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Laplace Transform Solutions for Heat and Mass Transfer Problem Using Caputo–Fabrizio Fractional Derivative

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Abstract. The novelty of this study lies in the development of closed-form analytical solutions for coupled heat and mass transfer processes using the Caputo–Fabrizio fractional derivative, a non-singular operator that realistically captures memory-dependent diffusion without the limitations of classical fractional models. Unlike existing works that mainly focus on integer-order or singular-kernel fractional formulations, this research introduces a non-singular, exponential-kernel fractional framework to investigate transient thermal and solutal transport in viscous media. More exactly, this study investigates the analytical solutions of coupled heat and mass transfer problems in a time-fractional framework employing the Caputo–Fabrizio fractional derivative operator. The model considers transient transport phenomena in a viscous fluid medium under generalized thermal and concentration gradients. The use of the Caputo–Fabrizio derivative, characterized by its non-singular exponential kernel, enables the realistic modeling of memory effects without mathematical singularities inherent in classical fractional operators. The governing equations are transformed into the Laplace domain to derive closed-form expressions for temperature and concentration distributions. The inverse Laplace transform is then applied to obtain the time-domain solutions. The influence of the fractional parameter on heat and mass transfer characteristics is examined in detail, highlighting the role of fractional-order differentiation in controlling diffusion and relaxation behaviors. Graphical results demonstrate that decreasing fractional order slows the rate of thermal and solutal diffusion, reflecting stronger memory effects. The findings offer valuable insights into the design and optimization of fractional-order heat and mass transfer systems applicable in thermal engineering, material processing, and porous media transport.

2020 Mathematics Subject Classifications: 76A05, 76A10, 76U05, 76S05, 76W05, 26A33, 35R11, 35A22

Key Words and Phrases: Caputo–Fabrizio fractional derivative, Laplace transform, heat and mass transfer, non-singular kernel, memory effect, analytical solution.

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1. Introduction

The Casson fluid model is a well-known non-Newtonian formulation commonly applied in hemodynamics. It effectively describes shear-thinning behavior, where viscosity decreases at high shear rates and increases significantly at low shear rates. Human blood is often modeled as a Casson fluid due to its non-Newtonian nature. At low shear rates, blood behaves like an elastic solid, demonstrating yield stress characteristics that align with the Casson model. Natural convection and buoyancy-driven flow of Casson fluids play an important role in biomedical processes such as blood movement in microvessels, thermal effects during hyperthermia treatments, and heat transfer in synovial fluids within joints [1–3]

In recent years, fractional calculus has become a powerful mathematical tool for describing complex systems. By extending traditional calculus to include derivatives and integrals of non-integer order, it provides a flexible framework for modeling real-world phenomena. Among various fractional derivative definitions, the Caputo and Caputo–Fabrizio forms are especially significant for physical and engineering applications. These operators are widely used to represent nonlocal and memory-dependent processes, particularly in non-Newtonian fluid flow, viscoelasticity, turbulence, and transport through porous media. Unlike integer-order derivatives, fractional derivatives take into account the system's past behavior, which makes them well-suited for describing materials with memory effects [4–10].

The Caputo derivative involves a singular kernel, which can create challenges in practical modeling. To address this, Caputo and Fabrizio introduced a non-singular exponential kernel that simplifies the description of physical processes [11]. This Caputo–Fabrizio derivative has proven more effective for studying fluid flow problems involving heat and mass transfer. Fractional calculus has since been widely applied in rheological and fluid mechanics studies. For instance, Akhtar et al. [12] analyzed rotational flow in cylindrical geometries for fractional viscoelastic fluids, while Hayat et al. [13] investigated the fractional Maxwell model for periodic unidirectional flows. Khan [14] explored oscillatory flow of fractional Jeffrey fluids with partial slip in porous media, and Khan et al. [15] examined magnetohydrodynamic (MHD) Oldroyd-B flows under modified Darcy's law. Qi et al. [16] further studied unsteady helical motion of fractional Oldroyd-B fluids, and many other researchers have continued exploring fractional derivatives in diverse fluid flow problems [17–25] The Casson model, in particular, has been used to analyze a wide range of practical problems. Asma et al. [16] studied oscillatory Casson fluid flow with heat transfer over a vertical plate, while Hussanan et al. [26] examined boundary layer behavior under Newtonian heating conditions. Similar studies have focused on free convection with slip effects [27] and temperature-dependent Casson fluid motion using Wright functions [28]. Recent investigations in 2025 have expanded Casson fluid research across various configurations. Nima et al. [29] analyzed Casson fluid in porous inclined surfaces with heat generation and microorganisms; Venkat Reddy et al. [30] considered double-diffusive convection in porous media; and Rehman et al. [31] modeled MHD Casson nanofluids using neural networks. Abbas et al. [32] introduced Caputo fractional modeling for MHD Casson fluids with buoyancy and radiation effects, while Alwuthaynani [33] studied fractional Casson flow over a Riga plate. Other works explored entropy generation [34], variable thermal properties [35], and unsteady Casson nanofluids with microorganisms and magnetic effects [36]. Abdoalrahman [37] examined numerically the Marangoni convection in hybrid nanofluid flow over a disk. Bhatti et al. [38] investigated magnetized hyperbolic tangent nanofluid flow incorporating thermo-diffusion and diffusion-thermo effects, emphasizing the role of Hall current and nonlinear stretching on heat and mass transport. Zeeshan et al. [39] extended this line of research by examining non-similar boundary-layer flow of a non-Newtonian hybrid nanofluid over a cylinder with viscous dissipation, revealing how hybrid nanoparticles and dissipative heating modify thermal performance. Hatami et al. [40] contributed further by analyzing three-dimensional condensation nanofluid films on rotating inclined disks, demonstrating the capability of analytical methods to capture microscale heat-transfer behaviors in rotating systems. The current study focuses on heat and mass transfer in blood flow modeled through fractional differential equations using the Caputo–Fabrizio operator. The flow occurs over a vertically oscillating plate. Analytical solutions for velocity, temperature, and concentration fields are derived using the integral transform method. The physical interpretation of the results is supported by numerical simulations and graphical analysis, highlighting how the memory parameter affects the fluid's velocity, temperature, and concentration distributions.

2. Flow Configuration Description

Consider the unsteady free convection flow involving heat and mass transfer of an incompressible, electrically conducting, and heat-radiating Casson fluid.

The flow occurs along an infinite, non-conducting, vertical, oscillating flat plate.

- The flow occurs along an infinite, non-conducting, vertical, oscillating flat plate.
    - The x-axis is oriented along the plate in the upward direction.
    - The y-axis is normal to the plate.
    - The z-axis be normal to the x–y plane.
- Initially ($t \leq 0$):
    - Both the fluid and the plate are stationary.
    - The temperature and species concentration are uniform and constant, denoted by T_∞ and C_∞, respectively.
- For ($t > 0$)
    - The plate begins to oscillate within its own plane with velocity $U_0 \sin(\omega t)$, where U_0 is a positive constant representing the characteristic velocity.
    - The surface temperature T_w is directly proportional to the heat transfer from the plate to the fluid.
    - The concentration near the plate increases from C_∞ to C_w .

- Because the plate is infinite in both x and z directions and electrically non-conducting, all physical quantities (except pressure) depend only on y and t.

- Based on these assumptions, the governing equations for free convection heat and mass transfer of a viscous, incompressible Casson fluid are derived as shown in [41].

$$\frac{\partial u}{\partial t} = v\left(1+\frac{1}{\beta}\right)\frac{\partial u^2}{\partial y^2} + g\beta_T(T - T_\infty) + g\beta_c(C - C_\infty), \tag{1}$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_\rho}\frac{\partial^2 T}{\partial y^2} + \frac{1}{\rho C_\rho}\frac{16\sigma' T_\infty^3}{3k'}\frac{\partial^2 T}{\partial y^2} \tag{2}$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2}, \tag{3}$$

where u, T, and C are, respectively, the fluid velocity, temperature, species concentration and $v, \beta g, \beta_T, \beta_C, k, \rho, c_\rho, q_r, D$ are, respectively, the kinematic viscosity, Casson fluid parameter, acceleration due to gravity, fluid density, thermal expansion coefficient, fluid constant density, specific heat at constant pressure, thermal conductivity, radiative heat flux, chemical molecular diffusivity.

The associate initial and boundary conditions are

$$u = 0, T = T_\infty, C = C_\infty, for \;\; y > 0 \;\; and \;\; t = 0, \tag{4}$$

$$u = U_0 \sin(\omega t), T(0,t) = T_\infty + T_w(1 - ae^{-bt}), C = C_w, \; at \;\; y = 0 \;\; for \;\; t > 0 \tag{5}$$

$$u = 0, T = T_\infty, C = C_\infty, for \;\; y \longleftrightarrow \infty \tag{6}$$

Introducing dimensionless variables and parameters

$$y^* = \frac{U_0 y}{v}, u^* = \frac{u}{U_0}, t^* = \frac{tU_0^2}{v}, T^* = \frac{T - T_\infty}{T_\infty}, C^* = \frac{C - C_\infty}{C_w - C_\infty}$$

$$Gr = \frac{g\beta_T v T_\infty}{U_0^3}, Gm = \frac{g\beta_c v(C_w - C_\infty)}{U_0^3}, Pr = \frac{\rho v C_\rho}{k}, Sc = \frac{v}{D}, N_r = \frac{16\sigma' T_\infty^3}{3kk'} \tag{7}$$

into Eqs.(1) to (6) and dropping star notation, we obtain the following initial-boundary values problem

$$\frac{\partial u(y,t)}{\partial t} = \left(1+\frac{1}{\beta}\right)\frac{\partial u^2(y,t)}{\partial y^2} + G_r T(y,t) + GmC(y,t), \tag{8}$$

$$\frac{\partial T(y,t)}{\partial t} = \frac{(1+N_r)}{Pr}\frac{\partial^2 T(y,t)}{\partial y^2}, \tag{9}$$

$$\frac{\partial C(y,t)}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C(y,t)}{\partial y^2}, \tag{10}$$

where Gr is the thermal Grashof number, Gm is the mass Grashof number, Pr is the Prandtl number, Sc is the Schmidt number, N_r is the thermal radiation parameter.

The corresponding non-dimensional initial and boundary conditions are

$$\mathrm{u(y,t) = 0, T(y,0) = 0, C(y,0) = 0, y > 0} \tag{11}$$

$$\mathrm{u(0,t) = \sin(\omega t), T(0,t) = 1 - ae^{-bt}, C(0,t) = 1, t > 0} \tag{12}$$

$$\mathrm{u(y,t) \longrightarrow 0, T(y,t) \longrightarrow 0, C(y,t) \longrightarrow 0} as \mathrm{y} \longrightarrow 0 \tag{13}$$

In order to develop a model with time-fractional derivatives, we replace the time derivative of order one with Caputo-Fabrizio time-fractional derivative of order $\alpha \in$ (0,1).

Eqs. (8), (9) and (10) are written as:

$$\mathrm{D_t^\alpha u(y,t)} = \left(1 + \frac{1}{\beta}\right)\frac{\partial^2 \mathrm{T(y,t)}}{\partial \mathrm{y}^2} + \mathrm{GrT(y,t) + GmC(y,t)}, \tag{14}$$

$$\mathrm{D_t^\alpha T(y,t)} = \left(\frac{1 + \mathrm{N_r}}{\mathrm{Pr}}\right)\frac{\partial^2 \mathrm{T(y,t)}}{\partial \mathrm{y}^2}, \tag{15}$$

$$\mathrm{D_t^\alpha C(y,t)} = \frac{1}{\mathrm{Sc}}\frac{\partial^2 \mathrm{T(y,t)}}{\partial \mathrm{y}^2}, \tag{16}$$

where Caputo-Fabrizio time-fractional derivative is defined by

$$\mathrm{D_t^\alpha u(y,t)} = \frac{1}{1-\alpha}\int_0^{\mathrm{t}} \exp\left(\frac{-\alpha(\mathrm{t}-\tau)}{(1-\alpha)}\right)\frac{\partial \mathrm{u}(\mathrm{y},\tau)}{\partial \tau}\mathrm{d}\tau, 0 < \alpha < 1. \tag{17}$$

3. Solution of the problem

3.1. Heat Transfer

Applying the Laplace transform to temperature equation under the corresponding conditions, we get:

$$\frac{\partial^2 \overline{\mathrm{T}}(\mathrm{y,q})}{\partial \mathrm{y}^2} = \frac{\mathrm{Pr}}{1 + \mathrm{Nr}}\frac{\mathrm{q}}{(1-\alpha)\mathrm{q} + \alpha}\overline{\mathrm{T}}(\mathrm{y,q}), 0 < \alpha < 1. \tag{18}$$

$$\overline{\mathrm{T}}(0,\mathrm{q}) = \frac{1}{\mathrm{q}} - \frac{\mathrm{a}}{\mathrm{q+b}}, \quad \overline{\mathrm{T}}(\mathrm{y,q}) \longrightarrow 0, \;\; as \;\; y \longrightarrow \infty \tag{19}$$

where $\overline{\mathrm{T}}(\mathrm{t,q}) = \int_0^\infty \mathrm{T(y,t)}\exp(-\mathrm{qt})\mathrm{dt}$ is the Laplace transform of the function T(y,t)

$$\frac{\partial^2 \overline{\mathrm{T}}(\mathrm{y,s})}{\partial \mathrm{y}^2} = \frac{\mathrm{Pr}}{1 + \mathrm{Nr}}\frac{\gamma \mathrm{q}}{\mathrm{q} + \alpha\gamma}\overline{\mathrm{T}}(\mathrm{y,q}); \quad \gamma = \frac{1}{1-\alpha} \tag{20}$$

$$\overline{\mathrm{T}}(0,\mathrm{q}) = \frac{1}{\mathrm{q}} - \frac{\mathrm{a}}{\mathrm{q+b}}, \quad \overline{\mathrm{T}}(\mathrm{y,q}) \longrightarrow 0, \;\; as \;\; y \longrightarrow \infty \tag{21}$$

The solution of the above problem is:

$$\overline{\mathrm{T}}(\mathrm{y},\mathrm{q}) = \left(\frac{1}{\mathrm{q}} - \frac{\mathrm{a}}{\mathrm{q}+\mathrm{b}}\right)\exp\left(-\mathrm{y}\sqrt{\frac{\mathrm{Pr}}{1+\mathrm{Nr}}\frac{\gamma\mathrm{q}}{\mathrm{q}+\alpha\gamma}}\right)$$

$$= (1-\mathrm{a})\Phi\left(\mathrm{y},\mathrm{q};\frac{\mathrm{Pr}\gamma}{1+\mathrm{Nr}},\alpha\gamma\right) - \mathrm{a}\overline{\psi}\left(\mathrm{y},\mathrm{q};\frac{\mathrm{Pr}\gamma}{1+\mathrm{Nr}},\alpha\gamma,-\mathrm{b}\right) \tag{22}$$

Equations (22) for the classical fluid takes the form: $(\alpha = 1)$

$$\overline{\mathrm{T}}(\mathrm{y},\mathrm{q}) = \frac{1}{\mathrm{q}}\exp\left(-\mathrm{y}\sqrt{\frac{\mathrm{Pr}}{1+\mathrm{Nr}}\mathrm{q}}\right) - \frac{\mathrm{a}}{\mathrm{q}+\mathrm{b}}\exp\left(-\mathrm{y}\sqrt{\frac{\mathrm{Pr}}{1+\mathrm{Nr}}\mathrm{q}}\right) \tag{23}$$

3.2. Mass transfer:

Applying Laplace transform to mass transfer equation under the imposed conditions gives:

$$\frac{\mathrm{Sc}\gamma\mathrm{q}}{\mathrm{q}+\alpha\gamma}\overline{\mathrm{C}}(\mathrm{y},\mathrm{q}) = \frac{\partial^2\overline{\mathrm{C}}(\mathrm{y},\mathrm{q})}{\partial\mathrm{y}^2} \tag{24}$$

$$\overline{\mathrm{C}}(\mathrm{y},\mathrm{q}) = \frac{1}{\mathrm{q}}, \quad \overline{\mathrm{C}}(\mathrm{y},\mathrm{q}) \longrightarrow 0, \quad as \quad \mathrm{y} \longrightarrow \infty \tag{25}$$

where $\overline{\mathrm{C}}(\mathrm{y},\mathrm{q}) = \int_0^\infty \mathrm{C}(\mathrm{y},\mathrm{t})\exp(-\mathrm{qt})\mathrm{dt}$ is the Laplace transform of the function $\mathrm{C}(\mathrm{y},\mathrm{t})$
The solution of the above mass transfer problem is:

$$\overline{\mathrm{C}}(\mathrm{y},\mathrm{q}) = \frac{1}{\mathrm{q}}\exp\left(-\mathrm{y}\sqrt{\frac{\mathrm{Sc}\gamma\mathrm{q}}{\mathrm{q}+\alpha\gamma}}\right) = \Phi(\mathrm{y},\mathrm{q};\mathrm{Sc}\gamma,\alpha\gamma) \tag{26}$$

3.3. Velocity field

The transformed shape of fluid velocity equations is:

$$\frac{\delta\gamma\mathrm{q}}{\mathrm{q}+\alpha\gamma}\overline{\mathrm{u}}(\mathrm{y},\mathrm{q}) = \frac{\partial^2\overline{\mathrm{u}}(\mathrm{y},\mathrm{q})}{\partial\mathrm{y}^2} + \delta\mathrm{Gr}\left(\frac{1}{\mathrm{q}} - \frac{\mathrm{a}}{\mathrm{q}+\mathrm{b}}\right)\exp\left(-\mathrm{y}\sqrt{\frac{\mathrm{a}_1\gamma\mathrm{q}}{\mathrm{q}+\alpha\gamma}}\right) + \delta Gm\frac{1}{q}\exp\left(-y\sqrt{\frac{Sc\gamma q}{q+\alpha\gamma}}\right), \tag{27}$$

$$\overline{\mathrm{u}}(0,\mathrm{q}) = \frac{\omega}{\mathrm{q}^2+\omega^2}, \quad \overline{\mathrm{u}}(\mathrm{y},\mathrm{q}) \longrightarrow 0 \quad \text{as} \quad \mathrm{y} \longrightarrow \infty,$$

where $\frac{1}{\delta} = \left(1 + \frac{1}{\beta}\right)$ and $\mathrm{a}_1 = \left(\frac{\mathrm{Pr}}{1+\mathrm{Nr}}\right)$.
The above problem gives the following transformed solution:

$$\overline{\mathrm{u}}(\mathrm{y},\mathrm{q}) = \frac{\omega}{\mathrm{q}^2+\omega^2}\exp\left(-\mathrm{y}\sqrt{\frac{\delta\gamma\mathrm{q}}{\mathrm{q}+\alpha\gamma}}\right) - \frac{\delta\mathrm{Gr}}{\gamma(1-\mathrm{a}_1)}\left[\mathrm{b}_1\frac{1}{\mathrm{q}} + \alpha\gamma\frac{1}{\mathrm{q}^2}\mathrm{b}_2\frac{1}{\mathrm{q}+\mathrm{b}}\right]\exp\left(-\mathrm{y}\sqrt{\frac{\delta\gamma\mathrm{q}}{\mathrm{q}+\alpha\gamma}}\right) -$$

$$-\frac{\delta \mathrm{Gm}}{\gamma(1-\mathrm{a}_1)}\left(\frac{1}{\mathrm{q}}+\alpha\gamma\frac{1}{\mathrm{q}^2}\right)\exp\left(-\mathrm{y}\sqrt{\frac{\delta\gamma \mathrm{q}}{\mathrm{q}+\alpha\gamma}}\right)+\frac{\delta \mathrm{Gr}}{\gamma(1-\mathrm{a}_1)}\left[\mathrm{b}_1\frac{1}{\mathrm{q}}+\alpha\gamma\frac{1}{\mathrm{q}^2}\mathrm{b}_2\frac{1}{\mathrm{q}+\mathrm{b}}\right]\times$$
$$\times\exp\left(-\mathrm{y}\sqrt{\frac{\delta\gamma \mathrm{q}}{\mathrm{q}+\alpha\gamma}}\right)+\frac{\delta \mathrm{Gm}}{\gamma(1-\mathrm{a}_1)}\left(\frac{1}{\mathrm{q}}+\alpha\gamma\frac{1}{\mathrm{q}^2}\right)\exp\left(-\mathrm{y}\sqrt{\frac{\mathrm{Sc}\gamma \mathrm{q}}{\mathrm{q}+\alpha\gamma}}\right), \tag{28}$$

where $\mathrm{b}_1=\left(\frac{1-\mathrm{a}\alpha\gamma}{\mathrm{b}}\right)$, $\mathrm{b}_2=\mathrm{a}\left(\frac{\alpha\gamma}{\mathrm{b}}-1\right)$, $\mathrm{a}_1\neq 0$.
For the classical fluid model: $(\alpha=1)$

$$\overline{\mathrm{u}}(\mathrm{y},\mathrm{q})=\frac{\omega}{\mathrm{q}^2+\omega^2}\exp(-\mathrm{y}\sqrt{\delta \mathrm{q}})-\frac{\delta \mathrm{Gr}}{1-\mathrm{a}_1}\left[-\frac{\mathrm{a}}{\mathrm{b}}\frac{1}{\mathrm{q}}+\frac{1}{\mathrm{q}^2}\frac{\mathrm{a}}{\mathrm{b}}\frac{1}{\mathrm{q}+\mathrm{b}}\right]\exp(-\mathrm{y}\sqrt{\delta \mathrm{q}})-\frac{\delta \mathrm{Gm}}{(1-\mathrm{a}_1)}\frac{1}{\mathrm{q}^2}\exp(-\mathrm{y}\sqrt{\delta \mathrm{q}})+$$
$$+\frac{\delta \mathrm{Gr}}{(1-\mathrm{a}_1)}\left[-\frac{\mathrm{a}}{\mathrm{b}}\frac{1}{\mathrm{q}}+\frac{1}{\mathrm{q}^2}\frac{\mathrm{a}}{\mathrm{b}}\frac{1}{\mathrm{q}+\mathrm{b}}\right]\exp(-\mathrm{y}\sqrt{\mathrm{a}_1 \mathrm{q}})+\frac{\delta \mathrm{Gm}}{(1-\mathrm{a}_1)}\frac{1}{\mathrm{q}^2}\exp(-\mathrm{y}\sqrt{\mathrm{Scq}}) \tag{29}$$

4. Discussion of Results

To gain deeper physical insight into the behavior of fluid temperature, solute concentration, and velocity, numerical evaluations were performed for various parameter values using Mathcad. The corresponding results are illustrated in Figures 1–6. All quantities presented are nondimensional, and each profile is plotted with respect to the spatial coordinate y. Unless otherwise specified, the parameter values a = 0.75 and b = 0.15 are used throughout the graphical analysis.

Our initial focus was to examine the effect of the fractional parameter α on the dimensionless temperature, concentration, and velocity fields. The study further considers the influence of the Casson fluid parameter β on the velocity distribution. For the ordinary fluid, corresponding to the unit fractional parameter α, results are also computed. The Figure 1 (a) and (b) are plotted to discuss the influence of the fractional parameter α on the fluid temperature {temp(y,t)} for a fixed value of the Prandtl number (Pr = 0.71) and the thermal radiation parameter (N_r = 0.6). The curves are sketched for different values of time t (for small and large times) and of fractional parameter α. It is observed from these figures that, at small values of the time t = 0.1, by increasing the value of fractional parameter, the temperature {temp(y,t)} decreases and tends to zero quickly, but for large values of the time t = 2 by increasing the value of fractional parameter α the temperature {temp(y,t)} increases and tends to zero slowly. Also, from the comparison of the two Figure 1(a) and 1(b) by increasing values of the time t, the plate temperature {temp(y,t)} increases.

Figure 2 (a) and (b) are plotted to discuss the influence of the fractional parameter α on the fluid temperature {temp(y,t)} for the fixed value of the Prandtl number Pr = 0.71. The curves are sketched for different values of time (for small t = 0.1 and large time t = 2) and for the fractional parameter α when (N = 0.6). It is observed that, at small values of the time t = 0.1, by increasing the value of fractional parameter, the

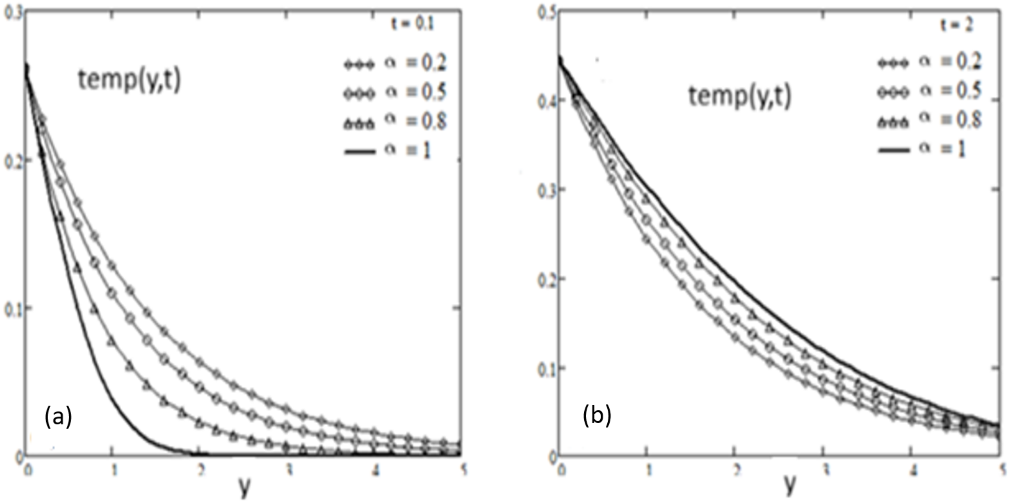

Figure 1: (a) Profiles of temperature versus y variation and small values of time t. (b) Profiles of temperature y versus α for α variation and large values of time t

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temperature {temp(y,t)} decreases whereas for large time t = 2, the temperature increases with increasing values of fractional parameter. From the comparison of Figure 2 (a) and (b), it is easy to see that the increase in temperature {temp(y,t)} is significant for large value of time.

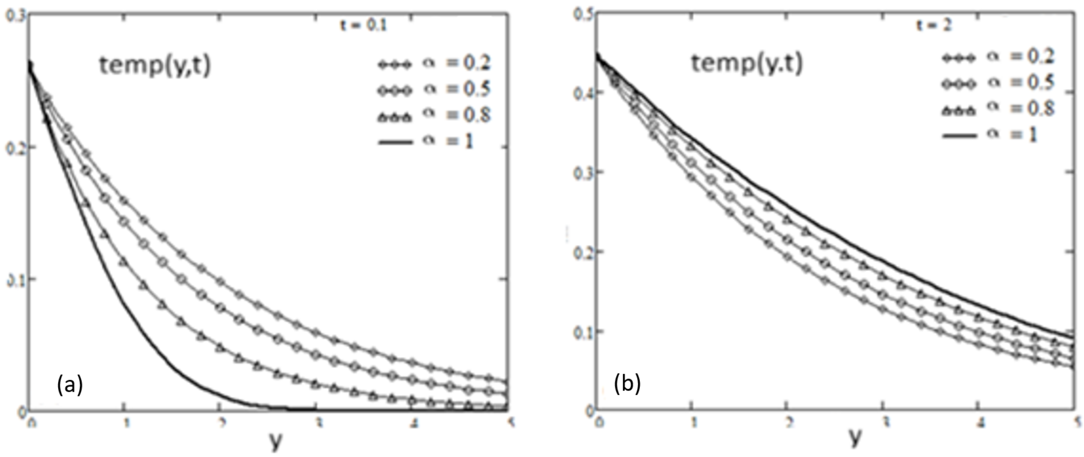

Figure 2: (a) Profiles of temperature for α variation and for different values of Pr at t = 0.1. (b) Profiles of temperature versus y for α variation and for different values of Pr at t = 2.

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The diagrams of Figure 3 (a) and (b) are plotted to discuss the influence of the fractional parameter α on the fluid concentration {conc(y,t)}, at a fixed Schmidt number (Sc = 0.2). The curves are sketched for different values of time t (for small and large times) and of fractional parameter α. The fluid concentration {conc(y,t)} decreases with increasing fractional parameter as shown in Figure 3 (a) whereas an opposite effect is noted in Figure 3 (b).

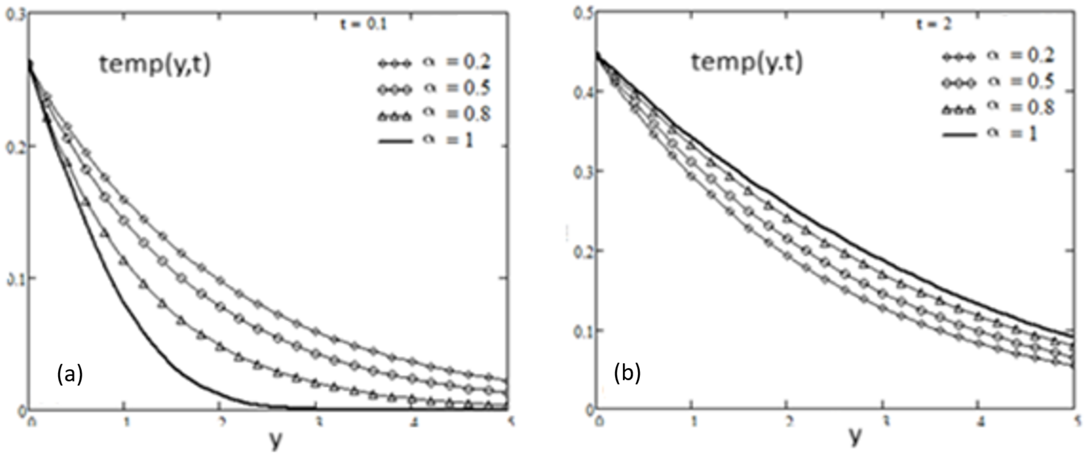

Figure 3: (a) Profiles of concentration versus y for α variation and small values of time t. (b) Profiles of concentration versus y for α variation and large values of time t.

It is seen in Figure 4 (a) and (b) that the concentration has different behavior at small values of the time t, respectively, at large values of the time t. For small values of the time t, by increasing the value of fractional parameter, the concentration {conc(y,t)} decreases, while, for large values of the time t, by increasing the value of fractional parameter α the concentration increases

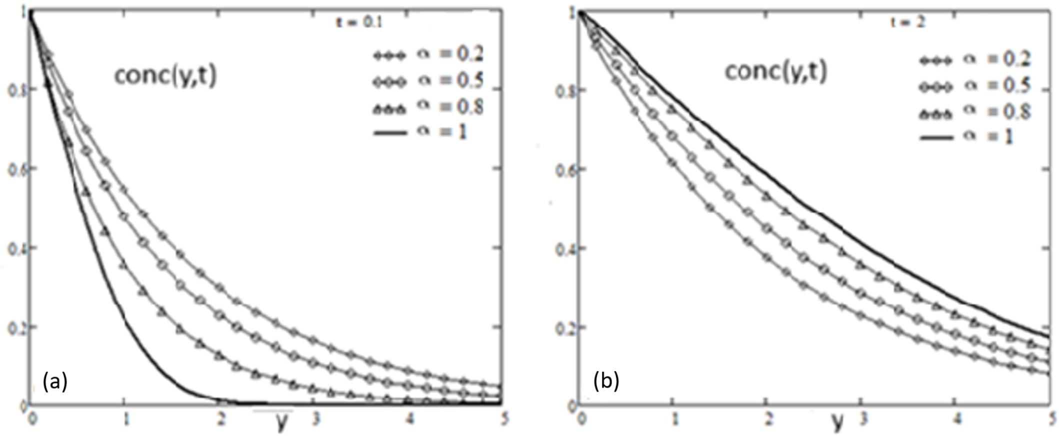

Figure 4: (a) Profiles of concentration for α variation and for t = 0.1. (b) Profiles of concentration versus α variation and for t = 2.

The diagrams from Figure 5 (a) and (b) are plotted to discuss the influence of the fractional parameter α on the fluid flow velocity {vel(y,t)} for fixed value of the Grashof mass parameter Gm $= 0.1$. The curves are sketched at different instants (for small t $= 0.1$ and large time t $= 3$) and of fractional parameter α. It is observed that for small time t $=$ 0.1 by increasing the fractional parameter the velocity {vel(y,t)} decreases. For large time t $= 3$ the fluid velocity {vel(y,t)} increases by increasing the value of fractional parameter and, the ordinary fluid has greater velocity {vel(y,t)} than fractional model. In Figure 6,

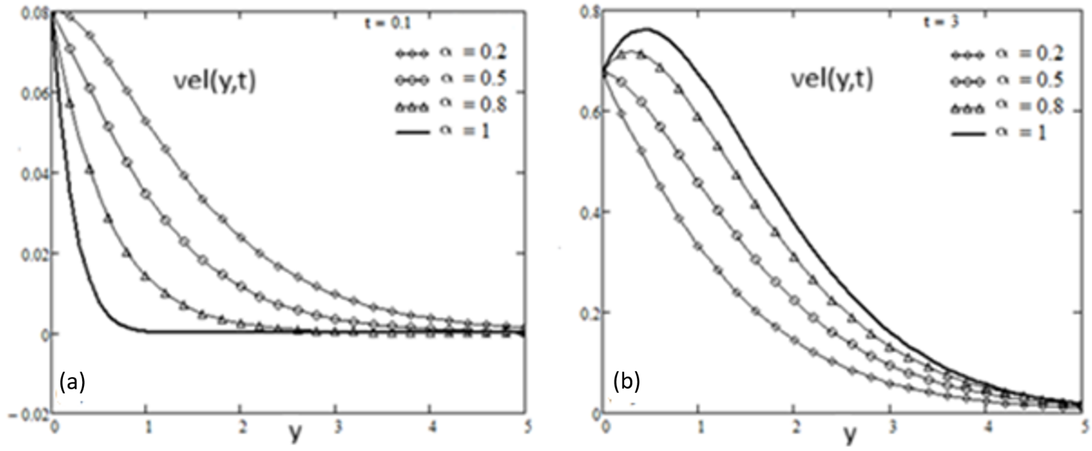

Figure 5: (a) Profiles of velocity for α variation and for t $= 0.1$. (b) Profiles of velocity for α variation and for t $= 3$.

the effect of Casson fluid parameter β is examined on the fluid velocity {vel(y,t)}. The other parameters are kept fixed. For increasing values of Casson fluid parameter β, the fluid velocity {vel(y,t)} decreases. Indeed, for large β such that $1/\beta$ approaches to zero, the present fluid model reduces to that of Newtonian fluid model, and it is easy to see from Figure 6 that the more we increase the Casson parameter, the fluid velocity {vel(y,t)} decreases, and for the case when $1/\beta$ approaches to zero, the fluid changes to Newtonian fluid shows smaller velocity compare to the Casson fluid. That is when the non-Newtonian behavior increases by increasing β and hence the fluid motion decreases.

5. Conclusion

In this study, Caputo–Fabrizio fractional derivatives were employed to develop a generalized fractional mathematical model describing the optimized blood flow of a Casson fluid over an infinite oscillating plate subject to exponential heating and constant solute concentration. Through the application of the Laplace transform technique, closed-form analytical expressions for velocity, temperature, and concentration distributions were successfully derived. The classical Casson fluid model ($\alpha = 1$) was shown to emerge naturally as a limiting case of the fractional formulation, demonstrating the consistency of the present model with established fluid mechanics theory. The numerical simulations performed using Mathcad, along with the graphical analysis, reveal several important physical insights. The fractional-order parameter significantly influences the rate of thermal and

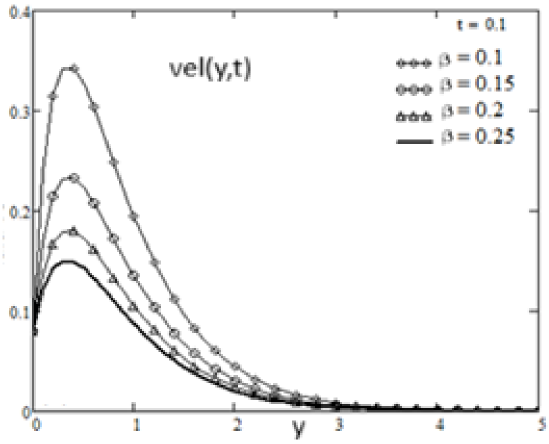

Figure 6: Profiles of velocity for Casson fluid parameter β variation for different time t.

solutal diffusion, where lower fractional orders produce enhanced memory effects that slow down diffusion processes. Variations in Prandtl and Schmidt numbers highlight the sensitivity of thermal and mass boundary layers to fluid properties, while the behavior of the Casson parameter illustrates the transition between non-Newtonian and Newtonian regimes. The combined impact of heat generation, viscous effects, and fractional-order dynamics provides a more realistic representation of blood flow, particularly in biomedical systems where viscoplastic behavior and memory-dependent transport phenomena are prominent. Overall, the findings demonstrate that fractional calculus, particularly the Caputo–Fabrizio derivative with its non-singular kernel, offers a powerful and flexible framework for modeling complex physiological flows. The present results are useful in the design and optimization of biomedical devices, targeted drug delivery mechanisms, and thermal therapies, where accurate prediction of heat and mass transfer in non-Newtonian fluids is essential. Future work may further extend this model by incorporating magnetic fields, porous medium effects, reactive solute transport, or time-dependent boundary conditions to capture more intricate real-world scenarios.

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