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Performance Assessment of Logistics Ports Using Order Theory: A Case Study of Egyptian Ports

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Abstract. A crucial first step in increasing port productivity, improving service delivery and guaranteeing greater integration with neighboring sectors is the accurate and scientific assessment of port performance. In this study, a novel approach to the multi-criteria decision analysis (MCDA) paradigm is presented. Our new approach's key characteristic is that it uses partially ordered sets whose components are structured from a relational viewpoint rather than a computational one as in earlier approaches. We will also examine the current state of Egyptian ports in this study and assess them using the suggested methodology. The suggested strategy yielded outstanding computational results for analyzing 12 Egyptian ports. Furthermore, this study advances strategic knowledge on how operational characteristics and infrastructure affect port performance in developing countries. It provides a repeatable framework for decision makers seeking to prioritize investments, increase port competitiveness, and promote sustainable economic growth through increased maritime logistics efficiency by utilizing a data-driven, multi-criteria evaluation methodology.

2020 Mathematics Subject Classifications: 06A06, 06A07, 90B06

Key Words and Phrases: Port logistics, partial ordering set, evaluation, multi-criteria decision making

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1. Introduction

Ports are vital nodes in the global logistics and supply chain network, contributing significantly to economic growth by providing logistics and transportation services. Beyond their basic operational roles, ports add value through services that enhance trade efficiency and regional connectivity. Optimizing port operations and management techniques is therefore essential to boost efficiency, improve service delivery, and strengthen integration with neighboring industries—all of which advance the broader logistics sector [1].

Accurate and scientific assessment of port performance is a crucial step toward achieving these objectives. It enables ports to identify weaknesses, apply managerial and technological improvements, and strategically guide their development. Recent studies have proposed analytical models and comprehensive indicator systems to examine the relationship between port operations and regional economic development [2].

In port evaluation, traditional multi-criteria decision analysis (MCDA) techniques remain widely applied. The Analytic Hierarchy Process (AHP) is commonly used for determining criteria weights, while outranking and utility-based techniques such as TOPSIS and VIKOR are employed for ranking alternatives. Because of their interpretability and simplicity, these "AHP + ranking" combinations continue to dominate port studies and influence how researchers define criteria and assess port performance [3].

A major methodological evolution involves the use of fuzzy, interval, and other uncertainty-aware extensions of MCDA to capture ambiguous expert judgments and linguistic assessments. Methods like fuzzy AHP, fuzzy TOPSIS, and hybrid fuzzy frameworks are frequently adopted for port selection, service quality, and safety evaluations. These approaches better reflect real-world decision settings where criteria such as attractiveness and safety depend on subjective or uncertain assessments [4-6].

Sustainability and green port evaluation have become distinct and rapidly growing research areas. Scholars now employ multi-dimensional indicator systems covering economic, environmental, social, and operational aspects. MCDA provides an effective framework for balancing trade-offs between emissions, energy consumption, community impacts, and operational efficiency, supporting ports in their transition toward sustainability [7, 8].

Integrating MCDA with quantitative tools such as Data Envelopment Analysis (DEA), Principal Component Analysis (PCA), Geographic Information Systems (GIS), and simulation models has become common. These hybrid approaches enhance analytical robustness, reduce indicator redundancy, and enable spatially informed assessments of port efficiency and hinterland connectivity [9].

Recent research also applies MCDA to multi-stakeholder assessments of port service quality and risk management. By combining fuzzy AHP with Importance–Performance Analysis and other hybrid tools, studies have prioritized improvements from different stakeholder perspectives. In addition, MCDA-based frameworks are increasingly used to evaluate cybersecurity threats, occupational safety, and resilience to disruptions by integrating quantitative and qualitative risk indicators [6, 10].

Emerging studies explore integrating artificial intelligence and data-driven methods with MCDA. Examples include using machine learning to discover or weight criteria and combining agent-based simulations with entropy-based AHP. These approaches aim to automate parts of the decision-making process and scale MCDA to complex, data-rich port systems, though interpretability and stakeholder validation remain ongoing challenges [10 - 14].

Despite their usefulness, most MCDA methods depend on relative weights, which can introduce compensatory effects and uncertainty in representing qualitative knowledge. To address these limitations, this study proposes a novel MCDA approach based on partially ordered sets—called LAPOS—that structures alternatives relationally rather than computationally. The proposed method is applied to evaluate twelve Egyptian ports, providing robust computational outcomes and valuable insights into how operational and infrastructural factors influence port performance in developing countries.

The remainder of the paper follows this format: Results of earlier research on multi-criteria decision evaluation techniques are presented in Section 2. The fundamental definitions of partially ordered set theory, along with some earlier research, are presented in Section 3. Section 4 presents the proposed method, which is called LAPOS. Twelve Egyptian ports are the subject of a case study employing the suggested methodology in Section 5. Section 6 presents the results' interpretation and analysis. In Section 7, final thoughts and future directions are discussed.

2. Related Works

The use of Multi-Criteria Decision Analysis (MCDA) in marine and port studies has grown dramatically, resulting in various different and advanced research trends. This evolution reflects the industry's increasing complexity and the need for more powerful, sophisticated decision-support technologies. The connected work can be classified as follows.

2.1. Classical MCDA Foundations

MCDA techniques continue to provide key frameworks for evaluating port operations and stakeholder decision-making. Rezaei et al. [15] found that an integrated MCDA framework effectively connects port performance measurements and carrier selection decisions, providing a transparent and systematic approach to evaluating key performance indicators (KPIs) that have a direct impact on port choice. Their research found that this methodology enables port managers to identify strategic improvement opportunities in order to boost competitiveness and meet stakeholder expectations. In addition to this research, Lorencic et al. [16] established that the MCDA approach provides a comprehensive framework for evaluating cruise port performance from the perspective of the port authority, emphasizing the importance of balancing operational efficiency with stakeholder satisfaction in specialized port contexts. Their study provides useful information for port officials looking to enhance service quality and allocate resources as efficiently as possible in the cruise terminal industry. Collectively, these studies show how MCDA techniques utilize structured multi-criteria decision-making frameworks to handle both traditional container port performance and specialized cruise port operations.

2.2. Fuzzy and Uncertainty-Aware Extensions

The evaluation of environmental performance in ports across various geographical settings has greatly advanced with the use of fuzzy MCDA approaches.
In order to assess sustainable practices in Egyptian ports, Elzarka and Elgazzar [17] created a green port performance index using fuzzy AHP. They came to the conclusion that this method offers a useful tool for measuring and tracking sustainability measures in underdeveloped nations. Their study showed how fuzzy logic provides port authorities with a useful framework to evaluate and enhance their ecological performance while effectively addressing the subjectivity and complexity of environmental criteria.
In addition to this study, Akbari et al. [18] evaluated port environmental performance using the fuzzy analytic hierarchy methodology, creating a thorough hierarchical structure of environmental indicators that allows for a methodical evaluation of port sustainability measures. Their study demonstrated that FAHP offers a strong approach for evaluating and rating ports according to their environmental performance while also successfully managing linguistic uncertainties in environmental evaluations. When taken as a whole, these studies highlight the benefits of fuzzy MCDA techniques in creating uniform but flexible frameworks for port sustainability evaluation in a variety of operational and geographic contexts.

2.3. ustainability and Green Port Evaluation

The use of MCDA for port sustainability has grown to handle complex environmental concerns using advanced approaches.
Hosseini and Kaneko [19] concluded that the Fuzzy AHP framework efficiently converts qualitative environmental assessments into quantifiable measures, giving port managers a reliable decision-making tool for prioritizing sustainability activities in the face of uncertainty.
Roh et al. [20] demonstrated that successful sustainable port development necessitates a comprehensive approach that balances technological innovation, stakeholder engagement, and policy support, and their case study revealed that green port initiatives rely on adaptive management strategies tailored to local contexts.

Chiu et al. [21] shown that integrating fuzzy logic with AHP methodology effectively addresses the uncertainty inherent in environmental performance evaluations, providing port authorities with a systematic strategy to assessing green port variables and allocating resources efficiently. Collectively, these studies highlight the importance of fuzzy MCDA approaches in enabling thorough and context-aware sustainability assessments for modern ports.

2.4. Integration with Quantitative and Spatial Methods

The combination of specialized quantitative and geographical analysis tools has substantially accelerated the development of MCDA techniques. Isbaex et al. [22] conducted a comprehensive systematic review of GIS applications in the maritime-port sector, concluding that geographic information systems provide powerful spatial analysis capabilities that allow for better port planning, environmental management, and operational efficiency. Their findings emphasized the growing importance of spatial data integration in promoting sustainable port development and decision-making processes.
Andrejić and Kilibarda [23] developed an integrated PCA-DEA approach for distribution channel selection. This approach reduces data dimensionality while maintaining essential information for efficiency measurement in logistics systems. Their methodology was particularly useful in evaluating and optimizing distribution networks connected to port operations and supply chain management. These studies show how combining spatial analysis tools like GIS with quantitative methods like PCA-DEA can provide comprehensive decision-support frameworks for port management, taking into account both geographical considerations and operational efficiency metrics in maritime logistics systems.

2.5. Service Quality and Customer Focus

The development of sophisticated fuzzy MCDA techniques has substantially increased port service quality evaluation by addressing both the complexity of service qualities and the inherent uncertainties in customer perceptions. Nguyen et al. [24] used an integrated approach to assess port service quality, integrating extension fuzzy AHP with importance-performance analysis (IPA), and concluded that this hybrid methodology efficiently translates subjective service quality assessments into actionable insights for port management. Their findings showed that the fuzzy AHP component handles linguistic uncertainties in customer evaluations, but the IPA framework gives clear visual assistance for identifying improvement areas based on their importance and existing performance levels.
Tatar [25] built on this basis by introducing the fermatean fuzzy AHP approach for analyzing port service quality dimensions, creating a more advanced mathematical framework for dealing with higher levels of uncertainty and ambiguity in service quality evaluations. Their technique proved greater capacity to manage complicated and uncertain decision-making contexts, especially when dealing with imprecise and ambiguous consumer feedback. These studies show the gradual evolution of fuzzy MCDA techniques in port service quality assessment, from basic fuzzy sets to more advanced fermatean fuzzy systems, giv-

ing port authorities increasingly sophisticated tools for measuring and improving customer satisfaction and service performance in competitive maritime environments.

2.6. Risk, Resilience, and Safety Assessment

The use of fuzzy Multi-Criteria Decision-Making (MCDM) techniques has proven very useful in solving complicated risk and safety concerns in port environments characterized by subjective assessments and uncertain conditions. Rane and Choudhary [26] conducted a detailed investigation on the efficacy of integrated fuzzy AHP and fuzzy TOPSIS techniques, indicating that this combination methodology provides a strong foundation for dealing with subjective judgments in selection processes under uncertain settings. Their findings showed that combining both methods improves decision-making accuracy by leveraging fuzzy AHP for criterion weighting and fuzzy TOPSIS for alternative ranking, effectively resolving the inherent ambiguity in expert judgments.
In addition to this methodological improvement, Hsu et al. [27] constructed a comprehensive risk assessment model for work safety in container dry ports, using fuzzy MCDM approaches to evaluate occupational hazards and safety performance. Their research developed a structured methodology for detecting, prioritizing, and reducing safety hazards in port operations, highlighting the vital role of proactive risk management in ensuring operational continuity and safeguarding staff well-being. These studies highlight the importance of fuzzy MCDM approaches in port risk management, providing port authorities with scientifically grounded yet practical tools for improving safety performance, supporting resilient operations, and ensuring long-term port management through evidence-based decision-making under uncertainty.

2.7. Stakeholder Participation and Robustness Analysis.

The incorporation of collaborative decision-making frameworks and extensive performance analysis has greatly improved the strength of maritime transport research and port management techniques. Arof [28] conducted a systematic assessment of the combined Delphi-AHP method in maritime transport research and concluded that this integrated strategy efficiently fosters stakeholder consensus and improves decision-making dependability through organized expert participation. The study found that the Delphi method's iterative feedback loop, paired with AHP's systematic pairwise comparisons, provides a strong mechanism for pooling various expert perspectives and lowering subjectivity in complicated maritime decisions.
To complement this analytical perspective, Ha and Yang [29] undertook a comparative analysis of port performance indicators, looking at both the independence and interdependence of important performance parameters. Their study demonstrated that multivariate statistical techniques can successfully capture the independent contributions and synergistic effects of various performance measures, and that the development of accurate port evaluation systems requires an understanding of the intricate relationships between performance indicators. In order to improve the robustness and reliability of maritime transport research, these studies collectively emphasize the significance of fusing advanced

analytical techniques with participatory decision-making methods. This approach offers researchers and port authorities comprehensive frameworks for multidimensional performance assessment and stakeholder-inclusive decision-making that take into account both independent and interdependent factors in port operations and management.

2.8. AI and Data-Driven Hybridization

The combination of artificial intelligence and fuzzy MCDA methodologies continues to enhance port digitization and smart transformation.
According to Karlı et al. [30], the fuzzy AHP approach efficiently prioritizes essential elements of smart ports, including operational efficiency, environmental sustainability, and technological integration. Their case study of Filyos Port shows that this technique provides a systematic framework for port authorities to create strategic roadmaps for smart port transformation that balance both technological and operational requirements.
Sim et al. [31] extended this research to cutting-edge technologies by proposing an artificial intelligence-based smart port logistics metaverse architecture that combines digital twin technology with port operations. Their research on Busan Port showed how this novel strategy, which uses real-time simulation and predictive analytics in a virtual setting, greatly improves safety management, environmental monitoring, and productivity. From conventional fuzzy MCDA applications to sophisticated AI-powered metaverse environments, these studies collectively demonstrate the changing face of port digitalization and provide all-inclusive solutions for the creation of next-generation smart ports.

Table (1): Major Trends in Port Evaluation Using MCDA

Trend	Representative Studies	Typical MCDA Methods	Application Domains
Classical MCDA dominance (AHP + outranking/utility)	Rezaei et al. (2019) Lorencic et al. (2022)	AHP, TOPSIS, VIKOR, PROMETHEE	Port selection, competitiveness ranking, operational benchmarking
Fuzzy/uncertainty-aware extensions	Elzarka & Elgazzar (2014) Akbari et al. (2021)	Fuzzy AHP, Interval type-2 Fuzzy AHP, Fuzzy TOPSIS	Service quality, safety risk, attractiveness evaluation
Sustainability and green ports	Hosseini & Kaneko (2021) Roh et al (2023) Chiu et al. (2014)	AHP, Fuzzy TOPSIS, Hybrid fuzzy models	Environmental performance, energy use, social impact, green transition
Integration with DEA, PCA, GIS, simulation	Isbaex et al. (2025) Andrejić & Kilibarda (2015)	DEA–MCDA hybrids, PCA + MCDA, GIS-based MCDA	Efficiency analysis, spatial accessibility, terminal performance
Service quality and customer-focused evaluation	Nguyen et al. (2022) Tatar (2023)	Fuzzy AHP, IPA + MCDA hybrids	Port service quality (PSQ), customer satisfaction, stakeholder perception
Risk, resilience, and safety assessment	Rane & Choudhary (2023) Hsu et al. (2023)	Fuzzy AHP, Fuzzy TOPSIS, DEMATEL + MCDA	Occupational safety, cybersecurity, disaster resilience
Stakeholder participation and robustness	Arof, A. M. (2015) Ha, M. H & Yang (2017)	Group AHP, Delphi-AHP, Sensitivity & scenario analysis	Policy evaluation, strategic planning, stakeholder consensus
AI, simulation and data-driven hybridization	Karl et al (2021) Sim et al. (2024)	Entropy-AHP, ML-assisted MCDA, Agent-based simulation + MCDA	Big-data-driven evaluation, emergent risks, predictive port performance

3. Preliminaries

In the first section, we introduced the importance of multi-criteria decision analysis methods in general and the importance of port evaluation in particular. In the second section, we presented previous work on port evaluation using various approaches. Here, we will present the basic ideas of the partially ordered sets associated with the proposed method.

A partially-ordered set (poset) $P = (X, \prec)$ is a set of elements X, along with a binary relation, $\prec$, having the property that $\prec$ is transitive and anti-symmetric for all elements in $\mathcal{P}$. Two elements a_i and a_j are comparable if $a_i \prec a_j$ otherwise are **incomparable** if $a_i \nprec a_j$. A non-empty subset $C = \{a_1, a_2, \ldots, a_k\} \subseteq X$ is called a chain in P if $a_1 \prec a_2 \prec \cdots \prec a_k$. A cover relation for $P = (X, \prec)$ is the set of pairs (a, b) such that $a, b \in X$, and b covers a whenever there exists no element $c \in X$ such that $a \prec c \prec b$. An antichain is a non-empty set in which there are no comparable pairs of elements.

Posets are graphically visualized using Hasse diagrams. A Hasse diagram can be derived from a directed acyclic graph, where the vertices represent the objects and a line relates object a with b whenever $a \prec b$. In the case of a transitivity relation whenever $a \prec b$ and $b \prec c$ then it suffices to draw a line only for $a \prec b$ and $b \prec c$ and not for $a \prec c$. A convenient way to code a partial order is the ζ matrix: The rows and columns of this matrix are labeled with the object names. If $a \prec b$, then the corresponding cell gets a_1, in all other cases a_0. The cover matrix is obtained from the zeta matrix by removing all transitive relations from it. It is always feasible to divide the elements of X into disjoint chains since every single element in X is a chain in and of itself. Such a partition is referred to as a decomposition, and the minimum decomposition is one that has the fewest possible disjoint chains. The size of a minimum decomposition is equivalent to the size of a maximum antichain, according to Dilworth [**?**]. Badr et al. proposed a new Integer Linear Programming model (ILPM) for Dilworth's Decomposition theorem [**?**]. On the other hand, Badr et al. introduced an efficient algorithm to compute the width of a given partially ordered set P according to Dilworth's Theorem [**?**]. The complete list of jump-critical ordered sets with jump number four was introduced [**?**].

4. The proposed method

In this section, the description of the proposed method, called level analysis partially ordered sets (LAPOS), is introduced. LAPOS depends on the properties of the partially ordered sets. Get all ordered pairs of each column (features) with relation ($\leq$) between each row in the columns. Get intersection between columns ordered pairs and make a relation matrix (zeta matrix) and cover matrix, Draw Hasse diagram, according to levels of Hasse diagram select k features from top to down and from left to right level then ignore the others, as shown in Algorithm1.

Algorithm 1: level analysis partially ordered sets (LAPOS)

Step 1: Load the dataset $A[\,]\,[\,]$
Step 2: Apply one scaling techniques (MinMax)
Step 3: Get all ordered pairs of each column with relation ($\leq$) between each row in the columns.
Step 4: Get intersection between columns ordered pairs and make a relation matrix (zeta matrix) and cover matrix
Step 5: Draw Hasse diagram
Step 6: According to levels of Hasse diagram select k features from top to down and from left to right level then ignore the others.

4.1. Complexity of the Proposed Technique (LAPOS)

Here, we analyze the complexity of LAPOS. Suppose the size of the dataset (m, n) such that m is the number of rows and n is the number of features. The complexity of first step is $O(mn)$ and also the second step has complexity $O(mn)$. Finally, the third step has the complexity $O(mn^2)$.
The complexity for the proposed technique (LAPOS) is $\approx O(mn) + O(mn) + O(mn^2) \approx O(mn^2)$

5. Case Study

It is essential to adhere to core criteria including scientific rigor, pragmatism, and comprehensiveness while creating an evaluation indicator system for port logistics vitality. This study creates a comprehensive and well-organized indicator framework for evaluating the vitality of port logistics by drawing on prior research and pertinent evaluation standards. Alexandria Port, Dekheila Port, Damietta Port, Port Said Port, Suez Port, Sokhna Port, Safaga Port, Nuweiba Port, Hurghada Port, Arish Port, East Port Said Port, and El Tor Port are among the twelve ports in the Arab Republic of Egypt that we examine in this part. In order to evaluate these ports as indicated in Table (1), 14 indicators were developed, including: the number of berths, annual (million tons), allocation length (meters), maximum draft station (meters), number of stations for a period (TEU), number of witnesses, land connectivity (1 = yes, 0 = no), storage area (km2), number of logistics services (1 = yes, 0 = no), industrial neighborhoods (1 = yes, 0 = no), temporary handling speed (container/hour), free zone (1 = yes, 0 = no), and water availability (1 = yes, 0 = no)), "Maritime Transport and Logistics Sector - Official Data for Damietta Port"[†], "Ministry of Transport"[‡], "Red Sea Ports Authority"[§], and "the Suez Canal Economic Zone Authority"[¶]

[†] https://www.mts.gov.eg/ar/
[‡] https://www.mts.gov.eg/ar/
[§] http://www.rspa.gov.eg/
[¶] www.sczone.eg

Table (2): Evaluation Criteria of Port Logistics Efficiency Across Major Egyptian Ports

Port Name	Number of Berths	Annual Capacity (Million Tons)	Berth Length (Meters)	Maximum Draft (Meters)	Annual Number of Containers (TEU)	Number of Cranes	Railway Connectivity (1=Yes, 0=No)	Warehouse Area (Km²)	Number of Vessels Annually	Availability of Logistics Services (1=Yes, 0=No)	Nearby Industrial Zones (1=Yes, 0=No)	Container Handling Speed (Containers/Hour)	Presence of a Free Zone (1=Yes, 0=No)	Presence of a Water Treatment Plant (1=Yes, 0=No)
Alexandria	67	30	12000	13.5	1200000	35	1	2.5	3000	1	1	35	1	1
Dekheila	15	25	7000	16	1000000	20	1	1.8	2000	1	1	30	1	1
Damietta	22	25	9000	14.5	1300000	28	1	3	2500	1	1	32	1	1
Port Said	37	20	8500	15	2500000	40	0	1.5	2800	1	1	40	1	1
Suez	12	10	4500	12	300000	12	1	1	1800	1	0	20	0	0
Sokhna	10	15	3200	17	1800000	24	0	5.5	2200	1	1	38	1	1
Safaga	4	6	1800	11.5	100000	6	0	0.6	700	0	0	15	0	0
Nuweiba	3	1.5	1200	9	60000	4	0	0.4	400	0	0	10	0	0
Hurghada	2	1	900	7.5	20000	2	0	0.2	350	0	0	8	0	0
Arish	2	0.5	800	6.5	15000	2	0	0.2	300	0	0	6	0	0
East Port Said	5	18	6000	17.5	2200000	30	1	4	2700	1	1	45	1	1
El-Tor	2	0.7	950	8	10000	2	0	0.3	150	0	0	5	0	0

5.1. Implementation of the proposed method on the case study

This section will include a case study that demonstrates the suggested LAPOS approach that was introduced in Algorithm 1. Next, load the original dataset, which has 14 characteristics and 12 instances (ports), as seen in Table 2. Use the MinMax scaling approach. Use the relation (¡=) between each row in the columns to retrieve all ordered pairs of each column. Create a cover matrix and a relation matrix (zeta matrix) by finding the intersection of column-ordered pairs.

Table (3): Zeta matrix

	Alex	Dekh	Dam	Said	suez	Sokh	Safa	Nuwe	Hurg	Aris	Esaid	Tor
Alex	1	0	0	0	1	0	1	1	1	1	0	1
Dekh	0	1	0	0	1	0	1	1	1	1	0	1
Dam	0	0	1	0	1	0	1	1	1	1	0	1
Said	0	0	0	1	0	0	1	1	1	1	0	1
suez	0	0	0	0	1	0	1	1	1	1	0	1
Sokh	0	0	0	0	0	1	1	1	1	1	0	1
Safa	0	0	0	0	0	0	1	1	1	1	0	1
Nuwe	0	0	0	0	0	0	0	1	1	1	0	1
Hurg	0	0	0	0	0	0	0	0	1	1	0	0
Aris	0	0	0	0	0	0	0	0	0	1	0	0
Esaid	0	0	0	0	0	0	1	1	1	1	1	1
Tor	0	0	0	0	0	0	0	0	0	0	0	1

Table (4): Cover matrix

	Alex	Dekh	Dam	Said	suez	Sokh	Safa	Nuwe	Hurg	Aris	Esaid	Tor
Alex	0	0	0	0	1	0	0	0	0	0	0	0
Dekh	0	0	0	0	1	0	0	0	0	0	0	0
Dam	0	0	0	0	1	0	0	0	0	0	0	0
Said	0	0	0	0	0	0	1	0	0	0	0	0
suez	0	0	0	0	0	0	1	0	0	0	0	0
Sokh	0	0	0	0	0	0	1	0	0	0	0	0
Safa	0	0	0	0	0	0	0	1	0	0	0	0
Nuwe	0	0	0	0	0	0	0	0	1	0	0	1
Hurg	0	0	0	0	0	0	0	0	0	1	0	0
Aris	0	0	0	0	0	0	0	0	0	0	0	0
Esaid	0	0	0	0	0	0	1	0	0	0	0	0
Tor	0	0	0	0	0	0	0	0	0	0	0	0

Draw Hasse diagram as shown in Figure 1:

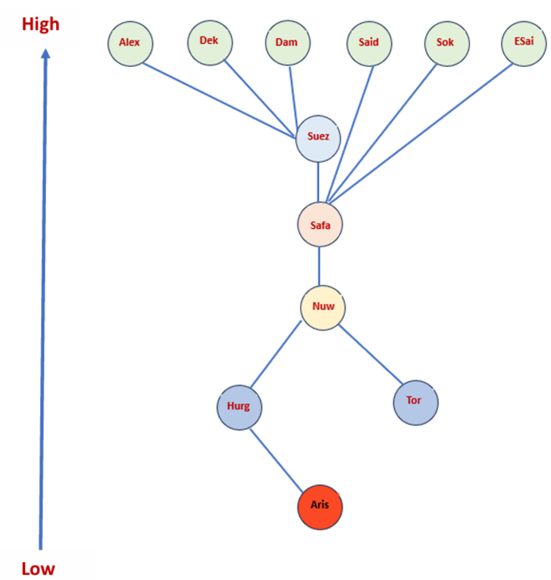

Figure 1: Hierarchical Visualization of Port Logistics Vitality Using a Hasse Diagram

According to levels of Hasse diagram select k features from top to down and from left to right level.

Table 5: Result of the proposed method for arranging the twelve ports according to the priority

Ports	Level	Rank
Alexandria	1	1
Dekheila	1	2
Damietta	1	3
Port Said	1	4
Sokhna	1	5
East Port Said	1	6
Suez	2	7
Safaga	3	8
Nuweiba	4	9
Hurghada	5	10
El-Tor	5	11
Arish	6	12

Table 5 presents the final ranking of Egyptian logistics ports obtained from the LAPOS analysis. The results indicate that six ports — Alexandria, Dekheila, Damietta, Port Said, Sokhna, and East Port Said — appear in Level 1 of the Hasse diagram. Within this level, the ports are ordered from left to right according to their positions in the diagram, reflecting the traversal rule adopted by the LAPOS method.

The lower levels (2 to 6) represent ports with gradually decreasing performance compared to those in higher levels, illustrating a clear hierarchical structure derived from the multi-criteria indicators used in the study.

6. Results Analysis and Interpretation

The Hasse diagram presented in Figure 1 illustrates a clear hierarchical structure among the evaluated logistics ports, categorizing the twelve Egyptian ports into six distinct levels based on their priorities. The first level comprises Alexandria, Dekheila, Damietta, Port Said, Sokhna, and East Port Said ports—indicating the highest vitality tier. The second level includes Suez Port, followed by Safaga in the third level, Nuweiba in the fourth, Hurghada and El-Tor in the fifth, and finally, Arish Port occupies the sixth and lowest level. This hierarchical arrangement aligns with the fundamental principle of order theory, where elements occupying higher positions in the Hasse diagram are considered superior in terms of the evaluated criteria compared to those positioned below. Thus, an upward movement in levels signifies an improvement in logistics performance, while a downward shift reflects a decline. Table 3 shows the ranking of the twelve Egyptian ports according to priority. The levels derived from Table 3 further substantiate this ranking, providing a quantitative measure of logistics vitality across the ports. Alexandria Port, Dekheila Port, Damietta Port, Port Said Port, Sokhna Port and East Port Said Port demonstrated the highest vitality scores, underscoring their superior performance, while Arish Port ranked the lowest, reflecting relatively limited logistics capabilities. These findings offer a valuable foundation for assessing and comparing the competitiveness of logistics ports in Egypt. The outstanding performance of Alexandria, Dekheila, and Damietta ports can be attributed to their strength across multiple indicators—most notably in cargo throughput, container handling volume, and the degree of logistics information system automation. Such advantages contribute to their elevated development status and enhanced market competitiveness. In contrast, the relatively low ranking of Arish Port is primarily due to its limited operational scale and underdeveloped logistics digitalization infrastructure. This highlights the necessity of strategic investments and targeted improvements in both management practices and infrastructure development to enhance the port's logistics vitality and competitive position in the future.

7. Verification of Results

The Analytic Hierarchy Process (AHP) and the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) are two more Multi-Criteria Decision-Making (MCDM) techniques that were used in a quantitative verification to guarantee the precision and resilience of the outcomes produced by the LAPOS method. The same dataset and evaluation standards utilized in the initial study of Egyptian logistics ports were used for both approaches.

7.1. Application of the AHP Method

First, the Analytic Hierarchy Process (AHP) was used to identify the relative relevance weights of the evaluation criteria. The Entropy Weighting Method was first used to determine objective weights based on the degree of variation in each criterion across all ports because a comprehensive expert-based pairwise comparison matrix was not available. These weights were then used to build a pairwise comparison matrix, from which the eigenvector corresponding to the final weights was taken. After calculating the Consistency Ratio (CR) and Consistency Index (CI), the CR value was 0.072, below the permissible threshold ($CR < 0.1$). This attests to the matrix's internal consistency and the dependability of the weights that were determined. These weights were used to determine each port's weighted score, which was then used to rank the ports based on their logistics performance. According to the data, Arish Port came in last, indicating its limited operational and digital capabilities, whereas Alexandria, Dekheila, and Damietta rated highest.

7.2. Application of the TOPSIS Method

The same criteria and entropy-based weights were then used to apply the TOPSIS approach. To unify the scales of measurement, the decision matrix was first normalized. Next, each normalized value was multiplied by its corresponding weight to create the weighted normalized matrix. Each criterion's positive and negative ideal solutions (A^+ and A^-) were identified, and the Euclidean distances between each port and these two ideal locations were computed. After that, the ports were arranged in descending order of relative closeness coefficient (Ci) values, where larger values denote greater performance (closer to the ideal solution). Alexandria, East Port Said, and Sokhna were in the top three, while Arish Port was in bottom place, according to the results, which were in good agreement with the rating that LAPOS had acquired.

7.3. Comparison and Correlation of Results

The Spearman's Rank Correlation Coefficient (ρ) was computed in order to compare and assess the level of consistency amongst the three evaluation techniques (LAPOS, AHP, and TOPSIS). Prior to that, the port rankings derived from each approach were directly compared, as may be seen below:

Table 6: Comparison of Egyptian Port Rankings Using LAPOS, AHP, and TOPSIS

Rank	LAPOS	AHP	TOPSIS
1	Alexandria	Alexandria	Alexandria
2	Dekheila	Dekheila	East Port Said
3	Damietta	Damietta	Sokhna
4	Port Said	East Port Said	Dekheila
5	Sokhna	Sokhna	Damietta
6	East Port Said	Suez	Port Said
7	Suez	Safaga	Suez
8	Safaga	Nuweiba	Safaga
9	Nuweiba	Hurghada	Nuweiba
10	Hurghada	El-Tor	Hurghada
11	El-Tor	Arish	El-Tor
12	Arish	—	Arish

Table (6) illustrates how consistently ports are ranked using the three approaches, especially at the top and bottom. Arish Port continuously stayed at the bottom in every way, while Alexandria, Dekheila, and Damietta continued to hold their top spots. The Spearman's correlation coefficient (ρ), which quantifies the degree of correlation between the ranking outcomes, was calculated as follows:

Table (7): Spearman's Rank Correlation Coefficients among the Three Evaluation Methods

Comparison	Spearman's ρ	Correlation Strength
LAPOS – TOPSIS	0.89	Very Strong
LAPOS – AHP	0.83	Strong
AHP – TOPSIS	0.91	Very Strong

When compared to conventional mathematical procedures, these numbers show a very significant correlation between the three ways' results, indicating the high consistency of ranking directions and the dependability of the results obtained using the LAPOS methodology. In great agreement with the findings of both AHP and TOPSIS, the verification results verify that the LAPOS technique offered a rational and impartial assessment of Egyptian ports' performance. This consistency supports the validity of the LAPOS technique as a useful tool for rating ports according to a variety of performance parameters and shows how strong the analytical framework employed in the study was. These figures demonstrate a highly significant correlation between the outcomes of the three methods when compared to traditional mathematical procedures, demonstrating the reliability of the outcomes produced by the LAPOS methodology as well as the high consistency of ranking directions. The verification results confirm that the LAPOS technique provided a logical and unbiased evaluation of Egyptian ports' performance, which is in excellent accord with the conclusions of both AHP and TOPSIS. This consistency demonstrates the strength of the analytical framework used in the study and validates the LAPOS technique as a practical tool for assessing ports based on a range of performance indicators.

8. Conclusion

Using multi-criteria indicators, this study devised and implemented the LAPOS (Level Analysis based on Partially Ordered Sets) technique to evaluate the performance of Egyptian logistics ports. The method avoided the drawbacks of traditional total-ranking techniques such as AHP or TOPSIS by incorporating the concepts of Order Theory to represent both hierarchical and incomparable relationships among ports.

The findings verified that several ports, including Alexandria, Dekheila, Damietta, Port Said, Sokhna, and East Port Said, form an antichain of nodes without any cover relation in Level 1 of the Hasse diagram. They should be regarded as functionally equivalent in performance since they cannot be compared within the same level. Any sequential numbering (1, 2, 3, . . .) in Table 5 represents only a tie-breaking rule for practical presentation and does not imply true dominance or superiority. This recognition of incomparability allows decision-makers to respect genuine multi-criteria trade-offs instead of forcing arbitrary total rankings—one of the key advantages of the LAPOS approach.

Another significant contribution of this study lies in its feature selection process. Algorithm 1 introduces a dedicated phase that identifies a subset of key performance indicators based on their position in the poset structure (top-to-bottom and left-to-right traversal). This enables LAPOS to function not only as a ranking method but also as a dimensionality-reduction tool, determining which of the fourteen indicators used in this study are most influential. This dual capability enhances both the interpretability and efficiency of port performance analysis.

From a managerial perspective, the results suggest that all ports within the same LAPOS level should be given equal investment priority, as their operational performance is statistically indistinguishable under the chosen criteria. This relational approach offers a more transparent and balanced framework for benchmarking and resource allocation in port management.

Future studies may expand this methodology to a broader set of ports or to longitudinal (time-series) data to assess performance evolution over time. Moreover, comparing LAPOS with fuzzy or probabilistic ranking models could help develop hybrid frameworks that combine order-theoretic and uncertainty-based reasoning. Moreover, we intend to develop a more rigorous mathematical model to enhance the proposed method, enabling a more objective and theoretically consistent mechanism for determining the selection and preference among ports (or elements) that belong to the same level. Overall, the LAPOS technique provides a flexible, non-parametric, and data-driven approach for multi-criteria port performance evaluation. By capturing both the hierarchical structure and the incomparability among ports, it ensures that strategic decisions are grounded in a realistic and comprehensive understanding of the underlying logistics system.

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