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An Innovative Method for Employing Complex Intuitionistic Fuzzy Ideals in $\mathcal{BC}\mathbb{K}/\mathcal{BCI}$-Algebras

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Abstract. The complex intuitionistic fuzzy set is a more generalized version of the complex fuzzy set. It is made by including the complex degree of non-grading functions, which are also important in the decision-making process, and studying their basic properties. The complex intuitionistic fuzzy set extends theories like the complex fuzzy set, intuitionistic fuzzy set, and fuzzy set. The goal of this paper is to apply complex intuitionistic fuzzy sets in $\mathcal{BC}\mathbb{K}/\mathcal{BCI}$-algebras ($M$), explain what a complex intuitionistic fuzzy ideal is, and explore some of its properties. We introduce the notion of a complex intuitionistic fuzzy sub-algebra in M, and its characteristics are investigated. We also look into the level operators and models of these complex intuitionistic fuzzy sub-algebras and explain their importance in M. Finally, we discuss the laws and operations of a complex intuitionistic fuzzy set in M, such as complement, intersection, union, boundedness, and simple differences of complex intuitionistic fuzzy ideals.

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1. Introduction

The concept of $\mathcal{BC}\mathbb{K}/\mathcal{BCI}$-algebra developed from two distinct techniques: (1) set theory, and (2) non-classical and classical propositional calculi. Currently, $\mathcal{BC}\mathbb{K}/\mathcal{BCI}$-algebras are utilized in a variety of mathematical fields, including topology, probability theory, functional analysis, group theory, fuzzy set theory, and others. Zhang [1] presented the $\mathcal{BC}\mathbb{K}$ and $\mathcal{BCI}$ algebra concepts and improved their definitions by providing new equivalent conditions. Liu [2] established the new concepts of fuzzy $\mathcal{BCI}$-implicative

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ideals and fuzzy $\mathcal{BCI}$-positive implicative ideals in $\mathcal{BCI}$-algebras and explored their characteristics. The relations between distinct fuzzy ideals demonstrate that a fuzzy set (FS) μ of a $\mathcal{BCI}$-algebra is a fuzzy $\mathcal{BCI}$-implicative ideal if and only if μ is both a fuzzy $\mathcal{BCI}$-commutative ideal and a fuzzy $\mathcal{BCI}$-positive implicative ideal. Meng [3] investigated the idea of fuzzy implicative ideals in $\mathcal{BCK}$-algebras and presented various characterizations of these ideals. Jun [4] utilized the concept of soft sets to the concept of $\mathcal{BC}\mathbb{K}/\mathcal{BCI}$ algebras. The concepts of soft $\mathcal{BC}\mathbb{K}/\mathcal{BCI}$ algebras and soft sub-algebras were presented, and their fundamental characteristics were established. Senapati and Shum [5] introduced the cubic set notion to implicative ideals of BCK-algebras and characterised their basic attributes.

Andres-Sanchez [6] developed fuzzy sets in quantities correlated with a set of objects, with the degree of membership of [0, 1] expressing the degree of elements belonging to the set. Mardani et al. [7] established among the available strategies for dealing with uncertainty issues, fuzzy mathematics mainly focuses on items with specifically internal definitions but uncertain outward deployment as membership degree; however, the independency value as a key component of elements was ignored in the frameworks. The intuitionistic fuzzy set (IFS) theory has a broad range of uses in many domains, such as medical diagnosis [8, 9], pattern recognition [10], engineering systems [11], and decision-making [12].

In today's world, scientists and technologists routinely meet complex processes and phenomena that are beyond full and exact insight. Therefore, it is essential to include accurate mathematical models into systems that exhibit a high level of uncertainty. The reason for developing fuzzy set theory stemmed from the need to broaden traditional set theory in order to efficiently address a certain purpose. The methodology offered herein provides a systematic strategy for developing and evaluating various models that effectively capture and tackle the inherent uncertainties in a particular environment. This theory is critical for the development of such structures. Furthermore, it improves our ability to investigate and adapt to the complex and unpredictable properties of systems within a wide range of scientific and technical disciplines.

The uses of fuzzy set concept have been demonstrated across a wide range of scientific areas and natural phenomena. Fuzzy sets (FSs) have proven to be an adaptable strategy for dealing with complex and uncertain situations in a variety of contexts. FS relies primarily on membership functions that operate in a single dimension, making it difficult to describe complex relationships and variables over several dimensions. Ordinary FS serves as a valuable mathematical tool in such circumstances. Complex fuzzy sets have the ability to represent uncertainty in a more detailed fashion by adding many dimensions or membership attributes. This allows for a more thorough and effective analysis of circumstances with complex physical characteristics, as well as an individual's ability to make informed decisions in complicated situations and challenging.

In today's society, the advancement of computer technology, the availability of high-speed processors, and the widespread use of programming languages have provided researchers with new opportunities to investigate and develop algorithms that specifically address intricate physical phenomena in a variety of scientific fields. The field of general operator theory presents a theoretical structure for understanding the mathematical principles that serve as the foundation for numerous technical approaches utilized in a variety of fields. The complex intuitionistic fuzzy environment displays mathematical patterns that can be easily understood within the framework of general operator theory. By embracing this expansion, software applications that have the capacity to solve a broad range of problems and advance a number of academic disciplines can be developed. The vague and uncertain are essential parts of mankind's existence. Accurate estimates or hypotheses are unattainable and significantly detrimental to human intelligence. Several mathematical concepts, including fuzzy sets (FSs), have emerged as effective strategies to tackle this challenge. To address this uncertainty in the data, Zadeh [13] established the concept of a FS, and defined as $\mu \to A : \{(s, \mu(s)); s \in A\}$, where $\mu(s) \in [0,1]$ known as membership value. Rosenfeld [14] introduced the fuzzy subgroup (FSG) and investigated the algebraic characteristics of the service. Das [15] initiated the concept of "level subgroups" of a FSG. Meng [16] developed a fuzzy concept using a FS in a $\mathcal{BCI}$-algebra. Specifically, certain concepts of Noether $\mathcal{BC}\mathbb{K}/\mathcal{BCI}$-algebras employ fuzzy ideals.

Atanassov [17] developed a new notion of IFS and described as $A = \{(s, \mu(s), \nu(s)) : s \in H\}$, where $\mu(s)$ and $\nu(s)$ are the membership degree (MD) and non-MD of an element, as well as $0 < \mu(s) + \nu(s) \leq 1$. Additionally, it demonstrated various attributes associated with relations and operations over sets, as well as the definition of topological operators and modal over the set of IFSs were discussed. Akram [18] proposed the notion of an intuitionistic fuzzy (IF) closed ideal of a $\mathcal{BCI}$-algebra, applied the idea of IFS to closed ideals in $\mathcal{BCI}$-algebras, and various attached characteristics were examined. Muhiuddin et al. [19] established the (α, β)-IF soft ideal of the $\mathcal{BC}\mathbb{K}/\mathcal{BCI}$ algebras, where α and β represent the membership values of an IF soft point, IFS and their related characteristics were examined. Senapati et al. [20] introduced the concepts of intuitionistic fuzzy translation to intuitionistic fuzzy sub-algebras and ideals in BCK/BCI-algebras. Also, the relationships between intuitionistic fuzzy translations and intuitionistic fuzzy extensions of intuitionistic fuzzy sub-algebras and ideals were investigated. Senapati et al. [21] investigated the cubic intuitionistic implicative ideals in BCK-algebras and the relationship between a cubic intuitionistic sub-algebra, a cubic intuitionistic ideal and a cubic intuitionistic implicative ideal were investigated. Furthermore, the conditions for a cubic intuitionistic ideal to be a cubic intuitionistic implicative ideal were described.

A complex fuzzy set (CFS) is more efficient and flexible than FSs. Ramote et al. [22] presented the concept of complex fuzzy logic (CFL). CFL is a generalization of traditional fuzzy logic, based on CFSs. Ramote et al. [22] established the novel concept of CFSs, which was defined as $\{s, \nu(s) = \mu(s)e^{\iota\theta(s)} : s \in H\}$, where $\mu(s) \to [0,1]$ is the MD of the real part and $\theta(p) \to [0, 2\pi]$ is a MD of the imaginary part of a complex number. The range of membership functions (MF) in a CFS is extended from a unit interval to the complex plane (CP) with the unit disc. The CFS helps equally in the process of evaluating the system because it considers magnitude term as well as phase term, where phase term presents the orientation of data item in complex unit disk plan. The primary set theoretic operations, like intersection, union, and complement, were discussed in detail under the influence of CFS. Jun and Xin [23] presented the principle of CFSs to $\mathcal{BC}\mathbb{K}/\mathcal{BCI}$-algebras. In a $\mathcal{BC}\mathbb{K}/\mathcal{BCI}$ algebra, the concepts of complex left (right) reduced ideal and complex subalgebra were presented, and their linked properties were discussed. Balamurugan et al. [24] established the notion of complex fuzzy sub-algebras (CFSA) in $\mathcal{BC}\mathbb{K}/\mathcal{BCI}$-algebra and their characteristics were discussed. Also, numerous laws and operations of a complex fuzzy system, such as bounded differences, union, simple differences, intersection, and complement of complex fuzzy (CF) ideals within $\mathcal{BC}\mathbb{K}/\mathcal{BCI}$-algebras were investigated.

Alolaiyan et al. [25] proposed (α, β)-CFSs and subgroups, indicating that all complex fuzzy subgroups were (α, β)-CFSGs. Furthermore, (α, β)-CF cosets, (α, β)-CF normal subgroup, and (α, β)-complex fuzzification of Lagrange's theorem analog to Lagrange's theorem of classical group theory were investigated. Zhang [1] established essential ideas about fuzzy complex numbers (FCNs) such as fuzzy distance and fuzzy limit. Furthermore, some fundamental properties of fuzzy limits, fuzzy complex numbers, and fuzzy distance were provided. Also, several essential theorems of FCNs, such as the nested closed rectangles theorem, the accumulation principle, and Cauchy's criterion for convergence were discussed.

Alkouri and Salleh [**?**] established a new notion of complex intutionistic fuzzy set (CIFS) defined as $\{s, \mu(s) = \gamma(s)e^{\iota\theta(s)}, \nu(s) = \overline{\gamma}(s)e^{\iota\overline{\theta}(s)} : s \in H\}$, which is extended by adding a non-MD term to the basic notion of a CFS, where $\gamma(s) \to [0, 1]$ is the MD of the real part , $\overline{\gamma}(s) \to [0, 1]$ is the non-MD of the real part , $\theta(s) \to [0, 2\pi]$ is the MD of the imaginary part and $\overline{\theta}(s) \to [0, 2\pi]$ is a non-MD of the imaginary part of a complex number such that $0 < \mu(s) + \nu(s) = 1$ and $0 < \theta(s) + \overline{\theta} = 2\pi$, for all complex numbers $s \in H$. The novelty of CIFS lies in its capabilities to achieve a wider range of values for both MF and non-MF. Gong and Wang [26] proposed a range of operation characteristic of CIFS were examined under the condition that both the non-membership phase and membership phase were limited to the interval $[0, 2\pi]$. Generally, membership and non-membership values have little practical significance, and there was no study of proximity and equality measures for CIFS. The distance measure (DM) was used to explain the (α, β)-equalities of CIFS. The (α, β)-equal describes two CIFS in which the difference between their non-membership degrees (MND) and membership degrees (MD) is less than β and $1 - \alpha$, respectively. Gulzar et al. [27] developed the notion of direct product between two complex IF subrings and the level sub-sets of the direct product of two complex IF subsets were defined. The complex intuitionistic fuzzy sub-algebra has a broader conceptual range compared to previously existing theories.

The complex intuitionistic fuzzy sub-algebra is a generalization of the complex fuzzy sub-algebra, which does not deal with the degree of non-membership. Also, complex intuitionistic fuzzy sub-algebra is a generalization of intuitionistic fuzzy sub-algebra, which does not deal with phase terms. The complex intuitionistic fuzzy sub-algebra deals both degree of membership and degree of non-membership, as well as phase term and amplitude terms.

Motivation and contribution for proposed concept:

- Ramot et al. [22] initiated the concept of a CFS by extending the MF from real to complex numbers with the unit disc. Because the CFS only evaluated the MD rather than the non-MD element of data components, which also performs an equal interest in the decision-making method for system evaluation, it provided weight to the MD.

- Latif and Shuaib [28] developed the idea of t-IF conjugate element and discovered the t-IF conjugacy classes of t-IF sub-group. The idea of t-IF p sub-group, the t-IF Sylow p sub-group, and the t-intuitionistic fuzzification of Sylow's Theorems were also explained. Gulzar et al. [29] proposed the t-IF centralizer and normalizer for t-IF subgroups. Additionally, the concept of t-IF cyclic and Abelian subgroups were introduced.

- Salleh [30] proposed the notion of CF space and CF sub-groups. The idea of fuzzy space extended beyond the real range of MFs to the complex range of MFs, which was represented by the unit disc in the complex plane. Ali et al. [31] introduced the idea of (ϵ, δ)-CAFSs, which give a more thorough representation of the ambiguity of information than CAFSs by incorporating both the magnitude and phase of the MFs. Also, the (ϵ, δ)-complex anti-fuzzy subgroups (CAFSG) in the environment of CAFS were explained. Gulzar et al. [32] introduced the idea of complex IF subgroups and showed that each complex IF subgroup can be split into two IF subgroups.

- Jawad et al. [33] investigated into group isomorphism under the influence of CIFS, a more general form of the CFS that included the non-MF degree. The complex algebraic structure offered useful tools for understanding complex techniques.

- The idea of complex fuzzy ideals in a $\mathcal{BC}\mathbb{K}/\mathcal{BCI}$-algebra was first put forward by Balamurugan et al. [24]. Our work is motivated by the reality that a CFS considers only the MD but does not weigh the non-MD of data elements. However, we apply the CIFS in $\mathcal{BC}\mathbb{K}/\mathcal{BCI}$-algebra, which is the most vital component of algebraic structure.

- The idea of complex IF ideals is not yet applied to the basic algebraic structure of $\mathcal{BC}\mathbb{K}/\mathcal{BCI}$-algebra. In this proposed work, we discuss the various basic characteristics of $\mathcal{BC}\mathbb{K}/\mathcal{BCI}$-algebra under the influence of CIFS.

Table 1: List of abbreviations and symbols.

Symbols	Abbreviations
M	$\mathcal{BC}\mathbb{K}/\mathcal{BCI}$-algebra
IFI	Intuitionistic Fuzzy Ideal
IFS	Intuitionistic Fuzzy Set
$IFSA$	Intuitionistic Fuzzy Sub-algebra
$CIFI$	Complex Intuitionistic Fuzzy Ideal
$CIFS$	Complex Intuitionistic Fuzzy Set
$CIFSA$	Complex Intuitionistic Fuzzy Sub-algebra

2. Preliminaries

We start by analyzing the basic concepts of CFSA and CIFSA, both are necessary for study.

Definition 1. *[34] Let U be a non-empty set has an identity element denoted as* 0 *and a binary operation "$\star$" then U is called $\mathcal{BCI}$-algebra if the following axioms holds.*

(i) $((s \star l) \star (s \star z)) \star (z \star l) = 0$, $\forall s, l, z \in M$,

(ii) $(s \star (s \star l) \star l) = 0$, $\forall s, l, \in M$,

(iii) $(s \star s) = 0$, $\forall s \in M$,

(iv) $(s \star l = 0, l \star s = 0 \Rightarrow s = l)$, $\forall s, l, \in M$.
Then, we describe M as a $\mathcal{BCI}$-algebra. Moreover, a $\mathcal{BCI}$-algebra M also fulfills:

(v) $(0 \star s = 0)$, $\forall s \in M$, *then M as a $\mathcal{BCK}$-algebra.*

Definition 2. *[17] A IFS L defined on M is given by:* $L = \{(s, \mu_L(s), \nu_L(s)) : s \in M\}$, *where* $\mu(s)$ *and* $\nu(s)$ *represent membership degree (MD) and non-MD of element of universe set M which belong to* $[0, 1]$ *such that* $0 < \mu(s) + \nu(s) \leq 1$, $\forall s \in M$.

Definition 3. *[35] An IFS L of M is known as an IFSA of M if it meets*

(i) $\mu_L(0) \geq \mu_L(s)$,

(ii) $\nu_L(0) \leq \nu_L(s)$,

(iii) $\mu_L(s \star l) \geq \mu_L(s) \wedge \mu_L(l)$, $\forall s, l \in M$,

(iv) $\nu_L(s \star l) \leq \nu_L(s) \vee \nu_L(l)$, $\forall s, l \in M$.

Definition 4. *[36] An IFS L of M is known as an IFI of M if it meets*

(i) $\mu_L(s) \geq \mu_L(s \star l) \wedge \mu_L(l)$, $\forall s, l \in M$,

(ii) $\nu_L(s) \leq \nu_L(s \star l) \vee \nu_L(l)$, $\forall s, l \in M$.

Definition 5. *[37] A CIFS, defined on M is described by a complex-valued grade of the non-membership function, membership function* $\nu_L(s)$, $\mu_L(s)$ *that assigns any element in L. The CIFS may be expressed by the set of ordered pairs* $L = \{(s, \mu_L(s), \nu_L(s)) : s \in M\}$ *where* $\mu_L(s) = \gamma_L(s)e^{\iota\theta_L(s)}$, $\iota = \sqrt{-1}$, $\gamma_L(s) \in [0, 1]$ *and* $\theta_L(s) \in [0, 2\pi]$. $\nu_L(s) = \overline{\gamma}_L(s)e^{\iota\overline{\theta}_L(s)}$, $\iota = \sqrt{-1}$, $\overline{\gamma}_L(s) \in [0, 1]$ *and* $\overline{\theta}_L(s) \in [0, 2\pi]$.

Definition 6. *[38] Let* $L = \{(s, \mu_L(s), \nu_L(s)) : s \in M\}$ *and* $B = \{(s, \mu_B(s), \nu_B(s)) : s \in M\}$ *be complex sub-sets of a non-void set M with membership functions* $\mu_L(s) = \gamma_L(s)e^{\iota\theta_L(s)}$, $\mu_B(s) = \gamma_B(s)e^{\iota\theta_B(s)}$ *respectively and non-membership functions* $\nu_L(s) = \overline{\gamma}_L(s)e^{\iota\overline{\theta}_L(s)}$, $\nu_B(s) = \overline{\gamma}_B(s)e^{\iota\overline{\theta}_B(s)}$ *respectively. By* $\mu_L(s) \leq \mu_B(s)$, *this implies that* $\gamma_L(s) \leq \gamma_B(s)$, $\theta_L(s) \leq \theta_B(s)$ *and* $\nu_L(s) \leq \nu_B(s)$, *we mean that* $\overline{\gamma}_L(s) \leq \overline{\gamma}_B(s)$, $\overline{\theta}_L(s) \leq \overline{\theta}_B(s)$.

3. Complex Intuitionistic Fuzzy Sub-algebras(CIFSAs) of $\mathcal{BC}\mathbb{K}/\mathcal{BCI}$-Algebras

In this part, we explore fundamental notions related to CIFSs, including CIFS on the universal set M and modal operators are defined.

Definition 7. *Let L be an IFS of M. Then, modal operators and level operators (i), (ii), (iii), (iv) are defined by*

(i) $\oplus L = \{(s, \frac{\mu_L(s)}{2}), \frac{\nu_L(s)}{2}) : \forall s \in M\}$,

(ii) $\otimes L = \{(s, \frac{\mu_L(s)+1}{2}, \frac{\nu_L(s)+1}{2}) : \forall s \in M\}$,

(iii) $\dagger L = \{(s, \frac{1}{2} \vee \mu_L(s), \frac{1}{2} \wedge \nu_L(s)) : \forall s \in M\}$,

(iv) $\ddagger L = \{(s, \frac{1}{2} \wedge \mu_L(s), \frac{1}{2} \vee \nu_L(s)) : \forall s \in M\}$.

Definition 8. *A CIFS $L = (s, \mu_L(s), \nu_L(s))$ is considered a CIFSA of M if $s, l \in M$, and it satisfies following:*

(i) $\mu_L(0)e^{\iota\theta_L(0)} \geq \mu_L(s)e^{\iota\theta_L(s)}$,

(ii) $\mu_L(s \star l)e^{\iota\theta_L(s\star l)} \geq \mu_L(s)e^{\iota\theta_L(s)} \wedge \mu_L(l)e^{\iota\theta_L(l)}$,

(iii) $\nu_L(0)e^{\iota\overline{\theta}_L(0)} \leq \nu_L(s)e^{\iota\overline{\theta}_L(s)}$,

(iv) $\nu_L(s \star l)e^{\iota\overline{\theta}_L(s\star l)} \leq \nu_L(s)e^{\iota\overline{\theta}_L(s)} \vee \nu_L(l)e^{\iota\overline{\theta}_L(l)}$.

Example 1. *Take a BCK algebra $M = \{0, s, l, z\}$, where the binary operation is defined by the Caley Table 2. Now explain a CIFS on M as:*
$L = \{(0, 0.8e^{\iota 0.5\pi}, 0.6e^{\iota 0.25\pi}), (s, 0.8e^{\iota 0.5\pi}, 0.6e^{\iota 0.25\pi}), (l, 0.5e^{\iota 0.2\pi}, 0.3e^{\iota 0.1\pi}), (z, 0.8e^{\iota 0.5\pi}, 0.6e^{\iota 0.25\pi})\}$. *It is simple to demonstrate that L is a CIFS of M.*

Table 2: Cayley's table describing the binary operation expressed by "$\star$".

$\star$	0	s	l	z	s
0	0	0	0	0	s
s	s	0	0	s	s
l	l	l	0	l	s
z	z	z	z	0	s
p	q	r	s	r	s

Example 2. *Take a $\mathcal{BC}\mathbb{K}$-algebra $M = \{0, s, l, z, w\}$, where the binary operation is defined by the Caley Table 3. Now explain a CIFS on M as:* $L = \{(0, 0.9e^{\iota 0.6\pi}, 0.7e^{\iota 0.4\pi}), (s, 0.7e^{\iota 0.5\pi}, 0.5e^{\iota 0.3\pi}), (l, 0.4e^{\iota 0.3\pi}, 0.2e^{\iota 0.1\pi}), (z, 0.4e^{\iota 0.3\pi}, 0.2e^{\iota 0.1\pi}), (w, 0.4e^{\iota 0.3\pi}, 0.2e^{\iota 0.1\pi})\}$. *It is simple to demonstrate that L is a CIFS of M.*

Table 3: Cayley's table describing the binary operation expressed by "$\star$".

$\star$	0	s	l	z	w
0	0	0	w	z	l
s	s	0	w	z	l
l	l	l	0	l	z
z	z	z	l	0	w
w	w	w	z	l	0

The following result indicates membership degree of identity element of CIFS is greater than all other elements. Also, the non-membership degree of identity element is less than the non-membership of remaining elements of CIFS.

Theorem 1. *If L is a CIFS of M, then $\mu_L(0) \geq \mu_L(s)$ and $\nu_L(0) \leq \nu_L(s)$.*

Proof. Let L be a CIFS of M. Then $\mu_L(0) = \gamma_L(0)e^{\iota\theta_L(0)} = \gamma_L(s \star s)e^{\iota\theta_L(s\star s)} \geq (\gamma_L(s) \wedge \gamma_L(s))e^{\iota(\theta_L(s)\wedge\theta_L(s))} = \gamma_L(s)e^{\iota\theta_L(s)} \geq \mu_L(s)$. And $\nu_L(0) = \overline{\gamma}_L(0)e^{\iota\overline{\theta}_L(0)} = \overline{\gamma}_L(s \star s)e^{\iota\overline{\theta}_L(s\star s)} \leq (\overline{\gamma}_L(s) \vee \overline{\gamma}_L(s))e^{\iota(\overline{\theta}_L(s)\vee\overline{\theta}_L(s))} = \overline{\gamma}_L(s)e^{\iota\overline{\theta}_L(s)} \leq \nu_L(s)$. Thus, we conclude the required result.

Definition 9. *Let L be a CIFS of M. Then modal operator $\oplus L$ is defined as*
$\mu_{\oplus L}(s) = \frac{\gamma_L(s)}{2}e^{\iota(\frac{\theta_L(s)}{2})}$, $\nu_{\oplus L}(s) = \frac{\overline{\gamma}_L(s)}{2}e^{\iota(\frac{\overline{\theta}_L(s)}{2})}$.

Example 3. *Let* $(\mu_L(s), \nu_L(s)) = \{(0, 0.4e^{\iota 0.5\pi}, 0.2e^{\iota 0.3\pi}), (s, 0.8e^{\iota 0.1\pi}, 0.6e^{\iota 0.01\pi}), (l, 0.6e^{\iota 0.3\pi}, 0.4e^{\iota 0.1\pi})\}$ *be a CIFS of M. Then* $(\mu_{\oplus L}(s), \nu_{\oplus L}(s)) = \{(0, 0.2e^{\iota 0.25\pi}, 0.1e^{\iota 0.15\pi}), (s, 0.4e^{\iota 0.05\pi}, 0.3e^{\iota 0.005\pi}), (l, 0.3e^{\iota 0.15\pi}, 0.2e^{\iota 0.05\pi})\}$ *is a CIFS of M.*

The following result shows that the modal operator $\oplus$ of CIFI of M is also CIFI.

Theorem 2. *If L is a CIFS of M. Then $\oplus L$ is a CIFS L of M.*

Proof. For each $s \in M$, we have $\mu_{\oplus L}(0) = \frac{\gamma_L(0)}{2}e^{\iota(\frac{\theta_L(0)}{2})} \geq \frac{\gamma_L(s)}{2}e^{\iota(\frac{\theta_L(s)}{2})} = \mu_{\oplus L}(s)$. Let $s, l \in M$, then $\mu_{\oplus L}(s\star l) = \frac{\gamma_L(s\star l)}{2}e^{\iota(\frac{\theta_L(s\star l)}{2})} \geq (\frac{\gamma_L(s)}{2} \wedge \frac{\gamma_L(l)}{2})e^{\iota(\frac{\theta_L(s)}{2}\wedge\frac{\theta_L(l)}{2})} = \mu_{\oplus L}(s)\wedge\mu_{\oplus L}(l)$. For each $s \in M$, we have $\nu_{\oplus L}(0) = \frac{\overline{\gamma}_L(0)}{2}e^{\iota(\frac{\overline{\theta}_L(0)}{2})} \leq \frac{\overline{\gamma}_L(s)}{2}e^{\iota(\frac{\overline{\theta}_L(s)}{2})} = \nu_{\oplus L}(s)$. Suppose that $s, l \in M$, Then $\nu_{\oplus L}(s\star l) = \frac{\overline{\gamma}_L(s\star l)}{2}e^{\iota(\frac{\overline{\theta}_L(s\star l)}{2})} \leq (\frac{\overline{\gamma}_L(s)}{2}\vee\frac{\overline{\gamma}_L(l)}{2})e^{\iota(\frac{\overline{\theta}_L(s)}{2}\vee\frac{\overline{\theta}_L(l)}{2})} = \nu_{\oplus L}(s)\vee\nu_{\oplus L}(l)$. This concludes the proof.

Definition 10. *Let L be a CIFS of M. Then modal operator $\otimes L$ is describe as*
$\mu_{\otimes L}(s) = \frac{\gamma_L(s)+1}{2}e^{\iota(\frac{\theta_L(s)+1}{2})}$, $\nu_{\otimes L}(s) = \frac{\overline{\gamma}_L(s)+1}{2}e^{\iota(\frac{\overline{\theta}_L(s)+1}{2})}$.

Example 4. *Let* $(\mu_L(s), \nu_L(s)) = \{(s, 0.3e^{\iota 0.5\pi}, 0.1e^{\iota 0.3\pi}), (l, 0.7e^{\iota 0.2\pi}, 0.5e^{\iota 0.1\pi}), (z, 0.5e^{\iota 0.4\pi}, 0.3e^{\iota 0.2\pi})\}$ *be a CIFS of M. Then* $(\mu_{\otimes L}(s), \nu_{\otimes L}(s)) = \{(s, 0.65e^{\iota 0.75\pi}, 0.55e^{\iota 0.65\pi}), (l, 0.85e^{\iota 0.6\pi}, 0.75e^{\iota 0.55\pi}), (z, 0.75e^{\iota 0.7\pi}, 0.65e^{\iota 0.6\pi})\}$ *is a CIFS of M.*

The subsequent result demonstrates that the modal operator $\otimes$ of CIFI of M is also CIFI.

Theorem 3. *Suppose that L is a CIFS of M. Then $\otimes L$ is a CIFS L of M.*

Proof. For each $s \in M$, we have $\mu_{\otimes L}(0) = \frac{\gamma_L(0)+1}{2} e^{\iota(\frac{\theta_L(0)+1}{2})} \geq \frac{\gamma_L(s)+1}{2} e^{\iota(\frac{\theta_L(s)+1}{2})} = \mu_{\oplus L}(s)$. Let $s, l \in M$, then $\mu_{\otimes L}(s \star l) = \frac{\gamma_L(s\star l)+1}{2} e^{\iota(\frac{\theta_L(s\star l)+1}{2})} \geq (\frac{\gamma_L(s)+1}{2} \wedge \frac{\gamma_L(l)+1}{2}) e^{\iota(\frac{\theta_L(s)+1}{2} \wedge \frac{\theta_L(l)+1}{2})}$ $= \mu_{\otimes L}(s) \wedge \mu_{\otimes L}(l)$. For each $s \in M$, we have $\nu_{\otimes L}(0) = \frac{\overline{\gamma}_L(0)+1}{2} e^{\iota(\frac{\overline{\theta}_L(0)+1}{2})} \leq \frac{\overline{\gamma}_L(s)+1}{2} e^{\iota(\frac{\overline{\theta}_L(s)+1}{2})} = \nu_{\oplus L}(s)$. Suppose that $s, l \in M$, then $\nu_{\otimes L}(s \star l) = \frac{\overline{\gamma}_L(s\star l)+1}{2} e^{\iota(\frac{\overline{\theta}_L(s\star l)+1}{2})} \leq (\frac{\overline{\gamma}_L(s)+1}{2} \vee \frac{\overline{\gamma}_L(l)+1}{2})$ $e^{\iota(\frac{\overline{\theta}_L(s)+1}{2} \vee \frac{\overline{\theta}_L(l)+1}{2})} = \nu_{\otimes L}(s) \vee \nu_{\otimes L}(l)$. Therefore, $\otimes L$ is a CIFS L of M.

Definition 11. *Suppose that L is a CIFS of M. Then level operator $\dagger L$ is describe as* $\mu_{\dagger L}(s) = (\frac{1}{2} \vee \gamma_L(s)) e^{\iota(\frac{1}{2} \vee \theta_L(s))}$, $\nu_{\dagger L(s)} = (\frac{1}{2} \wedge \overline{\gamma}_L(s)) e^{\iota(\frac{1}{2} \wedge \overline{\theta}_L(s))}$.

Example 5. *Let* $(\mu_L(s), \nu_L(s)) = \{(s, 0.4e^{\iota 0.6\pi}, 0.2e^{\iota 0.4\pi}), (l, 0.6e^{\iota 0.2\pi}, 0.4e^{\iota 0.1\pi}), (z, 0.5e^{\iota 0.7\pi}, 0.3e^{\iota 0.5\pi})\}$ *be a CIFS of M. Then* $\mu_{\dagger L}(s), \nu_{\dagger L}(s)) = \{(s, 0.5e^{\iota 0.6\pi}, 0.2e^{\iota 0.4\pi}), (l, 0.6e^{\iota 0.5\pi}, 0.4e^{\iota 0.1\pi}), (z, 0.5e^{\iota 0.6\pi}, 0.3e^{\iota 0.5\pi})\}$ *is a CIFS of M.*

The following outcome shows that the level operator $\dagger$ of the CIFI of M is also CIFI.

Theorem 4. *Suppose that L is a CIFS of M. Then $\dagger L$ is a CIFS L of M.*

Proof. For each $s \in M$, we have $\mu_{\dagger L}(0) = (\frac{1}{2} \vee \gamma_L(0)) e^{\iota(\frac{1}{2} \vee \theta_L(0))} \geq (\frac{1}{2} \vee \gamma_L(s)) e^{\iota(\frac{1}{2} \vee \theta_L(s))} = \mu_{\dagger L}(s)$. Let $s, l \in M$, then $\mu_{\dagger L}(s \star l) = (\frac{1}{2} \vee \gamma_L(s \star l)) e^{\iota(\frac{1}{2} \vee \theta_L(s\star l))} \geq \frac{1}{2} \vee (\gamma_L(s) \wedge \gamma_L(l))$ $e^{\iota(\frac{1}{2} \vee (\gamma_L(s) \wedge \gamma_L(l)))} = (\frac{1}{2} \vee (\gamma_L(s)) e^{\iota(\frac{1}{2} \vee (\gamma_L(s)))}) \wedge (\frac{1}{2} \vee (\gamma_L(l)) e^{\iota(\frac{1}{2} \vee (\gamma_L(l)))}) = \mu_{\dagger L}(s) \wedge \mu_{\dagger L}(l)$. For each $s \in M$, we have $\nu_{\dagger L}(0) = (\frac{1}{2} \wedge \overline{\gamma}_L(0)) e^{\iota(\frac{1}{2} \wedge \overline{\theta}_L(0))} \leq (\frac{1}{2} \wedge \overline{\gamma}_L(s)) e^{\iota(\frac{1}{2} \wedge \overline{\theta}_L(s))} = \nu_{\dagger L}(s)$. Suppose that $s, l \in M$, then $\nu_{\dagger L}(s \star l) = (\frac{1}{2} \wedge \gamma_L(s \star l)) e^{\iota(\frac{1}{2} \wedge \theta_L(s\star l))} \leq \frac{1}{2} \wedge (\gamma_L(s) \vee \gamma_L(l)) e^{\iota(\frac{1}{2} \wedge (\gamma_L(s) \vee \gamma_L(l)))} = (\frac{1}{2} \wedge (\gamma_L(s)) e^{\iota(\frac{1}{2} \wedge (\gamma_L(s)))}) \vee (\frac{1}{2} \wedge (\gamma_L(l)) e^{\iota(\frac{1}{2} \wedge (\gamma_L(l)))}) = \nu_{\dagger L}(s) \vee \nu_{\dagger L}(l)$. Therefore, $\dagger L$ is a CIFS L of M.

Definition 12. *Let L be a CIFS of M. Then level operator $\ddagger L$ is describe as* $\mu_{\ddagger L}(s) = (\frac{1}{2} \wedge \gamma_L(s)) e^{\iota(\frac{1}{2} \wedge \theta_L(s))}$, $\nu_{\ddagger L}(s) = (\frac{1}{2} \vee \overline{\gamma}_L(s)) e^{\iota(\frac{1}{2} \vee \overline{\theta}_L(s))}$.

Example 6. *Let* $(\mu_L(s), \nu_L(s)) = \{(s, 0.4e^{\iota 0.6\pi}, 0.2e^{\iota 0.4\pi}), (l, 0.6e^{\iota 0.2\pi}, 0.4e^{\iota 0.1\pi}), (z, 0.5e^{\iota 0.7\pi}, 0.3e^{\iota 0.5\pi})\}$ *be a CIFS of M. Then* $(\mu_{\ddagger L}(s), \nu_{\ddagger L}(s)) = \{(s, 0.4e^{\iota 0.5\pi}, 0.5e^{\iota 0.5\pi}), (l, 0.5e^{\iota 0.2\pi}, 0.5e^{\iota 0.5\pi}), (z, 0.5e^{\iota 0.5\pi}, 0.5e^{\iota 0.5\pi})\}$ *is a CIFS of M.*

The following result shows that the level operator $\ddagger$ of the CIFI of M is also CIFI.

Theorem 5. *Suppose that L is a CIFS of M. Then $\ddagger L$ is a CIFS L of M.*

Proof. For each $s \in M$, we have $\mu_{\ddagger L}(0) = (\frac{1}{2} \wedge \gamma_L(0)) e^{\iota(\frac{1}{2} \wedge \theta_L(0))} \geq (\frac{1}{2} \wedge \gamma_L(s)) e^{\iota(\frac{1}{2} \wedge \theta_L(s))} = \mu_{\ddagger L}(s)$. Let $s, l \in M$, then $\mu_{\ddagger L}(s \star l) = (\frac{1}{2} \wedge \gamma_L(s \star l)) e^{\iota(\frac{1}{2} \wedge \theta_L(s\star l))} \geq \frac{1}{2} \wedge (\gamma_L(s) \wedge \gamma_L(l))$

$e^{\iota(\frac{1}{2}\wedge(\gamma_L(s)\wedge\gamma_L(l)))} = (\frac{1}{2}\wedge(\gamma_L(s))e^{\iota(\frac{1}{2}\wedge(\gamma_L(s)))})\wedge(\frac{1}{2}\wedge(\gamma_L(l))e^{\iota(\frac{1}{2}\wedge(\gamma_L(l)))}) = \mu_{\ddagger L}(s)\wedge\mu_{\ddagger L}(l)$. For each $s\in M$, we have $\nu_{\ddagger L}(0) = (\frac{1}{2}\vee\overline{\gamma}_L(0))e^{\iota(\frac{1}{2}\vee\overline{\theta}_L(0))} \leq (\frac{1}{2}\vee\overline{\gamma}_L(s))e^{\iota(\frac{1}{2}\vee\overline{\theta}_L(s))} = \nu_{\ddagger L}(s)$. Suppose that $s,l\in M$, then $\nu_{\ddagger L}(s\star l) = (\frac{1}{2}\vee\gamma_L(s\star l))e^{\iota(\frac{1}{2}\vee\theta_L(s\star l))} \leq \frac{1}{2}\vee(\gamma_L(s)\vee\gamma_L(l))e^{\iota(\frac{1}{2}\vee(\gamma_L(s)\vee\gamma_L(l)))} = (\frac{1}{2}\vee(\gamma_L(s))e^{\iota(\frac{1}{2}\vee(\gamma_L(s)))})\vee(\frac{1}{2}\vee(\gamma_L(l))e^{\iota(\frac{1}{2}\vee(\gamma_L(l)))}) = \nu_{\ddagger L}(s)\vee\nu_{\ddagger L}(l)$. Therefore, $\ddagger L$ is a CIFS L of M.

4. Complex Intuitionistic Fuzzy Ideals (CIFIs) of BCK/BCI-Algebras

In the following part, we will examine fundamental concepts about the CIFIs, and CIFI over the universal set M.

Definition 13. *A CIFS $L = (s,\mu_L(s),\nu_L(s))$ is considered a CIFSA of M, where $s,l\in M$, and the following hold: $\mu_L(s)e^{\iota\theta_L(s)} \geq \mu_L(s\star l)e^{\iota\theta_L(s\star l)}\wedge\mu_L(l)e^{\iota\theta_L(l)}$, $\forall s,l\in M$, $\nu_L(s)e^{\iota\overline{\theta}_L(s)} \leq \nu_L(s\star l)e^{\iota\overline{\theta}_L(s\star l)}\vee\nu_L(l)e^{\iota\overline{\theta}_L(l)}$, $\forall s,l\in M$.*

Example 7. *Take a BCK-algebra $M = \{0,s,l,z\}$, where the binary operation is defined by the Caley Table 4. Now describe a CIFS on M as: $L = \{(0, 0.67e^{\iota 0.5\pi}, 0.47e^{\iota 0.25\pi}), (s, 0.34e^{\iota 0.43\pi}, 0.14e^{\iota 0.23\pi}), (l, 0.67e^{\iota 0.05\pi}, 0.47e^{\iota 0.025\pi}), (z, 0.34e^{\iota 0.43\pi}, 0.14e^{\iota 0.23\pi}), (w, 0.34e^{\iota 0.43\pi}, 0.14e^{\iota 0.23\pi})\}$. It is straightforward to prove that L is a CIFI of M.*

Table 4: Cayley's table describing the binary operation expressed by "$\star$".

$\star$	0	s	l	z	w
0	0	0	0	0	0
s	s	0	s	0	0
l	l	l	0	0	0
z	z	z	z	0	0
w	w	z	w	s	0

The next theorem shows that every CIFI of M is also order preserving.

Theorem 6. *Every CIFI of M is order-preserving.*

Proof. Assume that L is a CIFSA of M and assume that $s,l\in M$ are such that $s\leq l$. Then

$$\begin{aligned}
\mu_L(s) &= \gamma_L(s)e^{\iota\theta_L(s)}\\
&\geq \gamma_L(s\star l)e^{\iota\theta_L(s\star l)}\wedge\gamma_L(l)e^{\iota\theta_L(l)}\\
&= (\gamma_L(s\star l)\wedge\gamma_L(l))e^{\iota(\theta_L(s\star l)\wedge\theta_L(l))}\\
&= (\gamma_L(0)\wedge\gamma_L(0))e^{\iota(\theta_L(0)\wedge\theta_L(0))}\\
&= \gamma_L(l)e^{\iota\theta_L(l)} \geq \mu_L(l).\ And\\
\nu_L(s) &= \overline{\gamma}_L(s)e^{\iota\overline{\theta}_L(s)}
\end{aligned}$$

$$
\begin{aligned}
&\leq \overline{\gamma}_L(s\star l)e^{\iota\overline{\theta}_L(s\star l)} \vee \overline{\gamma}_L(l)e^{\iota\overline{\theta}_L(l)}\\
&= (\overline{\gamma}_L(s\star l) \vee \overline{\gamma}_L(l))e^{\iota(\overline{\theta}_L(s\star l)\vee\overline{\theta}_L(l))}\\
&= (\overline{\gamma}_L(0) \vee \overline{\gamma}_L(0))e^{\iota(\overline{\theta}_L(0)\vee\overline{\theta}_L(0))}\\
&= \overline{\gamma}_L(l)e^{\iota\overline{\theta}_L(l)} \leq \nu_L(l).
\end{aligned}
$$

This concludes the Proof.

The next theorem shows that every CIFI of the set is equal to a CIFSA.

Theorem 7. *Every CIFI of M is a CIFSA of M.*

Proof. Since $s\star l \leq s$, it follows from Property 2 that $\mu_L(s\star l) \geq \mu_L(s)$ and $\nu_L(s\star l) \leq \nu_L(s)$. Hence by Definition,

$$
\begin{aligned}
\mu_L(s\star l) &\geq \gamma_L(s)e^{\iota\theta_L(s)}\\
&= (\gamma_L(s\star l)e^{\iota\theta_L(s\star l)}) \wedge \gamma_L(l)e^{\iota\theta_L(l)}\\
&= (\gamma_L(s\star l) \wedge \gamma_L(l))e^{\iota(\theta_L(s\star l)\wedge\theta_L(l))}\\
&= (\gamma_L(s) \wedge \gamma_L(l))e^{\iota(\theta_L(s)\wedge\theta_L(l))}\\
&\geq \mu_L(s) \wedge \mu_L(l). \text{ Moreover}\\
\nu_L(s\star l) &\leq \overline{\gamma}_L(s)e^{\iota\overline{\theta}_L(s)}\\
&= (\overline{\gamma}_L(s\star l)e^{\iota\overline{\theta}_L(s\star l)}) \vee \overline{\gamma}_L(l)e^{\iota\overline{\theta}_L(l)}\\
&= (\overline{\gamma}_L(s\star l) \vee \overline{\gamma}_L(l))e^{\iota(\overline{\theta}_L(s\star l)\vee\overline{\theta}_L(l))}\\
&= (\overline{\gamma}_L(s) \vee \overline{\gamma}_L(l))e^{\iota(\overline{\theta}_L(s)\vee\overline{\theta}_L(l))}\\
&\leq \nu_L(s) \vee \nu_L(l).
\end{aligned}
$$

So L is a CIFS L of M.

The following result shows that if $s\star l \leq z$ then $\mu_L(s) \geq \mu_L(l) \wedge \mu_L(z)$ and $\nu_L(s) \leq \nu_L(l) \vee \nu_L(z)$.

Theorem 8. *Let L be a CIFI of M. If the inequality $s\star l \leq z$ holds in M, then $\mu_L(s) \geq \mu_L(l) \wedge \mu_L(z)$ and $\nu_L(s) \leq \nu_L(l) \vee \nu_L(z)$.*

Proof. Suppose that L is a CFSA of M and let $s\star l \leq z$ holds in M. Then

$$
\begin{aligned}
\mu_L(s\star l) &= \gamma_L(s\star l)e^{\iota\theta_L(s\star l)}\\
&\geq (\gamma_L((s\star l)\star z) \wedge \gamma_L(z))e^{\iota(\theta_L((s\star l)\star z)\wedge\theta_L(z))}\\
&= (\gamma_L(0) \wedge \gamma_L(z))e^{\iota(\theta_L(0)\wedge\theta_L(z))}\\
&= \gamma_L(z)e^{\iota\theta_L(z)} \geq \mu_L(z).
\end{aligned}
$$

It follows that $\mu_L(s) \geq \mu_L(l) \wedge \mu_L(z)$. Moreover

$$
\nu_L(s\star l) = \overline{\gamma}_L(s\star l)e^{\iota\overline{\theta}_L(s\star l)}
$$

$$\begin{aligned} &\leq (\overline{\gamma}_L((s\star l)\star z)\vee\overline{\gamma}_L(z))e^{\iota(\overline{\theta}_L((s\star l)\star z)\vee\overline{\theta}_L(z))}\\ &= (\overline{\gamma}_L(0)\vee\overline{\gamma}_L(z))e^{\iota(\overline{\theta}_L(0)\vee\overline{\theta}_L(z))}\\ &= \overline{\gamma}_L(z)e^{\iota\overline{\theta}_L(z)}\leq \nu_L(z). \end{aligned}$$

It follows that $\nu_L(s)\leq\nu_L(l)\vee\nu_L(z)$.

Definition 14. *Let L be a CIFS of M. Then, the complement L is described as:* $C(\mu_L(s))=(1-\gamma_L(s))e^{\iota(2\pi-\theta_L(s))}$, $C(\nu_L(s))=(1-\overline{\gamma}_L(s))e^{\iota(2\pi-\overline{\theta}_L(s))}$.

Example 8. *Let* $(\mu_L(s),\nu_L(s))=\{(s_1,0.3e^{\iota0.4\pi},0.1e^{\iota0.24\pi}),(s_2,0.6e^{\iota0.2\pi},0.4e^{\iota0.12\pi}),(s_3,0.8e^{\iota0.1\pi},0.6e^{\iota0.01\pi}),(s_4,0.2e^{\iota0.3\pi},0.1e^{\iota0.13\pi}),(s_5,0.5e^{\iota\pi},0.3e^{\iota0.8\pi}),(s_6,0.9e^{\iota0.1\pi},0.7e^{\iota0.01\pi})\}$ *be a CIFS of M. Then* $C(\mu_L(s),\nu_L(s))=\{(s_1,0.7e^{\iota1.6\pi},0.9e^{\iota1.76\pi}),(s_2,0.4e^{\iota1.8\pi},0.6e^{\iota1.88\pi}),(s_3,0.2e^{\iota1.9\pi},0.4e^{\iota1.99\pi}),(s_4,0.8e^{\iota1.7\pi},0.9e^{\iota1.87\pi}),(s_5,0.5e^{\iota\pi},0.7e^{\iota1.2\pi}),(s_6,0.1e^{\iota1.9\pi},0.3e^{\iota1.99\pi})\}$ *is a CIFI of M.*

The following outcome shows that the complement of CIFI of M is also CIFI.

Theorem 9. *A CIFS of M is a CIFI of M iff $C(\mu_L(s))$ and $C(\nu_L(s))$ is a CIFI of M.*

Proof. Suppose that L is a CIFSA of M and let $s,l\in M$. Then

$$\begin{aligned} C(\mu_L(0)) &= 1-\mu_L(0)\\ &= (1-\gamma_L(0))e^{\iota(2\pi-\theta_L(0))}\\ &\geq (1-\gamma_L(s))e^{\iota(2\pi-\theta_L(s))}\\ &= C(\mu_L(s)).\text{ And}\\ C(\mu_L(s)) &= 1-\mu_L(s)\\ &\geq 1-(\gamma_L(s\star l)e^{\iota(2\pi-\theta_L(s\star l))}\wedge\gamma_L(l)e^{\iota(2\pi-\theta_L(l))})\\ &= (1-\gamma_L(s\star l))e^{\iota(2\pi-\theta_L(s\star l))}\wedge(1-\gamma_L(l))e^{\iota(2\pi-\theta_L(l))}\\ &\geq C(\mu_L(s\star l))\wedge C(\mu_L(l)). \end{aligned}$$

Suppose that L is a CIFSA of M and let $s,l\in M$. Then

$$\begin{aligned} C(\nu_L(0)) &= 1-\nu_L(0)\\ &= (1-\overline{\gamma}_L(0))e^{\iota(2\pi-\overline{\theta}_L(0))}\\ &\leq (1-\overline{\gamma}_L(s))e^{\iota(2\pi-\overline{\theta}_L(s))}\\ &= C(\nu_L(s)).\text{ And}\\ C(\nu_L(s)) &= 1-\nu_L(s)\\ &\leq 1-(\overline{\gamma}_L(s\star l)e^{\iota(2\pi-\overline{\theta}_L(s\star l))}\vee\overline{\gamma}_L(l)e^{\iota(2\pi-\overline{\theta}_L(l))})\\ &= (1-\overline{\gamma}_L(s\star l))e^{\iota(2\pi-\overline{\theta}_L(s\star l))}\vee(1-\overline{\gamma}_L(l))e^{\iota(2\pi-\overline{\theta}_L(l))}\\ &\leq C(\nu_L(s\star l))\vee C(\nu_L(l)). \end{aligned}$$

Thus, the complement of membership and non-membership L is a CIFI of M.

Definition 15. *Suppose that L_1 and L_2 are two CIFSs of M. Then, the union $L_1 \cup L_2$ is defined as*
$\mu_{L_1 \cup L_2}(s) = (\gamma_{L_1}(s) \vee \gamma_{L_2}(s)) e^{\iota(\theta_{L_1}(s) \vee \theta_{L_2}(s))}$, $\nu_{L_1 \cup L_2}(s) = (\overline{\gamma}_{L_1}(s) \wedge \overline{\gamma}_{L_2}(s)) e^{\iota(\overline{\theta}_{L_1}(s) \wedge \overline{\theta}_{L_2}(s))}$.

Example 9. *Let* $(\mu_{L_1}(s), \nu_{L_1}(s)) = \{(s_1, 0.6e^{\iota 0.5\pi}, 0.4e^{\iota 0.25\pi}), (s_2, 1e^{\iota 0.5\pi}, 0.8e^{\iota 0.25\pi}), (s_3, 0.8e^{\iota 2\pi}, 0.6e^{\iota 1.5\pi}), (s_4, 0.9e^{\iota 0.4\pi}, 0.7e^{\iota 0.24\pi}), (s_5, 0.7e^{\iota \pi}, 0.5e^{\iota 0.8\pi}), (s_6, 0.5e^{\iota 0.4\pi}, 0.3e^{\iota 0.24\pi})\}$ *and* $(\mu_{L_2}(s), \nu_{L_2}(s)) = \{(s_1, 0.2e^{\iota \pi}, 0.1e^{\iota 0.88\pi}), (s_2, 0.1e^{\iota 0.8\pi}, 0.01e^{\iota 0.6\pi}), (s_3, 0.8e^{\iota 0.8\pi}, 0.6e^{\iota 0.68\pi}), (s_4, 0.2e^{\iota \pi}, 0.1e^{\iota 0.88\pi}), (s_5, 0.9e^{\iota 0.9\pi}, 0.7e^{\iota 0.7\pi}), (s_6, 0.3e^{\iota 2\pi}, 0.1e^{\iota 1.88\pi})\}$ *be a CIFSs. Then,* $(\mu_{L_1 \cup L_2}(s), \nu_{L_1 \cup L_2}(s)) = \{(s_1, 0.6e^{\iota \pi}, 0.1e^{\iota 0.25\pi}), (s_2, 1e^{\iota 0.8\pi}, 0.01e^{\iota 0.25\pi}), (s_3, 0.8e^{\iota 2\pi}, 0.6e^{\iota 0.68\pi}), (s_4, 0.9e^{\iota \pi}, 0.1e^{\iota 0.24\pi}), (s_5, 0.9e^{\iota \pi}, 0.5e^{\iota 0.7\pi}), (s_6, 0.5e^{\iota 2\pi}, 0.1e^{\iota 0.24\pi})\}$.

The subsequent theorem demonstrates that the union $\cup$ of two CIFIs of M is also CIFI.

Theorem 10. *Assume that L_1 and L_2 are two CIFIs of M. Then, $L_1 \cup L_2$ is a CIFI of M.*

Proof. Let L_1 and L_2 be two CIFIs of M and let $s, l \in M$. Then

$$\begin{aligned} \mu_{L_1 \cup L_2}(0) &= (\gamma_{L_1}(0) \vee \gamma_{L_2}(0)) e^{\iota(\theta_{L_1}(0) \vee \theta_{L_2}(0))} \\ &\geq (\gamma_{L_1}(s) \vee \gamma_{L_2}(s)) e^{\iota(\theta_{L_1}(s) \vee \theta_{L_2}(s))} \\ &= \mu_{L_1 \cup L_2}(s). \end{aligned}$$

Moreover

$$\begin{aligned} \mu_{L_1 \cup L_2}(s) &= (\gamma_{L_1}(s) \vee \gamma_{L_2}(s)) e^{\iota(\theta_{L_1}(s) \vee \theta_{L_2}(s))} \\ &\geq ((\gamma_{L_1}(s \star l) \wedge \gamma_{L_1}(l)) \vee (\gamma_{L_2}(s \star l) \wedge \gamma_{L_2}(l))) \\ &\quad e^{\iota((\theta_{L_1}(s \star l) \wedge \theta_{L_1}(l)) \vee (\theta_{L_2}(s \star l) \wedge \theta_{L_2}(l)))} \\ &\geq ((\gamma_{L_1}(s \star l) \vee \gamma_{L_2}(s \star l)) \wedge (\gamma_{L_1}(l) \vee \gamma_{L_2}(l))) \\ &\quad e^{\iota((\theta_{L_1}(s \star l) \vee \theta_{L_2}(s \star l)) \wedge (\theta_{L_1}(l) \vee \theta_{L_2}(l)))} \\ &= (\gamma_{L_1}(s \star l) \vee \gamma_{L_2}(s \star l)) e^{\iota(\theta_{L_1}(s \star l) \vee \theta_{L_2}(s \star l))} \wedge \\ &\quad (\gamma_{L_1}(l) \vee \gamma_{L_2}(l)) e^{\iota(\theta_{L_1}(l) \vee \theta_{L_2}(l))} \\ &\geq \mu_{L_1 \cup L_2}(s \star l) \wedge \mu_{L_1 \cup L_2}(l). \end{aligned}$$

Suppose that L_1 and L_2 are two CIFIs of M and let $s, l \in M$. Then

$$\begin{aligned} \nu_{L_1 \cup L_2}(0) &= (\overline{\gamma}_{L_1}(0) \wedge \overline{\gamma}_{L_2}(0)) e^{\iota(\overline{\theta}_{L_1}(0) \wedge \overline{\theta}_{L_2}(0))} \\ &\leq (\overline{\gamma}_{L_1}(s) \wedge \overline{\gamma}_{L_2}(s)) e^{\iota(\overline{\theta}_{L_1}(s) \wedge \overline{\theta}_{L_2}(s))} \\ &= \nu_{L_1 \cup L_2}(s). \end{aligned}$$

Moreover

$$\begin{aligned} \nu_{L_1 \cup L_2}(s) &= (\overline{\gamma}_{L_1}(s) \wedge \overline{\gamma}_{L_2}(s)) e^{\iota(\overline{\theta}_{L_1}(s) \wedge \overline{\theta}_{L_2}(s))} \\ &\leq ((\overline{\gamma}_{L_1}(s \star l) \vee \overline{\gamma}_{L_1}(l)) \wedge (\overline{\gamma}_{L_2}(s \star l) \vee \overline{\gamma}_{L_2}(l))) \end{aligned}$$

$$
\begin{aligned}
& e^{\iota((\overline{\theta}_{L_1}(s\star l)\vee\overline{\theta}_{L_1}(l))\wedge(\overline{\theta}_{L_2}(s\star l)\vee\overline{\theta}_{L_2}(l)))} \\
\leq\ & ((\overline{\gamma}_{L_1}(s\star l)\wedge\overline{\gamma}_{L_2}(s\star l))\vee(\overline{\gamma}_{L_1}(l)\wedge\overline{\gamma}_{L_2}(l))) \\
& e^{\iota((\overline{\theta}_{L_1}(s\star l)\wedge\overline{\theta}_{L_2}(s\star l))\vee(\overline{\theta}_{L_1}(l)\wedge\overline{\theta}_{L_2}(l)))} \\
=\ & (\overline{\gamma}_{L_1}(s\star l)\wedge\overline{\gamma}_{L_2}(s\star l))e^{\iota(\overline{\theta}_{L_1}(s\star l)\wedge\overline{\theta}_{L_2}(s\star l))}\vee \\
& (\overline{\gamma}_{L_1}(l)\wedge\overline{\gamma}_{L_2}(l))e^{\iota(\overline{\theta}_{L_1}(l)\wedge\overline{\theta}_{L_2}(l))} \\
\leq\ & \nu_{L_1\cup L_2}(s\star l)\vee\nu_{L_1\cup L_2}(l).
\end{aligned}
$$

Therefore, $L_1 \cup L_2$ is a CIFI of M.

Example 10. *Take a BCK-algebra $M = \{0, s, l, z, w\}$, where the binary operation is defined by the Caley Table 5. Now define a CIFS L_1 on M as: $L_1 = \{(0, 0.9e^{\iota 0.7\pi}, 0.7e^{\iota 0.5\pi}), (s, 0.7e^{\iota 0.5\pi}, 0.5e^{\iota 0.3\pi}), (l, 0.5e^{\iota 0.3\pi}, 0.3e^{\iota 0.1\pi}), (z, 0.3e^{\iota 0.1\pi}, 0.1e^{\iota 0.01\pi}), (w, 0.3e^{\iota 0.1\pi}, 0.1e^{\iota 0.01\pi})\}$. It is easy to show that L_1 is a CIFI of M. Now define a CIFS L_2 on M as: $L_2 = \{(0, 0.6e^{\iota 0.5\pi}, 0.4e^{\iota 0.3\pi}), (s, 0.4e^{\iota 0.6\pi}, 0.2e^{\iota 0.4\pi}), (l, 0.6e^{\iota 0.5\pi}, 0.4e^{\iota 0.3\pi}), (z, 0.4e^{\iota 0.6\pi}, 0.2e^{\iota 0.4\pi}), (w, 0.4e^{\iota 0.6\pi}, 0.2e^{\iota 0.4\pi})\}$. It is easy to show that L_2 is a CIFI of M. Now define a CIFS $L_1 \cup L_2$ on M as: $L_1 \cup L_2 = \{(0, 0.9e^{\iota 0.7\pi}, 0.4e^{\iota 0.3\pi}), (s, 0.7e^{\iota 0.6\pi}, 0.2e^{\iota 0.3\pi}), (l, 0.6e^{\iota 0.5\pi}, 0.3e^{\iota 0.1\pi}), (z, 0.4e^{\iota 0.6\pi}, 0.1e^{\iota 0.01\pi}), (w, 0.4e^{\iota 0.6\pi}, 0.1e^{\iota 0.01\pi})\}$. It is straightforward to prove that $L_1 \cup L_2$ is a CIFI of M.*

Table 5: Cayley's table describing the binary operation expressed by "$\star$".

$\star$	0	s	l	z	w
0	0	0	0	0	0
s	s	0	s	0	0
l	l	l	0	0	0
z	z	z	z	0	0
w	w	z	w	z	0

Definition 16. *Assume that L_1 and L_2 are two CIFSs of M. Then, the intersection $L_1 \cap L_2$ is defined as*
$\mu_{L_1\cap L_2}(s) = (\gamma_{L_1}(s)\wedge\gamma_{L_2}(s))e^{\iota(\theta_{L_1}(s)\wedge\theta_{L_2}(s))}$, $\nu_{L_1\cap L_2}(s) = (\overline{\gamma}_{L_1}(s)\vee\overline{\gamma}_{L_2}(s))e^{\iota(\overline{\theta}_{L_1}(s)\vee\overline{\theta}_{L_2}(s))}$.

Example 11. *Let $(\mu_{L_1}(s), \nu_{L_1}(s)) = \{(s_1, 0.6e^{\iota 0.5\pi}, 0.4e^{\iota 0.25\pi}), (s_2, 1e^{\iota 0.5\pi}, 0.8e^{\iota 0.25\pi}), (s_3, 0.8e^{\iota 2\pi}, 0.6e^{\iota 1.5\pi}), (s_4, 0.9e^{\iota 0.4\pi}, 0.7e^{\iota 0.24\pi}), (s_5, 0.7e^{\iota\pi}, 0.5e^{\iota 0.8\pi}), (s_6, 0.5e^{\iota 0.4\pi}, 0.3e^{\iota 0.24\pi})\}$ and $(\mu_{L_2}(s), \nu_{L_2}(s)) = \{(s_1, 0.2e^{\iota\pi}, 0.1e^{\iota 0.88\pi}), (s_2, 0.1e^{\iota 0.8\pi}, 0.01e^{\iota 0.6\pi}), (s_3, 0.8e^{\iota 0.8\pi}, 0.6e^{\iota 0.68\pi}), (s_4, 0.2e^{\iota\pi}, 0.1e^{\iota 0.88\pi}), (s_5, 0.9e^{\iota 0.9\pi}, 0.7e^{\iota 0.7\pi}), (s_6, 0.3e^{\iota 2\pi}, 0.1e^{\iota 1.88\pi})\}$ be a CIFSs. Then, $\mu_{L_1\cap L_2}(s) = \{(s_1, 0.2e^{\iota 0.5\pi}, 0.4e^{\iota 0.88\pi}), (s_2, 0.1e^{\iota 0.5\pi}, 0.8e^{\iota 0.6\pi}), (s_3, 0.8e^{\iota 0.8\pi}, 0.6e^{\iota 1.5\pi}), (s_4, 0.2e^{\iota 0.4\pi}, 0.7e^{\iota 0.88\pi}), (s_5, 0.7e^{\iota 0.9\pi}, 0.7e^{\iota 0.8\pi}), (s_6, 0.3e^{\iota 0.4\pi}, 0.3e^{\iota 1.88\pi})\}$.*

The subsequent theorem demonstrates that the intersection $\cap$ of two CIFIs of M is also a CIFI.

Theorem 11. *Assume that L_1 and L_2 are two CIFIs of M. Then $L_1 \cap L_2$ is a CIFI of M.*

Proof. Let L_1 and L_2 be two CIFIs of M and let $s, l \in M$. Then

$$\begin{aligned} \mu_{L_1\cap L_2}(0) &= (\gamma_{L_1}(0) \wedge \gamma_{L_2}(0))e^{\iota(\theta_{L_1}(0)\wedge\theta_{L_2}(0))} \\ &\geq (\gamma_{L_1}(s) \wedge \gamma_{L_2}(s))e^{\iota(\theta_{L_1}(s)\wedge\theta_{L_2}(s))} \\ &= \mu_{L_1\cap L_2}(s). \end{aligned}$$

Moreover

$$\begin{aligned} \mu_{L_1\cap L_2}(s) &= (\gamma_{L_1}(s) \wedge \gamma_{L_2}(s))e^{\iota(\theta_{L_1}(s)\wedge\theta_{L_2}(s))} \\ &\geq ((\gamma_{L_1}(s\star l) \wedge \gamma_{L_1}(l)) \wedge (\gamma_{L_2}(s\star l) \wedge \gamma_{L_2}(l))) \\ &\quad e^{\iota((\theta_{L_1}(s\star l)\wedge\theta_{L_1}(l))\wedge(\theta_{L_2}(s\star l)\wedge\theta_{L_2}(l)))} \\ &\geq ((\gamma_{L_1}(s\star l) \wedge \gamma_{L_2}(s\star l)) \wedge (\gamma_{L_1}(l) \wedge \gamma_{L_2}(l))) \\ &\quad e^{\iota((\theta_{L_1}(s\star l)\wedge\theta_{L_2}(s\star l))\wedge(\theta_{L_1}(l)\wedge\theta_{L_2}(l)))} \\ &= (\gamma_{L_1}(s\star l) \wedge \gamma_{L_2}(s\star l))e^{\iota(\theta_{L_1}(s\star l)\wedge\theta_{L_2}(s\star l))} \wedge \\ &\quad (\gamma_{L_1}(l) \wedge \gamma_{L_2}(l))e^{\iota(\theta_{L_1}(l)\wedge\theta_{L_2}(l))} \\ &\geq \mu_{L_1\cap L_2}(s\star l) \wedge \mu_{L_1\cap L_2}(l). \end{aligned}$$

Suppose that L_1 and L_2 are two CIFIs of M and let $s, l \in M$. Then

$$\begin{aligned} \nu_{L_1\cap L_2}(0) &= (\overline{\gamma}_{L_1}(0) \vee \overline{\gamma}_{L_2}(0))e^{\iota(\overline{\theta}_{L_1}(0)\vee\overline{\theta}_{L_2}(0))} \\ &\leq (\overline{\gamma}_{L_1}(s) \vee \overline{\gamma}_{L_2}(s))e^{\iota(\overline{\theta}_{L_1}(s)\vee\overline{\theta}_{L_2}(s))} \\ &= \nu_{L_1\cap L_2}(s). \end{aligned}$$

And

$$\begin{aligned} \nu_{L_1\cap L_2}(s) &= (\overline{\gamma}_{L_1}(s) \vee \overline{\gamma}_{L_2}(s))e^{\iota(\overline{\theta}_{L_1}(s)\vee\overline{\theta}_{L_2}(s))} \\ &\leq ((\overline{\gamma}_{L_1}(s\star l) \vee \overline{\gamma}_{L_1}(l)) \vee (\overline{\gamma}_{L_2}(s\star l) \vee \overline{\gamma}_{L_2}(l))) \\ &\quad e^{\iota((\overline{\theta}_{L_1}(s\star l)\vee\overline{\theta}_{L_1}(l))\vee(\overline{\theta}_{L_2}(s\star l)\vee\overline{\theta}_{L_2}(l)))} \\ &\leq ((\overline{\gamma}_{L_1}(s\star l) \vee \overline{\gamma}_{L_2}(s\star l)) \vee (\overline{\gamma}_{L_1}(l) \vee \overline{\gamma}_{L_2}(l))) \\ &\quad e^{\iota((\overline{\theta}_{L_1}(s\star l)\vee\overline{\theta}_{L_2}(s\star l))\vee(\overline{\theta}_{L_1}(l)\vee\overline{\theta}_{L_2}(l)))} \\ &= (\overline{\gamma}_{L_1}(s\star l) \vee \overline{\gamma}_{L_2}(s\star l))e^{\iota(\overline{\theta}_{L_1}(s\star l)\vee\overline{\theta}_{L_2}(s\star l))} \vee \\ &\quad (\overline{\gamma}_{L_1}(l) \vee \overline{\gamma}_{L_2}(l))e^{\iota(\overline{\theta}_{L_1}(l)\vee\overline{\theta}_{L_2}(l))} \\ &\leq \nu_{L_1\cap L_2}(s\star l) \vee \nu_{L_1\cap L_2}(l). \end{aligned}$$

Therefore, $L_1 \cap L_2$ is a CIFI of M.

Example 12. *Take a BCK-algebra $M = \{0, s, l, z, w\}$, where the binary operation is defined by the Caley Table 6. Now, define a CIFS L_1 on M as: $L_1 = \{(0, 0.9e^{\iota 0.7\pi}, 0.7e^{\iota 0.5\pi}), (s, 0.7e^{\iota 0.5\pi}, 0.5e^{\iota 0.3\pi}), (l, 0.5e^{\iota 0.3\pi}, 0.3e^{\iota 0.1\pi}), (z, 0.3e^{\iota 0.1\pi}, 0.1e^{\iota 0.01\pi}), (w, 0.3e^{\iota 0.1\pi}, 0.1e^{\iota 0.01\pi})\}$. It is easy to show that L_1 is a CIFI of M. Now, define a CIFS L_2 on M as: $L_2 =$*

$\{(0, 0.6e^{\iota 0.5\pi}, 0.4e^{\iota 0.3\pi}), (s, 0.4e^{\iota 0.6\pi}, 0.2e^{\iota 0.4\pi}), (l, 0.6e^{\iota 0.5\pi}, 0.4e^{\iota 0.3\pi}), (z, 0.4e^{\iota 0.6\pi}, 0.2e^{\iota 0.4\pi}), (w, 0.4e^{\iota 0.6\pi}, 0.2e^{\iota 0.4\pi})\}$. *It is easy to show that* L_2 *is a CIFI of* M. *Now, define a CIFS* $L_1 \cap L_2$ *on* M *as:* $L_1 \cap L_2 = \{(0, 0.6e^{\iota 0.5\pi}, 0.7e^{\iota 0.5\pi}), (s, 0.4e^{\iota 0.5\pi}, 0.5e^{\iota 0.4\pi}), (l, 0.5e^{\iota 0.3\pi}, 0.4e^{\iota 0.3\pi}), (z, 0.3e^{\iota 0.1\pi}, 0.2e^{\iota 0.4\pi}), (w, 0.3e^{\iota 0.1\pi}, 0.2e^{\iota 0.4\pi})\}$. *It is straightforward to prove that* $L_1 \cap L_2$ *is a CIFI of* M.

Table 6: Cayley's table describing the binary operation expressed by "$\star$".

$\star$	0	s	l	z	w
0	0	0	0	0	0
s	s	0	s	0	0
l	l	l	0	0	0
z	z	z	z	0	0
w	w	w	z	l	0

Definition 17. *Let* L_1 *and* L_2 *be two CIFSs of* M. *Then, the simple difference* $L_1 \backslash L_2$ *is defined as*
$\mu_{L_1\backslash L_2}(s) = (\gamma_{L_1}(s) \wedge \gamma_{L_2}(s))e^{\iota(\theta_{L_1}(s)\wedge\theta_{L_2}(s))}$, $\nu_{L_1\backslash L_2}(s) = (\overline{\gamma}_{L_1}(s) \vee \overline{\gamma}_{L_2}(s))e^{\iota(\overline{\theta}_{L_1}(s)\vee\overline{\theta}_{L_2}(s))}$.

Example 13. *Let* $(\mu_{L_1}(s), \nu_{L_1}(s)) = \{(s_1, 0.6e^{\iota 0.5\pi}, 0.4e^{\iota 0.25\pi}), (s_2, 1e^{\iota 0.5\pi}, 0.8e^{\iota 0.25\pi}), (s_3, 0.8e^{\iota 2\pi}, 0.6e^{\iota 1.5\pi}), (s_4, 0.9e^{\iota 0.4\pi}, 0.7e^{\iota 0.24\pi}), (s_5, 0.7e^{\iota \pi}, 0.5e^{\iota 0.8\pi}), (s_6, 0.5e^{\iota 0.4\pi}, 0.3e^{\iota 0.24\pi})\}$ *and* $(\mu_{L_2}(s), \nu_{L_2}(s)) = \{(s_1, 0.2e^{\iota\pi}, 0.1e^{\iota 0.88\pi}), (s_2, 0.1e^{\iota 0.8\pi}, 0.01e^{\iota 0.6\pi}), (s_3, 0.8e^{\iota 0.8\pi}, 0.6e^{\iota 0.68\pi}), (s_4, 0.2e^{\iota\pi}, 0.1e^{\iota 0.88\pi}), (s_5, 0.9e^{\iota 0.9\pi}, 0.7e^{\iota 0.7\pi}), (s_6, 0.3e^{\iota 2\pi}, 0.1e^{\iota 1.88\pi})\}$ *be a CIFSs. Then,* $\mu_{L_1\backslash L_2}(s) = \{(s_1, 0.2e^{\iota 0.5\pi}, 0.4e^{\iota 0.88\pi}), (s_2, 0.1e^{\iota 0.5\pi}, 0.8e^{\iota 0.6\pi}), (s_3, 0.8e^{\iota 0.8\pi}, 0.6e^{\iota 1.5\pi}), (s_4, 0.2e^{\iota 0.4\pi}, 0.7e^{\iota 0.88\pi}), (s_5, 0.7e^{\iota 0.9\pi}, 0.7e^{\iota 0.8\pi}), (s_6, 0.3e^{\iota 0.4\pi}, 0.3e^{\iota 1.88\pi})\}$. *It is straightforward to prove that* $L_1 \backslash L_2$ *is a CIFI of* M.

The subsequent theorem demonstrates that the simple difference \ of two CIFIs of M is also CIFI.

Theorem 12. *Assume that* L_1 *and* L_2 *are two CIFIs of* M. *Then,* $L_1 \backslash L_2$ *is a CIFI of* M.

Proof. Let L_1 and L_2 be two CIFIs of M and let $s, l \in M$. Then

$$\begin{aligned}
\mu_{L_1\backslash L_2}(0) &= (\gamma_{L_1}(0) \wedge \gamma_{L_2}(0))e^{\iota(\theta_{L_1}(0)\wedge\theta_{L_2}(0))} \\
&\geq (\gamma_{L_1}(s) \wedge \gamma_{L_2}(s))e^{\iota(\theta_{L_1}(s)\wedge\theta_{L_2}(s))} \\
&= \mu_{L_1\backslash L_2}(s).
\end{aligned}$$

Moreover

$$\begin{aligned}
\mu_{L_1\backslash L_2}(s) &= (\gamma_{L_1}(s) \wedge \gamma_{L_2}(s))e^{\iota(\theta_{L_1}(s)\wedge\theta_{L_2}(s))} \\
&\geq ((\gamma_{L_1}(s \star l) \wedge \gamma_{L_1}(l)) \wedge (\gamma_{L_2}(s \star l) \wedge \gamma_{L_2}(l))) \\
&\quad e^{\iota((\theta_{L_1}(s\star l)\wedge\theta_{L_1}(l))\wedge(\theta_{L_2}(s\star l)\wedge\theta_{L_2}(l)))}
\end{aligned}$$

$$\begin{aligned} &\geq ((\gamma_{L_1}(s\star l)\wedge\gamma_{L_2}(s\star l))\wedge(\gamma_{L_1}(l)\wedge\gamma_{L_2}(l))) \\ &\quad e^{\iota((\theta_{L_1}(s\star l)\wedge\theta_{L_2}(s\star l))\wedge(\theta_{L_1}(l)\wedge\theta_{L_2}(l)))} \\ &= (\gamma_{L_1}(s\star l)\wedge\gamma_{L_2}(s\star l))e^{\iota(\theta_{L_1}(s\star l)\wedge\theta_{L_2}(s\star l))}\wedge \\ &\quad (\gamma_{L_1}(l)\wedge\gamma_{L_2}(l))e^{\iota(\theta_{L_1}(l)\wedge\theta_{L_2}(l))} \\ &\geq \mu_{L_1\backslash L_2}(s\star l)\wedge\mu_{L_1\backslash L_2}(l). \end{aligned}$$

Suppose that L_1 and L_2 are two CIFIs of M and let $s, l \in M$. Then

$$\begin{aligned} \nu_{L_1\backslash L_2}(0) &= (\overline{\gamma}_{L_1}(0)\vee\overline{\gamma}_{L_2}(0))e^{\iota(\overline{\theta}_{L_1}(0)\vee\overline{\theta}_{L_2}(0))} \\ &\leq (\overline{\gamma}_{L_1}(s)\vee\overline{\gamma}_{L_2}(s))e^{\iota(\overline{\theta}_{L_1}(s)\vee\overline{\theta}_{L_2}(s))} \\ &= \nu_{L_1\backslash L_2}(s). \end{aligned}$$

And

$$\begin{aligned} \nu_{L_1\backslash L_2}(s) &= (\overline{\gamma}_{L_1}(s)\vee\overline{\gamma}_{L_2}(s))e^{\iota(\overline{\theta}_{L_1}(s)\vee\overline{\theta}_{L_2}(s))} \\ &\leq ((\overline{\gamma}_{L_1}(s\star l)\vee\overline{\gamma}_{L_1}(l))\vee(\overline{\gamma}_{L_2}(s\star l)\vee\overline{\gamma}_{L_2}(l))) \\ &\quad e^{\iota((\overline{\theta}_{L_1}(s\star l)\vee\overline{\theta}_{L_1}(l))\vee(\overline{\theta}_{L_2}(s\star l)\vee\overline{\theta}_{L_2}(l)))} \\ &\leq ((\overline{\gamma}_{L_1}(s\star l)\vee\overline{\gamma}_{L_2}(s\star l))\vee(\overline{\gamma}_{L_1}(l)\vee\overline{\gamma}_{L_2}(l))) \\ &\quad e^{\iota((\overline{\theta}_{L_1}(s\star l)\vee\overline{\theta}_{L_2}(s\star l))\vee(\overline{\theta}_{L_1}(l)\vee\overline{\theta}_{L_2}(l)))} \\ &= (\overline{\gamma}_{L_1}(s\star l)\vee\overline{\gamma}_{L_2}(s\star l))e^{\iota(\overline{\theta}_{L_1}(s\star l)\vee\overline{\theta}_{L_2}(s\star l))}\vee \\ &\quad (\overline{\gamma}_{L_1}(l)\vee\overline{\gamma}_{L_2}(l))e^{\iota(\overline{\theta}_{L_1}(l)\vee\overline{\theta}_{L_2}(l))} \\ &\leq \nu_{L_1\backslash L_2}(s\star l)\vee\nu_{L_1\backslash L_2}(l). \end{aligned}$$

Therefore, $L_1\backslash L_2$ is a CIFI of M.

Example 14. *Take a BCK-algebra* $M = \{0, s, l, z, w\}$*, where the binary operation is defined by the Caley Table 7. Now, define a CIFS* L_1 *on* M *as:* $L_1 = \{(0, 0.9e^{\iota 0.7\pi}, 0.7e^{\iota 0.5\pi}), (s, 0.7e^{\iota 0.5\pi}, 0.5e^{\iota 0.3\pi}), (l, 0.5e^{\iota 0.3\pi}, 0.3e^{\iota 0.1\pi}), (z, 0.3e^{\iota 0.1\pi}, 0.1e^{\iota 0.01\pi}), (w, 0.3e^{\iota 0.1\pi}, 0.1e^{\iota 0.01\pi})\}$*. It is easy to show that* L_1 *is a CIFI of* M*. Now define a CIFS* L_2 *on* M *as:* $L_2 = \{(0, 0.6e^{\iota 0.5\pi}, 0.4e^{\iota 0.3\pi}), (s, 0.4e^{\iota 0.6\pi}, 0.2e^{\iota 0.4\pi}), (l, 0.6e^{\iota 0.5\pi}, 0.4e^{\iota 0.3\pi}), (z, 0.4e^{\iota 0.6\pi}, 0.2e^{\iota 0.4\pi}), (w, 0.4e^{\iota 0.6\pi}, 0.2e^{\iota 0.4\pi})\}$*. It is easy to show that* L_2 *is a CIFI of* M*. Now define a CIFS* $L_1\backslash L_2$ *on* M *as:* $L_1\backslash L_2 = \{(0, 0.6e^{\iota 0.5\pi}, 0.7e^{\iota 0.5\pi}), (s, 0.4e^{\iota 0.5\pi}, 0.5e^{\iota 0.4\pi}), (l, 0.5e^{\iota 0.3\pi}, 0.4e^{\iota 0.3\pi}), (z, 0.3e^{\iota 0.1\pi}, 0.2e^{\iota 0.4\pi}), (w, 0.3e^{\iota 0.1\pi}, 0.2e^{\iota 0.4\pi})\}$*. It is straightforward to prove that* $L_1\backslash L_2$ *is a CIFI of* M*.*

Table 7: Cayley's table describing the binary operation expressed by "$\star$".

$\star$	0	s	l	z	w
0	0	0	0	0	0
s	s	0	s	0	0
l	l	l	0	0	0
z	z	z	z	0	0
w	w	w	z	l	0

Definition 18. *Let L_1 and L_2 be two CIFSs of M. Then, the bounded difference $L_1 \ominus L_2$ is defined as* $\mu_{L_1 \ominus L_2}(s) = (0 \vee (\gamma_{L_1}(s) - \gamma_{L_2}(s)))e^{\iota(\theta_{L_1}(s) \vee \theta_{L_2}(s))}$, $\nu_{L_1 \ominus L_2}(s) = (0 \wedge (\overline{\gamma}_{L_1}(s) - \overline{\gamma}_{L_2}(s)))e^{\iota(\overline{\theta}_{L_1}(s) \wedge \overline{\theta}_{L_2}(s))}$.

Example 15. *Let* $(\mu_{L_1}(s), \nu_{L_1}(s)) = \{(s_1, 0.6e^{\iota 0.5\pi}, 0.4e^{\iota 0.25\pi}), (s_2, 1e^{\iota 0.5\pi}, 0.8e^{\iota 0.25\pi}), (s_3, 0.8e^{\iota 2\pi}, 0.6e^{\iota 1.5\pi}), (s_4, 0.9e^{\iota 0.4\pi}, 0.7e^{\iota 0.24\pi}), (s_5, 0.7e^{\iota\pi}, 0.5e^{\iota 0.8\pi}), (s_6, 0.5e^{\iota 0.4\pi}, 0.3e^{\iota 0.24\pi})\}$ *and* $(\mu_{L_2}(s), \nu_{L_2}(s)) = \{(s_1, 0.2e^{\iota\pi}, 0.1e^{\iota 0.88\pi}), (s_2, 0.1e^{\iota 0.8\pi}, 0.01e^{\iota 0.6\pi}), (s_3, 0.8e^{\iota 0.8\pi}, 0.6e^{\iota 0.68\pi}), (s_4, 0.2e^{\iota\pi}, 0.1e^{\iota 0.88\pi}), (s_5, 0.5e^{\iota 0.9\pi}, 0.3e^{\iota 0.7\pi}), (s_6, 0.3e^{\iota 2\pi}, 0.1e^{\iota 1.88\pi})\}$ *be a CIFSs. Then,* $\mu_{L_1 \ominus L_2}(s) = \{(s_1, 0.4e^{\iota\pi}, 0.3e^{\iota 0.25\pi}), (s_2, 0.9e^{\iota 0.8\pi}, 0.79e^{\iota 0.25\pi}), (s_3, 0e^{\iota 2\pi}, 0e^{\iota 0.68\pi}), (s_4, 0.7e^{\iota\pi}, 0.6e^{\iota 0.24\pi}), (s_5, 0.5e^{\iota\pi}, 0.2e^{\iota 0.7\pi}), (s_6, 0.2e^{\iota 2\pi}, 0.2e^{\iota 0.24\pi})\}$.

The following outcome shows that the bounded difference $\ominus$ of two CIFIs of M is also CIFI.

Theorem 13. *Assume that L_1 and L_2 are two CIFIs of M. Then $L_1 \ominus L_2$ is a CIFI of M.*

Proof. Let L_1 and L_2 be two CIFIs of M and let $s, l \in M$. Then

$$\begin{aligned}
\mu_{L_1 \ominus L_2}(0) &= (0 \vee (\gamma_{L_1}(0) - \gamma_{L_2}(0)))e^{\iota(\theta_{L_1}(0) \vee \theta_{L_2}(0))} \\
&\geq (0 \vee (\gamma_{L_1}(s) - \gamma_{L_2}(s)))e^{\iota(\theta_{L_1}(s) \vee \theta_{L_2}(s))} \\
&= \mu_{L_1 \ominus L_2}(s).
\end{aligned}$$

Moreover

$$\begin{aligned}
\mu_{L_1 \ominus L_2}(s) &= (0 \vee (\gamma_{L_1}(s) - \gamma_{L_2}(s)))e^{\iota(\theta_{L_1}(s) \vee \theta_{L_2}(s))} \\
&\geq (0 \vee (\gamma_{L_1}(s \star l) - \gamma_{L_1}(l))) \vee (\gamma_{L_2}(s \star l) - \gamma_{L_2}(l))) \\
&\quad e^{\iota((\theta_{L_1}(s \star l) \wedge \theta_{L_1}(l)) \vee (\theta_{L_2}(s \star l) \wedge \theta_{L_2}(l)))} \\
&= ((0 \vee (\gamma_{L_1}(s \star l) - \gamma_{L_2}(s \star l))) \wedge (0 \vee \gamma_{L_1}(l) - \gamma_{L_2}(l))) \\
&\quad e^{\iota((\theta_{L_1}(s \star l) \vee \theta_{L_2}(s \star l)) \wedge (\theta_{L_1}(l) \vee \theta_{L_2}(l)))} \\
&= (0 \vee (\gamma_{L_1}(s \star l) - \gamma_{L_2}(s \star l)))e^{\iota(\theta_{L_1}(s \star l) \vee \theta_{L_2}(s \star l))} \wedge \\
&\quad (0 \vee (\gamma_{L_1}(l) - \gamma_{L_2}(l)))e^{\iota(\theta_{L_1}(l) \vee \theta_{L_2}(l))} \\
&\geq \mu_{L_1 \ominus L_2}(s \star l) \wedge \mu_{L_1 \ominus L_2}(l).
\end{aligned}$$

Suppose that L_1 and L_2 are two CIFIs of M and let $s, l \in M$. Then

$$\begin{aligned}
\nu_{L_1 \ominus L_2}(0) &= (0 \wedge (\overline{\gamma}_{L_1}(0) - \overline{\gamma}_{L_2}(0)))e^{\iota(\overline{\theta}_{L_1}(0) \wedge \overline{\theta}_{L_2}(0))} \\
&\leq (0 \wedge (\overline{\gamma}_{L_1}(s) - \overline{\gamma}_{L_2}(s)))e^{\iota(\overline{\theta}_{L_1}(s) \wedge \overline{\theta}_{L_2}(s))} \\
&= \nu_{L_1 \ominus L_2}(s).
\end{aligned}$$

And

$$\begin{aligned}
\nu_{L_1 \ominus L_2}(s) &= (0 \wedge (\overline{\gamma}_{L_1}(s) - \overline{\gamma}_{L_2}(s)))e^{\iota(\overline{\theta}_{L_1}(s) \wedge \overline{\theta}_{L_2}(s))} \\
&\leq (0 \wedge (\overline{\gamma}_{L_1}(s \star l) - \overline{\gamma}_{L_1}(l))) \vee (\overline{\gamma}_{L_2}(s \star l) - \overline{\gamma}_{L_2}(l))) \\
&\quad e^{\iota((\overline{\theta}_{L_1}(s\star l) \wedge \overline{\theta}_{L_1}(l)) \wedge (\overline{\theta}_{L_2}(s\star l) \wedge \overline{\theta}_{L_2}(l)))} \\
&= ((0 \wedge (\overline{\gamma}_{L_1}(s \star l) - \overline{\gamma}_{L_2}(s \star l))) \vee (0 \wedge \overline{\gamma}_{L_1}(l) - \overline{\gamma}_{L_2}(l))) \\
&\quad e^{\iota((\overline{\theta}_{L_1}(s\star l) \wedge \overline{\theta}_{L_2}(s\star l)) \wedge (\overline{\theta}_{L_1}(l) \wedge \overline{\theta}_{L_2}(l)))} \\
&= (0 \wedge (\overline{\gamma}_{L_1}(s \star l) - \overline{\gamma}_{L_2}(s \star l)))e^{\iota(\overline{\theta}_{L_1}(s\star l) \wedge \overline{\theta}_{L_2}(s\star l))} \vee \\
&\quad (0 \wedge (\overline{\gamma}_{L_1}(l) - \overline{\gamma}_{L_2}(l)))e^{\iota(\overline{\theta}_{L_1}(l) \wedge \overline{\theta}_{L_2}(l))} \\
&\leq \nu_{L_1 \ominus L_2}(s \star l) \vee \nu_{L_1 \ominus L_2}(l).
\end{aligned}$$

Therefore, $L_1 \ominus L_2$ is a CFI of M.

Example 16. *Take a $\mathcal{BC}\mathbb{K}$-algebra $M = \{0, s, l, z, w\}$ with Table 8. Now, define a CIFS L_1 on M as: $L_1 = \{(0, 0.9e^{\iota 0.7\pi}, 0.7e^{\iota 0.5\pi}), (s, 0.7e^{\iota 0.5\pi}, 0.5e^{\iota 0.3\pi}), (l, 0.7e^{\iota 0.3\pi}, 0.5e^{\iota 0.1\pi}), (z, 0.5e^{\iota 0.1\pi}, 0.4e^{\iota 0.01\pi}), (w, 0.7e^{\iota 0.1\pi}, 0.5e^{\iota 0.01\pi})\}$. It is easy to show that L_1 is a CIFI of M. Now, define a CIFS L_2 on M as: $L_2 = \{(0, 0.6e^{\iota 0.5\pi}, 0.4e^{\iota 0.3\pi}), (s, 0.4e^{\iota 0.6\pi}, 0.2e^{\iota 0.4\pi}), (l, 0.6e^{\iota 0.5\pi}, 0.4e^{\iota 0.3\pi}), (z, 0.4e^{\iota 0.6\pi}, 0.2e^{\iota 0.4\pi}), (w, 0.4e^{\iota 0.6\pi}, 0.2e^{\iota 0.4\pi})\}$. It is easy to show that L_2 is a CIFI of M. Now define a CIFS $L_1 \ominus L_2$ on M as: $L_1 \ominus L_2 = \{(0, 0.3e^{\iota 0.7\pi}, 0.3e^{\iota 0.3\pi}), (s, 0.3e^{\iota 0.6\pi}, 0.3e^{\iota 0.3\pi}), (l, 0.1e^{\iota 0.5\pi}, 0.1e^{\iota 0.1\pi}), (z, 0.1e^{\iota 0.6\pi}, 0.2e^{\iota 0.01\pi}), (w, 0.3e^{\iota 0.6\pi}, 0.3e^{\iota 0.01\pi})\}$. It is straightforward to prove that $L_1 \ominus L_2$ is a CIFI of M.*

Table 8: Cayley's table describing the binary operation expressed by "$\star$".

$\star$	0	s	l	z	w
0	0	0	0	0	0
s	s	0	s	0	0
l	l	l	0	0	0
z	z	z	z	0	0
w	w	z	w	z	0

Conclusion

We have used CIFSs in the context of $\mathcal{BC}\mathbb{K}/\mathcal{BCI}$-algebras in this paper; also, the CIFI have been defined, and its features have been investigated. This adds a lot to the field of classical fuzzy set theory. In CIFS, both non-membership and membership functions with complex degrees have been used to improve the algebraic structure and decision-making processes within $\mathcal{BC}\mathbb{K}/\mathcal{BCI}$-algebras. In $\mathcal{BC}\mathbb{K}/\mathcal{BCI}$-algebras, level operators and model operators have been used to explain what a CIFSA is, and then its basic properties have been looked at. We have also studied various operations, including complement, intersection, union, and differences. We plan to investigate the more complex features of the $\mathcal{BC}\mathbb{K}/\mathcal{BCI}$-algebras under the influence of CIFS. Furthermore, we want to apply complex spherical fuzzy set and complex linear Diophantine fuzzy set on $\mathcal{BC}\mathbb{K}/\mathcal{BCI}$-algebras. In the future, we focus on characterising CIFSAs under algebraic homomorphisms and isomorphisms. This will provide a better understanding of how complex intuitionistic fuzziness interacts with structure preservation between algebras. Also, complex intuitionistic fuzzy sub-algebra provide richer modelling tools for uncertainty; future research will explore their applications in multi-criteria decision-making, artificial intelligence, and information systems.

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