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Optimized Nonlinear Grey Bernoulli Model for Nowcasting the Philippine Gross Domestic Product

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Abstract. This study advances an optimized Nonlinear Grey Bernoulli Model (NGBM(1,1)) for nowcasting the gross domestic product of the Philippines. Two parameter optimization strategies — Particle Swarm Optimization (PSO) and an exponential background value — were employed to minimize the out-of-sample mean absolute percentage error (MAPE). A harmonic function simulation identified a 70/30 training-to-testing ratio as the most effective data-splitting scheme. Applied to quarterly GDP data from the first quarter of 2021 to the fourth quarter of 2024, the PSO-optimized NGBM(1,1) yielded the lowest out-of-sample MAPE of 5.45 percent and the lowest root mean square error, outperforming benchmark models. Results indicate that PSO provides substantial improvements in forecasting performance, whereas the exponential background method yielded smaller gains.

2020 Mathematics Subject Classifications: 90B50, 62P20, 91B84, 62M10

Key Words and Phrases: Gross domestic product, particle swarm optimization, nonlinear grey Bernoulli model, Philippines

1. Introduction

Gross Domestic Product (GDP) plays a crucial role in understanding economic health. Representing the total monetary or market value of all finished goods and services produced within a country's borders over a specific period, GDP serves as a broad indicator of a country's overall economic activity and acts as a comprehensive measure of its economic health [1].

Over the past decades, advancements in time-series econometrics have enabled the development of automated platforms for monitoring macroeconomic conditions in real time [2]. Giannone, Reichlin, and Small [3] pioneered the first consistent statistical framework for this purpose, termed nowcasting. It is defined as the prediction of the present, the very near future, and the very recent past. Nowcasting is particularly valuable for economic planning because key indicators of economic health, such as GDP and its components, are released quarterly, often with a significant delay of up to two months. These delays disrupt

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the planning processes of various stakeholders, including the central bank, legislators, fiscal planners, and financial and business firms, all of whom are heavily affected by the business cycle [4].

The Nonlinear Grey Bernoulli Model (NGBM(1,1)) was introduced by Chen, Chen, and Chen [5] to address this need, offering a framework more suitable for nonlinear systems. Studies have shown that NGBM(1,1) can capture complex patterns in economic data, but its performance heavily depends on the precise estimation of model parameters, particularly the power index and background value coefficient.

This study seeks to improve estimation accuracy by incorporating Particle Swarm Optimization (PSO), a metaheuristic algorithm inspired by the collective behavior of bird flocks. PSO has been demonstrated to effectively optimize parameters in nonlinear grey models by minimizing forecasting errors. A further refinement, proposed by Cheng et al. [6], is also explored; this approach reformulates the background value using an exponential curve to enhance model responsiveness to dynamic patterns.

Given the importance of GDP for economic policymaking, producing accurate and timely forecasts is essential. The primary aim of this research is to enhance the predictive performance of the NGBM(1,1) model for nowcasting the quarterly GDP of the Philippines by optimizing its parameters using PSO and an exponential background value.

2. Methodology

2.1. Benchmark Models

Benchmark models serve as a standard reference to evaluate the performance and improvement of a new model. In this study, the standard Grey Model (GM(1,1)) and the Nonlinear Grey Bernoulli Model (NGBM(1,1)) serve as benchmarks to assess the performance of the optimized NGBM(1,1) variant.

2.1.1. Grey Model

The Grey Model GM(1,1) serves as a benchmark in this study. This class of models relies on the most recent data, where the window size determines the historical scope. A fundamental assumption of grey models is that the input data must be strictly non-negative and exhibit a fixed frequency. Furthermore, as they do not require large sample sizes, grey models are particularly suitable for nowcasting.

The first step in the GM(1,1) modeling process is to define the initial non-negative sequence, denoted as $x^{(0)}$. This sequence comprises the original baseline data points over time i, forming the foundation for all subsequent calculations:

$$x^{(0)} = \left\{ x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n) \right\}. \tag{1}$$

The second step involves generating the first-order Accumulated Generating Operation (AGO) sequence, denoted as $x^{(1)}$. This is done by performing a cumulative sum on the initial sequence $x^{(0)}$, where each term $x^{(1)}(k)$ is calculated as the sum of all previous terms $x^{(0)}$. The AGO sequence smoothens the data, making the series' behavior easier to model:

$$x^{(1)} = \left\{ x^{(1)}(1), x^{(1)}(2), x^{(1)}(3), \ldots, x^{(1)}(n) \right\}. \tag{2}$$

The next step is to compute the mean of the first-order AGO sequence to create a background value sequence $z^{(1)}$:

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1) \quad \text{for } k = 2, 3, \ldots, n. \tag{3}$$

The fourth step defines the basic form of the GM(1,1) model. This equation incorporates the parameters a and b:

$$x^{(0)}(k) + az^{(1)}(k) = b. \tag{4}$$

The fifth step involves estimating the parameters a and b using the least-squares method. The parameters are determined by solving the following equation:

$$(a, b)' = (B'B)^{-1}B'Y, \tag{5}$$

where

$$Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} \qquad \text{and} \qquad B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}.$$

The next step solves the whitening equation using the determined parameters to generate the predicted AGO values $\hat{x}^{(1)}$, also called as the time response sequence:

$$\hat{x}^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a} \right] e^{-ak} + \frac{b}{a}. \tag{6}$$

Finally, the predicted AGO sequence is transformed back to its original scale to compute the predicted original sequence $\hat{x}^{(0)}$:

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1). \tag{7}$$

2.1.2. Nonlinear Grey Bernoulli Model

The Nonlinear Grey Bernoulli Model, NGBM(1,1), is derived from grey systems theory and is used to analyze systems with small samples and uncertain information. It represents an extension of the basic GM(1,1) model, incorporating the Bernoulli differential equation to introduce nonlinearity via a power coefficient n [7].

From (1), the background value sequence of the NGBM(1,1) is given by:

$$z^{(1)}(k) = \alpha x^{(1)}(k) + (1-\alpha)x^{(1)}(k-1), \tag{8}$$

where $k = 2, 3, \ldots, n$ and $\alpha \in (0, 1)$.

The model is then defined by its whitening equation, a first-order Bernoulli differential equation that incorporates the power index m alongside parameters a and b:

$$x^{(0)}(k) + az^{(1)}(k) = b\left[z^{(1)}(k)\right]^m, \quad m \neq 1. \tag{9}$$

The parameters a and b are estimated using the least-squares method. The solution in matrix form is given by:

$$(a, b)' = (B'B)^{-1}B'Y, \tag{10}$$

where

$$Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} \qquad \text{and} \qquad B = \begin{bmatrix} -z^{(1)}(2) & \left[z^{(1)}(2)\right]^m \\ -z^{(1)}(3) & \left[z^{(1)}(3)\right]^m \\ \vdots & \vdots \\ -z^{(1)}(n) & \left[z^{(1)}(n)\right]^m \end{bmatrix}.$$

The whitening equation is solved using these estimated parameters to generate the predicted AGO values $\hat{x}^{(1)}$:

$$\hat{x}^{(1)}(k) = \left[\left(x^{(0)}(1)\right)^{1-m} - \frac{b}{a}\right]e^{-a(1-m)(k-1)} + \frac{b}{a}. \tag{11}$$

Note: The equation has been simplified to a standard, more recognizable form. Please verify this matches your intended derivation.

Finally, the predicted AGO sequence is transformed back to its original scale using the inverse accumulated generating operation (IAGO) to obtain the forecasted values $\hat{x}^{(0)}$:

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1). \tag{12}$$

2.2. Parameter Optimization of Nonlinear Grey Bernoulli Model

2.2.1. Optimization with the Background Value in the form of the Exponential Curve

Cheng et al. [6] introduced a method where the background value function $z^{(1)}(t)$ is modeled by an exponential curve between consecutive time points $k-1$ and k. Starting from the NGBM(1,1) whitening equation,

$$\frac{dx^{(1)}}{dt} + ax^{(1)}(t) = b\left[x^{(1)}(t)\right]^m, \tag{13}$$

the integral over the interval $[k-1,k]$ is taken on both sides:

$$\int_{k-1}^{k} \frac{dx^{(1)}}{dt}dt + a\int_{k-1}^{k} x^{(1)}(t)dt = b\int_{k-1}^{k} \left[x^{(1)}(t)\right]^{m} dt. \tag{14}$$

According to [6], the integrals are defined as:

$$z^{(1)}(k) = \int_{k-1}^{k} x^{(1)}(t)dt = x^{(1)}(\xi), \quad (k-1 \leq \xi \leq k), \tag{15}$$

and

$$z^{(2)}(k) = \int_{k-1}^{k} \left[x^{(1)}(t)\right]^{m} dt = \left[x^{(1)}(\eta)\right]^{m}, \quad (k-1 \leq \eta \leq k). \tag{16}$$

The key innovation is modeling the background value with an exponential curve:

$$x^{(1)}(t) = x^{(1)}(k-1)\left[\frac{x^{(1)}(k)}{x^{(1)}(k-1)}\right]^{t-(k-1)}. \tag{17}$$

From this, the background values $z^{(1)}(k)$ and $z^{(2)}(k)$ are derived as:

$$z^{(1)}(k) = x^{(1)}(k-1)\left[\frac{x^{(1)}(k)}{x^{(1)}(k-1)}\right]^{1-\alpha} \tag{18}$$

and

$$z^{(2)}(k) = \left\{x^{(1)}(k-1)\left[\frac{x^{(1)}(k)}{x^{(1)}(k-1)}\right]^{1-\beta}\right\}^{m}. \tag{19}$$

In this formulation, α and β are background coefficients that determine the relative weighting of the accumulated values $x^{(1)}(k)$ and $x^{(1)}(k-1)$ in the construction of the background sequences $z^{(1)}(k)$ and $z^{(2)}(k)$, respectively. The inclusion of both parameters allows the model to flexibly adjust its responsiveness to nonlinear and dynamic patterns in the data.

For given values of α, β, and m, the parameters $(a,b)'$ are estimated using the least squares method:

$$(a,b)' = (B'B)^{-1}B'H, \tag{20}$$

where

$$H = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -z^{(1)}(2) & z^{(2)}(2) \\ -z^{(1)}(3) & z^{(2)}(3) \\ \vdots & \vdots \\ -z^{(1)}(n) & z^{(2)}(n) \end{bmatrix}.$$

The optimal values of α, β, and m are determined by solving the following optimization problem:

$$\begin{aligned} \min_{\alpha,\beta,m} \quad & \text{MAPE} = \frac{1}{n-1}\sum_{t=2}^{n}\left|\frac{x^{(0)}(t)-\hat{x}^{(0)}(t)}{x^{(0)}(t)}\right| \times 100\% \\ \text{s.t.} \quad & 0 \le \alpha \le 1, \\ & 0 \le \beta \le 1, \\ & m \ge 0, \quad m \ne 1. \end{aligned} \tag{21}$$

The time response function, which is the solution to the whitening equation, is given by:

$$\hat{x}^{(1)}(k) = \left[\left(x^{(1)}(1)\right)^{1-m} - \frac{b}{a}\right] e^{-a(1-m)(k-1)} + \frac{b}{a}. \tag{22}$$

Note: The exponent in the exponential term has been corrected to $-a(1-m)(k-1)$ for consistency with standard NGBM literature.

Finally, the predicted AGO sequence is transformed back to the original scale using the inverse accumulated generating operation (IAGO) to obtain the forecasted values:

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1). \tag{23}$$

A grid-search approach is used to evaluate combinations of α, β, and m within the specified bounded parameter space to solve the optimization problem.

2.2.2. Particle Swarm Optimization

The procedure begins by initializing the NGBM(1,1) parameters, specifically the power index m and the background value coefficient α. These initial values are used by the model to generate a forecast sequence based on the historical data. The forecasted values are then compared to the actual observations, and the prediction error is quantified using the out-of-sample Mean Absolute Percentage Error (MAPE). This MAPE value serves as the fitness function for the PSO algorithm.

The PSO algorithm then iteratively refines the parameter values by simulating the social behavior of a particle swarm. Each particle represents a candidate solution, characterized by its position in the parameter space, and its performance is evaluated based on the corresponding out-of-sample MAPE. The particles update their positions based on their own historical best position and the global best position discovered by the entire swarm. This process continues until a convergence criterion is satisfied or a predefined maximum number of iterations is reached.

Zhou et al. [7] describe the main elements of the PSO algorithm as follows:

Particle: A candidate solution defined by the decision variables α and m. The i-th particle is represented as a vector:

$$X_i = \{\alpha_i, m_i\}, \tag{24}$$

where the parameters are random real numbers constrained within appropriate ranges: $\alpha \in (0,1)$ and $m \neq 1$.

Fitness: The performance of each particle is evaluated using the MAPE, which is the objective function to be minimized.

Personal Best: After k iterations, particle i retains the best position it has personally visited, denoted as PB_i^k. This position satisfies:

$$\mathrm{MAPE}(PB_i^k) = \min\left\{\mathrm{MAPE}\left(X_i^\tau\right)\right\}, \quad \tau = 1, 2, \ldots, k. \tag{25}$$

Global Best: After k iterations, the global best position, denoted as GB^k, is the best position discovered by any particle in the entire swarm. It satisfies:

$$\mathrm{MAPE}(GB^k) = \min\left\{\mathrm{MAPE}\left(PB_i^k\right)\right\}, \quad i = 1, 2, \ldots, S, \tag{26}$$

where S is the swarm size.

In this study, the PSO algorithm was implemented in R using the package's default parameter settings. The swarm size was automatically set to 40, and the maximum number of iterations was 1000. The inertia weight (w) was set to $1/(2\log 2)$, while the cognitive and social coefficients (local and global exploration constants, respectively) were both assigned the default value of $0.5 + \log 2$.

Preliminary tests with alternative parameter values did not yield substantial improvements in convergence behavior or forecast accuracy, confirming that the default settings were sufficient for achieving stable and efficient optimization in this application.

2.3. Model Evaluation

The prediction accuracy of the competing models was evaluated using the out-of-sample mean absolute percentage error, defined as:

$$\mathrm{MAPE} = \frac{1}{n-1}\sum_{t=2}^{n}\left|\frac{x^{(0)}(t) - \hat{x}^{(0)}(t)}{x^{(0)}(t)}\right| \times 100\%. \tag{27}$$

The forecasting performance based on MAPE values can be graded according to the criteria proposed by [8], as shown in Table 1.

Table 1: Forecast accuracy grades based on MAPE [8].

MAPE (%)	**Forecasting Power**
≤ 10	Excellent
10–20	Good
20–50	Reasonable

In addition to MAPE, the out-of-sample Root Mean Square Error (RMSE) is reported as a supplementary accuracy metric. The RMSE is defined as the square root of the average squared differences between the actual and predicted values:

$$\mathrm{RMSE} = \sqrt{\frac{1}{n-1}\sum_{t=2}^{n}\left(x^{(0)}(t) - \hat{x}^{(0)}(t)\right)^2}. \tag{28}$$

While MAPE expresses the forecast error as a percentage, RMSE measures the absolute magnitude of the error in the original units of the data, thereby providing a complementary perspective on model accuracy.

2.4. Data

The Gross Domestic Product (GDP) data used in this study were calculated using the expenditure approach, one of the standard methods for measuring GDP. This approach defines GDP as the sum of all final expenditures in the economy:

$$\mathrm{GDP} = C + I + G + (X - M), \tag{29}$$

where C is household consumption, I is gross capital formation (investment), G is government spending, and $(X - M)$ represents net exports (exports minus imports).

The data consist of quarterly Philippine GDP figures from the first quarter of 2021 to the fourth quarter of 2024. The values are expressed in constant 2018 US dollars and were sourced from the OpenSTAT portal maintained by the Philippine Statistics Authority (PSA).

3. Results and Discussions

A simulation study was conducted to determine the optimal data-splitting strategy for nowcasting quarterly GDP with grey models. Due to the limited size of the available dataset, it is crucial to select a partitioning scheme that minimizes prediction errors.

The simulation design is informed by previous research that uses harmonic regression to model cyclical patterns in GDP. For instance, Bujosa et al. [9] applied linear dynamic harmonic regression to Spanish economic indicators, including GDP, and De Groot et al. [10] integrated harmonic regression with Fourier and GARCH methods to analyze GDP cycles in OECD countries.

Both studies represented cyclical dynamics using sinusoidal components, modeled by the general harmonic regression form:

$$y(t) = \sum_{j=1}^{R}\left[a_j \cos(\omega_j t) + b_j \sin(\omega_j t)\right] + e_t. \tag{30}$$

In this equation, p_j and $\omega_j = 2\pi/p_j$ denote the period and frequency of the jth cycle, respectively. The coefficients a_j and b_j determine the amplitude and phase of the cycle, while e_t represents the error term.

Building on this framework, the simulation study employs a specific case of the harmonic regression model:

$$y(t) = \beta_0 t + \beta_1 \cos(\gamma t) + \beta_2 \sin(\gamma t), \tag{31}$$

where $\gamma = 2\pi/4$ captures the quarterly cyclical frequency, and $t = 1, 2, \ldots, 16$ indexes the 16 quarterly observations. A linear growth trend is introduced with $\beta_0 = 0.15$, while the coefficients β_1 and β_2 are systematically varied over the set $\{-0.1, 0, 0.1, 0.2\}$ to simulate different cyclical patterns in amplitude and phase.

Table 2: Best-performing models based on out-of-sample MAPE under a 70/30 data split.

(β_1, β_2)	**Best Performing Model**
(-0.1, -0.1)	PSO-NGBM(1,1)
(0, -0.1)	PSO-NGBM(1,1)
(0.1, -0.1)	PSO-NGBM(1,1)
(0.2, -0.1)	PSO-NGBM(1,1)
(-0.1, 0)	PSO-NGBM(1,1)
(0.1, 0)	PSO-NGBM(1,1)
(0.2, 0)	PSO-NGBM(1,1)
(-0.1, 0.1)	PSO-NGBM(1,1)
(0, 0.1)	PSO-NGBM(1,1)
(0.1, 0.1)	PSO-NGBM(1,1)
(0.2, 0.1)	Exponential NGBM(1,1)
(-0.1, 0.2)	PSO-NGBM(1,1)
(0, 0.2)	PSO-NGBM(1,1)
(0.1, 0.2)	Exponential NGBM(1,1)
(0.2, 0.2)	Exponential NGBM(1,1)

As shown in Table 2, the PSO-NGBM(1,1) model demonstrated superior performance, achieving the lowest MAPE in 12 of the 15 parameter combinations. The Exponential NGBM(1,1) model performed best only in the three scenarios with the strongest cyclical amplitudes—specifically, (0.2, 0.1), (0.1, 0.2), and (0.2, 0.2). This suggests that the exponential background value offers an advantage for data with highly pronounced cyclical patterns. However, this advantage was limited to a small subset of cases, and the performance gain over the PSO-optimized model was modest.

Under the 80/20 data split, PSO-NGBM(1,1) achieved the lowest MAPE in 6 of the 15 parameter combinations, while the Exponential NGBM(1,1) performed best in 5 cases, and the traditional NGBM(1,1) in 4. Consistent with earlier results, the Exponential NGBM(1,1) excelled under parameter settings associated with stronger cyclical behavior. The traditional NGBM(1,1) performed well in a limited number of cases, particularly at (0, -0.1), (0, 0.1), and (0.1, 0.1). By contrast, GM(1,1) consistently showed the weakest performance across all scenarios.

Table 3: Best-performing models based on out-of-sample MAPE under an 80/20 data split.

(β_1, β_2)	**Best Performing Model**
(-0.1, -0.1)	PSO-NGBM(1,1)
(0, -0.1)	NGBM(1,1)
(0.1, -0.1)	Exponential NGBM(1,1)
(0.2, -0.1)	Exponential NGBM(1,1)
(-0.1, 0)	PSO-NGBM(1,1)
(0.1, 0)	PSO-NGBM(1,1)
(0.2, 0)	PSO-NGBM(1,1)
(-0.1, 0.1)	PSO-NGBM(1,1)
(0, 0.1)	NGBM(1,1)
(0.1, 0.1)	NGBM(1,1)
(0.2, 0.1)	Exponential NGBM(1,1)
(-0.1, 0.2)	Exponential NGBM(1,1)
(0, 0.2)	NGBM(1,1)
(0.1, 0.2)	PSO-NGBM(1,1)
(0.2, 0.2)	Exponential NGBM(1,1)

The simulation aimed to determine whether a 70/30 or 80/20 training–testing split yields superior predictive performance for grey models. A direct comparison shows that the model producing the lowest MAPE under the 70/30 split achieved a lower absolute MAPE value in 8 of the 15 parameter combinations, compared to 7 for the 80/20 split. Although the margin is narrow, the results favor the 70/30 split when prediction accuracy is the primary objective.

The 80/20 split performed relatively better under moderate cyclical patterns, whereas the 70/30 split provided lower MAPE values in settings characterized by stronger cyclical behavior. Across both splits, PSO-NGBM(1,1) was the most frequent top performer, with the Exponential NGBM(1,1) also showing competitive results, particularly for pronounced cycles. In contrast, GM(1,1) consistently lagged behind and failed to achieve the lowest MAPE in any scenario.

Overall, while both splitting schemes are viable, the 70/30 partitioning provides a slight advantage in forecasting accuracy. Accordingly, the 70/30 split was adopted for the subsequent application to the actual GDP dataset.

3.1. Comparative Study

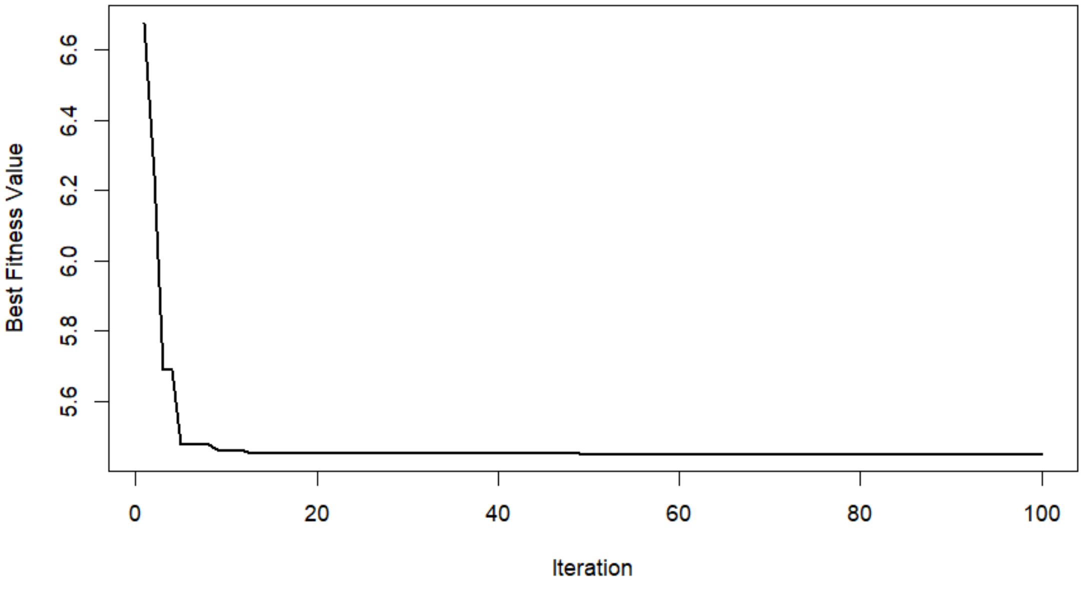
Before presenting the full forecast results, the optimization behavior of the PSO algorithm is examined in the figure below.

The convergence plot in Figure 1 illustrates the minimization of the out-of-sample MAPE across iterations. The PSO algorithm rapidly reduced the error during the initial iterations, stabilizing after approximately 30 iterations. This indicates efficient

Table 4: Comparison of best-performing models under 70/30 and 80/20 data splits.

(β_1, β_2)	**70/30 Split**	**80/20 Split**
$(-0.1, -0.1)$	**PSO-NGBM(1,1)**	PSO-NGBM(1,1)
$(0, -0.1)$	**PSO-NGBM(1,1)**	NGBM(1,1)
$(0.1, -0.1)$	**PSO-NGBM(1,1**)	Exponential NGBM(1,1)
$(0.2, -0.1)$	PSO-NGBM(1,1)	**Exponential NGBM(1,1)**
$(-0.1, 0)$	PSO-NGBM(1,1)	**PSO-NGBM(1,1)**
$(0.1, 0)$	PSO-NGBM(1,1)	**PSO-NGBM(1,1)**
$(0.2, 0)$	PSO-NGBM(1,1)	**PSO-NGBM(1,1)**
$(-0.1, 0.1)$	PSO-NGBM(1,1)	**PSO-NGBM(1,1)**
$(0, 0.1)$	PSO-NGBM(1,1)	**NGBM(1,1)**
$(0.1, 0.1)$	**PSO-NGBM(1,1**)	NGBM(1,1)
$(0.2, 0.1)$	**Exponential NGBM(1,1)**	Exponential NGBM(1,1)
$(-0.1, 0.2)$	PSO-NGBM(1,1)	**Exponential NGBM(1,1)**
$(0, 0.2)$	**PSO-NGBM(1,1)**	NGBM(1,1)
$(0.1, 0.2)$	**Exponential NGBM(1,1)**	PSO-NGBM(1,1)
$(0.2, 0.2)$	**Exponential NGBM(1,1)**	Exponential NGBM(1,1)

Figure 1: Convergence plot of PSO for NGBM(1,1)



convergence to a near-optimal solution with no substantial improvements thereafter.

The results reveal a consistent pattern across all models, with prediction errors peaking during the fourth quarter of each year in both in-sample and out-of-sample periods.

For PSO-NGBM(1,1), Q4 prediction errors were 8.72% (2021), 10.48% (2022), 8.72% (2023), and 7.73% (2024). Similar patterns occurred in the Exponential NGBM(1,1) and

Table 5: Forecast results of PSO-NGBM, Exponential NGBM, NGBM(1,1), and GM(1,1)

Time	Actual	PSO-NGBM(1,1) $\alpha = 0.88$, $m = -0.03$		Exponential NGBM(1,1) $\alpha = 0.11$, $m = 0.01$, $\beta = 0.99$		NGBM(1,1) $\alpha = 0.5$, $m = 0.013$		GM(1,1) $\alpha = 0.5$	
		Pred	PE(%)	Pred	PE(%)	Pred	PE(%)	Pred	PE(%)
2021 Q1	4,266,797	–	–	–	–	–	–	–	–
2021 Q2	4,641,825	4,696,840	1.19	4,640,146	0.04	4,641,810	0.0003	4,664,919	0.50
2021 Q3	4,426,630	4,711,694	6.44	4,713,355	6.48	4,721,235	6.66	4,723,610	6.71
2021 Q4	5,204,832	4,751,109	8.72	4,778,371	8.19	4,789,839	7.97	4,783,039	8.10
2022 Q1	4,611,219	4,802,210	4.14	4,839,870	4.96	4,853,679	5.26	4,843,217	5.03
2022 Q2	4,990,934	4,860,519	2.61	4,899,553	1.83	4,914,964	1.52	4,904,151	1.74
2022 Q3	4,767,549	4,923,948	3.28	4,958,237	4.00	4,974,757	4.35	4,965,852	4.16
2022 Q4	5,575,902	4,991,362	10.48	5,016,381	10.03	5,033,651	9.72	5,028,329	9.82
2023 Q1	4,907,013	5,062,082	3.16	5,074,269	3.41	5,092,018	3.77	5,091,592	3.76
2023 Q2	5,203,455	5,135,673	1.30	5,132,090	1.37	5,150,100	1.03	5,155,651	0.92
2023 Q3	5,051,396	5,211,843	3.18	5,189,978	2.74	5,208,070	3.10	5,220,516	3.35
In-sample MAPE			**4.45**		**4.31**		**4.34**		**4.41**
2023 Q4	5,884,528	5,371,179	8.72	5,306,319	9.83	5,266,053	10.51	5,286,198	10.17
2024 Q1	5,195,588	5,454,102	4.98	5,364,904	3.26	5,324,144	2.47	5,352,705	3.02
2024 Q2	5,539,083	5,539,090	0.0001	5,423,833	2.08	5,382,416	2.83	5,420,049	2.15
2024 Q3	5,316,054	5,626,090	5.83	5,483,143	3.14	5,440,929	2.35	5,488,241	3.24
2024 Q4	6,193,631	5,715,068	7.73	5,542,866	10.51	5,499,730	11.20	5,557,291	10.27
Out-of-sample MAPE			**5.45**		**5.76**		**5.87**		**5.77**

benchmark models. Nevertheless, PSO-NGBM(1,1) achieved the lowest Q4 errors in the out-of-sample forecasts among all grey models.

PSO-NGBM(1,1) also achieved a near-perfect prediction in 2024 Q2 with a PE of 0.0001%. The traditional NGBM(1,1) produced the lowest errors in 2024 Q1 (2.47%) and Q3 (2.35%), while Exponential NGBM(1,1) and GM(1,1) showed competitive performance.

For the in-sample period (2021 Q2-2023 Q3), all models achieved MAPE values below 5%. Exponential NGBM(1,1) attained the lowest in-sample MAPE (4.31%), followed by NGBM(1,1) (4.34%), GM(1,1) (4.41%), and PSO-NGBM(1,1) (4.45%). However, in out-of-sample forecasting, PSO-NGBM(1,1) achieved the lowest MAPE (5.45%), demonstrating superior generalization.

Figure 2 shows that GM(1,1) and Exponential NGBM(1,1) closely follow the general GDP trajectory but fail to capture short-term fluctuations. The traditional NGBM(1,1) shows more variation but still underestimates cyclical amplitudes.

The PSO-optimized NGBM(1,1) (green line) exhibits a steeper upward trajectory from 2023 Q3 onward, reflecting a stronger emphasis on long-term trends at the expense of short-term variations.

All models approximate the upward GDP trend with varying accuracy, but none fully capture the seasonal dynamics, highlighting the need for explicit seasonal modeling in future grey model extensions.

3.2. Out-of-Sample Forecasts

As shown in Figure 3, none of the grey models fully captured the sharp fluctuations in the actual GDP series, a limitation consistent with the broader pattern observed in Figure 2.

Figure 2: Forecasts of the grey models

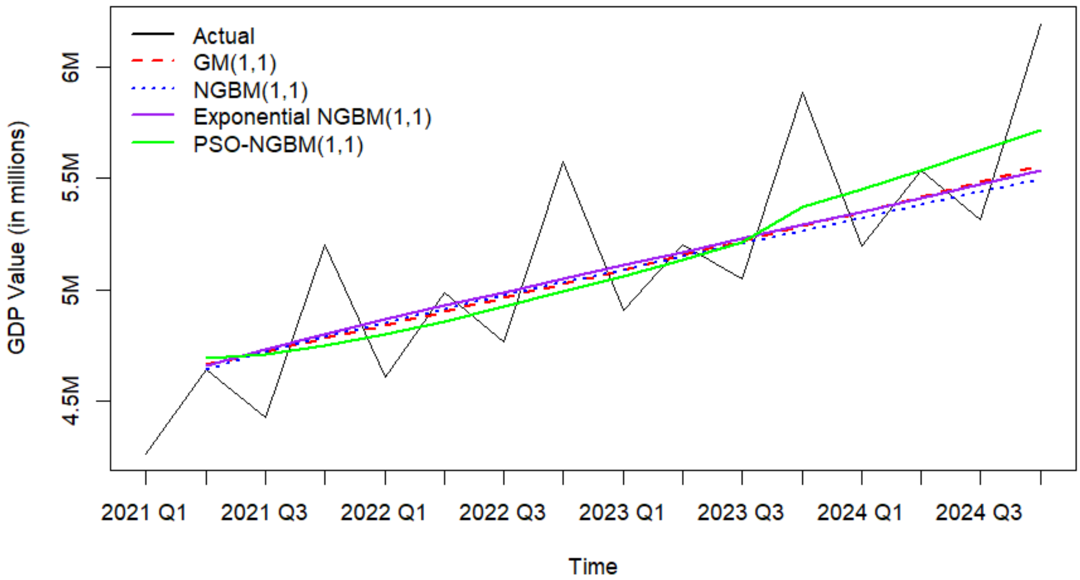
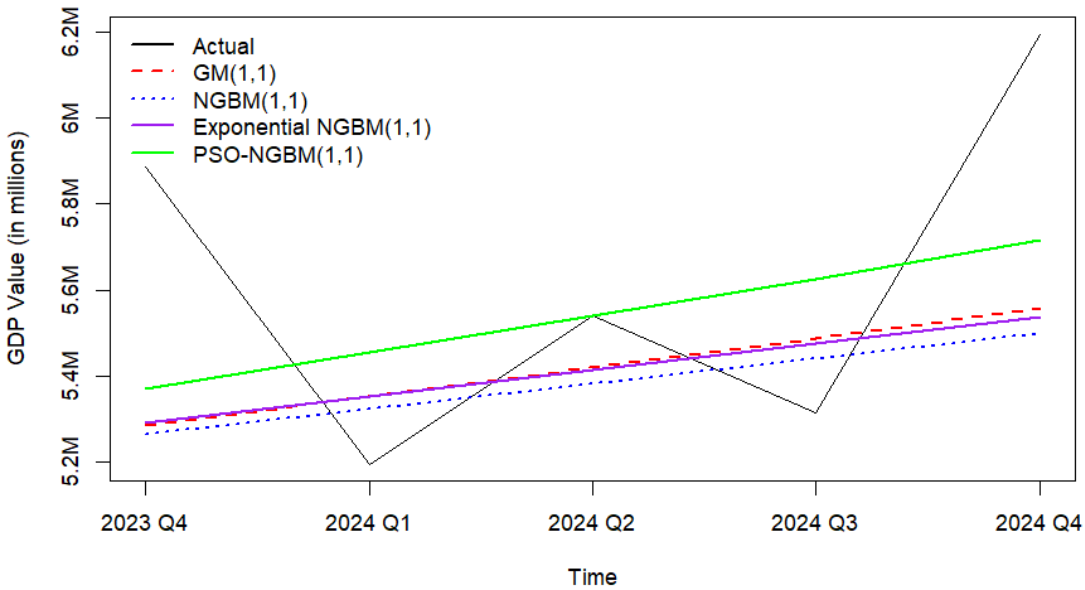


Figure 3: Out-of-sample forecasts of the grey models (2023 Q4–2024 Q4).



Among the models, PSO-NGBM(1,1) produced forecasts closest to the actual GDP in several quarters, most notably in 2024 Q2 where its prediction was nearly identical to the observed value. Although it overestimated GDP in 2024 Q1 and 2024 Q3 — periods characterized by sharp declines — it maintained relatively small deviations overall.

The Exponential NGBM(1,1) and GM(1,1) produced similar trajectories but consistently underestimated GDP in the fourth quarter. In contrast, the traditional NGBM(1,1) yielded the lowest forecast path and systematically underpredicted GDP throughout most of the evaluation period, with particularly pronounced errors in Q4.

These results underscore the challenge of modeling abrupt quarterly shifts in GDP and suggest that grey forecasting frameworks require additional mechanisms to capture short-term volatility.

Table 6: Out-of-sample forecast accuracy metrics.

	PSO-NGBM(1,1)	**Exp NGBM(1,1)**	**NGBM(1,1)**	**GM(1,1)**
RMSE	362,077.8	412,050.9	429,109.5	407,781.9
MAPE (%)	5.45	5.76	5.87	5.77

Note: RMSE values are expressed in millions of Philippine Pesos (PHP).

The out-of-sample period from 2023 Q4 to 2024 Q4 was used to evaluate predictive performance. Table 6 reports the Root Mean Square Error (RMSE), which measures the average magnitude of forecast errors in the original units.

PSO-NGBM(1,1) achieved the lowest RMSE (362,077.8), indicating smaller absolute errors compared to other models. The Exponential NGBM(1,1) recorded an RMSE of 412,050.9, followed by GM(1,1) at 407,781.9 and the traditional NGBM(1,1) at 429,109.5. Although the differences are modest, PSO-NGBM(1,1) maintained a consistent advantage in absolute error terms.

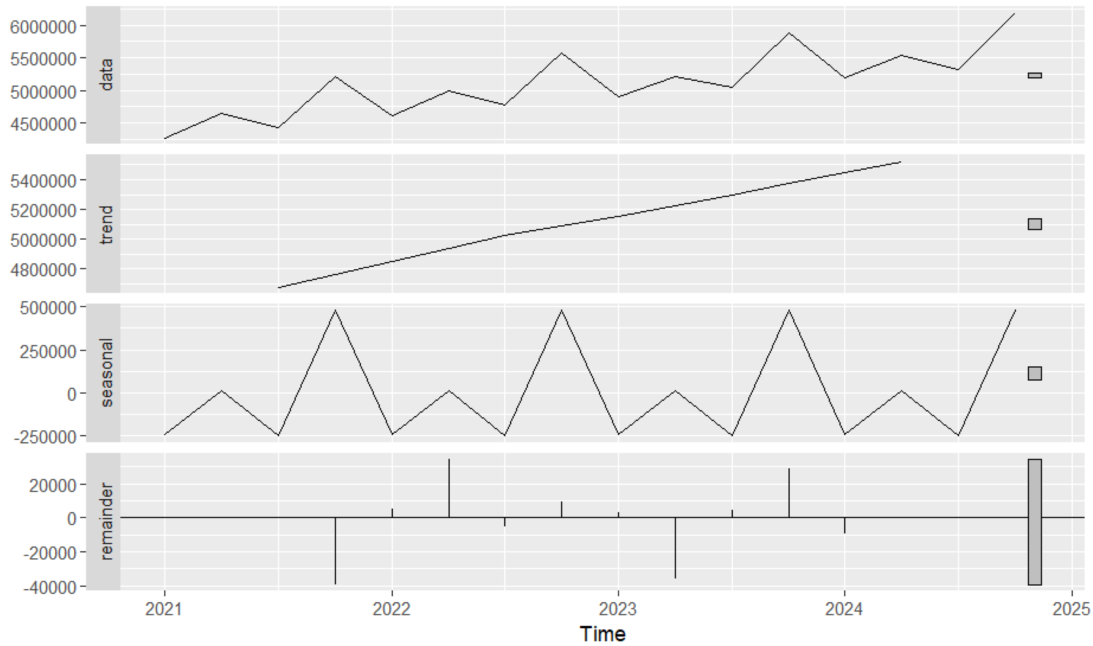
In terms of out-of-sample MAPE, PSO-NGBM(1,1) again performed best at 5.45%. While the Exponential NGBM(1,1) achieved the lowest in-sample MAPE, its accuracy declined slightly out-of-sample to 5.76%. Similarly, the forecasting accuracy of both benchmark models deteriorated in the out-of-sample period.

Overall, both RMSE and MAPE results indicate that PSO-NGBM(1,1) produced comparatively lower errors in both magnitude and percentage terms. These findings align with the simulation study, reinforcing that PSO-NGBM(1,1) provides a consistent, though modest, advantage in forecasting unseen data for short-term GDP nowcasting with limited observations.

3.3. Seasonal Adjustment

The recurring fourth-quarter prediction errors observed in the initial forecasts indicate a fundamental limitation of the standard grey models used in this study, pointing to unmodeled seasonal patterns. To investigate this, a seasonal decomposition of the quarterly GDP series was performed to assess whether removing these fluctuations could enhance forecasting consistency. This post-modeling analysis was particularly relevant given the systematic Q4 errors, which suggested that seasonality contributed significantly to forecast instability.

Figure 4: Seasonal-trend decomposition of Philippine GDP.



The decomposition analysis in Figure 4 reveals a strong and consistent seasonal pattern, characterized by a first-quarter decline followed by a sharp fourth-quarter surge. This confirms the presence of a systematic seasonal structure that was not adequately captured by the initial grey model specifications.

To address this limitation, the GDP series was seasonally adjusted by removing the identified seasonal component, and the grey models were reapplied to this adjusted series. Forecasting accuracy was then reassessed and compared with the results from the original data.

Table 7: Out-of-sample forecast performance on seasonally adjusted GDP.

		PSO-NGBM(1,1)		**Exp NGBM(1,1)**		**NGBM(1,1)**		**GM(1,1)**	
Time	**Actual**	Pred	PE(%)	Pred	PE(%)	Pred	PE(%)	Pred	PE(%)
2023 Q4	5,884,528	5,868,679	0.27	5,890,093	0.09	5,856,314	0.48	5,851,976	0.55
2024 Q1	5,195,588	5,210,630	0.29	5,240,289	0.86	5,206,173	0.20	5,206,605	0.21
2024 Q2	5,539,083	5,539,075	0.0001	5,577,771	0.70	5,543,085	0.07	5,548,962	0.18
2024 Q3	5,316,054	5,338,762	0.43	5,387,230	1.34	5,351,767	0.67	5,363,714	0.90
2024 Q4	6,193,631	6,136,223	0.94	6,195,159	0.02	6,158,733	0.56	6,177,333	0.26
MAPE (%)			**0.38**		**0.60**		**0.40**		**0.42**

The results in Table 7 demonstrate a dramatic improvement in forecasting accuracy after seasonal adjustment. All models achieved exceptional performance, with MAPE values consistently below 1%. PSO-NGBM(1,1) achieved the lowest overall MAPE (0.38%), followed closely by NGBM(1,1) (0.40%) and GM(1,1) (0.42%). In contrast, the Exponential NGBM(1,1) produced the highest MAPE (0.60%), indicating that the exponential background value provided minimal benefit in this context.

Notably, the effectiveness of PSO optimization persisted even after removing seasonal effects, whereas the exponential background value approach contributed little to forecast improvement. This finding underscores the robustness of PSO-NGBM(1,1) in adapting to both raw and seasonally adjusted data, suggesting that optimization techniques play a more critical role than background value adjustments in enhancing forecast performance.

These results confirm that seasonal adjustment substantially improves the forecasting accuracy of grey models for Philippine GDP data. The systematic reduction in prediction errors highlights the importance of explicitly accounting for seasonal patterns when applying grey models to strongly seasonal economic time series.

4. Conclusion and Recommendations

This study demonstrates that the PSO-optimized NGBM(1,1) model outperforms benchmark grey models in nowcasting Philippine GDP, achieving the lowest out-of-sample MAPE (5.45%) and RMSE. While all models produced acceptable forecasts, PSO-NGBM(1,1) delivered consistently superior performance.

Forecast accuracy improved dramatically after seasonal adjustment, with MAPE values falling from over 5% to below 1%. This improvement is economically significant; a 1% forecasting error for GDP, measured in millions, represents a substantial monetary deviation that can directly impact the accuracy of policy assessments and subsequent economic interventions.

The findings suggest that developing hybrid grey models with integrated seasonal components could enhance nowcasting accuracy without requiring separate preprocessing. Future research should also explore optimization strategies beyond the exponential background value to improve parameter estimation. In addition to standard accuracy metrics like MAPE and RMSE, rigorous statistical testing is recommended to establish the significance of performance differences between models. Furthermore, combining closed-form estimation with metaheuristic algorithms like PSO may yield more stable and computationally efficient forecasts.

Ongoing refinement of these models is vital for generating timely and reliable GDP estimates to guide policy. In dynamic economic environments where official statistics are released with considerable delay, accurate nowcasts can strengthen evidence-based governance and support more responsive economic planning.

Overall, this study confirms the promise of PSO-optimized grey models as effective and reliable tools for short-term GDP forecasting in data-constrained environments.

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Declaration

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