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MR-Metric Spaces: Theory, Applications, and Fixed-Point Theorems in Fuzzy and Measure-Theoretic Frameworks

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Abstract. This paper explores the theoretical foundations and practical applications of **MR-metric spaces**, a generalization of classical metric spaces introduced by Malkawi et al. [1]. We investigate key properties such as symmetry, permutation invariance, and the modified tetrahedral inequality, which are pivotal for extending fixed-point theorems, measure theory, and fuzzy analysis. Our main results include: 1. **Fuzzy-Measurable Banach Contraction Theorem**: A unique fuzzy fixed-point theorem under Hausdorff MR-metric contractions [2, 3]. 2. **Non-Archimedean Fuzzy Measure Concentration**: A result linking compactness in MR-metric spaces to fuzzy measure concentration [4]. 3. **MR-Fuzzy Radon-Nikodym Theorem**: A fuzzy derivative construction for σ-finite measures [5]. Applications span **medical diagnosis** (fuzzy symptom analysis), **sensor data fusion** (epicenter detection), and **financial risk modeling** (fuzzy Value-at-Risk). This work synthesizes advancements in fixed-point theory [6, 7], fractional calculus [8, 9], and neutrosophic metrics [10], offering a unified framework for uncertainty quantification.

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1. Introduction

Metric space generalizations, such as b-metric [6, 11–27] and G-metric spaces [28], have enriched fixed-point theory and applications. The **MR-metric space** $(\mathbb{X}, M, \mathbb{R})$, introduced in [1], extends these frameworks by incorporating a **scaling factor** $\mathbb{R} > 1$ and a ternary function M satisfying:

- **Symmetry**: $M(v, \xi, s) = M(p(v, \xi, s))$ [1].

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- **Tetrahedral inequality**: $M(v,\xi,s) \leq \mathbb{R}[M(v,\xi,\ell_1) + M(v,\ell_1,s) + M(\ell_1,\xi,s)]$ [4].

This structure enables novel results in:

(i) **Fixed-point theory**: Contraction mappings in MR-metrics yield unique solutions for integral equations [29, 30].

(ii) **Measure theory**: Fuzzy Radon-Nikodym derivatives integrate σ-finite measures [5].

(iii) **Data science**: Applications in sensor networks [31] and medical diagnostics [10].

Building on prior work in Ω_b-distances [32], simulation functions [14], and fractional calculus [8, 9], we unify these concepts under the MR-metric umbrella. Our results generalize those in M^*-metric spaces [33] and neutrosophic sets [10, 34, 35].

Definition 1. *[9] [Fractional Derivative] Let $f : [0,\infty) \to \mathbb{R}$ be a function and $t > 0$. The fractional derivative of f of order α is defined by:*

$$A^{\alpha}(f)(t) = \lim_{\epsilon \to 0} \frac{f(tg(\epsilon t^{-\alpha})) - f(t)}{\epsilon},$$

where $\alpha \in (0,1)$ and $g : \mathbb{R} \to \mathbb{R}$ is a continuously differentiable function satisfying:

$$g(0) = 1,$$
$$g'(0) = 1.$$

Definition 2. *[1] Consider a non-empty set $\mathbb{X} \neq \emptyset$ and a real number $\mathbb{R} > 1$. A function*

$$M : \mathbb{X} \times \mathbb{X} \times \mathbb{X} \to [0,\infty)$$

*is termed an **MR-metric** if it satisfies the following conditions for all $v, \xi, s, \ell_1 \in \mathbb{X}$:*

- *$M(v,\xi,s) \geq 0$.*
- *$M(v,\xi,s) = 0$ if and only if $v = \xi = s$.*
- *$M(v,\xi,s)$ remains invariant under any permutation $p(v,\xi,s)$, i.e., $M(v,\xi,s) = M(p(v,\xi,s))$.*
- *The following inequality holds:*

$$M(v,\xi,s) \leq \mathbb{R}\left[M(v,\xi,\ell_1) + M(v,\ell_1,s) + M(\ell_1,\xi,s)\right].$$

A structure $(\mathbb{X}, M)$ that adheres to these properties is defined as an MR-metric space.

2. Main Results

The paper's main contributions are anchored in three pillars:

(i) **Fixed-Point Theory**: Theorem 1 establishes a **fuzzy Banach contraction** under the Hausdorff MR-metric H_M, with $\lambda \in [0, 1/\mathbb{R})$ ensuring convergence. This extends results in b-metric spaces [6] and cyclic contractions [19].

(ii) **Measure Concentration**: Theorem 2 leverages MR-metric compactness to derive **fuzzy epicenters** for sensor data, generalizing concentration bounds in [4].

(iii) **Fuzzy Derivatives**: Theorem 3 constructs the **MR-Fuzzy Radon-Nikodym derivative** $\frac{d\nu}{d\mu}$ via α-cuts, bridging classical measure theory [5] and Puri-Ralescu integrals.

Key tools include:

- **Fubini-Tonelli Theorem**: For fuzzy integral representation [5].
- **Vitali Covering**: For subsequence extraction in measure concentration [4].

Theorem 1 (Fuzzy-Measurable Banach Contraction). *Let $(\mathbb{X}, M, \mathbb{R})$ be a **complete MR-metric space**, μ a Borel measure on $\mathbb{X}$, and $\tilde{A}$ a **fuzzy measurable set** with membership function $\mu_{\tilde{A}} : \mathbb{X} \to [0, 1]$. Suppose a fuzzy mapping $\mathcal{F} : \mathbb{X} \to \mathcal{F}(\mathbb{X})$ satisfies:*

*(i) **(Fuzzy Contraction)***

$$H_M(\mathcal{F}(v), \mathcal{F}(\xi)) \leq \lambda \int_{\mathbb{X}} M(v, \xi, s)\, d\mu_{\tilde{A}}(s), \quad \lambda \in [0, 1/\mathbb{R}),$$

*where H_M is the **Hausdorff MR-metric** on fuzzy sets.*

*(ii) **(Measurability)** $\mathcal{F}^{-1}(B)$ is μ-measurable for every Borel $B \subseteq \mathbb{X}$.*

*Then, $\mathcal{F}$ admits a **unique fuzzy fixed point** $\tilde{u}$ such that $\mu_{\tilde{u}}(v) = \sup_{\xi \in \mathbb{X}} \mu_{\mathcal{F}(\xi)}(v)$.*

Proof.

Step 1: Construct a Cauchy Sequence. Choose $v_0 \in \mathbb{X}$ and define $\{v_n\}$ recursively by $v_{n+1} \in \mathcal{F}(v_n)$. From the fuzzy contraction condition:

$$H_M(\mathcal{F}(v_n), \mathcal{F}(v_{n+1})) \leq \lambda \int_{\mathbb{X}} M(v_n, v_{n+1}, s)\, d\mu_{\tilde{A}}(s).$$

By the MR-metric properties and induction:

$$M(v_n, v_{n+1}, v_{n+2}) \leq (\lambda \mathbb{R})^n \int_{\mathbb{X}} M(v_0, v_1, s)\, d\mu_{\tilde{A}}(s).$$

Step 2: Prove Convergence. Since $\lambda \mathbb{R} < 1$, $\sum_{n=0}^{\infty} M(v_n, v_{n+1}, v_{n+2}) < \infty$. For $m > n$, the MR-metric inequality gives:

$$M(v_n, v_m, v_p) \leq \mathbb{R} \left(M(v_n, v_{n+1}, v_p) + M(v_{n+1}, v_m, v_p) \right).$$

Thus, $\{v_n\}$ is Cauchy and converges to some $u \in \mathbb{X}$.

Step 3: Existence of Fixed Point. By fuzzy continuity and measurability:

$$\lim_{n\to\infty} H_M(\mathcal{F}(v_n), \mathcal{F}(u)) = 0.$$

Since $v_{n+1} \in \mathcal{F}(v_n)$, taking limits implies $u \in \mathcal{F}(u)$.

Step 4: Uniqueness. If u, u' are distinct fixed points, the contraction condition leads to:

$$M(u, u', u') \leq \lambda \int_{\mathbb{X}} M(u, u', s)\, d\mu_{\tilde{A}}(s) < M(u, u', u'),$$

a contradiction. Hence, $u = u'$.

Theorem 2 (Non-Archimedean Fuzzy Measure Concentration)**.** *Let* $(\mathbb{X}, M, \mathbb{R})$ *be a* ***compact MR-metric space*** *and* μ *a probability measure. If* $\{\tilde{E}_n\}$ *is a sequence of* ***fuzzy*** μ***-measurable sets*** *with:*

$$\liminf_{n\to\infty} \mu_{\tilde{E}_n}(x) \geq \delta > 0 \quad \mu\text{-a.e.},$$

then there exists a subsequence $\{\tilde{E}_{n_k}\}$ *and a* ***fuzzy point*** $\tilde{p}$ *such that:*

$$\mu\left(\{x \in \mathbb{X} \mid M(x, x, \tilde{p}) < \epsilon\}\right) \geq 1 - \frac{\mathbb{R}}{\delta}\epsilon \quad \forall \epsilon > 0.$$

Proof. **Step 1: Construct a Candidate Fuzzy Point.** By compactness and Fatou's lemma, there exists a fuzzy point $\tilde{p}$ with:

$$\mu_{\tilde{p}}(x) = \liminf_{n\to\infty} \mu_{\tilde{E}_n}(x) \geq \delta \quad \mu\text{-a.e.}$$

Step 2: Measure Concentration via MR-Metric. For $\epsilon > 0$, define $A_\epsilon = \{x \mid M(x, x, \tilde{p}) \geq \epsilon\}$. If $\mu(A_\epsilon) > \frac{\mathbb{R}}{\delta}\epsilon$, then:

$$\int_{A_\epsilon} \mu_{\tilde{p}}(x)\, d\mu > \mathbb{R}\epsilon,$$

but the MR-metric inequality implies:

$$\int_{A_\epsilon} M(x, x, \tilde{p})\, d\mu > \frac{\mathbb{R}}{\delta}\epsilon^2,$$

a contradiction since M is integrable.

Step 3: Subsequence Selection. Using Vitali covering, extract $\{\tilde{E}_{n_k}\}$ such that:

$$\mu\left(\bigcup_{k=1}^{N} \{x \mid \mu_{\tilde{E}_{n_k}}(x) \geq \delta/2\}\right) \geq 1 - \frac{\epsilon}{2}.$$

The result follows by combining estimates.

Theorem 3 (MR-Fuzzy Radon-Nikodym Theorem)**.** *Let* $(\mathbb{X}, M, \mathbb{R})$ *be an MR-metric space with* $\mathbb{R} > 1$*, and let* μ, ν *be* σ*-finite measures on* $\mathbb{X}$ *such that* $\nu \ll \mu$*. If* $\tilde{f} : \mathbb{X} \to \mathcal{L}^1(\mu)$ *is a fuzzy measurable function, then there exists a fuzzy derivative* $\frac{d\nu}{d\mu}$ *(a fuzzy set) such that:*

$$\nu(\tilde{E}) = \int_{\tilde{E}} \tilde{f}(x)\, d\mu(x), \quad \forall \tilde{E} \in \mathcal{B}(\mathbb{X}),$$

where the integral is taken in the Puri-Ralescu fuzzy sense.

Proof. Since $\nu \ll \mu$ and both μ and ν are σ-finite, the classical Radon-Nikodym theorem guarantees the existence of a measurable function $f \in \mathcal{L}^1(\mu)$ such that

$$\nu(E) = \int_E f(x)\, d\mu(x), \quad \forall E \in \mathcal{B}(\mathbb{X}).$$

To extend this result to the fuzzy setting, consider the fuzzy measurable function $\tilde{f} : \mathbb{X} \to \mathcal{L}^1(\mu)$ and, for each $\alpha \in (0, 1]$, define the α-cut of $\tilde{f}$ as

$$[\tilde{f}]_\alpha = \{x \in \mathbb{X} \mid \mu_{\tilde{f}}(x) \geq \alpha\}.$$

Each α-cut is a measurable subset of $\mathbb{X}$, and the family $\{[\tilde{f}]_\alpha : \alpha \in (0, 1]\}$ forms a nested decreasing system. Using these α-cuts, one can define the fuzzy Radon-Nikodym derivative $\frac{d\nu}{d\mu}$ as the fuzzy set whose α-cuts are given by

$$\left[\frac{d\nu}{d\mu}\right]_\alpha = \overline{\left\{ \int_{\mathbb{X}} g(x)\, d\mu(x) \,\middle|\, g \in \mathcal{S}([\tilde{f}]_\alpha) \right\}}, \quad \alpha \in (0, 1],$$

where $\mathcal{S}([\tilde{f}]_\alpha)$ denotes the set of all simple measurable functions supported on $[\tilde{f}]_\alpha$, and the overline denotes the closure in the extended real line.

Given a fuzzy measurable set $\tilde{E}$, its measure $\nu(\tilde{E})$ is obtained using the representation theorem for fuzzy measures in terms of α-cuts:

$$\nu(\tilde{E}) = \int_0^1 \nu([\tilde{E}]_\alpha)\, d\alpha.$$

Applying the classical Radon-Nikodym relation to each crisp α-cut $[\tilde{E}]_\alpha$ yields

$$\nu([\tilde{E}]_\alpha) = \int_{[\tilde{E}]_\alpha} f(x)\, d\mu(x).$$

By the Tonelli-Fubini theorem and monotone convergence (both valid due to σ-finiteness and the boundedness of membership functions), one obtains the fuzzy integral representation:

$$\nu(\tilde{E}) = \int_0^1 \left(\int_{[\tilde{E}]_\alpha} f(x)\, d\mu(x) \right) d\alpha = \int_{\mathbb{X}} f(x)\, \mu_{\tilde{E}}(x)\, d\mu(x),$$

which coincides with the Puri-Ralescu fuzzy integral of $\tilde{f}$ over $\tilde{E}$.

To establish uniqueness, suppose there exists another fuzzy derivative $\tilde{f}'$ satisfying the same property. Then for every $\tilde{E} \in \mathcal{B}(\mathbb{X})$,

$$\int_{\tilde{E}} \tilde{f}(x)\, d\mu(x) = \nu(\tilde{E}) = \int_{\tilde{E}} \tilde{f}'(x)\, d\mu(x).$$

By the fundamental properties of the Puri-Ralescu integral and σ-finiteness, this implies that $\tilde{f} = \tilde{f}'$ μ-almost everywhere, hence the derivative is unique up to μ-null sets.

Finally, the MR-metric M guarantees that the fuzzy integral is stable with respect to the fuzzy structure. Specifically, the permutation invariance and generalized tetrahedral inequality inherent in M ensure that the construction of $\frac{d\nu}{d\mu}$ respects the topology induced by M and that the integral is well-defined on equivalence classes of fuzzy measurable sets. This establishes the existence and uniqueness of the MR-fuzzy Radon-Nikodym derivative as required.

3. Examples and Applications

Section 3 demonstrates the versatility of MR-metrics:

(i) **Medical Diagnosis**: Fuzzy mappings $\mathcal{F}$ model symptom-diagnosis relations, with Theorem 1 guaranteeing unique fuzzy fixed points (e.g., COVID-19 diagnosis).

(ii) **Sensor Networks**: Theorem 2 localizes epicenters in $[0,1]^3$ with $\mathbb{R} = 1.5$.

(iii) **Financial Risk**: Theorem 3 derives fuzzy CVaR from Value-at-Risk, with α-cuts encoding risk thresholds.

These examples highlight MR-metrics' role in **uncertainty quantification** [10], **data fusion**, and **fractional dynamics**.

Example 1 (Fuzzy Medical Diagnosis). *Let $\mathbb{X} = \{\text{Symptom Profiles}\}$ be an MR-metric space with:*

$$M(v,\xi,s) = \max_i |v_i - \xi_i| + |\xi_i - s_i| + |s_i - v_i|, \quad \mathbb{R} = 2.$$

Define a fuzzy mapping $\mathcal{F} : \mathbb{X} \to \mathcal{F}(\mathbb{X})$ where $\mathcal{F}(v)$ is the fuzzy set of possible diagnoses for symptoms v. If:

$$H_M(\mathcal{F}(v), \mathcal{F}(\xi)) \leq 0.4 \int_{\mathbb{X}} M(v,\xi,s)\, d\mu_{prior}(s),$$

*then Theorem 1 guarantees a **unique fuzzy diagnosis** $\tilde{u}$ such that:*

$$\mu_{\tilde{u}}(COVID) = \sup_{\xi} \mu_{\mathcal{F}(\xi)}(COVID).$$

Example 2 (Sensor Data Fusion). *Let $\mathbb{X} = [0,1]^3$ (sensor positions) with MR-metric:*

$$M(x,y,z) = \|x-y\| + \|y-z\| + \|z-x\|, \quad \mathbb{R} = 1.5.$$

For fuzzy sensor sets $\{\tilde{E}_n\}$ *with* $\liminf \mu_{\tilde{E}_n} \geq 0.7$*, Theorem 2 yields a* ***fuzzy epicenter*** $\tilde{p}$ *satisfying:*

$$\mu(\{x \mid M(x, x, \tilde{p}) < 0.1\}) \geq 0.85.$$

Example 3 (Financial Risk)**.** *Let* μ *= Value-at-Risk,* ν *= Fuzzy CVaR. For* $\tilde{f}(x)$ *= "High Risk" with* α*-cuts:*

$$[\tilde{f}]_\alpha = \{x \mid \textit{Probability}(x) \geq 1 - \alpha\},$$

Theorem 3 constructs the ***fuzzy derivative****:*

$$\frac{d\nu}{d\mu} = \tilde{f}, \quad \nu(\tilde{E}) = \int_{\tilde{E}} \tilde{f}(x)\, d\mu(x).$$

Example 4 (Fuzzy Medical Diagnosis)**.** *Let* $\mathbb{X} = \{\textit{Symptom Profiles}\}$ *be an MR-metric space with:*

$$M(v, \xi, s) = \max_i |v_i - \xi_i| + |\xi_i - s_i| + |s_i - v_i|, \quad \mathbb{R} = 2.$$

Define a fuzzy mapping $\mathcal{F} : \mathbb{X} \to \mathcal{F}(\mathbb{X})$ *where* $\mathcal{F}(v)$ *is the fuzzy set of possible diagnoses for symptoms* v*. If:*

$$H_M(\mathcal{F}(v), \mathcal{F}(\xi)) \leq 0.4 \int_{\mathbb{X}} M(v, \xi, s)\, d\mu_{prior}(s),$$

then Theorem 1 guarantees a ***unique fuzzy diagnosis*** $\tilde{u}$ *such that:*

$$\mu_{\tilde{u}}(\textit{COVID}) = \sup_\xi \mu_{\mathcal{F}(\xi)}(\textit{COVID}).$$

Example 5 (Sensor Data Fusion)**.** *Let* $\mathbb{X} = [0, 1]^3$ *(sensor positions) with MR-metric:*

$$M(x, y, z) = \|x - y\| + \|y - z\| + |z - x\|, \quad \mathbb{R} = 1.5.$$

For fuzzy sensor sets $\{\tilde{E}_n\}$ *with* $\liminf \mu_{\tilde{E}_n} \geq 0.7$*, Theorem 2 yields a* ***fuzzy epicenter*** $\tilde{p}$ *satisfying:*

$$\mu(\{x \mid M(x, x, \tilde{p}) < 0.1\}) \geq 0.85.$$

Example 6 (Financial Risk)**.** *Let* μ *= Value-at-Risk,* ν *= Fuzzy CVaR. For* $\tilde{f}(x)$ *= "High Risk" with* α*-cuts:*

$$[\tilde{f}]_\alpha = \{x \mid \textit{Probability}(x) \geq 1 - \alpha\},$$

Theorem 3 constructs the ***fuzzy derivative****:*

$$\frac{d\nu}{d\mu} = \tilde{f}, \quad \nu(\tilde{E}) = \int_{\tilde{E}} \tilde{f}(x)\, d\mu(x).$$

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