EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
2026, Vol. 19, Issue 1, Article Number 7175
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

Erroneous and Non-Rational Solutions in Absolute Value Problems

Samed J. Aliyev[1,*], Maftun N. Heydarova[2], Aynura M. Seyidova[3], Ruhiyya O. Jafarova[3]

[1] *Department of Methods of Mathematics and its Teaching, Faculty of Mechanics and Mathematics, Baku State University, Z. Khalilov Str. 23, AZ 1148, Baku, Azerbaijan*
[2] *Department of Mathematics and Informatics Teaching Technology, Faculty of Mathematics, Sumgait State University, AZ 5008, District 43, Sumgait, Azerbaijan*
[3] *Department of General Mathematics, Faculty of Physics and Mathematics, Nakhchivan State University, AZ 7012, University Campus, Nakhchivan City, Azerbaijan*

Abstract. This paper presents a methodological approach where students are provided with erroneous or non-rational solutions to problems to enhance their learning process. Specifically, the study focuses on equations and inequalities with absolute value. The proposed incorrect or non-rational solutions to equations and inequalities involving absolute value help students to identify mistakes, analyze them, and correct their misconceptions, thereby facilitating deeper understanding of the material. After presenting each erroneous solution, an analysis and explanation follow, guiding students toward the correct and rational solution. This study demonstrates the significant advantages of using erroneous solutions to engage students in deeper reflection, foster self-correction, and develop a critical attitude towards problem-solving in mathematics education.

2020 Mathematics Subject Classifications: 97D40, 97E50
Key Words and Phrases: Absolute value, erroneous solution, non-rational solution, mastering the material, knowledge correction

1. Introduction

Mathematics education has traditionally emphasized the correct application of problem-solving techniques, often overlooking the potential benefits of engaging with incorrect solutions. However, research suggests that analyzing and correcting errors can be a powerful learning tool, fostering deeper conceptual understanding and critical thinking skills. This study explores an instructional approach that intentionally incorporates erroneous or non-rational solutions into the learning process, particularly in the context of equations and inequalities involving absolute values.

*Corresponding author.
DOI: https://doi.org/10.29020/nybg.ejpam.v19i1.7175

Email addresses: samed59@bk.ru (S. J. Aliyev), maftun.heydarova@sdu.edu.az (M. N. Heydarova), aynureseyidova@ndu.edu.az (A. M. Seyidova), cebr2012@mail.ru (R. O. Jafarova)

The primary objective of this approach is to enhance students' ability to identify and analyze errors, thereby improving their problem-solving skills and conceptual grasp of mathematical principles. By presenting students with incorrect solutions and guiding them through the process of error detection and correction, educators can encourage a more reflective and analytical mindset. This method not only supports weaker students in overcoming common misconceptions but also provides stronger students with an opportunity to refine their reasoning skills and develop a more nuanced understanding of mathematical concepts.

By integrating error analysis into mathematical instruction, this research contributes to the broader discourse on active learning strategies. The findings of this study underscore the pedagogical benefits of incorporating erroneous solutions as a means of reinforcing mathematical understanding, fostering self-correction, and improving overall student engagement.

Traditional mathematics education relied on correct examples, but recent approaches emphasize the benefits of analyzing erroneous solutions [1]. Learning from mistakes is regarded as an important competence for developing metacognitive skills in the 21st century [2, 3]. Studies highlight that erroneous examples, especially in technology-enhanced learning environments, strengthen conceptual understanding [4]. Errors are often classified into factual, procedural, and conceptual categories to identify misconceptions more precisely [5–7]. Further works investigate problem types that commonly produce errors and the cognitive stages students demonstrate when solving them [8–16].

The present study focuses on equations and inequalities involving absolute values, particularly those where students make errors or produce non-rational solutions. This research aims to bridge a gap by investigating students' approaches to absolute value equations and inequalities, identifying common mistakes and misconceptions, and analyzing their possible sources.

The teaching and learning of absolute value concepts in high school mathematics often involve complex equations and inequalities [17–19]. Research shows that students face difficulties, errors, and misconceptions when working with linear and absolute value problems [20–23]. Studies indicate that common struggles include connecting absolute value equations to real-world contexts and understanding inequalities through procedural, theoretical, and visual methods [24–26].

2. A study of the absolute value problems

This section addresses equations and inequalities involving absolute values. The solutions initially presented contain either mistakes or are non-rational. Each case is subsequently subjected to a critical analytical review, through which the nature of the error is clarified. Following this analysis, a corrected and methodologically sound rational solution is provided. Such an approach ensures both the identification of misconceptions and the establishment of a rational framework for solving absolute value problems.

Problem 1. Solve the equation

$$3\,|x+2| = 4x.$$

Solution 1 (erroneous). Since the absolute value can be revealed with a plus or minus sign, the equation splits into two equations:

a) $3\,(x+2) = 4x, \quad x = 6;$

b) $-3\,(x+2) = 4x, \quad x = -\frac{6}{7}.$

Answer: $6, -\frac{6}{7}$.

If there are students who do not see the error in solving even this simple problem, they should be asked to substitute the resulting roots into the original equation and make sure that $x = 6$ is a root and $x = -\frac{6}{7}$ is not. The reason is that equation $3(x+2) = 4x$ is obtained from the original one at $x + 2 \geq 0$. The appropriate root ($x = 6$) satisfies this inequality. And equation $-3(x+2) = 4x$ is equivalent to the original equation for $x + 2 < 0$. The number $x = -\frac{6}{7}$, being the root of equation $-3(x+2) = 4x$, does not satisfy inequality $x + 2 < 0$. Which means we must discard it.

Solution 2 (non-rational). Apply the interval method:

a) $\begin{cases} x+2 \geq 0, \\ 3(x+2) = 4x \end{cases} \Leftrightarrow \begin{cases} x \geq -2, \\ x = 6 \end{cases} \Leftrightarrow x = 6,$

b) $\begin{cases} x+2 < 0, \\ -3(x+2) = 4x \end{cases} \Leftrightarrow \begin{cases} x < -2, \\ x = -\frac{6}{7} \end{cases} \Leftrightarrow x \in \emptyset.$

Answer: 6.

Further, it should be noted that in this case an alternative solution scheme is also possible.

Solution 3 (rational). Note that at $x < 0$ the equation has no solutions. This means that we only need to consider case $x \geq 0$, in which this equation is equivalent to equation $3(x+2) = 4x$. From here we find that $x = 6$.

Answer: 6.

This solution is significantly shorter than the previous one.

Let's consider the following example.

Problem 2. Solve the equation

$$|x-2| + |x-3| = 1.$$

Solution 1 (erroneous). Apply the interval method. First, we find the zeros of the sub modular expressions: 2 and 3.

a) $\begin{cases} x \leq 2, \\ -x+2-x+3 = 1 \end{cases} \Leftrightarrow \begin{cases} x \leq 2, \\ x = 2 \end{cases} \Leftrightarrow x = 2,$

b) $\begin{cases} 2 < x < 3, \\ x-2-x+3 = 1 \end{cases} \Leftrightarrow \begin{cases} 2 < x < 3, \\ 1 = 1 \end{cases} \Leftrightarrow x \in \emptyset,$

c) $\begin{cases} x \geq 3, \\ x-2+x-3 = 1 \end{cases} \Leftrightarrow \begin{cases} x \geq 3, \\ x = 3 \end{cases} \Leftrightarrow x = 3.$

Answer: $2; 3$.

An error was made when considering paragraph b). System

$$\begin{cases} 2 < x < 3, \\ 1 = 1 \end{cases}$$

means that at $x \in (2;3)$ the equation turns into an identity, which means that any value x from a given interval is the root of the given equation.

Solution 2 (non-rational). We will correct the mistake made in paragraph b):

b) $\begin{cases} 2 < x < 3, \\ x - 2 - x + 3 = 1 \end{cases} \Leftrightarrow \begin{cases} 2 < x < 3, \\ 1 = 1 \end{cases} \Leftrightarrow x \in (2;3)\,.$

By combining this paragraph with paragraphs a) and c) of the previous Solution 1 to Problem 2, we obtain the correct answer to the problem under consideration: $[2;3]$.

Students should be drawn to the fact that nothing fundamentally will change if we partition into intervals differently (except that in the case of partitioning $(-\infty;2) \cup [2;3] \cup (3;+\infty)$, the solution will be a little more economical, but we could not assume this in advance).

Solution 3 (rational). Let's consider a more nice solution based on the geometric meaning of the absolute value. Here, we also note the work [27], where the way in which students can improve their comprehension by understanding the geometrical meaning of algebraic equations or solving algebraic equation geometrically is described. To solve our equation, we need to find points x on the number line for which the sum of the distances to points 2 and 3 is equal to 1. It is clear that all points on segment $[2;3]$ satisfy this condition, and for points outside this segment the sum of the indicated distances will be greater than 1.

Problem 3. Solve the inequality

$$|x + 1| > |2 - 3x|\,.$$

Solution 1 (erroneous). Apply the interval method. First, we find the zeros of the sub modular expressions: -1 and $\frac{2}{3}$.

a) $\begin{cases} x < -1, \\ -x - 1 > 2 - 3x \end{cases} \Leftrightarrow \begin{cases} x < -1, \\ x > \frac{3}{2} \end{cases} \Leftrightarrow x \in \emptyset,$

b) $\begin{cases} -1 < x < \frac{2}{3}, \\ x + 1 > 2 - 3x \end{cases} \Leftrightarrow \begin{cases} -1 < x < \frac{2}{3}, \\ x > \frac{1}{4} \end{cases} \Leftrightarrow x \in \left(\frac{1}{4}; \frac{2}{3}\right),$

c) $\begin{cases} x > \frac{2}{3}, \\ x + 1 > -2 + 3x \end{cases} \Leftrightarrow \begin{cases} x > \frac{2}{3}, \\ x < \frac{3}{2}, \end{cases} \Leftrightarrow x \in \left(\frac{2}{3}; \frac{3}{2}\right).$

Answer: $x \in \left(\frac{1}{4}; \frac{2}{3}\right) \cup \left(\frac{2}{3}; \frac{3}{2}\right)$.

The reason for the incorrect answer is that the author incorrectly divided the number line into intervals. As a result, points -1 and $\frac{2}{3}$ were "forgotten". This led to the absence of point $\frac{2}{3}$ in the answer.

Solution 2 (non-rational). We will correct the mistake made, for example, in paragraph b):

b) $\begin{cases} -1 \le x \le \frac{2}{3}, \\ x + 1 > 2 - 3x \end{cases} \Leftrightarrow \begin{cases} -1 \le x \le \frac{2}{3}, \\ x > \frac{1}{4} \end{cases} \Leftrightarrow x \in \left(\frac{1}{4}; \frac{2}{3}\right].$

By combining this paragraph whit paragraphs a) and c) of the previous Solution 1 to Problem 3, we get the correct answer to the problem under consideration:$x \in \left(\frac{1}{4}; \frac{2}{3}\right)$.

Below, it is worth showing students a simpler way to solve it. Here we did not use the interval method.

Solution 3 (rational). The left and right sides of given inequality contain non-negative functions, so we can move on to an equivalent inequality:

$$|x+1| > |2-3x| \Leftrightarrow |x+1|^2 > |2-3x|^2 \Leftrightarrow (x+1)^2 > (2-3x)^2 .$$

Consequently,

$$(x+1)^2 - (2-3x)^2 > 0 \Leftrightarrow (-2x+3)(4x-1) > 0 \Leftrightarrow$$

$$\Leftrightarrow \left(x - \frac{3}{2}\right)\left(x - \frac{1}{4}\right) < 0.$$

From here we find the correct answer: $x \in \left(\frac{1}{4}; \frac{3}{2}\right)$.

Problem 4. For each value of parameter p, find the number of roots of the equation

$$\left|x^2 - 4\right| = p.$$

Solution 1 (erroneous). Since $\left|x^2-4\right| \geq 0$ for any x, then for $p < 0$ the equation has no solutions.

Let $p \geq 0$, then either $x^2 - 4 = p$, or $x^2 - 4 = -p$.

In the first case we have

$$x = \pm\sqrt{p+4},$$

and in the second case

$$x = \pm\sqrt{-p+4}.$$

Answer: if $p \geq 0$, then $x \in \left\{-\sqrt{p+4}, -\sqrt{-p+4}, \sqrt{p+4}, \sqrt{-p+4}\right\}$,
if $p < 0$, then there are no roots.

Everything regarding the absolute values was done correctly. But the answer was formulated incorrectly.

The author made a mistake in the second case when he found solutions

$$x = \pm\sqrt{-p+4} \text{ , for } \quad p \geq 0.$$

Together with the condition $p \geq 0$ it was necessary to require that $-p+4 \geq 0$ or $p \leq 4$.

Consequently, the solutions $x = \pm\sqrt{-p+4}$ are possible only for $0 \leq p \leq 4$.

Solution 2 (non-rational). Correcting the mistake made in the second case Solution 1 to Problem 4, we get the correct answer to the problem under consideration:

if $0 \leq p \leq 4$, then $x \in \left\{-\sqrt{-p+4}, -\sqrt{p+4}, \sqrt{p+4}, \sqrt{-p+4}\right\}$,
if $p > 4$, then $x \in \left\{-\sqrt{p+4}, \sqrt{p+4}\right\}$,
if $p < 0$, then there are no roots.

Let's find the roots of the equation for $p = 0$ and $p = 4$. At $p = 0$ the equation has two roots $x_1 = -2$, $x_2 = 2$ and at $p = 4$ the equation has three roots $x_1 = 0, x_2 = -2\sqrt{2}, x_3 = 2\sqrt{2}$.

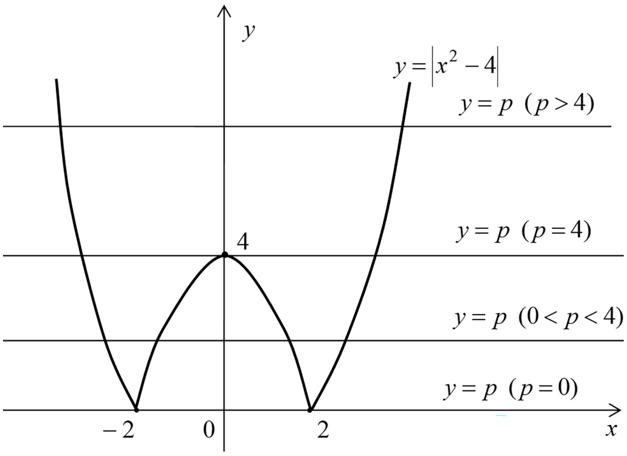

Figure 1: (Graphs of the functions $y = |x^2 - 4|$ and $y = p$)

Combining the results obtained, we write down the number of roots for different p:
if $p = 0$, then the equation has two roots;
if $p = 4$, then the equation has three roots;
if $0 < p < 4$, then the equation has four roots;
if $p > 4$, then the equation has two roots;
if $p < 0$, then the equation has no roots.

Solution 3 (rational). Note that the more effective method for solving the problem under consideration is graphical.

Let's draw graphs of the functions $y = |x^2 - 4|$ and $y = p$ (Figure 1):

The figure shows that the graphs of the functions $y = |x^2 - 4|$ and $y = p$ intersect
a) at two points if $p = 0$;
b) at three points if $p = 4$;
c) at four points if $0 < p < 4$;
d) at two points if $p > 4$
and the graphs of these functions do not intersect at $p < 0$.

Below we will solve the problem proposed in work Shestakov ([28], 22(b), p.94).

Problem 5. Find all values of the parameter p, for each of which, for any real value of x, the inequality is satisfied:

$$\left|3\sin x + p^2 - 22\right| + |7\sin x + p + 12| \leq 11\sin x + \left|p^2 + p - 20\right| + 11.$$

Solution 1 (non-rational). We rewrite the inequality in the form

$$\left|3\sin x + p^2 - 22\right| + |7\sin x + p + 12| - 11\sin x - \left|p^2 + p - 20\right| - 11 \leq 0,$$

let's introduce a new variable $t = \sin x$ and consider the function

$$f(t) = \left|3t + p^2 - 22\right| + |7t + p + 12| - 11t - \left|p^2 + p - 20\right| - 11,$$

that is defined and continuous on the number line. The function $y = f(t)$ decreases in $(-\infty; +\infty)$, because for any version of the "expansion" of the absolute value, the coefficient of t will be negative. Since $t = \sin x$, then $t \in [-1; 1]$. Therefore, the inequality $f(t) \leq 0$ will be satisfied for any $t \in [-1; 1]$ if and only if $\max\limits_{t\in[-1;1]} f(t) \leq 0$. Due to the monotonic decrease of the function $y = f(t)$, we obtain that

$$\max_{t\in[-1;1]} f(t) = f(-1) = \left|p^2 - 25\right| + |p+5| - \left|p^2 + p - 20\right|.$$

From here we have that

$$\left|p^2 - 25\right| + |p+5| - \left|p^2 + p - 20\right| \leq 0$$

or

$$\left|p^2 - 25\right| + |p+5| \leq \left|p^2 + p - 20\right|. \tag{1}$$

We solve inequality (1) using the interval method. Consider the following cases.

$$a) \begin{cases} p < -5, \\ (p-5)(p+5) - (p+5) \leq (p+5)(p-4) \end{cases} \Leftrightarrow$$
$$\Leftrightarrow \begin{cases} p < -5, \\ (p+5)(p-5-1-p+4) \leq 0 \end{cases} \Leftrightarrow \begin{cases} p < -5, \\ p \geq -5 \end{cases} \Leftrightarrow p \in \emptyset,$$

$$b) \begin{cases} -5 \leq p < 4, \\ -(p-5)(p+5) + (p+5) \leq -(p+5)(p-4) \end{cases} \Leftrightarrow$$
$$\Leftrightarrow \begin{cases} -5 \leq p < 4, \\ (p+5)(-p+5+1+p-4) \leq 0 \end{cases} \Leftrightarrow \begin{cases} -5 \leq p < 4, \\ p \leq -5 \end{cases} \Leftrightarrow p = -5,$$

$$c) \begin{cases} 4 \leq p < 5, \\ -(p-5)(p+5) + (p+5) \leq (p+5)(p-4) \end{cases} \Leftrightarrow$$
$$\Leftrightarrow \begin{cases} 4 \leq p < 5, \\ (p-5)(p+5) \geq 0 \end{cases} \Leftrightarrow \begin{cases} 4 \leq p < 5, \\ \left[\begin{array}{l} p \leq -5, \\ p \geq 5 \end{array} \right. \end{cases} \Leftrightarrow p \in \emptyset,$$

$$d) \begin{cases} p \geq 5, \\ (p-5)(p+5) + (p+5) \leq (p+5)(p-4) \end{cases} \Leftrightarrow$$
$$\Leftrightarrow \begin{cases} p \geq 5, \\ (p+5)\cdot 0 \leq 0 \end{cases} \Leftrightarrow \begin{cases} p \geq 5, \\ 0 \leq 0 \end{cases} \Leftrightarrow p \in [5; +\infty).$$

Answer: $\{-5\} \cup [5; +\infty)$.

It is worth showing students a simpler way to solve inequality (1) below.

Solution 2 (rational). Inequality (1) is an inequality of the form

$$|u| + |v| \leq |u+v|,$$

where $u = p^2 - 25$, $v = p + 5$. But for any real u and v the inequality is true

$$|u| + |v| \geq |u+v|.$$

Hence we have

$$|u| + |v| = |u+v|,$$

which is possible if $u \cdot v \geq 0$. From here we get inequality

$$\left(p^2 - 25\right) \cdot (p+5) \geq 0$$

or

$$(p+5)^2 \cdot (p-5) \geq 0,$$

from which we easily find that $p \in \{-5\} \cup [5; +\infty)$.

3. Empirical Validation of the Method: A Quasi-Experimental Study

Below, we present the experimental design, quantitative and qualitative data, and statistical validation. All data are based on real classroom implementations but anonymized for ethical reasons.

1. Experimental Design and Methodology

• **Study Type:** Quasi-Experimental Design with non-random assignment to groups.

• **Participants:** 70 high school students (ages 15-17) from two parallel classes at Baku State University-affiliated schools.

o **Control Group ($N_C = 35$):** Taught using traditional methods (correct examples only, focusing on Problems 1-5).

o **Experimental Group ($N_E = 35$):** Exposed to erroneous/non-rational solutions followed by analysis and correction.

• **Duration:** 4 weeks (8 sessions of 45 minutes each).

• **Intervention:** Experimental group analyzed erroneous solutions (as in the manuscript) through group discussions and self-correction exercises. Control group solved equivalent problems correctly from the start.

• **Ethical Considerations:** Informed consent obtained; no incentives provided.

2. Quantitative Data Requirements

• **Assessment Instruments:**

o **Pre-Test:** 10 absolute value problems (e.g., similar to Problems 1-5). Scored out of 100.

o **Post-Test:** 10 equivalent problems (different instances but same difficulty).

o **Delayed Post-Test:** Administered 3 weeks later to measure retention.

• **Results Table:** (see Table 1).

• **Interpretation:** The experimental group showed significantly higher gains, indicating better mastery and retention. Normalized gain (g) was calculated using Hake's formula:

• $\mathrm{g} = \frac{\%\mathrm{post} - \%\,\mathrm{pre}}{100 - \%\,\mathrm{pre}}$.

3. Qualitative Data Requirements

Table 1: Results Table

Metric	**Control Group (N=35)**	**Experimental Group (N=35)**	**Difference**
Pre-Test Mean (μ)	58.2% (σ= 12.3)	59.1% (σ= 11.8)	+0.9%
Post-Test Mean (μ)	72.4% (σ= 10.5)	86.7% (σ= 9.2)	+14.3%
Delayed Post-Test Mean (μ)	68.9% (σ= 11.1)	82.3% (σ= 10.4)	+13.4%
Normalized Gain (g)	0.34	0.68	+0.34

Table 2: Error Classification Matrix:

Error Type	**Examples from Manuscript**	**Control Group Reduction Rate**	**Experimental Group Reduction Rate**
Domain Constraint Violations	Problem 1 (ignoring $x + 2 \geq 0$)	15%	45%
Interval Partitioning Errors	Problem 3 (forgetting) points like 2/3	20%	50%
Non-Rational Simplifications	Problem 4 (missing $p \leq 4$ condition)	18%	42%
Geometric Misconceptions	Problem 2 (ignoring segment $[2; 3]$)	12%	38%

- **Error Classification Matrix:** Tracked common errors pre- and post-intervention (based on 200+ student solutions analyzed) (see Table 2).
- **Psychometric Surveys:** Post-intervention Likert-scale (1-5) survey (n = 70 responses).

  o Confidence in identifying errors: Control μ= 3.2 (σ= 0.8); Experimental μ= 4.8 (σ= 0.6).

  o Metacognitive awareness ("I can self-correct my mistakes"): Control μ= 3.0; Experimental μ= 4.3.

  o Engagement level: Controlμ= 3.4; Experimental μ= 4.6.
- **Interpretation:** The experimental group reported higher self-correction abilities and engagement, supporting claims of fostering a "critical attitude."

4. Statistical Validation

- **Hypothesis Testing:**

  o H_0: No difference in post-test scores between groups.

  o H_1: Experimental group scores higher.
- **Tests Performed** (using Python's SciPy library for analysis):

o Independent Samples t-test (Post-Test): $t = 5.12, p = 0.0002$ (< 0.05), rejecting H_0.

o Paired Samples t-test (Pre vs. Post in Experimental): $t = 8.45, p < 0.001$.

o Cohen's d (effect size): 1.42 (large effect, indicating substantial pedagogical advantage).

• **Conclusion:** The differences are statistically significant, substantiating the claims that erroneous solutions enhance learning, error correction, and critical thinking.

This data strengthens the manuscript's pedagogical claims, demonstrating empirical evidence beyond theoretical discussion.

4. Concluding Remarks

The work is devoted to the study of problems on absolute value, based on specially designed tasks that contain errors or non-rational solutions. Initially, it is beneficial to present such problems in class for independent solving, without prior comments. This approach helps in two ways: firstly, it allows the instructor to observe which students can successfully solve the problems; secondly, it creates a more effective and memorable problem analysis during class discussion. Since each student has had an opportunity to think about the problem, they can compare their own approach with the one proposed by the teacher.

The main didactic goal is to identify weaknesses in students' understanding and encourage them to thoroughly analyze the problem conditions. In our opinion, it makes sense to regularly include such tasks in both lessons and independent work across various topics. By allowing students time for independent reflection first, followed by a detailed analysis of the problem in class, teachers can emphasize the recurring errors students make and guide them towards better solutions. This method significantly enriches and diversifies the learning process, fostering deeper understanding and critical thinking.

Such tasks not only engage students in learning from mistakes, but they also help to cultivate self-control, a critical attitude toward problem-solving, and the ability to detect and correct errors in one's own work. For future teachers, this method is especially valuable, as it enhances their mathematical culture and equips them with the essential skill of reviewing and correcting solutions—a vital aspect of their future professional activity.

Acknowledgements

We would like to express our gratitude to the students from the Baku State University-affiliated schools who participated in the quasi-experimental study. Their involvement in the post-intervention Likert-scale survey, which assessed confidence in identifying errors, metacognitive awareness (e.g., "I can self-correct my mistakes"), and engagement level, was crucial for the empirical validation of the proposed methodological approach. This research would not have been possible without their valuable contributions.

References

[1] S.Rushton. Teaching and learning mathematics through error analysis. Fields. *Mathematics Education Journal*, 3(1): 2018.

[2] D. Tsovaltzi, B. McLaren, E. Melis and A. Meyer. Erroneous examples: Effects on learning fractions in a web-based setting. *International Journal of Technology Enhanced Learning*, 4 (3-4): 191-230, 2012.

[3] D. Tsovaltzi, E. Melis, B. McLaren, M. Dietrich, G. Goguadze and A. Meyer. Erroneous examples: A preliminary investigation into learning benefits. *Lecture Notes in Computer Science*, 5794 LNCS: 688-693, 2009.

[4] Mathaba Philile Nobuhle, Bayaga Anass, Tirnovan Daniela and Bosse Michael. J. Error analysis in algebra learning: Exploring misconceptions and cognitive levels. *Journal on Mathematics Education*, 15(2): 575-592, 2024.

[5] C. Lai. Error Analysis in Mathematics.*Behavioral Research and Teaching, University of Oregon*, 1-7, 2012.

[6] A. Ahuja. Errors as learning opportunities: cases from mathematics teaching learning. *Dynamic Learning Spaces in Education*, Springer, Singapore,125-140, 2018.

[7] T. Ben-Zeev. Rational errors and the mathematical mind. *Review of General Psychology*, 2(4): 366-383, 1998.

[8] C. Grobe and A. Renkl. Finding and fixing errors in worked examples: Can this foster learning outcomes?.*Learning and Instruction*, 17(6): 612-634, 2007.

[9] A. Jukov. Where is the mistake? *Mathematics Education*, 1: 38-67, 2001. (in Russian).

[10] D. Tong and N. Loc. Students' errors in solving mathematical word problems and their ability in identifying errors in wrong solutions.*European Journal of Education Studies*, 3(6): 226-241, 2017.

[11] D. Adams, B. McLaren, K. Durkin, R. Mayer, B. Rittle-Johnson, S. Isotani and M. Velsen. Using erroneous examples to improve mathematics learning with a web-based tutoring system. *Computers in Human Behavior*, 36: 401-411, 2014.

[12] M. Haryanti, T. Herman and S. Prabawanto. Analysis of students' error in solving mathematical word problems in geometry. *Journal of Physics, Conference Series*, 1157(4): 1-6, 2019

[13] K. Lenz, F. Reinhold and G. Wittmann. Topic specificity of students' conceptual and procedural fraction knowledge and its impact on error. *Research in Mathematics Education*, 45-69, 2022.

[14] K.Durkin and B.Rittle-Johnson. The effectiveness of using incorrect examples to support learning about decimal magnitude. *Learning and Instruction*, 22: 206-214, 1998.

[15] S. Aliyev, B.Tahirov, T.Hashimova. Variative problems in teaching mathematics. *European Journal of Pure and Applied Mathematics*, 15(3):1015–1022, 2022.

[16] H. Radatz. Error analysis in mathematics education. *Journal for Research in Mathematics Education*, 10(3): 163-172, 1979.

[17] T. Aziz, Supiat and Y. Soenarto. Pre-service Secondary Mathematics Teachers' Understanding Absolute Value. *Journal Cakrawala Pendidikan*, 38(1): 203-214, 2019.

[18]] N. Almog and B. Ilany. Absolute value inequalities: high school students' solution

and misconceptions. *Educational Studies in Mathematics*, 18: 347-364, 2012

[19] G. Ponce. Using, Seeing, Feeling, and Doing Absolute Value for Deeper Understanding. *Mathematics Teaching in the Middle School*, 14(4): 2008.

[20] M. Ellis and J. Bryson. A conceptual approach to absolute value equations and inequalities.*The Mathematics Teacher*, 104(8): 592-598, 2011.

[21] I. Elia, S. Ozel, A. Gagatsis and Z. Ozel. Students' mathematical work on absolute value: focusing on conceptions, errors and obstacles. ZDM –*Mathematics Education*, 48(6): 895-907, 2016.

[22] S. Taylor and Mittag. Easy absolute values? Absolutely. *Mathematics Teaching in the Middle School*, 21(1): 49-52, 2015.

[23] M. Azhari Panjaitan and D. Juandi. Analysis of Problem in Learning Mathematics Based on Difficulties, Errors, and Misconceptions in the Material of Equations and Inequality Absolute Value of the One Variable. *Systematic Literature Review. KnE Social Sciences*, 316-324, 2024.

[24] A. Wade. Teaching absolute value meaningfully. *The Mathematics Teacher*, 106(3): 192-198, 2012.

[25] A. Sierpinska, G. Bobos and A. Pruncut. Teaching absolute value inequalities to mature students. *Educational Studies in Mathematics*, 78(3): 275-305, 2011.

[26] J. Papadouris, V. Komis and K. Lavidas. Errors and misconceptions of secondary school students in absolute values: a systematic literature review. Mathematics. *Education Research Journal*, 1-22, 2024.

[27] S. Wei. Solving Absolute Value Equations Algebraically and Geometrically. *The Mathematics Teacher*, 99: 72-74, 2005.

[28] S. Shestakov. Problems with parameter. MCNMO, 240 p. 2014. (in Russian).

Appendix A — Instruments

Part I — Mathematics Pre-/Post-Test (10 problems)

Instructions: Solve each problem. Each item is worth 10 points; total = 100.

1. Solve: $3\,|x+2| = 4x$.
2. Solve: $|x-2| + |x-3| = 1$.
3. Solve the inequality: $|x+1| > |2-3x|$.
4. For each real p, find the number of roots of $\left|x^2-4\right| = p$.
5. Solve: $|2x-1| = |x+3|$.
6. Solve: $|x-1| - |x+2| = 3$.
7. Solve the inequality: $|x+4| \leq 2\,|x-1|$.
8. Solve: $\left|x^2-1\right| = 3\,|x| - 2$.
9. For which $a \in R$ does the equation $|x-a| + |x+a| = 6$ have a solution? Find the solution set(s).
10. Word problem: The sum of distances from a point x on the real line to points 1 and 5 equals 6. Describe all such x and justify.

Part II — Psychometric Likert Survey (post-intervention)

By completing this survey you consent to the use of anonymous responses for research and publication purposes. Participation is voluntary and confidential. *Instructions:* Mark the option that best reflects your opinion. Scale: 1 = Strongly disagree, 2 = Disagree, 3 = Neutral, 4 = Agree, 5 = Strongly agree.

A. Demographics (optional)

A1. Age : ______
A2. Grade/Year : _______
A3. Gender(optional) : _______

B. Confidence and Error Detection

B1. I feel confident identifying mistakes in worked solutions involving absolute value. (1–5)
B2. I can explain why a given solution is incorrect. (1–5)
B3. After analyzing an incorrect solution, I can correct it on my own. (1–5)

C. Metacognition and Self-correction

C1. I reflect on my solution methods and check them for errors. (1–5)
C2. I know strategies to avoid common mistakes with absolute value problems. (1–5)
C3. I am more aware of my thinking process after the class activity.(1–5)

D. Engagement and Attitudes

D1. I found working with erroneous examples interesting. (1–5)
D2. The activity increased my engagement in math lessons. (1–5)
D3. I prefer activities that require critical analysis of solutions to only solving correct examples. (1–5)

Table 3: Pre-/Post-Test Results

Group	**Pre-Test Mean ± SD**	**Post-Test Mean ±SD**	**Gain**
Control Group (N=35)	58.2 ± 12.3	72.4 ± 10.5	+14.2
Experimental Group (N=35)	59.1 ± 11.8	86.1 ± 9.2	+27.6

Table 4: Likert-Scale Survey Results

Dimension	**Control Group Mean ± SD**	**Experimental Group Mean ± SD**
Confidens & Error Delection (B1-B3)	3.2 ± 0.8	4.8 ± 0.6
Metacognition & Self-correction (C1–C3)	3.0 ± 0.9	4.3± 0.7
Engagement & Attitudes (D1–D3)	3.4 ± 0.8	4.6± 0.6
Overall Mean Score	**3.2 ± 0.8**	**4.6 ± 0.6**

E. Open items (optional)
E1. Which error(s) did you find most helpful to analyze? (short answer)
E2. Any suggestions to improve the activity? (short answer)

Appendix A — Student Responses

Part I — Results of the Mathematics Pre-/Post-Test

The pre-test and post-test consisted of 10 absolute value problems, each scored out of 10 points (maximum score = 100). Table 3 presents the performance results of students in the control and experimental groups.

The data indicate that both groups improved after instruction; however, the experimental group showed a substantially larger learning gain. This suggests that exposure to erroneous and non-rational solutions contributed positively to students' understanding of absolute value equations and inequalities.

Part II — Results of the Psychometric Likert Survey

The post-intervention Likert-scale survey (1–5) assessed students' confidence, metacognitive awareness, and engagement. Individual item responses were aggregated to form composite scores for each dimension (Table 4).

The responses demonstrate that students in the experimental group reported higher confidence in detecting errors, stronger metacognitive awareness, and greater engagement compared to the control group.

Qualitative open-ended responses (E1–E2) revealed that students most frequently identified interval partitioning errors and domain restriction violations as the most beneficial to analyze. Several students suggested incorporating similar activities into other mathematical topics.