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Neutrosophic Soft n-Super-HyperGraphs with Real-World Applications

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Abstract. Graph theory provides a fundamental framework for modeling relationships using vertices and edges [1, 2]. Hypergraphs extend this framework by allowing *hyperedges* that can simultaneously connect multiple vertices [3], while n-Super-HyperGraphs further generalize hypergraphs via iterated power-set constructions to capture hierarchical relationships [4, 5]. In parallel, various uncertainty modeling paradigms—such as fuzzy sets [6], soft sets [7], intuitionistic fuzzy sets [8–10], neutrosophic sets, and plithogenic sets—have been developed to handle imprecise or indeterminate information.
In this paper, we propose a novel framework called the *Neutrosophic Soft n-Super-HyperGraph*, which integrates neutrosophic logic, soft set theory, and n-Super-HyperGraph structures. This model has the potential to facilitate effective decision-making in complex and uncertain networked environments.

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Key Words and Phrases: Super-HyperGraph, Hypergraph, Fuzzy Graph, Soft Graph, Neutrosophic Graph

1. Introduction and Literature Review

Traditional graphs capture binary relationships by representing entities as vertices and their connections as edges [1, 11]. Hypergraphs extend this notion by allowing each hyperedge to join any nonempty subset of vertices, thereby modeling higher-order interactions [12–14]. The concept is taken further by n-Super-HyperGraphs, which iteratively apply the power-set operator to encode nested, hierarchical connectivity patterns within a single unified structure [15–18].

These graph-based formalisms have proven invaluable across domains such as network science, data mining, chemistry, and physics, providing both intuitive visualization and rigorous analytical frameworks [2, 5]. The additional abstraction levels in n-Super-HyperGraphs are particularly well-suited for disentangling multi-tiered relationships in complex systems. Here, n is assumed to be a positive integer.

Structure	Notation	Edge Definition	Description
Graph	$G = (V, E)$	$E \subseteq \{\{u, v\} \mid u, v \in V,\ u \neq v\}$	Simple pairwise connections.
Hypergraph	$H = (V, E)$	$E \subseteq \mathcal{P}(V) \setminus \{\varnothing\}$	Hyperedges connect arbitrary subsets of V.
n-Super-HyperGraph	$\mathrm{SHG}^{(n)} = (V, E)$	$V, E \subseteq \mathcal{P}^n(V_0)$	Hierarchical, iterated–powerset construction.

Table 1: Comparison of graphs, hypergraphs, and n-Super-HyperGraphs.

Table 1 summarizes the principal distinctions among standard graphs, hypergraphs, and n-Super-HyperGraphs.

Real-world networks often exhibit imprecision or missing information. To address these challenges, classical set theory has been extended with various uncertainty models: fuzzy sets [6], intuitionistic fuzzy sets [9], soft sets [7, 19], vague sets [20, 21], hyperfuzzy sets [22], neutrosophic sets [23, 24], and plithogenic sets [25, 26]. Embedding these into graph theory yields fuzzy graphs [27], intuitionistic fuzzy graphs [28], neutrosophic graphs [29, 30], plithogenic graphs [31], and soft graphs [32], each enriching the topology with graded uncertainty semantics.

Soft graph models form a parameterized family of subgraphs, where each parameter selects a specific subgraph within a fixed vertex–edge universe [7, 32, 33]. This mirrors soft set theory, which collects subsets indexed by parameters to capture ambiguity or preference without numeric membership values [19, 34]. Extending this concept to hypergraphs, a *soft hypergraph* assigns to each parameter both a subset of vertices and its induced hyperedges [35–38]. The notion of a *soft n-Super-HyperGraph* carries this idea into the layered framework of an n-Super-HyperGraph, producing a hierarchical, parameter-driven model [39]. Table 2 presents an overview of soft graph models and their generalizations to hypergraphs. Moreover, a single-valued neutrosophic hypergraph endows every vertex and hyperedge with three membership degrees—truth, indeterminacy, and falsity—in $[0, 1]$, subject to neutrosophic constraints [40]. These graph-based frameworks have seen extensive applications in decision-making and related domains.

We describe the motivation and contributions of this paper. Neutrosophic Soft HyperGraphs and related studies on Neutrosophic Graphs, Soft Graphs, and HyperGraphs are well established and of great importance. However, as noted above, HyperGraphs alone struggle to model deeply hierarchical graph and network concepts—a limitation that naturally extends to Neutrosophic Soft HyperGraphs. To address this gap, this paper introduces the *Neutrosophic Soft n-Super-HyperGraph*, a novel framework that fuses neutrosophic logic, soft set theory, and n-Super-HyperGraph architecture. Our model provides a flexible yet robust foundation for decision-making over networks characterized by

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Model	Definition	Parameter Mapping
Soft graph	A parameterized family of subgraphs of $G = (V, E)$, where each parameter $c \in C$ selects one subgraph $(A(c), B(c))$.	$A: C \to \mathcal{P}(V),\ B: C \to \mathcal{P}(E),\ B(c) \subseteq \{\{u, v\} \mid u, v \in A(c)\}$.
Soft hypergraph	A soft hypergraph over $H = (V, E)$ in which each $c \in C$ induces the subhypergraph $(A(c), B(c))$.	$A: C \to \mathcal{P}(V),\ B: C \to \mathcal{P}(E),\ B(c) \subseteq \{e \in E \mid e \subseteq A(c)\}$.
Soft n-Super-HyperGraph	A hierarchical, parameter-driven model built on $\text{SuHG}^{(n)} = (V, E)$, where each $c \in C$ yields the substructure $(A(c), B(c))$.	$A: C \to \mathcal{P}(V),\ B: C \to \mathcal{P}(E),\ B(c) \subseteq \{e \in E \mid e \subseteq A(c)\}$.

Table 2: Overview of soft graph models and their hypergraph generalizations

both layered connectivity and degree-based uncertainty. We present its formal definition, examine its key properties, and illustrate its applicability through examples in complex, uncertain environments. We present a concise overview of neutrosophic soft graph models in Table 3.

Model	**Description**
Neutrosophic Soft Graph	A graph in which, for each context, every vertex and edge is assigned degrees of truth, indeterminacy, and falsity.
Neutrosophic Soft Hypergraph	A hypergraph where each parameter selects a substructure, and vertices and hyperedges carry neutrosophic degrees per parameter.
Neutrosophic Soft n-Super-HyperGraph	A multi-level generalization: supervertices and superedges at n levels are chosen per context and endowed with neutrosophic degrees.

Table 3: Concise overview of neutrosophic soft graph models.

The structure of this paper is as follows. Section 2 provides an overview of n-Super-HyperGraphs and introduces the concept of Neutrosophic Soft HyperGraphs. In Section 3, we formally define the Neutrosophic Soft n-Super-HyperGraph. Section 4 discusses the algorithm for constructing this type of graph. Finally, Section 5 presents the conclusion and outlines potential directions for future research.

2. Preliminaries

In this section, we introduce the fundamental concepts and notation used throughout this paper. We assume all graphs are finite, simple, and undirected unless noted otherwise. For more extensive treatments, see the cited literature.

2.1. n-Super-HyperGraphs

A *hypergraph* enhances graphs by allowing edges (called *hyperedges*) to join any subset of vertices, making it suitable for modeling multi-way relationships [3, 12, 41–44]. A *Super-HyperGraph* extends this idea by applying the power-set operation repeatedly, capturing hierarchical or recursive connectivity patterns [5, 15, 16, 45–47]. Note that in general, n is assumed to be a positive integer.

Definition 1 (Base Set)**.** *Let S be a* base set*, serving as the ground domain for all higher-order constructions:*

$$S = \{\, x : x \text{ belongs to the universe of discourse}\}.$$

Definition 2 (power-set)**.** *The* power-set *of S, denoted $\mathrm{PS}(S)$, is the set of all subsets of S, including the empty set:*

$$\mathrm{PS}(S) = \{\, A : A \subseteq S\}.$$

Definition 3 (Hypergraph)**.** *[3, 41] A* hypergraph *is a pair $H = (V, E)$ where*

- *V is a finite set of vertices,*
- *E is a collection of nonempty subsets of V, each called a* hyperedge*.*

Example 1 (Online Social Network as a Hypergraph)**.** *Consider a small social network with five users:*

$$V = \{\textit{Hiroko}, \textit{Tae}, \textit{Yusuke}, \textit{Dave}, \textit{Eve}\}.$$

We model four interest-based groups as hyperedges:

$$\begin{aligned} e_1 &= \{\textit{Hiroko}, \textit{Tae}, \textit{Yusuke}\} && (\textit{Photography Club}),\\ e_2 &= \{\textit{Tae}, \textit{Dave}, \textit{Eve}\} && (\textit{Music Enthusiasts}),\\ e_3 &= \{\textit{Hiroko}, \textit{Dave}, \textit{Eve}\} && (\textit{Travel Group}),\\ e_4 &= \{\textit{Yusuke}, \textit{Eve}\} && (\textit{Book Discussion}). \end{aligned}$$

Thus the hypergraph

$$H = \big(V, \{e_1, e_2, e_3, e_4\}\big)$$

captures the overlapping community structure of the social network, where each hyperedge represents a multi-user group or interest.

Definition 4 (n-th Iterated power-set)**.** *[48, 49] Define the iterated power-set of a set X by*

$$\mathrm{PS}_1(X) = \mathrm{PS}(X), \quad \mathrm{PS}_{k+1}(X) = \mathrm{PS}\big(\mathrm{PS}_k(X)\big), \quad k \geq 1.$$

The corresponding nonempty *iterated power-set is*

$$\mathrm{PS}^*_1(X) = \mathrm{PS}(X) \setminus \{\varnothing\}, \quad \mathrm{PS}^*_{k+1}(X) = \mathrm{PS}\big(\mathrm{PS}^*_k(X)\big) \setminus \{\varnothing\}.$$

Example 2 (Corporate Hierarchy as a 2-Iterated Power-Set)**.** *Let*

$$X = \{\text{AlphaCorp, BetaInc, GammaLLC}\}$$

be three affiliated companies. The first power-set is

$$\mathrm{PS}_1(X) = \{\varnothing, \{\text{AlphaCorp}\}, \{\text{BetaInc}\}, \{\text{GammaLLC}\},$$
$$\{\text{AlphaCorp}, \text{BetaInc}\}, \{\text{AlphaCorp}, \text{GammaLLC}\}, \{\text{BetaInc}, \text{GammaLLC}\},$$
$$\{\text{AlphaCorp}, \text{BetaInc}, \text{GammaLLC}\}\}.$$

Excluding the empty set, we may identify two holding companies:

$$H_1 = \{\text{AlphaCorp}, \text{BetaInc}\}, \quad H_2 = \{\text{GammaLLC}\},$$
$$H_1, H_2 \in \mathrm{PS}_1^*(X).$$

The second iterated power-set (nonempty) is

$$\mathrm{PS}_2^*(X) = \mathrm{PS}\big(\mathrm{PS}_1^*(X)\big) \setminus \{\varnothing\}.$$

In particular, the set

$$C = \{\, H_1,\, H_2\} \quad \in \quad \mathrm{PS}_2^*(X)$$

represents a parent company C that directly controls two subsidiaries H_1 and H_2, each of which in turn holds its constituent firms—a clear two-level corporate hierarchy.

Definition 5 (n-Super-HyperGraph)**.** *[47, 50, 51] Let V_0 be a finite base set and define $\mathrm{PS}^k(V_0)$ by iterating the power-set k times. An* n-Super-HyperGraph *is a pair*

$$\mathrm{SuHG}^{(n)} = (V, E),$$

where

$$V \subseteq \mathrm{PS}^n(V_0), \quad E \subseteq \mathrm{PS}^n(V_0).$$

Members of V are called n-supervertices *and members of E are* n-superedges*.*

Example 3 (Autonomous Robot Operating Modes as a 2-Super-HyperGraph)**.** *Consider the core software modules of an autonomous robot:*

$$V_0 = \{\text{Perception, Planning, Actuation}\}.$$

At the first level ($n = 1$), we form two functional clusters (1-supervertices):

$$\begin{aligned} c_1 &= \{\text{Perception, Planning}\}, \\ c_2 &= \{\text{Planning, Actuation}\}, \end{aligned} \quad V_1 = \{c_1,\, c_2\}.$$

At the second level ($n = 2$), we combine these clusters into two operational modes (2-supervertices):

$$\begin{array}{ll} s_{\text{full}} = \{c_1, c_2\}, & \textit{(full operation mode)} \\ s_{\text{nav}} = \{c_1\}, & \textit{(navigation-only mode)} \end{array} \quad V_2 = \{ s_{\text{full}}, s_{\text{nav}} \}.$$

We then introduce two 2-superedges to represent available mode selections:

$$\begin{array}{l} e_1 = \{ s_{\text{full}} \}, \\ e_2 = \{ s_{\text{nav}} \}, \end{array} \quad E_2 = \{e_1, e_2\}.$$

Thus the 2*-Super-HyperGraph is*

$$\mathrm{SuHG}^{(2)} = (V_2, E_2).$$

Here:

- $\mathrm{SuHG}^{(2)}$ *captures two levels of grouping: first into functional clusters, then into high-level operating modes.*
- *The superedge e_1 allows selection of the full operation mode, while e_2 restricts the robot to navigation only.*

This concrete 2*-Super-HyperGraph models how an autonomous robot's low-level modules combine into clusters, which in turn form distinct modes of operation in real-world missions.*

2.2. Neutrosophic n-Super-HyperGraph

A single-valued neutrosophic set assigns to each element three membership degrees—truth, indeterminacy, and falsity—in $[0, 1]$, with their sum at most three [23, 52]. A single-valued neutrosophic graph extends this concept to graphs, equipping both vertices and edges with neutrosophic memberships to model uncertain relationships [29, 53, 54]. A single-valued neutrosophic hypergraph further generalizes to hypergraphs, where each vertex–hyperedge incidence is endowed with its own triple of membership degrees under the same constraint [55–58]. We begin by recalling the definition of a single-valued neutrosophic hypergraph and then extend it to the framework of n-Super-HyperGraphs (cf. [55–58]).

Definition 6 (Neutrosophic Set)**.** *[23] Let X be a non-empty set. A Neutrosophic Set (NS) A on X is characterized by three membership functions:*

$$T_A : X \to [0, 1], \quad I_A : X \to [0, 1], \quad F_A : X \to [0, 1],$$

where for each $x \in X$, the values $T_A(x)$, $I_A(x)$, and $F_A(x)$ represent the degrees of truth, indeterminacy, and falsity, respectively. These values satisfy the following condition:

$$0 \le T_A(x) + I_A(x) + F_A(x) \le 3.$$

Definition 7 (Single-Valued Neutrosophic Graph)**.** *A* single-valued neutrosophic graph *(SVNG) is an ordered septuple*

$$G_N = \big(V,\, E,\, T_V,\, I_V,\, F_V,\, T_E,\, I_E,\, F_E\big),$$

where

- V *is a nonempty set of* vertices*,*
- $E \subseteq V \times V$ *is the set of* edges*,*
- $T_V,\, I_V,\, F_V \colon V \to [0,1]$ *are the* truth-, indeterminacy-, *and* falsity-*membership functions on vertices,*
- $T_E,\, I_E,\, F_E \colon E \to [0,1]$ *are the* truth-, indeterminacy-, *and* falsity-*membership functions on edges,*

satisfying the following conditions:

(i) For every vertex $v \in V$*,*

$$0 \;\le\; T_V(v) \;+\; I_V(v) \;+\; F_V(v) \;\le\; 3.$$

(ii) For every edge $e \in E$*,*

$$0 \;\le\; T_E(e) \;+\; I_E(e) \;+\; F_E(e) \;\le\; 3.$$

(iii) For each edge $e = (u,v) \in E$*, the edge-vertex consistency constraints hold:*

$$\begin{aligned} T_E(e) \;&\le\; \min\{T_V(u),\, T_V(v)\},\\ I_E(e) \;&\le\; \min\{I_V(u),\, I_V(v)\},\\ F_E(e) \;&\ge\; \max\{F_V(u),\, F_V(v)\}. \end{aligned}$$

Definition 8 (Single-Valued Neutrosophic Hypergraph)**.** *(cf.[57, 58]) Let* $V = \{v_1, \dots, v_N\}$ *be a finite vertex set, and let* $\{E_i\}_{i=1}^{M}$ *be a collection of non-empty neutrosophic subsets of* V *such that* $V = \bigcup_{i=1}^{M} \operatorname{supp}(E_i)$*. Each hyperedge* E_i *is specified by three membership functions*

$$T_{E_i},\, I_{E_i},\, F_{E_i} : V \to [0,1],$$

assigning to each vertex $v \in V$ *its truth, indeterminacy, and falsity degrees, respectively, and satisfying*

$$0 \;\le\; T_{E_i}(v) + I_{E_i}(v) + F_{E_i}(v) \;\le\; 3 \quad \forall\, v \in V.$$

We represent E_i *as the set*

$$E_i \;=\; \big\{(v,\, T_{E_i}(v),\, I_{E_i}(v),\, F_{E_i}(v)) : v \in V\big\}.$$

The pair $H = (V, \{E_i\})$ *is called a* single-valued neutrosophic hypergraph*.*

Example 4 (Air Pollution Events as a Single-Valued Neutrosophic Hypergraph)**.** *Consider a monitoring network for four common air pollutants:*

$$V = \{\mathrm{NO}_2,\ \mathrm{CO},\ \mathrm{PM}_{2.5},\ \mathrm{O}_3\}.$$

We model three types of pollution episodes as neutrosophic hyperedges:

$$E_1 = \{\mathrm{NO}_2, \mathrm{CO}, \mathrm{PM}_{2.5}, \mathrm{O}_3\} \quad (\textit{Smog event}),$$

$$E_2 = \{\mathrm{NO}_2, \mathrm{CO}\} \quad (\textit{Traffic pollution}),$$

$$E_3 = \{\mathrm{PM}_{2.5}, \mathrm{O}_3\} \quad (\textit{Industrial emission}).$$

For each E_i, define membership functions

$$T_{E_i},\, I_{E_i},\, F_{E_i} : V \to [0,1],$$

satisfying $0 \le T_{E_i}(v) + I_{E_i}(v) + F_{E_i}(v) \le 3$. A plausible assignment is:

Smog event** (E_1)**:

$$\begin{aligned} T_{E_1}(\mathrm{NO}_2) = 0.80, \quad I_{E_1}(\mathrm{NO}_2) = 0.10, \quad F_{E_1}(\mathrm{NO}_2) = 0.10,\\ T_{E_1}(\mathrm{CO}) = 0.75, \quad I_{E_1}(\mathrm{CO}) = 0.15, \quad F_{E_1}(\mathrm{CO}) = 0.10,\\ T_{E_1}(\mathrm{PM}_{2.5}) = 0.85, \quad I_{E_1}(\mathrm{PM}_{2.5}) = 0.10, \quad F_{E_1}(\mathrm{PM}_{2.5}) = 0.05,\\ T_{E_1}(\mathrm{O}_3) = 0.70, \quad I_{E_1}(\mathrm{O}_3) = 0.20, \quad F_{E_1}(\mathrm{O}_3) = 0.10; \end{aligned}$$

Traffic pollution** (E_2)**:

$$\begin{aligned} T_{E_2}(\mathrm{NO}_2) = 0.90, \quad I_{E_2}(\mathrm{NO}_2) = 0.05, \quad F_{E_2}(\mathrm{NO}_2) = 0.05,\\ T_{E_2}(\mathrm{CO}) = 0.88, \quad I_{E_2}(\mathrm{CO}) = 0.07, \quad F_{E_2}(\mathrm{CO}) = 0.05; \end{aligned}$$

Industrial emission** (E_3)**:

$$\begin{aligned} T_{E_3}(\mathrm{PM}_{2.5}) = 0.95, \quad I_{E_3}(\mathrm{PM}_{2.5}) = 0.03, \quad F_{E_3}(\mathrm{PM}_{2.5}) = 0.02,\\ T_{E_3}(\mathrm{O}_3) = 0.82, \quad I_{E_3}(\mathrm{O}_3) = 0.10, \quad F_{E_3}(\mathrm{O}_3) = 0.08. \end{aligned}$$

Then

$$E_i = \big\{(v,\, T_{E_i}(v),\, I_{E_i}(v),\, F_{E_i}(v)) : v \in E_i\big\}, \quad i = 1, 2, 3,$$

and the pair

$$H = \big(V, \{E_1, E_2, E_3\}\big)$$

is a single-valued neutrosophic hypergraph capturing both the membership degrees and the uncertainty of each pollutant in different pollution events.

Definition 9 (Neutrosophic n-Super-HyperGraph)**.** *Let V_0 be a finite ground set. Define iterated power-sets by*

$$\mathrm{PS}^0(V_0) = V_0, \quad \mathrm{PS}^{k+1}(V_0) = \mathrm{PS}\big(\mathrm{PS}^k(V_0)\big), \quad k \geq 0.$$

An n-Super-HyperGraph *is a pair* $\mathrm{SuHG}^{(n)} = (V, E)$ *with*

$$V \subseteq \mathrm{PS}^n(V_0), \quad E \subseteq \mathrm{PS}^n(V_0).$$

A neutrosophic n-Super-HyperGraph *enhances this structure by equipping both supervertices and superedges with neutrosophic memberships:*

$$\big(V,\, E,\, T_V,\, I_V,\, F_V,\, T_E,\, I_E,\, F_E\big),$$

where

- $T_V, I_V, F_V : V \to [0,1]$ *assign to each supervertex v its truth, indeterminacy, and falsity degrees, subject to*

$$0 \;\leq\; T_V(v) + I_V(v) + F_V(v) \;\leq\; 3 \quad \forall\, v \in V.$$

- $T_E, I_E, F_E : E \times V \to [0,1]$ *assign to each pair (e, v) the corresponding neutrosophic membership values, satisfying*

$$0 \;\leq\; T_E(e,v) + I_E(e,v) + F_E(e,v) \;\leq\; 3 \quad \forall\, e \in E,\; v \in V.$$

These must obey the containment constraints:

$$T_E(e,v) \leq T_V(v), \quad I_E(e,v) \leq I_V(v), \quad F_E(e,v) \leq F_V(v), \quad \forall\, e \in E,\; v \in V.$$

Example 5 (Organizational Workflow as a Neutrosophic 2-Super-HyperGraph)**.** *Consider a small organization with four employees:*

$$V_0 = \{\textit{Hiroko},\ \textit{Yutaka},\ \textit{Haruko},\ \textit{Shinya}\}.$$

They form two departments:

$$\textit{DeptA} = \{\textit{Hiroko}, \textit{Yutaka}\}, \quad \textit{DeptB} = \{\textit{Haruko}, \textit{Shinya}\},$$

so that

$$\mathrm{PS}^1(V_0) \supseteq V_1 = \{\textit{DeptA},\ \textit{DeptB}\}.$$

At the next level, these two departments collaborate on a joint division:

$$\mathrm{PS}^2(V_0) \supseteq V_2 = \{\textit{Div1}\}, \quad \textit{Div1} = \{\textit{DeptA}, \textit{DeptB}\}.$$

A single superedge models their shared project:

$$E_2 = \{\textit{ProjectX}\}, \quad \textit{ProjectX} = \{\textit{DeptA},\ \textit{DeptB}\}.$$

We assign neutrosophic membership degrees to each supervertex $v \in V_2$*:*

$$T_V(Div1) = 0.9, \quad I_V(Div1) = 0.05, \quad F_V(Div1) = 0.05,$$

indicating strong confidence that Div1 is active, with slight uncertainty or doubt. Likewise, for the superedge ProjectX and each constituent department v*:*

$$T_E(ProjectX, DeptA) = 0.85, \quad I_E(ProjectX, DeptA) = 0.10, \quad F_E(ProjectX, DeptA) = 0.05,$$
$$T_E(ProjectX, DeptB) = 0.80, \quad I_E(ProjectX, DeptB) = 0.15, \quad F_E(ProjectX, DeptB) = 0.05.$$

These values satisfy

$$0 \le T_V(v) + I_V(v) + F_V(v) \le 3, \quad 0 \le T_E(e, v) + I_E(e, v) + F_E(e, v) \le 3,$$

and the containment constraints $T_E(e, v) \le T_V(v)$, $I_E(e, v) \le I_V(v)$, $F_E(e, v) \le F_V(v)$ *hold for all* $v \in V_2$, $e \in E_2$. *Thus the tuple*

$$\big(V_2,\, E_2,\, T_V,\, I_V,\, F_V,\, T_E,\, I_E,\, F_E\big)$$

constitutes a neutrosophic 2*-Super-HyperGraph capturing the organization's hierarchical structure and uncertainty in collaboration.*

Example 6 (Corporate Divisions as a Neutrosophic 3-Super-HyperGraph)**.** *Consider a small company with four employees:*

$$V_0 = \{Hiroko,\ Yutaka,\ Haruko,\ Shinya\}.$$

These employees form teams:

$$T_1 = \{Hiroko, Yutaka\}, \quad T_2 = \{Haruko\}, \quad T_3 = \{Shinya\},$$

so that $\mathrm{PS}^1(V_0) \supseteq V_1 = \{T_1, T_2, T_3\}$.

At the next level, teams assemble into departments:

$$D_A = \{T_1, T_2\}, \quad D_B = \{T_3\},$$

giving $\mathrm{PS}^2(V_0) \supseteq V_2 = \{D_A, D_B\}$.

Finally, departments are grouped into divisions (the third power-set):

$$\mathrm{PS}^3(V_0) \supseteq V_3 = \big\{\, \{D_A, D_B\},\ \{D_A\},\ \{D_B\}\big\}.$$

Choose the set of supervertices and superedges as

$$V = \big\{S_1, S_2, S_3\big\}, \quad E = \{\, S_1\},$$

where

$$S_1 = \{D_A, D_B\}, \quad S_2 = \{D_A\}, \quad S_3 = \{D_B\}.$$

Assign neutrosophic membership degrees to each supervertex S_i:

$$\begin{aligned} T_V(S_1) &= 0.90, \quad I_V(S_1) = 0.05, \quad F_V(S_1) = 0.05, \\ T_V(S_2) &= 0.75, \quad I_V(S_2) = 0.15, \quad F_V(S_2) = 0.10, \\ T_V(S_3) &= 0.80, \quad I_V(S_3) = 0.10, \quad F_V(S_3) = 0.10, \end{aligned}$$

all satisfying $0 \le T_V + I_V + F_V \le 3$.

Next, for the single superedge S_1 and each supervertex $S_i \in V$, define:

$$\begin{aligned} T_E(S_1, S_1) &= 0.95, \quad I_E(S_1, S_1) = 0.03, \quad F_E(S_1, S_1) = 0.02, \\ T_E(S_1, S_2) &= 0.85, \quad I_E(S_1, S_2) = 0.10, \quad F_E(S_1, S_2) = 0.05, \\ T_E(S_1, S_3) &= 0.88, \quad I_E(S_1, S_3) = 0.07, \quad F_E(S_1, S_3) = 0.05. \end{aligned}$$

These values obey

$$0 \le T_E + I_E + F_E \le 3, \quad T_E(e, v) \le T_V(v), \ I_E(e, v) \le I_V(v), \ F_E(e, v) \le F_V(v)$$

for all (e, v). Consequently, the tuple

$$\big(V, E, T_V, I_V, F_V, T_E, I_E, F_E\big)$$

is a valid neutrosophic 3-Super-HyperGraph, modeling the company's four-level hierarchy under uncertainty.

2.3. Soft Super-HyperGraphs

Soft graph models represent a family of subgraphs indexed by a parameter set, where each parameter selects a particular subgraph on a fixed vertex–edge universe [7, 32, 33]. This notion coincides with the theory of soft sets—a parameterized collection of subsets that encodes uncertainty or preference without numerical membership values [19, 34]. Extending this to hypergraphs, a *soft hypergraph* assigns to each parameter a vertex subset and its induced hyperedges [35–38]. A *soft n-Super-HyperGraph* carries this idea into the layered setting of an n-Super-HyperGraph, producing a hierarchical, parameter-driven model [39].

Definition 10 (Soft Set)**.** *[7, 19] Let U be a universal set and A be a set of attributes. A soft set over U is a pair $(\mathcal{F}, S)$, where $S \subseteq A$ and $\mathcal{F} : S \to \mathcal{P}(U)$. Here, $\mathcal{P}(U)$ denotes the power set of U. Mathematically, a soft set is represented as:*

$$(\mathcal{F}, S) = \{(\alpha, \mathcal{F}(\alpha)) \mid \alpha \in S, \mathcal{F}(\alpha) \in \mathcal{P}(U)\}.$$

Each $\alpha \in S$ is called a parameter, and $\mathcal{F}(\alpha)$ is the set of elements in U associated with α.

Definition 11 (Soft Hypergraph)**.** *[35–38] Let $H = (V, E)$ be a hypergraph and let C be a nonempty set of parameters. A* soft hypergraph *over H with parameter set C is a quadruple*

$$\big(H, C, A, B\big),$$

where

$$A : C \to \mathrm{PS}(V), \quad B : C \to \mathrm{PS}(E),$$

and for each $c \in C$,

$$B(c) \subseteq \{e \in E : e \subseteq A(c)\}.$$

The induced pair $\big(A(c), B(c)\big)$ is called the soft subhypergraph *at parameter c.*

Example 7 (Monthly Training Programs as a Soft Hypergraph)**.** *Consider a training center offering the following courses:*

$$V = \{\text{Intro to Python, Data Analysis, Machine Learning, Deep Learning}\}.$$

Three certificate programs are defined by their required courses:

$$E_1 = \{\text{Intro to Python, Data Analysis}\}, \quad E_2 = \{\text{Data Analysis, Machine Learning}\}, \quad E_3 = \{\text{Machine Learn}$$

Let the parameter set denote the monthly sessions:

$$C = \{\text{May, June}\}.$$

Define the mappings A and B by

$$A(\text{May}) = \{\text{Intro to Python, Data Analysis, Machine Learning}\}, \quad B(\text{May}) = \{E_1, E_2\},$$

$$A(\text{June}) = \{\text{Data Analysis, Deep Learning}\}, \quad B(\text{June}) = \varnothing.$$

Here $B(c) \subseteq \{e \in E : e \subseteq A(c)\}$ for each $c \in C$, so $\big(H = (V, E), C, A, B\big)$ is a soft hypergraph showing which programs can run in each month.

Definition 12 (Soft n-Super-HyperGraph)**.** *Let $\mathrm{SuHG}^{(n)} = (V, E)$ be an n-Super-HyperGraph and C a nonempty parameter set. A* soft n-Super-HyperGraph *is the structure*

$$\big(V, E, C, A, B\big),$$

with

$$A : C \to \mathrm{PS}(V), \quad B : C \to \mathrm{PS}(E),$$

such that for every $c \in C$,

$$A(c) \subseteq V, \quad B(c) \subseteq \{\, e \in E : e \subseteq A(c)\}.$$

Each $\big(A(c), B(c)\big)$ forms a sub-Super-HyperGraph *of* $\mathrm{SuHG}^{(n)}$ *corresponding to parameter c.*

Example 8 (Consulting Engagements as a Soft 2-Super-HyperGraph)**.** *Consider a small consultancy with four consultants:*

$$V_0 = \{\textit{Hiroko, Yutaka, Haruko, Shinya}\}.$$

They form two teams:

$$T_1 = \{Hiroko, Yutaka\}, \quad T_2 = \{Haruko, Shinya\},$$

and these teams jointly run a division:

$$D = \{T_1, T_2\}.$$

Thus the 2*-supervertices are*

$$V = \{T_1, T_2, D\},$$

and we define superedges that model collaborations:

$$E = \{E_1, E_2, E_3\},$$

where

$$E_1 = \{T_1\}, \quad E_2 = \{T_2\}, \quad E_3 = \{T_1, T_2\}.$$

This yields the 2*-Super-HyperGraph* $\mathrm{SuHG}^{(2)} = (V, E)$*.*

Next, let the parameter set represent two ongoing projects:

$$C = \{Alpha, Beta\}.$$

Define

$$A: C \to \mathrm{PS}(V), \quad B: C \to \mathrm{PS}(E),$$

by

$$A(Alpha) = \{T_1, D\}, \quad B(Alpha) = \{E_1, E_3\},$$
$$A(Beta) = \{T_2\}, \qquad B(Beta) = \{E_2\}.$$

Observe that

$$B(c) \subseteq \{e \in E : e \subseteq A(c)\} \quad \text{for each } c \in C,$$

so $(A(c), B(c))$ *is indeed a sub-Super-HyperGraph for each project. Concretely:*

- ***Project Alpha:*** *involves Team 1 and the Division D.*

$$A(Alpha) = \{T_1, D\}, \quad B(Alpha) = \{\{T_1\}, \{T_1, T_2\}\}.$$

- ***Project Beta:*** *involves only Team 2.*

$$A(Beta) = \{T_2\}, \quad B(Beta) = \{\{T_2\}\}.$$

Hence the tuple

$$(V, E, C, A, B)$$

is a soft 2-Super-HyperGraph *modeling how different projects select subsets of the company's hierarchical structure.*

Example 9 (Strategic Initiatives as a Soft 3-Super-HyperGraph)**.** *Consider a consultancy with four consultants:*

$$V_0 = \{Hiroko,\ Yutaka,\ Haruko,\ Shinya\}.$$

They form two teams:

$$T_1 = \{Hiroko, Yutaka\}, \quad T_2 = \{Haruko, Shinya\},$$

and these teams assemble into departments:

$$D_1 = \{T_1\}, \quad D_2 = \{T_2\}.$$

At the third level, departments are grouped into divisions:

$$V_3 \subseteq \mathrm{PS}^3(V_0) \;=\; \big\{\{D_1\}, \{D_2\}, \{D_1, D_2\}\big\}.$$

Select as our supervertices and superedges:

$$V = \{S_1, S_2, S_3\}, \quad E = \{E_1, E_2\},$$

with

$$S_1 = \{D_1, D_2\}, \quad S_2 = \{D_1\}, \qquad S_3 = \{D_2\},$$
$$E_1 = \{S_1, S_2\}, \quad E_2 = \{S_1, S_3\}.$$

Thus $\mathrm{SuHG}^{(3)} = (V, E)$ *models the firm's four-level hierarchy.*

Suppose the firm runs two concurrent strategic initiatives:

$$C = \{Alpha,\ Beta\}.$$

Define soft-selection maps

$$A : C \to \mathrm{PS}(V), \quad B : C \to \mathrm{PS}(E),$$

by

$$A(Alpha) = \{S_1,\ S_2\}, \quad B(Alpha) = \{E_1\},$$
$$A(Beta) = \{S_1,\ S_3\}, \quad B(Beta) = \{E_2\}.$$

One checks easily that for each $c \in C$*,*

$$B(c) \;\subseteq\; \{\, e \in E : e \subseteq A(c)\},$$

so $(A(c), B(c))$ *is a valid sub-Super-HyperGraph of* $\mathrm{SuHG}^{(3)}$*. Concretely:*

- ***Initiative Alpha:*** *involves Division* S_1 *(both departments) and* S_2 *(Dept. 1 only), modeling the kick-off phase:*

$$A(Alpha) = \{S_1, S_2\}, \quad B(Alpha) = \{\{S_1, S_2\}\}.$$

- ***Initiative Beta:*** *involves Division S_1 and S_3 (Dept. 2), representing a spin-off project:*

$$A(Beta) = \{S_1, S_3\}, \quad B(Beta) = \{\{S_1, S_3\}\}.$$

Hence the tuple

$$\big(V,\, E,\, C,\, A,\, B\big)$$

is a soft 3-Super-HyperGraph*, capturing how different initiatives select subsets of the organization's three-level superstructure.*

2.4. Neutrosophic Soft HyperGraph

A neutrosophic soft set is a parameterized collection of single-valued neutrosophic subsets, modeling uncertainty across distinct elements under each parameter (cf.[59–61]). A neutrosophic soft graph is a set of parameterized neutrosophic assignments to graph vertices and edges, capturing uncertain structural preferences (cf.[62–64]). We now introduce the notion of a neutrosophic soft hypergraph, which combines the ideas of single-valued neutrosophic sets and soft sets on the underlying hypergraph (cf.[37, 65]).

Definition 13 (Neutrosophic Soft Set)**.** *(cf.[59–61]) Let U be a nonempty* universe of discourse *and let E be a nonempty set of* parameters*. Fix a subset $A \subseteq E$. A* neutrosophic soft set *over U with parameter set A is a pair*

$$(F, A),$$

where

$$F\colon A \longrightarrow \mathrm{SNS}(U),$$

and for each $c \in A$,

$$F(c) = \big\{\, \big(u,\ T_F(c;u),\ I_F(c;u),\ F_F(c;u)\big) : u \in U \big\}.$$

Here

$$T_F,\ I_F,\ F_F\colon A \times U \ \to\ [0,1]$$

are the truth*,* indeterminacy*, and* falsity *membership functions, satisfying the neutrosophic constraint*

$$0 \ \le\ T_F(c;u) \ +\ I_F(c;u) \ +\ F_F(c;u) \ \le\ 3 \quad \forall\, c \in A,\ \forall\, u \in U.$$

Definition 14 (Neutrosophic Soft Graph)**.** *[62–64] Let $G^* = (V, E, T_V, I_V, F_V, T_E, I_E, F_E)$ be a single-valued neutrosophic graph, where*

$$T_V, I_V, F_V : V \to [0,1], \quad T_E, I_E, F_E : E \times V \to [0,1],$$

satisfy the usual neutrosophic constraints (see Definition 3). Let C be a nonempty set of parameters. A neutrosophic soft graph *over G^* with parameter set C is a 4-tuple*

$$\big(G^*,\, C,\, R,\, S\big),$$

where

$$R : C \longrightarrow \mathrm{SNS}(V), \quad S : C \longrightarrow \mathrm{SNS}(E),$$

and for each $c \in C$*:*

$$R(c) = \{(v, T_R(c; v), I_R(c; v), F_R(c; v)) : v \in V\},$$

$$S(c) = \{(e, T_S(c; e), I_S(c; e), F_S(c; e)) : e \in E\},$$

with

$$0 \le T_R(c; v) + I_R(c; v) + F_R(c; v) \le 3,$$

$$0 \le T_S(c; e) + I_S(c; e) + F_S(c; e) \le 3,$$

and the following consistency conditions for every $e = \{u, v\} \in E$*:*

$$T_S(c; e) \le \min\{T_R(c; u), T_R(c; v)\},$$

$$I_S(c; e) \le \min\{I_R(c; u), I_R(c; v)\},$$

$$F_S(c; e) \ge \max\{F_R(c; u), F_R(c; v)\}.$$

Finally, we require the coverage condition

$$\bigcup_{e \in E} \mathrm{supp}\big(S(c; e)\big) = V \quad \textit{for each } c \in C,$$

so that every vertex appears in at least one neutrosophic soft edge under each parameter.

Example 10 (Contextual Trust Network as a Neutrosophic Soft Graph)**.** *Let*

$$V = \{\textit{Hiroko}, \textit{Tae}, \textit{Yusuke}\}$$

and

$$E = \{ e_1 = \{\textit{Hiroko}, \textit{Tae}\}, e_2 = \{\textit{Tae}, \textit{Yusuke}\}\}.$$

Define two contexts $C = \{\textit{Work}, \textit{Friends}\}$*. For each context* $c \in C$*, we assign neutrosophic soft vertex- and edge-memberships* $R(c)$ *and* $S(c)$ *as follows:*

Work context $(c = \textbf{\textit{Work}})$***:***

$$R(\textit{Work}) = \{(\textit{Hiroko}, 0.90, 0.05, 0.05),$$

$$(\textit{Tae}, 0.85, 0.10, 0.05), (\textit{Yusuke}, 0.80, 0.15, 0.05)\},$$

$$S(\textit{Work}) = \{(e_1, 0.80, 0.10, 0.10), (e_2, 0.60, 0.20, 0.20)\}.$$

Friends context $(c = \textbf{\textit{Friends}})$***:***

$$R(\textit{Friends}) = \{(\textit{Hiroko}, 0.70, 0.20, 0.10),$$

$$(\textit{Tae}, 0.90, 0.05, 0.05), (\textit{Yusuke}, 0.95, 0.03, 0.02)\},$$

$$S(Friends) = \big\{(e_1, 0.75, 0.15, 0.10),$$
$$(e_2, 0.85, 0.10, 0.05)\big\}.$$

One verifies for each $c \in C$:

$$0 \le T_R(c;v) + I_R(c;v) + F_R(c;v) \le 3,$$

$$0 \le T_S(c;e) + I_S(c;e) + F_S(c;e) \le 3,$$

and for each edge $e = \{u, v\}$:

$$T_S(c;e) \le \min\{T_R(c;u), T_R(c;v)\},$$

$$I_S(c;e) \le \min\{I_R(c;u), I_R(c;v)\},$$

$$F_S(c;e) \ge \max\{F_R(c;u), F_R(c;v)\}.$$

Finally, every vertex appears in some soft edge under each context, so $\big(G^*, C, R, S\big)$ is a valid neutrosophic soft graph modeling context-dependent trust relationships.

Definition 15 (Neutrosophic Soft Hypergraph)**.** *(cf.[37, 65]) Let $H = (V, E)$ be a finite crisp hypergraph, and let C be a nonempty set of parameters. A* neutrosophic soft hypergraph *over H with parameters C is a triple*

$$\big(R,\ S,\ C\big),$$

together with mappings

$$R : C \longrightarrow SNS(V), \qquad S : C \longrightarrow SNS(E),$$

where $SNS(X)$ denotes the family of single-valued neutrosophic subsets of X. Concretely, for each $c \in C$:

- $R(c) = \{\,\big(v,\, T_R(c;v),\, I_R(c;v),\, F_R(c;v)\big) : v \in V\}$ *satisfies*

$$0 \ \le\ T_R(c;v) + I_R(c;v) + F_R(c;v) \ \le\ 3, \quad \forall\, v \in V.$$

- $S(c) = \{\,\big(e,\, T_S(c;e),\, I_S(c;e),\, F_S(c;e)\big) : e \in E\}$ *satisfies*

$$0 \ \le\ T_S(c;e) + I_S(c;e) + F_S(c;e) \ \le\ 3, \quad \forall\, e \in E,$$

  and moreover

$$T_S(c;e) \ \le\ \min_{v \in e} T_R(c;v),$$
$$I_S(c;e) \ \le\ \min_{v \in e} I_R(c;v),$$
$$F_S(c;e) \ \le\ \max_{v \in e} F_R(c;v).$$

Finally, we require the coverage condition

$$\bigcup_{e \in E} \operatorname{supp}\big(S(c;e)\big) \;=\; V \quad \textit{for each } c \in C,$$

so that every vertex appears in some neutrosophic soft hyperedge under each parameter.

Example 11 (Medical Diagnosis as a Neutrosophic Soft Hypergraph)**.** *Consider a medical-diagnosis hypergraph where*

$$V = \{\textit{Fever},\ \textit{Cough},\ \textit{Headache}\}$$

and two diseases are modeled as hyperedges:

$$E = \big\{\ \textit{Flu} = \{\textit{Fever}, \textit{Cough}, \textit{Headache}\},\ \ \textit{Cold} = \{\textit{Cough}, \textit{Headache}\}\big\}.$$

Let the parameter set C represent two patients:

$$C = \{\textit{Patient}_1,\ \textit{Patient}_2\}.$$

We define neutrosophic-soft mappings

$$R : C \to \mathrm{SNS}(V), \qquad S : C \to \mathrm{SNS}(E).$$

For each $c \in C$, $R(c)$ assigns to each symptom $v \in V$ a triple $\big(T_R(c;v), I_R(c;v), F_R(c;v)\big)$ and $S(c)$ assigns to each disease $e \in E$ a triple $\big(T_S(c;e), I_S(c;e), F_S(c;e)\big)$.

Patient_1:

$$R(\textit{Patient}_1): \begin{array}{l} (\textit{Fever},\ 0.90,\ 0.05,\ 0.05), \\ (\textit{Cough},\ 0.80,\ 0.10,\ 0.10), \\ (\textit{Headache},\ 0.70,\ 0.20,\ 0.10); \end{array} \qquad S(\textit{Patient}_1): \begin{cases} \textit{Flu}:\ (0.85,\ 0.10,\ 0.05), \\ \textit{Cold}:\ (0.60,\ 0.25,\ 0.15). \end{cases}$$

Patient_2:

$$R(\textit{Patient}_2): \begin{array}{l} (\textit{Fever},\ 0.20,\ 0.30,\ 0.50), \\ (\textit{Cough},\ 0.65,\ 0.20,\ 0.15), \\ (\textit{Headache},\ 0.50,\ 0.30,\ 0.20); \end{array} \qquad S(\textit{Patient}_2): \begin{cases} \textit{Flu}:\ (0.40,\ 0.30,\ 0.30), \\ \textit{Cold}:\ (0.55,\ 0.25,\ 0.20). \end{cases}$$

One checks for each c, each v, $0 \le T_R + I_R + F_R \le 3$, and similarly for S. Moreover,

$$T_S(c;e) \le \min_{v \in e} T_R(c;v), \quad I_S(c;e) \le \min_{v \in e} I_R(c;v), \quad F_S(c;e) \le \max_{v \in e} F_R(c;v).$$

Finally, every symptom appears in at least one soft hyperedge: $\bigcup_{e \in E} \operatorname{supp}(S(c;e)) = V$. Hence $\big(R, S, C\big)$ is a concrete neutrosophic soft hypergraph modeling uncertain symptom–disease relationships in two patient cases.

3. Main Results

This section presents the main results of the paper.

3.1. Neutrosophic Soft n-Super-HyperGraph

We provide below the formal definition of a *Neutrosophic Soft n-Super-HyperGraph.* This concept applies the principles of neutrosophic sets and soft sets to the framework of n-Super-HyperGraphs.

Definition 16 (Neutrosophic Soft n-Super-HyperGraph)**.** *Let* $\mathrm{SuHG}^{(n)} = (V, E)$ *be an n-Super-HyperGraph on base set* V_0*, and let* C *be a nonempty parameter set. A* neutrosophic soft n-Super-HyperGraph *is a septuple*

$$\big(V,\, E,\, C,\, A,\, B,\, T_V,\, I_V,\, F_V,\, T_E,\, I_E,\, F_E\big),$$

where

- $A : C \to \mathrm{PS}(V)$ *and* $B : C \to \mathrm{PS}(E)$ *satisfy*

$$A(c) \subseteq V, \quad B(c) \subseteq \{\, e \in E : e \subseteq A(c)\}, \quad \forall\, c \in C.$$

- $T_V, I_V, F_V : C \times V \to [0, 1]$ *assign to each* (c, v) *its truth, indeterminacy, and falsity degrees, with*

$$0 \leq T_V(c, v) + I_V(c, v) + F_V(c, v) \leq 3 \quad \forall\, c \in C,\ v \in V.$$

- $T_E, I_E, F_E : C \times E \times V \to [0, 1]$ *assign to each triple* (c, e, v) *its neutrosophic membership, subject to*

$$0 \leq T_E(c, e, v) + I_E(c, e, v) + F_E(c, e, v) \leq 3 \quad \forall\, c \in C,\ e \in E,\ v \in V.$$

Moreover, the following containment constraints *must hold for all* $c \in C$*,* $e \in E$*, and* $v \in V$*:*

$$T_E(c, e, v) \leq T_V(c, v), \quad I_E(c, e, v) \leq I_V(c, v), \quad F_E(c, e, v) \leq F_V(c, v).$$

Example 12 (Disaster Response Network as a Neutrosophic Soft 2-Super-HyperGraph)**.** *Consider an emergency response system composed of four local units:*

$$V_0 = \{\mathit{Unit\ A},\ \mathit{Unit\ B},\ \mathit{Unit\ C},\ \mathit{Unit\ D}\}.$$

They form two squads:

$$\mathit{Squad}_1 = \{\mathit{Unit\ A}, \mathit{Unit\ B}\}, \quad \mathit{Squad}_2 = \{\mathit{Unit\ C}, \mathit{Unit\ D}\},$$

so that $\mathrm{PS}^1(V_0) \supseteq V_1 = \{Squad_1, Squad_2\}$. *At the next level, these squads combine into a regional command:*

$$\mathrm{PS}^2(V_0) \supseteq V_2 = \{Region\}, \quad Region = \{Squad_1, Squad_2\}.$$

We take the single superedge to model a coordinated operation:

$$E_2 = \{Operation\}, \quad Operation = \{Squad_1, Squad_2\}.$$

Thus $\mathrm{SuHG}^{(2)} = (V_2, E_2)$ *is our* 2*-Super-HyperGraph.*

Next, let the parameter set represent two disaster scenarios:

$$C = \{Earthquake,\ Flood\}.$$

We define

$$A : C \to \mathrm{PS}(V_2), \quad B : C \to \mathrm{PS}(E_2),$$

by

$$A(Earthquake) = \{Region,\ Squad_1\}, \quad B(Earthquake) = \{Operation\},$$
$$A(Flood) = \{Region,\ Squad_2\}, \qquad B(Flood) = \{Operation\}.$$

Clearly $A(c) \subseteq V_2$ *and* $B(c) \subseteq \{e : e \subseteq A(c)\}$ *for each* c.

Finally, assign neutrosophic-soft membership degrees:

Vertex memberships (T_V, I_V, F_V)***:***

	Earthquake			*Flood*		
v	T_V	I_V	F_V	T_V	I_V	F_V
Region	0.85	0.10	0.05	0.80	0.15	0.05
$Squad_1$	0.90	0.08	0.02	0.60	0.30	0.10
$Squad_2$	0.70	0.20	0.10	0.88	0.10	0.02

Each row sums to at most 3, satisfying $0 \le T_V + I_V + F_V \le 3$.

Edge–vertex memberships (T_E, I_E, F_E)***:***

	Earthquake			*Flood*		
v	T_E	I_E	F_E	T_E	I_E	F_E
$Squad_1$	0.88	0.10	0.02	0	0	0
$Squad_2$	0.80	0.15	0.05	0.85	0.12	0.03

Entries for which $v \notin A(c)$ *are zero. One checks for each* (c, e, v)*:*

$$T_E(c, e, v) \le T_V(c, v), \quad I_E(c, e, v) \le I_V(c, v), \quad F_E(c, e, v) \le F_V(c, v).$$

Therefore, the tuple

$$\big(V_2,\ E_2,\ C,\ A,\ B,\ T_V,\ I_V,\ F_V,\ T_E,\ I_E,\ F_E\big)$$

is a concrete neutrosophic soft 2*-Super-HyperGraph modeling how different disaster scenarios select and evaluate subsets of the response network under uncertainty.*

Example 13 (Autonomous Vehicle Sensor Fusion as a Neutrosophic Soft 1-Super-HyperGraph)**.** *In an autonomous driving system, four sensor types collect environmental data:*

$$V_0 = \{Camera,\ Lidar,\ Radar,\ Microphone\}.$$

We group them into three sensor clusters*:*

$$\begin{aligned} S_{\text{vision}} &= \{Camera,\ Lidar\}, \\ S_{\text{radar}} &= \{Radar\}, \\ S_{\text{audio}} &= \{Microphone\}, \end{aligned}$$

so that our level-1 supervertices are

$$V = \{\, S_{\text{vision}},\, S_{\text{radar}},\, S_{\text{audio}}\,\}.$$

We model the fusion module as a single hyperedge linking all clusters:

$$E = \{\, e_{\text{fusion}}\}, \quad e_{\text{fusion}} = \{\, S_{\text{vision}},\, S_{\text{radar}},\, S_{\text{audio}}\,\}.$$

Thus $\text{SuHG}^{(1)} = (V, E)$.

Different driving scenarios induce uncertainty in sensor reliability. Let

$$C = \{Highway,\ Urban\}$$

be our parameter set. Define

$$A : C \to \text{PS}(V), \qquad B : C \to \text{PS}(E),$$

by

$$\begin{aligned} A(Highway) &= \{S_{\text{vision}},\, S_{\text{radar}}\}, \quad B(Highway) = \{e_{\text{fusion}}\}, \\ A(Urban) &= \{S_{\text{vision}},\, S_{\text{audio}}\}, \qquad B(Urban) = \{e_{\text{fusion}}\}. \end{aligned}$$

Clearly $B(c) \subseteq \{e \in E : e \subseteq A(c)\}$ *for each* c.

We now assign neutrosophic-soft memberships:

Vertex memberships (T_V, I_V, F_V)*:*

	Highway			*Urban*		
Cluster	T_V	I_V	F_V	T_V	I_V	F_V
S_{vision}	0.95	0.03	0.02	0.90	0.07	0.03
S_{radar}	0.92	0.05	0.03	0	0	0
S_{audio}	0	0	0	0.85	0.10	0.05

Each row meets $0 \le T_V + I_V + F_V \le 3$. *A zero entry reflects a cluster not used in that scenario.*

Edge–vertex memberships (T_E, I_E, F_E)***:*** *For the single fusion edge* e_{fusion}*, we record memberships only for clusters in* $A(c)$*:*

	Highway			*Urban*		
Cluster	T_E	I_E	F_E	T_E	I_E	F_E
S_{vision}	0.93	0.05	0.02	0.88	0.08	0.04
S_{radar}	0.90	0.07	0.03	0	0	0
S_{audio}	0	0	0	0.82	0.12	0.06

One checks $0 \le T_E + I_E + F_E \le 3$ *and the* containment constraints $T_E(c, e, v) \le T_V(c, v)$, $I_E(c, e, v) \le I_V(c, v)$, *and* $F_E(c, e, v) \le F_V(c, v)$ *for each* $(c, e_{\text{fusion}}, v)$.

Hence the tuple

$$\big(V,\, E,\, C,\, A,\, B,\, T_V,\, I_V,\, F_V,\, T_E,\, I_E,\, F_E\big)$$

constitutes a concrete neutrosophic soft 1-Super-HyperGraph *capturing how sensor clusters are selected and weighted under uncertainty in different driving scenarios.*

Example 14 (Autonomous Robot Control as a Neutrosophic Soft 2-Super-HyperGraph)**.** *We model an autonomous robot's software architecture under varying mission contexts.*

Base Modules (level 0):

$$V_0 = \{\textit{Perception},\ \textit{Planning},\ \textit{Communication}\}.$$

Level-1 supervertices (modules):

$$V_1 = \{\ \textit{Perception},\ \textit{Planning},\ \textit{Communication}\}.$$

Level-2 supervertices (functional clusters):

$$\begin{aligned} \text{Nav} &= \{\textit{Perception},\ \textit{Planning}\}, \\ \text{CommOps} &= \{\textit{Perception},\ \textit{Communication}\}. \end{aligned} \quad V_2 = \{\text{Nav},\ \text{CommOps}\}.$$

Level-2 superedges (operation modes):

$$\begin{aligned} e_{\text{full}} &= \{\text{Nav},\ \text{CommOps}\}, && \textit{(full system mode)} \\ e_{\text{nav}} &= \{\text{Nav}\}, && \textit{(navigation-only mode)} \end{aligned} \quad E_2 = \{\, e_{\text{full}},\, e_{\text{nav}}\,\}.$$

Thus $\text{SuHG}^{(2)} = (V_2, E_2)$ *is our* 2*-Super-HyperGraph.*

Parameters (missions):

$$C = \{\textit{Rescue},\ \textit{Survey}\}.$$

Define the selection maps

$$A : C \to \text{PS}(V_2), \quad B : C \to \text{PS}(E_2),$$

by

$$A(\mathit{Rescue}) = \{\text{Nav, CommOps}\}, \quad B(\mathit{Rescue}) = \{e_{\text{full}}\},$$
$$A(\mathit{Survey}) = \{\text{Nav}\}, \qquad B(\mathit{Survey}) = \{e_{\text{nav}}\}.$$

One checks $A(c) \subseteq V_2$ *and* $B(c) \subseteq \{e : e \subseteq A(c)\}$.

Neutrosophic-soft memberships (T_V, I_V, F_V)***:***

	Rescue			*Survey*		
v	T_V	I_V	F_V	T_V	I_V	F_V
Nav	0.90	0.05	0.05	0.80	0.15	0.05
CommOps	0.85	0.10	0.05	0	0	0

Each row satisfies $0 \le T_V + I_V + F_V \le 3$. *zeros reflect unused clusters.*

Neutrosophic-soft memberships (T_E, I_E, F_E)***:***

	Rescue, e_{full}			*Survey*, e_{nav}		
v	T_E	I_E	F_E	T_E	I_E	F_E
Nav	0.88	0.10	0.02	0.78	0.20	0.02
CommOps	0.82	0.15	0.03	0	0	0

One verifies for all (c, e, v)*:*

$$0 \le T_E + I_E + F_E \le 3, \quad T_E(c,e,v) \le T_V(c,v), \quad I_E(c,e,v) \le I_V(c,v), \quad F_E(c,e,v) \le F_V(c,v).$$

Hence the tuple

$$\bigl(V_2,\, E_2,\, C,\, A,\, B,\, T_V,\, I_V,\, F_V,\, T_E,\, I_E,\, F_E\bigr)$$

is a detailed neutrosophic soft 2-Super-HyperGraph *representing how an autonomous robot's functional clusters are selected and weighted under different mission scenarios.*

Theorem 1 (Generalization of Neutrosophic and Soft Models)**.** *Every neutrosophic n-Super-HyperGraph and every soft n-Super-HyperGraph can be realized as a special case of a neutrosophic soft n-Super-HyperGraph.*

Proof. We split the proof into two parts.

(i) Embedding a neutrosophic n-Super-HyperGraph. Suppose we are given a neutrosophic n-Super-HyperGraph

$$H_{\text{neut}} = (V,\, E,\, \widetilde{T}_V,\, \widetilde{I}_V,\, \widetilde{F}_V,\, \widetilde{T}_E,\, \widetilde{I}_E,\, \widetilde{F}_E).$$

Define a neutrosophic soft structure by choosing

$$C = \{*\}, \quad A(*) = V, \quad B(*) = E,$$

and let

$$T_V(*, v) = \widetilde{T}_V(v), \quad I_V(*, v) = \widetilde{I}_V(v), \quad F_V(*, v) = \widetilde{F}_V(v),$$

$$T_E(*, e, v) = \widetilde{T}_E(e, v), \quad I_E(*, e, v) = \widetilde{I}_E(e, v), \quad F_E(*, e, v) = \widetilde{F}_E(e, v).$$

All parameter-indexed constraints reduce to those of H_{neut}. In particular, since $A(*) = V$ and $B(*) = E$, the containment constraints

$$T_E(*, e, v) \leq T_V(*, v), \ldots$$

are exactly those in the original neutrosophic model. Thus H_{neut} embeds as a neutrosophic soft n-Super-HyperGraph with a single parameter.

(ii) Embedding a soft n-Super-HyperGraph. Now let

$$H_{\text{soft}} = (V,\, E,\, C,\, A,\, B)$$

be a soft n-Super-HyperGraph. We define neutrosophic memberships by

$$T_V(c, v) = \begin{cases} 1, & v \in A(c), \\ 0, & v \notin A(c), \end{cases} \quad I_V(c, v) = F_V(c, v) = 0,$$

and

$$T_E(c, e, v) = \begin{cases} 1, & e \in B(c) \text{ and } v \in e, \\ 0, & \text{otherwise}, \end{cases} \quad I_E(c, e, v) = F_E(c, e, v) = 0.$$

Then $0 \leq T_V + I_V + F_V \leq 3$ and $0 \leq T_E + I_E + F_E \leq 3$ hold trivially. Moreover, if $e \in B(c)$ and $v \in e \subseteq A(c)$, then $T_E(c, e, v) = 1 \leq T_V(c, v) = 1$; all other containment constraints hold since both sides vanish. Hence H_{soft} arises as a neutrosophic soft n-Super-HyperGraph with binary membership values.

Combining (i) and (ii), we see that the neutrosophic soft n-Super-HyperGraph framework simultaneously extends both the pure neutrosophic and the soft models.

Theorem 2 (Neutrosophic Soft n-Super-HyperGraph Generalizes Neutrosophic Soft Hypergraph)**.** *Let $\big(H = (V, E),\, C,\, R,\, S\big)$ be any neutrosophic soft hypergraph. Then it can be viewed as a neutrosophic soft 1-Super-HyperGraph by setting*

$$\text{SuHG}^{(1)} = (V, E), \quad A = C,$$

and defining the neutrosophic soft membership functions on the level-1 supervertices and superedges by

$$T_V(c; v) = T_R(c; v), \;\; I_V(c; v) = I_R(c; v), \;\; F_V(c; v) = F_R(c; v),$$

$$T_E(c; e, v) = \begin{cases} T_S(c; e), & v \in e, \\ 0, & v \notin e, \end{cases}$$

$$I_E(c; e, v) = \begin{cases} I_S(c; e), & v \in e, \\ 0, & v \notin e, \end{cases}$$

$$F_E(c; e, v) = \begin{cases} F_S(c; e), & v \in e, \\ 0, & v \notin e. \end{cases}$$

Then the resulting tuple

$$\big(V,\, E,\, C,\, T_V,\, I_V,\, F_V,\, T_E,\, I_E,\, F_E\big)$$

satisfies all the axioms of a neutrosophic soft 1*-Super-HyperGraph, and reproduces exactly the original neutrosophic soft hypergraph under the identification of level-1 superstructures with* (V, E)*.*

Proof. Let (H, C, R, S) be given. We check each requirement in the definition of a neutrosophic soft 1-Super-HyperGraph:

(i) Parameterized vertex sets. By construction, for every $c \in C$,

$$A(c) \;=\; V \quad \text{and}$$

$$T_V(c; v) = T_R(c; v),\; I_V(c; v) = I_R(c; v),\; F_V(c; v) = F_R(c; v),$$

so

$$0 \le T_V(c; v) + I_V(c; v) + F_V(c; v) \le 3$$
$$\forall\, v \in V.$$

(ii) Parameterized edge–vertex memberships. For each triple (c, e, v), define

$$T_E(c; e, v), I_E(c; e, v), F_E(c; e, v)$$

as above. If $v \notin e$, all three are zero, and hence

$$0 \le T_E + I_E + F_E \le 3.$$

If $v \in e$, then

$$T_E(c; e, v) = T_S(c; e) \le \min_{u \in e} T_R(c; u) = \min_{u \in e} T_V(c; u) \le T_V(c; v),$$

and similarly $I_E(c; e, v) \le I_V(c; v)$, $F_E(c; e, v) \le F_V(c; v)$. Thus the containment constraints hold.

(iii) Coverage condition. Since in the original neutrosophic soft hypergraph each parameter c satisfies $\bigcup_{e \in E} \operatorname{supp}\big(S(c; e)\big) = V$, it follows immediately that in the level-1 Super-HyperGraph each vertex v has $T_V(c; v) > 0$ for some edge–vertex pair (e, v).

All remaining axioms (non-negativity, upper bound 3, etc.) follow at once from the corresponding properties of R and S. Hence the construction yields a valid neutrosophic soft 1-Super-HyperGraph whose projections onto the original vertex and edge sets recover exactly the given neutrosophic soft hypergraph.

Algorithm 1 Constructing a Neutrosophic Soft n-Super-HyperGraph

Require: Base set V_0, integer $n > 0$, parameters C,
soft–selection $A, B\colon C \to \mathrm{PS}^n(V_0)$ with $B(c) \subseteq \{e \subseteq A(c)\}$,
initial neutrosophic maps $T_V^0, I_V^0, F_V^0 : V_0 \to [0,1]$,
$T_E^0, I_E^0, F_E^0 \colon E_n \times V_n \to [0,1]$.

Ensure: Neutrosophic soft n-Super-HyperGraph $H^* = (V_n, E_n, C, A, B, T_V, I_V, F_V, T_E, I_E, F_E)$.

// 1. build underlying n-super-hypergraph
$V' \leftarrow V_0$
for $k = 1$ to n **do**
  $V_k \leftarrow \mathrm{PS}(V'), \quad E_k \leftarrow V_k$
  $V' \leftarrow V_k$
end for
$V_n \leftarrow V_n$, $E_n \leftarrow E_n$
// 2. initialize neutrosophic memberships
for each $c \in C$ **do**
  for each $v \in V_n$ **do**
    $\big(T_V, I_V, F_V\big)(c,v) \leftarrow \begin{cases} (T_V^0, I_V^0, F_V^0)(v), & v \in A(c) \\ (0,0,0), & \text{otherwise.} \end{cases}$
  end for
  for each $e \in E_n$ **do**
    for each $v \in V_n$ **do**
      $(T_E, I_E, F_E)(c,e,v) \leftarrow \begin{cases} \big(\min(T_E^0(e,v), T_V(c,v)),\ \min(I_E^0(e,v), I_V(c,v)), & \\ \min(F_E^0(e,v), F_V(c,v))\big), & v \in e, \\ (0,0,0), & v \notin e. \end{cases}$
    end for
  end for
end for
// 3. enforce neutrosophic constraints
for each (c,v) **do**
  normalize if $T_V + I_V + F_V > 3$
end for
for each (c,e,v) **do**
  normalize if $T_E + I_E + F_E > 3$
end for
return H^*

4. Additional Result: Algorithm for Constructing a Neutrosophic Soft n-Super-HyperGraph

In this section, we present the algorithm for constructing a Neutrosophic Soft n-Super-HyperGraph. The algorithm 1 is given below.

Example 15 (Applying Algorithm 1 to a Disaster-Response Network)**.** *We illustrate each step of the construction algorithm in the context of a two–level emergency response system.*

1. Problem data.

$$V_0 = \{\text{UnitA, UnitB, UnitC, UnitD}\}, \quad n = 2, \quad C = \{\textit{Earthquake}, \textit{Flood}\}.$$

Define the soft-selection maps

$$A(c),\, B(c) \subseteq \mathrm{PS}^2(V_0)$$

by

$$\begin{aligned} &\textit{Earthquake}: \quad A = \{\text{Region, Squad}_1\}, \quad B = \{\text{Operation}\}, \\ &\textit{Flood}: \qquad\quad A = \{\text{Region, Squad}_2\}, \quad B = \{\text{Operation}\}, \end{aligned}$$

where $\text{Squad}_1 = \{\text{UnitA}, \text{UnitB}\}$, $\text{Squad}_2 = \{\text{UnitC}, \text{UnitD}\}$, $\text{Region} = \{\text{Squad}_1, \text{Squad}_2\}$, *and* $\text{Operation} = \text{Region}$.

Initial neutrosophic maps on V_0*:*

$$\begin{aligned} T_V^0(\text{UnitA}) = 0.90,\ I_V^0(\text{UnitA}) = 0.05,\ F_V^0(\text{UnitA}) = 0.05, \\ T_V^0(\text{UnitB}) = 0.85,\ I_V^0(\text{UnitB}) = 0.10,\ F_V^0(\text{UnitB}) = 0.05, \\ T_V^0(\text{UnitC}) = 0.80,\ I_V^0(\text{UnitC}) = 0.15,\ F_V^0(\text{UnitC}) = 0.05, \\ T_V^0(\text{UnitD}) = 0.75,\ I_V^0(\text{UnitD}) = 0.20,\ F_V^0(\text{UnitD}) = 0.05. \end{aligned}$$

On the unique 2-superedge Operation *and each supervertex* $v \in \{\text{Squad}_1, \text{Squad}_2\}$*, set for example*

$$\begin{aligned} T_E^0(\text{Operation}, v) &= T_V^0\big(\textit{any member of } v\big), \\ I_E^0(\text{Operation}, v) &= I_V^0\big(\textit{any member of } v\big), \\ F_E^0(\text{Operation}, v) &= F_V^0\big(\textit{any member of } v\big). \end{aligned}$$

2. Build underlying 2-Super-HyperGraph.
By iterating the power-set:

$$V_1 = \text{PS}(V_0), \quad E_1 = V_1, \quad V_2 = \text{PS}(V_1), \quad E_2 = V_2.$$

We restrict to the chosen supervertices $V_2^* = \{\text{Region, Squad}_1 \ldots\}$ *and superedges* $E_2^* = \{\text{Operation}\}$.

3. Initialize neutrosophic memberships.
For each scenario $c \in C$ *and each* $v \in V_2^*$*:*

$$(T_V, I_V, F_V)(c, v) = \begin{cases} (T_V^0, I_V^0, F_V^0)(v'), & v' \in v \subseteq A(c), \\ (0,0,0), & \textit{otherwise.} \end{cases}$$

For the single superedge Operation *and each supervertex* $v \in V_2^*$*:*

$$(T_E, I_E, F_E)(c, \text{Operation}, v) = \begin{cases} \big(\min(T_E^0, I_E^0, F_E^0), \min, \min\big), & v \in B(c), \\ (0,0,0), & \textit{otherwise.} \end{cases}$$

4. Enforce neutrosophic constraints.
Normalize any triple whose sum exceeds 3*. In our numeric assignments above, all sums remain* ≤ 3.

Result. *The algorithm yields*

$$H^* = \big(V_2^*,\, E_2^*,\, C,\, A,\, B,\, T_V, I_V, F_V,\, T_E, I_E, F_E\big),$$

a fully specified neutrosophic soft 2*-Super-HyperGraph modeling how different disaster scenarios select and weight response clusters under uncertainty.*

Theorem 3 (Correctness of Algorithm 1). *The output*

$$H^* = (V_n, E_n, C, A, B, T_V, I_V, F_V, T_E, I_E, F_E)$$

satisfies all the axioms of a neutrosophic soft n-Super-HyperGraph.

Proof. By construction:

(i) *Underlying n-Super-HyperGraph:* Step 1 computes

$$V_k = \mathrm{PS}(V_{k-1}), \quad E_k = V_k \quad (1 \le k \le n),$$

so that $V_n, E_n \subseteq \mathrm{PS}^n(V_0)$, as required.

(ii) *Soft selection maps:* We leave $A\colon C \to \mathrm{PS}^n(V_0)$ and $B\colon C \to \mathrm{PS}^n(V_0)$ unchanged, and the algorithm respects $B(c) \subseteq \{e : e \subseteq A(c)\}$ by hypothesis.

(iii) *Vertex memberships:* For each $c \in C$ and $v \in V_n$,

$$(T_V, I_V, F_V)(c, v) = \begin{cases} (T_V^0, I_V^0, F_V^0)(v), & v \in A(c), \\ (0, 0, 0), & v \notin A(c). \end{cases}$$

Hence $0 \le T_V(c, v) + I_V(c, v) + F_V(c, v) \le 3$.

(iv) *Edge–vertex memberships:* For each c, e, v,

$$(T_E, I_E, F_E)(c, e, v) =$$

$$\begin{cases} \big(\min(T_E^0(e, v), T_V(c, v)),\ \min(I_E^0(e, v), I_V(c, v)), \\ \min(F_E^0(e, v), F_V(c, v))\big), & v \in e, \\ (0, 0, 0), & v \notin e. \end{cases}$$

By construction, $T_E(c, e, v) \le T_V(c, v)$, $I_E(c, e, v) \le I_V(c, v)$, and $F_E(c, e, v) \le F_V(c, v)$, and each sum lies in $[0, 3]$.

(v) *Normalization:* Step 3 rescales any triple whose sum exceeds 3, restoring the bound $0 \le T + I + F \le 3$.

No other axioms are violated. Thus H^* is a valid neutrosophic soft n-Super-HyperGraph.

Theorem 4 (Time Complexity of Algorithm 1). *Let $N_0 = |V_0|$ and define $N_k = |\mathrm{PS}^k(V_0)|$ for $1 \le k \le n$. Then the algorithm runs in time*

$$O\Big(\sum_{k=1}^{n} N_{k-1}\, N_k \;+\; |C|\, (N_n + N_n^2)\Big),$$

which is exponential in n (and doubly exponential in N_0).

Proof.

- *Step 1:* Computing each power-set $\mathrm{PS}(V')$ of size N_{k-1} requires $O(N_{k-1}2^{N_{k-1}}) = O(N_{k-1}N_k)$ time. Summing over k gives the first term.
- *Step 2:* For each $c \in C$, assigning (T_V, I_V, F_V) to all N_n vertices costs $O(N_n)$, and assigning (T_E, I_E, F_E) to all N_n edges and their N_n vertices costs $O(N_n^2)$. Hence $O(|C|(N_n + N_n^2))$.
- *Step 3:* A final pass over all (c, v) and (c, e, v) again costs $O(|C|(N_n + N_n^2))$, which is absorbed into the previous bound.

Since $N_k = 2^{N_{k-1}}$, the overall complexity grows exponentially in n (and doubly exponentially in $|V_0|$).

5. Conclusion

In this paper, we have introduced the *Neutrosophic Soft Super-HyperGraph*, a novel framework that synergistically combines neutrosophic logic, soft set theory, and Super-HyperGraph structures to capture both hierarchical interdependencies and uncertainty in complex systems. By unifying hierarchical connectivity, parameterized substructure selection, and neutrosophic uncertainty, our model enables flexible multi-level representations, targeted subgraph analysis, and informed decision-making under uncertain and large-scale network conditions.

For future work, we intend to extend this framework to additional graph generalizations, including bidirected graphs [66–68], multigraphs [69, 70], pseudographs [71], and multidirected graphs [72, 73]. Further extensions of the Neutrosophic Soft Super-HyperGraph could also leverage HyperUncertain Sets[22, 74, 75], HyperRough Sets[76, 77], HyperWeighted Sets[78], and HyperSoft Sets[79, 80]. Moreover, we will perform empirical evaluations on real-world datasets using appropriate computational tools to assess the practical applicability, scalability, and robustness of the proposed approach.

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Data Availability

This paper is purely theoretical and does not involve any empirical data. We welcome future empirical studies that build upon and test the concepts presented here.

Ethical Approval

As this work is entirely conceptual and involves no human or animal subjects, ethical approval was not required.

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Disclaimer on Computational Tools

No computer-based tools—such as symbolic computation systems, automated theorem provers, or proof assistants (e.g., Mathematica, SageMath, Coq)—were employed in the development, analysis, or verification of the results contained in this paper. All derivations and proofs were conducted manually through analytical methods by the authors.

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The theoretical models and concepts proposed in this manuscript have not yet undergone empirical testing or practical deployment. Future work may investigate their utility in applied or experimental contexts. While the authors have taken care to maintain accuracy and provide appropriate citations, inadvertent errors or omissions may remain. Readers are encouraged to consult original references for confirmation and further study.

The authors assert that all mathematical results and justifications included in this work have been carefully reviewed and are believed to be correct. Should any inaccuracies or ambiguities be discovered, the authors welcome constructive feedback and will provide clarification upon request.

The conclusions presented are valid only within the specific theoretical framework and assumptions described in the text. Generalizing these results to other mathematical contexts may require further investigation. All opinions and interpretations expressed herein are solely those of the authors and do not necessarily reflect the views of their respective institutions.

Consent to Publish

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