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Mathematical Modeling for the Analysis and Optimization of Nutrient Dynamics in Edible Mushrooms

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Abstract. Edible mushrooms are a valuable source of essential nutrients, including proteins, vitamins, minerals, and dietary fibers, and are important for human health and nutrition. Their nutrient composition varies according to species, substrate type, and environmental conditions. This study presents a mathematical model describing nutrient absorption, retention, and degradation in edible mushrooms using differential equations derived from controlled growth experiments. The model captures the dynamic interactions between substrate nutrients and mushroom metabolism, helping to identify the key factors influencing nutrient accumulation. Parameter estimation and sensitivity analysis confirmed the accuracy of the model in predicting nutrient variations across growth conditions. The simulation results revealed the optimal environmental and substrate parameters that maximize the nutrient content and overall yield. The proposed framework provides a quantitative approach to improve mushroom cultivation efficiency and nutritional value, offering potential applications in sustainable agriculture, food processing, and nutritional science.

2020 Mathematics Subject Classifications: 03H05, 33F05, 92B05

Key Words and Phrases: Edible mushroom, essential nutrient, human health, mathematical model, substrate composition

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1. Introduction

Mushrooms have garnered increasing interest as functional foods and potential superfoods, driven by their complex nutritional profiles and bioactive compounds [1, 2]. Beyond their culinary appeal, mushrooms offer a wealth of nutrients, including high-quality proteins, essential vitamins (notably B-complex vitamins, vitamin D, and vitamin C), key minerals (such as selenium, potassium, phosphorus, iron, and copper), dietary fibers (including beta-glucans), and a diverse array of antioxidants [3, 4]. These components contribute to a range of potential health benefits, including immune system modulation [5], anti-inflammatory effects [6], cardiovascular protection [7], and potential anti-cancer properties [8]. Furthermore, their low caloric density and minimal fat content make mushrooms attractive components of various dietary strategies, including those aimed at weight management [9]. However, the nutritional composition of mushrooms is far from uniform, exhibiting significant variation due to factors such as species, substrate composition, cultivation practices, environmental conditions (temperature, humidity, light exposure, CO_2 levels), and harvesting time [10, 11]. This variability necessitates a thorough understanding of the factors influencing these changes to optimize the cultivation and maximize the nutritional value of edible mushrooms. For example, selenium content can be significantly enhanced through substrate supplementation [12], whereas light exposure can impact vitamin D levels [13].

Mathematical modeling in epidemiology provides a systematic framework for understanding the spread and control of infectious diseases through equations that describe population interactions. It helps predict outbreak dynamics, evaluate intervention strategies, and guide public health decision-making using analytical and numerical methods [14–23].

Mathematical modeling offers a powerful predictive framework for analyzing and optimizing the nutrient content of mushrooms under various environmental and cultivation scenarios. By constructing models that capture the critical processes of nutrient uptake, assimilation, metabolic transformation, and degradation, we can gain actionable insights to guide mushroom growers and nutritionists in enhancing the nutritional quality and yield of edible mushrooms [24]. These models range from simple empirical relationships to complex systems of differential equations that describe the underlying biological processes. The accuracy and efficiency of numerical solutions for such models are crucial [25, 26]. Edible mushrooms, including popular varieties such as Shiitake (Lentinula edodes), Oyster (Pleurotus ostreatus), Button (Agaricus bisporus), and Enoki (Flammulina velutipes), are celebrated for their diverse and valuable nutritional profiles. These species contain variable amounts of proteins, carbohydrates, vitamins, minerals, and a wide spectrum of bioactive compounds, which contribute to a multitude of potential health benefits, including immune system support and antioxidant activity [27, 28].

The nutrient content of mushrooms is influenced by a complex interplay of several factors, including:

(i) **Substrate Composition**: The richness and composition of the substrate plays a fundamental role in determining the nutrient uptake and profile of the developing mushroom [29]. Substrates abundant in organic matter tend to facilitate enhanced mineral and nutrient absorption in plants.

(ii) **Environmental Conditions**: Factors such as temperature, relative humidity, light intensity and spectrum, and CO_2 concentrations exert a significant influence on mushroom growth and nutrient accumulation [30].

(iii) **Harvesting Time/Maturity Stage**: The precise stage of maturity at harvest significantly affects the nutrient composition, with premature harvesting often resulting in lower overall nutrient levels and over-mature mushrooms potentially experiencing nutrient losses due to degradation processes.

(iv) **Species and Genetic Variability**: The inherent genetic makeup of different mushroom species (and even different strains within the same species) dictates their capacity for nutrient uptake, metabolic processing, and accumulation [31].

Differential equation models have been widely and effectively applied to analyze nutrient dynamics across a broad spectrum of biological systems, including plants and microbial cultures [32]. This study seeks to adapt and extend these modeling approaches to the specific context of mushroom nutrient dynamics, providing a novel and potentially powerful approach for optimizing cultivation strategies for enhanced nutritional value.

To develop an effective and predictive model, this study draws upon established numerical methods applicable to systems of ordinary differential equations, including those commonly used for stiff systems. By integrating insights derived from mathematical modeling with empirical data obtained from controlled mushroom cultivation experiments, we aim to create a robust and practical framework for optimizing nutrient content and enhancing the overall quality and health benefits of edible mushrooms. This study aimed to develop and validate a mathematical model for the analysis of nutrient dynamics in edible mushrooms, thereby providing a basis for optimizing growth conditions with the specific goal of maximizing nutrient retention and enhancing the overall nutritional profile. We anticipate that this approach will enable more precise and efficient strategies for mushroom cultivation, ultimately contributing to the production of more nutritious and sustainable food sources. The remainder of this paper is organized as follows: Section 2 presents the methodology and formulation of the proposed model. Section 3 discusses the basic analysis, and Section 4 establishes the existence of equilibrium points. Section 5 derives the basic reproduction number (R_0), and Section 6 provides a sensitivity analysis. Section 7 presents the numerical simulations, and Section 8 concludes the study.

2. Methodology

2.1. Model Assumptions

To develop the mathematical model, several assumptions were made: 1. Controlled Environment: Mushrooms are grown in a controlled environment with constant temperature, humidity, and light exposure. 2. Proportional Nutrient Absorption: The rate of nutrient absorption by mushrooms is proportional to the concentration of available nutrients in the substrate. 3. Constant Nutrient Degradation Rate: Nutrient degradation within mushrooms follows a first-order kinetic process with a constant degradation rate. 4. Negligible External Factors: Factors such as pests, diseases, and pollutants are assumed to be negligible and do not affect the nutrient content.

2.2. Model Formulation

The nutrient content of the mushroom was modeled using a differential equation that captures the dynamic balance between nutrient absorption from the substrate and nutrient degradation within the mushroom. The model integrates nutrient dynamics, environmental influences, biomass growth, nutrient distribution and waste production.

Nutrient Uptake Dynamics

Let:

- $N(t)$ be the nutrient concentration in the substrate at time t.
- $U(t)$ be the nutrient uptake rate by the mushroom.
- $M(t)$ be the total biomass of the mushroom at time t.
- $E(t)$ be the environmental factors influencing nutrient uptake.

The nutrient uptake rate is as follows:

$$U(t) = k_1 N(t) f(E(t)).$$

where k_1 is the nutrient uptake coefficient, and $f(E(t))$ represents the influence of environmental factors, modeled as:

$$f(E(t)) = \exp\left(-\frac{(T(t) - T_{\text{opt}})^2}{\sigma_T^2}\right) \times \exp\left(-\frac{(H(t) - H_{\text{opt}})^2}{\sigma_H^2}\right).$$

where $T(t)$ and $H(t)$ are the temperature and humidity at time t, T_{opt} and H_{opt} are their optimal values, and σ_T and σ_H represent the sensitivity to deviations from these optima. The nutrient concentration in the substrate changes as follows:

$$\frac{dN(t)}{dt} = -U(t).$$

Biomass Growth

Biomass growth is driven by nutrient uptake:

$$\frac{dM(t)}{dt} = \alpha U(t).$$

where α is the conversion efficiency of the nutrients into biomass.

Nutrient Distribution

The nutrient distribution between the cap $C(t)$ and stem $S(t)$ is modeled as:

$$\frac{dC(t)}{dt} = \beta U(t) - \gamma C(t),$$

$$\frac{dS(t)}{dt} = (1-\beta)U(t) - \delta S(t).$$

where β is the fraction of nutrients allocated to the cap, γ and δ are the rates of nutrient utilization or degradation in the cap and stem, respectively.

Waste Production

Waste or by-products $W(t)$ are produced as a result of metabolic processes:

$$\frac{dW(t)}{dt} = \epsilon U(t).$$

where ϵ is the waste generation efficiency.

Complete Combined Model

The complete set of differential equations governing the system is as follows:

$$\begin{aligned}
\frac{dN(t)}{dt} &= -k_1 N(t) f(E(t)), \\
\frac{dM(t)}{dt} &= \alpha k_1 N(t) f(E(t)), \\
\frac{dC(t)}{dt} &= \beta k_1 N(t) f(E(t)) - \gamma C(t), \\
\frac{dS(t)}{dt} &= (1-\beta) k_1 N(t) f(E(t)) - \delta S(t), \\
\frac{dW(t)}{dt} &= \epsilon k_1 N(t) f(E(t)).
\end{aligned} \tag{1}$$

3. Boundedness Analysis

3.1. Positivity and Invariant Region of the Model

Theorem 1. *(Positivity): Let* $\Omega = \{(N, M, C, S, W) \in \mathbb{R}^5_+ : N(0) > 0, M(0) > 0, C(0) > 0, S(0) > 0, W(0) > 0\}$. *Then, the solutions of* $\{N(t), M(t), C(t), S(t), W(t)\}$ *are positive for* $t \geq 0$.

We begin by analyzing the first equation of the system (1):

$$\frac{dN(t)}{dt} = -k_1 N(t) f(E(t)),$$

Since $f(E(t)) > 0$, we have:

$$\frac{dN(t)}{dt} \geq -k_1 N(t),$$

By separating the variables:

$$\frac{dN}{N} \geq -k_1\, dt,$$

Integrating both sides from 0 to t:

$$\int \frac{dN}{N} \geq \int -k_1\, dt,$$

This results in:

$$\ln N \geq -k_1 t + \ln N(0),$$

Applying the exponential function to both sides yields:

$$N(t) \geq N(0)e^{-k_1 t}.$$

Similarly, applying the same technique to all other state variables M, C, S, and W, in the system (1), we get: $M(t) \geq M(0)e^{-(\alpha k_1 f(E(t))+\mu)t}$, $C(t) \geq C(0)e^{-(\beta k_1 f(E(t))+\gamma)t}$, $S(t) \geq S(0)e^{-((1-\beta)k_1 f(E(t))+\delta)t}$, $W(t) \geq W(0)e^{-(\epsilon k_1 f(E(t))+\mu)t}$.

This proves that all state variables remain positive for $t \geq 0$, ensuring the positivity of the system as follows:

$$\Omega = \left\{(N, M, C, S, W) \in \mathbb{R}^5_+ : N, M, C, S, W \geq 0\right\}.$$

Theorem 2. *(Invariant region): All solutions $N(t), M(t), C(t), S(t), W(t)$ of the system for any initial conditions in the domain are bounded in the region Ω.*

To analyze the boundedness of the system, we define the total population $P(t)$ as:

$$P(t) = N(t) + M(t) + C(t) + S(t) + W(t).$$

Taking the derivative with respect to time, we obtain

$$\frac{dP(t)}{dt} = \frac{dN(t)}{dt} + \frac{dM(t)}{dt} + \frac{dC(t)}{dt} + \frac{dS(t)}{dt} + \frac{dW(t)}{dt}.$$

By substituting the system of equations:

$$\begin{aligned}\frac{dP(t)}{dt} &= -k_1 N(t) f(E(t)) + \alpha k_1 N(t) f(E(t)) + \beta k_1 N(t) f(E(t)) - \gamma C(t) \\ &\quad + (1-\beta) k_1 N(t) f(E(t)) - \delta S(t) + \epsilon k_1 N(t) f(E(t)).\end{aligned}$$

Simplifying:

$$\frac{dP(t)}{dt} = k_1 N(t) f(E(t))(\alpha + \beta + (1-\beta) + \epsilon - 1) - \gamma C(t) - \delta S(t).$$

Since $C(t), S(t) \geq 0$, the rate of change of $P(t)$ is bounded as follows: Therefore, by integrating and simplifying, we obtain

$$\lim_{t\to\infty} \sup P \leq \frac{\Omega}{\mu}.$$

Thus, the total population $P(t)$ is bounded above and approaches a finite value as $t \to \infty$:

$$\Omega = \left\{ (N, M, C, S, W) \in \mathbb{R}_+^5 : 0 \leq P(t) \leq \frac{\Omega}{\mu} \right\}.$$

This ensures the boundedness of the solution.

4. Existence of Equilibrium

To determine the equilibrium points for the combined mushroom nutrient dynamics model, we solve for the steady-state values of $N(t)$, $M(t)$, $C(t)$, $S(t)$, and $W(t)$ by setting the time derivatives of the system to zero. This results in two main equilibria: nutrient-free equilibrium (NFE) and endemic equilibrium (EE).

4.1. Nutrient-Free Equilibrium (NFE)

Nutrient-free equilibrium occurs when the nutrient concentration in the substrate becomes depleted, meaning that there are no available nutrients for uptake by the mushroom. At this point, all variables related to growth and nutrient use settled at a steady state with no additional biomass growth or waste production. This equilibrium is denoted by E_0 and given by:

$$E_0 = \left(N^0, M^0, C^0, S^0, W^0\right) = (0, 0, 0, 0, 0).$$

4.2. Endemic Equilibrium (EE)

Endemic equilibrium occurs when the system stabilizes with a non-zero nutrient concentration in the substrate, allowing for steady nutrient uptake, biomass growth, and waste production. To find the endemic equilibrium, we set the time derivatives of the system to zero and solve for the equilibrium values of N^*, M^*, C^*, S^*, and W^*. The system of equations for the endemic equilibrium is as follows:

$$\frac{dN(t)}{dt} = 0 \quad \Rightarrow \quad N^* f(E^*) = 0,$$

$$\frac{dM(t)}{dt} = 0 \quad \Rightarrow \quad M^* = \alpha k_1 N^* f(E^*),$$

$$\frac{dC(t)}{dt} = 0 \quad \Rightarrow \quad \beta k_1 N^* f(E^*) = \gamma C^*,$$

$$\frac{dS(t)}{dt} = 0 \quad \Rightarrow \quad (1-\beta) k_1 N^* f(E^*) = \delta S^*,$$

$$\frac{dW(t)}{dt} = 0 \quad \Rightarrow \quad W^* = \epsilon k_1 N^* f(E^*).$$

These equations can be solved step-by-step:
Nutrient Uptake: From $N^* f(E^*) = 0$, the equilibrium value of N^* depends on the environmental factors. Assuming that the environmental function $f(E^*)$ is non-zero (i.e., conditions are suitable for growth), the system stabilizes with some positive value N^*.
Biomass Growth: The biomass at equilibrium M^* is proportional to the nutrient concentration N^*:

$$M^* = \alpha k_1 N^* f(E^*),$$

Nutrient Distribution:

$$C^* = \frac{\beta k_1 N^* f(E^*)}{\gamma}, \quad S^* = \frac{(1-\beta) k_1 N^* f(E^*)}{\delta},$$

Waste Production:

$$W^* = \epsilon k_1 N^* f(E^*),$$

Final Endemic Equilibrium (EE) expression:

$$E_1 = (N^*, M^*, C^*, S^*, W^*) = \left(N^*, \alpha k_1 N^* f(E^*), \frac{\beta k_1 N^* f(E^*)}{\gamma}, \frac{(1-\beta) k_1 N^* f(E^*)}{\delta}, \epsilon k_1 N^* f(E^*) \right).$$

This endemic equilibrium describes a steady state in which nutrient uptake, biomass growth, nutrient distribution, and waste production occur at constant rates determined by environmental factors and nutrient availability.

5. Basic reproduction number

To determine the basic reproduction number R_0 for the combined mathematical model of nutrient dynamics in edible mushrooms, we can use the next-generation matrix method. The transmission matrix F represents the rate of new infections or changes in the state due to nutrient uptake. This model captures how nutrient uptake $U(t)$ translates into changes in biomass, nutrient distribution, and waste. The matrix F for new nutrient uptake effects and the transfer matrix V describes the changes in state due to the distribution and degradation of nutrients, and waste production. Based on the equations given:

$$F = \begin{bmatrix} -k_1 f(E(t)) & 0 & 0 & 0 \\ \alpha k_1 f(E(t)) & 0 & 0 & 0 \\ \beta k_1 f(E(t)) & 0 & 0 & 0 \\ (1-\beta) k_1 f(E(t)) & 0 & 0 & 0 \\ \epsilon k_1 f(E(t)) & 0 & 0 & 0 \end{bmatrix}, \quad V = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\gamma & 0 & 0 \\ 0 & 0 & -\delta & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The Jacobian matrices J_F and J_V are as follows:

$$J_F = \begin{bmatrix} -k_1 f(E(t)) & 0 & 0 & 0 \\ \alpha k_1 f(E(t)) & 0 & 0 & 0 \\ \beta k_1 f(E(t)) & 0 & 0 & 0 \\ (1-\beta) k_1 f(E(t)) & 0 & 0 & 0 \\ \epsilon k_1 f(E(t)) & 0 & 0 & 0 \end{bmatrix}, \quad J_V = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\gamma & 0 & 0 \\ 0 & 0 & -\delta & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Using the next-generation matrix method, R_0 is given by the largest eigenvalue of $F \cdot V^{-1}$.

$$F \cdot V^{-1} = \begin{bmatrix} -k_1 f(E(t)) & 0 & 0 & 0 \\ \alpha k_1 f(E(t)) & 0 & 0 & 0 \\ \beta k_1 f(E(t)) & 0 & 0 & 0 \\ (1-\beta) k_1 f(E(t)) & 0 & 0 & 0 \\ \epsilon k_1 f(E(t)) & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} -\frac{1}{\gamma} & 0 & 0 & 0 \\ 0 & -\frac{1}{\delta} & 0 & 0 \end{bmatrix}.$$

Simplifying,

$$F \cdot V^{-1} = \begin{bmatrix} \frac{k_1 f(E(t))}{\gamma} & 0 \\ \frac{\alpha k_1 f(E(t))}{\gamma} & 0 \\ \frac{\beta k_1 f(E(t))}{\delta} & 0 \\ \frac{(1-\beta) k_1 f(E(t))}{\delta} & 0 \\ \frac{\epsilon k_1 f(E(t))}{\gamma} & 0. \end{bmatrix}$$

The largest eigenvalue is:

$$R_0 = \frac{k_1 f(E(t))}{\gamma + \delta}.$$

Here, the basic reproduction number R_0 is derived by considering the effects of nutrient uptake on biomass growth and waste production relative to degradation rates. Where $f(E(t))$ is given by the environmental influence function. If $R_0 < 1$, the system is stable with diminishing effects of nutrient uptake, whereas $R_0 > 1$ indicates growth and the need for management of nutrient and waste dynamics.

6. Sensitivity analysis

In epidemiology, the average number of secondary infections in a population that is completely susceptible to infection due to a single infected individual is known as the basic reproduction number R_0. The normalized forward sensitivity index of the reproduction number R_0 with respect to parameter γ is expressed as

$$\Psi_{\omega}^{R_0} = \frac{\partial R_0}{\partial \gamma} \times \frac{\gamma}{R_0}. \tag{2}$$

This methodology can be extended to calculate the sensitivity index of R_0 for all parameters within the editable mushroom model. This analysis enables an evaluation of the

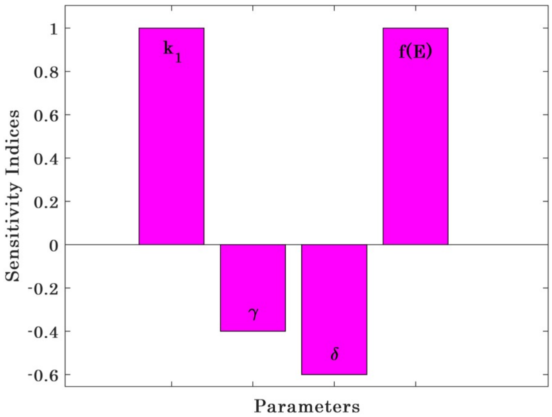


Figure 1: Normalized forward sensitivity indices of R_0 concerning various model parameters

relationships between the reproduction number and various parameters, as depicted in Figure 1. The results of the normalized forward sensitivity indices highlighted significant trends. Parameters with negative sensitivity indices, such as $\Psi_{\gamma}^{R_0}$ and $\Psi_{\delta}^{R_0}$, exhibited an inverse relationship with R_0, suggesting that increasing these parameters can aid in controlling the editable mushroom. Conversely, parameters with positive sensitivity indices, such as $\Psi_{\kappa_1}^{R_0}$ and $\Psi_{f(E)}^{R_0}$, demonstrate a direct relationship with R_0, indicating that reducing these parameters is necessary to effectively suppress editable mushrooms.

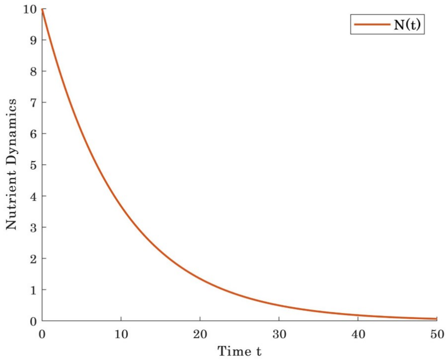
7. Numerical simulation

Figure 2 illustrates the key components of the Editable Mushroom model, focusing on the dynamics of the interactions between nutrients and biomass. Each subplot provides insights into specific aspects of the model, contributing to a comprehensive understanding of the interconnections between nutrients, biomass growth, and waste production within the system. The following is a detailed explanation of each figure.

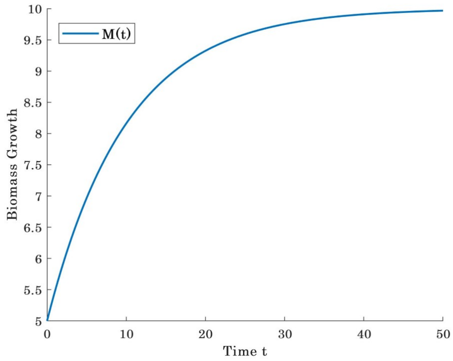
- **Nutrient Dynamics Class** $N(t)$ (Figure 2a): This figure depicts the dynamics of nutrient availability over time, illustrating how the concentration or level of nutrients changes. Understanding $N(t)$ is essential for assessing the effectiveness of nutrient utilization within the system and its impact on overall mushroom growth.

- **Biomass Growth Class** $M(t)$ (Figure 2b): This graph represents the growth of biomass in the mushroom model, showing how biomass increases over time in response to the availability of nutrients and environmental conditions. Monitoring $M(t)$ is crucial for evaluating the health and productivity of the mushroom cultivation process.

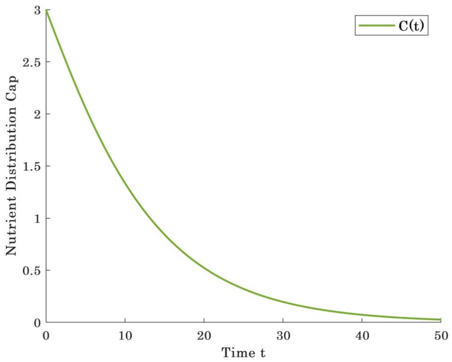
- **Nutrient Distribution Cap Class** $C(t)$ (Figure 2c): This subplot illustrates the



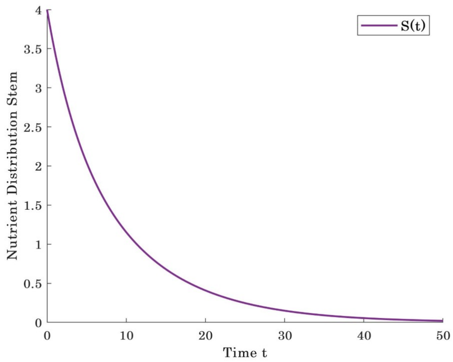
(a) Nutrient Dynamics class $N(t)$



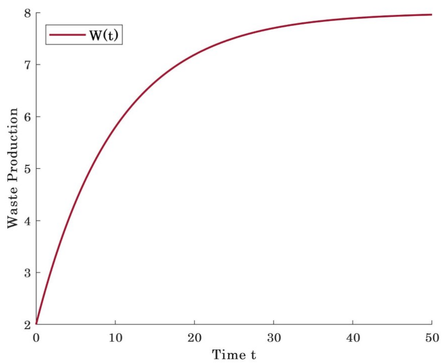
(b) Biomass Growth class $M(t)$



(c) Nutrient Distribution Cap class $C(t)$



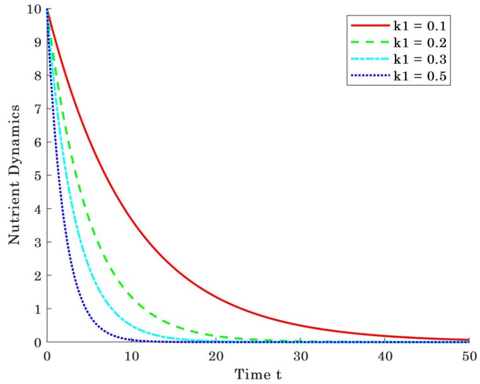
(d) Nutrient Distribution Stem class $S(t)$



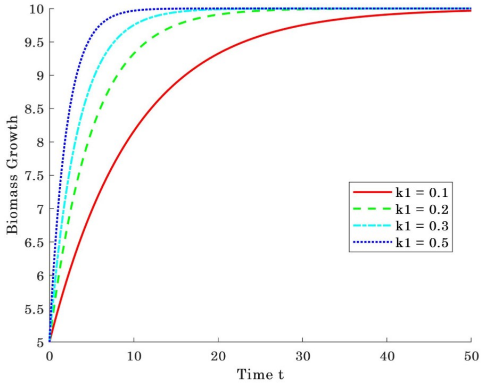
(e) Waste Production class $W(t)$

Figure 2: Graphical representation of the Nutrients in Editable Mushroom model

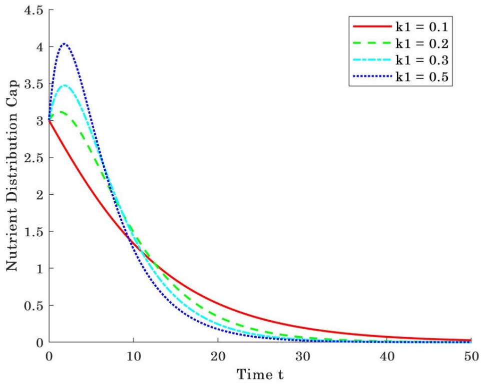
distribution of nutrients specifically in the cap of the mushrooms, providing insights into how nutrients are allocated within different parts of the mushroom, which can influence growth rates and overall quality.



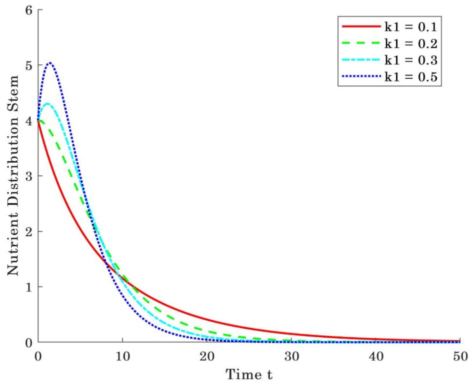
(a) Impact of varying k_1 on $N(t)$



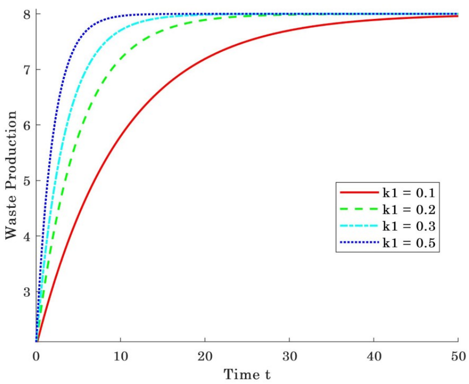
(b) Impact of varying k_1 on $M(t)$



(c) Impact of varying k_1 on $I(t)$



(d) Impact of varying k_1 on $S(t)$



(e) Impact of varying k_1 on $W(t)$

Figure 3: Effect of different values of k_1 on all the compartments in Editable Mushroom model

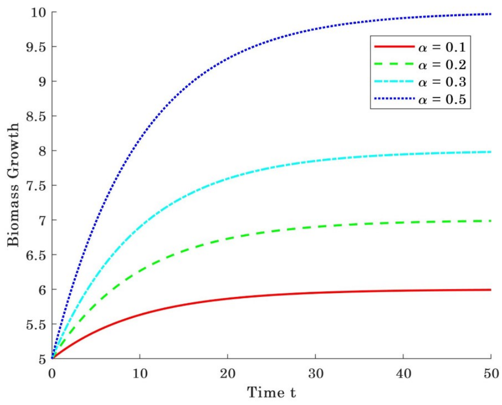
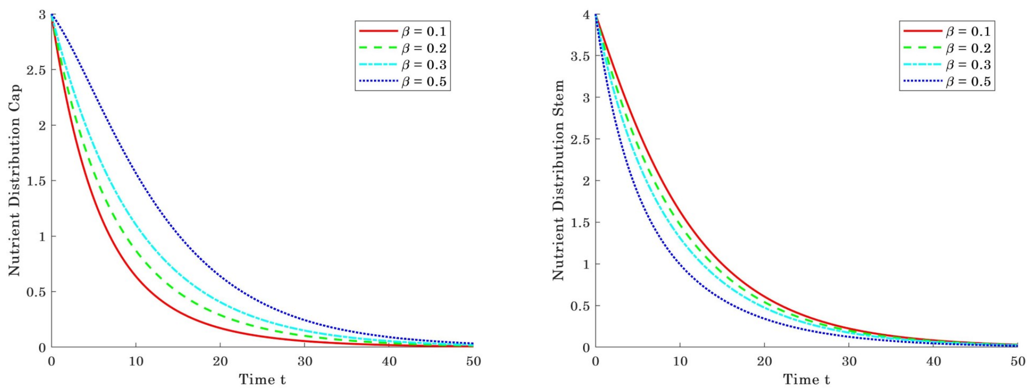


Figure 4: Effect of different values of α on the Biomass Growth



(a) Impact of varying β on $C(t)$ (b) Impact of varying β on $S(t)$

Figure 5: Effect of different values of β on the Nutrient Distribution Cap and Nutrient Distribution Stem classes

- **Nutrient Distribution Stem Class** $S(t)$ (Figure 2d): Similar to $C(t)$, this figure focuses on the nutrient distribution in the stem of the mushrooms, revealing how nutrients are transported and utilized in this specific part of the mushroom structure.
- **Waste Production Class** $W(t)$ (Figure 2e): This graph outlines the production of waste over time in the mushroom cultivation process, highlighting how waste is generated in relation to nutrient consumption and biomass growth. Monitoring $W(t)$ is crucial for effective waste management and ensuring sustainable practices in mushroom cultivation.

Figure 3 illustrates the effects of varying the parameter k_1 on different compartments of the Editable Mushroom model. Each subplot provides insights into how changes in k_1 influence the nutrient dynamics, biomass growth, and waste production. Below is a detailed explanation of each figure:

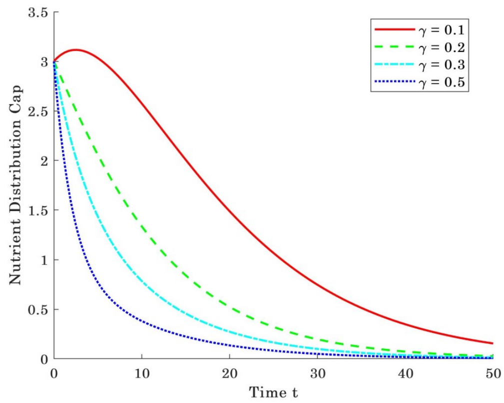


Figure 6: Effect of different values of γ on the Nutrient Distribution Cap

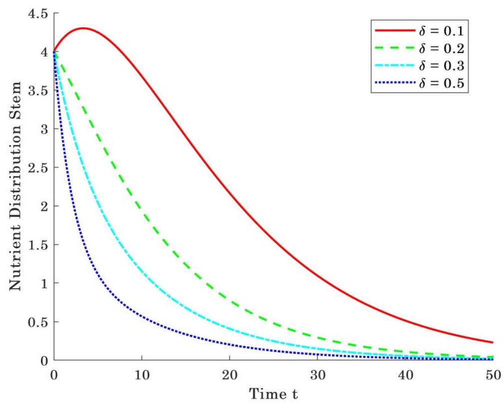


Figure 7: Effect of different values of δ on the Nutrient Distribution Stem

- **Impact of varying k_1 on Nutrient Dynamics** $N(t)$ (Figure 3a): This figure shows how different values of the parameter k_1 affect the nutrient dynamics over time. Variations in k_1 can influence the rate at which nutrients are consumed or replenished in the system, highlighting the importance of this parameter in maintaining the nutrient balance essential for mushroom growth.

- **Impact of varying k_1 on Biomass Growth** $M(t)$ (Figure 3b): Here, the effect of k_1 on biomass growth is depicted. This graph illustrates how changes in k_1 affect the growth rate of mushrooms. A higher k_1 may correlate with increased growth, whereas lower values could result in slower biomass accumulation, emphasizing the significance of nutrient availability.

- **Impact of varying k_1 on Nutrient Distribution** $C(t)$ (Figure 3c): This subplot focuses on the distribution of nutrients in the cap of the mushrooms. This reveals how varying k_1 affects the nutrient allocation to this specific part of the mushroom, which can influence its development and quality.

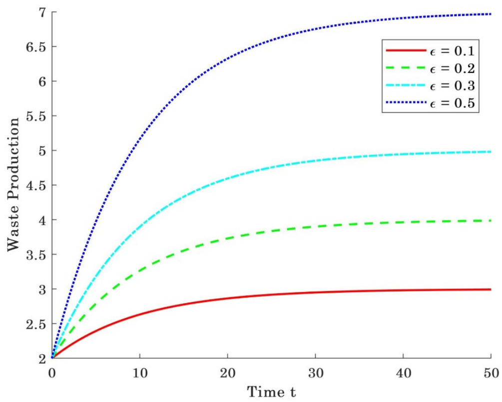


Figure 8: Effect of different values of ϵ on the Waste Production

- **Impact of varying k_1 on Nutrient Distribution** $S(t)$ (Figure 3d): Similar to the previous figure, this graph examines how k_1 affects nutrient distribution in the stem of the mushrooms. These results provide insights into how nutrient transport and allocation in the stem respond to changes in k_1, thereby impacting overall mushroom health.

- **Impact of varying k_1 on Waste Production** $W(t)$ (Figure 3e): This figure illustrates the relationship between k_1 and waste production over time. This indicates how different values of k_1 can lead to varying amounts of waste, which is crucial for understanding the sustainability of the mushroom cultivation process and managing waste effectively.

Figure 4 illustrates the effect of varying the parameter α on the biomass growth $M(t)$ in the Editable Mushroom model. The graph depicts how different values of α influence the biomass accumulation rate over time. This parameter, typically associated with growth efficiency or the conversion rate of nutrients into biomass, plays a crucial role in determining the effectiveness of mushrooms in utilizing available resources. Variations in α can represent changes in environmental conditions, such as temperature, moisture, or nutrient availability, which directly affect mushroom growth dynamics. For instance, higher values of α may indicate a more favorable environment for growth, leading to a rapid increase in biomass, whereas lower values could signify suboptimal conditions resulting in slower growth. Understanding the relationship between α and biomass growth is vital for mushroom cultivation practices. By adjusting the factors that influence α, cultivators can optimize the conditions to enhance mushroom growth rates and yields. This knowledge can inform decisions related to substrate preparation, nutrient supplementation, and environmental control. The insights gained from analyzing the effects of α can be used to validate the Editable Mushroom model. If the model accurately predicts biomass growth under varying α values, it reinforces the reliability of the model for simulating mushroom growth dynamics in real-world scenarios.

Figure 5 presents two subplots that explore the effects of varying the parameter β on the nutrient distribution in the Editable Mushroom model. Each subplot focuses on a different component of the nutrient distribution:

- **Impact of Varying β on Nutrient Distribution Cap $C(t)$ (Figure 5a)**: This subplot illustrates how changes in β influence nutrient allocation, specifically to the mushroom cap. β can be associated with the rate at which nutrients are absorbed and utilized by the mushroom cap. As β varies, the graph shows the corresponding changes in the nutrient concentration over time. A higher β value may indicate more efficient nutrient uptake or enhanced growth conditions, resulting in increased nutrient levels in the capsule. Conversely, a lower β could signify reduced efficiency, affecting the overall growth and health.
- **Impact of Varying β on Nutrient Distribution Stem $S(t)$ (Figure 5b)**: This subplot examines how varying β affects nutrient distribution in the stem of the mushroom. The stem is crucial for providing structural support and nutrient transport. The effects of β in this context can provide insights into how nutrient allocation impacts the growth dynamics of the stem. Similar to the cap, higher β values may facilitate better nutrient distribution to the stem, supporting overall mushroom development. Understanding these dynamics is vital for optimizing growth conditions and improving the yield.

The effects depicted in Figure 5 underscore the importance of parameter β in managing nutrient distribution within the Editable Mushroom model. By understanding how this parameter influences both the cap and stem, cultivators can make informed decisions to enhance mushroom growth and optimize their cultivation practices. Effective manipulation of β can lead to improved nutrient usage efficiency, ultimately supporting higher yields and healthier mushrooms.

Figure 6 illustrates the effect of varying the parameter γ on the nutrient distribution specifically in the cap class $C(t)$ of the Editable Mushroom model. In the context of the model, γ typically represents a rate or efficiency factor associated with the nutrient distribution mechanisms that affect the cap. This could reflect aspects such as the transport rate of nutrients or the cap's ability to absorb them from the substrate. The graph shows how changes in γ lead to variations in the nutrient concentration over time within the cap. When γ increases, it may indicate a more efficient nutrient uptake process. This could lead to a higher nutrient concentration in the cap, which is critical for optimal growth and development. The graph likely depicts a steeper increase in nutrient levels, suggesting that the cap thrives under these conditions. Conversely, lower values of γ may hinder nutrient transport and absorption efficiency. The graph reflects a slower increase in nutrient concentration or potentially a plateau, indicating that the cap does not receive sufficient nutrients for optimal growth. The insights gained from Figure 6 highlight the significance of parameter γ in managing the nutrient distribution within the mushroom cap. Understanding how γ influences nutrient dynamics is crucial for optimizing the cultivation strategies. By effectively manipulating this parameter, cultivators can enhance nutrient availability in the cap, leading to improved mushroom quality and yields. This underscores the importance of precise parameter tuning in agricultural models to achieve the desired production outcomes.

Figure 7 illustrates the effect of varying the parameter δ on the nutrient distribution within the stem class $S(t)$ of the Editable Mushroom model. In the context of this model, δ typically represents a decay rate or dilution factor that affects how nutrients are allocated and retained within the stem. It can influence the efficiency of nutrient transport and storage in the stem. The graph displays how changes in δ affect the nutrient concentration over time within the stem. An increase in δ may indicate faster nutrient decay or lower retention capability in the stem. Consequently, the graph may show a decline in nutrient levels, suggesting that the stem does not effectively maintain nutrient stores. Conversely, lower values of δ could lead to a slower decay rate and improved retention of nutrients. The graph likely illustrates a gradual increase or stabilization of nutrient concentration, indicating that the stem can better support growth by retaining essential nutrients. The insights gained from Figure 7 underscore the importance of parameter δ in managing nutrient distribution within mushroom stems. Understanding how δ influences nutrient dynamics is critical for optimizing the cultivation practices. By effectively adjusting this parameter, cultivators can enhance the stem's ability to retain nutrients, promoting healthier growth and higher yields. This highlights the necessity of precise parameter tuning in agricultural models to achieve desirable outcomes in the production of mushrooms.

Figure 8 illustrates the effect of varying the parameter ϵ on waste production $W(t)$ in the Editable Mushroom model. In this model, ϵ often represents a waste production rate or efficiency factor that affects the amount of waste generated relative to nutrient uptake and growth processes. This indicates the metabolic efficiency of the mushroom, where higher values signify increased waste relative to biomass. The graph shows how different values of ϵ influence the waste quantity produced over time. When ϵ is increased, it may lead to a higher waste production rate. This can be represented in the graph as a steeper increase in the waste curve, indicating that the mushroom generates more waste relative to its nutrient uptake. This may be due to suboptimal growth conditions or inefficiencies in metabolic processes. Conversely, lower values of ϵ may reduce waste production. The graph might show a flatter curve, suggesting that the mushroom manages its resources more effectively, leading to less waste generation relative to its growth. The insights gained from Figure 8 highlight the significance of parameter ϵ in managing waste production during mushroom cultivation. Understanding how ϵ affects waste dynamics is essential for optimizing growth conditions and enhancing the sustainability of mushroom-farming practices. By effectively tuning this parameter, cultivators can minimize waste production, improve metabolic efficiency, and ultimately achieve better yields and environmental outcomes. This emphasizes the importance of parameter optimization in agricultural models for driving efficient and sustainable production.

8. Conclusions

In this study, we present a comprehensive mathematical model that focuses on analyzing nutrient absorption, retention, and degradation in edible mushrooms, with the aim of optimizing growth conditions and enhancing nutrient content. The model is based on a system of differential equations that integrates key components, such as nutrient uptake, biomass growth, nutrient distribution between the mushroom cap and stem, and waste production.

The nutrient uptake rate, $U(t)$, is modeled as being proportional to the nutrient concentration in the substrate, $N(t)$, with environmental factors such as temperature and humidity affecting the uptake efficiency. These environmental factors were incorporated using Gaussian functions that accounted for deviations from the optimal conditions. Biomass growth is driven by nutrient uptake, with the conversion efficiency parameter α linking nutrient absorption to biomass production. Nutrient distribution is further modeled through separate equations for the cap and stem, accounting for the different rates of allocation and degradation. Waste production, as a by-product of metabolic processes, is also included in the model, providing a holistic understanding of nutrient dynamics.

Moreover, the mathematical analysis of the model focuses on key aspects such as boundedness, equilibrium points, and the basic reproduction number (R_0), which is critical for determining the stability and long-term behavior of the system. The boundedness analysis ensured that the solutions remained within biologically realistic ranges, preventing unbounded or non-physical growth. Equilibrium analysis helps identify the stable states of a system under given environmental conditions, and the basic reproduction number provides insights into the threshold conditions necessary for sustainable nutrient absorption and growth. In addition, a sensitivity analysis was performed to evaluate the impact of various parameters on nutrient dynamics. This analysis helps identify the most influential factors, allowing targeted interventions to improve mushroom growth and nutrient content. Finally, numerical simulations were conducted to illustrate the model's behavior under different scenarios, providing practical insights and guidance for optimizing mushroom cultivation practices.

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