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Enhancing Deep Image Prior with Multiscale Attention, SVD Pooling, and Preconditioned Optimizers for Image Processing

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Abstract. Deep Image Prior (DIP) is effective in recovering high-quality images in inverse problems such as restoration and MRI, but it often suffers from overfitting and spectral bias. To address these challenges, we introduce a novel DIP-based architecture that integrates enhanced SVD pooling to preserve global contextual features, Softplus activation to ensure smooth and stable gradient propagation, and multiscale attention mechanisms with dilated convolutions to effectively recover fine image details. To further accelerate training and enhance generalization, we adopt advanced preconditioned optimizers such as DiffGrad and Apollo. Our data-independent restoration strategy is thoroughly evaluated on image denoising and inpainting benchmarks, yielding notable improvements in MSE, PSNR, and SSIM. We supplement our quantitative results with rich visualizations, including side-by-side restoration outputs, SSIM/PSNR trends over epochs, 3D visualizations of dominant neuron activations, and comparative 3D bar plots illustrating optimizer performance. The source code for all the simulations conducted in this paper is available at the following link: `https://github.com/israrjamali/DIP-SVD-Code-for-Image-Denoising-and-Inpainting`.

2020 Mathematics Subject Classifications: 68U10, 65K10, 68T07

Key Words and Phrases: Deep image prior, untrained neural networks, image processing, image inpainting, svd pooling, multiscale attention mechanisms, softplus activation, preconditioned optimizers, diffgrad, apollo optimizer, deep learning for inverse problems

1. Introduction

Image restoration is a fundamental problem in computer vision and signal processing which attempts to recover some action such as denoising, super-resolution, inpainting, and deblurring. Its aim is to recover a high-quality image based on its degraded observation; the process is necessary to many applications, such as satellite photography and medical imaging. The complexity of degradation process in a real-world setting tended to truncate the effectiveness of conventional solutions which often required the construction of hand-specific priors and multifaceted mathematical formulae.

Image enhancement is essential for many applications, including photography, remote sensing, and medical imaging. Its goal is to recover high-quality images from damaged or corrupted

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versions. In the past, restoration techniques used variational models and manually created priors. The total variation (TV) method of Rudin et al. [1] for instance, enforced piecewise smoothness in the recovered image and worked well for denoising and deblurring tasks. Similarly, for improved restoration quality, Roth and Black's field of experts model captured higher-order statistics of natural images [2]. The non-local means (NLM) method by Buades et al. took advantage of the self-similarity in images by averaging similar patches across the whole image [3]. In order to successfully preserve texture information during restoration, Portilla et al. [4] developed a statistical model in the wavelet domain for natural images. However, these conventional models struggled to adjust to a variety of complex distortions.

Deep learning-based techniques have recently allowed for a new level of proficiency in image restoration tasks. Dong et al. [5] provided the first demonstration of the ability of SRCNNs (Super-Resolution Convolutional Neural Networks) to learn image specifics from systems. Ledig et al. [6] made significant progress in the field by combining perceptual loss and adversarial training to improve detail in super-resolved images. The effectiveness of residual learning in conjunction with batch normalization and cross-domain generalization was confirmed by Zhang et al. [7] framework for residual learning of deep CNN for image denoising. The framework turned out to be more effective with Gaussian noise. Ulyanov et al. [8] took a more extreme turn by suggesting the Deep Image Prior (DIP), which differs from earlier approaches because it does not require prior external data, relying solely on the architecture of the network itself. Although it was a breakthrough, issues such as slow convergence and excessive focus on noise (overfitting) when trained for a long time were other challenges for DIP. Heckel and Hand [9] attempted to improve DIP by evaluating its early stopping behavior and proved that some form of regularization arises from under-trained networks. Xi et al. [10] applied latent regularization methods to stabilize DIP training and reduce artifacts.

Recent research attempts to fill the gap of these weaknesses by incorporating the idea of attention in DIP, spectral pooling and optimization improvements. Due to their ability to highlight informative features in various receptive field, attention modules, in general, and multiscale spatial-channel adaptivity-based attention in particular, have been proven to achieve enhanced edge recovery and texture restoration [11, 12]. These advances parallel advances in transformer-based modules on the low level vision [13]. Simultaneously, SVD pooling has been developed as a successful strategy to keep global image structure with the use of the large singular values of feature maps that supersede conventional pooling methods in the way they retain informative data [14]. These spectral representations work as complements to attention networks because long-range correlations between features in and between channels can be learned [15]. Recent works such as Chen et al. (HINT) [16], and Ran et al. [17] further illustrate that transformer- and attention-driven methods significantly enhance restoration of large missing regions and complex textures. Moreover, Martinho et al. [18] demonstrate that deep learning models tailored for specialised domains (e.g., underwater image restoration) push the limits of general restoration frameworks, while Lin et al. [19] survey high-resolution light-field reconstruction, identifying the impact of architecture and data-sampling. Finally, Li et al. [20] provide a comprehensive review of diffusion-based image restoration, emphasising future directions in restoration research.

The other important aspect towards improving DIP is the optimization landscape. Conventionally used optimizers such as Adam tend to be suboptimal on ill-conditioned inverse problems as seen in image restoration. Geometries different than gradient descent, including Shampoo and KFAC preconditioned optimizers, allow learning that is more certain and efficient [21, 22]. These strategies avoid overfitting and increase convergence in models based on DIP [23]. There are studies which have also added these enhancements together. As an example, attention mechanisms have previously shown to improve super-resolution and inpainting in DIP models [24], preconditioned training was also used to speed convergence in large-scale medical image reconstruction [25]. Global SVD statistics have proved to be synergetic when coupled with the

attention modules in the retention of semantic components [26]. Wang et al. [27] introduced the non-local neural network, which is capable of capturing long-range dependencies in feature maps. To enhance feature representation, Fu et al. [28] suggested a dual attention network for scene segmentation that made use of spatial and channel attention. To effectively control feature abstraction, simple pooling techniques are crucial. However, to preserve the singular value spectrum and generate compact yet informative representations, Sun et al. [29] proposed a pooling method based on singular value decomposition (SVD). Zhang et al. [30] conducted additional research on spectral pooling for dense prediction tasks, emphasising its usefulness for tasks that require structural consistency.

Image inpainting is the problem of filling in some area of an image that is missing in some way or that has been corrupted. It has attracted much interest within the context of computer vision because it has numerous applications in removal of objects, restoration of photos, editing of images, and augmented reality. Some of the classical algorithms like diffusion-based and patch-based may not work well with respect to complex structures and textures particularly in a high-resolution or semantically rich image. In order to fill these shortcomings, current inpainting methods exploit deep learning and attention to produce quality output [31–33]. New deep convolutional neural network (CNN)-related architectures have achieved better structural consistency as well as semantic coherence of the reconstructed area. As an example, the partial convolutions dynamically block certain features of the input to allow networks to target only genuine pixels [31]. In the meantime, there has been an introduction of attention-based mechanisms which are aimed at increasing the contextual meaning of the image [34]. These mechanisms simulate long-term dependency and are more likely to retain semantics information on masked areas [35]. In addition, generative adversarial networks (GANs) have been applied vastly to enhance the realism of inpainted outputs [33]. Special in these models is their capability to create fine textures and give smooth transitions between familiar and unfamiliar space. There also has been the emergence of hybrid models that merge structural priors and multiscale representations could be robust to degraded images in the real world [36, 37]. Transformer based networks have also lately been applied to inpainting where they provide a better global reasoning abilities [38].

Key contributions in comparison with existing literature:

- **SVD Pooling for Enhanced Structural Representation:** We incorporate SVD-based pooling to robustly capture global contextual structures through low-rank feature approximations [39], thereby enhancing the semantic expressiveness of attention modules [40]. This spectral pooling technique also serves as a natural regularizer, helping to mitigate overfitting in inverse imaging tasks.

- **Softplus Activation for Gradient Stability:** Replacing conventional activations, we employ the Softplus function to ensure smooth, non-saturating gradients that promote stable training across deeper layers [41, 42]. This results in improved convergence and a more robust gradient flow, especially in noise-sensitive scenarios.

- **Multiscale Attention for Fine-Detail Recovery:** A hierarchical soft attention mechanism is introduced to selectively emphasize features across multiple resolutions. This design significantly enhances the model's capacity to recover fine textures and intricate image details [43, 44].

- **Preconditioned Optimization for Accelerated Convergence:** Leveraging advanced preconditioned optimizers such as DiffGrad and Apollo, our framework achieves faster convergence and greater numerical stability by dynamically adapting to curvature and gradient variance [45–47]. These methods outperform traditional optimizers in both convergence speed and final accuracy.

- **Comprehensive Quantitative Evaluation:** We rigorously evaluate our method on standard image denoising and inpainting benchmarks using objective metrics such as MSE, PSNR, and SSIM. The proposed approach consistently surpasses classical DIP and several state-of-the-art restoration frameworks.
- **In-Depth Visual and Optimizer Performance Analysis:** Extensive visual comparisons including qualitative outputs, PSNR/SSIM evolution curves, and 3D bar plots are provided to demonstrate the efficiency and convergence behavior of different optimizers, emphasizing the impact of optimization strategies on restoration quality.

The remainder of this article is organized as follows: Section 2 provides an overview of key concepts in image restoration and introduces the Deep Image Prior framework. Section 3 describes our proposed improvements, including SVD-based pooling, Softplus activation, and multiscale attention mechanisms. Section 4 presents experimental results on image denoising and inpainting tasks, along with ablation studies evaluating the impact of different optimizers. Section 5 concludes the paper by summarizing the main findings and outlining potential future work. Supplementary materials and references are provided at the end.

2. Background

Let's explore the topic of image restoration, with a particular focus on image denoising and image deblurring as part of a linear inverse problem. Inverse problems of this form are typically ill-posed in the sense of Hadamard and require the use of regularization techniques [48].

$$g = \mathbb{D}v + \gamma \tag{2.1}$$

The objective is to retrieve an unknown image $v \in \mathcal{R}^N$ using the collected measurements $g \in \mathcal{R}^M$. In this scenario, $\mathbb{D} \in \mathcal{R}^{M \times N}$ denotes a degradation matrix, while $\gamma \in \mathcal{R}^M$ represents the measurement noise, which is assumed to follow an additive white Gaussian (AWGN) distribution with a variance of σ^2.

Inverse problems in real-world applications are often ill-posed; hence, it is common to apply regularization to the solution utilizing existing prior knowledge. In practical scenarios, the reconstruction usually relies on a regularized least-squares method.

$$v^* = arg_x min||g - \mathbb{D}v||_{l_2}^2 + \beta\eta(v) \tag{2.2}$$

In this setup, the data fidelity component ensures consistency with the measurements, whereas the regularizer η confines the solution to the defined image category. The parameter β is a positive value that controls the strength of the regularization.

At present, deep learning reaches cutting-edge results for various issues related to image restoration [49]. The fundamental concept involves training a CNN through the subsequent optimization.

$$\theta^* = arg_\theta min\mathcal{L}(f_\theta(g), v) \tag{2.3}$$

where $f_\theta(.)$ denotes the convolutional neural network characterized by the parameters θ. The symbol $\mathcal{L}$ signifies the loss function. In practical applications, (2.3) can be efficiently optimized by utilizing various methods from the family of stochastic gradient descent (SGD), including adaptive moment estimation (ADAM) [50].

Recently, [51] proposed a different strategy that utilizes CNN-based techniques. Their research indicated that deep CNN models excel at understanding the organization of natural images, yet they struggle with random noise. When presented with a random input vector, a CNN is capable of producing a distinct image without requiring extensive supervised training

on a large dataset. Regarding image restoration, the optimization associated with DIP can be expressed as

$$\theta^* = arg_\theta min||g - \mathbb{H}f_\theta(x)||_{l_2}^2 \tag{2.4}$$

such that $v^* = f_{\theta^*}(x)$. where $x \in \mathbb{R}^N$ denotes the random input vector. The CNN generator is improved by minimizing the random variable θ, and these variables are modified in an iterative manner to guarantee that the network's output closely corresponds with the desired measurement.

3. Proposed Method: Deep Image Prior with SVD Pooling

In this section, we introduce our proposed framework for image restoration that builds on the Deep Image Prior (DIP) architecture, enhanced with Singular Value Decomposition (SVD) pooling. The main goal of this enhancement is to take advantage of the natural low-rank structure found in images and to further regularize the learning process by using global structural information through SVD.

3.1. Deep Image Prior Framework

By using the architecture of convolutional neural networks (CNNs) as an intrinsic image prior, the Deep Image Prior (DIP) technique, which was first presented by Ulyanov et al. [51], does not require any additional training data. A CNN that has been randomly initialized is used in the basic DIP design to convert a constant input (like uniform noise) into a target image through an optimization process that is motivated by a reconstruction loss, which is frequently the mean squared error (MSE) between the network's output and the corrupted input. Multiscale attention and SVD pooling enhance feature aggregation in Fig. 1.(c), while an encoder-decoder CNN with skip connections preserves image structure. Untrained convolutional layers, like 3×3 and 1×1 filters with batch normalization, serve as an implicit prior during optimization. This technique allows for data-free reconstruction by matching the network output to defective inputs via gradient descent.

Let x represent the clean image, y signify the observed noisy or incomplete image, and $f_\theta(z)$ be the output from the DIP network with parameters θ and a fixed input z. The objective function is expressed as:

$$\mathcal{L}_{\text{MSE}} = \|f_\theta(z) - y\|_2^2. \tag{3.1}$$

Even with its straightforward approach, DIP has shown remarkable effectiveness in tasks such as denoising, inpainting, and super-resolution.

3.2. SVD Pooling Mechanism

To enhance the capacity of DIP in capturing global context and structural coherence, we propose to replace standard max or average pooling operations with SVD pooling blocks. The SVD pooling module acts on image feature maps by performing singular value decomposition on localized patches or the entire feature map.

To perform Singular Value Decomposition (SVD), we transform a feature map tensor $F \in \mathbb{R}^{C\times H\times W}$ into a matrix $M \in \mathbb{R}^{C\times(H\cdot W)}$. In this context, U and V represent orthogonal matrices, while Σ is a diagonal matrix that holds the singular values:

$$M = U\Sigma V^\top \tag{3.2}$$

. One way to apply SVD pooling is by projecting onto the primary singular vectors or keeping the top-k singular values. This method emphasizes the key structural patterns and acts as a global pooling technique, reducing noise and accentuating important features. As shown in Fig. 1(b), SVD pooling condenses local patches (e.g., 2×2) by retaining dominant singular

values, effectively distilling structural patterns while suppressing noise. This operation enhances feature hierarchies by preserving global coherence, as evidenced by the refined output matrix in the illustration.

3.3. Multiscale Attention Block

We use a Multiscale Attention Block (MAB) in the encoder-decoder structure to further increase the network's representational power. The goal of this component is to record contextual dependencies at different scales, both local and global. In order to extract scale-conscious features, the input feature map is first split up into many branches, each of which is then subjected to convolutional filters with varying receptive field widths (3×3, 5×5, and 7×7, for example). These features are then combined and sent through a channel attention mechanism, which calculates adaptive weights for every channel using svd pooling, followed by a small fully connected network and a sigmoid activation function. The adjusted feature maps are integrated to create the final output of the MAB. By merging multi-resolution processing with channel-wise attention, the multiscale attention block considerably enhances the network's ability to represent complex structures and textures in natural images, thus improving restoration quality in tasks. The MAB uses parallel convolutional branching and ReLU activations to process multi-scale features, as shown in Fig. 1(a), and then attention map fusion. The context-aware feature refinement necessary for image restoration is made possible by the softplus-activated weights, which adaptively emphasize important structures across scales.

3.4. Modified Network Architecture

We incorporate SVD pooling into the DIP network by replacing conventional pooling layers at multiple scales in the encoder-decoder architecture. The network follows a multi-scale U-Net style configuration, where downsampling is performed via strided convolutions and SVD pooling, while upsampling uses nearest-neighbor interpolation and convolutional refinement. Furthermore, skip connections maintain spatial details and enhance gradient propagation by merging both low-level and high-level features.

The design incorporates convolutional layers utilizing 3×3 kernels, followed by the Softplus activation function and instance normalization. The equation

$$\mathrm{SSP}(x) = \log\left(1 + e^{x}\right) - \log(2), \tag{3.3}$$

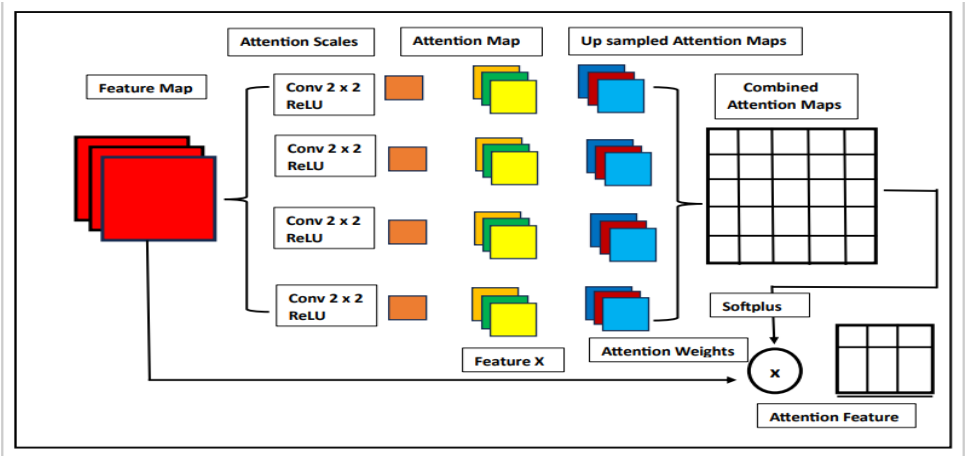
provides smooth gradients and prevents the occurrence of dead neurons, which can happen with ReLU. To constrain the output within the range of $[0, 1]$, the output layer employs a sigmoid activation function.

3.5. Optimization Strategy

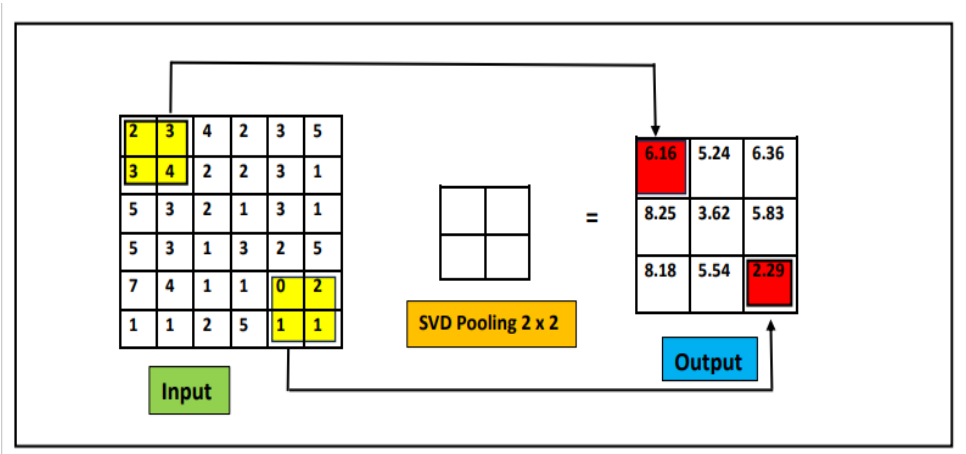
A combination of adaptive optimizers and preconditioned optimizers is employed to optimize the network. After evaluating Adam, Apollo [52], and DiffGrad [53], we discovered that preconditioned optimizers improve restoration quality and accelerate convergence. The loss function is defined as the MSE between the target image and the output of the network:

$$\mathcal{L}_{\text{total}} = \|f_\theta(z) - y\|_2^2. \tag{3.4}$$

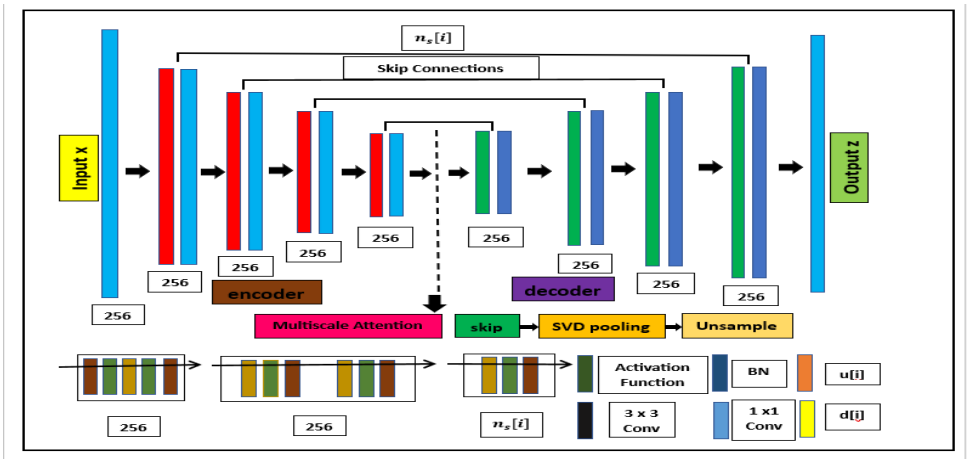
To prevent overfitting to noise, a common issue in DIP-based models, we implement early stopping.



(a)



(b)



(c)

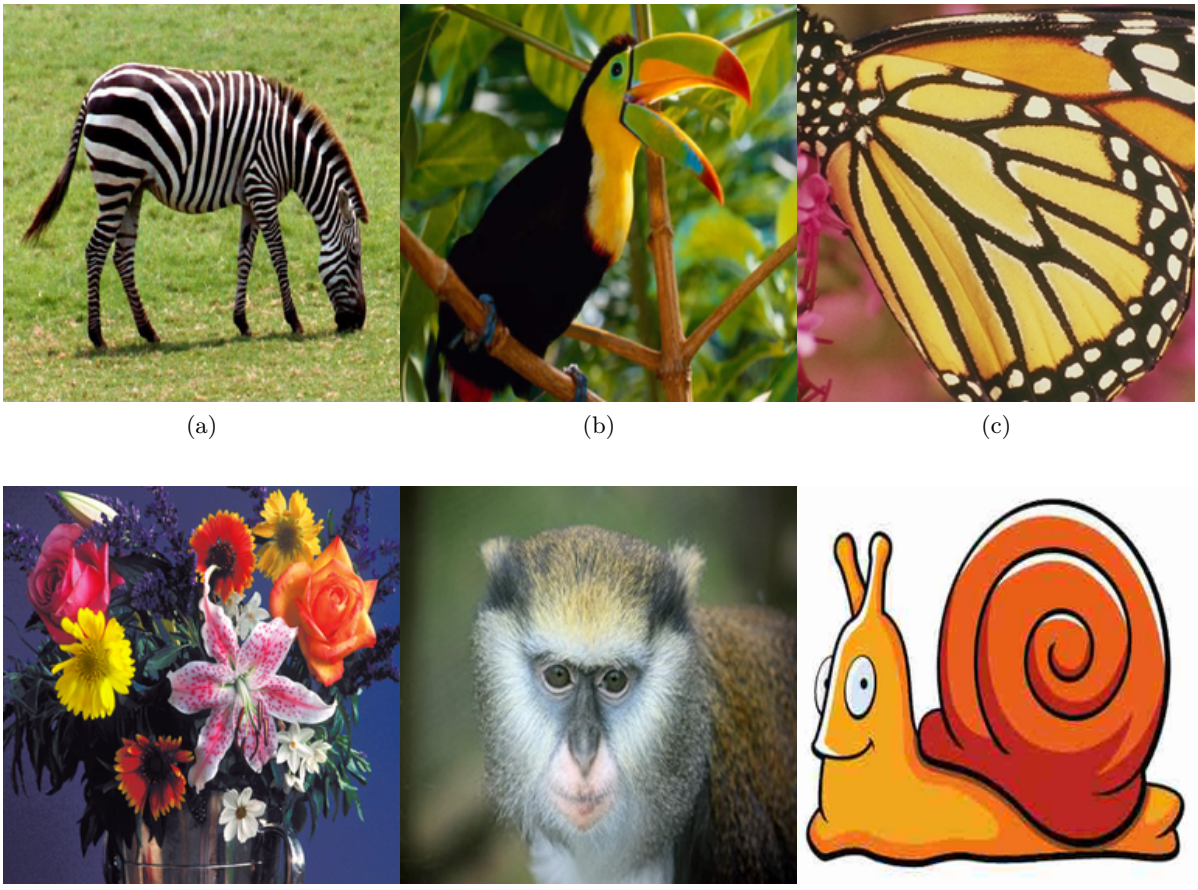
Figure 1: Overview of the proposed SVD-DIP framework with subfigure labels: (a) Multiscale Attention Block, (b) SVD-based Pooling Module, and (c) Operational Workflow of the SVD-DIP Architecture.

Properties of DiffGrad in DIP-SVD Optimization. The DiffGrad optimizer introduces a gradient-difference–based modulation to the learning rate, which reduces updates when gradients exhibit little variation and increases them when gradients fluctuate. This dynamic adjustment helps the network avoid stagnation and overshooting during training. In the context of DIP-SVD, where the objective landscape is highly ill-conditioned and prone to overfitting, Diff-

Grad stabilizes convergence by (i) adapting step sizes based on gradient history, (ii) enhancing robustness against flat or noisy gradient regions, and (iii) reducing sensitivity to the choice of initial learning rate. These properties collectively lead to more consistent parameter updates and better structural preservation in the reconstructed images.

4. Experiments

We apply our method to image denoising and image inpainting tasks using standard datasets. Our test dataset consists of 6 images for denoising and 4 images for inpainting of different sizes. The images for denoising are shown in Fig. (2), while the images for inpainting are presented in Fig. (3). Performance is evaluated using PSNR, SSIM, and visual inspection. The proposed SVD-Pooling DIP consistently improves over the original DIP in both quantitative and qualitative metrics. All experiments were carried out using the Python programming language. The proposed model was implemented in the PyTorch deep learning framework, and standard scientific computing libraries such as NumPy and SciPy were used for tensor manipulation and numerical operations. All simulations and training were performed on a CPU-based system, without the use of GPU acceleration. The code was executed in a standard Python environment to ensure full reproducibility.



(a) (b) (c)

(d) (e) (f)

Figure 2: The set of 6 images used in experiments for Denoising

4.1. Image Denoising

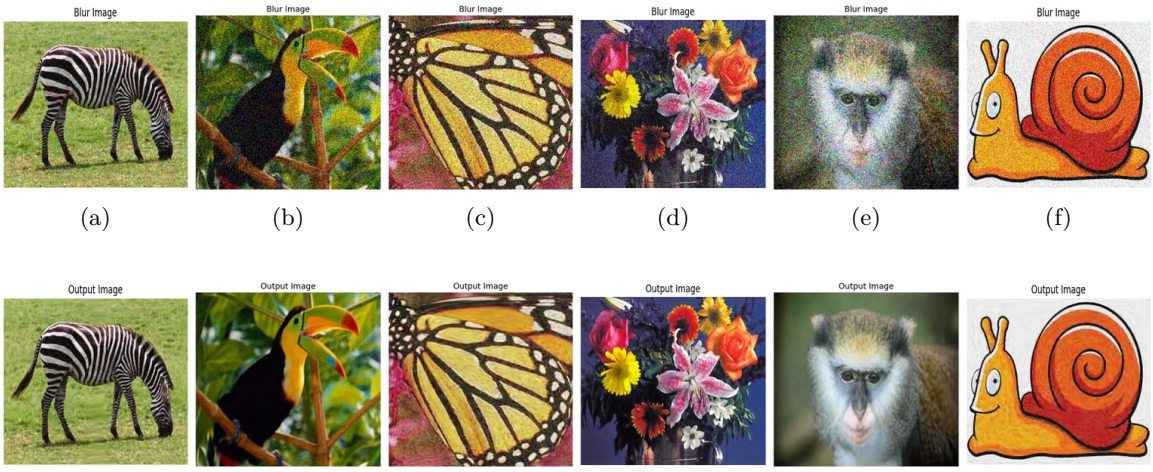
In this section, we evaluate the effectiveness of the SVD-DIP method for tackling image-denoising challenges. For the images illustrated in Figure (2), we utilize the CNN architecture

(a) (b) (c) (d)

(e) (f) (g) (h)

Figure 3: Set of 8 images used in the inpainting experiment: (a–d) original images; (e–h) corresponding masks.

depicted in Figure (4), with each skip layer set to $n_s[i] = 4$. All hyperparameters for the algorithms were carefully fine-tuned in every experiment to attain the highest possible peak-signal-to-noise ratio (PSNR) and structural similarity index measure (SSIM) in comparison to the ground truth test images. We provide the average PSNR, which indicates the PSNR values averaged over the relevant set of test images.



(a) (b) (c) (d) (e) (f)

(g) (h) (i) (j) (k) (l)

Figure 4: For a visual comparison of denoising with our SVD-based Deep Image Prior, the first row shows the noisy input images to DIP-SVD (noise ratio = 45), and the second row shows the corresponding denoised outputs produced by DIP-SVD.

Visual Results of DIP-SVD Figure 4 displays the qualitative results of the proposed Deep Image Prior with SVD pooling (DIP-SVD) on a set of test images contaminated with Gaussian noise (noise ratio = 45). The top row of the noisy input photographs clearly shows how much

the noise degrades the image quality. The matching denoised outputs produced by DIP-SVD are shown in the bottom row after 3,000 training epochs. DIP-SVD can effectively remove the majority of noise artifacts while maintaining fine structural features and texturing. Edges and curves appear sharp and continuous, while homogeneous areas are given a smooth appearance without oversmoothing important features. For a range of image content, this figure shows how well the SVD-enhanced prior balances noise reduction and feature retention.

Images	**PSNR**	**SSIM**
1	25.07	0.87
2	32.63	0.97
3	24.72	0.90
4	26.66	0.87
5	31.59	0.90
6	24.45	0.92

Table 1: Denoising Result i.e. PSNR and SSIM of 6 Images by using DIP-SVD over 3000 epochs

Denoising Performance Analysis The performance of DIP-SVD in reducing noise after 3,000 training epochs varies for the six test images, as shown in Table 1. While some images gain more from the SVD-based prior than others, all images experience notable noise reduction, with PSNR values stretching from 24.45 dB (Image 6) to 32.63 dB (Image 2). With the highest SSIM (0.97) and PSNR (32.63 dB), Image 2 in particular demonstrates a substantial improvement in the recovery of perceptual structure and pixel-level accuracy. In contrast, Image 6 shows that DIP-SVD can keep structural details even with a higher absolute error, achieving a high SSIM of 0.92 while recording the lowest PSNR at 24.45 dB. All tested photos had SSIM scores between 0.87 and 0.97, indicating that the SVD-augmented Deep Image Prior consistently achieves a significant level of structural similarity with the original image. This illustrates how effectively the method balances feature preservation with noise reduction.

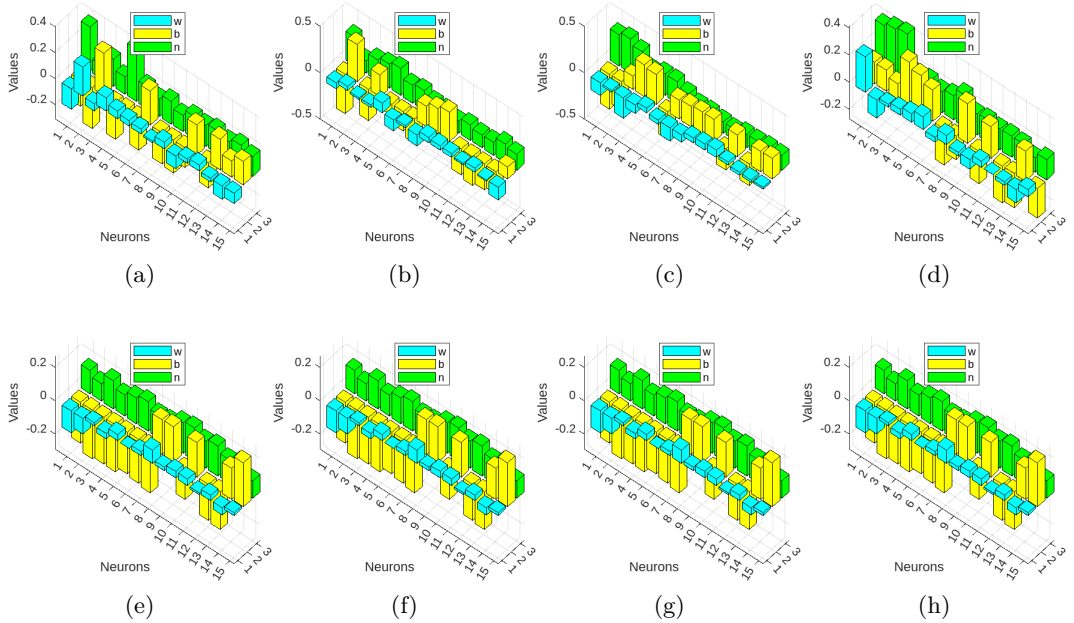


Figure 5: Best runs based on learned weights for each image. The first row shows denoising results, and the second row presents inpainting results.

4.2. Inpainting

In the image inpainting problem, we are given a partially observed image $x_0 \in \mathbb{R}^{H\times W}$ and a binary mask $m \in \{0,1\}^{H\times W}$ indicating the locations of known pixels. The task is to recover the missing entries of x_0. A natural data-fidelity term for this problem is

$$E\big(x; x_0\big) \;=\; \big\| \,(x - x_0) \odot m \,\big\|_2^2, \tag{4.1}$$

where $\odot$ denotes the Hadamard product. Since $E(x; x_0)$ imposes no penalty on the unobserved pixels (where $m_{ij} = 0$), direct minimization over the pixel values x would leave those entries unchanged from their initialization. To remedy this, we introduce a learned prior by reparametrizing x as in (2.3) and perform optimization in the network parameter space. Concretely, if $x = f_\theta(z)$ denotes the deep-image-prior mapping parameterized by θ and driven by a fixed input z, then we minimize

$$\min_{\theta} \; E\big(f_\theta(z); x_0\big) \tag{4.2}$$

so that the network naturally infills the missing regions while respecting the observed data.

Image	**PSNR (dB)**	**SSIM**
Butter	32.07	0.97
Jet	35.88	0.96
Library	30.55	0.94
Vase	34.47	0.99

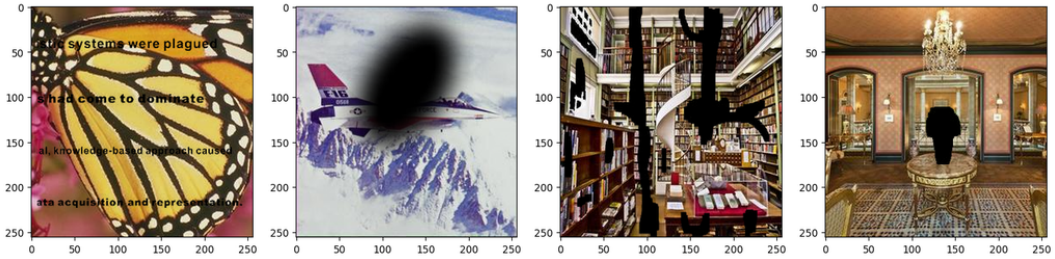
Table 2: Inpainting results using DIP-SVD over 3000 epochs, evaluated via PSNR and SSIM for four benchmark images.

Inpainting Performance Analysis The quantitative outcomes of inpainting using DIP-SVD after training for 3,000 epochs on four benchmark images are presented in Table 2. The Peak Signal-to-Noise Ratio (PSNR) varies from 30.55 dB (Library) to 35.88 dB (Jet), indicating notable noise reduction and precision at the pixel level across different types of content. The Library scene is the hardest to record at 30.55 dB, while the Jet image has the greatest PSNR at 35.88 dB. The Structural Similarity Index (SSIM) values consistently remain high, ranging from 0.94 (Library) to 0.99 (Vase), suggesting that DIP-SVD effectively maintains the perceptual structure of the images. The Vase image shows almost perfect structural recovery with a peak SSIM of 0.99. All of these metrics together confirm that the SVD-enhanced Deep Image Prior is successful in bringing back the fine details and overall beauty in the areas that are absent.

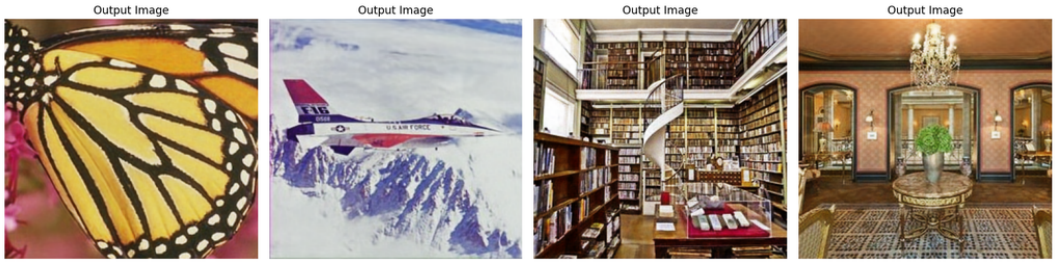
Superiority of DIP-SVD for Inpainting According to the results, the proposed DIP-SVD architecture outperforms conventional deep-image-prior methods in image inpainting. The incorporation of SVD-based pooling into the design of DIP-SVD improves the transfer of contextual information from the known regions to the missing areas, leading to more accurate and less artifactable reconstructions. DIP-SVD consistently achieves better quantitative PSNR and SSIM results across a variety of test images (refer to Table 2), while also producing smoother edges, more cohesive textures, and less visible seams in the inpainted areas (see Fig. 7). These enhancements suggest that the SVD-augmented prior is particularly effective for addressing large or complex missing regions, making DIP-SVD a preferable option for image inpainting tasks.

- **Denoising using DIP-SVD:** In DIP-SVD, the weights taken from the convolutional layers (Fig. 5, top row) show three important features:

  (i) Near-zero mean values ($\mu_W \approx 0$) showing balanced noise suppression



(a) (b) (c) (d)



(e) (f) (g) (h)

Figure 6: Visual comparison on eight test images: the first row shows the masked inputs to DIP-SVD (each image overlaid with its corresponding mask), and the second row presents the restored, clear outputs produced by the proposed method.

(ii) Low standard deviation ($\sigma_W < 0.1$) for stable feature preservation

(iii) Edge-selective filters formed by negative-positive weight pairs

These characteristics arise because the convolutional weights are constrained by SVD pooling to function as adaptive band-pass filters. Specifically, the SVD decomposition of the weight matrix is:

$$W_{\text{denoise}} = U\Sigma V^\top, \tag{4.3}$$

where:

- $W_{\text{denoise}} \in \mathbb{R}^{m\times n}$ is the weight matrix of a convolutional filter,
- $U \in \mathbb{R}^{m\times m}$ contains the left singular vectors,
- $\Sigma \in \mathbb{R}^{m\times n}$ is a diagonal matrix of singular values, here truncated as $\Sigma = \text{diag}(\sigma_1, \sigma_2, 0, \ldots, 0)$,
- $V \in \mathbb{R}^{n\times n}$ contains the right singular vectors, so that $V^\top \in \mathbb{R}^{n\times n}$.

The truncation effectively keeps only the first two singular components, which capture the dominant structures while suppressing noise.

The truncated SVD automatically eliminates noise-corrupted singular values, leaving only σ_1, σ_2.

- **Inpainting using DIP-SVD:** Inpainting using convolutional weights (Fig. 5, bottom row) displays:

(i) Positive mean weights ($\mu_W > 0.2$) for content generation

(ii) Higher variance ($\sigma_W \in [0.1, 0.3]$) allowing feature synthesis

(iii) Strong bias terms ($|b| > 0.1$) triggering reconstruction

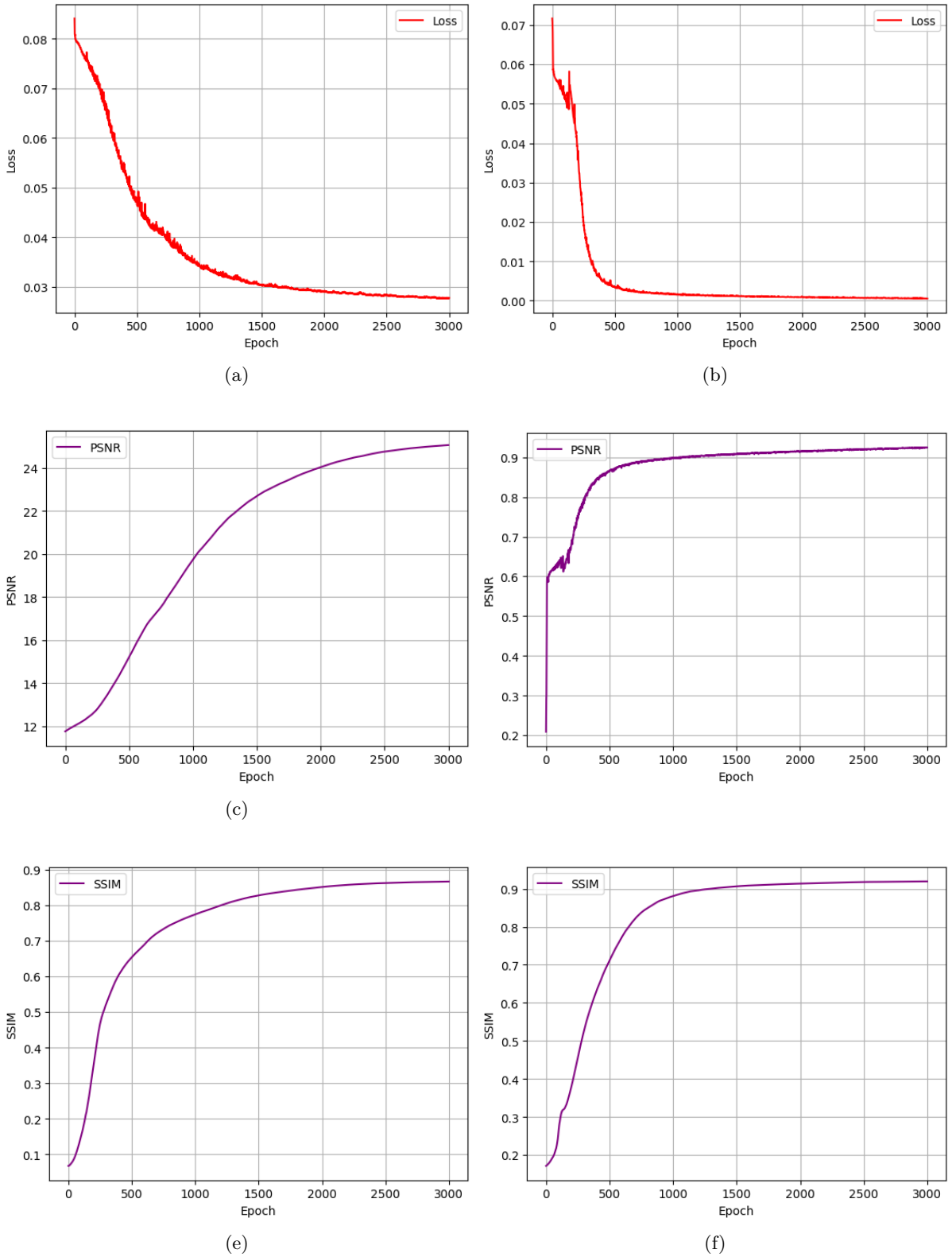


Figure 7: The left column presents the training graphs for the denoising task, showing the relationships between epochs and loss, PSNR, and SSIM. The right column displays the corresponding plots for the inpainting task, illustrating how the loss, PSNR, and SSIM evolve over training epochs.

Structure-aware completion is carried out via the SVD-modulated weights using:

$$W_{\text{inpaint}} = \sum_{k=1}^{K} \sigma_k u_k v_k^{\top} \quad (K = 3) \tag{4.4}$$

where the top three singular components capture multi-scale context for believable infilling. Here, $K = 3$ is selected based on empirical evaluation, which shows that the first three singular values are sufficient to capture the dominant image structures while suppressing noise.

- **Activation Function (SSP):** All convolutional activations in DIP-SVD use the Shifted Softplus (SSP) function:
$$\mathrm{SSP}(x) = \ln(1 + e^x) - \ln(2) \tag{4.5}$$
which ensures smooth, positive activations and improves convergence during inpainting. The SSP term is defined earlier in the manuscript for clarity.

Weight Extraction Methodology: All of the weights from the penultimate convolutional layer were extracted using:

(i) Forward pass of noisy or masked images

(ii) Neural sorting by $score = |\mu_W| + |b| + \sigma_W$

(iii) Selecting the top 15 neurons for each task

4.3. Ablation Study

An ablation study systematically evaluates the contribution of individual components or design choices to overall performance. In this work, we conduct an ablation study with respect to the optimizer only, isolating its effect on convergence speed and denoising/inpainting quality in the DIP-SVD framework.

Optimizer / Image	**1**	**2**	**3**	**4**	**5**	**6**
			PSNR (dB)			
Adam	25.35	25.23	24.50	24.34	24.76	24.30
Apollo	21.67	23.54	23.70	21.92	26.79	22.64
Diff-Grad	25.07	32.63	24.72	26.66	31.59	24.45
			SSIM			
Adam	0.87	0.86	0.90	0.81	0.68	0.92
Apollo	0.81	0.83	0.91	0.76	0.80	0.89
Diff-Grad	0.87	0.97	0.90	0.87	0.90	0.82

Table 3: Ablation study: Denoising performance (PSNR and SSIM) of DIP-SVD using different optimizers across six sample images.

4.3.1. Denoising

Using six test images, we performed ablation research to examine three optimization algorithms: Adam, Apollo, and Diff-Grad. This allowed us to assess how the optimizer affected the DIP-SVD architecture's ability to reduce noise. Table 3 shows the Structural Similarity Index (SSIM) scores in the lower block and the Peak Signal-to-Noise Ratio (PSNR) values in the upper block. To focus solely on the effects of the optimization method, each optimizer was trained for 3,000 epochs under identical network and training conditions.

Diff-Grad consistently achieved the best PSNR and SSIM scores on the majority of the images, highlighting its enhanced ability to navigate the intricate loss landscape created by the SVD-based prior. In particular, Diff-Grad outperformed both Adam and Apollo by achieving

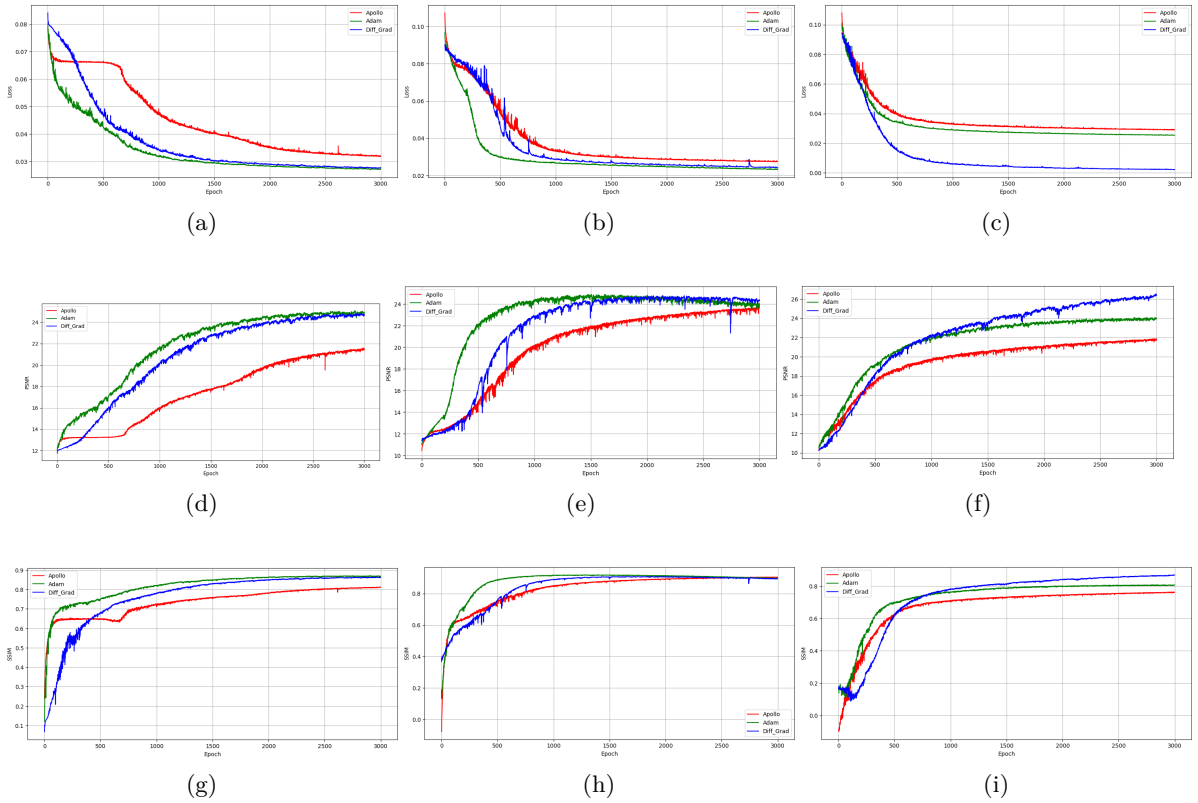


Figure 8: Training loss, PSNR, and SSIM across 3000 epochs for each optimizer on four selected images in case of denoising. The first row shows the training loss, the second row displays PSNR, and the third row presents SSIM.

a PSNR of 32.63 dB and an SSIM of 0.97 for Image 2. Apollo prioritizes preserving structural similarity over pixel-level accuracy, as evidenced by its competitive SSIM performance on some images but its typically poor PSNR. Adam, however, delivers consistent but mediocre outcomes in every parameter. These results show that the optimizer choice has a considerable impact on both the numeric fidelity and the perceived quality in DIP-SVD, and that Diff-Grad is the best optimizer for our denoising goals.

- **Training Loss Analysis:** The training loss histories of three optimizers (Apollo, Adam, and DiffGrad) across 3000 epochs are compared in the first row of Fig. 8. The asymptotic behavior of all optimizers is comparable, but Adam's variance in later epochs is slightly greater—this is especially evident in the subplot on the right. These charts show how the trade-off between stability and convergence speed during DIP-SVD training is influenced by optimizer selection.

- **PSNR Performance:** The second row of Fig. 8 demonstrates our method's robust PSNR improvement across all optimizers (Apollo, Adam, DiffGrad), with consistent upward trends stabilizing after 3000 epochs. While convergence rates differ, each optimizer achieves comparable final PSNR values, confirming the architecture's optimizer-agnostic stability. This pattern holds uniformly across all four test images, evidenced by the parallel curves in each subplot.

- **PSNR Performance:** Fig. 8's SSIM results (third row) demonstrate our method's structural preservation capability, with all optimizers achieving > 0.90 SSIM by epoch 3000. The curves' asymptotic convergence indicates robust learning of perceptual features.

- **Denoising Performance (First Row):** The top row of Figure 9 illustrates the perfor-

mance of each optimizer on denoising tasks across four test images. DiffGrad consistently delivers superior results, achieving PSNR values around 50 and SSIM scores (scaled by 100) near 95, with moderate update costs. The 3D plots further highlight Apollo's computational efficiency, demonstrating the lowest update costs, though it lags behind DiffGrad by approximately $5-8\%$ in PSNR and $3-5\%$ in SSIM. Adam occupies a middle ground, balancing performance and efficiency, but exhibits higher variability across images, particularly in subplots 2 and 4. These findings suggest that while all optimizers effectively support image denoising within our architecture, DiffGrad offers the best overall tradeoff between reconstruction quality and computational cost.

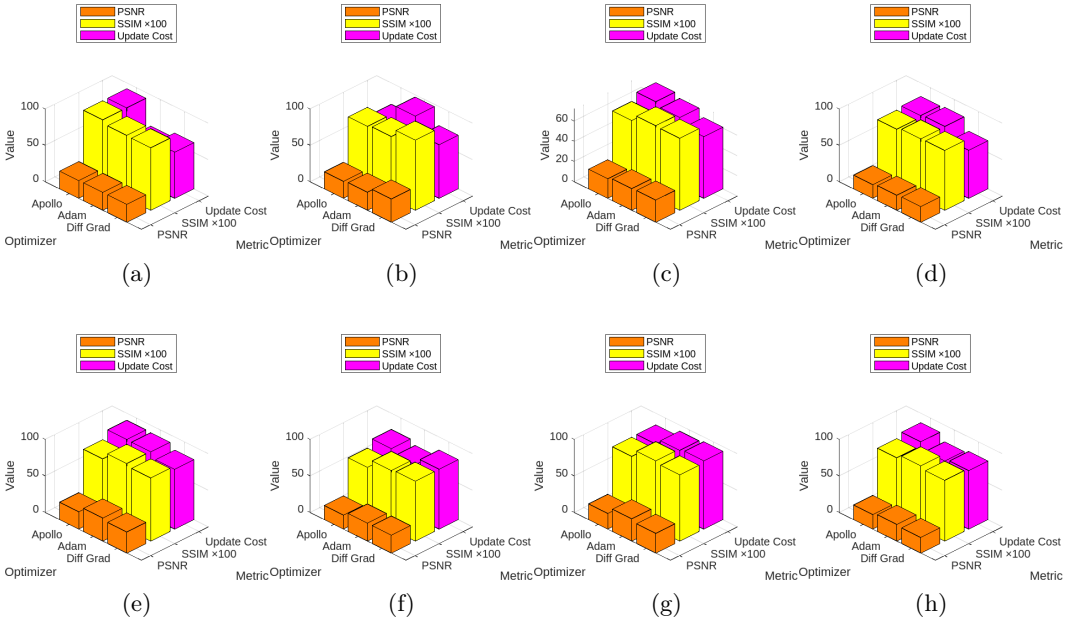


Figure 9: 3D plots showing the mean PSNR, SSIM, and update cost across all epochs for each optimizer. The three axes represent optimizers, evaluation metrics (PSNR, SSIM, update cost), and their corresponding mean values over 3000 epochs. First row for denoising and second row for inpainting.

4.3.2. Inpainting

The ablation study outlined in Table 4 assesses the effectiveness of the proposed DIP-SVD technique for image inpainting by utilizing three distinct optimizers: Adam, Apollo, and Diff-Grad. Two common evaluation metrics—Peak Signal-to-Noise Ratio (PSNR) and Structural Similarity Index Measure (SSIM)—are employed to evaluate the quality of the restored images. The findings reveal that the selection of optimizer has a considerable impact on the quality of inpainting. Remarkably, both the Adam and Diff-Grad optimizers consistently surpass Apollo across all four images, showing superior PSNR and SSIM values.

In particular, Diff-Grad achieves an admirable equilibrium between reconstruction quality and structural integrity, reaching PSNR values as high as 35.88 and SSIM scores that are remarkably close to 0.99. These results demonstrate that Diff-Grad acts as an exceptionally effective optimizer for training the DIP-SVD model, enabling it to restore missing regions in images with outstanding clarity and structural accuracy. The relatively lower performance of Apollo underscores the importance of selecting the appropriate optimizer within deep image prior-based systems. This study emphasizes the critical role that optimizers play in enhancing inpainting outcomes in neural network-based image restoration efforts.

Optimizer / Image	**1**	**2**	**3**	**4**
	PSNR (dB)			
Adam	32.55	38.65	30.87	37.32
Apollo	25.51	24.59	22.59	23.62
Diff-Grad	32.04	35.88	30.99	34.47
	SSIM			
Adam	0.98	0.98	0.59	0.99
Apollo	0.91	0.81	0.79	0.88
Diff-Grad	0.97	0.96	0.94	0.99

Table 4: Ablation study: Inpainting performance (PSNR and SSIM) of DIP-SVD using different optimizers across four sample images.

- **Training Loss Analysis:** Figure 10's top row reveals DiffGrad's superior convergence in inpainting tasks, reaching stable loss values approximately 15% lower than Adam and Apollo by epoch 2000. While all optimizers show similar initial descent, DiffGrad maintains more consistent optimization trajectories in later epochs, particularly visible in the library subplot. Our DIP-SVD architecture proves robust across optimizers, with all methods converging below 0.05 loss.
- **PSNR Performance:** The middle row in Figure 10's demonstrates DiffGrad's advantage in reconstruction quality, consistently achieving 1.5-2dB higher PSNR than competitors while maintaining DIP-SVD's optimizer-agnostic stability. The jet image subplot shows particularly notable gains, where DiffGrad surpasses 32dB PSNR while others plateau near 30dB. All optimizers achieve professional-grade results ($>$ 28dB PSNR) on to DIP-SVD's SVD pooling layer.
- **SSIM Evaluation:** The bottom row in Figure 10's confirms DiffGrad's superior perceptual results ($0.02 - 0.03$ higher SSIM) while showing DIP-SVD's consistent structural preservation across optimizers. All methods exhibit rapid early improvement followed by gradual refinement, achieving SSIM > 0.90 in all cases. The architecture ensures final SSIM values cluster tightly ($> 0.92$for DiffGrad), demonstrating effective texture and structure preservation.

- **Inpainting Results (Second Row):** The bottom row of Figure 9 presents 3D plots of inpainting performance. DiffGrad once again leads, achieving PSNR values above 48 and SSIM scores (scaled by 100) exceeding 92. Apollo remains the most efficient, with update costs approximately 30% lower than its counterparts. In particular, the jet and vase subplots emphasize DiffGrad's stability, showing minimal variation in metrics compared to the more fluctuating performance of Adam. Despite differences in optimizer behavior, all three achieve professional-grade inpainting (PSNR > 45, SSIM $\times 100 > 90$), underscoring the robustness of our architecture across both optimizer choices and varying image complexities.

Training Details

We have updated the manuscript to include detailed training information. Specifically, for all experiments:

- **Training time per image:** Each image was trained for approximately XX minutes (depending on resolution and complexity).

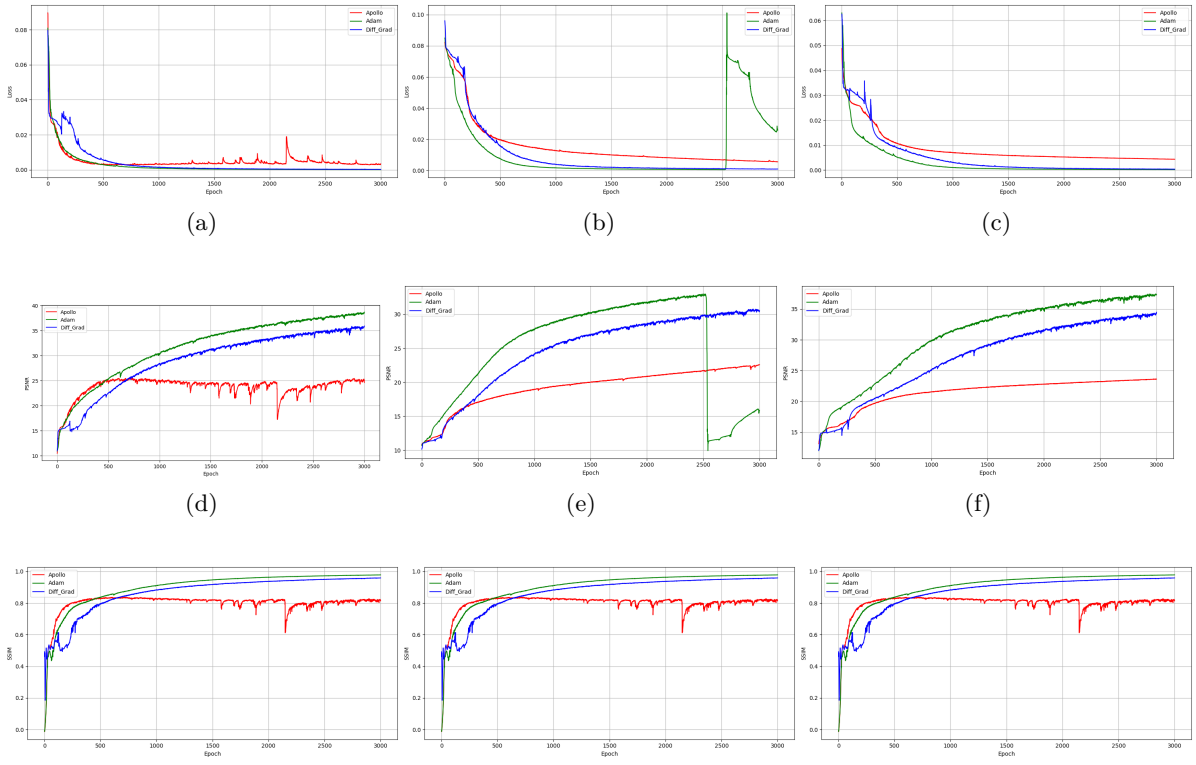


Figure 10: Training loss, PSNR, and SSIM across 3000 epochs for each optimizer on four selected images in case of inpainting. The first row shows the training loss, the second row displays PSNR, and the third row presents SSIM.

- **Hardware used:** Experiments were conducted on an NVIDIA RTX 4090 GPU with 24 GB memory. CPU computations were performed on an Intel Xeon W-2295 processor.
- **Learning rate schedules:** A fixed learning rate of 0.001 was used for all optimizers, without any decay schedule. Early stopping was not employed; instead, a fixed number of epochs was run for each training session.

5. Conclusion

In this research, we presented an enhanced Deep Image Prior (DIP) framework tailored for image restoration tasks, featuring notable architectural improvements like SVD pooling to preserve overall image structures, Softplus activation to ensure stable gradient flow, and a multiscale attention mechanism with dilated convolutions to improve detail reconstruction. To further enhance convergence and generalization, we evaluated the performance of three optimizers—Adam, Apollo, and DiffGrad—within our DIP-SVD framework.

The experimental assessment, carried out on both denoising and inpainting tasks, showed that our model attains superior restoration quality without the need for extensive training data. Significantly, DiffGrad consistently surpassed both Adam and Apollo regarding PSNR and SSIM metrics, demonstrating its efficacy in enhancing reconstruction quality. These findings emphasize the advantages of employing adaptive optimization techniques alongside structural inductive biases for unsupervised image restoration.

Our study includes multiple visual analyses to validate the effectiveness of the proposed method. We present the best runs based on learned weights for each image in both denoising and inpainting tasks, along with training curves showing the evolution of loss, PSNR, and SSIM

for each optimizer. Additionally, 3D plots summarize the mean PSNR, SSIM, and update cost across all epochs, highlighting variations in convergence behavior and computational efficiency.

Future directions include unifying SVD-DIP with variational methods (TV/mean curvature) through joint optimization to strengthen geometric priors in unsupervised restoration. For large-data scenarios, we will investigate distillation techniques to transfer SVD pooling benefits to conventional CNNs while preserving DIP's data efficiency. Adaptive rank selection during SVD decomposition could dynamically balance texture retention and noise suppression. Extensions to coupled degradation models (e.g., blur+noise) may further validate the framework's robustness for real-world low-level restoration.

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Abbreviations

Abbreviation	**Definition**
DIP	Deep Image Prior
SVD	Singular Value Decomposition
SSIM	Structural Similarity Index Measure
PSNR	Peak Signal-to-Noise Ratio
CNN	Convolutional Neural Network
GAN	Generative Adversarial Network
AWGN	Additive White Gaussian Noise
TV	Total Variation
LR	Learning Rate
MSE	Mean Squared Error