\ 3, \textit{if}\ n = 3. \\ n-2, \textit{if}\ n \geq 4. \end{cases}$

(iv) $Z_p^2(K_{m,n}) = Z_p^2(\bar{K}_m + \bar{K}_n) = \begin{cases} 2 \ , \textit{if}\ m = n = 1 \\ m+n-1, \textit{if}\ 2 \leq m \leq n \ \textit{or}\ 2 \leq n \leq m. \\ n, \textit{if}\ m = 1 \ \textit{and}\ n \geq 2. \end{cases}$

4. Conclusion

Perfect 2-distance zero forcing, a new variant of zero forcing has been introduced and studied in this paper. It is observed that every graph admits perfect 2-distance zero forcing. It is shown that the parameter for a perfect 2-distance zero forcing is always greater than the parameter for standard 2-distance zero forcing on any simple and undirected graph. Moreover, a certain variant of zero forcing called perfect co-zero forcing was defined to solve the perfect 2-distance zero forcing number of the join of any two graphs.

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References

[1] F. Barioli, W. Barrett, S. Fallat, H. Hall, L. Hogben, H. van der Holst, and B. Shader. Zero forcing parameters and minimum rank problems. *Linear Algebra and its Applications*, 433:401–411, 2010.

[2] K. Benson, D. Ferrero, M. Flagg, V. Furst, L. Hogben, V. Vasilevska, and B. Wissman. Zero forcing and power domination for graph products. *Australasian Journal of Combinatorics*, 70:221–235, 2018.

[3] R. Davila, T. Kalinowski, and S. Stephen. A lower bound on the zero forcing number. *Discrete Applied Mathematics*, 250:363–367, 2018.

[4] S. M. Fallat and L. Hogben. Minimum rank, maximum nullity, and zero forcing number of graphs. In *Handbook of Linear Algebra*, pages 775–810. CRC Press, Boca Raton, FL, 2 edition, 2013.

[5] M. Gentner, L. D. Penso, D. Rautenbach, and U. S. Souza. Extremal values and bounds for the zero forcing number. *Discrete Applied Mathematics*, 214:196–200, 2016.

[6] J. Hassan and L. Laja. Vertex cover zero forcing sets in graphs. *International Journal of Mathematics and Computer Science*, 19(4):999–1003, 2024.

[7] J. Hassan, M. A. Bonsocan, M. Langamin, V. Bilar, S. D. Aming, and B. Amiruddin. Zero forcing domination in some graphs: Characterizations and derived formulas. *European Journal of Pure and Applied Mathematics*, 17(4):3772–3780, 2024.

[8] J. Hassan, L. Laja, and H. Copel. 2-domination zero forcing in graphs. *International Journal of Mathematics and Computer Science*, 19(4):1065–1070, 2024.

[9] T. Kalinowski, N. Kamcev, and B. Sudakov. The zero forcing number of graphs. *SIAM Journal on Discrete Mathematics*, 33(1):95–115, 2019.

[10] J. Manditong, A. Tapeing, J. Hassan, A. R. Bakkang, N. H. Mohammad, and S. U. Kamdon. Some properties of zero forcing hop dominating sets in a graph. *European Journal of Pure and Applied Mathematics*, 17(1):324–337, 2024.

[11] J. Hassan, L. Udtohan, and L. Laja. 2-distance zero forcing in graphs. *European Journal of Pure and Applied Mathematics*, 17(2):1283–1293, 2024.