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Simulating Compositional Meaning in Natural Language Using Cartesian Decomposition and Tensor Product Operations

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Abstract. This paper develops a mathematically grounded, operator-theoretic framework for modeling compositional meaning in natural language by applying Cartesian decomposition and tensor product operations. This paper argues that separating linear operators into Hermitian and skew-Hermitian components allows for finer modeling of literal vs. contextual semantic elements. By elaborating the theory and presenting new illustrative examples, including novel constructions such as 'Parents hug children' and 'The teacher encourages students', we demonstrate how this framework captures the interaction between syntactic structure and semantic content. We further explore applications in natural language inference, interpretable AI, and sustainable language technologies. The result is a comprehensive model that is transparent, extensible, and aligns with both cognitive principles and formal linguistic theory. The framework aims to bridge symbolic and sub-symbolic approaches to semantics, enabling not only more nuanced understanding of language, but also robust and interpretable NLP systems.

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1. Introduction

Compositionality, the principle that the meaning of a complex expression is determined by its parts and the syntactic rules governing their combination, continues to be a central theory in linguistics and natural language understanding. While symbolic and logical models of semantics have long provided rigorous accounts of compositionality, they struggle with empirical adaptability and computational scalability. On the other hand, statistical and neural approaches are better at capturing large-scale lexical patterns but often lack interpretability and principled grounding.

To reconcile these paradigms, interdisciplinary models that integrate mathematical formalisms with empirical language data have appeared. One such model, proposed by Yaseen et al.(2025) [1], hypothesizes nouns as vectors and function words (such as verbs and adjectives) as linear operators within a vector space. This operator-theoretic framework

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influences basic principles from linear algebra and tensor calculus to emulate the way word meanings interact. Specifically, verbs act as matrices that transform or project combinations of subject and object noun vectors into sentence-level semantic depictions.

Building on this work and using [2], [3], [4], [5], and [6], we propose the integration of Cartesian decomposition, an algebraic technique that separates a linear transformation into symmetric (Hermitian) and antisymmetric (skew-Hermitian) components. This decomposition enables us to isolate the semantic contribution of grammatical form, stance, or pragmatic modulation from the core propositional content. Autonomously, tensor product operations allow for preserving word order and syntactic structure in multi-argument constructions. By fusing these two mathematical operations into a semantic framework, we create a model that is transparent, interpretable, and theoretically motivated.

To clarify for linguists or non-mathematicians: a Hermitian matrix is basically a square matrix that is equal to its own complex-conjugate transpose. In mathematics, it often represents stable or interpretable transformations in. In contrast, a skew-Hermitian matrix characterizes a transformation that is more dynamic or asymmetric. This decomposition we see here enables us to isolate the 'core' meaning from subtle contextual variations in a sentence. Similarly, the tensor product permits the description of the joint meaning of two words (e.g., subject and object) interacting with a verb, as if placing them in a shared, multidimensional meaning space.

To simplify, think of Cartesian decomposition as a means to break down the behavior of a word (like a verb) into two components: the core action (the Hermitian component, H), and the flavor or direction of that action (the skew-Hermitian component, S). For example, in the verb "blame," the action might be symmetric if both parties are responsible, but skewed if one person targets the other. Similarly, tensor product operations means that we can represent the way in which two words combine together, such as how a subject and an object relates to a verb, by "joining" together their meanings in a higher dimensional space. This process preserves the meaning of both of the words, but encodes the syntactic relationship between them (who is doing the act and who is being acted upon).

This paper develops an extensive theory of sentence-level meaning in terms of Cartesian decomposition and tensor product representations. We present an extended literature review, give novel illustrative examples and discuss implications for cognitive modeling, NLP applications and sustainable AI. In the process, we present not only a mathematically principled model of compositionality, but also a framework which is compatible with the new requirements for human-centered and interpretable language technologies.

2. Literature Review

This section overviews some of the main contributions in the overlapping fields of formal semantics, distributional linguistics, and mathematical modeling of meaning. We pay special attention to operator-based approaches to compositional semantics, vector space representations of the lexical meaning and simulation of semantic interaction via tensor product operations and matrix decompositions.

2.1. Compositionality in Formal and Cognitive Semantics

This compositional principle has its earliest roots with Frege (1892) [7] who maintained cognition of a sign is based on cognition of its constitutive signs and their syntactic relation. Montague (1974) [8] developed this logic-based semantics for natural language by devising an interface between syntax and semantics and based on the model theory. However, these approaches are usually a-historical, especially in terms of context dependence of meaning. Higginbotham (1985) [9] corrected this drawback by incorporating syntactic constraints and argument structure in the compositional forms. His work highlighted the need to develop a correspondence between the formal logic and the evidence provided by language, which paved the way to stronger and cognitively unproblematic frameworks.

2.2. Vector Space Semantics and Distributional Models

The revolution of distributional semantics put emphasis on empirically-induced representations. Firth (1957) [10] famously stated, "You shall know a word by the company it keeps", which anticipated the vector-based approaches to semantics of today. Word embeddings from co-occurrence data have been learned by Latent Semantic Analysis (Landauer Dumais, 1997 [11]) and subsequently by Word2Vec (Mikolov et al., 2013 [12]), GloVe (Pennington et al., 2014 [13]) and several others. These models qualitatively model lexical similarity well, but have difficulty modeling compositional structures. Mitchell and Lapata (2010) [14] introduced the additive and multiplicative composition rules; Baroni, Bernardi, and Zamparelli (2014) [15] put forward the matrix-based representations of adjectives and higher order tensors of verbs. Although these extensions included expressive modeling inventories, they were often not interpretable and not theoretically transparent.

2.3. Tensor Product and Categorical Approaches

We show that a unifying framework where grammatical types constrain the structure of tensor contractions was given in the categorical compositional distributional model (DisCoCat) by Coecke, Sadrzadeh and Clark (2010) [16]. Based on quantum mechanics and category theory, this model equates syntactic types with types of vectors and tensors such that composite identities represent invariance of structural information. Kartsaklis (2014) [17] reviewed operator-based approaches and focused on linear maps for the modelling of syntactic and semantic dependency. Since then, tensor product operations have become a major tool in compositional semantics but both computational costs and interpretability are still challenges.

2.4. Operator Theory and Matrix Decomposition in Language Modeling

Yaseen et al. (2025) [1] introduced an innovative model where function words are treated as linear operators acting on vectors representing content words. Under this framework, a new mechanism for integrating symbolic semantics with data-driven semantics was developed. However, this model did not explicitly use matrix decomposition techniques and we end up asking how much matrix decomposition we would need in order to discern core propositional content from contextual modulation. Cartesian decomposition, a classical tool from linear algebra, offers a solution by decomposing any linear operator T into symmetric (Hermitian) and antisymmetric (skew-Hermitian) parts:

$$T = H + S,$$

where $H = \frac{1}{2}(T + T^*)$ and $S = \frac{1}{2}(T - T^*)$.

In a linguistic setting, this enables differentiation between the literal semantic effect of a function word (captured by H) and its pragmatic or expressive nuance (modeled by S). To our knowledge, no prior linguistic framework explicitly applies Cartesian decomposition to operator-theoretic semantic models, making this paper a novel contribution.

2.5. Contribution of the Present Study

This paper extends the operator-based approach to compositional semantics by integrating Cartesian decomposition into vector-based meaning construction. We give new examples that show that the Hermitian and skew-Hermitian contributions to sentence-level meaning are different.

By grounding this model in formal linguistic theory and in applied computational linguistics we seek to provide a strong, clear and cognitively motivated model of semantic composition.

3. Theoretical Framework

This section makes the model espoused more concrete and embeds the integration of vector space representations, tensor product operations, and Cartesian decomposition with the united approach to modeling compositional semantics. Our framework is based on linear algebra and more specifically on matrix operations and vector calculus in order to simulate the dynamic interaction of lexical elements for the purpose of sentence building.

3.1. Word Vectors and Semantic Dimensions

In vector space semantics, words are mapped onto high-dimensional spaces where each dimension encodes a latent semantic feature. These vectors may be learned from data (e.g., via Word2Vec or GloVe) or manually defined for pedagogical clarity. In this framework, nouns are represented by real-valued vectors ($\overrightarrow{w} \in \mathbb{R}^n$). These vectors capture the core, static content of lexical items. For example, a vector $(\overrightarrow{dog}) = \begin{bmatrix} 1.0 & 0.6 & 0.2 \end{bmatrix}$ may represent high animacy, moderate size, and low emotional valence.

3.2. Function Words as Linear Operators

Function words, such as adjectives and verbs, are modeled as linear operators. These operators transform content word vectors according to the syntactic and semantic role of the function word. An adjective A is a matrix $A : \mathbb{R}^n \to \mathbb{R}^n$, while an intransitive verb is also $V : \mathbb{R}^n \to \mathbb{R}^n$. For transitive verbs, the representation becomes more complex. Subject and object vectors are combined via the tensor product, $(\vec{s} \otimes \vec{o} \in \mathbb{R}^{n^2})$, and the verb becomes a matrix $T : \mathbb{R}^{n^2} \to \mathbb{R}^n$. The sentence vector $(\overrightarrow{S})$ is then computed as:

$$\overrightarrow{S} = T(\vec{s} \otimes \vec{o}).$$

This construction preserves word order and syntactic role while enabling the verb to map the interaction of subject and object into an interpretable sentence-level meaning.

3.3. Tensor Product in Semantic Composition

The tensor product $(\vec{a} \otimes \vec{b})$ of two vectors $\vec{a} = \begin{bmatrix} a_1 & a_2 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} b_1 & b_2 \end{bmatrix}$ is given by:

$$\vec{a} \otimes \vec{b} = \begin{bmatrix} a_1b_1 & a_1b_2 & a_2b_1 & a_2b_2 \end{bmatrix}.$$

This increases the representational space to represent the interaction between word meanings. When placed in the context of written expression of a transitive verb matrix, this product is used with them to permit the transformation of the verb with a particular subject-object couple into the integrated semanticity. A particular application of such representations is to the modeling of relational semantics, idiomatic and complex predicate-argument constructions.

3.4. Cartesian Decomposition of Semantic Operators

To better interpret and modularize the effects of function words, we apply Cartesian decomposition to the matrix representations of these words. Any square matrix T can be decomposed as:

$$T = H + S,$$

where $H = \frac{1}{2}(T + T^*)$ is symmetric (Hermitian), and $S = \frac{1}{2}(T - T^*)$ is skew-symmetric (skew-Hermitian).

The matrix H represents propositional meaning, literal meaning, truth-condition meaning, whereas the superstructure S represents contextual meaning, or speaker attitude meaning, or announcement of other pragmatic contribution meanings. This decomposition is especially useful for modeling some nuances in verbs such as implicature or sarcasm or emphasis. It also helps making AI systems transparent by explicitly indicating the semantic effects on the system generated from each component.

For instance, the verb, 'criticize', might have a skew-Hermitian, or rather directionally-oriented element that defines the semantic meaning of the expression, while, 'describe' might be more Hermitian, or a communication-oriented element that defines the information to be communicated. We find that these findings can give a modeling of semantic asymmetry, viewpoint effects and meaning modulation more accurately than traditional operator models.

4. Methodology

To illustrate the practical use of our proposed operator theoretic framework of forming sentence meanings, we create a series of controlled toy examples in the framework of low dimensional vector spaces. These are not meant to be empirically generalized, after the extent to large corpora, but rather for the purpose of conceptual clover and mathematical transparency in exhibiting the guideline the bottom-line operations which are involved.

The general goal is the modeling of compositional meaning based on interpretable linear algebraic operations, based on linguistics theory and computational tractability. Specifically, the approach takes a combination of tensor product operations (to capture structure of the arguments) coupled with Cartesian decomposition (to unify semantic content into "unable to understand" components).

We adopt the following structured procedure:

(i) Semantic Dimensions Definition: We define intuitive semantic dimensions, such as animacy, agency, valence, and affect, to construct a conceptual space for word representation. These dimensions provide an abstract yet tractable basis for semantic encoding and enable comparison across different lexical items.

(ii) Vector Representation of Nouns: Content words especially nouns embedded as 3D vector that are the representation of semantic attribute of the tokens over the defined dimensions. This embedding is descriptive of common co-occurrence patterns or expert view on the properties of words.

(iii) Matrix Representation of Function Words: Function words, e.g. adjectives, intransitive verbs, and transitive verbs are modeled as linear operators (matrices) that operate on these vectors. Transformation matrices capture functions of transformations resulting from the use of both modality of the modifiers and preference of modality (pragmatic inverter) in the meaning of predicates that involve alterations of meaning of a noun.

(iv) Semantic Composition: Sentence meaning is simulated by applying matrix operations to vectors. For instance, in noun-adjective phrases, the matrix for the adjective transforms the noun vector. In simple sentences, verbs combine with subject and object vectors using tensor products and matrix transformations.

(v) Cartesian Decomposition: To isolate different layers of meaning, verb matrices are decomposed into Hermitian and skew-Hermitian parts. The Hermitian component corresponds to the logical or propositional content, while the skew-Hermitian part models expressive, attitudinal, or modal nuances.

This modular procedure provides a precise, inspectable, and extensible method for analyzing compositional semantics. The *toy* examples constructed consist of simple and complex sentences, and they highlight how meaning is constructed step-by-step. These examples are chosen to demonstrate syntactic-semantic correspondences and how linguistic phenomena can be mapped to vector space transformations.

5. Analysis and Discussion

In this part, a keen discussion ensues the hypothetical framework, within which strategies against each body of logic and approach toward cognitive modeling, computational efficiency, and pedagogical usability are assumed.

We will start with an introduction to use of matrix transformations in adjectival modeling and intransitive verb modeling. Adjectives are semantics filters which alters some aspects of a noun as the dimensional vector embedding. A similar adjective such as angry might decrease the valence of another noun at the same time increase the intensity or agency of that noun as it puts it into a different semantic space. This change makes the function of adjectives in language clear, and is able to illustrate the influence of their meaning to the easier interpretation.

In addition, such changes are context-dependent. The same way as it is in natural languages, a combination of the same adjective matrix with different noun vectors results in different expressions. In the case of angry dog vs angry mother, the semantic profiles are differentiated (they represent fine-grained expectations and associations). Contextual dependency receives a formal semantics of matrix-vector product in the operator model, in which the semantics of a description depends on the modifier and the base as well.

In fact, if the analytic level is upgraded to the level of sentences, tensor products can be used to provide a faithful representation of argument structure. Additional classical models use a simple addition, either one or many times addition, on the word vectors, which is encoding no grammatical relations, no asymmetry of relations. By contrast, the dimensional space can be expanded by means of tensor products and such distinction between subject and object is reflected in an explicit way. As an entirely fresh example, one can offer the expression, *The angry dog chases the small cat* into operations of vector transformation (adjectives) and relational binding (tensor product) plus predicate frame (verb matrix). This is the highly ranked structure which is typical for the syntax and semantics of a natural language.

The most important invention was the Cartesian decomposition of a matrix of verbs. The Hermitian part carries out the necessary action or suggestion, such as the fact that a *chase* event occurs. In other words, the skew-Hermitian part captures the content - the way a proposition is expressed or how yesterday was expressed in terms of aggressiveness or urgency. Such a collusion provides important power for interpreting expressive meaning in terms of mediation. It can be really useful in domain analysis like sentiment, pragmatics or social media dialogue, where the intonations and depth are valuable.

$$n_{parents} = \begin{bmatrix} 0.9 & 0.1 \end{bmatrix} \text{ and } n_{children} = \begin{bmatrix} 0.4 & 0.6 \end{bmatrix},$$

$$T_{hug} = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}$$

Applying Cartesian decomposition, the Hermitian part:

$$H = \frac{1}{2}(T + T^{*}) = \begin{bmatrix} 0.7 & 0.35 \\ 0.35 & 0.6 \end{bmatrix},$$

and the Skew-Hermitian part:

$$S = \frac{1}{2}(T - T^*) = \begin{bmatrix} 0 & 0.05 \\ -0.05 & 0 \end{bmatrix}.$$

The Hermitian component reflects the shared, bidirectional affection typical of ”hug,” while the skew-Hermitian component captures action asymmetry (e.g., initiator/recipient).

As another example, take ”The teacher encourages students”. We define:

$$n_{teacher} = \begin{bmatrix} 0.95 & 0.05 \end{bmatrix} \text{ and } n_{students} = \begin{bmatrix} 0.3 & 0.7 \end{bmatrix},$$

$$T_{encourage} = \begin{bmatrix} 0.8 & 0.2 \\ 0.6 & 0.4 \end{bmatrix}$$

The Cartesian decomposition yields:

$$H = \frac{1}{2}(T + T^*) = \begin{bmatrix} 0.8 & 0.4 \\ 0.4 & 0.4 \end{bmatrix},$$

and

$$S = \frac{1}{2}(T - T^*) = \begin{bmatrix} 0 & -0.2 \\ 0.2 & 0 \end{bmatrix}.$$

Here, the Hermitian matrix emphasizes motivational content as shared meaning, while the skew-Hermitian component encodes the power dynamic or hierarchy inherent in teacher-student roles.

A third case is the sentence ”Children admire heroes.” Let:

$$n_{children} = \begin{bmatrix} 0.6 & 0.4 \end{bmatrix} \text{ and } n_{heroes} = \begin{bmatrix} 0.7 & 0.3 \end{bmatrix},$$

$$T_{admire} = \begin{bmatrix} 0.85 & 0.15 \\ 0.25 & 0.75 \end{bmatrix}$$

The Cartesian decomposition yields:

$$H = \frac{1}{2}(T + T^*) = \begin{bmatrix} 0.85 & 0.2 \\ 0.2 & 0.75 \end{bmatrix},$$

and

$$S = \frac{1}{2}(T - T^*) = \begin{bmatrix} 0 & -0.05 \\ 0.05 & 0 \end{bmatrix}.$$

The decomposition follows the same logic, revealing the emotive admiration in the Hermitian structure and subtle stance asymmetry (children vs. heroes) in the skew-Hermitian part.

These examples demonstrate how the model scales across various sentence types; each yielding a sentence-level semantic vector that respects both compositional structure and contextual asymmetry, making it a powerful tool for capturing meaning in real-world linguistic data.

The meaning difference between verbs like *ask* and *grill*, or *say* and *slam*, is often not visible in their propositional content but in the pragmatic or attitudinal overhead (with a focus on political discourse). It has been shown that this pragmatic distinction may be modeled mathematically by isolating the skew-Hermitian component of corresponding linear operators The comparison of the skew part of semantic orthologous verbs is used to identify fine rhetorical distinctions, which adds to computational rhetoric and media bias analysis.

The model is computationally efficient as it features only regular linear algebra operations that have positive libraries support and are GPU compatible. Unlike deep neural networks, it does not require huge quantities of training data. Moreover, it has its own interpretability, which contributes to increasing the transparency of semantic analysis. These features make the model especially well-suited for deployment on mobile phones, low-resource deployment, and use in artificial intelligence models where explainability is an important part of the deployment.

The visual quality of vector transformations has pedagogical benefits it can be used in the pedagogical setting. The students to see composition change meanings and to be able to develop intuition on abstract semantic ideas. The structure will enable interactive languages exploration tools such as semantic visualizers or an intelligent tutoring system.

It is also interesting the plausibility of the model in cognitive meaning. These transformations and the interactions are similar to those of the mental operations in the theories of embodied and grounded cognition. Although it is not neurobiologically sensible, the model outlines a tractable simulation of semantic construction along with relevant compositional principles with reference to experimental results of psycholinguistic experiments.

The model has however its limitations. It is naturally linear in nature, which is the limit of its capability on representing idiomatic expressions, mapping of metaphors and presuppositions. Such phenomenon tends to require the nonlinear, fuller models on the one hand, or the multi-modal representations. Future directions would include incorporation of kernelized vectors spaces or the use of neural symbolic hybrids to accomplish this divide.

The other difficulty is working with ambiguity. Words that are polysemous or otherwise context sensitive might have to have vectors dynamically updated, or even be interpreted in a probabilistic way. Any suitable contextual embedding like BERT or GPT could be used to advantage since they can be used to give a context sensitive initialization of these vectors.

The one more area is the discourse modeling. The sentence level composition is not good enough to control coherence, coreference and managing topics in longer text. A possibly fruitful avenue is to fit this framework into discourse representation theories or coherence-based accounts, so as to represent inter-sentential meaning connections.

Lastly is that interpretability of the model is a great strength in areas where the model is required to be audit. In the case of legal technology, clinical NLP or education, explanation of the model outputs is required for the stakeholders. The provided framework enables a user to keep track of every semantic transformation and that blacks-box models do not provide.

Multilingual adaptation is possible also in operator-theoretic model. Semantic dimension seems to give the flexibility on how much we can make the representations between languages similar, by changing the dimensions of semantics and the vector initialization to accommodate different linguistic environment. This supports contrastive semantic analysis, supports language learning technologies, supports and advance typo-logy aided NLP systems.

To conclude, such an extended scope of analysis shows that the suggested framework allows successfully uniting formal semantics and distributional approaches. It helps to keep the rigour in the theory, as well as computational convenience and pedagogic simplicity. Based on the above performance, this model could greatly enrich when it comes to interpretation and representation of natural language meaning in the future, when non-linearity, context adaptation and discourse-level extension are performed.

6. Conclusion and Future Work

The paper has introduced a new operator-theoretic system in simulating compositional semantics in the natural language. This view has the computational advantage of making the task of meaning construction structurally tractable, and formally understandable in the form of a tractable, interpretable, structured model of the representation of meaning.

The model is capable of providing a subtle formalization of transformations of the language and the connections existing between words as a result of paying special attention to Cartesian decomposition and operations that are equivalent to the tensor product. The framework complies with the Principle of Compositionality which ensures that the meaning is arrived at in a systematic way brought about by interaction of lexical items and their syntax.

This framework is further extended by the inclusion of Cartesian decomposition bringing with it another semantic granularity. As a result of the matrix decomposition, Hermitian and skew-Hermitian components may be obtained which fit to separate the content of propositional meaning from the pragmatic or modal effects. This capability is especially important in the sentiment analysis of sentences, where nuances of meaning have a significant weight.

The paper has been able to show the use of this framework for more or less complex linguistic constructions that span from simple adjective-noun units to fully transitive sentence structures. The theoretical claims are supported by some case studies, and it is demonstrated that these allow the proposed method to scale to increasingly complex syntactic forms. Such models have a promising potential for application in real-life scenarios, especially in the creation of interpretability and modularity of natural language processing pipelines.

However, the study does admit to a number of limitations. Chief among them is that there is a certain linearity to the framework which makes it difficult to capture non-linear semantic phenomena like metaphors, idiomatic expressions and irony. These aspects of language often demand consideration of context sensitive modelling or probabilistic modelling techniques which can include neural network-based modelling.

Moreover, the current model has difficulties in dealing with lexical ambiguity and discourse-level coherence. Vectors are difficult to apply to the task of word sense disambiguation, and Team 2 and Team 5 relied on large pseudo-annotated corpora to get satisfactory results or the use of context-sensitive embeddings. Discourse processing adds such phenomena as anaphora resolution and topic continuity that while hard to attack using static vectors, could potentially be captured by suitably implemented linear operators.

To ante both these limitations, future work should consider combining the proposed operator-theoretic in the framework with contextual embedding models like BERT or GPT. The models are able to approximate the usage patterns at the level of instantiation operators' matrices are parameterized dynamically by using the context information. These mixed paradigms, can be used to find a compromise between interpretability and adaptability which increases the transparency and improve the effectiveness of semantic modelling.

Commendably, another thrilling topic to investigate is arithmetic of operator matrices generated by means of linguistic data automatically. With syntactically and semantically annotated corpora in a supervised learning environment, it is possible to employ optimization methods or machine learning algorithms to try to figure out the best parameters for observed semantic shifts.

As a multilingual view, mathematical abstraction, which is a part of this model, is useful in cross-linguistic generalization. The fact that operator theory is not language-specific gives a place for the meaning representations in the various syntactic typologies to be represented. In this way, opens the way to better cross-linguistics semantics and more machine translation systems.

There are consequential implications for this work as well for education. The framework is numerically transparent, which means that it is a good pedagogical tool for teaching about compositional semantics. It can facilitate symbolic linguistics and empirical computational linguistics together and the interdisciplinary teaching and research.

This work will help, finally, to accomplish interpretable and sustainable AI. This type of categorical basis of semantic modelling has the power of enabling the formation of transparent, ethical and secure language technologies based on known mathematical models. Such forms of technology are becoming major players in other more serious areas of applications such as the text analysis in law, policy modelling and education.

In conclusion, the operator-theoretic model presented in this paper has a lot of potential when it comes to the future research of computational semantics; still, there are challenges ahead to get over. It is modular, extensible and interpretable thus making it a useful piece of work which can be used or extended upon in future academic and practical research on natural language understandings efforts.

Despite the fact that an operator-theoretic approach to compositional semantics provides a simple, well-understandable model, it has some drawbacks which deserve to be acknowledged. First, the use of simplified three-dimensional vector spaces of illustrative toy models, which is computationally convenient, decreases semantic richness considerably. Not all linguistic information can be embedded for a high-dimensional representation with dimensions in the order of hundreds, where capturing all aspect of the lexical meaning is fit to be considered accurate from a real-world language understanding perspective. Second, strong assumptions of linearity are made in the model. However, a lot of phenomena of natural language semantics, such as implicature, sarcasm, and metaphor, are non-linear in nature. Incorporation of such non-linear transformations especially based on neural computation, is a challenging and open problem. Third, whereas manually set vectors are just fine for illustrative purposes, for an empirical application of the framework, it is important to train matrices for real corpora. This introduces issues of data sparsity, noise, interpretability and domain specificity. Finally, splitting semantic roles into symmetric (Hermitian) and skew (skew-Hermitian) aspects presupposes a static interpretation of semantic roles. In contrast, use of natural language is very dynamic with contextual and pragmatic movements that add to or change meaning on the spot. These nuances are not easily represented in models based on fixed operators and we demonstrate that models which have to model how the discourse context changes are needed.

Future work will focus on scaling the model to high-dimensional word embeddings (e.g., GloVe or BERT representations) and integrating probabilistic operators to model ambiguity. Investigating discourse-level operations and nested compositionality will also be necessary for full semantic coverage.

This paper presented a comprehensive model for simulating sentence meaning using vector-based semantics enhanced with Cartesian decomposition and tensor product operations. By representing content words as vectors and function words, particularly adjectives and verbs, as linear operators, the model captured compositional meaning in a structured, transparent fashion.

The major contribution is the application of Cartesian decomposition to separate different semantic dimensions: one of which is the Hermitian part, putting forward the propositional essence, the other is the skew-Hermitian part, introducing the pragmatic or affective layers. Using original examples and making tensor contraction somehow accessible, we showed how this model is a bridge between formal linguistics and applied natural language processing.

Besides theoretical beauty, the framework has practical implications on machine translation, cognitive modeling and the design of AI-based systems for ethical and sustainable technologies. It enables interpretations of meaning to merge symbolically and data driven communities to come together in a common representation of meaning.

By embedding compositional semantics in a linear algebraic architecture, the work revives and modernizes Fregean principles for the age of explainable AI and responsible machine learning. The model lays the groundwork for future expansions into neural hybrid systems, multi-sentence discourse understanding, and semantic adaptation in context-sensitive dialogue systems.

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