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Weak Forms of $\mu(\sigma_1, \sigma_2)$-Continuity for Multifunctions

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Abstract. A new class of continuous multifunctions between a generalized topological space and a bitopological space, namely upper (lower) weakly $\mu(\sigma_1, \sigma_2)$-continuous multifunctions, has been defined and studied. Moreover, several characterizations and some properties concerning upper weakly $\mu(\sigma_1, \sigma_2)$-continuous multifunctions and lower weakly $\mu(\sigma_1, \sigma_2)$-continuous multifunctions are established. Furthermore, the relationships between almost $\mu(\sigma_1, \sigma_2)$-continuity and weak $\mu(\sigma_1, \sigma_2)$-continuity are considered.

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1. Introduction

In 1961, Levine [1] introduced and investigated the notion of weakly continuous functions. Husain [2] introduced and studied the concept of almost continuous functions. Janković [3] introduced almost weak continuity as a generalization of both weak continuity and almost continuity. Noiri [4] investigated several characterizations of almost weakly continuous functions. Rose [5] introduced the notion of subweakly continuous functions and investigated the relationships between subweak continuity and weak continuity. Popa and Noiri [6] introduced the concept of weakly (τ, m)-continuous functions as functions from a topological space into a set satisfying some minimal conditions and investigated several characterizations of weakly (τ, m)-continuous functions. In 2002, Császár [7] introduced the concepts of generalized topological spaces and generalized neighborhood systems. The classes of topological spaces and neighborhood systems are contained in the classes of generalized topological spaces and generalized neighborhood systems, respectively. Furthermore, Császár [7] introduced two kinds of generalized continuous

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functions by utilizing the notions of generalized topological spaces and generalized neighborhood systems. In 2009, Kanibir and Reilly [8] extended the concept of generalized continuous functions to multifunctions and defined upper semi generalized continuous multifunctions and lower semi generalized continuous multifunctions. On the other hand, the present authors introduced and investigated four classes of multifunctions defined from a generalized topological space into a generalized topological space, namely upper $\alpha(\mu_X, \mu_Y)$-continuous multifunctions [9], lower $\alpha(\mu_X, \mu_Y)$-continuous multifunctions [9], upper weakly $\beta(\mu_X, \mu_Y)$-continuous multifunctions [10] and lower weakly $\beta(\mu_X, \mu_Y)$-continuous multifunctions [10]. Pue-on et al. [11] introduced and studied new classes of multifunctions between bitopological spaces, namely upper (τ_1, τ_2)-continuous multifunctions and lower (τ_1, τ_2)-continuous multifunctions. Klanarong et al. [12] introduced and investigated the notions of upper almost (τ_1, τ_2)-continuous multifunctions and lower almost (τ_1, τ_2)-continuous multifunctions. Thongmoon et al. [13] extended the concept of weakly continuous functions to multifunctions and presented two classes of multifunctions defined from a bitopological space into a bitopological space, called upper weakly (τ_1, τ_2)-continuous multifunctions and lower weakly (τ_1, τ_2)-continuous multifunctions. Quite recently, Pue-on et al. [14] introduced new classes of continuous multifunctions between an ideal topological space and a bitopological space, namely upper almost $\tau^\star(\sigma_1, \sigma_2)$-continuous multifunctions and lower almost $\tau^\star(\sigma_1, \sigma_2)$-continuous multifunctions. Moreover, several characterizations and some properties of upper almost $\tau^\star(\sigma_1, \sigma_2)$-continuous multifunctions and lower almost $\tau^\star(\sigma_1, \sigma_2)$-continuous multifunctions were discussed in [14]. In this paper, we introduce new classes of multifunctions between a generalized topological space and a bitopological space, namely upper weakly $\mu(\sigma_1, \sigma_2)$-continuous multifunctions and lower weakly $\mu(\sigma_1, \sigma_2)$-continuous multifunctions. We also investigate several characterizations of upper weakly $\mu(\sigma_1, \sigma_2)$-continuous multifunctions and lower weakly $\mu(\sigma_1, \sigma_2)$-continuous multifunctions.

2. Preliminaries

Throughout the present paper, spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or simply X and Y) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a bitopological space (X, τ_1, τ_2). The closure of A and the interior of A with respect to τ_i are denoted by τ_i-Cl(A) and τ_i-Int(A), respectively, for $i = 1, 2$. A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-*closed* [15] if $A = \tau_1\text{-Cl}(\tau_2\text{-Cl}(A))$. The complement of a $\tau_1\tau_2$-closed set is called $\tau_1\tau_2$-*open*. The intersection of all $\tau_1\tau_2$-closed sets of X containing A is called the $\tau_1\tau_2$-*closure* [15] of A and is denoted by $\tau_1\tau_2$-Cl(A). The union of all $\tau_1\tau_2$-open sets of X contained in A is called the $\tau_1\tau_2$-*interior* [15] of A and is denoted by $\tau_1\tau_2$-Int(A).

Lemma 1. [15] *Let A and B be subsets of a bitopological space (X, τ_1, τ_2). For the $\tau_1\tau_2$-closure, the following properties hold:*

(1) $A \subseteq \tau_1\tau_2\text{-}Cl(A)$ and $\tau_1\tau_2\text{-}Cl(\tau_1\tau_2\text{-}Cl(A)) = \tau_1\tau_2\text{-}Cl(A)$.

(2) If $A \subseteq B$, then $\tau_1\tau_2\text{-}Cl(A) \subseteq \tau_1\tau_2\text{-}Cl(B)$.

(3) $\tau_1\tau_2$-*Cl*(A) *is* $\tau_1\tau_2$-*closed.*

(4) A *is* $\tau_1\tau_2$-*closed if and only if* $A = \tau_1\tau_2$-*Cl*(A).

(5) $\tau_1\tau_2$-*Cl*$(X - A) = X - \tau_1\tau_2$-*Int*$(A)$.

A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)r$-*open* [16] (resp. $(\tau_1, \tau_2)s$-*open* [17], $(\tau_1, \tau_2)p$-*open* [17], $(\tau_1, \tau_2)\beta$-*open* [17]) if $A = \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A))$ (resp. $A \subseteq \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A))$, $A \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A))$, $A \subseteq \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A)))$). The complement of a $(\tau_1, \tau_2)r$-open (resp. $(\tau_1, \tau_2)s$-open, $(\tau_1, \tau_2)p$-open, $(\tau_1, \tau_2)\beta$-open) set is said to be $(\tau_1, \tau_2)r$-*closed* (resp. $(\tau_1, \tau_2)s$-*closed*, $(\tau_1, \tau_2)p$-*closed*, $(\tau_1, \tau_2)p$-*closed*). For a subset A of a bitopological space (X, τ_1, τ_2), a point $x \in X$ is called a $(\tau_1, \tau_2)\theta$-*cluster point* [16] of A if $\tau_1\tau_2\text{-Cl}(U) \cap A \neq \emptyset$ for every $\tau_1\tau_2$-open set U containing x. The set of all $(\tau_1, \tau_2)\theta$-cluster points of A is called the $(\tau_1, \tau_2)\theta$-*closure* [16] of A and is denoted by $(\tau_1, \tau_2)\theta\text{-Cl}(A)$. A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)\theta$-*closed* [16] if $(\tau_1, \tau_2)\theta\text{-Cl}(A) = A$. The complement of a $(\tau_1, \tau_2)\theta$-closed set is said to be $(\tau_1, \tau_2)\theta$-*open*. The union of all $(\tau_1, \tau_2)\theta$-open sets of X contained in A is called the $(\tau_1, \tau_2)\theta$-*interior* [16] of A and is denoted by $(\tau_1, \tau_2)\theta\text{-Int}(A)$.

Lemma 2. [16] *For a subset* A *of a bitopological space* (X, τ_1, τ_2)*, the following properties hold:*

(1) If A *is* $\tau_1\tau_2$-*open in* X*, then* $\tau_1\tau_2$-*Cl*$(A) = (\tau_1, \tau_2)\theta$-*Cl*$(A)$.

(2) $(\tau_1, \tau_2)\theta$-*Cl*(A) *is* $\tau_1\tau_2$-*closed in* X.

Let X be a nonempty set, and denote $\mathcal{P}(X)$ the power set of X. We call a class $\mu \subseteq \mathcal{P}(X)$ a *generalized topology* (briefly, GT) if $\emptyset \in \mu$, and an arbitrary union of elements of μ belongs to μ [7]. A set X with a GT μ on it is said to be a *generalized topological space* (briefly, GTS) and is denoted by (X, μ). For a GTS (X, μ), the elements of μ are called μ-*open sets* and the complements of μ-open sets are called μ-*closed sets*. For $A \subseteq X$, we denote by $c_\mu(A)$ the intersection of all μ-closed sets containing A and by $i_\mu(A)$ the union of all μ-open sets contained in A. Then, we have $i_\mu(i_\mu(A)) = i_\mu(A)$, $c_\mu(c_\mu(A)) = c_\mu(A)$, and $i_\mu(A) = X - c_\mu(X - A)$. According to [18], for $A \subseteq X$ and $x \in X$, we have $x \in c_\mu(A)$ if and only if $x \in M \in \mu$ implies $M \cap A \neq \emptyset$.

By a multifunction $F : X \to Y$, we mean a point-to-set correspondence from X into Y, and we always assume that $F(x) \neq \emptyset$ for all $x \in X$. For a multifunction $F : X \to Y$, we shall denote the upper and lower inverse of a set B of Y by $F^+(B)$ and $F^-(B)$, respectively, that is, $F^+(B) = \{x \in X \mid F(x) \subseteq B\}$ and $F^-(B) = \{x \in X \mid F(x) \cap B \neq \emptyset\}$. In particular, $F^-(y) = \{x \in X \mid y \in F(x)\}$ for each point $y \in Y$. For each $A \subseteq X$, $F(A) = \cup_{x \in A} F(x)$.

3. Upper and lower weakly $\mu(\sigma_1, \sigma_2)$-continuous multifunctions

In this section, we introduce the concepts of upper weakly $\mu(\sigma_1, \sigma_2)$-continuous multifunctions and lower weakly $\mu(\sigma_1, \sigma_2)$-continuous multifunctions. Furthermore, several characterizations of upper weakly $\mu(\sigma_1, \sigma_2)$-continuous multifunctions and lower weakly $\mu(\sigma_1, \sigma_2)$-continuous multifunctions are discussed.

Definition 1. *A multifunction $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$ is said to be upper weakly $\mu(\sigma_1, \sigma_2)$-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open set V of Y such that $F(x) \subseteq V$, there exists a μ-open set U of X containing x such that $F(U) \subseteq \sigma_1\sigma_2\text{-}Cl(V)$. A multifunction $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$ is said to be upper weakly $\mu(\sigma_1, \sigma_2)$-continuous if F is upper weakly $\mu(\sigma_1, \sigma_2)$-continuous at each point x of X.*

Theorem 1. *For a multifunction $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is upper weakly $\mu(\sigma_1, \sigma_2)$-continuous;

(2) $F^+(V) \subseteq i_\mu(F^+(\sigma_1\sigma_2\text{-}Cl(V)))$ for every $\sigma_1\sigma_2$-open set V of Y;

(3) $c_\mu(F^-(\sigma_1\sigma_2\text{-}Int(K))) \subseteq F^-(K)$ for every $\sigma_1\sigma_2$-closed set K of Y;

(4) $c_\mu(F^-(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(B)))) \subseteq F^-(\sigma_1\sigma_2\text{-}Cl(B))$ for every subset B of Y;

(5) $F^+(\sigma_1\sigma_2\text{-}Int(B)) \subseteq i_\mu(F^+(\sigma_1\sigma_2\text{-}Cl(\sigma_1\sigma_2\text{-}Int(B))))$ for every subset B of Y;

(6) $c_\mu(F^-(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))) \subseteq F^-(\sigma_1\sigma_2\text{-}Cl(V))$ for every $\sigma_1\sigma_2$-open set V of Y;

(7) $c_\mu(F^-(V)) \subseteq F^-(\sigma_1\sigma_2\text{-}Cl(V))$ for every $\sigma_1\sigma_2$-open set V of Y;

(8) $c_\mu(F^-(\sigma_1\sigma_2\text{-}Int(K))) \subseteq F^-(K)$ for every $(\sigma_1, \sigma_2)r$-closed set K of Y.

Proof. (1) ⇒ (2): Let V be any $\sigma_1\sigma_2$-open set of Y such that $x \in F^+(V)$. Then, we have $F(x) \subseteq V$ and by (1), there exists a μ-open set U of X containing x such that $F(U) \subseteq \sigma_1\sigma_2\text{-Cl}(V)$. Thus, $U \subseteq F^+(\sigma_1\sigma_2\text{-Cl}(V))$. Since U is μ-open, we have $x \in i_\mu(F^+(\sigma_1\sigma_2\text{-Cl}(V)))$ and so $F^+(V) \subseteq i_\mu(F^+(\sigma_1\sigma_2\text{-Cl}(V)))$.

(2) ⇒ (3): Let K be any $\sigma_1\sigma_2$-closed set of Y. Then, $Y - K$ is $\sigma_1\sigma_2$-open in Y. By (2), $X - F^-(K) = F^+(Y - K) \subseteq i_\mu(F^+(\sigma_1\sigma_2\text{-Cl}(Y - K))) = X - c_\mu(F^-(\sigma_1\sigma_2\text{-Int}(K)))$. Thus, $c_\mu(F^-(\sigma_1\sigma_2\text{-Int}(K))) \subseteq F^-(K)$.

(3) ⇒ (4): Let B be any subset of Y. Then, $\sigma_1\sigma_2\text{-Cl}(B)$ is a $\sigma_1\sigma_2$-closed set of Y and by (3), $c_\mu(F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(B)))) \subseteq F^-(\sigma_1\sigma_2\text{-Cl}(B))$.

(4) ⇒ (5): Let B be any subset of Y. Thus by (4), we have

$$\begin{aligned} X - i_\mu(F^+(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(B)))) &= c_\mu(X - F^+(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(B)))) \\ &= c_\mu(F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(Y - B)))) \\ &\subseteq F^-(\sigma_1\sigma_2\text{-Cl}(Y - B)) \\ &= X - F^+(\sigma_1\sigma_2\text{-Int}(B)) \end{aligned}$$

and hence $F^+(\sigma_1\sigma_2\text{-Int}(B)) \subseteq i_\mu(F^+(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(B))))$.

(5) ⇒ (1): Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y such that $F(x) \subseteq V$. By (5), $x \in F^+(V) \subseteq i_\mu(F^+(\sigma_1\sigma_2\text{-Cl}(V)))$ and there exists a μ-open set U of X containing x

such that $U \subseteq F^+(\sigma_1\sigma_2\text{-Cl}(V))$. Thus, $F(U) \subseteq \sigma_1\sigma_2\text{-Cl}(V)$ and hence F is upper weakly $\mu(\sigma_1, \sigma_2)$-continuous.

(4) ⇒ (6) and (6) ⇒ (7): The proofs are obvious.

(7) ⇒ (8): Let K be any $(\sigma_1, \sigma_2)r$-closed set of Y. Thus by (7),

$$c_\mu(F^-(\sigma_1\sigma_2\text{-Int}(K))) \subseteq F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))) = F^-(K).$$

(8) ⇒ (3): Let K be any $\sigma_1\sigma_2$-closed set of Y. Then, $\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))$ is $(\sigma_1, \sigma_2)r$-closed in Y and $\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))) = \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(K)) = \sigma_1\sigma_2\text{-Int}(K)$. By (8),

$$\begin{aligned} c_\mu(F^-(\sigma_1\sigma_2\text{-Int}(K))) &= c_\mu(F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))))) \\ &\subseteq F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))) \\ &\subseteq F^-(K). \end{aligned}$$

Definition 2. *A multifunction $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$ is said to be lower weakly $\mu(\sigma_1, \sigma_2)$-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open set V of Y such that $V \cap F(x) \neq \emptyset$, there exists a μ-open set U of X containing x such that $\sigma_1\sigma_2$-Cl$(V) \cap F(z) \neq \emptyset$ for every $z \in U$. A multifunction $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$ is said to be lower weakly $\mu(\sigma_1, \sigma_2)$-continuous if F is lower weakly $\mu(\sigma_1, \sigma_2)$-continuous at each point x of X.*

Theorem 2. *For a multifunction $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is lower weakly $\mu(\sigma_1, \sigma_2)$-continuous;

(2) $F^-(V) \subseteq i_\mu(F^-(\sigma_1\sigma_2\text{-Cl}(V)))$ for every $\sigma_1\sigma_2$-open set V of Y;

(3) $c_\mu(F^+(\sigma_1\sigma_2\text{-Int}(K))) \subseteq F^+(K)$ for every $\sigma_1\sigma_2$-closed set K of Y;

(4) $c_\mu(F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(B)))) \subseteq F^+(\sigma_1\sigma_2\text{-Cl}(B))$ for every subset B of Y;

(5) $F^-(\sigma_1\sigma_2\text{-Int}(B)) \subseteq i_\mu(F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(B))))$ for every subset B of Y;

(6) $c_\mu(F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) \subseteq F^+(\sigma_1\sigma_2\text{-Cl}(V))$ for every $\sigma_1\sigma_2$-open set V of Y;

(7) $c_\mu(F^+(V)) \subseteq F^+(\sigma_1\sigma_2\text{-Cl}(V))$ for every $\sigma_1\sigma_2$-open set V of Y;

(8) $c_\mu(F^+(\sigma_1\sigma_2\text{-Int}(K))) \subseteq F^+(K)$ for every $(\sigma_1, \sigma_2)r$-closed set K of Y.

Proof. The proof is similar to that of Theorem 1.

Definition 3. [19] *A multifunction $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$ is said to be upper almost $\mu(\sigma_1, \sigma_2)$-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open set V of Y such that $F(x) \subseteq V$, there exists a μ-open set U of X containing x such that $F(U) \subseteq \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))$. A multifunction $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$ is said to be upper almost $\mu(\sigma_1, \sigma_2)$-continuous if F is upper almost $\mu(\sigma_1, \sigma_2)$-continuous at each point x of X.*

Definition 4. [19] *A multifunction $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$ is said to be lower almost $\mu(\sigma_1, \sigma_2)$-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open set V of Y such that $V \cap F(x) \neq \emptyset$, there exists a μ-open set U of X containing x such that $\sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V)) \cap F(z) \neq \emptyset$ for every $z \in U$. A multifunction $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$ is said to be lower almost $\mu(\sigma_1, \sigma_2)$-continuous if F is lower almost $\mu(\sigma_1, \sigma_2)$-continuous at each point x of X.*

Remark 1. *For a multifunction $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$, the following implication holds:*

upper almost $\mu(\sigma_1, \sigma_2)$-continuity $\Rightarrow$ upper weakly $\mu(\sigma_1, \sigma_2)$-continuity.

The converse of the implication is not true in general. We give an example for the implication as follows.

Example 1. *Let $X = \{1, 2, 3\}$ with a generalized topology $\mu = \{\emptyset, \{2\}, \{1, 3\}, X\}$. Let $Y = \{a, b, c\}$ with topologies $\sigma_1 = \{\emptyset, \{a\}, \{a, b\}, Y\}$ and $\sigma_2 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, Y\}$. A multifunction $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$ is defined as follows: $F(1) = \{a\}$, $F(2) = \{b\}$ and $F(3) = \{a, c\}$. Then, F is upper weakly $\mu(\sigma_1, \sigma_2)$-continuous but F is not upper almost $\mu(\sigma_1, \sigma_2)$-continuous.*

Theorem 3. *For a multifunction $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is upper weakly $\mu(\sigma_1, \sigma_2)$-continuous;

(2) $c_\mu(F^-(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))) \subseteq F^-(\sigma_1\sigma_2\text{-}Cl(V))$ for every $(\sigma_1, \sigma_2)\beta$-open set V of Y;

(3) $c_\mu(F^-(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))) \subseteq F^-(\sigma_1\sigma_2\text{-}Cl(V))$ for every $(\sigma_1, \sigma_2)s$-open set V of Y.

Proof. (1) $\Rightarrow$ (2): This follows from (4) of Theorem 1.

(2) $\Rightarrow$ (3): The proof is obvious since every $(\sigma_1, \sigma_2)s$-open set is $(\sigma_1, \sigma_2)\beta$-open.

(3) $\Rightarrow$ (1): Since every $\sigma_1\sigma_2$-open set is $(\sigma_1, \sigma_2)s$-open, the proof is obvious by (7) of Theorem 1.

Theorem 4. *For a multifunction $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is lower weakly $\mu(\sigma_1, \sigma_2)$-continuous;

(2) $c_\mu(F^+(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))) \subseteq F^+(\sigma_1\sigma_2\text{-}Cl(V))$ for every $(\sigma_1, \sigma_2)\beta$-open set V of Y;

(3) $c_\mu(F^+(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))) \subseteq F^+(\sigma_1\sigma_2\text{-}Cl(V))$ for every $(\sigma_1, \sigma_2)s$-open set V of Y.

Proof. The proof is similar to that of Theorem 3.

Theorem 5. *For a multifunction $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is upper weakly $\mu(\sigma_1, \sigma_2)$-continuous;

(2) $c_\mu(F^-(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))) \subseteq F^-(\sigma_1\sigma_2\text{-}Cl(V))$ for every $(\sigma_1, \sigma_2)p$-open set V of Y;

(3) $c_\mu(F^-(V)) \subseteq F^-(\sigma_1\sigma_2\text{-}Cl(V))$ for every $(\sigma_1, \sigma_2)p$-open set V of Y;

(4) $F^+(V) \subseteq i_\mu(F^+(\sigma_1\sigma_2\text{-}Cl(V)))$ for every $(\sigma_1, \sigma_2)p$-open set V of Y.

Proof. (1) ⇒ (2): Let V be any $(\sigma_1, \sigma_2)p$-open set of Y. Since $\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))$ is $\sigma_1\sigma_2$-open, by Theorem 1(7)

$$\begin{aligned} c_\mu(F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) &\subseteq F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) \\ &\subseteq F^-(\sigma_1\sigma_2\text{-Cl}(V)). \end{aligned}$$

(2) ⇒ (3): Let V be any $(\sigma_1, \sigma_2)p$-open set of Y. By (2), we have

$$c_\mu(F^-(V)) \subseteq c_\mu(F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) \subseteq F^-(\sigma_1\sigma_2\text{-Cl}(V)).$$

(3) ⇒ (4): Let V be any $(\sigma_1, \sigma_2)p$-open set of Y. Thus by (3),

$$\begin{aligned} X - i_\mu(F^+(\sigma_1\sigma_2\text{-Cl}(V))) &= c_\mu(X - F^+(\sigma_1\sigma_2\text{-Cl}(V))) \\ &= c_\mu(F^-(Y - \sigma_1\sigma_2\text{-Cl}(V))) \\ &\subseteq F^-(\sigma_1\sigma_2\text{-Cl}(Y - \sigma_1\sigma_2\text{-Cl}(V))) \\ &= X - F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))) \\ &\subseteq X - F^+(V) \end{aligned}$$

and hence $F^+(V) \subseteq i_\mu(F^+(\sigma_1\sigma_2\text{-Cl}(V)))$.

(4) ⇒ (1): Let V be any $\sigma_1\sigma_2$-open set of Y. Then, V is $(\sigma_1, \sigma_2)p$-open in Y and by (4), $F^+(V) \subseteq i_\mu(F^+(\sigma_1\sigma_2\text{-Cl}(V)))$. By Theorem 1(2), F is upper weakly $\mu(\sigma_1, \sigma_2)$-continuous.

Theorem 6. *For a multifunction $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is lower weakly $\mu(\sigma_1, \sigma_2)$-continuous;

(2) $c_\mu(F^+(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))) \subseteq F^+(\sigma_1\sigma_2\text{-}Cl(V))$ for every $(\sigma_1, \sigma_2)p$-open set V of Y;

(3) $c_\mu(F^+(V)) \subseteq F^+(\sigma_1\sigma_2\text{-}Cl(V))$ for every $(\sigma_1, \sigma_2)p$-open set V of Y;

(4) $F^-(V) \subseteq i_\mu(F^-(\sigma_1\sigma_2\text{-}Cl(V)))$ for every $(\sigma_1, \sigma_2)p$-open set V of Y.

Proof. The proof is similar to that of Theorem 5.

Lemma 3. *If $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$ is lower weakly $\mu(\sigma_1, \sigma_2)$-continuous, then for each $x \in X$ and each subset B of Y with $(\sigma_1, \sigma_2)\theta$-Int$(B) \cap F(x) \neq \emptyset$, there exists a μ-open set U of X containing x such that $U \subseteq F^-(B)$.*

Proof. Since $(\sigma_1, \sigma_2)\theta\text{-Int}(B) \cap F(x) \neq \emptyset$, there exists a nonempty $\sigma_1\sigma_2$-open set V of Y such that $\sigma_1\sigma_2\text{-Cl}(V) \subseteq B$ and $V \cap F(x) \neq \emptyset$. Since F is lower weakly $\mu(\sigma_1, \sigma_2)$-continuous, there exists a μ-open set U of X containing x such that $\sigma_1\sigma_2\text{-Cl}(V) \cap F(z) \neq \emptyset$ for each $z \in U$ and hence $U \subseteq F^-(B)$.

Theorem 7. *For a multifunction $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is lower weakly $\mu(\sigma_1, \sigma_2)$-continuous;

(2) $c_\mu(F^+(B)) \subseteq F^+((\sigma_1, \sigma_2)\theta\text{-}Cl(B))$ for every subset B of Y;

(3) $F(c_\mu(A)) \subseteq (\sigma_1, \sigma_2)\theta\text{-}Cl(F(A))$ for every subset A of X.

Proof. (1) ⇒ (2): Let B be any subset of Y. Suppose that $x \notin F^+((\sigma_1, \sigma_2)\theta\text{-Cl}(B))$. Then, we have $x \in F^-(Y - (\sigma_1, \sigma_2)\theta\text{-Cl}(B)) = F^-((\sigma_1, \sigma_2)\theta\text{-Int}(Y - B))$. By Lemma 3, there exists a μ-open set U of X containing x such that $U \subseteq F^-(Y - B) = X - F^+(B)$. Therefore, $U \cap F^+(B) = \emptyset$. Thus, $x \notin \tau_1\tau_2\text{-Cl}(F^+(B))$ and hence

$$c_\mu(F^+(B)) \subseteq F^+((\sigma_1, \sigma_2)\theta\text{-Cl}(B)).$$

(2) ⇒ (3): Let A be any subset of X. By (2), we have

$$c_\mu(A) \subseteq c_\mu(F^+(F(A))) \subseteq F^+((\sigma_1, \sigma_2)\theta\text{-Cl}(F(A))).$$

Thus, $F(c_\mu(A)) \subseteq (\sigma_1, \sigma_2)\theta\text{-Cl}(F(A))$.

(3) ⇒ (1): Let V be any $\sigma_1\sigma_2$-open set of Y. Then, $\sigma_1\sigma_2\text{-Cl}(V) = (\sigma_1, \sigma_2)\theta\text{-Cl}(V)$ and by (3), $F(c_\mu(F^+(V))) \subseteq (\sigma_1, \sigma_2)\theta\text{-Cl}(F(F^+(V))) \subseteq (\sigma_1, \sigma_2)\theta\text{-Cl}(V) = \sigma_1\sigma_2\text{-Cl}(V)$. Thus, $c_\mu(F^+(V)) \subseteq F^+(\sigma_1\sigma_2\text{-Cl}(V))$ and by Theorem 1, F is lower weakly $\mu(\sigma_1, \sigma_2)$-continuous.

Theorem 8. *For a multifunction $F : (X, \mu) \to (Y, \sigma_1, \tau_2)$, the following properties are equivalent:*

(1) F is upper weakly $\mu(\sigma_1, \sigma_2)$-continuous;

(2) $c_\mu(F^-(\sigma_1\sigma_2\text{-}Int((\sigma_1, \sigma_2)\theta\text{-}Cl(B)))) \subseteq F^-((\sigma_1, \sigma_2)\theta\text{-}Cl(B))$ for every subset B of Y;

(3) $c_\mu(F^-(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(B)))) \subseteq F^-((\sigma_1, \sigma_2)\theta\text{-}Cl(B))$ for every subset B of Y.

Proof. (1) ⇒ (2): Let B be any subset of Y. Then, $(\sigma_1, \sigma_2)\theta\text{-Cl}(B)$ is $\sigma_1\sigma_2$-closed in Y and by Theorem 2, $c_\mu(F^-(\sigma_1\sigma_2\text{-Int}((\sigma_1, \sigma_2)\theta\text{-Cl}(B)))) \subseteq F^-((\sigma_1, \sigma_2)\theta\text{-Cl}(B))$.

(2) ⇒ (3): The proof is obvious.

(3) ⇒ (1): Let K be any $(\sigma_1, \sigma_2)r$-closed set of Y. Then, we have

$$(\sigma_1, \sigma_2)\theta\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K)) = \sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K)) = K$$

and hence

$$\begin{aligned} c_\mu(F^-(\sigma_1\sigma_2\text{-Int}(K))) &= c_\mu(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K)))) \\ &\subseteq F^-((\sigma_1, \sigma_2)\theta\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))) \\ &= F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))) \\ &= F^-(K). \end{aligned}$$

Thus by Theorem 2, F is upper weakly $\mu(\sigma_1, \sigma_2)$-continuous.

Theorem 9. *For a multifunction $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is lower weakly $\mu(\sigma_1, \sigma_2)$-continuous;

(2) $c_\mu(F^+(\sigma_1\sigma_2\text{-}Int((\sigma_1, \sigma_2)\theta\text{-}Cl(B)))) \subseteq F^+((\sigma_1, \sigma_2)\theta\text{-}Cl(B))$ for every subset B of Y;

(3) $c_\mu(F^+(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(B)))) \subseteq F^+((\sigma_1, \sigma_2)\theta\text{-}Cl(B))$ for every subset B of Y.

Proof. The proof is similar to that of Theorem 10.

Definition 5. [20] *A multifunction $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$ is said to be upper $\mu(\sigma_1, \sigma_2)$-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open set V of Y such that $F(x) \subseteq V$, there exists a μ-open set U of X containing x such that $F(U) \subseteq V$. A multifunction $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$ is said to be upper $\mu(\sigma_1, \sigma_2)$-continuous if F is upper $\mu(\sigma_1, \sigma_2)$-continuous at each point x of X.*

Definition 6. [20] *A multifunction $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$ is said to be lower $\mu(\sigma_1, \sigma_2)$-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open set V of Y such that $F(x) \cap V \neq \emptyset$, there exists a μ-open set U of X containing x such that $F(z) \cap V \neq \emptyset$ for every $z \in U$. A multifunction $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$ is said to be lower $\mu(\sigma_1, \sigma_2)$-continuous if F is lower $\mu(\sigma_1, \sigma_2)$-continuous at each point x of X.*

Recall that a subset A of a bitopological space (X, τ_1, τ_2) is said to be $\tau_1\tau_2$*-paracompact* [15] if every cover of A by $\tau_1\tau_2$-open sets of X is refined by a cover of A which consists of $\tau_1\tau_2$-open sets of X and is $\tau_1\tau_2$-locally finite in X. A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\tau_1\tau_2$*-regular* [15] if for each $x \in A$ and each $\tau_1\tau_2$-open set U of X containing x, there exists a $\tau_1\tau_2$-open set V of X such that $x \in V \subseteq \tau_1\tau_2\text{-Cl}(V) \subseteq U$.

Lemma 4. [15] *If A is a $\tau_1\tau_2$-regular $\tau_1\tau_2$-paracompact set of a bitopological space (X, τ_1, τ_2) and U is a $\tau_1\tau_2$-open neighbourhood of A, then there exists a $\tau_1\tau_2$-open set V of X such that $A \subseteq V \subseteq \tau_1\tau_2\text{-}Cl(V) \subseteq U$.*

Theorem 10. *For a multifunction $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$ such that $F(x)$ is a $\sigma_1\sigma_2$-regular $\sigma_1\sigma_2$-paracompact set of Y for each point $x \in X$, the following properties are equivalent:*

(1) F is upper $\mu(\sigma_1, \sigma_2)$-continuous;

(2) F is upper almost $\mu(\sigma_1, \sigma_2)$-continuous;

(3) F is upper weakly $\mu(\sigma_1, \sigma_2)$-continuous.

Proof. We show only the implication (3) ⇒ (1) since the others are obvious. Suppose that F is upper weakly $\mu(\sigma_1, \sigma_2)$-continuous. Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y such that $F(x) \subseteq V$. Since $F(x)$ is $\sigma_1\sigma_2$-regular $\sigma_1\sigma_2$-paracompact, by Lemma 4 there exists a $\sigma_1\sigma_2$-open set W of Y such that $F(x) \subseteq W \subseteq \sigma_1\sigma_2\text{-Cl}(W) \subseteq V$. Since F is upper weakly $\mu(\sigma_1, \sigma_2)$-continuous, there exists a μ-open set U of X containing x such that $F(U) \subseteq \sigma_1\sigma_2\text{-Cl}(W)$; hence $F(U) \subseteq V$. This shows that F is upper $\mu(\sigma_1, \sigma_2)$-continuous.

Recall that a bitopological space (X, τ_1, τ_2) is said to be $\tau_1\tau_2$-*compact* [15] if every cover of X by $\tau_1\tau_2$-open sets of X has a finite subcover.

Definition 7. [21] *A bitopological space (X, τ_1, τ_2) is said to be (τ_1, τ_2)-regular if for each $\tau_1\tau_2$-closed set F and each $x \in X - F$, there exist disjoint $\tau_1\tau_2$-open sets U and V such that $x \in U$ and $F \subseteq V$.*

Corollary 1. *Let $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$ be a multifunction such that $F(x)$ is $\sigma_1\sigma_2$-compact for each point $x \in X$ and (Y, σ_1, σ_2) is (σ_1, σ_2)-regular. Then, the following properties are equivalent:*

(1) F is upper $\mu(\sigma_1, \sigma_2)$-continuous;

(2) F is upper almost $\mu(\sigma_1, \sigma_2)$-continuous;

(3) F is upper weakly $\mu(\sigma_1, \sigma_2)$-continuous.

Lemma 5. [22] *If A is a $\tau_1\tau_2$-regular set of a bitopological space (X, τ_1, τ_2), then for each $\tau_1\tau_2$-open set G which intersect A, there exists a $\tau_1\tau_2$-open set W such that $A \cap W \neq \emptyset$ and $\tau_1\tau_2$-Cl$(W) \subseteq G$.*

Theorem 11. *For a multifunction $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$ such that $F(x)$ is a $\sigma_1\sigma_2$-regular set of Y for each point $x \in X$, the following properties are equivalent:*

(1) F is lower $\mu(\sigma_1, \sigma_2)$-continuous;

(2) F is lower almost $\mu(\sigma_1, \sigma_2)$-continuous;

(3) F is lower weakly $\mu(\sigma_1, \sigma_2)$-continuous.

Proof. We show only the implication (3) $\Rightarrow$ (1) since the others are obvious. Suppose that F is lower weakly $\mu(\sigma_1, \sigma_2)$-continuous. Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y such that $V \cap F(x) \neq \emptyset$. Since $F(x)$ is $\sigma_1\sigma_2$-regular, by Lemma 5 there exists a $\sigma_1\sigma_2$-open set W of Y such that $F(x) \cap W \neq \emptyset$ and $\sigma_1\sigma_2$-Cl$(W) \subseteq V$. Since F is lower weakly $\mu(\sigma_1, \sigma_2)$-continuous, there exists a μ-open set U of X containing x such that $\sigma_1\sigma_2$-Cl$(W) \cap F(z) \neq \emptyset$; hence $F(z) \cap V \neq \emptyset$ for each $z \in U$. This shows that F is lower $\mu(\sigma_1, \sigma_2)$-continuous.

Definition 8. [23] *A bitopological space (X, τ_1, τ_2) is said to be (τ_1, τ_2)-normal if for each pair of disjoint $\tau_1\tau_2$-closed sets F and F', there exist disjoint $\tau_1\tau_2$-open sets U and V such that $F \subseteq U$ and $F' \subseteq V$.*

Theorem 12. *If $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$ is a multifunction such that $F(x)$ is $\sigma_1\sigma_2$-closed in Y for each $x \in X$ and (Y, σ_1, σ_2) is a (σ_1, σ_2)-normal space, then the following properties are equivalent:*

(1) F is upper $\mu(\sigma_1, \sigma_2)$-continuous;

(2) F is upper almost $\mu(\sigma_1, \sigma_2)$-continuous;

(3) F is upper weakly $\mu(\sigma_1, \sigma_2)$-continuous.

Proof. As in Theorem 10, we prove only the implication (3) $\Rightarrow$ (1). Suppose that F is upper weakly $\mu(\sigma_1, \sigma_2)$-continuous. Let $x \in X$ and G be any $\sigma_1\sigma_2$-open set of Y containing $F(x)$. Since $F(x)$ is $\sigma_1\sigma_2$-closed in Y, by the (σ_1, σ_2)-normality of Y there exists a $\sigma_1\sigma_2$-open set V of Y such that $F(x) \subseteq V \subseteq \sigma_1\sigma_2$-Cl$(V) \subseteq G$. Since F is upper weakly $\mu(\sigma_1, \sigma_2)$-continuous, there exists a μ-open set U of X containing x such that $F(U) \subseteq \sigma_1\sigma_2$-Cl$(V) \subseteq G$. This shows that F is upper $\mu(\sigma_1, \sigma_2)$-continuous.

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