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Malaria and Malnutrition Dynamics in Children Using the Caputo–Fabrizio Fractional Derivative

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Abstract. Malaria poses a significant global public health challenge as an infectious disease transmitted by vectors, particularly affecting young children, with substantial morbidity and mortality rates. This study formulates criteria ensuring the stability, uniqueness, and existence of a fractional-order malaria-malnutrition framework utilizing the Caputo-Fabrizio differential operator, employing the fixed-point methodology. The adoption of this fractional differentiation technique is an innovative approach within such biological contexts. Furthermore, we obtain the earliest approximate solutions for the formulated model through the iterative Laplace transform procedure. At last, numerical simulations are performed using the selected model parameter values. Our findings indicate that ensuring a well-rounded diet is crucial for mitigating the spread of malaria among young children, thereby reducing both morbidity and mortality.

2020 Mathematics Subject Classifications: 26A33, 33E12, 34A08, 65R10

Key Words and Phrases: Mathematical models, malaria fever, Caputo-Fabrizio derivative, existence, uniqueness and stability, numerical simulations

1. Introduction

Malaria is a disease spread by infected female Anopheles mosquitoes transmitting the Plasmodium parasite. It is a significant health issue in numerous tropical and subtropical areas. The parasite first grows in the liver and then attacks Erythrocytes, resulting in symptoms like tiredness, muscle pain, headaches, sweating, chills and fever. Malaria does not spread directly from person to person. It is only transmitted through mosquito bites, when an infected person is bitten by a mosquito, it picks up the infectious agent and can pass it on to others. Within the mosquito, these parasites undergo a complex life cycle. While stopping malaria could potentially make children grow heavier, this improvement might only be temporary [1–4]. Historically, public health initiatives prioritized communicable diseases and malnutrition due to their significant health risks. Yet, understanding the interplay

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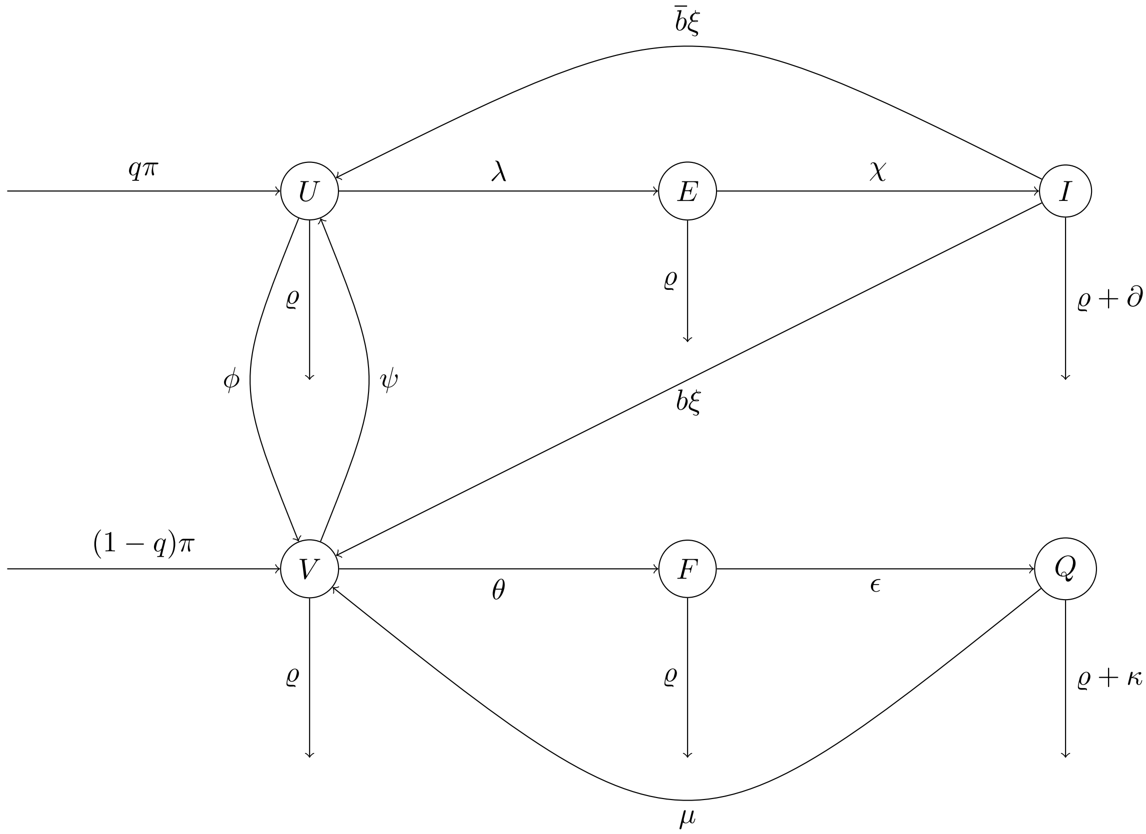
between infection and nutrition remains complex, making it challenging to determine the order of addressing these issues. This study aimed to find out how many children in the area suffer from malnutrition. It also examined whether treating diseases, especially malaria, helps children grow and develop properly. Researchers examined the relationship between malaria and malnutrition in children under five in a region heavily affected by malaria[5]. Animal studies suggested that a poor diet might help reduce malaria. This led to the idea that malnourished children could be less likely to get malaria, experience severe symptoms, or die from the disease [6–9] Malaria and malnutrition are key factors in child mortality in sub-Saharan Africa. Repeated malaria infections weaken children's nutrition and overall well-being [10, 11].

In this article, we summarize modelling in malaria research to help more researchers in epidemiology, transmission, and other areas understand it better[12]. A mathematical model was developed and analyzed to better understand how malaria spreads and identify effective ways to curb and oversee[13]. Recent papers have started considering environmental factors and the development of drug resistance in malaria [14–18]. Ngwa and Shu [19], as well as Ngwa [20], introduced an ODE compartmental model for malaria spread. Addo [21] and Tuwiine Mugisha, and Luboobi [22] have developed compartmental models for malaria transmission, using susceptible-infected-recovered-susceptible (SIRS) patterns for humans and susceptible-infected (SI) patterns for mosquitoes respectively. Yang, Wei, and Li [23]proposed a compartment model with SIR for humans and SI for vectors. These studies establish the reproduction number and investigate the conditions for the existence and stability of both disease-free and endemic states.

Mathematics, especially fractional calculus, is vital for modeling epidemics and biological processes. This study uses fractional operators and numerical methods to analyze disease dynamics, compute key parameters, and support decision-making, with potential for future hybrid approaches[24]. This study models poliomyelitis transmission using fractional-order ABC models with vaccination and post-paralytic compartments, analyzes stability and convergence, and integrates deep neural networks for accurate simulation and prediction of disease dynamics[25]. In this paper, we developed criteria for the uniqueness, stability and existence of a fractional-order typhoid fever model using the Caputo–Fabrizio operator and fixed-point theory[26]. Infectious diseases can be accurately modelled using the non-local Caputo–Fabrizio fractional derivative operator. Additionally, we have verified the stability conditions for steady-state solutions and established their existence using Banach fixed-point theory[27]. In this paper, we created a model to study how the new coronavirus spreads. This model helps us understand how the disease spreads over time, both in the short term and long term[28]. A fractional-order hepatitis B model with vaccine effects is analyzed using Caputo derivatives, simulations, and ANN to understand disease dynamics and support public health strategies[29]. This paper presents a fractional SIQR model with Caputo–Fabrizio derivative, proving key properties, simulating memory effects, and applying neural networks for analysis[30].

2. Model Information

We determine a Susceptible-Exposed-Infectious-Susceptible (SEIS) model of ailment to place the institutions under concern and teenagers under age five into two groups, similarly by their digestive rank: the first group is those the individuals are well augmented, named for individual follow-up 1, and the second group is composed of those the individuals are thin, named for each follow-up 2. The total babies organization is conveyed by N. Influx into the youthful category progresses at the rate η following a disposal p entering the well augment childlike U, and $1-p$ into the thin childlike V. Note that we only trust drafts into the unprotected division. A well-augmented childlike U because a feeble diet permits an action to reinforce a thin, unprotected V at a rate ψ, and a thin, trusting following position/correct diet acknowledges feasibility and embellishes a well-augmented trusting at a measure ϕ. Commonly, a trusting individual, for that reason, can take a wound caused by extending scourge with nausea; therefore, it can improve at the rate, λ, or fix and embellish unprotected recurring at a measure α. Concerning the well-augmented epidemic, a percentage b that restores holds improves thin, unprotected belongings; thus, $\overline{b} = 1 - b$ improves well-augmented, trusting belongings. Individuals exit the infected compartment by recovery or malaria-induced death, common obliteration n, or by erasure from the ailment e_1 for the class C_1 and e_2 for the class C_2. The nausea broadcast probabilities per pest bite on U and V are likely individually by γ_1 and γ_2. both trusting groups of youth are defenseless to sickness and move separately to the K_1 or K_2 classes. The strength of contamination, or the measure at which naive belongings get ailment, is ρ_1, ρ_2 and ρ_3 for the contagion-heading folk. The compartments include susceptible (U) individuals who are at risk of contracting malaria, exposed (E) individuals who have been infected but are not yet infectious, infected (I) individuals capable of transmitting the disease, vaccinated (V) individuals with partial or full immunity, fully infected or febrile (F) individuals displaying severe symptoms, and quarantined (Q) individuals who are isolated for treatment or control. By presenting these compartments clearly and linking them directly to the biological processes they represent, readers can better understand the dynamics of malaria transmission.



From the model flowcharts of the affliction broadcast described in the above Figure, we evolve the following equations.

$$\begin{cases} \dfrac{dU}{dt} = q\pi + (1-b)\xi I + \phi V - (\sigma + \psi + \varrho)U \\ \\ \dfrac{dE}{dt} = \sigma U - (\chi + \varrho)E \\ \\ \dfrac{dI}{dt} = \chi E - (\xi + \varrho + \partial)I \\ \\ \dfrac{dV}{dt} = (1-q)\pi + \mu Q + \psi U + b\xi I - (\theta + \phi + \varrho)V \\ \\ \dfrac{dF}{dt} = \theta V - (\varepsilon + \varrho)F \\ \\ \dfrac{dQ}{dt} = \epsilon F - (\mu + \varrho + \kappa)Q \end{cases}$$

Motivated by the previously listed works of literature, the Malaria fever model proposed in [11] is analyzed using the Caputo-Fabrizio operator of order η where $\eta \in (0,1)$.

$$\begin{cases} {}^{CF}D_t^{\eta}U(t) = q\pi + (1-b)\xi I + \phi V - (\sigma + \psi + \varrho)U \\ {}^{CF}D_t^{\eta}E(t) = \sigma U - (\chi + \varrho)E \\ {}^{CF}D_t^{\eta}I(t) = \chi E - (\xi + \varrho + \partial)I \\ {}^{CF}D_t^{\eta}V(t) = (1-q)\pi + \mu Q + \psi U + b\xi I - (\theta + \phi + \varrho)V \\ {}^{CF}D_t^{\eta}F(t) = \theta V - (\varepsilon + \varrho)F \\ {}^{CF}D_t^{\eta}Q(t) = \epsilon F - (\mu + \varrho + \kappa)Q \end{cases} \tag{2.1}$$

with initial conditions

$$U(0) \geq 0, E(0) \geq 0, I(0) \geq 0, V(0) \geq 0, F(0) \geq 0, Q(0) \geq 0. \tag{2.2}$$

The classic model given in [11] is accomplished for $\eta = 1$. Given that the Caputo-Fabrizio fractional derivative accurately illustrates the previously discussed occurrences. The quantitative findings are obtained repeatedly using the Laplace transform approach. To validate our results, we assign random values to the parameters and initial conditions.

The remainder of the paper is organized as follows: Section 3 provides some definitions and preliminary information on fractional calculus. The existence and uniqueness of the model's solution with the method and operator under consideration are covered in Section 4. Section 5 presents the stability of the approximated solution. The solution's graphical representation is related to Section 6. In section 7, numerical results are discussed. Finally, section 8 provides a summary of the study's main findings and observations.

3. Preliminaries

Definition 1. *([27]) Let $u \in H^1(0,d)$, $d > 0, 0 < \eta < 1$, then time-fractional Caputo-Fabrizio differential operator the time fractional Caputo–Fabrizio fractional differential operator (C-FFDO) is expressed as*

$${}^{CF}D_t^{\eta}u(t) = \frac{N(\eta)}{(1-\eta)} \int_0^t exp\Big[-\frac{\eta(t-s)}{1-\eta} \Big] u'(s)ds, \quad t \geq 0, \qquad 0 < \eta < 1, \tag{3.1}$$

where $N(\eta)$ is a normalization function dependent on η, ensuring that $N(0) = N(1) = 1$.

Definition 2. *([27]) The Caputo-Fabrizio integral operator of fractional order $0 < \eta < 1$ is given by*

$${}^{CF}J_t^{\eta}u(t) = \frac{2(1-\eta)}{(2-\eta)N(\eta)} u(t) + \frac{2\eta}{(2-\eta)N(\eta)} \int_0^t u(\aleph)d\aleph, \tag{3.2}$$

Like the traditional Caputo derivative, this new operator offers ${}^{CF}D_t^{\eta}u(t) = 0$, provided that u does not change.

The key advantage of the Caputo-Fabrizio operator applied to the traditional Caputo operator is that the updated kernel avoids singularity at $t = s$.

Definition 3. *([27]) The Laplace transformation of the Caputo-Fabrizio fractional operator of order $0 < \eta \leq 1$ and $m \in \mathbb{N}$ is given by*

$$\begin{aligned}\mathcal{L}\big({}^{CF}D_t^{m+\eta}u(t)\big)(\mathfrak{s}) &= \frac{1}{1-\eta}\mathcal{L}(u^{(m+1)}(t))\mathcal{L}\Bigg(exp\Big(-\frac{\eta}{1-\eta}t\Big)\Bigg) \\ &= \frac{\mathfrak{s}^{m+1}L(u(t)) - \mathfrak{s}^m u(0) - \mathfrak{s}^{m-1}u'(0) - \cdots - u^{(m)}(0)}{\mathfrak{s} + \eta(1-\mathfrak{s})}. \end{aligned} \tag{3.3}$$

Specifically, we have

$$\begin{aligned}\mathcal{L}\big({}^{CF}D_t^{\eta}u(t)\big)(\mathfrak{s}) &= \frac{\mathfrak{s}\mathcal{L}(u(t)) - u(0)}{\mathfrak{s} + \eta(1-\mathfrak{s})}, \qquad m = 0. \\ \mathcal{L}\big({}^{CF}D_t^{\eta+1}u(t)\big)(\mathfrak{s}) &= \frac{\mathfrak{s}^2\mathcal{L}(u(t)) - \mathfrak{s}u(0) - u'(0)}{\mathfrak{s} + \eta(1-\mathfrak{s})}, \qquad m = 1.\end{aligned}$$

4. Existence and Uniqueness

In this section, we examine the existence and uniqueness of the solution for the system (2.1) using the fixed-point theory.
Considering equation (3.2), we get

$$\begin{aligned}
U(t) &= U(0) + \frac{2(1-\eta)}{2\eta N(\eta)}(q\pi + \bar{b}\xi I(t) + \phi V(t) - (\sigma + \psi + \varrho)U(t)) \\
&\quad + \frac{2(1-\eta)}{(2-\eta)N(\eta)}\int_0^t (q\pi + \bar{b}\xi I(\aleph) + \phi V(\aleph) - (\sigma + \psi + \varrho)U(\aleph))d\aleph \\
E(t) &= E(0) + \frac{2(1-\eta)}{(2\eta)N(\eta)}(\sigma U(t) - (\chi + \varrho)E(t)) \\
&\quad + \frac{2(1-\eta)}{(2-\eta)N(\eta)}\int_0^t (\sigma U(\aleph) - (\chi + \varrho)E(\aleph))d\aleph \\
I(t) &= I(0) + \frac{2(1-\eta)}{(2\eta)N(\eta)}(\chi E(t) - (\xi + \varrho + \partial)I(t)) \\
&\quad + \frac{2(1-\eta)}{(2-\eta)N(\eta)}\int_0^t (\chi E(\aleph) - (\xi + \varrho + \partial)I(\aleph))d\aleph \\
V(t) &= V(0) + \frac{2(1-\eta)}{2\eta N(\eta)}((1-q)\pi + \mu Q(t) + \psi U(t) + b\xi I(t) - (\theta + \phi + \varrho)V(t)) \\
&\quad + \frac{2(1-\eta)}{(2-\eta)N(\eta)}\int_0^t ((1-q)\pi + \mu Q(\aleph) + \psi U(\aleph) + b\xi I(\aleph) - (\theta + \phi + \varrho)V(\aleph))d\aleph \\
F(t) &= F(0) + \frac{2(1-\eta)}{(2\eta)N(\eta)}(\theta V(t) - (\mu + n)F(t)) \\
&\quad + \frac{2(1-\eta)}{(2-\eta)N(\eta)}\int_0^t (\theta V(\aleph) - (\mu + \varrho)F(\aleph))d\aleph \\
Q(t) &= Q(0) + \frac{2(1-\eta)}{(2\eta)N(\eta)}(\epsilon F(t) - (\mu + \varrho + \kappa)Q(t)) \\
&\quad + \frac{2(1-\eta)}{(2-\eta)N(\eta)}\int_0^t (\epsilon F(\aleph) - (\mu + \varrho + \kappa)Q(\aleph))d\aleph
\end{aligned}$$

Let's now examine the subsequent kernels:

$$\begin{aligned}
\psi_1(t,U(t)) &= q\pi + \bar{b}\xi I + \phi V(t) - (\sigma+\psi+\varrho)U(t) \\
\psi_2(t,E(t)) &= \sigma U(t) - (\chi+\varrho)E(t) \\
\psi_3(t,I(t)) &= \chi E(t) - (\xi+\varrho+\partial)I(t) \\
\psi_4(t,V(t)) &= (1-q)\pi + \mu Q(t) + \psi U(t) + b\xi I(t) - (\theta+\phi+\varrho)V(t) \\
\psi_5(t,F(t)) &= \theta V(t) - (\mu+\varrho)F(t) \\
\psi_6(t,Q(t)) &= \epsilon F(t) - (\mu+\varrho+\kappa)Q(t)
\end{aligned} \tag{4.1}$$

Lemma 1. *The kernels $\psi_1, \psi_2, \psi_3, \psi_4, \psi_5,$ and ψ_6 in* (4.1) *satisfy the Lipschitz condition, i.e., for any two state vectors $X_1 = (U_1, E_1, I_1, V_1, F_1, Q_1)$ and $X_2 = (U_2, E_2, I_2, V_2, F_2, Q_2)$, there exist constants $0 < \Lambda_i < 1$ such that $\|\psi_i(t, X_1) - \psi_i(t, X_2)\| \leq \Lambda_i \|X_1 - X_2\|, \quad i = 1, 2, \ldots, 6$.*

Proof: Let U_1 and U_2, for the kernel ψ_1, E_1 and E_2, for the kernel ψ_2, I_1 and I_2, for the kernel ψ_3, V_1 and V_2, for the kernel ψ_4, F_1 and F_2, for the kernel ψ_5 and Q_1 and Q_2, the respective functions associated with the kernel ψ_6 correspond to the following:

$$\begin{aligned}
\|\psi_1(t,U_1(t)) - \psi_1(t,U_2(t))\| &= \|(-(\sigma+\psi+\varrho)U_1(t)) + (\sigma+\psi+\varrho)U_2(t))\|, \\
\|\psi_2(t,E_1(t)) - \psi_2(t,E_2(t))\| &= \|(-((\chi+\varrho))E_1(t)) + ((\chi+\varrho))E_2(t))\|, \\
\|\psi_3(t,I_1(t)) - \psi_3(t,I_2(t))\| &= \|(-(\xi+\varrho+\partial)I_1(t)) + (\xi+\varrho+\partial)I_2(t))\|, \\
\|\psi_4(t,V_1(t)) - \psi_4(t,V_2(t))\| &= \|(-(\theta+\phi+\varrho)V_1(t)) + (\theta+\phi+\varrho)V_2(t))\|, \\
\|\psi_5(t,F_1(t)) - \psi_5(t,F_2(t))\| &= \|(-(\mu+\varrho)F_1(t)) + (\mu+\varrho)F_2(t))\|, \\
\|\psi_6(t,Q_1(t)) - \psi_6(t,Q_2(t))\| &= \|(-(\mu+\varrho+\kappa)Q_1(t)) + (\mu+\varrho+\kappa)Q_2(t))\|,
\end{aligned} \tag{4.2}$$

Consider

$$\begin{aligned}
\|\psi_1(t,U_1(t)) - \psi_1(t,U_2(t))\| &= \|(-(\sigma+\psi+\varrho)U_1(t)) + (\sigma+\psi+\varrho)U_2(t))\|, \\
&\leq \|((\sigma+\psi+\varrho)(U_1(t)) - U_2(t)))\|, \\
&\leq \Lambda_1 \|(U_1(t)) - U_2(t)\|,
\end{aligned} \tag{4.3}$$

where $\Lambda_1 = \sigma + \psi + \varrho$. Similarly, we can obtain

$$\begin{aligned}
\|\psi_2(t,E_1(t)) - \psi_2(t,E_2(t))\| &\leq \Lambda_2 \|(E_1(t)) - E_2(t)\|, \\
\|\psi_3(t,I_1(t)) - \psi_3(t,I_2(t))\| &\leq \Lambda_3 \|(I_1(t)) - I_2(t)\|, \\
\|\psi_4(t,V_1(t)) - \psi_4(t,V_2(t))\| &\leq \Lambda_4 \|(V_1(t)) - V_2(t)\|, \\
\|\psi_5(t,F_1(t)) - \psi_5(t,F_2(t))\| &\leq \Lambda_5 \|(F_1(t)) - F_2(t)\|, \\
\|\psi_6(t,Q_1(t)) - \psi_6(t,Q_2(t))\| &\leq \Lambda_6 \|(Q_1(t)) - Q_2(t)\|,
\end{aligned} \tag{4.4}$$

where $\Lambda_2 = \chi + \varrho, \Lambda_3 = \xi + \varrho + \aleph$, $\Lambda_4 = \theta + \phi + \varrho$, $\Lambda_5 = \mu + \varrho$, $\Lambda_6 = \mu + \varrho + \kappa$. Utilising the subsequent recursive formula, we obtain

$$U_n(t) = \frac{2(1-\eta)}{(2-\eta)N(\eta)}\psi_1(t, U_{n-1}(t)) + \frac{2\eta}{(2-\eta)N(\eta)}\int_0^t \psi_1(\aleph, U_{n-1}(\aleph))d\aleph,$$

$$\begin{aligned}
E_n(t) &= \frac{2(1-\eta)}{(2-\eta)N(\eta)}\psi_2(t, E_{n-1}(t)) + \frac{2\eta}{(2-\eta)N(\eta)}\int_0^t \psi_2(\aleph, E_{n-1}(\aleph))d\aleph,\\
I_n(t) &= \frac{2(1-\eta)}{(2-\eta)N(\eta)}\psi_3(t, I_{n-1}(t)) + \frac{2\eta}{(2-\eta)N(\eta)}\int_0^t \psi_3(\aleph, I_{n-1}(\aleph))d\aleph,\\
V_n(t) &= \frac{2(1-\eta)}{(2-\eta)N(\eta)}\psi_4(t, V_{n-1}(t)) + \frac{2\eta}{(2-\eta)N(\eta)}\int_0^t \psi_4(\aleph, V_{n-1}(\aleph))d\aleph,\\
F_n(t) &= \frac{2(1-\eta)}{(2-\eta)N(\eta)}\psi_5(t, F_{n-1}(t)) + \frac{2\eta}{(2-\eta)N(\eta)}\int_0^t \psi_5(\aleph, F_{n-1}(\aleph))d\aleph,\\
Q_n(t) &= \frac{2(1-\eta)}{(2-\eta)N(\eta)}\psi_6(t, Q_{n-1}(t)) + \frac{2\eta}{(2-\eta)N(\eta)}\int_0^t \psi_6(\aleph, Q_{n-1}(\aleph))d\aleph.
\end{aligned}$$

Now, by triangle inequality, we get

$$\begin{aligned}
\|\Delta_{1n}(t)\| = \|U_n(t) - U_{n-1}(t)\| &\le \frac{2(1-\eta)}{(2-\eta)N(\eta)}\|\psi_1\big(t, U_{n-1}(t)\big) - \psi_1\big(t, U_{n-2}(t)\big)\|\\
&+ \frac{2\eta}{(2-\eta)N(\eta)}\left\|\int_0^t \Big[\psi_1\big(\aleph, U_{n-1}(\aleph)\big) - \psi_1\big(\aleph, U_{n-2}(\aleph)\big)\Big]d\aleph\right\|,\\
\|\Delta_{2n}(t)\| = \|E_n(t) - E_{n-1}(t)\| &\le \frac{2(1-\eta)}{(2-\eta)N(\eta)}\|\psi_2\big(t, E_{n-1}(t)\big) - \psi_2\big(t, E_{n-2}(t)\big)\|\\
&+ \frac{2\eta}{(2-\eta)N(\eta)}\left\|\int_0^t \Big[\psi_2\big(\aleph, E_{n-1}(\aleph)\big) - \psi_2\big(\aleph, E_{n-2}(\aleph)\big)\Big]d\aleph\right\|,\\
\|\Delta_{3n}(t)\| = \|I_n(t) - I_{n-1}(t)\| &\le \frac{2(1-\eta)}{(2-\eta)N(\eta)}\|\psi_3\big(t, I_{n-1}(t)\big) - \psi_3\big(t, I_{n-2}(t)\big)\|\\
&+ \frac{2\eta}{(2-\eta)N(\eta)}\left\|\int_0^t \Big[\psi_3\big(\aleph, I_{n-1}(\aleph)\big) - \psi_3\big(\aleph, I_{n-2}(\aleph)\big)\Big]d\aleph\right\|,\\
\|\Delta_{4n}(t)\| = \|V_n(t) - V_{n-1}(t)\| &\le \frac{2(1-\eta)}{(2-\eta)N(\eta)}\|\psi_4\big(t, V_{n-1}(t)\big) - \psi_4\big(t, V_{n-2}(t)\big)\|\\
&+ \frac{2\eta}{(2-\eta)N(\eta)}\left\|\int_0^t \Big[\psi_4\big(\aleph, V_{n-1}(\aleph)\big) - \psi_4\big(\aleph, V_{n-2}(\aleph)\big)\Big]d\aleph\right\|,\\
\|\Delta_{5n}(t)\| = \|F_n(t) - F_{n-1}(t)\| &\le \frac{2(1-\eta)}{(2-\eta)N(\eta)}\|\psi_5\big(t, F_{n-1}(t)\big) - \psi_5\big(t, F_{n-2}(t)\big)\|\\
&+ \frac{2\eta}{(2-\eta)N(\eta)}\left\|\int_0^t \Big[\psi_5\big(\aleph, F_{n-1}(\aleph)\big) - \psi_5\big(\aleph, F_{n-2}(\aleph)\big)\Big]d\aleph\right\|,\\
\|\Delta_{6n}(t)\| = \|Q_n(t) - Q_{n-1}(t)\| &\le \frac{2(1-\eta)}{(2-\eta)N(\eta)}\|\psi_6\big(t, Q_{n-1}(t)\big) - \psi_6\big(t, Q_{n-2}(t)\big)\|\\
&+ \frac{2\eta}{(2-\eta)N(\eta)}\left\|\int_0^t \Big[\psi_6\big(\aleph, Q_{n-1}(\aleph)\big) - \psi_6\big(\aleph, Q_{n-2}(\aleph)\big)\Big]d\aleph\right\|,
\end{aligned} \tag{4.5}$$

$$\begin{aligned}
U_n(t) &= \sum_{j=0}^{\infty}\Delta_{1j}(t), \quad E_n(t) = \sum_{j=0}^{\infty}\Delta_{2j}(t), \quad I_n(t) = \sum_{j=0}^{\infty}\Delta_{3j}(t),\\
V_n(t) &= \sum_{j=0}^{\infty}\Delta_{4j}(t), \quad F_n(t) = \sum_{j=0}^{\infty}\Delta_{5j}(t), \quad Q_n(t) = \sum_{j=0}^{\infty}\Delta_{6j}(t).
\end{aligned} \tag{4.6}$$

Since the kernels $\psi_1,\ \psi_2,\ \psi_3,\ \psi_4, \psi_5$ and ψ_6 fulfils the Lipschitz condition, we get

$$\begin{aligned}
\|\Delta_{1n}(t)\| = \|U_n(t) - U_{n-1}(t)\| &\leq \frac{2(1-\eta)}{(2-\eta)N(\eta)}\Lambda_1\|U_{n-1}(t) - U_{n-2}(t)\| \\
&+ \frac{2\eta}{(2-\eta)N(\eta)}\Lambda_1 \int_0^t \|U_{n-1}(\aleph) - U_{n-2}(\aleph)\|d\aleph, \\
\|\Delta_{2n}(t)\| = \|E_n(t) - E_{n-1}(t)\| &\leq \frac{2(1-\eta)}{(2-\eta)N(\eta)}\Lambda_2\|E_{n-1}(t) - E_{n-2}(t)\| \\
&+ \frac{2\eta}{(2-\eta)N(\eta)}\Lambda_2 \int_0^t \|E_{n-1}(\aleph) - E_{n-2}(\aleph)\|d\aleph, \\
\|\Delta_{3n}(t)\| = \|I_n(t) - I_{n-1}(t)\| &\leq \frac{2(1-\eta)}{(2-\eta)N(\eta)}\Lambda_3\|I_{n-1}(t) - I_{n-2}(t)\| \\
&+ \frac{2\eta}{(2-\eta)N(\eta)}\Lambda_3 \int_0^t \|I_{n-1}(\aleph) - I_{n-2}(\aleph)\|d\aleph, \\
\|\Delta_{4n}(t)\| = \|V_n(t) - V_{n-1}(t)\| &\leq \frac{2(1-\eta)}{(2-\eta)N(\eta)}\Lambda_4\|V_{n-1}(t) - V_{n-2}(t)\| \\
&+ \frac{2\eta}{(2-\eta)N(\eta)}\Lambda_4 \int_0^t \|V_{n-1}(\aleph) - V_{n-2}(\aleph)\|d\aleph, \\
\|\Delta_{5n}(t)\| = \|F_n(t) - F_{n-1}(t)\| &\leq \frac{2(1-\eta)}{(2-\eta)N(\eta)}\Lambda_5\|F_{n-1}(t) - F_{n-2}(t)\| \\
&+ \frac{2\eta}{(2-\eta)N(\eta)}\Lambda_5 \int_0^t \|F_{n-1}(\aleph) - F_{n-2}(\aleph)\|d\aleph, \\
\|\Delta_{6n}(t)\| = \|Q_n(t) - Q_{n-1}(t)\| &\leq \frac{2(1-\eta)}{(2-\eta)N(\eta)}\Lambda_6\|Q_{n-1}(t) - Q_{n-2}(t)\| \\
&+ \frac{2\eta}{(2-\eta)N(\eta)}\Lambda_6 \int_0^t \|Q_{n-1}(\aleph) - Q_{n-2}(\aleph)\|d\aleph,
\end{aligned} \tag{4.7}$$

it validates the outcome.

Theorem 1. *Prove that the system* (2.1) *admits solution.*

Proof. We have arrived at the following using the equation (4.7) and in accordance with the recursive formula:

$$\begin{aligned}
\|\Delta_1^n(t)\| &\leq \|U(0)\| + \left[\left(\frac{2(1-\eta)}{(2-\eta)N(\eta)}\Lambda_1\right) + \left(\frac{2\eta}{(2-\eta)N(\eta)}\Lambda_1 t\right)\right]^n, \\
\|\Delta_2^n(t)\| &\leq \|E(0)\| + \left[\left(\frac{2(1-\eta)}{(2-\eta)N(\eta)}\Lambda_2\right) + \left(\frac{2\eta}{(2-\eta)N(\eta)}\Lambda_2 t\right)\right]^n, \\
\|\Delta_3^n(t)\| &\leq \|I(0)\| + \left[\left(\frac{2(1-\eta)}{(2-\eta)N(\eta)}\Lambda_3\right) + \left(\frac{2\eta}{(2-\eta)N(\eta)}\Lambda_3 t\right)\right]^n, \\
\|\Delta_4^n(t)\| &\leq \|V(0)\| + \left[\left(\frac{2(1-\eta)}{(2-\eta)N(\eta)}\Lambda_4\right) + \left(\frac{2\eta}{(2-\eta)N(\eta)}\Lambda_4 t\right)\right]^n, \\
\|\Delta_5^n(t)\| &\leq \|F(0)\| + \left[\left(\frac{2(1-\eta)}{(2-\eta)N(\eta)}\Lambda_5\right) + \left(\frac{2\eta}{(2-\eta)N(\eta)}\Lambda_5 t\right)\right]^n,
\end{aligned}$$

$$\|\Delta_6^n(t)\| \leq \|Q(0)\| + \left[\left(\frac{2(1-\eta)}{(2-\eta)N(\eta)}\Lambda_6\right) + \left(\frac{2\eta}{(2-\eta)N(\eta)}\Lambda_6 t\right)\right]^n. \tag{4.8}$$

The recursive inequalities (4.8) can be written an

$$\|\Delta_i^n(t)\| \leq \|X_i(0)\| + \left[\frac{(2-\eta)N(\eta)}{2(1-\eta)}\Lambda_i + \frac{(2-\eta)N(\eta)}{2\eta}\,t\,\Lambda_i\right]^n, \quad i = 1, \dots, 6.$$

The recursive bound can be written as

$$\|\Delta_i^n(t)\| \leq \|X_i(0)\| + C_i^n, \qquad i = 1, 2, \dots, 6,$$

where C_i is a constant independent of the iteration, given by

$$C_i = \frac{(2-\eta)N(\eta)}{2(1-\eta)}\Lambda_i + \frac{(2-\eta)N(\eta)}{2\eta}\,t\,\Lambda_i, \qquad i = 1, 2, \dots, 6,$$

where $X_1 = U,\ X_2 = E,\ X_3 = I,\ X_4 = V,\ X_5 = F,\ X_6 = Q.$
Therefore, (4.8) exists. Additionally, we demonstrate that the functions in (4.8) constitute a solution system for (2.1) under the assumption that

$$\begin{aligned}
U(t) &= U_n(t) - \Upsilon_{1n}(t),\\
E(t) &= E_n(t) - \Upsilon_{2n}(t),\\
I(t) &= I_n(t) - \Upsilon_{3n}(t),\\
V(t) &= V_n(t) - \Upsilon_{4n}(t),\\
F(t) &= F_n(t) - \Upsilon_{5n}(t),\\
Q(t) &= Q_n(t) - \Upsilon_{6n}(t),
\end{aligned} \tag{4.9}$$

where $\Upsilon_{1n}(t), \Upsilon_{2n}(t), \Upsilon_{3n}(t), \Upsilon_{4n}(t), \Upsilon_{5n}(t)$ and $\Upsilon_{6n}(t)$ are the leftover terms of the solution. Therefore, we derive

$$\begin{aligned}
U(t) - U_n(t) &= \frac{2(1-\eta)}{(2-\eta)N(\eta)}\psi_1\big(t, U(t) - U_n(t)\big) + \frac{2\eta}{(2-\eta)N(\eta)}\int_0^t \psi_1\big(\aleph, U(\aleph) - U_n(\aleph)\big)d\aleph\\
&= \frac{2(1-\eta)}{(2-\eta)N(\eta)}\psi_1\big(t, -\Upsilon_{1n}(t)\big) + \frac{2\eta}{(2-\eta)N(\eta)}\int_0^t \psi_1\big(\aleph, -\Upsilon_{1n}(\aleph)\big)d\aleph\\
E(t) - E_n(t) &= \frac{2(1-\eta)}{(2-\eta)N(\eta)}\psi_2\big(t, E(t) - E_n(t)\big) + \frac{2\eta}{(2-\eta)N(\eta)}\int_0^t \psi_2\big(\aleph, E(\aleph) - E_n(\aleph)\big)d\aleph\\
&= \frac{2(1-\eta)}{(2-\eta)N(\eta)}\psi_2\big(t, -\Upsilon_{2n}(t)\big) + \frac{2\eta}{(2-\eta)N(\eta)}\int_0^t \psi_2\big(\aleph, -\Upsilon_{2n}(\aleph)\big)d\aleph\\
I(t) - I_n(t) &= \frac{2(1-\eta)}{(2-\eta)N(\eta)}\psi_3\big(t, I(t) - I_n(t)\big) + \frac{2\eta}{(2-\eta)N(\eta)}\int_0^t \psi_3\big(\aleph, I(\aleph) - I_n(\aleph)\big)d\aleph\\
&= \frac{2(1-\eta)}{(2-\eta)N(\eta)}\psi_3\big(t, -\Upsilon_{3n}(t)\big) + \frac{2\eta}{(2-\eta)N(\eta)}\int_0^t \psi_3\big(\aleph, -\Upsilon_{3n}(\aleph)\big)d\aleph\\
V(t) - V_n(t) &= \frac{2(1-\eta)}{(2-\eta)N(\eta)}\psi_4\big(t, V(t) - V_n(t)\big) + \frac{2\eta}{(2-\eta)N(\eta)}\int_0^t \psi_4\big(\aleph, V(\aleph) - V_n(\aleph)\big)d\aleph
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2(1-\eta)}{(2-\eta)N(\eta)}\psi_4\big(t,-\Upsilon_{4n}(t)\big) + \frac{2\eta}{(2-\eta)N(\eta)}\int_0^t \psi_4\big(\aleph,-\Upsilon_{4n}(\aleph)\big)d\aleph \\
F(t)-F_n(t) &= \frac{2(1-\eta)}{(2-\eta)N(\eta)}\psi_5\big(t,F(t)-F_n(t)\big) + \frac{2\eta}{(2-\eta)N(\eta)}\int_0^t \psi_5\big(\aleph,F(\aleph)-F_n(\aleph)\big)d\aleph \\
&= \frac{2(1-\eta)}{(2-\eta)N(\eta)}\psi_5\big(t,-\Upsilon_{5n}(t)\big) + \frac{2\eta}{(2-\eta)N(\eta)}\int_0^t \psi_5\big(\aleph,-\Upsilon_{5n}(\aleph)\big)d\aleph \\
Q(t)-Q_n(t) &= \frac{2(1-\eta)}{(2-\eta)N(\eta)}\psi_6\big(t,Q(t)-Q_n(t)\big) + \frac{2\eta}{(2-\eta)N(\eta)}\int_0^t \psi_6\big(\aleph,Q(\aleph)-Q_n(\aleph)\big)d\aleph \\
&= \frac{2(1-\eta)}{(2-\eta)N(\eta)}\psi_6\big(t,-\Upsilon_{6n}(t)\big) + \frac{2\eta}{(2-\eta)N(\eta)}\int_0^t \psi_6\big(\aleph,-\Upsilon_{6n}(\aleph)\big)d\aleph
\end{aligned} \tag{4.10}
$$

By taking the norm on both sides, we get

$$
\Big\|U(t)-U_n(0)-\frac{2(1-\eta)}{(2-\eta)N(\eta)}\psi_1\big(t,U(t)-U_n(t)\big)-\frac{2\eta}{(2-\eta)N(\eta)}\int_0^t \psi_1\big(\aleph,U(\aleph)-U_n(\aleph)\big)d\aleph\Big\|
$$

$$
\leq \|\Upsilon_{1n}(t)\|\left\{1+\left(\frac{2(1-\eta)}{(2-\eta)N(\eta)}\Lambda_1\right)+\left(\frac{2\eta}{(2-\eta)N(\eta)}\Lambda_1 t\right)\right\} \tag{4.11}
$$

Now take limit $n\to\infty$ in an equation (4.11), we get $\|\Upsilon_{1n}(t)\|\to 0$. Hence, we get

$$
U(t) = U(0) - \frac{2(1-\eta)}{(2-\eta)N(\eta)}\psi_1\big(t,U(t)\big) - \frac{2\eta}{(2-\eta)N(\eta)}\int_0^t \psi_1\big(\aleph,U(\aleph)\big)d\aleph \tag{4.12}
$$

Similarly, as limit $n\to\infty$, we get $\|\Upsilon_{2n}(t)\|\to 0$, $\|\Upsilon_{3n}(t)\|\to 0$, $\|\Upsilon_{4n}(t)\|\to 0$, $\|\Upsilon_{5n}(t)\|\to 0$, $\|\Upsilon_{6n}(t)\|\to 0$. Consequently, like (4.12) there exists solutions of (2.1)

Theorem 2. *Show that the system* (2.1) *has a only one solution.*

Proof. Let there is another solution of the system (2.1), say $U^*(t)$, $E^*(t)$, $I^*(t)$, $V^*(t)$, $F^*(t)$, and $Q^*(t)$, then we get

$$
\begin{aligned}
U(t)-U^*(t) &= \frac{2(1-\eta)}{(2-\eta)N(\eta)}\Big[\psi_1\big(t,U(t)\big)-\psi_1\big(t,U^*(t)\big)\Big] \\
&\quad + \frac{2\eta}{(2-\eta)N(\eta)}\int_0^t \Big[\psi_1\big(\aleph,U(\aleph)\big)-\psi_1\big(\aleph,U^*(\aleph)\big)\Big]d\aleph, \\
E(t)-E^*(t) &= \frac{2(1-\eta)}{(2-\eta)N(\eta)}\Big[\psi_2\big(t,E(t)\big)-\psi_2\big(t,E^*(t)\big)\Big] \\
&\quad + \frac{2\eta}{(2-\eta)N(\eta)}\int_0^t \Big[\psi_2\big(\aleph,E(\aleph)\big)-\psi_2\big(\aleph,E^*(\aleph)\big)\Big]d\aleph, \\
I(t)-I^*(t) &= \frac{2(1-\eta)}{(2-\eta)N(\eta)}\Big[\psi_3\big(t,I(t)\big)-\psi_3\big(t,I^*(t)\big)\Big] \\
&\quad + \frac{2\eta}{(2-\eta)N(\eta)}\int_0^t \Big[\psi_3\big(\aleph,I(\aleph)\big)-\psi_3\big(\aleph,I^*(\aleph)\big)\Big]d\aleph, \\
V(t)-V^*(t) &= \frac{2(1-\eta)}{(2-\eta)N(\eta)}\Big[\psi_4\big(t,V(t)\big)-\psi_4\big(t,V^*(t)\big)\Big]
\end{aligned}
$$

$$
\begin{aligned}
&\quad + \frac{2\eta}{(2-\eta)N(\eta)} \int_0^t \Big[\psi_4\big(\aleph, V(\aleph)\big) - \psi_4\big(\aleph, V^*(\aleph)\big)\Big] d\aleph, \\
F(t) - F^*(t) &= \frac{2(1-\eta)}{(2-\eta)N(\eta)} \Big[\psi_5\big(t, F(t)\big) - \psi_5\big(t, F^*(t)\big)\Big] \\
&\quad + \frac{2\eta}{(2-\eta)N(\eta)} \int_0^t \Big[\psi_5\big(\aleph, F(\aleph)\big) - \psi_5\big(\aleph, F^*(\aleph)\big)\Big] d\aleph, \\
Q(t) - Q^*(t) &= \frac{2(1-\eta)}{(2-\eta)N(\eta)} \Big[\psi_6\big(t, Q(t)\big) - \psi_6\big(t, Q^*(t)\big)\Big] \\
&\quad + \frac{2\eta}{(2-\eta)N(\eta)} \int_0^t \Big[\psi_6\big(\aleph, Q(\aleph)\big) - \psi_6\big(\aleph, Q^*(\aleph)\big)\Big] d\aleph,
\end{aligned} \tag{4.13}
$$

Again by using norm on (4.13), we get

$$
\begin{aligned}
\|U(t) - U^*(t)\| &= \frac{2(1-\eta)}{(2-\eta)N(\eta)} \Big\|\psi_1\big(t, U(t)\big) - \psi_1\big(t, U^*(t)\big)\Big\| \\
&\quad + \frac{2\eta}{(2-\eta)N(\eta)} \int_0^t \Big\|\psi_1\big(\aleph, U(\aleph)\big) - \psi_1\big(\aleph, U^*(\aleph)\big)\Big\| d\aleph, \\
\|E(t) - E^*(t)\| &= \frac{2(1-\eta)}{(2-\eta)N(\eta)} \Big\|\psi_2\big(t, E(t)\big) - \psi_2\big(t, E^*(t)\big)\Big\| \\
&\quad + \frac{2\eta}{(2-\eta)N(\eta)} \int_0^t \Big\|\psi_2\big(\aleph, E(\aleph)\big) - \psi_2\big(\aleph, E^*(\aleph)\big)\Big\| d\aleph, \\
\|I(t) - I^*(t)\| &= \frac{2(1-\eta)}{(2-\eta)N(\eta)} \Big\|\psi_3\big(t, I(t)\big) - \psi_3\big(t, I^*(t)\big)\Big\| \\
&\quad + \frac{2\eta}{(2-\eta)N(\eta)} \int_0^t \Big\|\psi_3\big(\aleph, I(\aleph)\big) - \psi_3\big(\aleph, I^*(\aleph)\big)\Big\| d\aleph, \\
\|V(t) - V^*(t)\| &= \frac{2(1-\eta)}{(2-\eta)N(\eta)} \Big\|\psi_4\big(t, V(t)\big) - \psi_4\big(t, V^*(t)\big)\Big\| \\
&\quad + \frac{2\eta}{(2-\eta)N(\eta)} \int_0^t \Big\|(\psi_4(\aleph, V(\aleph))) - \psi_4\big(\aleph, V^*(\aleph)\big)\Big] d\aleph, \\
\|F(t) - F^*(t)\| &= \frac{2(1-\eta)}{(2-\eta)N(\eta)} \Big\|\psi_5\big(t, F(t)\big) - \psi_5\big(t, F^*(t)\big)\Big\| \\
&\quad + \frac{2\eta}{(2-\eta)N(\eta)} \int_0^t \Big\|\psi_5\big(\aleph, F(\aleph)\big) - \psi_5\big(\aleph, F^*(\aleph)\big)\Big\| d\aleph, \\
\|Q(t) - Q^*(t)\| &= \frac{2(1-\eta)}{(2-\eta)N(\eta)} \Big\|\psi_6\big(t, Q(t)\big) - \psi_6\big(t, Q^*(t)\big)\Big\| \\
&\quad + \frac{2\eta}{(2-\eta)N(\eta)} \int_0^t \Big\|\psi_6\big(\aleph, Q(\aleph)\big) - \psi_6\big(\aleph, Q^*(\aleph)\big)\Big\| d\aleph.
\end{aligned} \tag{4.14}
$$

Based on Theorems 4.1 and 4.2, the results are

$$
\begin{aligned}
\|U(t) - U^*(t)\| &= \Lambda_1 \frac{2(1-\eta)}{(2-\eta)N(\eta)} \|U(t) - U^*(t)\| \\
&\quad + \Lambda_1 \frac{2\eta}{(2-\eta)N(\eta)} t \Big\|U(t) - U^*(t)\Big\|,
\end{aligned}
$$

$$\begin{aligned}
\|E(t) - E^*(t)\| &= \Lambda_2 \frac{2(1-\eta)}{(2-\eta)N(\eta)} \|E(t) - E^*(t)\| \\
&\quad + \Lambda_2 \frac{2\eta}{(2-\eta)N(\eta)} t \Big\| E(t) - E^*(t) \Big\|, \\
\|I(t) - I^*(t)\| &= \Lambda_3 \frac{2(1-\eta)}{(2-\eta)N(\eta)} \|I(t) - I^*(t)\| \\
&\quad + \Lambda_3 \frac{2\eta}{(2-\eta)N(\eta)} t \Big\| I(t) - I^*(t) \Big\|, \\
\|V(t) - V^*(t)\| &= \Lambda_4 \frac{2(1-\eta)}{(2-\eta)N(\eta)} \|V(t) - V^*(t)\| \\
&\quad + \Lambda_4 \frac{2\eta}{(2-\eta)N(\eta)} t \Big\| V(t) - V^*(t) \Big\|, \\
\|F(t) - F^*(t)\| &= \Lambda_5 \frac{2(1-\eta)}{(2-\eta)N(\eta)} \|F(t) - F^*(t)\| \\
&\quad + \Lambda_5 \frac{2\eta}{(2-\eta)N(\eta)} t \Big\| F(t) - F^*(t) \Big\|, \\
\|Q(t) - Q^*(t)\| &= \Lambda_6 \frac{2(1-\eta)}{(2-\eta)N(\eta)} \|Q(t) - Q^*(t)\| \\
&\quad + \Lambda_6 \frac{2\eta}{(2-\eta)N(\eta)} t \Big\| Q(t) - Q^*(t) \Big\|.
\end{aligned} \tag{4.15}$$

The following is how the computed functions in (4.11) fulfill the non-equalities:

$$\begin{aligned}
\|U(t) - U^*(t)\| \left\{ 1 - \frac{2\Lambda_1}{(2-\eta)N(\eta)} \big(1 - \eta - \eta t\big) \right\} &\leq 0, \\
\|E(t) - E^*(t)\| \left\{ 1 - \frac{2\Lambda_2}{(2-\eta)N(\eta)} \big(1 - \eta - \eta t\big) \right\} &\leq 0, \\
\|I(t) - I^*(t)\| \left\{ 1 - \frac{2\Lambda_3}{(2-\eta)N(\eta)} \big(1 - \eta - \eta t\big) \right\} &\leq 0, \\
\|V(t) - V^*(t)\| \left\{ 1 - \frac{2\Lambda_4}{(2-\eta)N(\eta)} \big(1 - \eta - \eta t\big) \right\} &\leq 0, \\
\|F(t) - F^*(t)\| \left\{ 1 - \frac{2\Lambda_5}{(2-\eta)N(\eta)} \big(1 - \eta - \eta t\big) \right\} &\leq 0, \\
\|Q(t) - Q^*(t)\| \left\{ 1 - \frac{2\Lambda_6}{(2-\eta)N(\eta)} \big(1 - \eta - \eta t\big) \right\} &\leq 0.
\end{aligned} \tag{4.16}$$

From the final equation, we establish that

$$U(t) = U^*(t),\ E(t) = E^*(t),\ I(t) = I^*(t),\ V(t) = V^*(t),\ F(t) = F^*(t),\ Q(t) = Q^*(t). \tag{4.17}$$

5. Stability

This section examines the stability conditions of approximate solutions and explores the use of the iterative Laplace transform method in the fractional malaria fever model.

5.1. Iterative Laplace Transform Method

Examine the Malaria infection framework (2.1) with starting values (2.2). Utilizing the Laplace method on both ends of equation (2.1), we get

$$
\begin{aligned}
\frac{\mathfrak{s}\mathcal{L}(U(t)) - U(0)}{\mathfrak{s}+\eta(1-\mathfrak{s})} &= \mathcal{L}\big(q\pi + (1-b)\xi I + \phi V - (\sigma+\psi+\varrho)U\big)\\
\frac{\mathfrak{s}\mathcal{L}(E(t)) - E(0)}{p+\eta(1-\mathfrak{s})} &= \mathcal{L}\big(\sigma U - (\chi+\varrho)E\big)\\
\frac{\mathfrak{s}\mathcal{L}(I(t)) - I(0)}{\mathfrak{s}+\eta(1-\mathfrak{s})} &= \mathcal{L}\big(\chi E - (\xi+\varrho+\partial)I\big)\\
\frac{\mathfrak{s}\mathcal{L}(V(t)) - V(0)}{\mathfrak{s}+\eta(1-\mathfrak{s})} &= \mathcal{L}\big((1-q)\pi + \mu Q + \psi U + b\xi I - (\theta+\phi+\varrho)V\big)\\
\frac{\mathfrak{s}\mathcal{L}(F(t)) - F(0)}{\mathfrak{s}+\eta(1-\mathfrak{s})} &= \mathcal{L}\big(\theta V - (\varepsilon+\varrho)F\big)\\
\frac{\mathfrak{s}\mathcal{L}(Q(t)) - Q(0)}{\mathfrak{s}+\eta(1-\mathfrak{s})} &= \mathcal{L}\big(\epsilon F - (\mu+\varrho+\kappa)Q\big)
\end{aligned}
\tag{5.1}
$$

After reordering, we get

$$
\begin{aligned}
\mathcal{L}(U(t)) &= \frac{U(0))}{\mathfrak{s}} + \left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(q\pi + (1-b)\xi I + \phi V - (\sigma+\psi+\varrho)U\big),\\
\mathcal{L}(E(t)) &= \frac{E(0))}{\mathfrak{s}} + \left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(\sigma U - (\chi+\varrho)E\big),\\
\mathcal{L}(I(t)) &= \frac{I(0))}{\mathfrak{s}} + \left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(\chi E - (\xi+\varrho+\partial)I\big),\\
\mathcal{L}(V(t)) &= \frac{V(0))}{\mathfrak{s}} + \left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big((1-q)\pi + \mu Q + \psi U + b\xi I - (\theta+\phi+\varrho)V\big),\\
\mathcal{L}(F(t)) &= \frac{F(0))}{\mathfrak{s}} + \left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(\theta V - (\varepsilon+\varrho)F\big),\\
\mathcal{L}(Q(t)) &= \frac{Q(0))}{\mathfrak{s}} + \left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(\epsilon F - (\mu+\varrho+\kappa)Q\big).
\end{aligned}
\tag{5.2}
$$

In addition, the inverse Laplace transform applied to equation (5.2) produces

$$
\begin{aligned}
U(t) &= U(0) + \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(q\pi + (1-b)\xi I + \phi V - (\sigma+\psi+\varrho)U\big)\right],\\
E(t) &= E(0) + \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(\sigma U - (\chi+\varrho)E\big)\right],\\
I(t) &= I(0) + \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(\chi E - (\xi+\varrho+\partial)I\big)\right],\\
V(t) &= V(0) + \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big((1-q)\pi + \mu Q + \psi U + b\xi I - (\theta+\phi+\varrho)V\big)\right],
\end{aligned}
$$

$$
\begin{aligned}
F(t) &= F(0) + \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(\theta V - (\varepsilon+\varrho)F\big)\right],\\
Q(t) &= Q(0) + \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(\epsilon F - (\mu+\varrho+\kappa)Q\big)\right].
\end{aligned} \tag{5.3}
$$

The method's infinite series solutions are provided as,

$$
U = \sum_{n=0}^{\infty} U_n, \quad E = \sum_{n=0}^{\infty} E_n, \quad I = \sum_{n=0}^{\infty} I_n, \quad V = \sum_{n=0}^{\infty} V_n, \quad F = \sum_{n=0}^{\infty} F_n, Q = \sum_{n=0}^{\infty} Q_n \tag{5.4}
$$

The recursive formula that follows is then obtained by applying beginning conditions.

$$
\begin{aligned}
U_{n+1}(t) &= U(0) + \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(q\pi + (1-b)\xi I_n + \phi V_n - (\sigma+\psi+\varrho)U_n\big)\right],\\
E_{n+1}(t) &= E(0) + \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(\sigma U_n - (\chi+\varrho)E_n\big)\right],\\
I_{n+1}(t) &= I(0) + \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(\chi E_n - (\xi+\varrho+\partial)I_n\big)\right],\\
V_{n+1}(t) &= V(0) + \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big((1-q)\pi + \mu Q_n + \psi U_n + b\xi I_n - (\theta+\phi+\varrho)V_n\big)\right],\\
F_{n+1}(t) &= F(0) + \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(\theta V_n - (\varepsilon+\varrho)F_n\big)\right],\\
Q_{n+1}(t) &= Q(0) + \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(\epsilon F_n - (\mu+\varrho+\kappa)Q_n\big)\right].
\end{aligned} \tag{5.5}
$$

5.2. Analysis of iteration method

As a Banach space, consider $(\mathfrak{B}, \|\cdot\|)$. Define F as the self-map of B_1, whose the repeating process is indicated by $z_{n+1} = g(J, z_n)$. Let $J(F)$ represent the fixed-point set on F. Additionally, F must have At least one item for z_n to approach a point $i \in J(F)$. After defining $k_n = \|y_{n+1} - g(F, y_n)\|$, let $\{y_n \in B_1\}$. The iteration technique $z_{n+1} = g(F, z_n)$ is referred to as F-stable if $\lim_{n\to\infty} k^n = 0$ implies that $\lim_{n\to\infty} y^n = i$. In contrast, we declare that there is an upper constraint on the sequence $\{y_n\}$. If all these requirements are met for $z_{n+1} = Fz_n$, then this iteration-also referred to as Picard's iteration-is $F-$ stable.

Theorem 3. *Suppose* $(\mathfrak{B}, \|\cdot\|)$ *as a Banach space and let* $\mathcal{F}$ *be a self-map on* $\mathfrak{B}$ *fulfilling*

$$
\|\mathcal{F}_w - \mathcal{F}_z\| \leq M\|w - z\| + m\|w - z\|.
$$

For each $w, z \in \mathfrak{B}$ *with* $0 \leq M,\ 0 \leq m < 1$. *Assume* $\mathcal{F}$ *is Picard stable. Suppose the equations* (5.5)*, which are connected to* (2.1)*.*

$$
U_{n+1}(t) = U(0) + \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(q\pi + (1-b)\xi I_n + \phi V_n - (\sigma+\psi+\varrho)U_n\big)\right]
$$

$$E_{n+1}(t) = E(0) + \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(\sigma U_n - (\chi+\varrho)E_n\big)\right]$$

$$I_{n+1}(t) = I(0) + \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(\chi E_n - (\xi+\varrho+\partial)I_n\big)\right]$$

$$V_{n+1}(t) = V(0) + \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big((1-q)\pi + \mu Q_n + \psi U_n + b\xi I_n - (\theta+\phi+\varrho)V_n\big)\right]$$

$$F_{n+1}(t) = F(0) + \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(\theta V_n - (\varepsilon+\varrho)F_n\big)\right]$$

$$Q_{n+1}(t) = Q(0) + \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(\epsilon F_n - (\mu+\varrho+\kappa)Q_n\big)\right]$$

where $\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}$ is a fractional Lagrange multiplier

Theorem 4. *Let $\mathcal{F}$ be a mapping on itself defined as*

$$\mathcal{F}(U_n(t)) = U_{n+1}(t) = U(0) + \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(q\pi + (1-b)\xi I_n + \phi V_n - (\sigma+\psi+\varrho)U_n\big)\right]$$

$$\mathcal{F}(E_n(t)) = E_{n+1}(t) = E(0) + \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(\sigma U_n - (\chi+\varrho)E_n\big)\right]$$

$$\mathcal{F}(I_n(t)) = I_{n+1}(t) = I(0) + \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)L\big(\chi E_n - (\xi+\varrho+\partial)I_n\big)\right]$$

$$\mathcal{F}(V_n(t)) = V_{n+1}(t) = V(0) + \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big((1-q)\pi + \mu Q_n + \psi U_n + b\xi I_n - (\theta+\phi+\varrho)V_n\big)\right]$$

$$\mathcal{F}(F_n(t)) = F_{n+1}(t) = F(0) + \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(\theta V_n - (\varepsilon+\varrho)F_n\big)\right]$$

$$\mathcal{F}(Q_n(t)) = Q_{n+1}(t) = Q(0) + \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(\epsilon F_n - (\mu+\varrho+\kappa)Q_n\big)\right]$$

is F-stable in $L^1(a,b)$ if

$$\begin{cases} \big((1-b)\xi h_1(\mathfrak{s}) + \phi h_2(\mathfrak{s}) + (\sigma+\psi+\varrho)h_3(\mathfrak{s})\big) < 1 \\ \big(\sigma f_1(\mathfrak{s}) - (\chi+\varrho)f_2(\mathfrak{s})\big) < 1 \\ \big(\chi g_1(\mathfrak{s}) - (\xi+\varrho+\partial)g_2(\mathfrak{s})\big) < 1 \\ \big(q\pi + (1-b)\xi z_1(\mathfrak{s}) + \phi z_1(\mathfrak{s}) - (\sigma+\psi+\varrho)z_1(\mathfrak{s})\big) < 1 \\ \big(\theta x_1(\mathfrak{s}) - (\varepsilon+\varrho)x_2(\mathfrak{s})\big) < 1 \\ \big(\epsilon y_1(\mathfrak{s}) - (\mu+\varrho+\kappa)y_2(\mathfrak{s})\big) < 1 \end{cases}$$

Proof. Here, we shall demonstrate that F has a fixed point. So for any (m,n) in R

where $R = \mathbb{N} \times \mathbb{N}$, we evaluate the followings.

$$\mathcal{F}(U_n(t)) - \mathcal{F}(U_m(t)) = \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(q\pi + (1-b)\xi I_n + \phi V_n - (\sigma+\psi+\varrho)U_n\big)\right] - \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(q\pi + (1-b)\xi I_m + \phi V_m - (\sigma+\psi+\varrho)U_m\big)\right]$$

Taking norm

$$\|\mathcal{F}(U_n(t)) - \mathcal{F}(U_m(t))\| = \Bigg\|\mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(q\pi + (1-b)\xi I_n + \phi V_n - (\sigma+\psi+\varrho)U_n\big)\right] - \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(q\pi + (1-b)\xi I_m + \phi V_m - (\sigma+\psi+\varrho)U_m\big)\right]\Bigg\|$$

$$\|\mathcal{F}(U_n(t)) - \mathcal{F}(U_m(t))\| = \Bigg\|\mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(q\pi + (1-b)\xi I_n + \phi V_n - (\sigma+\psi+\varrho)U_n\big)\right] - \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\big(q\pi + (1-b)\xi I_m + \phi V_m - (\sigma+\psi+\varrho)U_m\big)\right]\Bigg\|$$

$$\|\mathcal{F}(U_n(t)) - \mathcal{F}(U_m(t))\| = \Bigg\|\mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\Bigg(\mathcal{L}\big((1-b)\xi I_n + \phi V_n - (\sigma+\psi+\varrho)U_n\big)\right] - \mathcal{L}\big((1-b)\xi I_m + \phi V_m - (\sigma+\psi+\varrho)U_m\big)\Bigg)\Bigg]\Bigg\|$$

$$\|\mathcal{F}(U_n(t)) - \mathcal{F}(U_m(t))\| \le \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\Big[\big\|((1-b)\xi(I_n - I_m))\big\| \tag{5.6}$$

$$+ \big\|(\phi(V_n - V_m))\big\| + \big\| - ((\sigma+\psi+\varrho)(U_n - U_m))\big\|\Big]\right] \tag{5.7}$$

As the obtained solutions perform a comparable role, we suppose that

$$\|U_n(t) - U_m(t)\| = \|V_n(t) - V_m(t)\|,$$
$$\|U_n(t) - U_m(t)\| = \|I_n(t) - I_m(t)\|$$

Also, $U_n,\ E_n,\ I_n,\ V_n$, and F_n are convergent sequences, thus they are bounded. Consider the equation in (5.6). We have.

$$\begin{aligned}\|\mathcal{F}(U_n(t)) - \mathcal{F}(U_m(t))\| &\le \mathcal{L}^{-1}\left[\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\mathcal{L}\Big[\Big((1-b)\xi + \phi + (\sigma+\psi+\varrho)\Big)\|U_n - U_m\|\Big]\right] \\ &\le \Big((1-b)\xi h_1(\mathfrak{s}) + \phi h_2(\mathfrak{s}) + (\sigma+\psi+\varrho)h_3(\mathfrak{s})\Big)\|U_n - U_m\|\end{aligned} \tag{5.8}$$

where h_1, h_2 and h_3 are functions from $\mathcal{L}^{-1}\left[\mathcal{L}\left(\frac{\mathfrak{s}+\eta(1-\mathfrak{s})}{\mathfrak{s}}\right)\right]$
In the same manner, we can get

$$\|\mathcal{F}(U_n(t)) - \mathcal{F}(U_m(t))\| \le \Big((1-b)\xi h_1(\mathfrak{s}) + \phi h_2(\mathfrak{s}) + (\sigma+\psi+\varrho)h_3(\mathfrak{s})\Big)\|U_n - U_m\|$$

$$\begin{aligned}
\|\mathcal{F}(E_n(t)) - \mathcal{F}(E_m(t))\| &\leq \big(\sigma f_1(\mathfrak{s}) - (\chi+\varrho) f_2(\mathfrak{s})\big)\|E_n - E_m\| \\
\|\mathcal{F}(I_n(t)) - \mathcal{F}(I_m(t))\| &\leq \big(\chi g_1(\mathfrak{s}) - (\xi+\varrho+\partial) g_2(\mathfrak{s})\big)\|I_n - I_m\| \\
\|\mathcal{F}(V_n(t)) - \mathcal{F}(V_m(t))\| &\leq \big(q\pi + (1-b)\xi z_1(\mathfrak{s}) + \phi z_1(\mathfrak{s}) - (\sigma+\psi+\varrho) z_1(\mathfrak{s})\big)\|V_n - V_m\| \\
\|\mathcal{F}(F_n(t)) - \mathcal{F}(F_m(t))\| &\leq \big(\theta x_1(\mathfrak{s}) - (\varepsilon+\varrho) x_2(\mathfrak{s})\big)\|F_n - F_m\| \\
\|\mathcal{F}(Q_n(t)) - \mathcal{F}(Q_m(t))\| &\leq \big(\epsilon y_1(\mathfrak{s}) - (\mu+\varrho+\kappa) y_2(\mathfrak{s})\big)\|Q_n - Q_m\|
\end{aligned} \tag{5.9}$$

Hence, the function $\mathcal{F}-$ has a fixed point. Now show that $\mathcal{F}$ meets every the conditions listed above. Theorem 4.1. Here (5.8) and (5.9) is valid, likewise by using
$\Phi = (0,0,0,0,0,0)$,

$$\Phi = \begin{cases}
\Big((1-b)\xi h_1(\mathfrak{s}) + \phi h_2(\mathfrak{s}) + (\sigma+\psi+\varrho) h_3(\mathfrak{s})\Big) < 1 \\
\big(\sigma f_1(\mathfrak{s}) - (\chi+\varrho) f_2(\mathfrak{s})\big) < 1 \\
\big(\chi g_1(\mathfrak{s}) - (\xi+\varrho+\partial) g_2(\mathfrak{s})\big) < 1 \\
\big(q\pi + (1-b)\xi z_1(\mathfrak{s}) + \phi z_1(\mathfrak{s}) - (\sigma+\psi+\varrho) z_1(\mathfrak{s})\big) < 1 \\
\big(\theta x_1(\mathfrak{s}) - (\varepsilon+\varrho) x_2(\mathfrak{s})\big) < 1 \\
\big(\epsilon y_1(\mathfrak{s}) - (\mu+\varrho+\kappa) y_2(\mathfrak{s})\big) < 1
\end{cases}$$

All of the requirements in Theorem 4.2 are met by $\mathcal{F}$. Thus, $\mathcal{F}$ is Picard $\mathcal{F}-$ stable.

6. Illustration

At this stage, we perform computational experiments of the Caputo-Fabrizio operator model for Malaria and Malnutrition, represented by equation (2.1), considering initial conditions $U_0 = 0,\ E_0 = 0,\ I_0 = 0,\ V_0 = 0,\ F_0 = 0,\ Q_0 = 0$, across various fractional order values $\eta \in (0,1)$. The corresponding physical factors together have values as provided.

Table 1: 1

Model parameters

Variable	Explanation	Value	Reference
π	The rate at which the host population is being recruited	1520	[31]
χ, ϵ	The rate of transition from infected to infectious	$0.05, 0.001333$	[11]
ξ, μ	The rate of translation from infectious to susceptible	$0.10333, 0.08333$	[11]
b	Proportion of people who successfully overcome malaria	0.027	Assumed
ϱ	Rate of mortality due to natural causes	$\dfrac{1}{365 \times 5}$	Assumed
∂	Malaria mortality rate	0.00183542	[32]
κ	Mortality rate caused by malaria and malnutrition	9.4	[32]
ϕ	Transition rate from malnourished susceptible individuals to properly nourished susceptible individuals	0.00000986	[33]
q	Fraction of new entries as properly nourished susceptible	$\frac{2}{3}$	

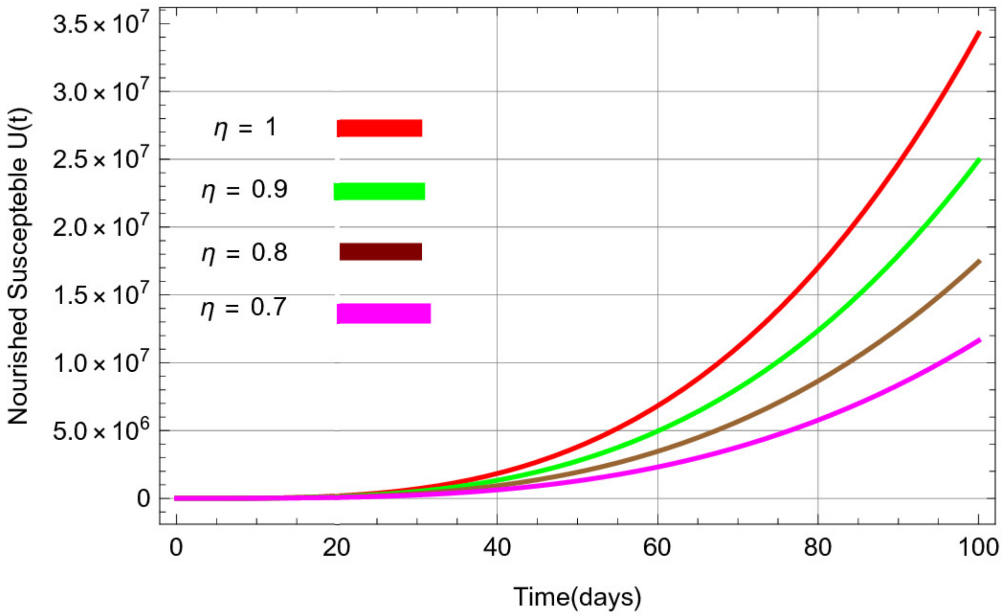


Figure 1: Near estimate of nourished susceptible individuals

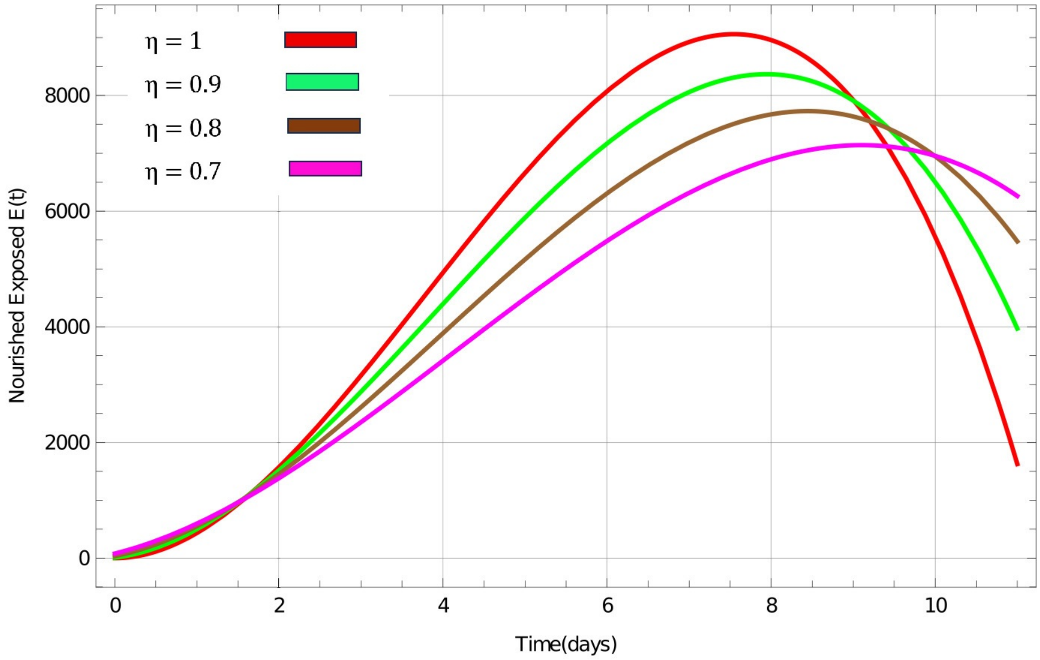


Figure 2: Near estimate of carrier exposed mosquitoes individuals

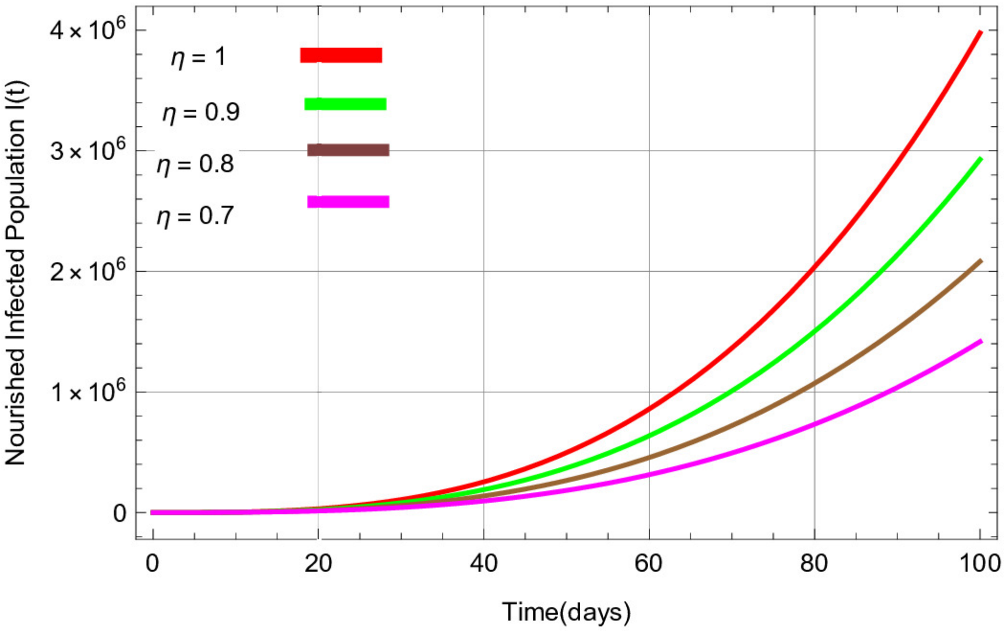


Figure 3: Near estimate of infectious mosquitoes individuals

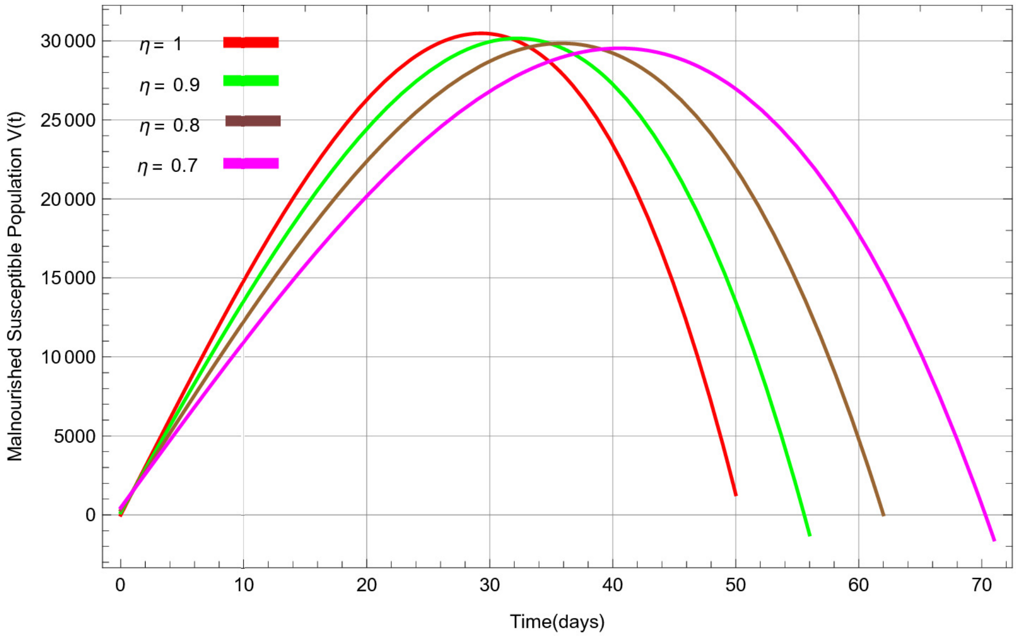


Figure 4: Near estimate of malnourished susceptible individuals

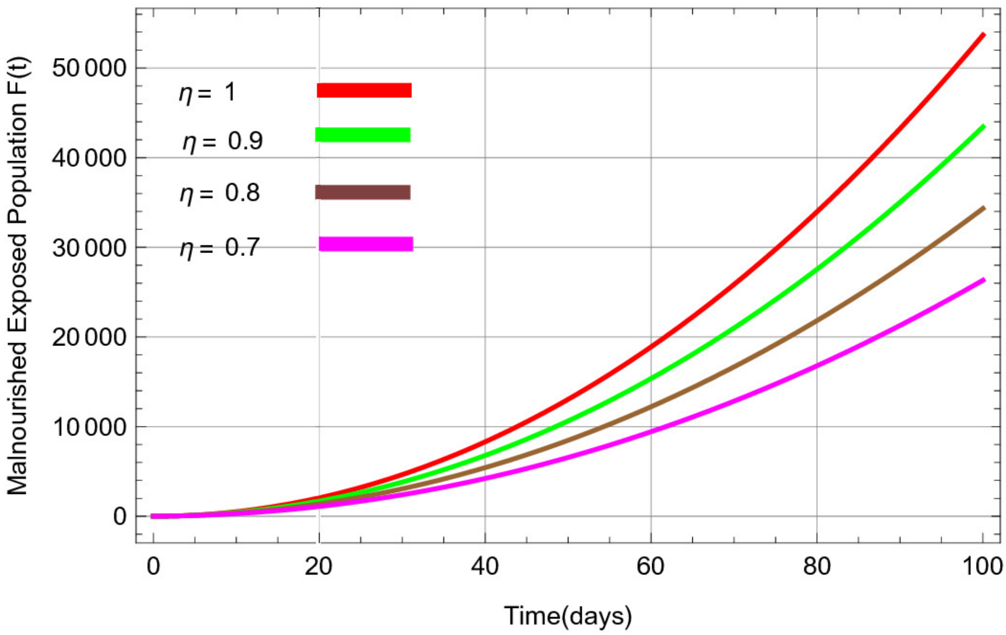


Figure 5: Near estimate of malnourished exposed individuals

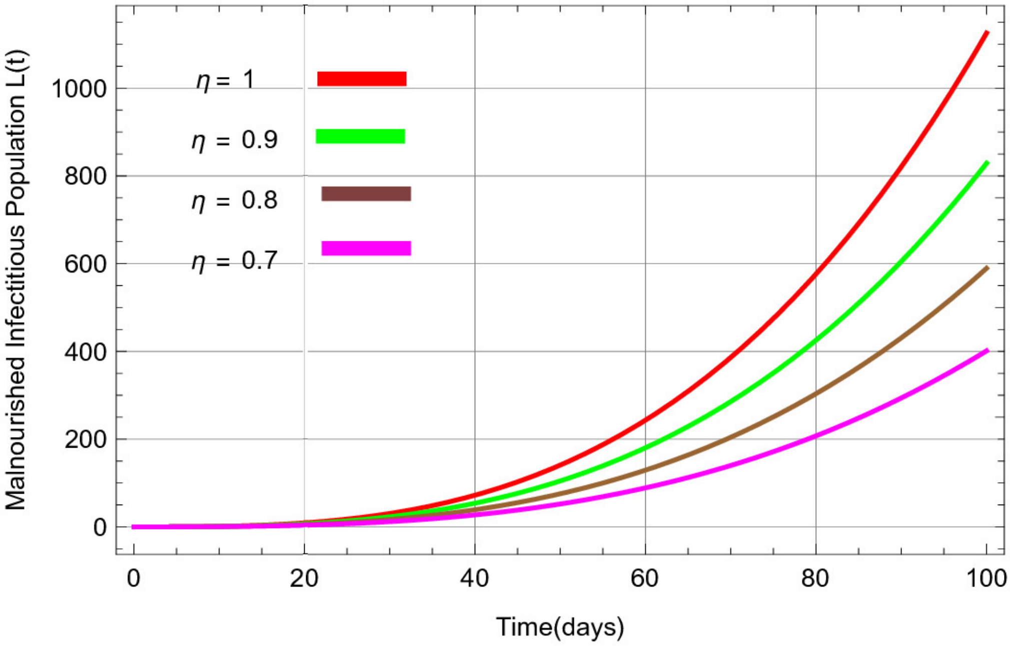


Figure 6: Near estimate of malnourished Infected individuals

7. Numerical Results and Discussion

Diagrams 1 through 6 depict the results of computational experiments illustrating nourished susceptible individuals $U(t)$, nourished exposed $E(t)$, nourished contagious persons $I(t)$, malnourished susceptible persons $V(t)$, malnourished exposed $F(t)$,and malnourished infectious individuals $Q(t)$ obtained for various values of $(\eta = 1, 0.9, 0.8, 0.7)$ utilizing the Iterative Laplace Transform Method(ILTM) applied to a fractional Malaria and Malnutrition model. These simulations demonstrate that ILTM accurately predicts the nature of these parameters within the specified area. Moreover, the simulations reveal that variations in the parameter values significantly impact the model dynamics. Specifically, fractional orders exhibit minimal influence on the spread patterns of the Malaria and Malnutrition model.

Given that mathematical models serve as symbolic representations of biological systems, they inherently inherit the loss of information during construction, potentially leading to imprecise predictions of model outcomes. Therefore, we opt to conduct a graphical sensitivity analysis of model parameters to delve into this issue further.

Figure 1 illustrates a sharp increase in nourished susceptible individuals $U(t)$ within the initial days across various values of the order η. In Figure 2, the graph depicting nourished exposed individuals $E(t)$ demonstrates a rapid increase in the early days followed by a decrease. Rapid growth of nourished infectious individuals $I(t)$ is observed in Figure 3, particularly with non-integer values of η. Conversely, Figure 4 shows a decline in the population of malnourished susceptible individuals $V(t)$ after a few days. Malnourished exposed individuals $F(t)$ in Figure 5 exhibit a rapid increase in the initial days, while malnourished infectious individuals $Q(t)$ in Figure 6 experience a slower increase with varying values of η. Notably, it is observed that computational outcomes consistently rely on the fractional-time derivative η, indicating the significant influence of specific fractional operators such as the

Caputo–Fabrizio operator in providing precise predictions with reduced noise. Furthermore, the hybrid nature of the Caputo–Fabrizio operator proves robust in capturing the complexity of the model and offering meaningful predictions. Malaria exacerbates the incidence of malnutrition. As anticipated, implementing protective strategies, including the application of mosquito repellent (periodic residual spraying) and insecticide-coated bed nets to decrease the daily count of bites from female mosquitoes will significantly reduce the number of children contracting malaria. Additionally, since malaria-infected children are vulnerable to malnutrition due to factors like imbalanced eating habits or reduced appetite, improving children's diets could be beneficial to alleviate the transmission of critical malaria episodes.

8. Conclusions

In this study, we investigated the Caputo-Fabrizio fractional-order system of malaria and malnutrition, examining the spread patterns through both direct and indirect transmission pathways of the disease through the iterative Laplace transform method. Additionally, by employing the Banach theorem, we established results concerning the presence, uniqueness, and stability of equilibrium solutions. The sequence solutions derived from this robust approach demonstrate a promising ability to mitigate the devastating impact of malaria and malnutrition over different time intervals and to combat a significant mortality factor. It is evident that the efficacy of this method can be significantly improved by streamlining processes and incorporating additional components. We utilized a randomized set of parameters in the behavior of the previously mentioned epidemic model.

Malaria and malnutrition stand out as the primary contributors to childhood mortality, particularly as repeated malaria exposure notably affects the nutritional well-being of children. The current analysis, while informative, is not comprehensive and offers potential avenues for extension. While malaria predominantly impacts children under five years old, malnutrition impacts the whole family. Looking ahead, research endeavors might explore expanded, intricate models that incorporate the dynamics of individuals aged five and above. malaria control, noting that innovative tools and climate adaptation strategies should be integrated into national programs with adequate funding, community engagement, and equitable implementation, alongside interventions such as nutrition programs and insecticide-treated nets.

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