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Certain Class of Analytic Functions Connected with the q-Analogue of the Le Roy-Type Mittag-Leffler Function

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Abstract. This article focuses on the introduction of a new subclass of analytic mappings, specifically involving the q-analog of the Le Roy-type Mittag-Leffler mapping. We derive coefficient inequalities and explore various properties, including growth and distortion, as well as the radii of close-to-convexity and starlikeness. Furthermore, we examine convex linear combinations, partial sums, convolutions, and neighborhood properties of this newly defined class.

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1. Introduction

Let A specify the category of analytic mappings $\aleph$ represent on the unit disk $\Delta = \{\omega : |\omega| < 1\}$ with normalization $\aleph(0) = 0$ and $\aleph'(0) = 1$, such a mapping possesses an extension of the Taylor series on the origin in the type

$$\aleph(\omega) = \omega + \sum_{\jmath=2}^{\infty} a_\jmath \omega^\jmath. \tag{1}$$

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S, demonstrates that the a category of A has been made up of univalent mappings in Δ.

If it delights the pursing, then a $\aleph(\omega)$ mapping of A is sometimes referred to as starlike and convex of order $\wp$.

$$\Re\left\{\frac{\omega\aleph'(\omega)}{\aleph(\omega)}\right\} > \wp, \quad (\omega \in \Delta),$$

$$\text{and} \quad \Re\left\{1 + \frac{\omega\aleph''(\omega)}{\aleph'(\omega)}\right\} > \wp, \quad (\omega \in \Delta),$$

The subclass of A can be illustrated by $S^*(\wp)$ and $K(\wp)$, respectively, with respect to particular $\wp(0 \leq \wp < 1)$. Another benefit is that, by T, designate the class under consideration of A composed of these kinds of mappings.

$$\aleph(\omega) = \omega - \sum_{\jmath=2}^{\infty} a_\jmath \omega^\jmath, \ (a_\jmath \geq 0, \ \omega \in \Delta) \tag{2}$$

and let $T^*(\wp) = T \cap S^*(\wp), C(\wp) = T \cap K(\wp)$. Silverman [1] and others complete a thorough analysis of the $T^*(\wp)$ and $C(\wp)$ classes, which contain intriguing properties.

Recently, q-calculus has attracted significant attention among researchers due to its wide range of mathematical and physical applications. Its advantages are the reason for its considerable attention in many branches of physics and mathematics. The significance of the $q-$ derivative operator D_q is clear from its potential uses in the analysis that has many subclasses of analytical mappings.

Researchers are increasingly intrigued by the exploration of q-calculus. It has attracted a lot of attention in many areas of mathematics and physics because of its benefits. The potential applications of the $q-$ derivative operator D_q in analysis, which has numerous subclasses of analytical mappings, make its significance evident. Ismail et al. [2] first introduced the idea of $q-$star mappings in 1990. Nevertheless, a solid foundation for applying the $q-$calculus was successfully established in Geometric Function Theory. For example, it is used to determine the velocity and stress in the rotational flow of Burge's fluid through an unbounded round channel [2].

Since then, numerous mathematics researchers have conducted excellent studies that have. Separately, apart from that, researchers and academics working on these themes may find value in a survey-cum-expository analytic piece just released by Srivastava [3]. This survey cum-expository analysis article [3] thoroughly examined the mathematical explanation and practical consequences of the fractional $q-$ calculus and fractional $q-$ derivative operators in Geometric Function Theory. Specifically, a couple of mapping groups of conical region-related $q-$ star-like mappings were also taken into consideration by Srivastava et al. [4]. To learn more about other recent studies using the $q-$calculus, see [5–14].

Definition 1. *([15]) Take into account that* $0 < q < 1$. *The fundamental (or q -) number* *is reflected by the* $[\jmath]_q$*, which is specified by*

$$[\jmath]_q = \begin{cases} \frac{1-q^\jmath}{1-q}, & \jmath \in \mathbb{C} \setminus \{0\}, \\ 0, & \jmath = 0, \\ 1 + q + \cdots + q^{n-1} = \sum_{i=0}^{n-1} q^i, & \jmath = n \in \mathbb{N} \end{cases}.$$

Definition 1 signifies in explicit that

$$\lim_{q \to 1^-} [n]_q = \lim_{q \to 1^-} \frac{1-q^n}{1-q} = n.$$

Definition 2. *([15]) The definition of the* $q-$*derivative is known as the q-difference operator, of a mapping* $\aleph$ *is*

$$\partial_q \aleph(\omega) = \begin{cases} \frac{\aleph(\omega)-\aleph(q\omega)}{\omega - q\omega}, & \omega \in \mathbb{C} \setminus \{0\}, \\ \aleph'(0), & \omega = 0 \end{cases}.$$

We look at that $\lim_{q \to 1^-} \partial_q \aleph(\omega) = \aleph'(\omega)$, if ω is differentiable at $\omega \in \mathbb{C}$.

With $\Re(\chi) > 0$ and $\Re(\varrho) > 0$, for a parameter $\chi, \varrho \in \mathbb{C}$, Wiman [16] invented the expanded Mittag-Leffler-type mapping, which can be described by

$$M_{\varrho,\chi}(\omega) = \sum_{\jmath=0}^{\infty} \frac{\omega^\jmath}{\Gamma(\varrho\jmath + \chi)'} \quad ,(\omega \in \mathbb{C}). \tag{3}$$

Schneider [17] and Garra and Polito [18] recently laid out the Le Roy-type Mittag-Leffler mapping, which has been defined as, for $\Re(\chi) > 0, \Re(\varrho) > 0$,

$$F^{\gamma}_{\varrho,\chi}(\omega) = \sum_{\jmath=0}^{\infty} \frac{\omega^\jmath}{[\Gamma(\varrho\jmath + \chi)]^\gamma}, \quad (\varrho, \chi, \gamma > 0, \omega \in \mathbb{C}),. \tag{4}$$

In 2014, [19], Sharma and Jain introduced the q -Mittag-Leffler-type mapping by

$$M^{\gamma}_{\varrho,\chi}(\omega; q) = \sum_{\jmath=0}^{\infty} \frac{(q^\gamma, q)}{(q;q)_n} . \frac{\omega^\jmath}{\Gamma_q(\varrho\jmath + \chi)}, \quad (\varrho, \chi, \gamma \in \mathbb{C}), \tag{5}$$

where $|q| < 1$ and Γ_q is the q -gamma mapping provided by

$$\Gamma_q(1 + \omega) = (1 - q^\omega)(1 - q)^{-1}\Gamma_q(\omega), \quad (q \in (0, 1), \omega \in \mathbb{C}).$$

Inspired by Gerhold [20], and Garra and Polito [18], we introduce the q -analogue of the Le Roy-type Mittag- Leffler mapping by

$$M^{\gamma}_{\varrho,\chi}(\omega; q) = \sum_{\jmath=0}^{\infty} \frac{\omega^\jmath}{(\Gamma_q(\varrho\jmath + \chi))^\gamma} \quad (\omega \in \Delta), \tag{6}$$

where $\Re(\varrho) > 0$, $\chi \in \mathbb{C} \setminus \{0, -1, -2, \dots\}$.

The Le Roy-type Mittag-Leffler mapping $M^{\gamma}_{\varrho,\chi}(\omega; q)$'s q -analog's normalization may be outlined as

$$\mathfrak{M}^{\gamma}_{\varrho,\chi}(\omega; q) = \omega \left(\Gamma_q(\chi)\right)^{\gamma} M^{\gamma}_{\varrho,\chi}(\omega; q) = \omega + \sum_{\jmath=1}^{\infty} \left(\frac{\Gamma_q(\chi)}{\Gamma_q(\varrho(\jmath - 1) + \chi)} \right)^{\gamma} \omega^{\jmath}, \tag{7}$$

where $\Re(\varrho) > 0$, $\chi \in \mathbb{C} \setminus \{0, -1, -2, \cdots\}$.

The corresponding new q-operators can be identified as follows: $H^{\gamma}_{\varrho,\chi;q}\aleph(\omega)$

$$\begin{aligned} H^{\gamma}_{\varrho,\chi;q}\aleph(\omega) &= \mathfrak{M}^{\gamma}_{\varrho,\chi}(\omega; q) * E(\omega) \\ &= \omega + \sum_{\jmath=2}^{\infty} \left(\frac{\Gamma_q(\chi)}{\Gamma_q(\varrho(\jmath - 1) + \chi)} \right)^{\gamma} a_{\jmath}\omega^{\jmath}, \quad (\omega \in \Delta) \\ &= \omega + \sum_{\jmath=2}^{\infty} \Theta(\jmath) a_{\jmath}\omega^{\jmath}, \end{aligned} \tag{8}$$

where

$$\Theta(\jmath) = \left(\frac{\Gamma_q(\chi)}{\Gamma_q(\varrho(\jmath - 1) + \chi)} \right)^{\gamma}.$$

Inspired by the early works of [21–23], we define a new class $\phi^{\gamma}_{\varrho,\chi,q}(\hbar, \wp)$ of A concerning the $q-$ analogue of the Le Roy-type Mittag-Leffler mapping is below:

Definition 1. *For $0 \le \hbar < 1, 0 \le \wp < 1, \varrho > 0, \chi > 0, \gamma > -1$ and $0 < q < 1$, we say $\aleph(w) \in A$ is in $\phi^{\gamma}_{\varrho,\chi,q}(\hbar, \wp)$ if it fulfills the requirement*

$$\Re \left(\frac{w \left(H^{\gamma}_{\varrho,\chi;q}\aleph(w)\right)' + \hbar w^2 \left(H^{\gamma}_{\varrho,\chi;q}\aleph(w)\right)''}{H^{\gamma}_{\varrho,\chi;q}\aleph(w)} \right) > \wp, \ (w \in \Delta).$$

Also, we indicate by $T\phi^{\gamma}_{\varrho,\chi,q}(\hbar, \wp) = \phi^{\gamma}_{\varrho,\chi,q}(\hbar, \wp) \cap T$.

2. Coefficient Inequalities

A acceptable condition for a mapping $\aleph$ given by (1) to be in $\phi^{\gamma}_{\varrho,\chi,q}(\hbar, \wp)$. is stipulated in this part.

Theorem 3. *A mapping $\aleph \in A$ is allocated to the class $\phi^{\gamma}_{\varrho,\chi,q}(\hbar, \wp)$ if*

$$\sum_{\jmath=2}^{\infty} [\jmath + \hbar\jmath(\jmath - 1) - \wp]\Theta(\jmath)|a_{\jmath}| \le 1 - \wp. \tag{9}$$

Proof. If we adopt $0 \leq \wp < 1$ and $\hbar \geq 0$, then

$$\varrho(w) = \frac{w\left(H^{\gamma}_{\varrho,\chi;q}\aleph(w)\right)' + \hbar w^2\left(H^{\gamma}_{\varrho,\chi;q}\aleph(w)\right)''}{H^{\gamma}_{\varrho,\chi;q}\aleph(w)}, \quad (w \in \Delta).$$

To prove this , we recall $|\varrho(w) - 1| < 1 - \wp, (w \in \Delta)$.
If $\aleph(w) = w \ (w \in \Delta)$, then we have $\varrho(w) = w(w \in \Delta)$.
It goes without saying that 9 is valid.
If $\aleph(w) \neq w \ (|w| = r < 1)$, as a result, a coefficient exists $\Omega_\jmath(\varrho, \chi, \gamma)a_\jmath \neq 0$ for some $\jmath \geq 2$.
The consequence is that $\sum\limits_{\jmath=2}^{\infty} \Theta(\jmath)|a_\jmath| > 0$. Additionally, take note of

$$\sum_{\jmath=2}^{\infty}[\jmath + \hbar\jmath(\jmath - 1) - \wp]\Theta(\jmath)|a_\jmath| > (1 - \wp)\sum_{\jmath=2}^{\infty}\Theta(\jmath)|a_\jmath|$$

$$\Rightarrow \quad \sum_{\jmath=2}^{\infty}\Theta(\jmath)|a_\jmath| < 1.$$

By (9), we acquire

$$\begin{aligned}
|\varrho(w) - 1| &= \left| \frac{\sum\limits_{\jmath=2}^{\infty}[\jmath + \hbar\jmath(\jmath - 1) - 1]\Theta(\jmath)a_\jmath w^{\jmath-1}}{1 + \sum\limits_{\jmath=2}^{\infty}\Theta(\jmath)a_\jmath w^{\jmath-1}} \right| \\
&< \frac{\sum\limits_{\jmath=2}^{\infty}[\jmath + \hbar\jmath(\jmath - 1) - 1]\Theta(\jmath)|a_\jmath|}{1 - \sum\limits_{\jmath=2}^{\infty}\Theta(\jmath)|a_\jmath|} \\
&\leq \frac{\sum\limits_{\jmath=2}^{\infty}[\jmath + \hbar\jmath(\jmath - 1) - \wp]\Theta(\jmath)|a_\jmath| - (1 - \wp)\Theta(\jmath)|a_\jmath|}{1 - \sum\limits_{\jmath=2}^{\infty}\Theta(\jmath)|a_\jmath|} \\
&\leq \frac{(1 - \wp) - (1 - \wp)\sum\limits_{\jmath=2}^{\infty}\Theta(\jmath)|a_\jmath|}{1 - \sum\limits_{\jmath=2}^{\infty}\Theta(\jmath)|a_\jmath|} \\
&= 1 - \wp, \quad (w \in \Delta).
\end{aligned}$$

Hence, we obtain

$$\Re\left(\frac{w\left(H^{\gamma}_{\varrho,\chi;q}\aleph(w)\right)' + \hbar w^2\left(H^{\gamma}_{\varrho,\chi;q}\aleph(w)\right)''}{H^{\gamma}_{\varrho,\chi;q}\aleph(w)}\right) = \Re(\varrho(w)) > 1 - (1 - \wp) = \wp.$$

Then $\aleph \in \phi^{\gamma}_{\varrho,\chi,q}(\hbar, \wp)$.

Theorem 4. *Let $\aleph$ be provided by (2). Then the mapping $\aleph \in T\phi^{\gamma}_{\varrho,\chi,q}(\hbar,\wp)$*

$$\Leftrightarrow \sum_{\jmath=2}^{\infty}[\jmath+\hbar\jmath(\jmath-1)-\wp]\Theta(\jmath)|a_\jmath| \leq 1-\wp. \tag{10}$$

Proof. Looking at Theorem 3, to examine it $\aleph \in T\phi^{\gamma}_{\varrho,\chi,q}(\hbar,\wp)$ meets the coefficient prerequisites inequality (9). If $\aleph \in T\phi^{\gamma}_{\varrho,\chi,q}(\hbar,\wp)$ then the mapping

$$\varrho(w) = \frac{w\left(H^{\gamma}_{\varrho,\chi;q}\aleph(w)\right)' + \hbar w^2\left(H^{\gamma}_{\varrho,\chi;q}\aleph(w)\right)''}{H^{\gamma}_{\varrho,\chi;q}\aleph(w)}, \quad (w \in \Delta)$$

satisfies $\Re(\varrho(w)) > \wp$. This suggests something else

$$H^{\gamma}_{\varrho,\chi;q}\aleph(w) = w - \sum_{\jmath=2}^{\infty}\Theta(\jmath)|a_\jmath|w^\jmath \neq 0, \quad (w \in \Delta\setminus\{0\}).$$

Recognising that $\frac{H^{\gamma}_{\varrho,\chi;q}\aleph(r)}{r}$ is in $(0,1)$, The true perpetual mapping is this with $\aleph(0)=1$, we have

$$\frac{H^{\gamma}_{\varrho,\chi;q}\aleph(r)}{r} = 1 - \sum_{\jmath=2}^{\infty}\Theta(k)|a_\jmath|r^{\jmath-1} > 0, \quad (0<r<1). \tag{11}$$

Now $\wp < \varrho(r) = \frac{1-\sum_{\jmath=2}^{\infty}[\jmath+\hbar\jmath(\jmath-1)]\Theta(\jmath)|a_\jmath|r^{\jmath-1}}{1-\sum_{\jmath=2}^{\infty}\Theta(\jmath)|a_\jmath|r^{\jmath-1}}$ and consequently by (11),

we get $\sum_{\jmath=2}^{\infty}[\jmath+\hbar\jmath(\jmath-1)-\wp]\Theta(\jmath)|a_\jmath|r^{\jmath-1} \leq 1-\wp$.

Setting $r \to 1$, we acquire $\sum_{\jmath=2}^{\infty}[\jmath+\hbar\jmath(\jmath-1)-\wp]\Theta(\jmath)|a_\jmath| \leq 1-\wp$.

This proves the converse part.

Remark 1. *If a mapping $\aleph \in T\phi^{\gamma}_{\varrho,\chi,q}(\hbar,\wp)$ then*

$$|a_\jmath| \leq \frac{1-\wp}{[\jmath+\hbar\jmath(\jmath-1)-\wp]\Theta(\jmath)}, \quad (\jmath \geq 2).$$

For the mappings, the equality is preserved

$$\aleph_\jmath(w) = w - \frac{1-\wp}{[\jmath+\hbar\jmath(\jmath-1)-\wp]\Theta(\jmath)}w^\jmath, (w \in \Delta, \jmath \geq 2). \tag{12}$$

3. Distortion Theorem

In this section, we discussed the distortion ranges of the class's mappings $T\phi^{\gamma}_{\varrho,\chi,q}(\hbar,\wp)$.

Theorem 5. *Let* $\aleph \in T\phi^{\gamma}_{\varrho,\chi,q}(\hbar,\wp)$ *and* $|w| = r < 1$. *Then*

$$r - \frac{1-\wp}{[2\hbar - \wp + 2]\Theta(2)} r^2 \leq |\aleph(\omega)| \leq r + \frac{1-\wp}{[2\hbar - \wp + 2]\Theta(2)} r^2 \tag{13}$$

and

$$1 - \frac{2(1-\wp)}{[2\hbar - \wp + 2]\Theta(2)} r \leq |\aleph'(\omega)| \leq 1 + \frac{2(1-\wp)}{[2\hbar - \wp + 2]\Theta(2)} r. \tag{14}$$

(12) *displays the* $\aleph_2(\omega)$ *extreme mapping, signifying an extreme approach.*

Proof. Since $\aleph \in T\phi^{\gamma}_{\varrho,\chi,q}(\hbar,\wp)$, We incorporate Theorem 4 to acquire

$$\begin{aligned}[2\hbar - \wp + 2]\Theta(2) \sum_{\jmath=2}^{\infty} |a_\jmath| &\leq \sum_{\jmath=2}^{\infty} [\jmath + \hbar\jmath(\jmath-1) - \wp]\Theta(\jmath)|a_\jmath| \\ &\leq 1 - \wp.\end{aligned}$$

$$\text{Thus,} \quad |\aleph(\omega)| \leq |\omega| + |\omega|^2 \sum_{\jmath=2}^{\infty} |a_\jmath| \leq r + \frac{1-\wp}{[2\hbar - \wp + 2]\Theta(2)}\, r^2.$$

$$\text{Also, we have} \quad |\aleph(\omega)| \leq |\omega| - |\omega|^2 \sum_{\jmath=2}^{\infty} |a_\jmath| \leq r - \frac{1-\wp}{[2\hbar - \wp + 2]\Theta(2)}\, r^2,$$

and (13) follows. In a related vein the inequalities for $\aleph'$,

$$|\aleph'(\omega)| \leq 1 + \sum_{\jmath=2}^{\infty} \jmath |a_\jmath| |\omega|^{\jmath-1} \leq 1 + |\omega| \sum_{\jmath=2}^{\infty} \jmath |a_\jmath|$$

and

$$\sum_{\jmath=2}^{\infty} \jmath |a_\jmath| \leq \frac{2(1-\wp)}{[2\hbar - \wp + 2]\Theta(2)}$$

are accomplished, resulting in (14).

Example 1. *Let us consider the analytic mapping*

$$\aleph(\omega) = \omega + a_2\omega^2 + a_3\omega^3 + \cdots,$$

which belongs to the class $T\phi^{\gamma}_{\varrho,\chi,q}(\hbar,\wp)$.
Choose the parameter values:

$$\hbar = 2, \quad \wp = 0.5, \quad \Theta(2) = 1, \quad r = 0.6.$$

Then, according to Theorem 13,

$$r - \frac{1-\wp}{[2\hbar - \wp + 2]\Theta(2)} r^2 \leq |\aleph(\omega)| \leq r + \frac{1-\wp}{[2\hbar - \wp + 2]\Theta(2)} r^2.$$

Substituting the chosen values, we have:

$$2\hbar - \wp + 2 = 2(2) - 0.5 + 2 = 5.5,$$

and hence

$$\frac{1-\wp}{[2\hbar - \wp + 2]\Theta(2)} = \frac{0.5}{5.5} \approx 0.0909.$$

Thus,

$$0.6 - 0.0909(0.6)^2 \leq |\aleph(\omega)| \leq 0.6 + 0.0909(0.6)^2,$$

$$0.6 - 0.0327 \leq |\aleph(\omega)| \leq 0.6 + 0.0327,$$

$$0.5673 \leq |\aleph(\omega)| \leq 0.6327.$$

Similarly, from inequality (14),

$$1 - \frac{2(1-\wp)}{[2\hbar - \wp + 2]\Theta(2)} r \leq |\aleph'(\omega)| \leq 1 + \frac{2(1-\wp)}{[2\hbar - \wp + 2]\Theta(2)} r.$$

Substituting the same values gives:

$$\frac{2(1-\wp)}{[2\hbar - \wp + 2]\Theta(2)} r = \frac{2(0.5)(0.6)}{5.5} = 0.1091.$$

Hence,

$$1 - 0.1091 \leq |\aleph'(\omega)| \leq 1 + 0.1091,$$

$$0.8909 \leq |\aleph'(\omega)| \leq 1.1091.$$

Therefore, for the given parameters, the mapping $\aleph(\omega)$ satisfies the distortion bounds:

$$0.5673 \leq |\aleph(\omega)| \leq 0.6327, \quad 0.8909 \leq |\aleph'(\omega)| \leq 1.1091.$$

4. Radii of close-to-convexity and starlikeness

This segment yields a near-convex and star-like radius of this category $T\phi^{\gamma}_{\varrho,\chi,q}(\hbar,\wp)$.

Theorem 6. *Let $\aleph \in T\phi^{\gamma}_{\varrho,\chi,q}(\hbar,\wp)$. The order of close-to-convex $\gamma(0 \leq \gamma < 1)$ is thus $\aleph$, where $|\omega| < t_1$ in the disc*

$$t_1 = \inf_{\jmath \geq 2} \left[\frac{(1-\gamma)[\jmath + \jmath\hbar(\jmath - 1) - \wp]\Upsilon_{\jmath,q}(\varrho,\chi,\gamma)}{\jmath(1-\wp)} \right]^{\frac{1}{\jmath - 1}}.$$

(12) *demonstrates that the estimated value is sharp with the extremal mapping $\aleph(\omega)$.*

Proof. If the order of $\aleph$ is near to convex γ and $\aleph \in T$, then we acquire

$$|\aleph'(\omega) - 1| \leq 1 - \gamma. \tag{15}$$

For the L.H.S of (15), we obtain

$$|\aleph'(\omega) - 1| \leq \sum_{\jmath=2}^{\infty} \jmath a_\jmath |\omega|^{\jmath-1} < 1 - \gamma$$

$$\Rightarrow \sum_{\jmath=2}^{\infty} \frac{\jmath}{1-\gamma} a_\jmath |\omega|^{\jmath-1} \leq 1.$$

We know that $\aleph(\omega) \in T\phi^{\gamma}_{\varrho,\chi,q}(\hbar, \wp) \Leftrightarrow$

$$\sum_{\jmath=2}^{\infty} \frac{[\jmath + \jmath\hbar(\jmath-1) - \wp]\Upsilon_{\jmath,q}(\varrho,\chi,\gamma)}{(1-\wp)} a_\jmath \leq 1.$$

Thus (15) holds true if

$$\frac{\jmath}{1-\gamma}|\omega|^{\jmath-1} \leq \frac{[\jmath + \jmath\hbar(\jmath-1) - \wp]\Upsilon_{\jmath,q}(\varrho,\chi,\gamma)}{(1-\wp)}$$

$$\Rightarrow \quad |\omega| \leq \left[\frac{(1-\gamma)[\jmath + \jmath\hbar(\jmath-1) - \wp]\Upsilon_{\jmath,q}(\varrho,\chi,\gamma)}{\jmath(1-\wp)}\right]^{\frac{1}{\jmath-1}}$$

hence the proof.

Theorem 7. *Let $\aleph \in T\phi^{\gamma}_{\varrho,\chi,q}(\hbar, \wp)$. The order of starlike $\aleph$ is then γ, $(0 \leq \gamma < 1)$ in the disc $|\omega| < t_2$, where*

$$t_2 = \inf_{\jmath \geq 2} \left[\frac{(1-\gamma)[\jmath + \jmath\hbar(\jmath-1) - \wp]\Upsilon_{\jmath,q}(\varrho,\chi,\gamma)}{(\jmath-\gamma)(1-\wp)}\right]^{\frac{1}{\jmath-1}}.$$

(12) *demonstrates that the calculated value is sharp with the extremal mapping $\aleph(\omega)$.*

Proof. We have $\aleph \in T$ and $\aleph$ is order of starlike γ, then

$$\left|\frac{\omega\aleph'(\omega)}{\aleph(\omega)} - 1\right| < 1 - \gamma. \tag{16}$$

For the L.H.S of (16), we have

$$\left|\frac{\omega\aleph'(\omega)}{\aleph(\omega)} - 1\right| \leq \frac{\sum\limits_{\jmath=2}^{\infty}(\jmath-1)a_\jmath|\omega|^{\jmath-1}}{1 - \sum\limits_{\jmath=2}^{\infty} a_\jmath|\omega|^{\jmath-1}}$$

$(1-\gamma)$ is bigger than the R.H.S of the left relation if

$$\sum_{\jmath=2}^{\infty} \frac{\jmath-\gamma}{1-\gamma} a_\jmath |\omega|^{\jmath-1} < 1.$$

We know that $\aleph \in T\phi^{\gamma}_{\varrho,\chi,q}(\hbar,\wp)$

$$\Leftrightarrow \sum_{\jmath=2}^{\infty} \frac{[\jmath+\jmath\hbar(\jmath-1)-\wp]\Upsilon_{\jmath,q}(\varrho,\chi,\gamma)}{(1-\wp)} a_\jmath \leq 1.$$

Thus (16) is true if

$$\frac{\jmath-\gamma}{1-\gamma}|\omega|^{\jmath-1} \leq \frac{[\jmath+\jmath\hbar(\jmath-1)-\wp]\Upsilon_{\jmath,q}(\varrho,\chi,\gamma)}{(1-\wp)}$$

$$\Rightarrow \quad |\omega| \leq \left[\frac{(1-\gamma)[\jmath+\jmath\hbar(\jmath-1)-\wp]\Upsilon_{\jmath,q}(\varrho,\chi,\gamma)}{(\jmath-\gamma)(1-\wp)}\right]^{\frac{1}{\jmath-1}}.$$

It gives the family the starlikeness.

5. Convex Linear combinations

Theorem 8. *Let* $\aleph_1(\omega)=\omega$ *and*

$$\aleph_\jmath(\omega) = \omega - \frac{1-\wp}{[\jmath+\hbar\jmath(\jmath-1)-\wp]\Theta(\jmath)}\omega^\jmath, \qquad (\omega \in \Delta,\ \jmath \geq 2).$$

Then $\aleph \in T\phi^{\gamma}_{\varrho,\chi,q}(\hbar,\wp) \Leftrightarrow \aleph$ *in the way it can be articulated*

$$\aleph(\omega) = \sum_{\jmath=1}^{\infty} \mu_\jmath \aleph_\jmath(\omega), \quad (\mu_\jmath \geq 0) \tag{17}$$

and $\sum_{\jmath=1}^{\infty} \mu_\jmath = 1$.

Proof. If a mapping $\aleph$ is of the type $\aleph(\omega) = \sum_{\jmath=1}^{\infty} \mu_\jmath \aleph_\jmath(\omega)$, $\mu_\jmath \geq 0$ and $\sum_{\jmath=1}^{\infty} \mu_\jmath = 1$ then

$$\begin{aligned} &\sum_{\jmath=2}^{\infty}[\jmath+\hbar\jmath(\jmath-1)-\wp]\Theta(\jmath)|a_\jmath| \\ =&\sum_{\jmath=2}^{\infty}[\jmath+\hbar\jmath(\jmath-1)-\wp]\Theta(\jmath)\frac{(1-\wp)\mu_\jmath}{[\jmath+\hbar\jmath(\jmath-1)-\wp]\Theta(\jmath)} \end{aligned}$$

$$= \sum_{\jmath=2}^{\infty}(1-\wp)\mu_\jmath = (1-\mu_1)(1-\wp)$$
$$\leq (1-\wp)$$

which provides (10), hence $\aleph \in T\phi^{\gamma}_{\varrho,\chi,q}(\hbar,\wp)$, by Theorem 4.

On the other hand, if $\aleph \in T\phi^{\gamma}_{\varrho,\chi,q}(\hbar,\wp)$, then we may set

$$\mu_\jmath = \frac{[\jmath + \hbar\jmath(\jmath-1) - \wp]\Theta(\jmath)}{1-\wp}|a_\jmath|, \quad (\jmath \geq 2),$$

and $\mu_1 = 1 - \sum_{\jmath=2}^{\infty} \mu_\jmath$.

Then the mapping $\aleph$ is of the type (17) .

6. Partial Sums

The mapping of $\aleph \in A$ given by (1), was established using partial sums $\aleph$, which Silverman [24] examined.

$$\aleph_1(\omega) = \omega \text{ and } \aleph_m(\omega) = \omega + \sum_{\jmath=2}^{m} a_\jmath \omega^\jmath, \ m = 2, 3, 4, \cdots .$$

The class $\phi^{\gamma}_{\varrho,\chi,q}(\hbar,\wp)$ can consider partial mapping sums and sharp lower limits. True component ratios of $\aleph$ to $\aleph_m$ and $\aleph'$ to $\aleph'_m$.

Theorem 9. *Let* $\aleph \in \phi^{\gamma}_{\varrho,\chi,q}(\hbar,\wp)$ *and fulfills (9). Then*

$$\Re\left(\frac{\aleph(\omega)}{\aleph_m(\omega)}\right) \geq 1 - \frac{1}{d_{m+1}}, \ (\omega \in \Delta, \ m \in \mathbb{N}),$$

where

$$d_\jmath = \frac{[\jmath + \hbar\jmath(\jmath-1) - \wp]}{1-\wp}.$$

Proof. Clearly, $d_{\jmath+1} > d_\jmath > 1, \jmath = 2, 3, 4, \cdots$.

Thus, by Theorem 3 we acquire,

$$\sum_{\jmath=2}^{\infty} |a_\jmath| + d_{m+1}\sum_{\jmath=2}^{\infty} |a_\jmath| \leq \sum_{\jmath=2}^{\infty} d_\jmath |a_\jmath| \leq 1. \tag{18}$$

$$\text{Setting } \vartheta(\omega) = d_{m+1}\left\{\frac{\aleph(\omega)}{\aleph_m(\omega)} - \left(1 - \frac{1}{d_{m+1}}\right)\right\}$$

$$\vartheta(\omega) = 1 + \frac{d_{m+1}\sum_{\jmath=m+1}^{\infty} a_\jmath \omega^{\jmath-1}}{1 + \sum_{\jmath=2}^{m} a_\jmath \omega^{\jmath-1}} \tag{19}$$

It's important that it's good enough to be exhibited $\Re(\vartheta(\omega)) > 0, \ (\omega \in \Delta)$. Applying (18), we have the opinion that

$$\left|\frac{\vartheta(\omega)-1}{\vartheta(\omega)+1}\right| \leq \frac{d_{m+1}\sum\limits_{\jmath=2}^{\infty}|a_\jmath|}{2-2\sum\limits_{\jmath=2}^{m}|a_\jmath| - d_{m+1}\sum\limits_{\jmath=m+1}^{\infty}|a_\jmath|} \leq 1$$

which gives,

$$\Re\left(\frac{\aleph(\omega)}{\aleph_m(\omega)}\right) \geq 1 - \frac{1}{d_{m+1}},$$

hence the proof.

Theorem 10. *Let* $\aleph$ *in* $T\phi^{\gamma}_{\varrho,\chi,q}(\hbar,\wp)$ *and fulfills (9). Then*

$$\Re\left(\frac{\aleph_m(\omega)}{\aleph(\omega)}\right) \geq \frac{d_{m+1}}{1+d_{m+1}}, \quad (\omega \in \Delta, m \in \mathbb{N}),$$

where

$$d_\jmath = \frac{[\jmath + \hbar\jmath(\jmath-1) - \wp]}{1-\wp}.$$

Proof. Clearly, $d_{\jmath+1} > d_\jmath > 1, \ \jmath = 2, 3, 4, \cdots$.
Thus, by Theorem 3 we get,

$$\sum_{\jmath=2}^{\infty}|a_\jmath| + d_{m+1}\sum_{\jmath=m+1}^{\infty}|a_\jmath| \leq \sum_{\jmath=2}^{\infty} d_\jmath|a_\jmath| \leq 1. \tag{20}$$

Setting $h(\omega) = (1+d_{m+1})\left\{\frac{\aleph_m(\omega)}{\aleph(\omega)} - \left(\frac{d_{m+1}}{1+d_{m+1}}\right)\right\}$

$$h(\omega) = 1 - \frac{(1+d_{m+1})\sum\limits_{\jmath=m+1}^{\infty} a_\jmath\omega^{\jmath-1}}{1+\sum\limits_{\jmath=2}^{m} a_\jmath\omega^{\jmath-1}} \tag{21}$$

to demonstrate $\Re(h(\omega)) > 0, \ \ (\omega \in \Delta)$. Implementing (20) we gain

$$\left|\frac{h(\omega)-1}{h(\omega)+1}\right| \leq \frac{(1+d_{m+1})\sum\limits_{\jmath=2}^{\infty}|a_\jmath|}{2-2\sum\limits_{\jmath=2}^{m}|a_\jmath| - (1+d_{m+1})\sum\limits_{\jmath=m+1}^{\infty}|a_\jmath|} \leq 1$$

which gives,

$$\Re\left(\frac{\aleph_m(\omega)}{\aleph(\omega)}\right) \geq \frac{d_{m+1}}{1+d_{m+1}},$$

and hence the results.

Theorem 11. *Let* $\aleph$ *in* $T\phi^{\gamma}_{\varrho,\chi,q}(\hbar,\wp)$ *and fulfills (9). Then*

$$\Re\left(\frac{\aleph'(\omega)}{\aleph'_m(\omega)}\right) \geq 1-\frac{m+1}{d_{m+1}}, \quad (\omega \in \Delta, m \in \mathbb{N}),$$

and

$$\Re\left(\frac{\aleph'_m(\omega)}{\aleph'(\omega)}\right) \geq \frac{d_{m+1}}{m+1+d_{m+1}}, \quad (\omega \in \Delta, m \in \mathbb{N})$$

where

$$d_\jmath = \frac{[\jmath+\hbar\jmath(\jmath-1)-\wp]}{1-\wp}.$$

Proof. By Setting

$$\vartheta(\omega) = d_{m+1}\left\{\frac{\aleph('\omega)}{\aleph'_m(\omega)} - \left(1-\frac{m+1}{d_{m+1}}\right)\right\}, \quad (\omega \in \Delta)$$

$$\text{and} \quad h(\omega) = (m+1+d_{m+1})\left\{\frac{\aleph'_m(\omega)}{\aleph('\omega)} - \left(\frac{d_{m+1}}{m+1+d_{m+1}}\right)\right\}, \quad (\omega \in \Delta).$$

The evidence is similar to that of Theorems 9 and 10, so the proofs are omitted.

7. Convolution properties

We will prove in this section that the $T\phi^{\gamma}_{\varrho,\chi,q}(\hbar,\wp)$ class is closed by convolution.

Theorem 12. *Let* $\vartheta(\omega)$ *be of the form*

$$\vartheta(\omega) = \omega - \sum_{\jmath=2}^{\infty} b_\jmath \omega^\jmath,$$

which is analytic in Δ*, and suppose that* $|b_\jmath| \leq 1$ *for all* $\jmath \geq 2$*. If* $\aleph \in T\phi^{\gamma}_{\varrho,\chi,q}(\hbar,\wp)$*, then*

$$\aleph * \vartheta \in T\phi^{\gamma}_{\varrho,\chi,q}(\hbar,\wp),$$

where the symbol $*$ *denotes the Hadamard product (convolution).*

Proof. Since $\aleph \in T\phi^{\gamma}_{\varrho,\chi,q}(\hbar,\wp)$, we have

$$\sum_{\jmath=2}^{\infty}[\jmath+\hbar\jmath(\jmath-1)-\wp]\Theta(\jmath)|a_\jmath| \leq 1-\wp.$$

Noting that

$$(\aleph * \vartheta)(\omega) = \omega - \sum_{\jmath=2}^{\infty} a_\jmath b_\jmath \omega^\jmath,$$

we obtain

$$\begin{aligned}\sum_{\jmath=2}^{\infty}[\jmath + \hbar\jmath(\jmath-1) - \wp]\Theta(\jmath)|a_\jmath b_\jmath| &\leq \sum_{\jmath=2}^{\infty}[\jmath + \hbar\jmath(\jmath-1) - \wp]\Theta(\jmath)|a_\jmath||b_\jmath| \\ &\leq \sum_{\jmath=2}^{\infty}[\jmath + \hbar\jmath(\jmath-1) - \wp]\Theta(\jmath)|a_\jmath| \\ &\leq 1 - \wp.\end{aligned}$$

Hence, by Theorem 3, it follows that $\aleph * \vartheta \in T\phi^{\gamma}_{\varrho,\chi,q}(\hbar, \wp)$.

8. Neighborhood property

We defined the mapping of $\aleph(\omega) \in T$ based on [25, 26], and the $\alpha-$ neighbourhood.

$$F_\alpha(\aleph) = \left\{ g \in T : \vartheta(\omega) = \omega - \sum_{\jmath=2}^{\infty} b_\jmath \omega^\jmath \text{ and } \sum_{\jmath=2}^{\infty} \jmath|a_\jmath - b_\jmath| \leq \alpha \right\}.$$

Definition 2. *The mapping* $\aleph \in T\phi^{\gamma}_{\varrho,\chi,q}(\hbar, \wp)$ *if the mapping* $h \in T\phi^{\gamma}_{\varrho,\chi,q}(\hbar, \wp)$ *takes place in a way that allows for mapping* $h \in T\phi^{\gamma}_{\varrho,\chi,q}(\hbar, \wp)$

$$\left| \frac{\aleph(\omega)}{h(\omega)} - 1 \right| < 1 - \gamma, \quad (\omega \in \Delta,\ 0 \leq \gamma < 1).$$

Theorem 13. *If* $h \in T\phi^{\gamma}_{\varrho,\chi,q}(\hbar, \wp)$ *and*

$$\gamma = 1 - \frac{\alpha(2\hbar - \wp + 2)\Upsilon_{2,q}(\varrho, \chi, \gamma)}{2(2\hbar - \wp + 2)\Upsilon_{2,q}(\varrho, \chi, \gamma) - (1 + \wp)}$$

then $F_\alpha(h) \subseteq T\phi^{\gamma;\gamma}_{\varrho,\chi,q}(\hbar, \wp)$.

Proof. Let $\aleph \in F_\alpha(h)$. After that, we come across that

$$\sum_{\jmath=2}^{\infty} \jmath|a_\jmath - b_\jmath| \leq \alpha,$$

which easily implies the coefficient inequality

$$\sum_{\jmath=2}^{\infty} |a_\jmath - b_\jmath| \leq \frac{\alpha}{\jmath}.$$

Since $h \in T\phi^{\gamma}_{\varrho,\chi,q}(\hbar,\wp)$, we have from equation (9) that

$$\sum_{\jmath=2}^{\infty} |a_\jmath| \le \frac{1-\wp}{(2\hbar-\wp+2)\Upsilon_{2,q}(\varrho,\chi,\gamma)}$$

and

$$\begin{aligned}
\left|\frac{\aleph(\omega)}{h(\omega)}-1\right| &< \frac{\sum\limits_{\jmath=2}^{\infty} \jmath|a_\jmath-b_\jmath|}{1-\sum\limits_{\jmath=2}^{\infty} b_\jmath z} \\
&\le \frac{\alpha}{2}\ \frac{(2\hbar-\wp+2)\Upsilon_{2,q}(\varrho,\chi,\gamma)}{(2\hbar-\wp+2)\Upsilon_{2,q}(\varrho,\chi,\gamma)-(1+\wp)} \\
&= 1-\gamma,
\end{aligned}$$

hence the proof.

9. Conclusions

In this paper, we introduced and investigated a certain class of analytic functions associated with the q-analogue of the Le Roy-type Mittag-Leffler function. The study focused on exploring the fundamental characteristics of these functions within the framework of geometric function theory. Several significant results were derived, including coefficient estimates, growth and distortion properties, convex linear combinations, partial sums, radii of close-to-convexity and starlikeness, convolution properties, and neighborhood results.

A noteworthy aspect of the present investigation lies in the limiting behavior of the proposed functions. Specifically $q \to 1^-$, the q- analogue of the Le Roy-type Mittag-Leffler function, defined by

$$E^{(q)}_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma_q(\alpha n+\beta)}$$

tends to classical part

$$\lim_{q\to 1^-} E^{(q)}_{\alpha,\beta}(z) = E_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n+\beta)}.$$

This limiting form reveals that deep connections between the q- analogue and several well known functions.

For particular parameter choices, the Le Roy-type Mittag-Leffler function reduces to classical functions of mathematical physics. For instance:

(i) when $\alpha = 1$ and $\beta = \nu + 1$ it corresponds to the Bessel function of the first kind $I_\nu(z)$ through the relationship $E_{1,\nu+1}(z) = z^{-\nu/2} I_\nu(2\sqrt{z})$.

(ii) More generally, the Le Roy-type Mittag-Leffler function can be expressed in terms of the Wright function, defined by

$$W_{\lambda,\mu}(z) = \sum_{n=0}^{\infty} \frac{z^n}{n!\Gamma(\lambda n + \mu)}.$$

This connection demonstrates that the q-analogue introduced in this study serves as a unifying framework linking various special functions under a single generalized structure.

Therefore, the limiting transition $q \to 1$ not only bridges the gap between the q-calculus framework and the classical analysis but also offers valuable insights into the analytic and geometric behavior of these related functions. This perspective opens promising directions for future research, including the study of asymptotic expansions, fractional calculus representations, integral transforms, and further extensions to multivariable or higher-order analogues of the q- Le Roy-type Mittag-Leffler function.

Author Contributions

This work was created by a team of authors who all contributed equally. The final version of the manuscript has received approval from all the authors.

Conflict of Interest

The authors assert that they do not have any conflicts of interest.

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